D-branes and Closed String Field Theory

N. Ishibashi (University of Tsukuba)

In collaboration with
Y. Baba(Tsukuba) and K. Murakami(KEK)

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理研シンポジウム「弦の場の理論07」
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§ 1 Introduction

D-branes in string field theory

- D-branes can be realized as soliton solutions in open string field theory

- D-branes in closed string field theory?

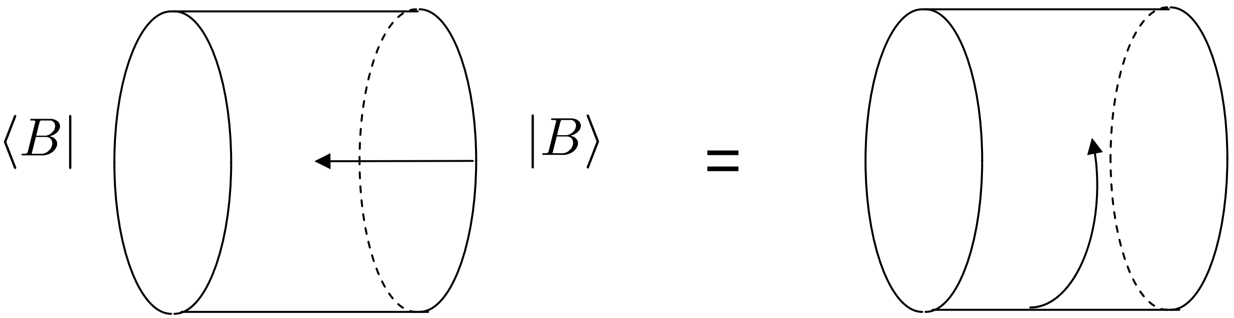
Hashimoto and Hata

HIKKO

$$I = \Psi K \Psi + g\Psi^3 + \underline{\langle B | \Psi \rangle}$$

♦A BRS invariant source term
♦tension of the brane cannot be fixed

Tensions of D-branes



Can one fix the normalization of the boundary states without using open strings?

For noncritical strings, the answer is yes.

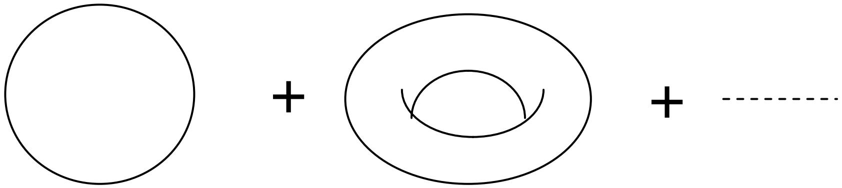
Fukuma and Yahikozawa

Let us describe their construction using SFT for noncritical strings.

c=0 noncritical strings

$$\int dM e^{-\mathrm{Tr}V(M)} \longrightarrow$$

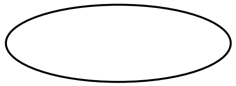
double scaling limit



$$V(M) = \frac{1}{2}M^2 + \frac{g}{4}M^4$$

$$M : N \times N \text{ matrix}$$

String Field $\quad \psi(l) \sim \mathrm{Tr}e^{lM} \sim$



l

Describing the matrix model in terms of this field, we obtain the string field theory for c=0 noncritical strings

Kawai, N.I.

Stochastic quantization of the matrix model

t: fictitious time

$$H = -\mathrm{Tr}\left[\left(\frac{\partial}{\partial M}\right)^2 - \frac{\partial V}{\partial M}\frac{\partial}{\partial M}\right]$$

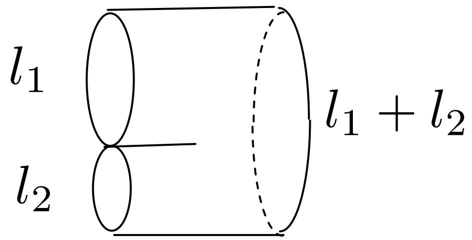
Jevicki and Rodrigues

string field $\quad \psi(l) = \mathrm{Tr}e^{lM}$

$$[\psi(l), \bar{\psi}(l')] = \delta(l - l')$$

$$\psi(l)|0\rangle = \langle 0|\bar{\psi}(l) = 0$$

$$\begin{aligned} H &= \int_0^\infty dl_1 \int_0^\infty dl_2 (l_1 + l_2)\bar{\psi}(l_1 + l_2)\psi(l_1)\psi(l_2) \\ &\quad + g_s^2 \int_0^\infty dl_1 \int_0^\infty dl_2 l_1 l_2 \bar{\psi}(l_1)\bar{\psi}(l_2)\psi(l_1 + l_2) + \int_0^\infty dl \rho(l)\bar{\psi}(l) \end{aligned}$$



joining-splitting interactions

correlation functions

$$\langle \psi(l_1)\cdots\psi(l_n)\rangle = \langle 0|\psi(l_1)\cdots\psi(l_n)|\Psi\rangle \quad \left(|\Psi\rangle = \lim_{t\to\infty} e^{-tH}|0\rangle\right)$$

Virasoro constraints

FKN, DVV

$$T(l)|\Psi\rangle = 0$$

$$T(l) = \int_0^l dl'\psi(l')\psi(l-l') + g_s^2\int_0^\infty dl' l'\bar{\psi}(l')\psi(l+l') + \rho(l)$$

Virasoro constraints $\longrightarrow$ Schwinger-Dyson equations for the correlation functions

various solutions $|\Psi\rangle$
to the Virasoro constraint ~ various vacua
$T(l)|\Psi\rangle = 0$

Solitonic operators

Fukuma and Yahikozawa
Hanada, Hayakawa, Kawai, Kuroki, Matsuo, Tada and N.I.

$$\int d\zeta \mathcal{V}_\pm(\zeta)$$

$$\mathcal{V}_\pm(\zeta) = \exp\left(\underline{\pm g_s} \int_0^\infty dl e^{-\zeta l} \bar{\psi}(l)\right) \exp\left(\mp \frac{2}{\underline{g_s}} \int_0^\infty \frac{dl}{l} e^{\zeta l} \psi(l)\right)$$

These coefficients are chosen so that

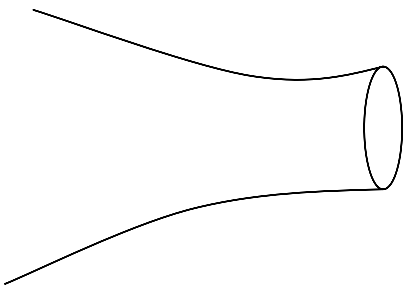
$$\left.\begin{array}{r} [T(l), \mathcal{V}_\pm(\zeta)] = \partial_\zeta(\cdot) \\ T(l)|\Psi\rangle = 0 \end{array}\right\} \longrightarrow T(l) \int d\zeta \mathcal{V}_\pm(\zeta)|\Psi\rangle = 0$$

$$\begin{aligned} T &\sim -\frac{1}{2}(\partial\varphi(\zeta))^2 \\ \mathcal{V}_\pm(\zeta) &\sim \; : e^{\pm\sqrt{2} i\varphi(\zeta)} : \end{aligned}$$

From one solution to the Virasoro constraint,
one can generate another by the solitonic operator.

These solitonic operators correspond to D-branes

$$\mathcal{V}_{\pm}(\zeta)|\Psi\rangle = \exp\left(\pm g_s \int_0^{\infty} dl e^{-\zeta l}\bar{\psi}(l)\right) \exp\left(\underline{\mp\frac{2}{g_s}\int_0^{\infty}\frac{dl}{l}e^{\zeta l}\psi(l)}\right)|\Psi\rangle$$



$$\langle\psi(l_1)\cdots\psi(l_n)\rangle = \langle 0|\psi(l_1)\cdots\psi(l_n)\int d\zeta\mathcal{V}_{\pm}(\zeta)|\Psi\rangle$$

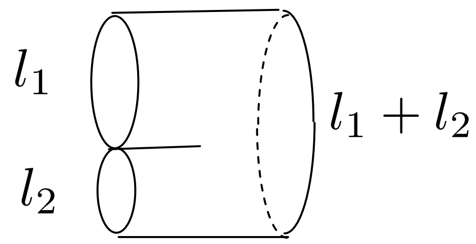
$\longrightarrow$ amplitudes with ZZ-branes

$\int d\zeta\mathcal{V}_{\pm}(\zeta)|\Psi\rangle$ = state with D(-1)-brane ghost D-brane

Okuda,Takayanagi

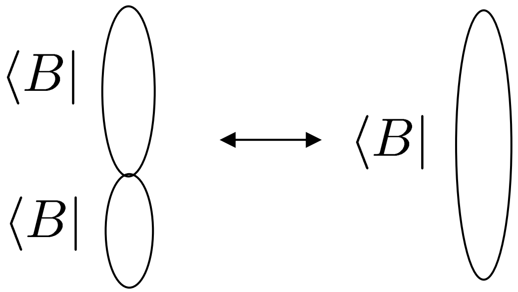
critical strings?

noncritical case is simple



idempotency equation

Kishimoto, Matsuo, Watanabe



For boundary states, things may not be so complicated

If we have

- SFT with length variable
- nonlinear equation like the Virasoro constraint

for critical strings, similar construction will be possible.

