AdS/CFT対応とグルーオン散乱振幅計算の最近の進展について

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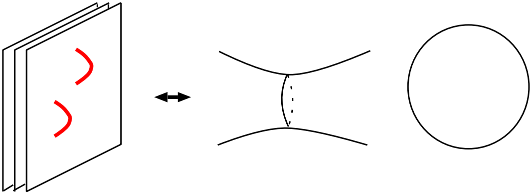
Outline

Introduction

AdS/CFT correspondence[Maldacena]

$U(N)$ $\mathcal{N}=4$ SYM $\Longleftrightarrow$ typer IIB Superstrings on $AdS_5 \times S^5$



$SO(2,4)$ conformal symmetry =isometry of AdS_5

$SO(6)$ R-symmetry = isometry of S^5 (radius R)

strong-weak coupling duality

$$\boxed{\lambda = g_{YM}^2 N \sim \frac{R^4}{(\alpha')^2}}$$

Evidences of AdS/CFT correspondence

correspondence between gauge invariant observable

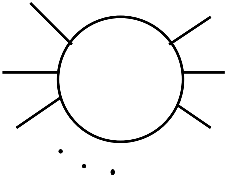
- correlation functions [Gubser-Klebanov-Polyakov, Witten,...]
- anomalous dimensions of gauge invariant operators
  Bethe Ansatz, spin chain[Minahan-Zarembo,...]
- BMN limit

new issue: Gluon Scattering amplitudes (colored states)
Alday-Maldacena arXiv:0705.0303

$\mathcal{N} = 4$ Super Yang-Mills Theory

Gluon Scattering Amplitudes in $\mathcal{N} = 4$ SYM

- UV finite (SUSY)
- IR divergent (massless gluons)
- Perturbatively simple structure $\Longrightarrow$ BDS ansatz
  Bern-Dixon-Smirnov, hep-th/0505205
- Applications
  - QCD (similar short distance physics)
  - finiteness of $N = 8$ SUGRA
  - MHV amplitudes, twistor string

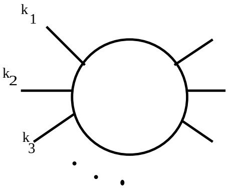


BDS conjecture

$\mathcal{N}=4$ SYM $SU(N_c)$
Planar Contribution to the L-loop, n-point amplitudes

$$\mathcal{A}_n^{(L)} = g^{n-2}\left(\frac{2e^{-\epsilon\gamma}g^2N_c}{(4\pi)^{2-\epsilon}}\right)\sum_{\rho}\mathrm{Tr}(T^{a_{\rho(1)}}\cdots T^{a_{\rho(n)}})A_n^{(L)}(\rho(1),\cdots,\rho(n))$$

- ρ: non-cyclic permutations
- $\lambda = g^2 N_c$: ' t Hooft coupling
- dimensional reduction (IR divergence)
  $D=4-2\epsilon$, γ: Euler's constant



ratio of loop amplitude and tree amplitude (color singlet)

$$\mathcal{M}_n^{(L)}(\epsilon) \equiv \frac{\mathcal{A}_n^{(L)}(\epsilon)}{\mathcal{A}_n^{(0)}}$$

is written in terms of 1-loop amplitudes.

$$\mathcal{M}_n(\epsilon) \equiv 1+\sum_{L=1}^{\infty} a^L \mathcal{M}_n^{(L)}(\epsilon) = \exp\left(\sum_{\ell=1}^{\infty} a^\ell (f^\ell(\epsilon) M_n^{(1)}(\ell\epsilon) + C^{(\ell)} + O(\epsilon)\right.$$

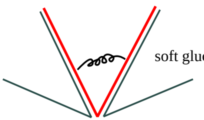
$$a = \frac{N_c g^2 \mu^{2\epsilon}}{8\pi^2}(4\pi e^{-\gamma})^\epsilon, \quad f^{(\ell)}(\epsilon) = f_0^{(\ell)} + f_1^{(\ell)}\epsilon + f_2^{(\ell)}\epsilon^2$$

IR singular part + finite remainder part

$$\mathcal{M}_n = \exp\left[\sum_{\ell=1}^{\infty} a^\ell f^{(\ell)}(\epsilon)\hat{I}_n^{(1)}(\ell\epsilon)\right]\tilde{h}_n(\epsilon)$$

resummation of soft gluons$\Longrightarrow$ IR singularity

contribute from adjacent momenta



$\hat{I}_n^{(1)}(\epsilon) = -\frac{1}{2\epsilon^2}\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^\epsilon$

