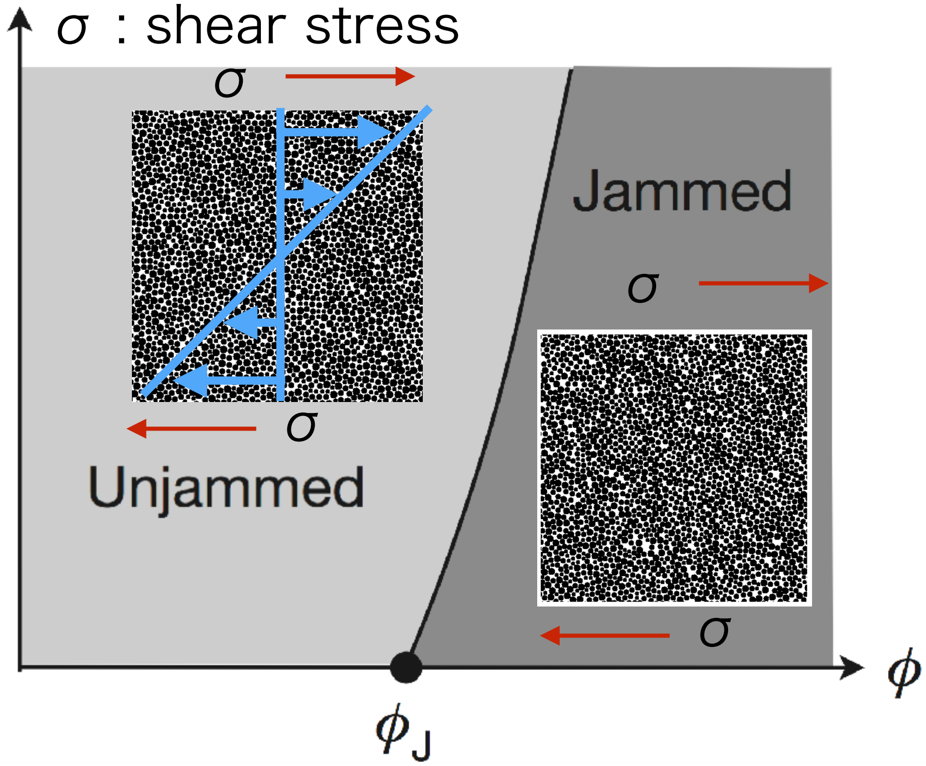
History-dependent shear jamming of frictional grains under oscillatory shear

Michio Otsuki (Osaka Univ.),
Hisao Hayakawa (Kyoto Univ.)

Jamming phase diagram (Frictionless)



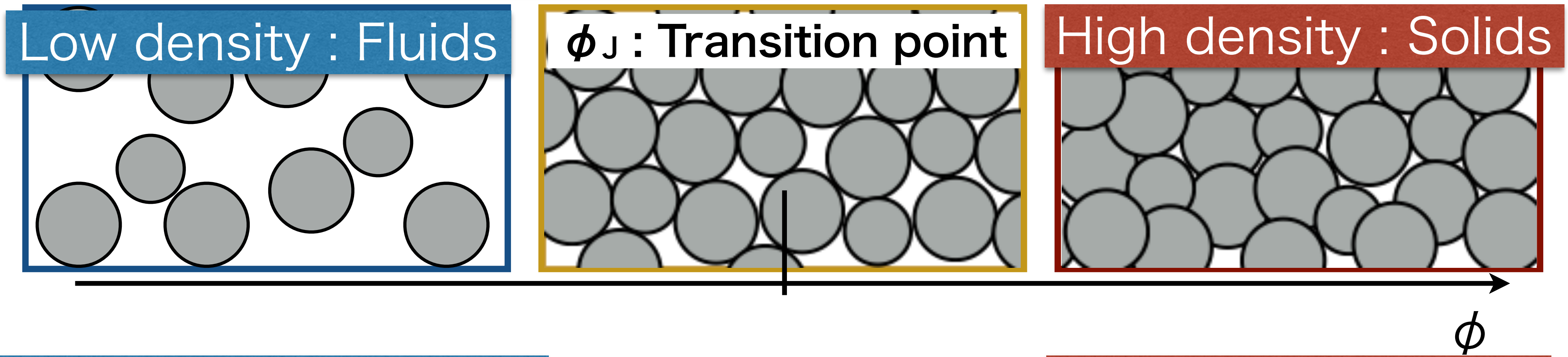
$\phi < \phi_J$

- Granular materials behave like fluids.
- There is no contact network.

$\phi > \phi_J$

- They behave as solids with ever-lasting contacts under small σ.
- They can flow if σ is larger than the yield stress.

ϕ : Packing fraction

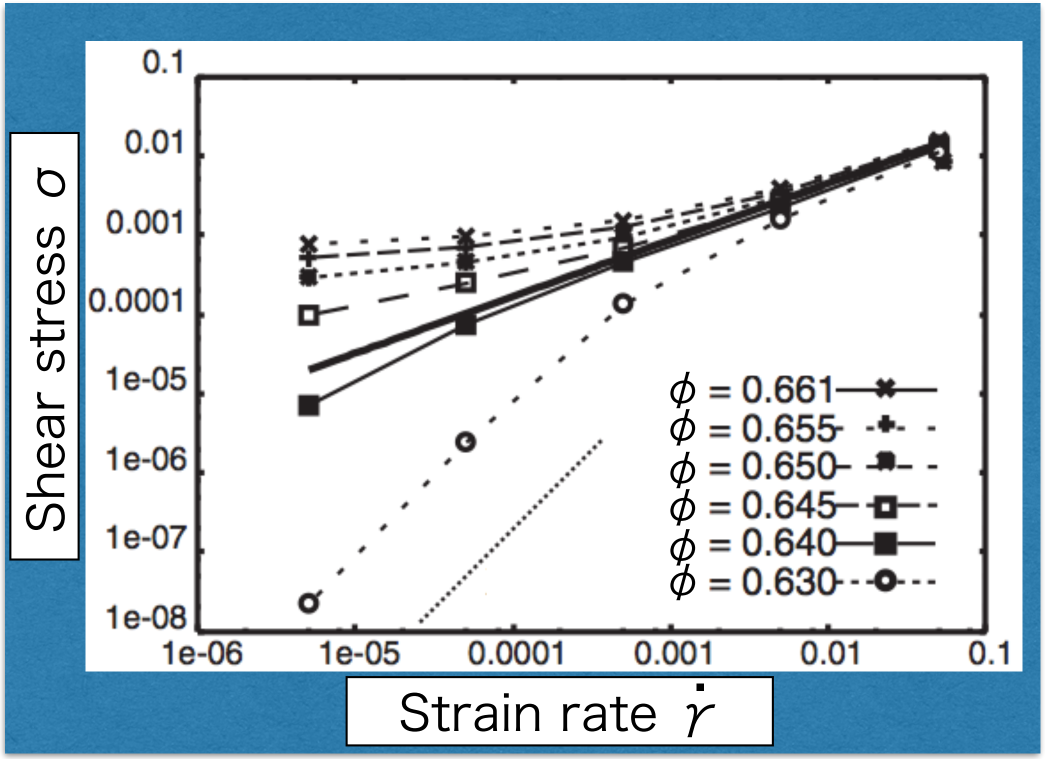


Unnamed : Liquid-like phase

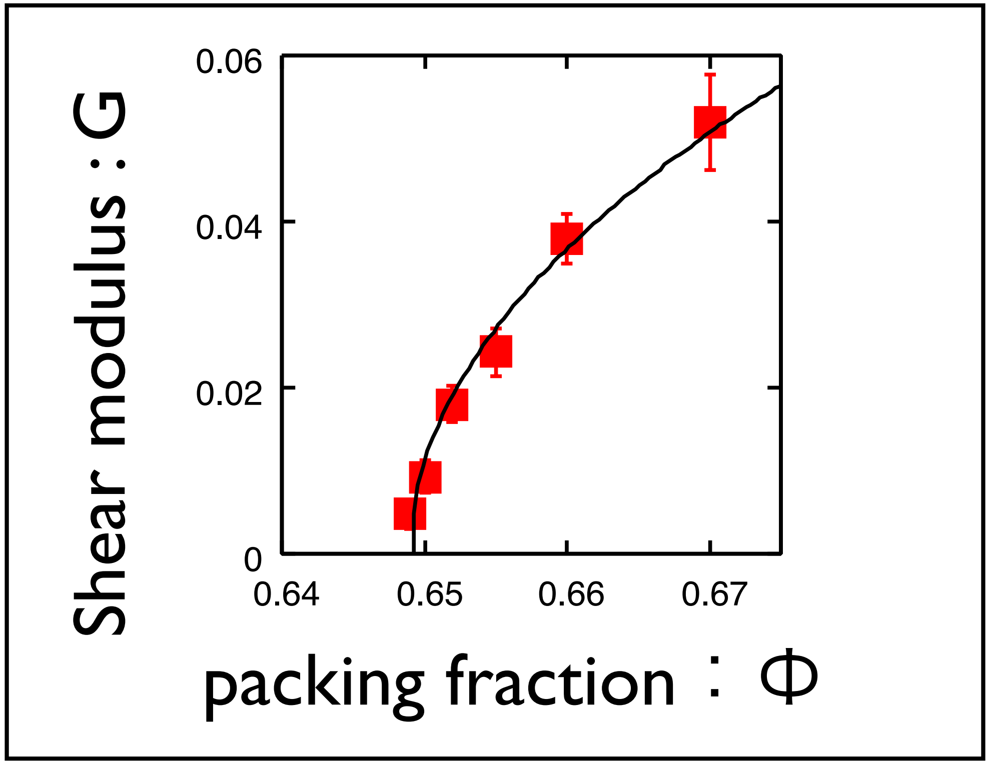
Jammed : Solid-like phase

Rheology near ϕ_J (Frictionless)