We will show that

- for OSp invariant SFT for the critical bosonic strings
- we can construct BRS invariant observables using the boundary states for D-branes

imitating the construction of the solitonic operators for the noncritical string case.

♦the BRS invariant observables

→BRS invariant source terms~D-branes

♦the tensions of the branes are fixed

Plan of the talk

§ 2 OSp Invariant String Field Theory

light-cone gauge SFT (t, α, X^i) $(t = x^+,\ \alpha = 2p^+,\ i = 1, \cdots, 24)$

$$I = \int dt \left[\frac{1}{2} \int \langle \Phi | \left(i \frac{\partial}{\partial t} - H \right) | \Phi \rangle + \frac{2g}{3} \int \langle V_3(1,2,3) | \, \Phi \rangle_1 | \Phi \rangle_2 | \Phi \rangle_3 \right]$$

O(25,1) symmetry

$\downarrow$

OSp invariant SFT =light−cone SFT with $(t, \alpha, X^i, X^{25}, X^{26}, \underline{C, \bar{C}})$

Grassmann

Siegel, Uehara, Neveu, West,
Zwiebach, Kugo, Kawano,….

OSp(27,1|2) symmetry

$$X^{25}, X^{26}, C, \bar{C} \to c = 0$$

variables

$$X^M = \left(X^\mu, C, \bar{C}\right) \colon OSp(26|2) \text{ vector}$$

$$\eta_{MN} = \begin{array}{cc} & \begin{array}{ccc} \qquad & C & \bar{C} \end{array} \\ \begin{array}{c} \\ C \\ \bar{C} \end{array} & \left(\begin{array}{c|cc} \delta_{\mu\nu} & & \\ \hline & 0 & -i \\ & i & 0 \end{array}\right) \end{array} = \eta^{MN}$$

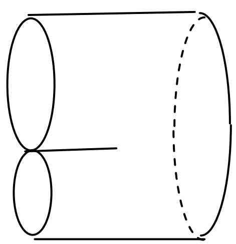
$$\begin{aligned} X^\mu(\tau,\sigma) &= x^\mu - 2ip^\mu\tau + i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-n(\tau+i\sigma)} + \tilde{\alpha}_n^\mu e^{-n(\tau-i\sigma)}\right) \\ C(\tau,\sigma) &= C_0 + 2i\pi_0\tau - i\sum_{n\neq 0}\frac{1}{n}\left(\gamma_n e^{-n(\tau+i\sigma)} + \tilde{\gamma}_n e^{-n(\tau-i\sigma)}\right) \\ \bar{C}(\tau,\sigma) &= \bar{C}_0 - 2i\bar{\pi}_0\tau + i\sum_{n\neq 0}\frac{1}{n}\left(\bar{\gamma}_n e^{-n(\tau+i\sigma)} + \tilde{\bar{\gamma}}_n e^{-n(\tau-i\sigma)}\right) \end{aligned}$$

action

$$S = \int dt \left[\frac{1}{2} \int d1d2 \; \langle R(1,2) | \Phi\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)} + \tilde{L}_0^{(2)} - 2}{\alpha_2} \right) |\Phi\rangle_2 \right.$$
$$\left. + \frac{2g}{3} \int d1d2d3 \; \langle V_3^0(1,2,3) | \, \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$

α_1

α_2



$\alpha_1 + \alpha_2$

$$\left\{ \begin{aligned} |\Phi\rangle &= |\ \rangle \otimes f\left(p^\mu, \pi_0, \bar{\pi}_0, t, \alpha\right) \\ dr &\equiv \frac{\alpha_r d\alpha_r}{2} \frac{d^{26}p_r}{(2\pi)^{26}} \underline{i d\bar{\pi}_0^{(r)} d\pi_0^{(r)}} \\ L_0 &= \frac{1}{2}p^2 + i\pi_0\bar{\pi}_0 + N \\ \tilde{L}_0 &= \frac{1}{2}p^2 + i\pi_0\bar{\pi}_0 + \tilde{N} \\ {}_2\langle\Phi| &\equiv \int d1 \langle R(1,2)|\Phi\rangle_1 \\ \left(|\Phi\rangle\right)^\dagger &= \langle\Phi| \ \text{(hermiticity)} \end{aligned} \right.$$

$$\langle R(1,2)| \equiv \delta(1,2)\ {}_{12}\langle 0|\, e^{E(1,2)}\ \frac{1}{\alpha_1}$$

$$\left\{\begin{array}{rcl} {}_{12}\langle 0| & = & {}_1\langle 0|{}_2\langle 0| \\ E(1,2) & = & -\sum_{n=1}^{\infty} \frac{1}{n}\left(\alpha_n^{N(1)}\alpha_n^{M(2)} + \tilde{\alpha}_n^{N(1)}\tilde{\alpha}_n^{M(2)}\right)\eta_{NM} \\ \delta(1,2) & = & 2\delta(\alpha_1+\alpha_2)(2\pi)^{26}\delta^{26}(p_1+p_2)i(\bar{\pi}_0^{(1)}+\bar{\pi}_0^{(2)})(\pi_0^{(1)}+\pi_0^{(2)}) \end{array}\right.$$

$$\left\langle V_3^0(1,2,3)\right| \equiv \delta(1,2,3)\ {}_{123}\langle 0| e^{E(1,2,3)}\mathcal{P}_{123}\,\frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3}$$

$$\left\{\begin{array}{rcl} {}_{123}\langle 0| & = & {}_1\langle 0|\,{}_2\langle 0|\,{}_3\langle 0| \\ \mathcal{P}_{123} & = & \mathcal{P}_1\mathcal{P}_2\mathcal{P}_3 \quad \mathcal{P}_r = \int_0^{2\pi}\frac{d\theta}{2\pi}e^{i\theta\left(L_0^{(r)}-\tilde{L}_0^{(r)}\right)} \\ \delta(1,2,3) & = & 2\delta\left(\sum_{s=1}^{3}\alpha_s\right)(2\pi)^{26}\delta^{26}\left(\sum_{r=1}^{3}p_r\right)i\left(\sum_{r'=1}^{3}\bar{\pi}_0^{(r')}\right)\left(\sum_{s'=1}^{3}\pi_0^{(s')}\right) \\ E(1,2,3) & = & \frac{1}{2}\sum_{n,m\geq 0}\sum_{r,s=1}^{3}\bar{N}_{nm}^{rs}\left(\alpha_n^{N(r)}\alpha_m^{M(s)}+\tilde{\alpha}_n^{N(r)}\tilde{\alpha}_m^{M(s)}\right)\eta_{NM} \\ \mu(1,2,3) & = & \exp\left(-\hat{\tau}_0\sum_{r=1}^{3}\frac{1}{\alpha_r}\right)\ ,\quad \hat{\tau}_0 = \sum_{r=1}^{3}\alpha_r\ln|\alpha_r| \end{array}\right.$$

OSp theory $\backsim$ covariant string theory with extra time and length

$$(t, \alpha, X^i, X^{25}, X^{26}, C, \bar{C}) \sim (t, \alpha, X^\mu, b, c, \tilde{b}, \tilde{c}) \qquad (\mu = 1, 2, \cdots, 26)$$

$$C(\tau,\sigma) = C_0 + 2i\pi_0\tau - i\sum_{n\neq 0}\tfrac{1}{n}\left(\gamma_n e^{-n(\tau+i\sigma)} + \tilde{\gamma}_n e^{-n(\tau-i\sigma)}\right)$$
$$\bar{C}(\tau,\sigma) = \bar{C}_0 - 2i\bar{\pi}_0\tau + i\sum_{n\neq 0}\tfrac{1}{n}\left(\bar{\gamma}_n e^{-n(\tau+i\sigma)} + \tilde{\bar{\gamma}}_n e^{-n(\tau-i\sigma)}\right)$$

$$\gamma_n = in\alpha c_n,\ \tilde{\gamma}_n = in\alpha\tilde{c}_n,$$
$$\bar{\gamma}_n = \frac{1}{\alpha}b_n,\ \tilde{\bar{\gamma}}_n = \frac{1}{\alpha}\tilde{b}_n$$
$$C_0 = 2\alpha(c_0 + \tilde{c}_0),\ \bar{\pi}_0 = \frac{1}{2\alpha}(b_0 + \tilde{b}_0)$$