$s_{i,i+1} = (k_i + k_{i+1})^2$

$$\ln\mathcal{M}_n^{IR}(\epsilon) = \frac{A_2}{\epsilon^2} + \frac{A_1}{\epsilon} + A_0 - \frac{1}{16}f(\lambda)\sum_{i=1}^{n}\left(\ln\left(\frac{\mu^2}{-s_{i,i+1}}\right)\right)^2 - \frac{g(\lambda)}{4}\sum_{i=1}^{n}\ln\left(\frac{\mu^2}{-s_{i,i+1}}\right)$$

$f(\lambda) = 4\sum_{l=1}^{\infty} f_0^{(l)} a^l$: cusp anomalous dimension

$g(\lambda) = 2\sum_{l=1}^{\infty} \frac{f_1^{(l)}}{l} a^l$: collinear anomalous dimension

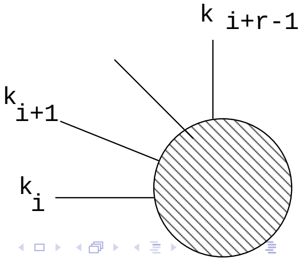
finite remainder part($\epsilon \to 0$)

$$\tilde{h}_n(0) = \exp\left(\frac{1}{4} f(\lambda) F_n^{(1)}(0) + C\right)$$

$F_n^{(1)}(0) = \sum_{i=1}^{n} g_{n,i}$

$$g_{n,i} = -\sum_{r=2}^{[n/2]-1} \ln\left(\frac{-t_i^{[r]}}{-t_i^{[r+1]}}\right) \ln\left(\frac{-t_{i+1}^{[r]}}{-t_i^{[r+1]}}\right) + D_{n,i} + L_{n,i} + \frac{3}{2}\zeta_2$$

Mandelstam variables:
$t_i^{[r]} \equiv (k_i + \ldots + k_{i+r-1})^2$



$\bullet n = 2m+1$

$$D_{2m+1} = -\sum_{r=2}^{m-1} \mathrm{Li}_2\left(1 - \frac{t_i^{[r]} t_{i-1}^{[r+2]}}{t_i^{[r+1]} t_{i-1}^{[r+1]}}\right)$$

$$L_{2m+1} = -\frac{1}{2} \ln\left(\frac{-t_i^{[m]}}{-t_{i+m+1}^{[m]}}\right) \ln\left(\frac{-t_{i+1}^{[m]}}{-t_{i+m}^{[m]}}\right)$$

$\bullet n = 2m$

$$D_{2m} = -\sum_{r=2}^{m-2} \mathrm{Li}_2\left(1 - \frac{t_i^{[r]} t_{i-1}^{[r+2]}}{t_i^{[r+1]} t_{i-1}^{[r+1]}}\right) - \frac{1}{2}\mathrm{Li}_2\left(1 - \frac{t_i^{[m-1]} t_{i-1}^{[m+1]}}{t_i^{[m]} t_{i-1}^{[m]}}\right)$$

$$L_{2m} = -\frac{1}{4} \ln\left(\frac{-t_i^{[m]}}{-t_{i+m+1}^{[m]}}\right) \ln\left(\frac{-t_{i+1}^{[m]}}{-t_{i+m}^{[m]}}\right)$$

$\mathrm{Li}_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}$: dilogarithmic function

4-point amplitude

$$\begin{aligned}\ln \mathcal{M}_4 &= \exp\{-\frac{f(\lambda)}{8}(\ln^2 \frac{\mu^2}{-s} + \ln^2 \frac{\mu^2}{-t}) \\ &\quad -\frac{g(\lambda)}{2}(\ln \frac{\mu^2}{-s} + \ln \frac{\mu^2}{-t}) + \frac{f(\lambda)}{8} \ln^2 \frac{s}{t} + const.\}\end{aligned}$$

$f(\lambda)$: universal function controlling leading divergence
anomalous dimension of twist two operators($\leftarrow$ spin chain)

$$f(\lambda) = \frac{\lambda}{2\pi^2}\left(1 - \frac{\lambda}{48} + \frac{11}{11520}\lambda^2 + \cdots\right)$$

Resummantion of power series in λ: $f(\lambda) \sim \sqrt{\lambda}$
Kotikov-Lipatov-Onischenko-Velzhanin, hep-ph/0301021