Under steady shear



Under quasi-static shear



$\phi < \phi_J$ $\sigma \propto \dot{\gamma}^2$

$\phi > \phi_J$ $\sigma = \text{const.}$

$\phi < \phi_J$ $G = 0$

$\phi > \phi_J$ $G \propto (\phi - \phi_J)^{1/2}$

Continuous transition

Continuous transition

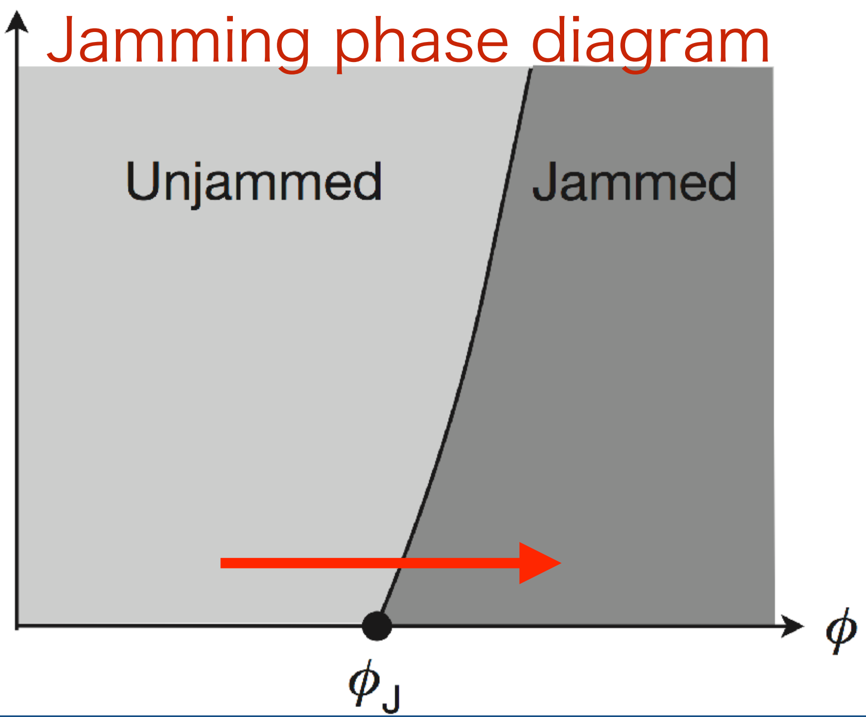
The coordination number exhibits discontinuous transition.

Effect of friction : Shear Jamming

D. Bi, J. Zhang, B. Chakraborty, and R. P. Behringer., Nature, 480, 355–358 (2011).

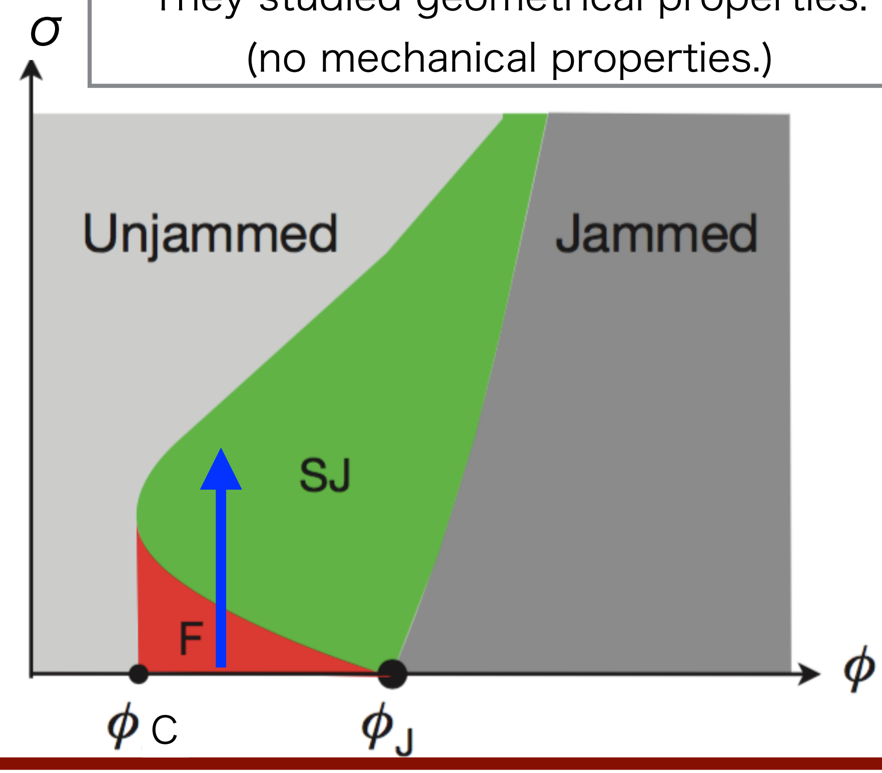
Frictionless grains

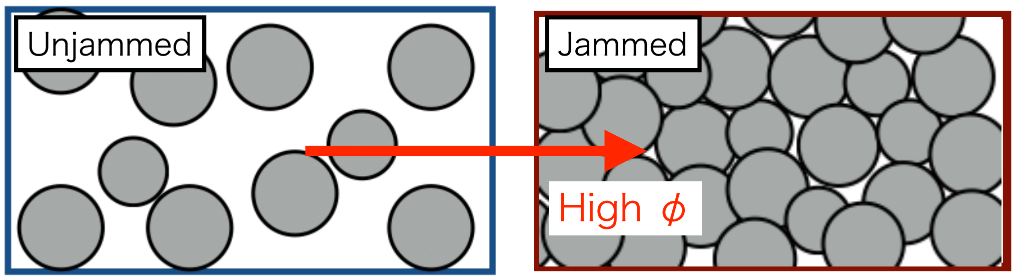
σ : shear stress

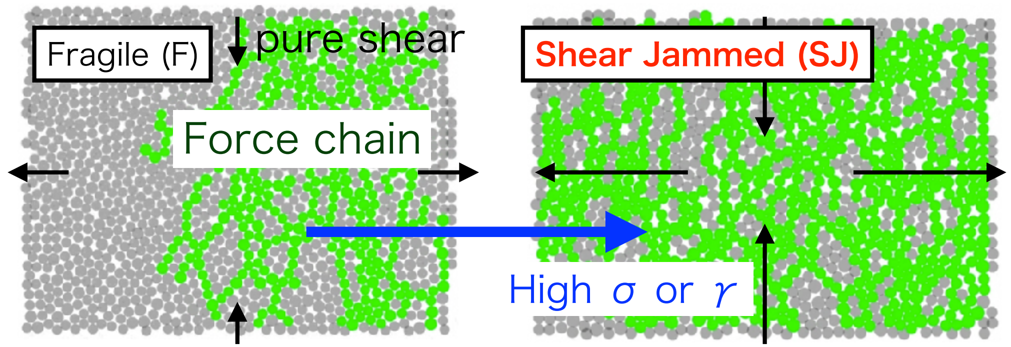


Frictional grains (Experiments)

They studied geometrical properties.
(no mechanical properties.)