HIKKO

- no $b_0 - \tilde{b}_0, c_0 - \tilde{c}_0$
- extra variables $\alpha, t, \pi_0, \bar{C}_0$

The “action” cannot be considered as the usual action

$$|\Phi\rangle = |\bar{\phi}\rangle + i\pi_0|\bar{\chi}\rangle + i\bar{\pi}_0|\chi\rangle + i\pi_0\bar{\pi}_0|\phi\rangle$$

$$\longrightarrow \quad S \sim \int dt d^{26}x \left(\bar{\phi}^2 + \bar{\phi}\left(\alpha\partial_t - p^2 - M^2\right)\phi + \cdots\right)$$

looks like the action for stochastic quantization

BRS symmetry

$$\text{BRS transformation} \sim J^{C-} \in OSp$$

$$\delta_{\mathrm{B}}\Phi = Q_{\mathrm{B}}\Phi + g\Phi * \Phi$$

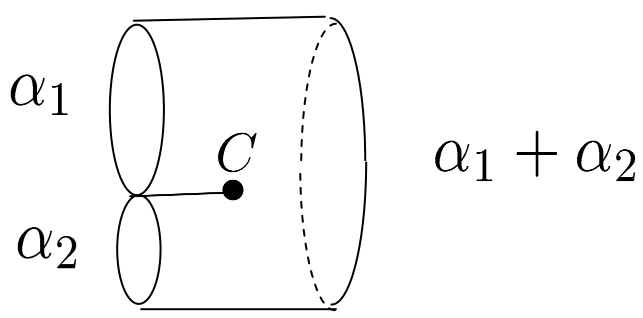
$$Q_{\mathrm{B}} \sim Q_{\mathrm{B}}^{\mathrm{KO}} - i\pi_0(\partial_\alpha + \tfrac{1}{\alpha})$$

the string field Hamiltonian is BRS exact

$$
\begin{aligned}
Q_{\rm B} \quad &= \quad \frac{C_0}{2\alpha}(L_0 + \tilde{L}_0 - 2) - i\pi_0 \frac{\partial}{\partial \alpha} \\
&\quad + \frac{i}{\alpha}\sum_{n=1}^{\infty}\left(\frac{\gamma_{-n}L_n - L_{-n}\gamma_n}{n} + \frac{\tilde{\gamma}_{-n}\tilde{L}_n - \tilde{L}_{-n}\tilde{\gamma}_n}{n}\right)
\end{aligned}
$$

$$
|\Phi * \Psi\rangle_4 = \int d1d2d3 \ \langle V_3(1,2,3)\,|\Phi\rangle_1 \ |\Psi\rangle_2 \,|R(3,4)\rangle
$$

$$
\langle V_3(1,2,3)| = \delta(1,2,3) \ {}_{123}\langle 0|e^{E(1,2,3)}\underline{C(\rho_I)}\mathcal{P}_{123}\frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3}
$$



OSp invariant SFT $\backsim$ stochastic formulation of string theory ?

Green's functions of BRS invariant observables $\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$

||

Green's functions in 26D

- Wick rotation $x^{26} = ix^0$
- 1 particle pole $\frac{1}{p^2+M^2}$

$\downarrow$

S-matrix elements in 26D

|| ?

S-matrix elements derived from the light-cone gauge SFT

§ 3 Observables and Correlation Functions

observables $\quad \mathcal{O} = \langle \ | \Phi \rangle$

$$\delta_{\mathrm{B}} \mathcal{O} = \langle \ | \Big(Q_{\mathrm{B}} | \Phi \rangle + g | \Phi * \Phi \rangle \Big) = 0$$

$\longrightarrow \quad Q_{\mathrm{B}} | \ \rangle = 0$

ignoring the multi-string contribution

if $\quad | \ \rangle = Q_{\mathrm{B}} | \ \rangle' \quad \Big((Q_{\mathrm{B}})^2 = 0 \Big)$

$\longrightarrow \quad \mathcal{O} \sim \delta_{\mathrm{B}}{}' \langle \ | \Phi \rangle$

we need the cohomology of Q_{B}

cohomology of $Q_{\rm B}$

$$\begin{cases} Q_{\rm B} \sim Q_{\rm B}^{\rm KO} - i\pi_0(\partial_\alpha + \frac{1}{\alpha}) \\ ||f||^2 = \int \alpha d\alpha |f(\alpha)|^2 \end{cases}$$

$$\longrightarrow \quad \begin{cases} \frac{1}{\alpha}|0\rangle_{C,\bar{C}} \otimes |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta(p-k) \\ \frac{1}{\alpha}\bar{\pi}_0\pi_0|0\rangle_{C,\bar{C}} \otimes |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta(p-k) \end{cases}$$ (on-shell)

$|\overline{\text{primary}}\rangle_X$: Virasoro primary state of CFT of X

$$\left(L_0 + \tilde{L}_0 - 2\right) |\overline{\text{primary}}\rangle_X \otimes |0\rangle_{C,\bar{C}} \, (2\pi)^{26}\delta^{26}(p-k)$$
$$= \left(k^2 + 2i\pi_0\bar{\pi}_0 + M^2\right) |\overline{\text{primary}}\rangle_X \otimes |0\rangle_{C,\bar{C}} \, (2\pi)^{26}\delta^{26}(p-k)$$

This state corresponds to a particle with mass M

Observables

$$\mathcal{O} = \langle \ |\Phi\rangle$$

$$|\ \rangle = \frac{1}{\alpha}|0\rangle_{C,\bar{C}} \otimes |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta(p-k)$$

$$|\ \rangle = \frac{1}{\alpha}\bar{\pi}_0\pi_0|0\rangle_{C,\bar{C}} \otimes |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta(p-k)$$

$$|\Phi\rangle = |\bar{\phi}\rangle + i\pi_0|\bar{\chi}\rangle + i\bar{\pi}_0|\chi\rangle + i\pi_0\bar{\pi}_0|\phi\rangle$$

$$S \sim \int dt d^{26}x \left(\bar{\phi}^2 + \bar{\phi}\left(\alpha\partial_t - p^2 - M^2\right)\phi + \cdots\right)$$

$$\boxed{\mathcal{O}(t,k) = \frac{i}{2}\int_{-\infty}^{\infty} d\alpha \int d\bar{\pi}_0 d\pi_0\, {}_{C,\bar{C}}\langle 0| \otimes {}_X\langle\overline{\text{primary}}|\Phi(t,\alpha,\pi_0,\bar{\pi}_0,k)\rangle}$$

Free propagator

$$\mathcal{O}(t,k) = \frac{i}{2}\int_{-\infty}^{\infty} d\alpha \int d\bar{\pi}_0 d\pi_0{}_{C,\bar{C}}\langle 0| \otimes {}_X\langle\overline{\text{primary}}|\Phi(t,\alpha,\pi_0,\bar{\pi}_0,k)\rangle$$

$${}_X\langle\overline{\text{primary}}|\overline{\text{primary}}\rangle_X = 1$$

<u>two point function</u>

$$\left\langle\!\!\left\langle \tilde{\mathcal{O}}(E_1,p_1)\tilde{\mathcal{O}}(E_2,p_2) \right\rangle\!\!\right\rangle$$
$$\equiv \int dt_1 dt_2\, e^{iE_1t_1+iE_2t_2}\, \langle\!\langle 0|\mathrm{T}\,\mathcal{O}(t_1,p_1)\mathcal{O}(t_2,p_2)|0\rangle\!\rangle$$

We would like to show that the lowest order contribution to this two point function coincides with the free propagator of the particle corresponding to this state.

light-cone quantization

$$|\Phi\rangle = |\psi\rangle + |\bar{\psi}\rangle$$

$|\psi\rangle : \ \alpha > 0$ annihilation

$|\bar{\psi}\rangle : \ \alpha < 0$ creation

$$(|\psi\rangle)^{\dagger} = \langle\bar{\psi}|$$

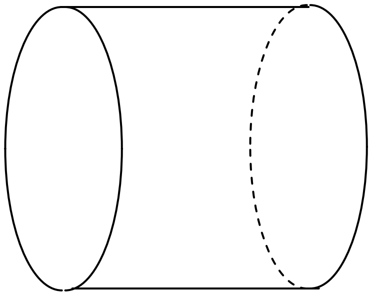
$$\left[|\psi\rangle_r, |\bar{\psi}\rangle_s\right] = |R(r,s)\rangle$$

$|0\rangle\!\rangle : \ $ vacuum

$$|\psi\rangle|0\rangle\!\rangle = \langle\!\langle 0|\langle\bar{\psi}| = 0$$

$$\langle\!\langle 0|\mathrm{T}\Phi(t_1,\alpha_1)\Phi(t_2,\alpha_2)|0\rangle\!\rangle \longleftrightarrow$$