- Recursive structure of the amplitudes
  IR singular part $\Longleftarrow$ IR finiteness of physical quantities
  In $\mathcal{N}=4$ SYM, finite parts obey the same iterative relations as the IR singular terms
- explict loop calculation
  - 4-point up to 3-loops [BDS]
  - 5-point up to 2-loops [Cachazo et. al. , Bern et. al.]
  - $n(\geq 6)$-point 1-loop [Bern et. al.]
- Discrepancy in 6-point 2-loop amplitude[Bern et. al. 0803.1465]
- Dual conformal symmetry [Drummond-Henn-Korchemsky-Sokatchev]

A review of Alday-Maldacena calculation

AdS_5:
embedding coordinates in $\mathbf{R}^{2,4}$: isometry $SO(2,4)$

$$-X_{-1}^2 - X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = -R^2$$

Poincaré coordinates: (x^μ, z) $\mu = 0, 1, 2, 3$

$$X^\mu = \frac{x^\mu}{z}, \quad X_{-1} + X_4 = \frac{1}{z}, \quad X_{-1} - X_4 = \frac{z^2 + x_\mu x^\mu}{z}$$

Poincaré metric

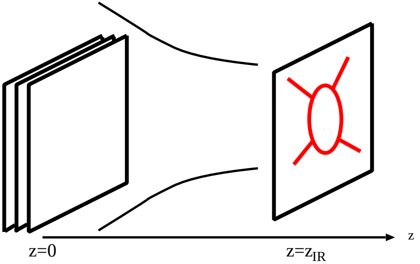
$$ds^2 = R^2 \frac{dx_\mu dx^\mu + dz^2}{z^2}$$

AdS/CFT correspondence: $R^4 = 4\pi(\alpha')^2 g_{YM}^2 N$

N_c D3-brane at $z=0$ $\rightarrow$ $\mathcal{N}=4\ U(N_c)$ SYM

gravity dual : AdS_5 geometry

- Place a D-brane at $z=z_{IR}$
  in AdS_5 geometry
  scattering of open strings on the IR D-brane
- proper momentum $\frac{kz_{IR}}{R}\gg 1$
  $(z_{IR}\rightarrow\infty)$
  high-momentum, fixed angle



Gross-Mende type analysis

Classical solution gives the leading contribution.

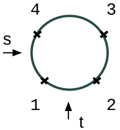
But the calculation is harder than in flat space.

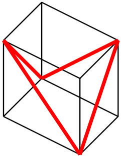
T-duality in the x-direction

T-duality

• open string on a cricle with radius R: $R \to \frac{\alpha'}{R}$
KK momentum $p = \frac{n}{R}$ $\leftrightarrow$ winding $2\pi w R$
$X(\tau,\sigma) = x - i2\alpha' p\tau + \cdots \iff X'(\tau,\sigma) = x' + 2\alpha' p\sigma + \cdots$
Neumann b.c. $\partial_\sigma X = 0 \iff$ Dirichlte b.c. $\partial_\tau X' = 0$

•T-duality in curved noncompact spacetime
metric: $ds^2 = w(z)^2 dx_\mu dx^\mu$,
$x^\mu \to y^\mu$
$\partial_\alpha y^\mu = i w(z)^2 \epsilon_{\alpha\beta}\partial_\beta x^\mu$
momentum $\leftrightarrow$ winding
$x^\mu = ik^\mu\tau + \ldots \longleftrightarrow y^\mu = k^\mu\sigma + \ldots$
Neumann b.c $\to$ Dirichlet b.c.
$\Delta y^\mu = 2\pi k^\mu$:
closed curve surrounded by light-like segments





T-duality in AdS

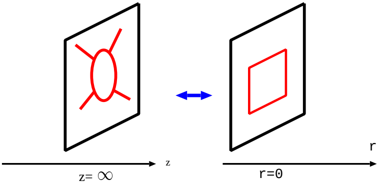
T-duality $x^\mu \to y^\mu$ $(\partial_\alpha y^\mu = iw(z)^2 \epsilon_{\alpha\beta}\partial_\beta x^\mu,\ w(z) = R^2/z^2)$

$$ds^2 = R^2 \frac{dx_\mu dx^\mu + dz^2}{z^2} \to d\tilde{s}^2 = \frac{z^2}{R^2} dy^\mu dy_\mu + R^2 \frac{dz^2}{z^2}$$

$$d\tilde{s}^2 = R^2 \frac{dy^\mu dy_\mu + dr^2}{r^2} \qquad \left(r = \frac{R^2}{z} \right) AdS_5 \text{ metric}$$