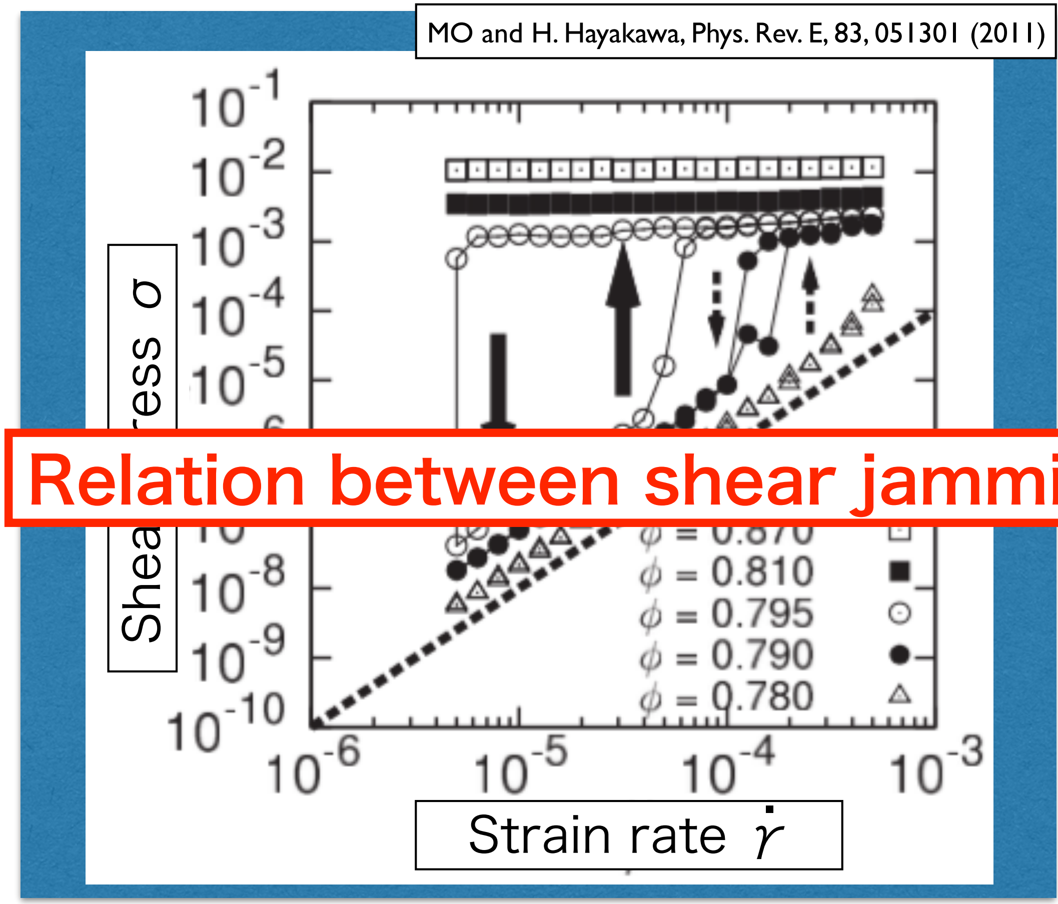




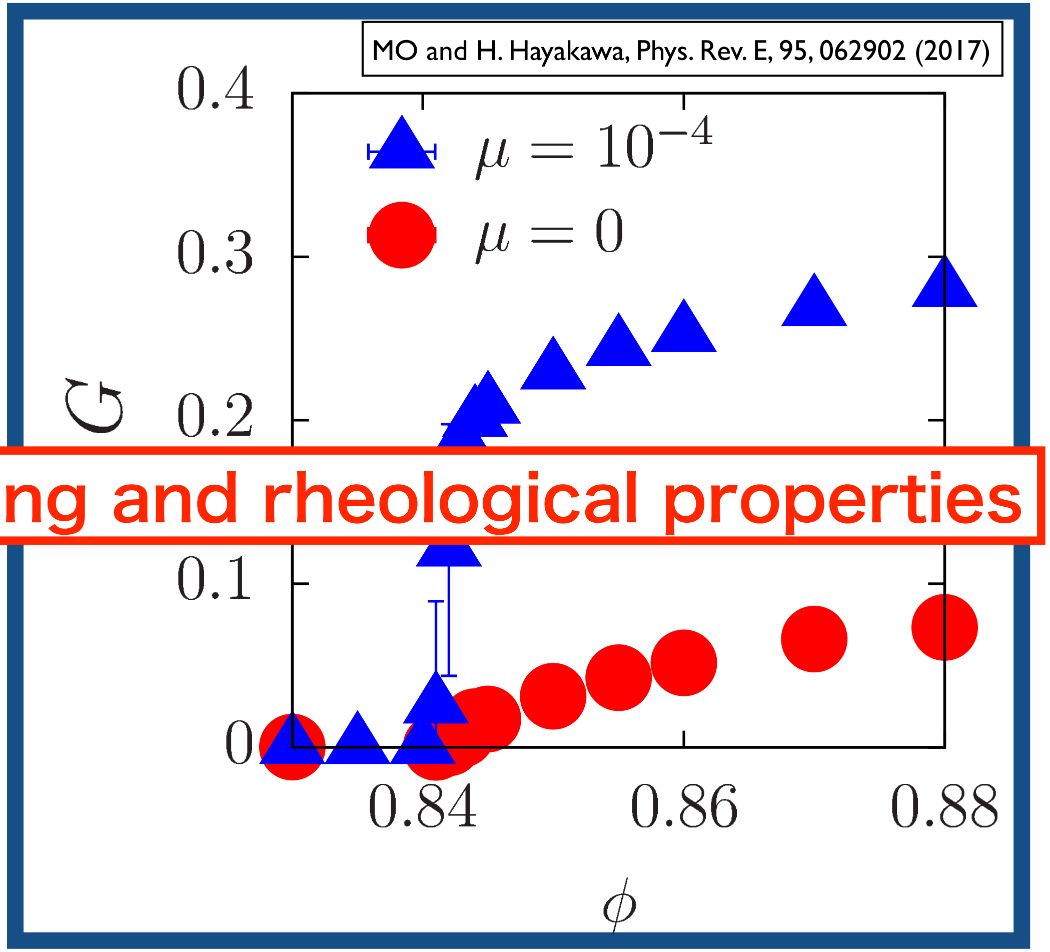


Rheology near ϕ_J (Frictional)

Under steady shear



Under quasi-static shear



Relation between shear jamming and rheological properties

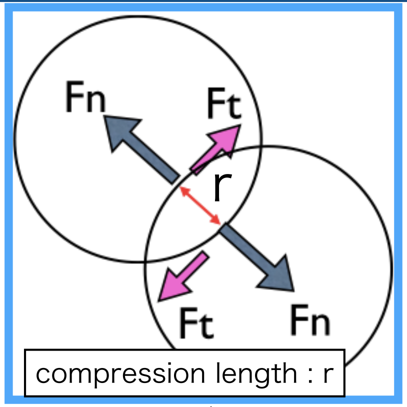
Discontinuous shear thickening (DST)

(They disappear in frictionless limit.)

$$G = G_0 + A(\phi - \phi_J)^{1/2}$$

Discontinuous transition | **Discontinuous transition**

Model : dry granular particles(DEM)



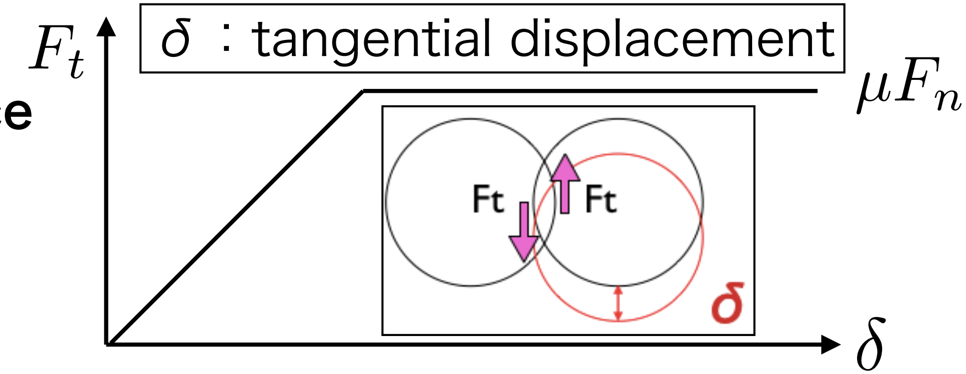
F_n : Normal repulsive force

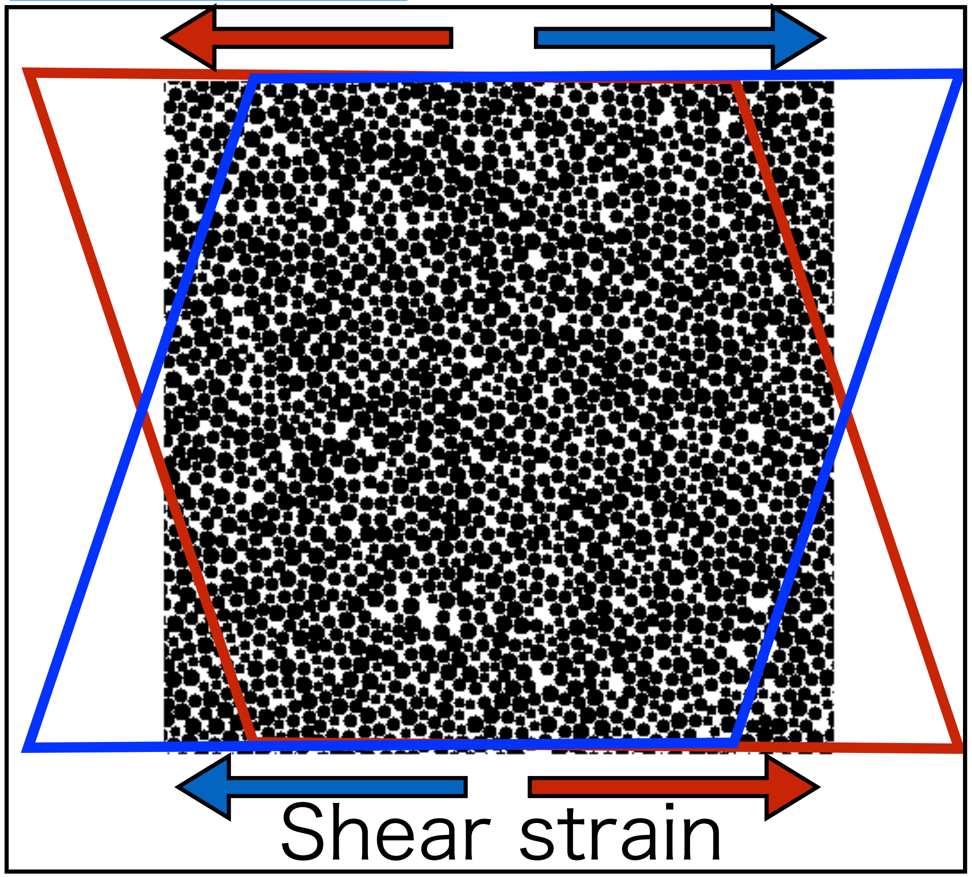
F_t : Tangential frictional force

$F_n \propto r$ + dissipative part

Coulomb law : $F_t \leq \mu F_n$

μ : friction coefficient





- Oscillatory shear strain :