$t_1 > t_2$

$\alpha_1 > 0$  $\alpha_2 < 0$

$$
\left\langle\!\!\left\langle \tilde{\mathcal{O}}(E_1,p_1)\tilde{\mathcal{O}}(E_2,p_2)\right\rangle\!\!\right\rangle_{\text{free}}
$$

$$
= \int dt_1 dt_2\, e^{iE_1t_1+iE_2t_2}(2\pi)^{26}\delta^{26}(p_1+p_2)
$$

$$
\times\Bigg[\theta(t_1-t_2)\boxed{\frac{i}{2}\int_0^\infty \frac{d\alpha_1}{\alpha_1}\int d\bar{\pi}_0^{(1)}d\pi_0^{(1)}e^{-i\frac{t_1-t_2}{\alpha_1}(p_1^2+M^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)})}}
$$

$$
+\,\theta(t_2-t_1)\frac{i}{2}\int_0^\infty \frac{d\alpha_2}{\alpha_2}\int d\bar{\pi}_0^{(2)}d\pi_0^{(2)}e^{-i\frac{t_2-t_1}{\alpha_2}(p_2^2+M^2+2i\pi_0^{(2)}\bar{\pi}_0^{(2)})}\Bigg]
$$

$$
\frac{i}{2}\int_0^\infty \frac{d\alpha_1}{\alpha_1}\int d\bar{\pi}_0^{(1)}d\pi_0^{(1)}e^{-i\frac{t_1-t_2}{\alpha_1}(p_1^2+M^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}-i\epsilon)}
$$

$$
=\int_0^\infty \frac{d\alpha_1}{\alpha_1}\, i\frac{t_1-t_2}{\alpha_1}e^{-i\frac{t_1-t_2}{\alpha_1}(p_1^2+M^2-i\epsilon)}
$$

$$
= i\int_0^\infty dt e^{-it(p_1^2+M^2-i\epsilon)}
$$

$$
=\frac{1}{p_1^2+M^2}
$$

$$\left\langle\!\!\left\langle \tilde{\mathcal{O}}(E_1, p_1)\tilde{\mathcal{O}}(E_2, p_2) \right\rangle\!\!\right\rangle_{\mathrm{free}}$$

$$= \int dt_1 dt_2 e^{iE_1 t_1 + iE_2 t_2} (2\pi)^{26} \delta^{26}(p_1 + p_2) \left[\frac{\theta(t_1 - t_2)}{p_1^2 + M^2} + \frac{\theta(t_2 - t_1)}{p_2^2 + M^2} \right]$$

$$= \frac{(2\pi)^{26}\delta^{26}(p_1 + p_2)}{p_1^2 + M^2} 2\pi\delta(E_1) 2\pi\delta(E_2)$$

the Hamiltonian is BRS exact

<u>a representative of the class</u>

$$\begin{aligned} \varphi(p) &\equiv \int \frac{dE}{2\pi} \tilde{\mathcal{O}}(E, p) \\ &= \mathcal{O}(t = 0, p) \\ &= \frac{i}{2} \int \frac{dE}{2\pi} \int_{-\infty}^{\infty} d\alpha \int d\bar{\pi}_0 d\pi_0 \ {}_{C,\bar{C}}\langle 0| \otimes {}_X\langle \overline{\mathrm{primary}} | \tilde{\Phi}(E, \alpha, \pi_0, \bar{\pi}_0, p) \rangle \end{aligned}$$

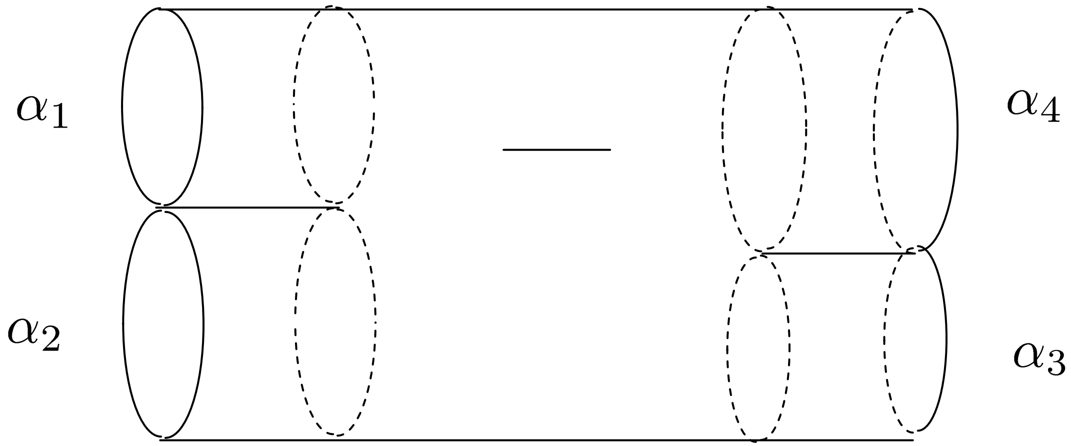
$$\langle\!\langle \varphi_1(p_1)\varphi_2(p_2) \rangle\!\rangle_{\mathrm{free}} = \frac{(2\pi)^{26}\delta^{26}(p_1 + p_2)}{p_1^2 + M^2}$$

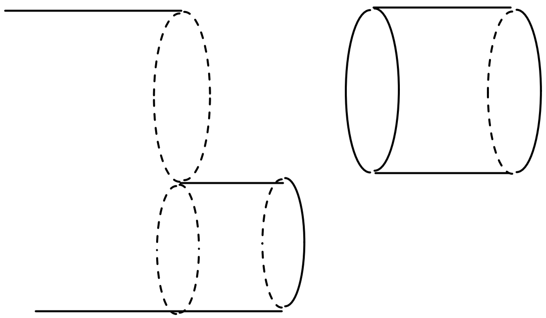
N point functions

$$\langle\langle 0|\, \mathrm{T} \prod_{r=1}^{N} \mathcal{O}_r(t_r, p_r)\, |0\rangle\rangle$$

we would like to look for the singularity $\frac{1}{p_r^2 + M_r^2}$

light-cone diagram

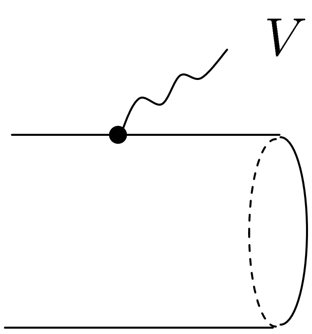




$$\frac{i}{2}\int_0^\infty \frac{d\alpha_r}{\alpha_r}\int d\bar{\pi}_0^{(r)} d\pi_0^{(r)}\, e^{-i\frac{t}{\alpha_r}(p_r^2+2i\pi_0^{(r)}\bar{\pi}_0^{(r)}+M_r^2)}\, f(\alpha_r,\pi_0^{(r)},\bar{\pi}_0^{(r)},p_r)$$

the singularity comes from $\alpha_r \sim 0$

$$=\frac{i}{2}\int_0^\infty \frac{d\alpha_r}{\alpha_r}\int d\bar{\pi}_0^{(r)} d\pi_0^{(r)}\, e^{-i\frac{t}{\alpha_r}(p_r^2+2i\pi_0^{(r)}\bar{\pi}_0^{(r)}+M_r^2)}\, [\underline{f(0,0,0,p_r)}+\cdots]$$



$$= \frac{1}{p_r^2 + M_r^2} f(\alpha_r, \pi_0^{(r)}, \bar{\pi}_0^{(r)}, p_r)\Big|_{p_r^2 + M_r^2 = \alpha_r = \pi_0^{(r)} = \bar{\pi}_0^{(r)} = 0} + \text{less singular terms}$$

Repeating this for $r = 1, \cdots, N-1$ we get

$$\left\langle\!\!\left\langle \prod_{r=1}^{N} \varphi_r(p_r) \right\rangle\!\!\right\rangle$$

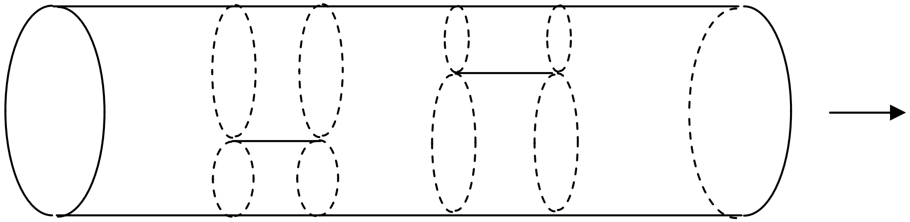
$$= \left(\prod_{r=1}^{N} \frac{1}{p_r^2 + M_r^2} \right) (-i)(2\pi)^{26} \delta^{26}\left(\sum_{r=1}^{N} p_r \right) G^{OSp}_{\text{amputated}}(p_r^{OSp})\Big|_{p_r^2 + M_r^2 = E_r = \alpha_r = \pi_0^{(r)} = \bar{\pi}_0^{(r)} = 0}$$

$$+ \text{less singular terms}$$

Rem.