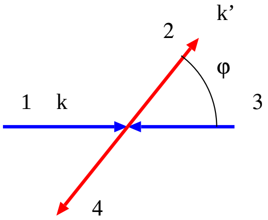
T-dualized surface in (y^μ, r)-space

- ends at $r = \frac{R^2}{z_{IR}} \to 0$ $(z_{IR} \to \infty)$
- y^μ: surrounded by the light-like segments



4-point amplitude: kinematics

center of mass frame



$$
\begin{aligned}
k_1 &= (k, \mathbf{k}) \\
k_2 &= (-k, -\mathbf{k}') \\
k_3 &= (k, -\mathbf{k}) \\
k_4 &= (-k, \mathbf{k}')
\end{aligned}
$$

$k = |\mathbf{k}| = |\mathbf{k}'|$, $\mathbf{k} \cdot \mathbf{k}' = k^2 \cos\varphi$

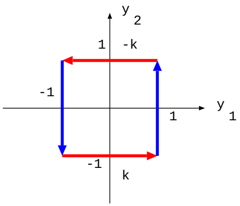
Mandelstam variables

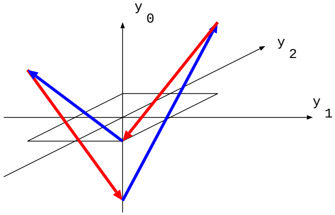
$$
\begin{aligned}
s &= -(k_1 + k_2)^2 = -4k^2 \sin^2 \frac{\varphi}{2} \\
t &= -(k_1 + k_4)^2 = -4k^2 \cos^2 \frac{\varphi}{2} \\
u &= -(k_1 + k_3)^2 = 4k^2
\end{aligned}
$$

$s + t + u = 0$, $s, t < 0$: space-like momentum transfer

surface in the T-dual picture

Poincaré coordinates (y^0, y^1, y^2, y^3, r)
Poinacaré metric $ds^2 = \frac{-dy_0^2+dy_1^2+dy_2^3+dy_3^2+dr^2}{r^2}$
surface parameterized by (y_1, y_2), $y_3 = 0$
specific momentum configuration $s = t$, $\varphi = \frac{\pi}{2}$
$\mathbf{k} = (1,0)$, $\mathbf{k}' = (0,1)$





boundary conditions: $r(\pm 1, y_2) = r(y_1, \pm 1) = 0$,
$y_0(\pm, y_2) = \pm y_2$, $y_0(y_1, \pm 1) = \pm y_1$

Nambu-Goto action

$h_{y_a y_b}$: induced metric

$$h_{y_a y_a} = \frac{-(\partial_a y_0)^2 + 1 + (\partial_a r)^2}{r^2}, \quad h_{y_1 y_2} = \frac{-\partial_1 y_0 \partial_2 y_0 + \partial_1 r \partial_2 r}{r^2}$$

$$\begin{aligned} S &= \frac{R^2}{2\pi} \int dy_1 dy_2 \sqrt{-\det(h_{y_a y_b})} \\ &= \frac{R^2}{2\pi} \int dy_1 dy_2 \frac{\sqrt{1 + (\partial_i r)^2 - (\partial_i y_0)^2 - (\partial_1 r \partial_2 y_0 - \partial_2 r \partial_1 y_0)^2}}{r^2} \end{aligned}$$

solution to the equation of motion:

$$y_0 = y_1 y_2, \quad r = \sqrt{(1 - y_1^2)(1 - y_2^2)}$$

general (s,t) solution

- Back to the embedding coordinates: $(y^\mu, r) \to (Y^m)$

$$Y^\mu = \frac{y^\mu}{r}, \quad Y_{-1} + Y_4 = \frac{1}{r}, \quad Y_{-1} + Y_4 = \frac{r^2 + y^\mu y_\mu}{r^2}$$

- $SO(2,4)$ conformal symmetry (boost in (0,4)-plane+overall scaling)

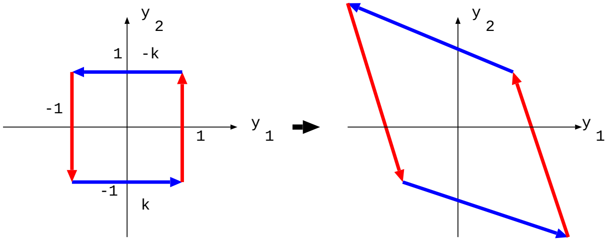
$$\begin{pmatrix} Y^0 \\ Y^4 \end{pmatrix} \to \begin{pmatrix} Y'^0 \\ Y'^4 \end{pmatrix} = \begin{pmatrix} \gamma & v\gamma \\ v\gamma & \gamma \end{pmatrix} \begin{pmatrix} Y^0 \\ Y^4 \end{pmatrix}$$