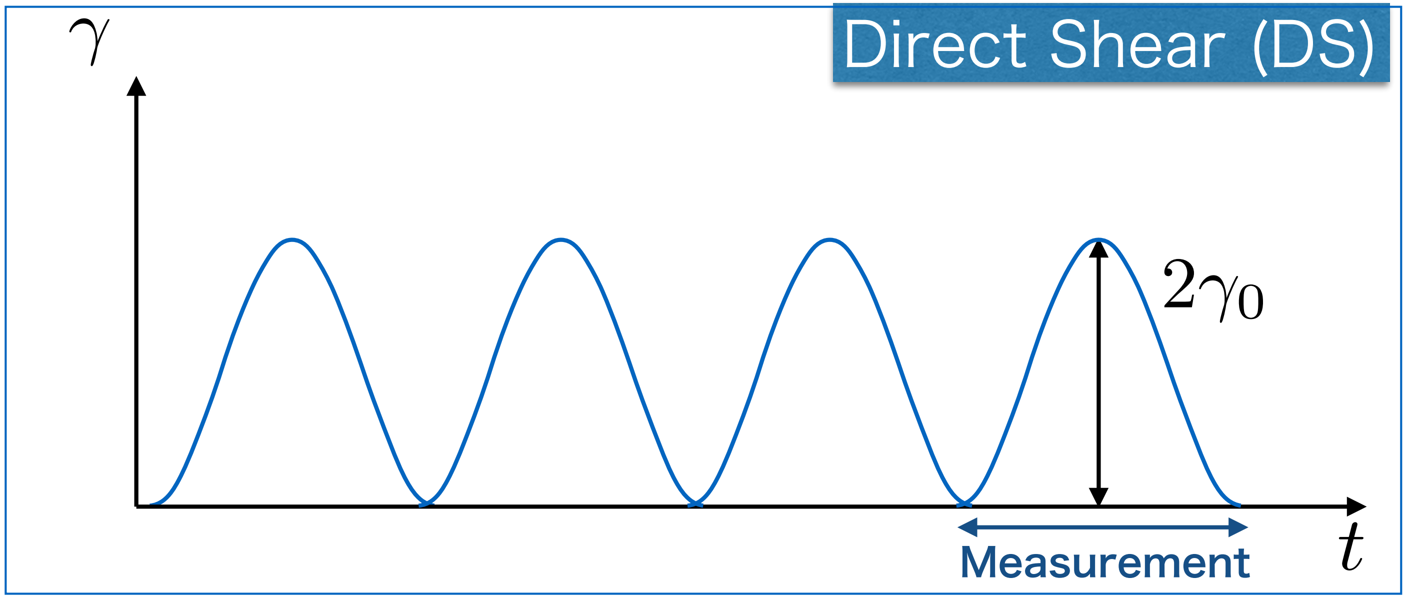
$$\gamma(t) = \gamma_0(1 - \cos\omega t)$$

- Small angular frequency : ω
- Strain amplitude : γ_0
- Shear stress : σ, Pressure : P

Shear storage modulus :

$$G' = -\frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\cos(\omega t)}{\gamma_0}$$

Protocols to measure G

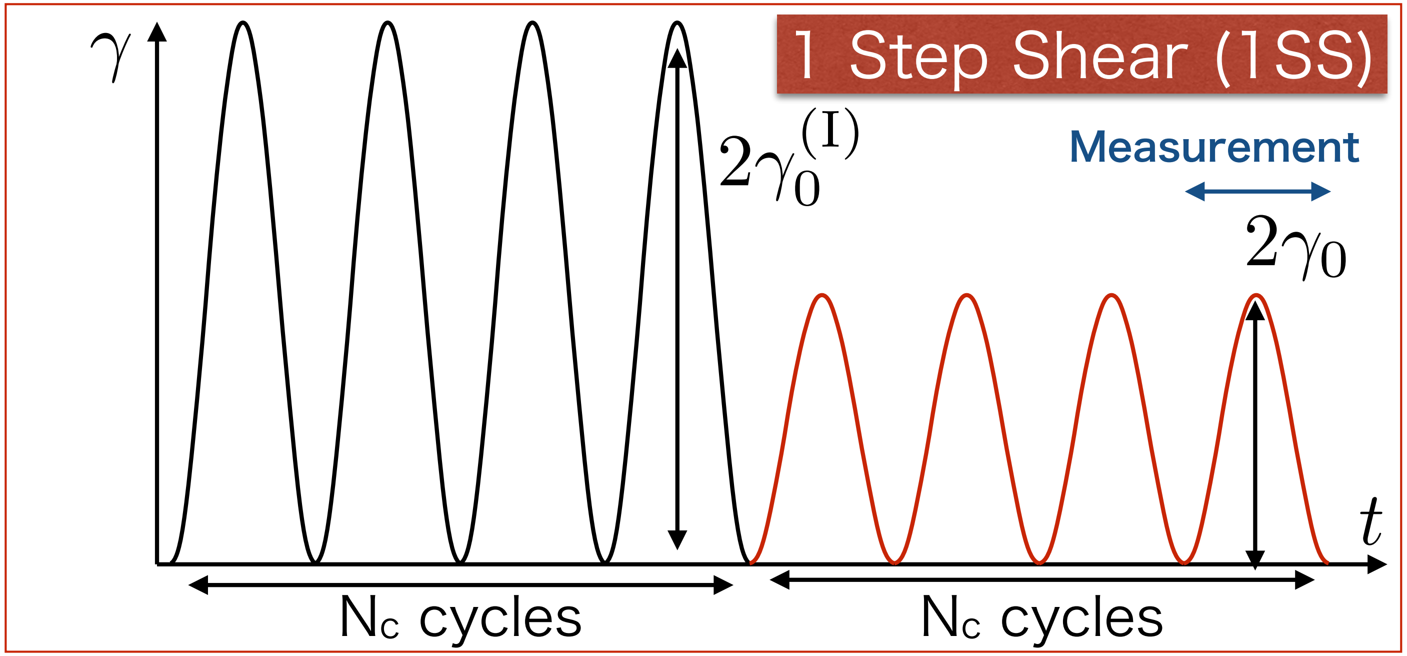


Time dependent strain : $\gamma(t)$

Amplitude for measurement:

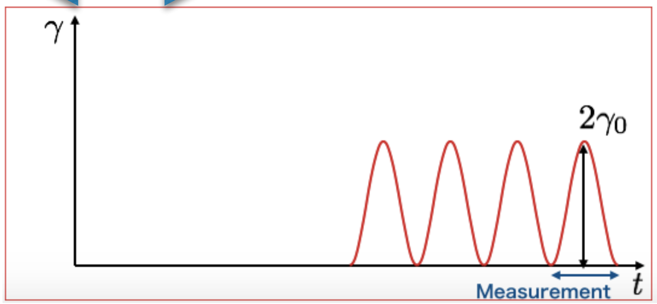
γ_0

Initial amplitude : $\gamma_0^{(\mathrm{I})}$

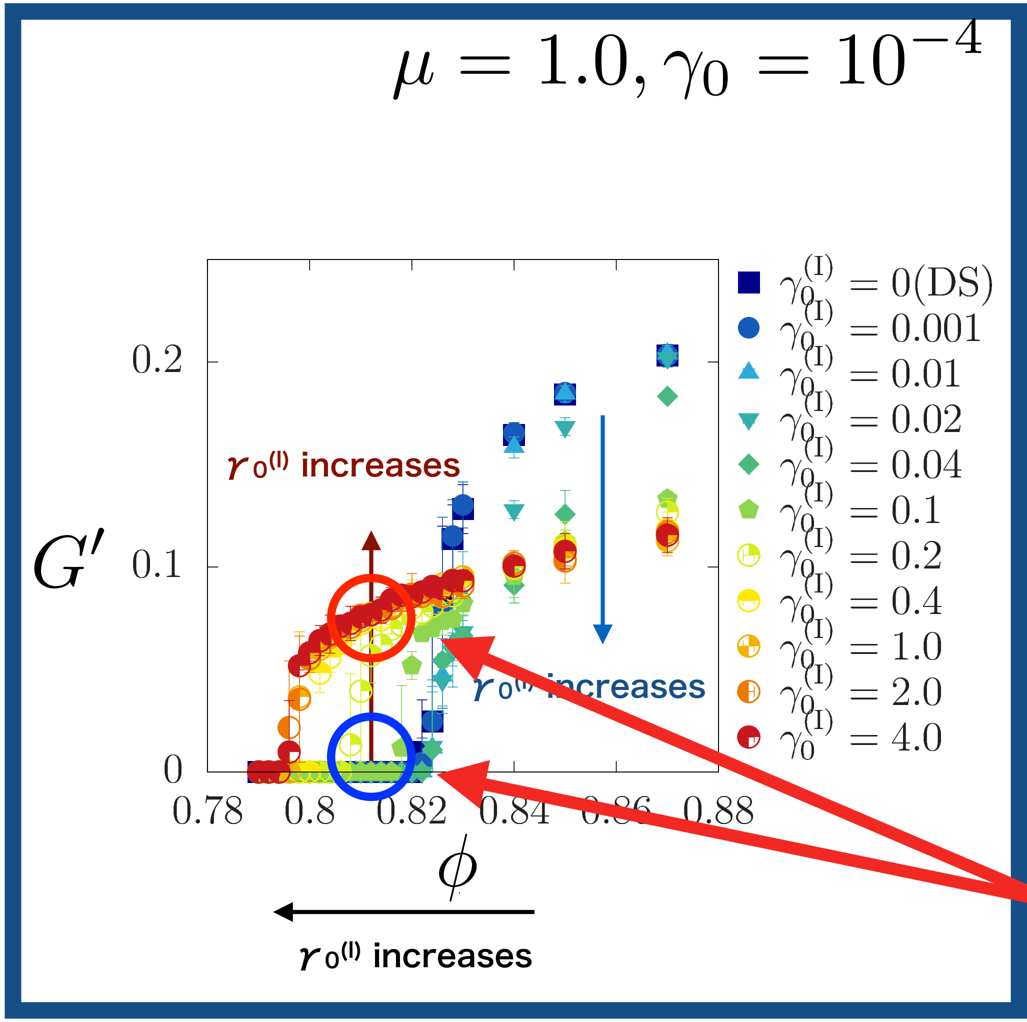


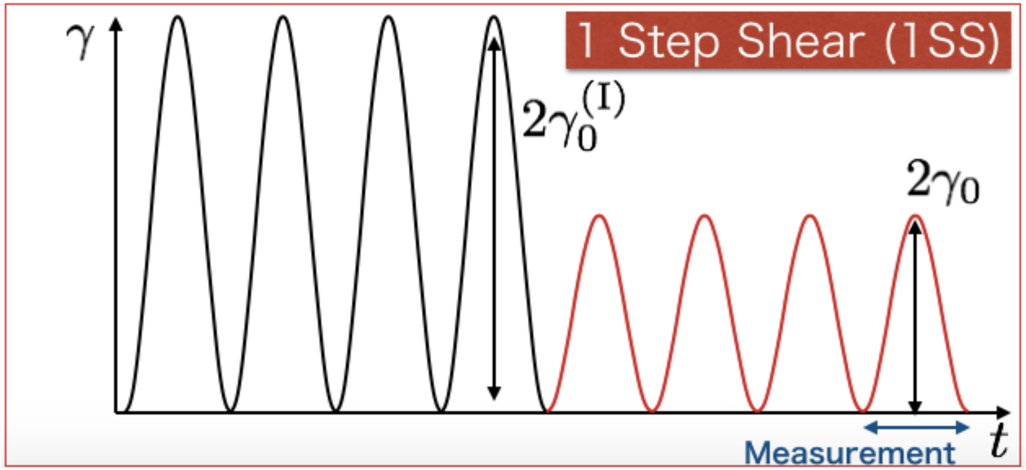
Remark

1SS with $\gamma_0^{(\mathrm{I})} = 0$ ⟷ DS



Effect of initial shear on shear modulus





γ_0 : Strain for measurement

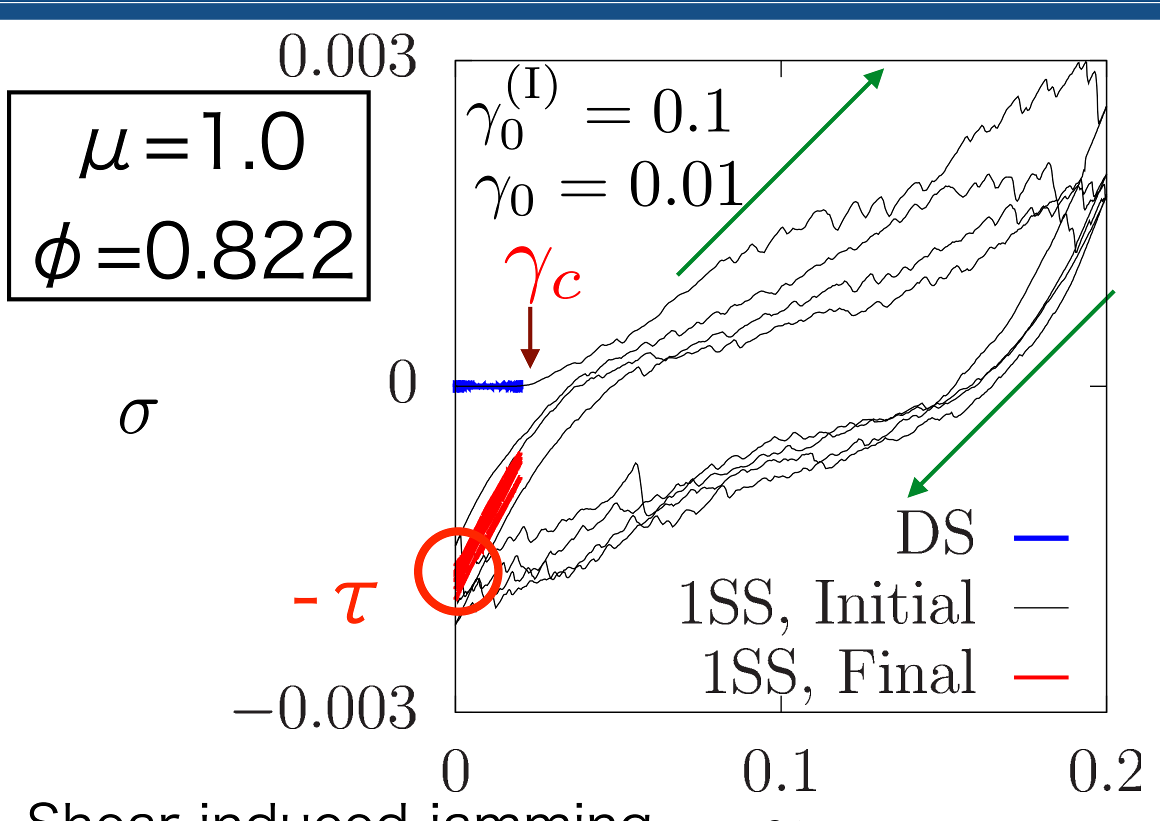
$\gamma_0^{(I)}$: **Initial strain**

Shear induced jamming

(unjammed without shear)

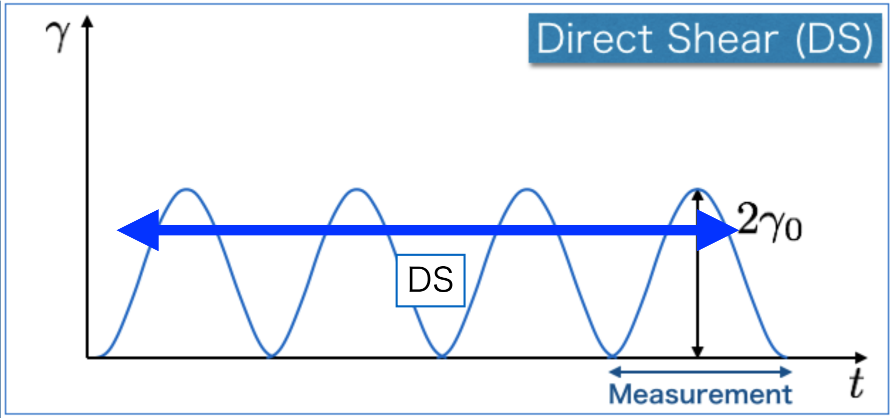
- Transition point ϕ_J decreases as the initial strain $\gamma_0^{(I)}$ increases.
- Shear modulus decreases as $\gamma_0^{(I)}$ increases for higher ϕ.
- Shear modulus increases as $\gamma_0^{(I)}$ increases for lower ϕ.