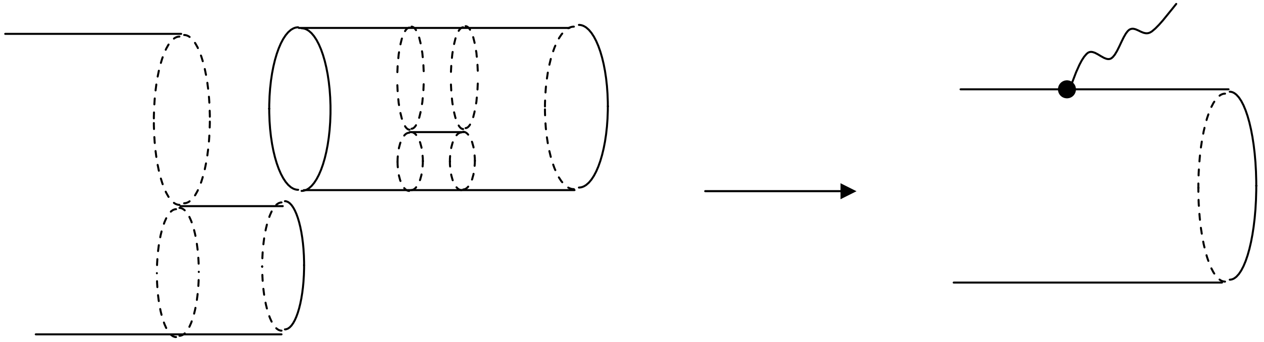
higher order corrections

- two point function



$$\frac{1}{p^2 + M^2 + \Sigma(p)}$$

- N point function



can also be treated in the same way
at least formally

Wick rotation $x^{26} = ix^0$

$$S(p_r) = (2\pi)^{26}\delta\left(\sum_{r=1}^{N} p_r\right) G^{OSp}_{\text{amputated}}(p_r^{OSp})\Big|_{p_r^2+M_r^2=E_r=\alpha_r=\pi_0^{(r)}=\bar{\pi}_0^{(r)}=0}$$

$$\boxed{G^{OSp}_{\text{amputated}}\,(p_r,\epsilon_r)\Big|_{p_r^2+M_r^2=E_r=\alpha_r=\pi_0^{(r)}=\bar{\pi}_0^{(r)}=0} = G^{LC}_{\text{amputated}}\,(p_r,\epsilon_r)\Big|_{p_r^2+M_r^2=0}}$$

- LHS is an analytic function of $p\cdot p,\ p\cdot\epsilon, \epsilon\cdot\epsilon$
- Wick rotation + OSp(27,1|2) trans.

$$G^{OSp}_{\text{amputated}}\,(p_r,\epsilon_r)\Big|_{p_r^2+M_r^2=E_r=\alpha_r=\pi_0^{(r)}=\bar{\pi}_0^{(r)}=0}$$
$$= G^{OSp}_{\text{amputated}}\,(p_r,\epsilon_r)\Big|_{p_r^2+M_r^2=p^{25}=p^{26}=\pi_0^{(r)}=\bar{\pi}_0^{(r)}=0}$$

- $(\alpha, E, X^1, \cdots, X^{24}, \underline{X^{25}, X^{26}, C, \bar{C}}) \sim (\alpha, E, X^1, \cdots, X^{24})$

$S(p_r)$ reproduces the light-cone gauge result

§ 3 D-brane States

Let us construct off-shell BRS invariant states, imitating the construction of the solitonic operator in the noncritical case.

$\boxed{\text{Boundary states in OSp theory}}$

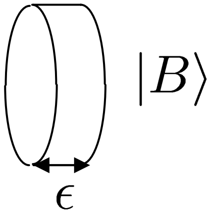
$|B_0\rangle$ boundary state for a flat Dp-brane

$$\begin{cases} P^\mu(\sigma)|B_0\rangle = X^i(\sigma)|B_0\rangle = 0 \quad (\mu = 1, \cdots p+1, \; i = p+1, \cdots, 26) \\ C(\sigma)|B_0\rangle = \bar{C}(\sigma)|B_0\rangle = 0 \quad (\text{Dirichlet in } C, \bar{C} \text{ directions}) \\ |B_0\rangle \text{ is } \alpha \text{ independent} \end{cases}$$

BRS invariant regularization

$$|B_0\rangle \to |B_0\rangle^{\epsilon} \equiv e^{-\frac{\epsilon}{|\alpha|}(L_0+\tilde{L}_0-2)}|B_0\rangle$$

$$|B\rangle \to e^{-\epsilon H}|B\rangle$$



states with one soliton

$$|D\rangle\!\rangle \equiv \lambda \int d\zeta \, \bar{\mathcal{O}}_D(\zeta)|0\rangle\!\rangle$$

$$\bar{\mathcal{O}}_D(\zeta) = \exp\left[a\int_{-\infty}^{0} dr \, \frac{e^{\zeta\alpha_r}}{\alpha_r} {}_r^{\epsilon}\langle B_0|\Phi\rangle_r + F(\zeta)\right]$$

$a, F(\zeta)$ will be fixed by requiring $\delta_{\mathrm{B}}|D\rangle\!\rangle = 0$

$$\begin{aligned}\bar{\mathcal{O}}_D(\zeta) &= \exp\left[a\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|\Phi\rangle_r + F(\zeta)\right] \\ &= \exp\left[a\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|\bar{\psi}\rangle_r + F(\zeta)\right]\end{aligned}$$

$$\begin{cases} \displaystyle\int_{-\infty}^{0} dr \equiv \int_{-\infty}^{0} \frac{\alpha_r d\alpha_r}{2} \int \frac{d^{26}p_r}{(2\pi)^{26}}\, i d\bar{\pi}_0^{(r)} d\pi_0^{(r)} \\ |\Phi\rangle = |\psi\rangle + \underline{|\bar{\psi}\rangle} \end{cases}$$

creation

$$\begin{aligned}\delta_{\mathrm{B}} \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|\bar{\psi}\rangle_r &= \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|Q_{\mathrm{B}}^{(r)}|\bar{\psi}\rangle_r \\ &\quad + g\int_0^{\infty} d3\, \frac{e^{-\zeta\alpha_3}}{\alpha_3} \left[\int_{-\infty}^{0} d1 \int_0^{\infty} d2\, \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |\psi\rangle_2 |B_0\rangle_3^{\epsilon}\right. \\ &\qquad + \int_0^{\infty} d1 \int_{-\infty}^{0} d2\, \langle V_3(1,2,3)|\psi\rangle_1 |\bar{\psi}\rangle_2 |B_0\rangle_3^{\epsilon} \\ &\qquad \left. + \int_{-\infty}^{0} d1 \int_{-\infty}^{0} d2\, \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |\bar{\psi}\rangle_2 |B_0\rangle_3^{\epsilon}\right]\end{aligned}$$

$\boxed{\mathcal{O}\left(g^0\right)}$

$$Q_{\rm B} \sim Q_{\rm B}^{\rm KO} - i\pi_0(\partial_\alpha + \tfrac{1}{\alpha})$$

$$\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|Q_{\rm B}^{(r)}|\bar{\psi}\rangle_r = \zeta \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|i\pi_0^{(r)}|\bar{\psi}\rangle_r$$

shorthand notation

$$\left\{ \begin{array}{lcl} \bar{\phi}(\zeta) & \equiv & \displaystyle\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|\bar{\psi}\rangle_r \\ \bar{\chi}(\zeta) & \equiv & \displaystyle\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|i\pi_0^{(r)}|\bar{\psi}\rangle_r \end{array} \right.$$

$\bar{\chi}$ and $\bar{\phi}$ commute with each other

$$\bar{\mathcal{O}}_D(\zeta) = \exp\left(a\bar{\phi}(\zeta) + F(\zeta)\right)$$

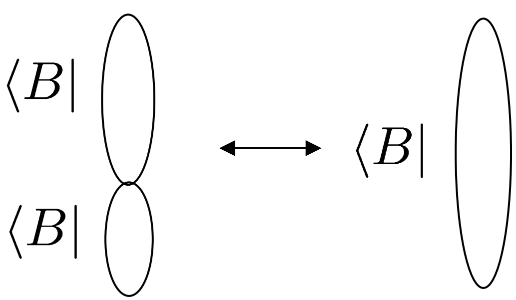
$$\boxed{\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r^{\epsilon}\langle B_0|Q_{\rm B}^{(r)}|\bar{\psi}\rangle_r = \zeta\bar{\chi}(\zeta)}$$