$\gamma = \frac{1}{\sqrt{1-v^2}}$

- map to the Poincaré coordinates $Y'^m \to (y'^\mu, r')$

$$r' = \frac{ar}{1+by_0}, \quad y'_0 = \frac{a\sqrt{1+b^2}y_0}{1+by_0}, \quad y'_1 = \frac{ay_1}{1+by_0}, \quad y'_2 = \frac{ay_2}{1+by_0}$$

$b = v\gamma(<1)$, a: overall scale parameter



$$(y_0, y_1, y_2) = \begin{cases} (1,-1,-1) \\ (-1,1,-1) \\ (1,1,1) \\ (-1,-1,1) \end{cases} \rightarrow (y'_0, y'_1, y'_2) = \begin{cases} (\frac{a\sqrt{1+b^2}}{1+b}, -\frac{a}{1+b}, -\frac{a}{1+b}) \\ (-\frac{a\sqrt{1+b^2}}{1-b}, \frac{a}{1-b}, -\frac{a}{1-b}) \\ (\frac{a\sqrt{1+b^2}}{1+b}, \frac{a}{1+b}, \frac{a}{1+b}) \\ (-\frac{a\sqrt{1+b^2}}{1-b}, -\frac{a}{1-b}, \frac{a}{1-b}) \end{cases}$$

$$-s(2\pi)^2 = \frac{8a^2}{(1-b)^2}, \; -t(2\pi)^2 = \frac{8a^2}{(1+b)^2}$$

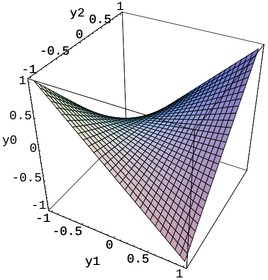
evaluation of the Nambu-Goto action

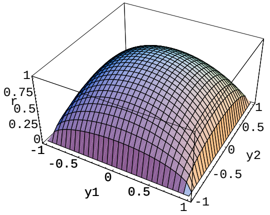
The action is divergent due to

- noncompactness of the target space
- divergence at the cusps (or corners)

We introduce

- Dimensional regularization of the gravity dual
  D3-brane$\Longrightarrow$ Dp-brane with $p = 3 - 2\epsilon$
- conformal gauge





Dimensional regularization

Dp-brane metric ($D = 1 + p = 4 - 2\epsilon$), μ: IR cutoff

$$ds^2 = f^{-\frac{1}{2}} dx_D^2 + f^{\frac{1}{2}}[dr^2 + r^2 d\Omega_{9-D}^2], \quad f = \frac{c_D \lambda_D}{r^{8-D}}$$

$$c_D = 2^{4\epsilon}\pi^{3\epsilon}\Gamma(2+\epsilon), \quad \lambda_D = g_D^2 N = \frac{\lambda \mu^{2\epsilon}}{(4\pi e^{-\gamma})^{\epsilon}}$$

T-dual metric

$$ds^2 = f^{\frac{1}{2}}(dy_D^2 + dr^2) = \sqrt{c_D \lambda_D} \frac{dy_D^2 + dr^2}{r^{2+\epsilon}}$$

the Nambu-Goto action

$$S = \frac{\sqrt{c_D \lambda_D}}{2\pi} \int \frac{\mathcal{L}_{\epsilon=0}}{r^{\epsilon}}$$

solution: $r_\epsilon \sim \sqrt{1 + \epsilon/2}\, r_{\epsilon=0}; \quad y_\epsilon^\mu \simeq y_{\epsilon=0}^\mu$

conformal gauge

induced metric on the worldsheet (y_1, y_2) for $s = t$

$$ds^2_{WS} = \frac{dy_1^2}{(1-y_1^2)^2} + \frac{dy_2^2}{(1-y_2^2)^2} = du_1^2 + du_2^2$$

where $y_i = \tanh u_i$ $(i = 1, 2)$. $(-\infty < u_i < +\infty)$. The NG-action becomes

$$iS = -\frac{R^2}{2\pi} \int du_1 du_2 \frac{1}{2} \frac{\partial r \partial r + \partial y_\mu \partial y^\mu}{r^2}$$

$$y_1 = \frac{\tanh u_1}{1 + b \tanh u_1 \tanh u_2} \quad y_2 = \frac{\tanh u_2}{1 + b \tanh u_1 \tanh u_2}$$

$$y_0 = \frac{\sqrt{1+b^2} \tanh u_1 \tanh u_2}{1 + b \tanh u_1 \tanh u_2} \quad r = \frac{1}{\cosh u_1 \cosh u_2 + b \sinh u_1 \sinh u_2}$$