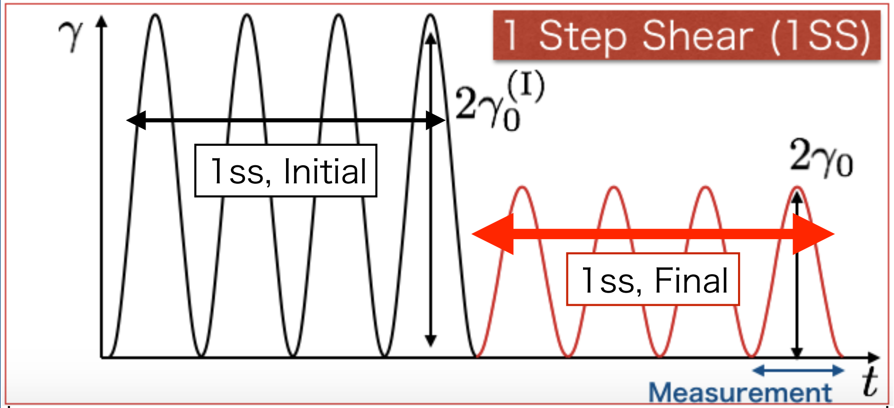
Stress-strain curve

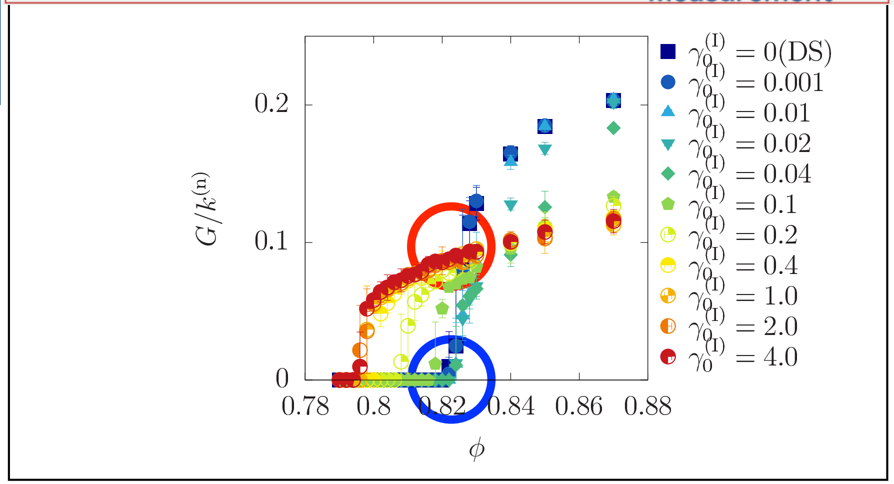


Shear induced jamming

(unjammed without shear)







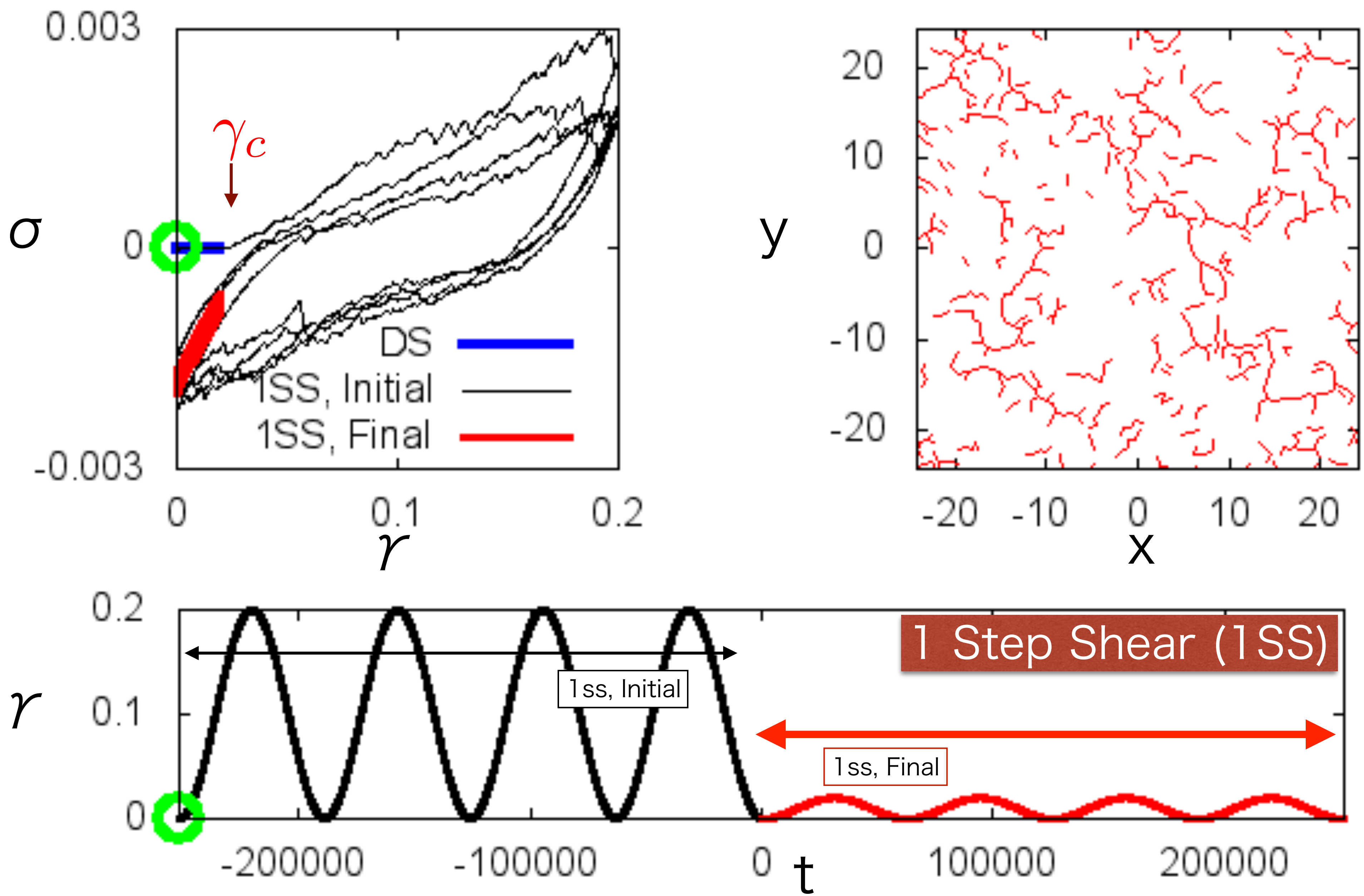
γ_c : Critical strain

$\gamma < \gamma_C \longrightarrow$ no rigidity (unjammed)

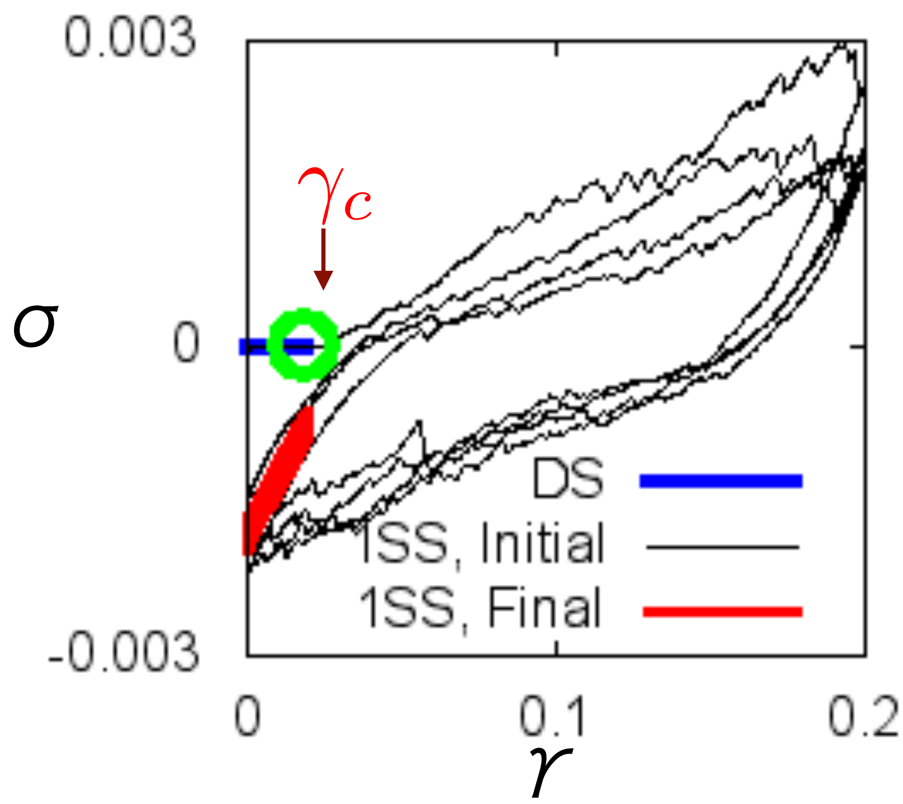
$\gamma > \gamma_C \longrightarrow$ rigidity (Shear jamming)