Using these, we obtain

$$
\begin{aligned}
\delta_{\mathrm{B}}|D\rangle\!\rangle \;&=\; \lambda \int d\zeta\, \exp\left(a\bar{\phi}(\zeta) + F(\zeta)\right) \\
&\quad \times \Bigg[a\zeta\bar{\chi}(\zeta) + g a^2 \int_{-\infty}^{0} d1 \int_{0}^{\infty} d2 \int_{0}^{\infty} d3 \frac{e^{\zeta\alpha_1}}{\alpha_2\alpha_3} \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |B_0\rangle_2^{\epsilon} |B_0\rangle_3^{\epsilon} \\
&\qquad + g a \int_{-\infty}^{0} d1 \int_{-\infty}^{0} d2 \int_{0}^{\infty} d3 \frac{e^{\zeta(\alpha_1+\alpha_2)}}{\alpha_3} \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |\bar{\psi}\rangle_2 |B_0\rangle_3^{\epsilon} \Bigg] |0\rangle\!\rangle
\end{aligned}
$$

$\boxed{\mathcal{O}\left(g\right)}$

We need the idempotency equation.



Idempotency equations

leading order in ϵ

$$\int d'3\langle V_3(1,2,3)|B_0\rangle_3^\epsilon \sim 2C_2\delta(\alpha_1+\alpha_2+\alpha_3)\,{}_1^\epsilon\langle B_0|\,{}_2^\epsilon\langle B_0|\left(\frac{i}{\alpha_1}\pi_0^{(1)}+\frac{i}{\alpha_2}\pi_0^{(2)}\right)\mathcal{P}_{12}$$

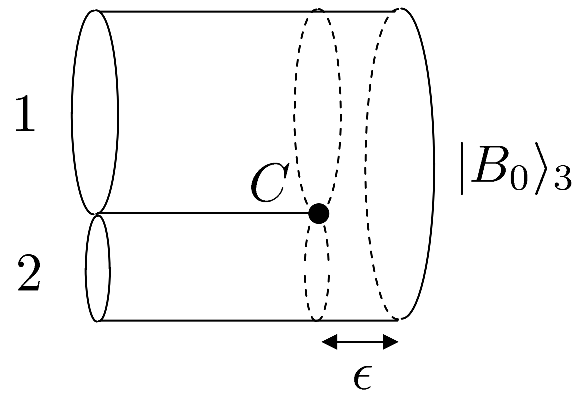
$$\int d'1d'2\langle V_3(1,2,3)|B_0\rangle_1^\epsilon|B_0\rangle_2^\epsilon \sim -2C_1\delta(\alpha_1+\alpha_2+\alpha_3)\,{}_3^\epsilon\langle B_0|\frac{2i}{\alpha_3}\pi_0^{(3)}\mathcal{P}_3$$

$$\left\{\begin{aligned} d'r &= \frac{d^{26}p_r}{(2\pi)^{26}}id\bar{\pi}_0^{(r)}d\pi_0^{(r)} \\ C_2 &\equiv \frac{1}{(16\pi)^{\frac{p+1}{2}}}\frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}} \\ C_1 &\equiv \frac{(4\pi^3)^{\frac{p+1}{2}}}{(2\pi)^{25}}\frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}} \end{aligned}\right.$$

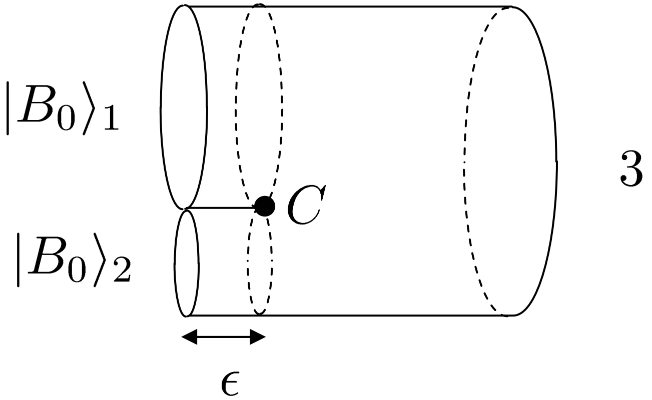
corrections
$$\begin{cases} \times\left(1+\mathcal{O}\left((-\ln\epsilon)^{-1}\right)\right) & (\text{for } p\neq -1) \\ \times\left(1+\mathcal{O}\left(\epsilon^2\right)\right) & (\text{for } p=-1) \end{cases}$$

Derivation of the idempotency equations

$$\int d'3 \langle V_3(1,2,3)|B_0\rangle_3^\epsilon$$

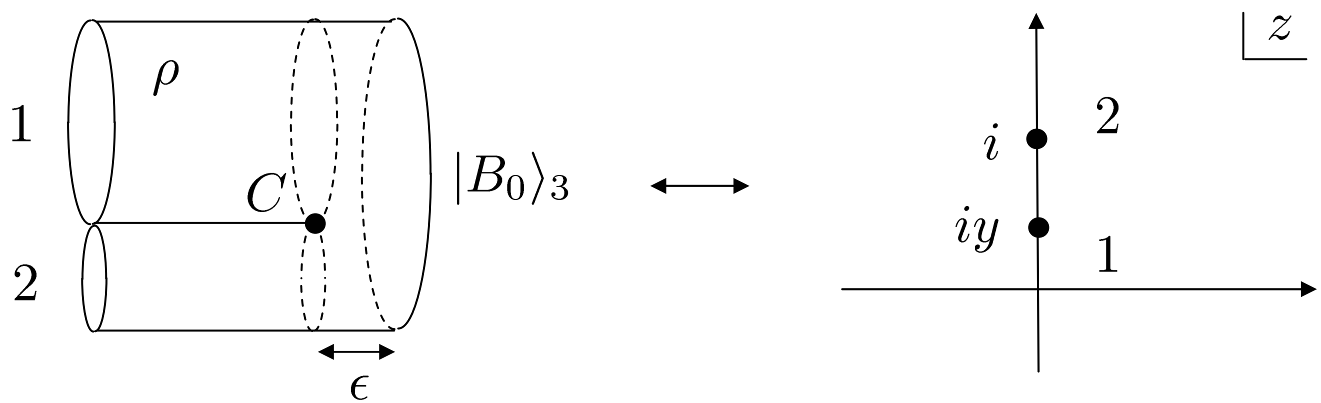


$$\int d'1 d'2 \langle V_3(1,2,3)|B_0\rangle_1^\epsilon |B_0\rangle_2^\epsilon$$



$$\int d'3\langle V_3(1,2,3)|B_0\rangle_3^{\epsilon} \sim {}_{12}\langle 0|\exp\left(E\left(\alpha_n^{M(r)},\tilde{\alpha}_n^{M(r)}\right)\right) \times F\left(\gamma_n^{(r)},\tilde{\gamma}_n^{(r)}\right) \times \mathcal{K}\left(\alpha_r,\epsilon\right)$$

$${}_{12}\langle 0|\exp\left(E\left(\alpha_n^{M(r)},\tilde{\alpha}_n^{M(r)}\right)\right) X^M(\rho)X^N(\rho')|0\rangle = G^{MN}(z,z')$$