Evaluate dimensionally regularized action in the conformal gauge:

$$-iS = B_\epsilon \int_{-\infty}^{\infty} du_1 du_2 \left(\frac{r}{a}\right)^{-\epsilon} (1 + \epsilon I_1 + \epsilon^2 I_2 + \cdots)$$

$B_\epsilon = \frac{\sqrt{c_D \lambda_D}}{2\pi}$, $I_1 = -\frac{\partial y^\mu \partial y_\mu}{4r^2}$, $I_2 = -\frac{\partial r \partial r}{8r^2}$

$$iS = -B_\epsilon \left(\frac{\pi \Gamma(-\frac{\epsilon}{2})^2}{\Gamma(\frac{1-\epsilon}{2})^2} {}_2F_1(\frac{1}{2}, -\frac{\epsilon}{2}, \frac{1-\epsilon}{2}; b^2) + 1 \right)$$

For small ϵ

$$\begin{aligned} & {}_2F_1(\frac{1}{2}, -\frac{\epsilon}{2}, \frac{1-\epsilon}{2}; b^2) \\ = \ & 1 + \frac{1}{2}\log(1-b^2)\epsilon + \frac{1}{2}\log(1-b)\log(1+b)\epsilon^2 \\ = \ & \frac{1}{2}\left(\frac{8a^2}{(-t)4\pi^2}\right)^{\frac{\epsilon}{2}} + \frac{1}{2}\left(\frac{8a^2}{(-s)4\pi^2}\right)^{\frac{\epsilon}{2}} - \frac{\epsilon^2}{4}\left(\frac{1}{2}\log\frac{s}{t}\right)^2 \end{aligned}$$

4-point function: result

$$
\begin{aligned}
iS &= -\frac{\sqrt{\lambda}}{2\pi}\Big\{\frac{1}{\epsilon^2}\left[2\left(\frac{\mu^2}{(-t)}\right)^{\frac{\epsilon}{2}}+2\left(\frac{\mu^2}{(-s)}\right)^{\frac{\epsilon}{2}}\right] \\
&\quad +\frac{1}{\epsilon}(1-\log 2)\left[\left(\frac{\mu^2}{(-t)}\right)^{\frac{\epsilon}{2}}+\left(\frac{\mu^2}{(-s)}\right)^{\frac{\epsilon}{2}}\right] \\
&\quad -\left(\frac{1}{2}\log\frac{s}{t}\right)^2 \\
&\quad +\frac{1}{6}(-\pi^2+3(-1-2\log 2+(\log 2)^2))+1\Big\}
\end{aligned}
$$

$\mathcal{M}=e^{iS}$ agrees with the BDS formula with

$$
f(\lambda)=\frac{\sqrt{\lambda}}{\pi},\quad g(\lambda)=\frac{\sqrt{\lambda}}{2\pi}(1-\ln 2)
$$

Test of BDS ansatz

test at strong coupling: higher-point amplitudes from strings on AdS

- attempt to find general solution
  Mironov-Morozov-Tomaras, Itoyama-Mironov-Morozov
- Large n limit
  Alday-Maldacena
- explicit calculation of 6-point and 8-point amplitudes with specific momentum configuration
  Astefanesei-KI-Dobashi-Nastase

test at weak coupling

- Wilson loop and BDS formula
  Drummond-Henn-Korchemsky-Sokatchev
- Regge limit Brower-Nastase-Schnitzer-Tan

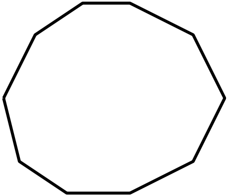
n-point amplitude

T-dual surface:

Plateau Problem: n-polygon surrounded light-like segments with $r=0$ on the boundary

eqs. of motion

$$\partial\left(\frac{\partial r}{r^2}\right) = -\frac{(\partial r)^2+(\partial y^\mu)^2}{r^3}, \quad \partial\left(\frac{\partial y^\mu}{r^2}\right)=0$$



Exact solution for n-point amplitude is not yet known for $n \geq 5$.