τ : Residual stress

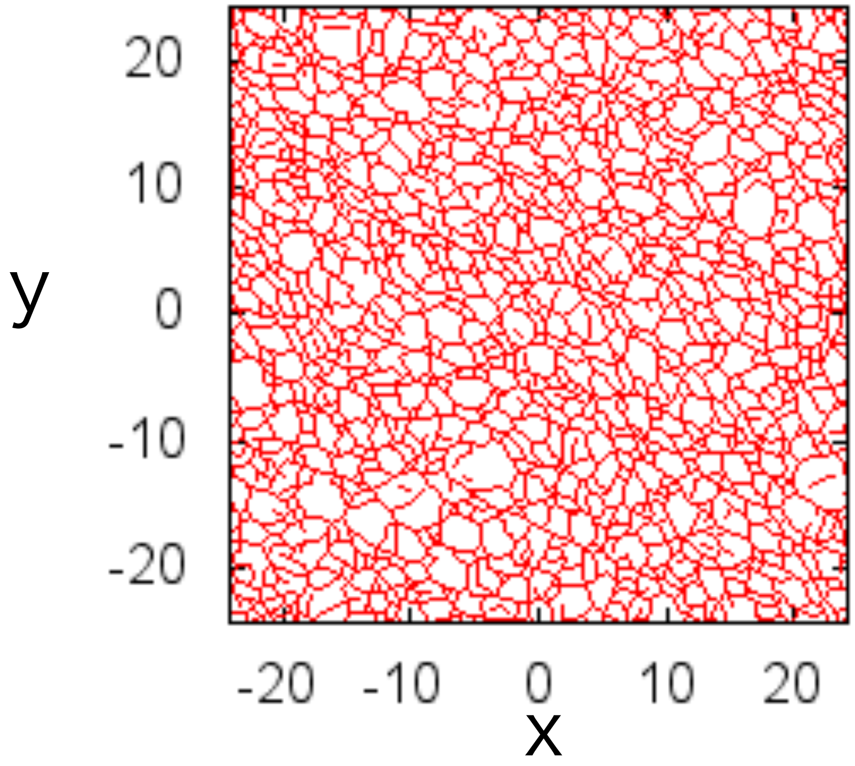
Contact network for 1SS

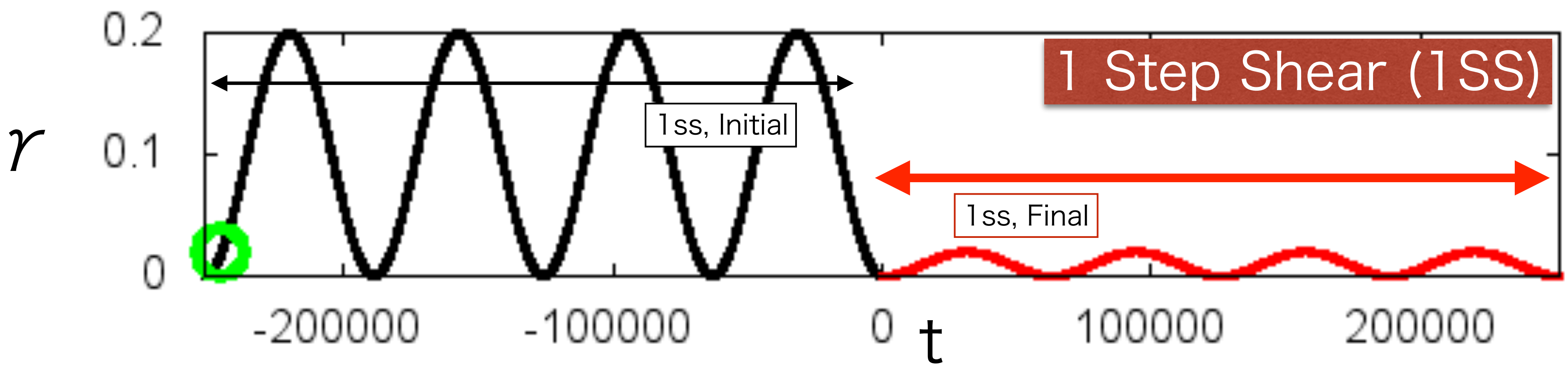


Contact network for 1SS

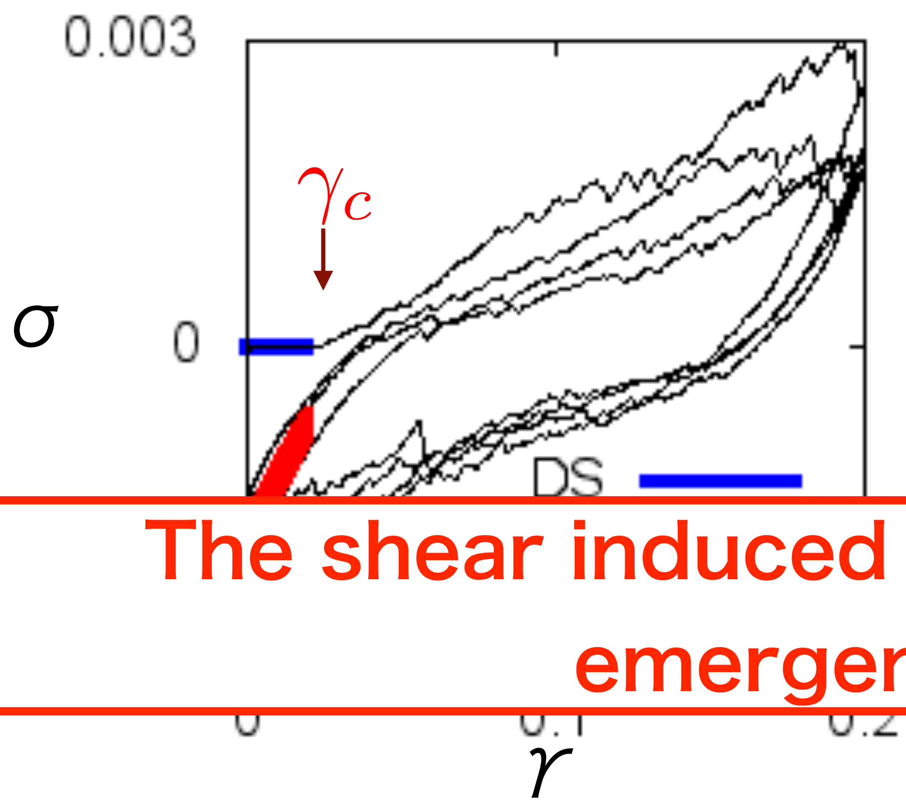


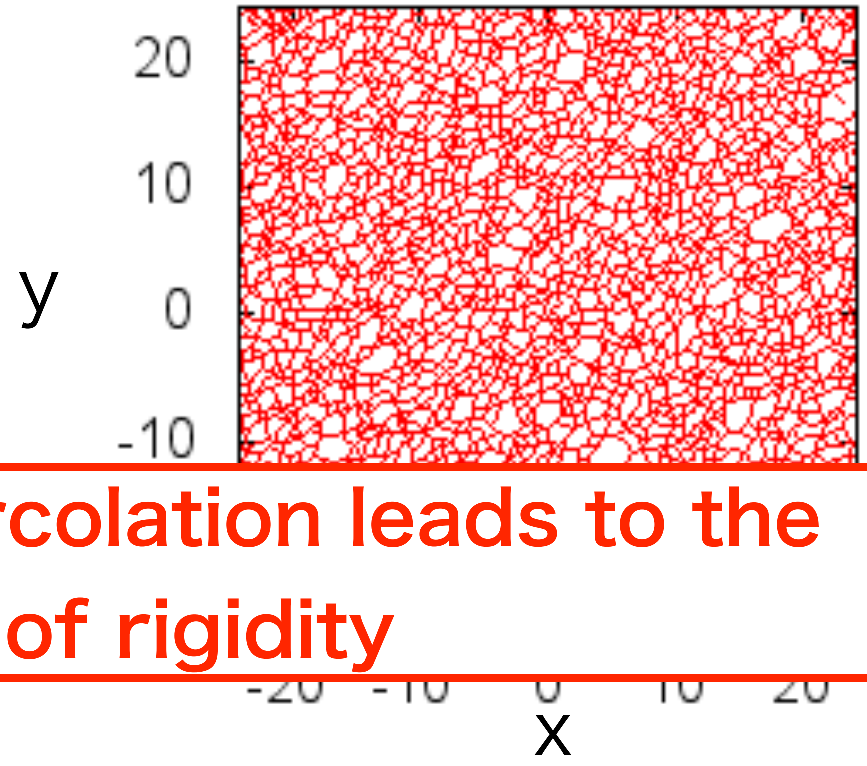
Shear induced percolation appears



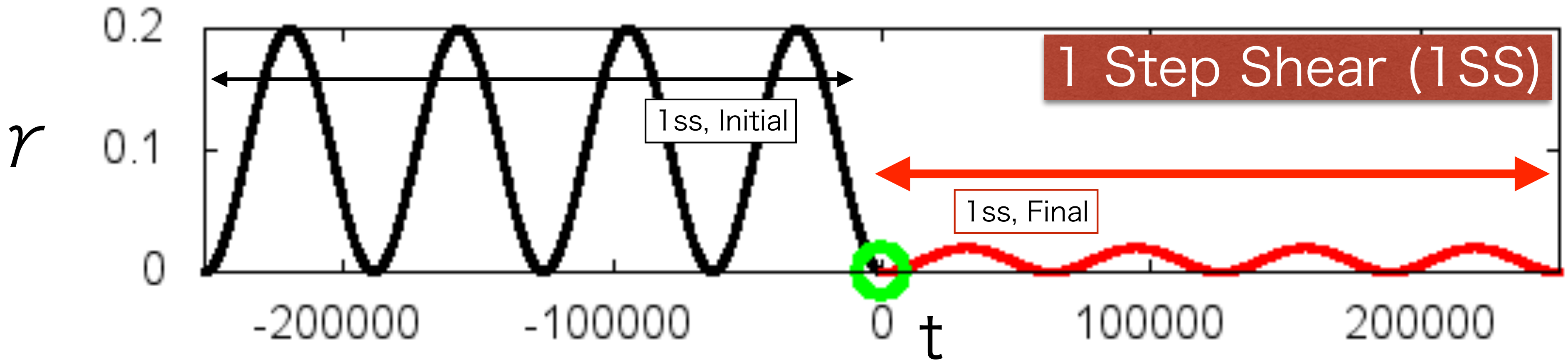


Contact network for 1SS





The shear induced percolation leads to the emergence of rigidity



Phase diagram on $\gamma_0^{(I)}$-ϕ plane

Storage modulus : $G'\left(\phi, \gamma_0^{(I)}\right)$

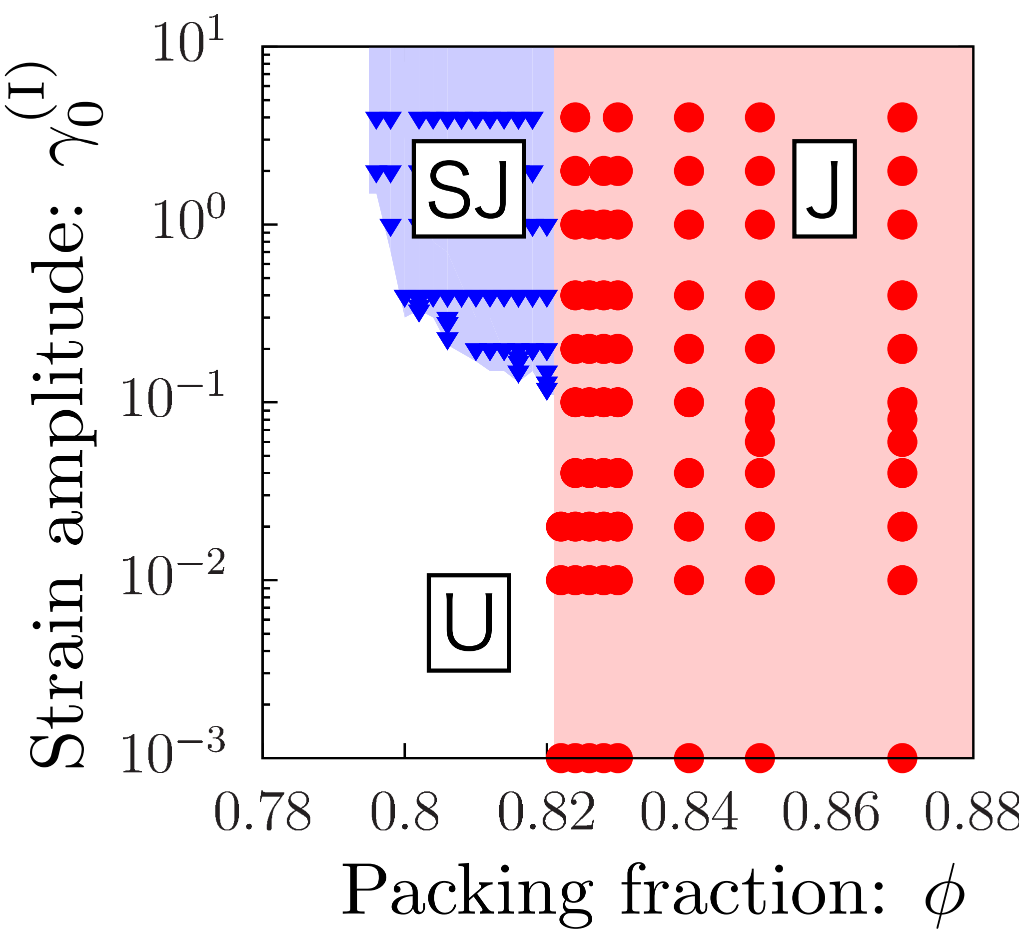
G' without shear:

$$G'_0(\phi) \equiv \lim_{\gamma_0^{(I)} \to 0} G'\left(\phi, \gamma_0^{(I)}\right)$$

Unjammed $G'\left(\phi, \gamma_0^{(I)}\right) = 0$