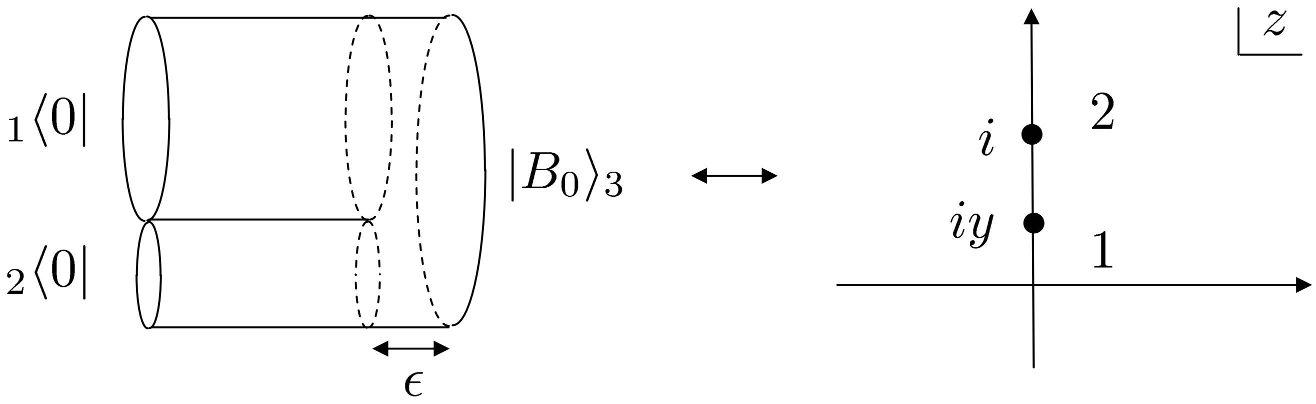


$$\rho(z) = \alpha_1 \ln\frac{z-iy}{z+iy} + \alpha_2 \ln\frac{z-i}{z+i}$$

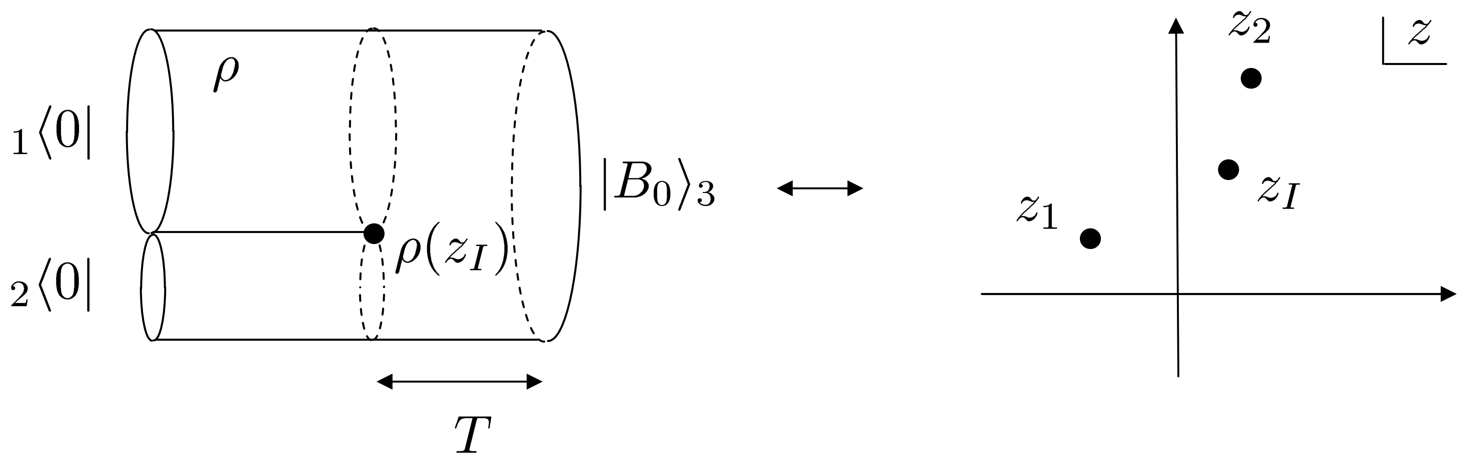
- $E\left(\alpha_n^{M(r)}, \tilde{\alpha}_n^{M(r)}\right)$: quadratic in $\alpha_n, \tilde{\alpha}_n$
  can be evaluated from the Neumann function

- $F\left(\gamma_n^{(r)}, \tilde{\gamma}_n^{(r)}\right)$ can also be evaluated from the Neumann function

- $\mathcal{K}\left(\alpha_r, \epsilon\right)$



$$\exp\left(-S_{\text{Liouville}}\right) \text{ for } ds^2 = d\rho d\bar{\rho}$$



$$\exp\left(-S_{\mathrm{Liouville}}\right) \propto \left|\partial^2\rho(z_I)\right|^{-1} \prod_{i=1,2} \left|\partial w_i(z_i)\right|^2$$

$$\rho(z) - \rho(z_I) = \alpha_i \ln w_i(z) \ (z \sim z_i)$$

we can fix the normalization in the limit $T \to \infty$

$\int d'1d'2\langle V_3(1,2,3)|B_0\rangle_1^\epsilon|B_0\rangle_2^\epsilon$ can be evaluated in the same way

Using these relations we obtain

$$\begin{aligned}
\delta_{\mathrm{B}}|D\rangle\rangle &= \lambda \int d\zeta \left[a\zeta\bar{\chi}(\zeta) + ga^2 C_1 \partial_\zeta \bar{\chi}(\zeta) + 2gaC_2\bar{\chi}(\zeta)\partial_\zeta\bar{\phi}(\zeta)\right] e^{a\bar{\phi}(\zeta)+F(\zeta)}\,|0\rangle\rangle \\
&= \lambda \int d\zeta ga^2 C_1 \partial_\zeta \left(\bar{\chi}(\zeta) e^{a\bar{\phi}(\zeta)+F(\zeta)}\right)|0\rangle\rangle
\end{aligned}$$

if

$$F(\zeta) = b\zeta^2$$

$$(a, b) = \pm(A, B)$$

$$A = \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}$$

$$B = \frac{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}$$

$$|D_\pm\rangle\rangle \equiv \lambda \int d\zeta \exp\left[\pm A\bar{\phi} \pm B\zeta^2\right]$$

Later we will show $\left\{\begin{array}{ll} |D_+\rangle\rangle: & \mathrm{one\ D-brane} \\ |D_-\rangle\rangle: & \mathrm{one\ ghost\ D-brane} \end{array}\right.$

States with N solitons

$$|D_{N+}\rangle\rangle \equiv \lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \, \bar{\mathcal{O}}_{D_{N+}}(\zeta_1, \cdots, \zeta_N)|0\rangle\rangle$$

$$\bar{\mathcal{O}}_{D_{N+}}(\zeta_1, \cdots, \zeta_N) = \exp\left[\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right) + F_N(\zeta_1, \cdots, \zeta_N)\right]$$

Imposing the BRS invariance, we obtain

$$F_N(\zeta_1, \cdots, \zeta_N) = 2\sum_{i>j} \ln(\zeta_i - \zeta_j)$$

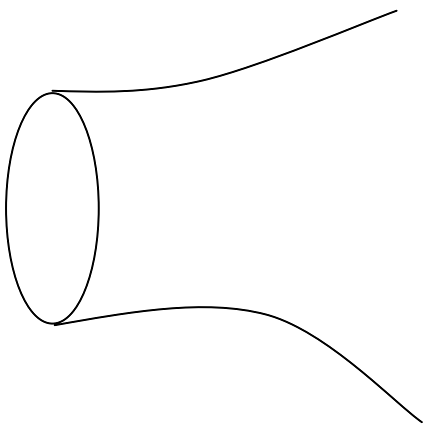
$$|D_{N+}\rangle\rangle = \lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \prod_{i>j} (\zeta_i - \zeta_j)^2 \exp\left[\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right)\right] |0\rangle\rangle$$

looks like the matrix model

ζ can be identified with the open string tachyon

$$\bar{\phi}(\zeta) \equiv \int_{-\infty}^{0} dr \, \frac{e^{\zeta \alpha_r}}{\alpha_r} \, {}^{\epsilon}_{r}\langle B_0 | \bar{\psi} \rangle_r$$

$$e^{-\zeta \times (\mathrm{length})} \langle B_0 |$$



$\zeta_i \ (i = 1, \cdots, N)$ may be identified with the eigenvalues of the matrix valued tachyon

α: length of the string $\longrightarrow$ open string tachyon

More generally we obtain

$$|D_{N+,M-}\rangle\rangle \equiv \lambda_{N+,M-} \int \prod_{i=1}^{N} d\zeta_i \prod_{\bar{\imath}=1}^{M} d\zeta_{\bar{\imath}} \prod_{i>j} (\zeta_i - \zeta_j)^2 \prod_{\bar{\imath}>\bar{\jmath}} (\zeta_{\bar{\imath}} - \zeta_{\bar{\jmath}})^2 \prod_{i,\bar{\jmath}} (\zeta_i - \zeta_{\bar{\jmath}})^{-2}$$
$$\times \exp\left[A\left(\sum_{i=1}^{N} \bar{\phi}(\zeta_i) - \sum_{\bar{\imath}=1}^{M} \bar{\phi}(\zeta_{\bar{\imath}})\right) + B\left(\sum_{i=1}^{N} \zeta_i^2 - \sum_{\bar{\imath}=1}^{M} \zeta_{\bar{\imath}}^2\right)\right] |0\rangle\rangle$$

- we can also construct $\langle\langle D_\pm|$ etc.