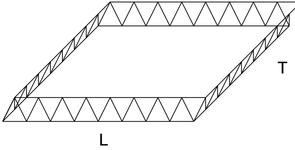
large n-point amplitude

[Alday-Maldacena 0710.1060]
Wilson-loop with many light-like boundaries
approximate it by smooth space-like one

AdS

$T \gg L$: heavy-quark potential
[Rey-Yee, Maldacena]

$$\log\langle W\rangle = \sqrt{\lambda}\frac{4\pi^2}{\Gamma(\frac{1}{4})^4}\frac{T}{L} \sim 0.228\sqrt{\lambda}\frac{T}{L}$$



BDS

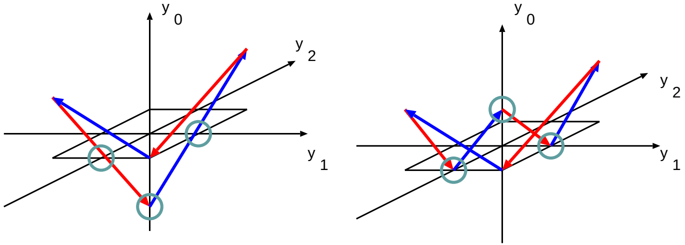
$$\frac{f(\lambda)}{4}\mathcal{F}_n \sim \frac{\sqrt{\lambda}}{4}\frac{T}{L}$$

wavy circular boundary [Itoyama-Mironov-Morozov,0803.1547

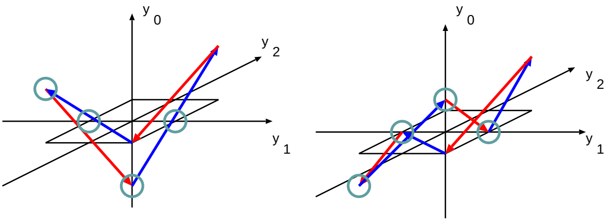
Cutting and Gluing

Construct 6-pt and 8-pt amplitudes from 4-pt by cutting and gluing.

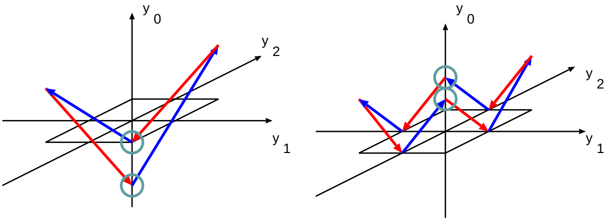
- $y = \pm y_1 y_2$ is also a solution of eom.
- cutting and gluing the surface such that $r = \sqrt{(1-y_1^2)(1-y_2^2)}$ remains unchanged.



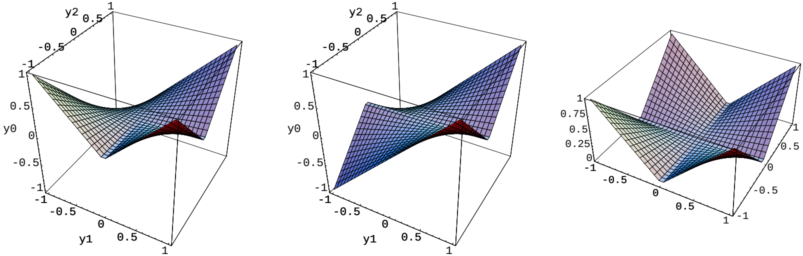
6-point function (sol 1) $\quad y_0 = \frac{1}{2}(|y_1 y_2| + y_1 y_2 - |y_1| y_2 + y_1 |y_2|)$



6-point function (sol 2) $y_0 = y_1|y_2|$



8-point function $y_0 = |y_1 y_2|$



- All results do not coincide with the BDS conjecture.
- AdS side: Leading IR singularities are the same as the 4-point amplitudes
  extra cusp: $O(\frac{1}{\epsilon})$ contributions
- The finite part is also different
- There are extra sources on the y-axis. This could not be a real solution to the Plateau problem.
- These AdS amplitudes might correspond to different field theory diagrams:

planar gluon amplitudes/Wilson loops duality

Drummond-Henn-Korchemsky-Sokatchev 0712.1223

light-like Wilson loop (weak coupling)

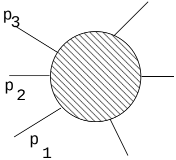
$$
\begin{aligned}
W(C_n) &= \frac{1}{N}\langle 0|\mathrm{Tr} P \exp\left(i \oint_{C_n} dx^\mu A_\mu(x) \right)|0\rangle \\
&= 1 + \frac{1}{2}(ig)^2 C_f(SU(N)) \oint_{C_n} dx^\mu \oint_{C_n} dy^\nu G_{\mu\nu}(x-y) + O(g^4)
\end{aligned}
$$