§ 4 Disk Amplitudes

$$\langle\!\langle 0|\, \mathrm{T} \prod_{r=1}^{N} \mathcal{O}_r(t_r, p_r)\, |0\rangle\!\rangle \;\rightarrow\; \langle\!\langle 0|\, \mathrm{T} \prod_{r=1}^{N} \mathcal{O}_r(t_r, p_r)\, |D_\pm\rangle\!\rangle$$

$$|D_\pm\rangle\!\rangle \equiv \lambda \int d\zeta \exp\left[\pm A\bar{\phi} \pm B\zeta^2\right]$$

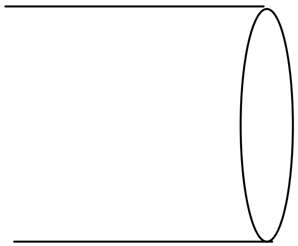
$|D_\pm\rangle\!\rangle \;$: $\mathrm{D-brane\ state}$

⟶ BRS invariant source term

Let us calculate amplitudes in the presence of such a source term.

saddle point approximation

$$
\begin{aligned}
|D_\pm\rangle\rangle &= \lambda \int d\zeta \exp\left[\pm \frac{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}\zeta^2 \pm \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\bar\phi(\zeta)\right]|0\rangle\rangle \\
&\simeq \lambda'_\pm \exp\left[\pm\frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\bar\phi(0)\right]|0\rangle\rangle \\
&= \lambda'_\pm \exp\left[\pm\frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\int_{-\infty}^{0}\frac{dr}{\alpha_r}{}^{\epsilon}_{r}\langle B_0|\bar\psi\rangle_r\right]|0\rangle\rangle
\end{aligned}
$$



$|B_0\rangle$

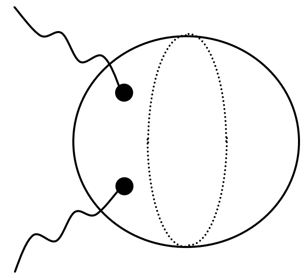
generate boundaries on the worldsheet

$$
\begin{aligned}
|D_+\rangle\rangle &: \ \text{unstable} \\
|D_-\rangle\rangle &: \ \text{stable}
\end{aligned}
$$

correlation functions

$$\left\langle\!\!\left\langle \mathcal{O}_1(t_1)\cdots\mathcal{O}_N(t_N) \right\rangle\!\!\right\rangle_{D_\pm} = \frac{\langle\!\langle 0|\mathrm{T}\mathcal{O}_1(t_1)\cdots\mathcal{O}_N(t_N)|D_\pm\rangle\!\rangle}{\langle\!\langle 0|D_\pm\rangle\!\rangle}$$

=



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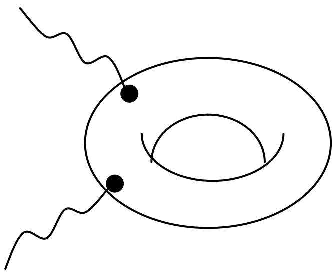

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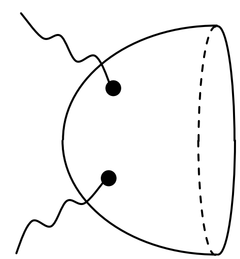
+



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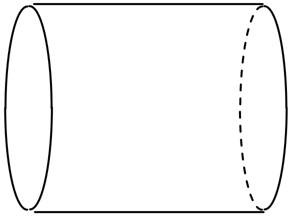
Let us calculate the disk amplitude

Rem.

In our first paper, we calculated the vacuum amplitude

$$\langle\!\langle D|e^{-iT\hat{H}}|D\rangle\!\rangle$$

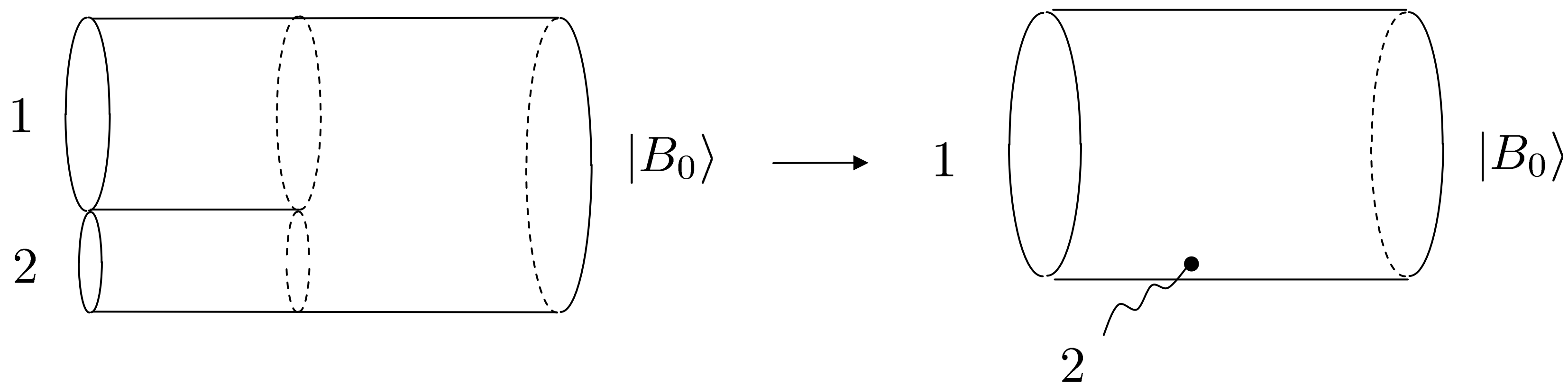


- we implicitly introduced two D-branes by taking the bra and ket

$\rightarrow$ states with even number of D-branes

but
- vacuum amplitude is problematic in light-cone type formulations
- we overlooked a factor of 2

disk two point function



$$
\begin{aligned}
\propto\ & \frac{1}{(k_2^2-2)} \int_{t_3}^{t_1} dT \int_{-\infty}^{0} \frac{d\alpha_1}{2\alpha_1} \int i\, d\bar{\pi}_0^{(1)} d\pi_0^{(1)}\, \frac{1}{\alpha_1}\, e^{-i\frac{t_1-T}{-\alpha_1}\left(k_1^2+M_1^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}\right)} \\
& \times {}_X\langle 1|\, V_2(k_2)\, e^{-i\frac{T-t_3}{-\alpha_1}\left(L_0^X+\tilde{L}_0^X+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}-2\right)} |B_0\rangle_X \\
=\ & \frac{1}{(k_2^2-2)}\, i \int_0^{\infty} dT' \int_0^{\infty} dT'' e^{-iT'\left(k_1^2-2\right)}\, {}_X\langle 1|\, V_2(k_2)\, e^{-iT''\left(L_0^X+\tilde{L}_0^X-2\right)} |B_0\rangle_X \\
=\ & \frac{1}{k_1^2-2}\, \frac{1}{k_2^2-2}\, {}_X\langle 1|\, V_2(k_2)\, \frac{-i}{L_0^X+\tilde{L}_0^X-2} |B_0\rangle_X
\end{aligned}
$$

tachyon-tachyon

$$\frac{1}{k_1^2-2}\frac{1}{k_2^2-2}\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\,{}_X\langle k_1|\, e^{ik_{2,\mu}X^\mu}(0)\,\frac{-i}{L_0^X+\tilde{L}_0^X-2}|B_0\rangle_X+\cdots$$

$$\downarrow$$

$$S_{TTD_\pm}=\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\,{}_X\langle k_1|\, e^{ik_{2,\mu}X^\mu}(0)\,\frac{1}{L_0^X+\tilde{L}_0^X-2}|B_0\rangle_X$$

coincides with the disk amplitude up to normalization

- more general disk amplitudes can be calculated in the same way

low-energy effective action

$$
\begin{aligned}
S \;=\;& \frac{1}{8g^2}\int d^{26}x\sqrt{-G}R + \int d^{26}x\sqrt{-G}\left(-\frac{1}{2}G^{\mu\nu}\partial_\mu T\partial_\nu T + T^2 + \frac{2g}{3}T^3\right) + \cdots \\
& \pm \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\int d^{26}x \prod_{i\in \mathrm{D}}\delta(x^i)\left[T(x) - 2\sum_{\mu,\nu\in \mathrm{N}} h_{\mu\nu}(x)\eta^{\mu\nu} + \cdots\right]
\end{aligned}
$$

$$G_{\mu\nu} = \eta_{\mu\nu} + 4gh_{\mu\nu}$$

Comparing this with the action for D-branes

$$
\boxed{
\begin{aligned}
&|D_+\rangle\!\rangle\ (\mathrm{unstable}):\ \ \mathrm{one\ D-brane} \\
&|D_-\rangle\!\rangle\ (\mathrm{stable}):\ \ \mathrm{one\ ghost\ D-brane}
\end{aligned}
}
$$

§ 5 Conclusion and Discussion

- D-brane $\longleftrightarrow$ BRS invariant source term
- other amplitudes
- similar construction for superstrings

    we need to construct OSp SFT for superstrings

- more fundamental formulation?