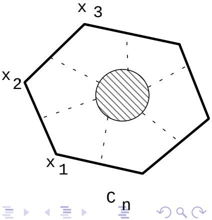
$$
G_{\mu\nu}(x) = -g_{\mu\nu}\frac{\Gamma(1-\epsilon_{UV})}{4\pi^2}(-x^2+i0)^{-1+\epsilon_{UV}}(\mu^2 e^{-\gamma})^{\epsilon_{UV}}
$$

$x_i - x_{i+1} \sim p^i$

cusp singularity: UV cutoff

$D = 4 - 2\epsilon_{UV}$ $(\epsilon_{UV} < 0)$





$$\ln W(C_n) = Z_n^{WL} + F_n^{WL} + O(\epsilon_{UV})$$

Z_n: cusp singularity, F_n^{WL}: finite part

$$\ln \mathcal{M}_n^{MHV} = \ln W(C_n) + const. \quad (F_n^{MHV} = F_n^{WL})$$

$$\epsilon_{IR} = -\epsilon_{UV}, \quad t_i^{[2]}/\mu_{IR}^2 = (x_{i,i+2})^2 \mu_{UV}^2 e^{\gamma(a)}, \quad \frac{t_i^{[j]}}{t_k^{[l]}} = \frac{x_{i,i+j}^2}{x_{k,k+l}^2}$$

$t_i^{[j]} = (p_i + \cdots + p_{i+j-1})$, $x_{i,j} = x_i - x_j$

- n-point 1-loop [Brandhuber-Heslop-Travaglini]
- anomalous conformal Ward indentities in x-space (dual conformal symmetry)

$$\sum_{i=1}^{n} (2x_i^\mu x_i \partial_i - x_i^2 \partial_i^\mu) F^{WL} = \frac{f(\lambda)}{2} \sum_{i=1}^{n} x_{i,i+1}^\mu \ln \left(\frac{{x_{i,i+2}}^2}{x_{i-1,i+2}^2} \right)$$

- BDS F_n satisfy anomalous Ward identities.

Assume the dual conformal symmetry

- the n-gon light-like Wilson loop
  non-trivial dependence might come from the function of the cross-ratio
  $$u_{ij,kl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$$
  But there is no such variable for $n = 4, 5$. This dual conformal symmetry fix the higher-loop corrections for $n = 4, 5$ point amplitudes.
- The two-loop $n = 6$ amplitude provides the non-trivial test.
  $u_{13,46}$, $u_{24,15}$, $u_{35,26}$

6-point 2-loop test

Bern-Dixon-Kosower-Roiban-Spradlin-Vergu-Volovich, 0803.1465
Drummond-Henn-Korchemsky-Sokatchev, 0803.1466
They checked numerically

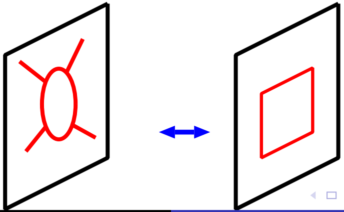
$$\ln M_6^{MHV} = \ln W(C_6) + const.$$

$$F_6^{WL} = F_6^{BDS} + R_6(u), \quad R_6 \neq 0$$

The gluon amplitudes/Wilson loops duality holds but the BDS ansatz does not hold for 6-point 2-loop amplitudes.
The Regge limit of amplitudes: the BDS formula needs to be corrected. [Brower et al.]

Summary

- Gluon scattering amplitudes at strong coupling are very interesting in order to check the AdS/CFT correspondence
- BDS ansatz is OK for
  - $n = 4, 5$-point amplitudes
  - $n \geq 6$ 1-loop amplitudes
- Discrepancy between 6-point 2-loop amplitude and the BDS formula
- gluon amplitudes/ Wilson loop duality (weak-strong, weak-weak)



Outlook

- exact formula for general n-point amplitudes
  How the BDS formula is corrected?
  dual conformal symmetry?
  Numerical analysis?
- quark scattering, add D7-branes
  Komargodski-Razamat MacGreevy-Sever
  n-point amplitudes$=$ $2n$-point gluon amplitudes
- non-AdS geometry (finite temperature, beta-deformation)
  K.I, H. Nastase and K. Iwasaki, arXiv:0711.3532
  Oz-Theisen-Yankielowicz