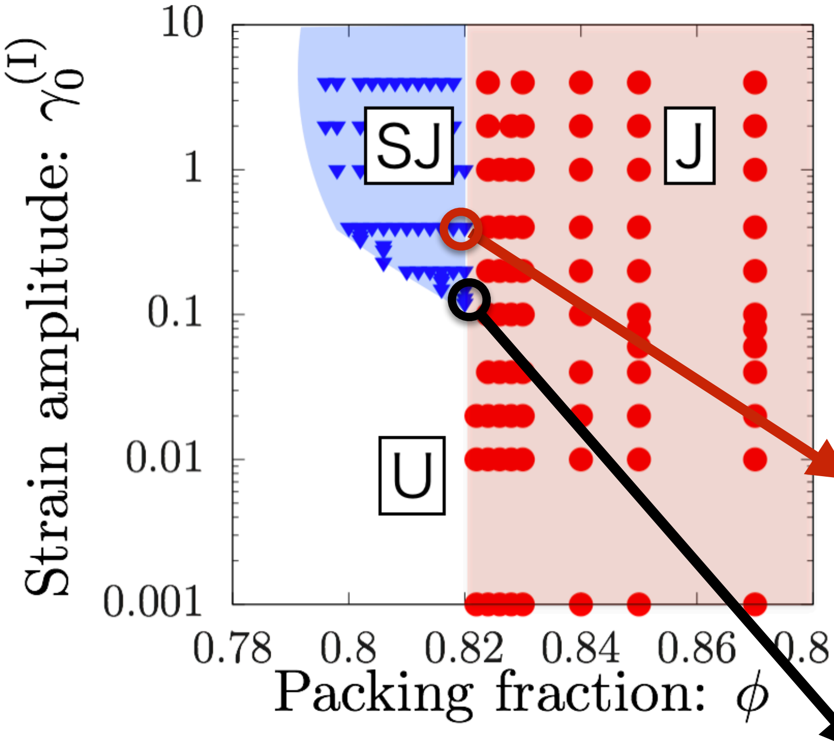
Jammed $G'_0(\phi) > 0$

Shear Jammed $G'_0(\phi) = 0,\ G'\left(\phi, \gamma_0^{(I)}\right) > 0$

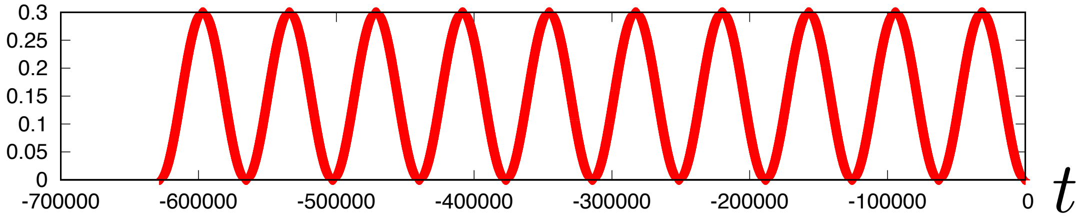


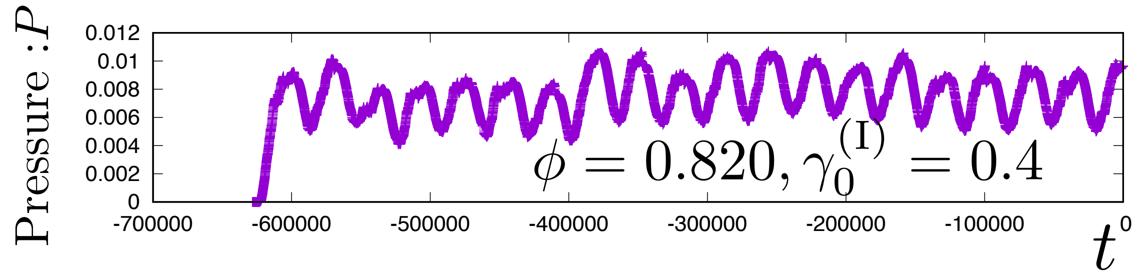
- SJ is above U.
- The critical strain for SJ is finite.

“Fragile” phase?

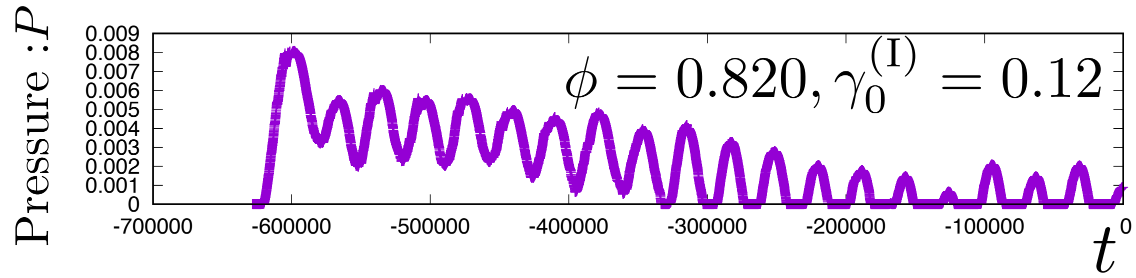


Initial strain: $\gamma(t)$





- Liquid-like state (P=0) before applying shear
- Solid-like state (P>0) after applying shear



- The state depends on the phase of the shear

- There is a **“Fragile”** phase **in SJ**.
- “Fragile” : There are solid-like and liquid like phase in a cycle.

Definition of Fragile state: Dependence on phase

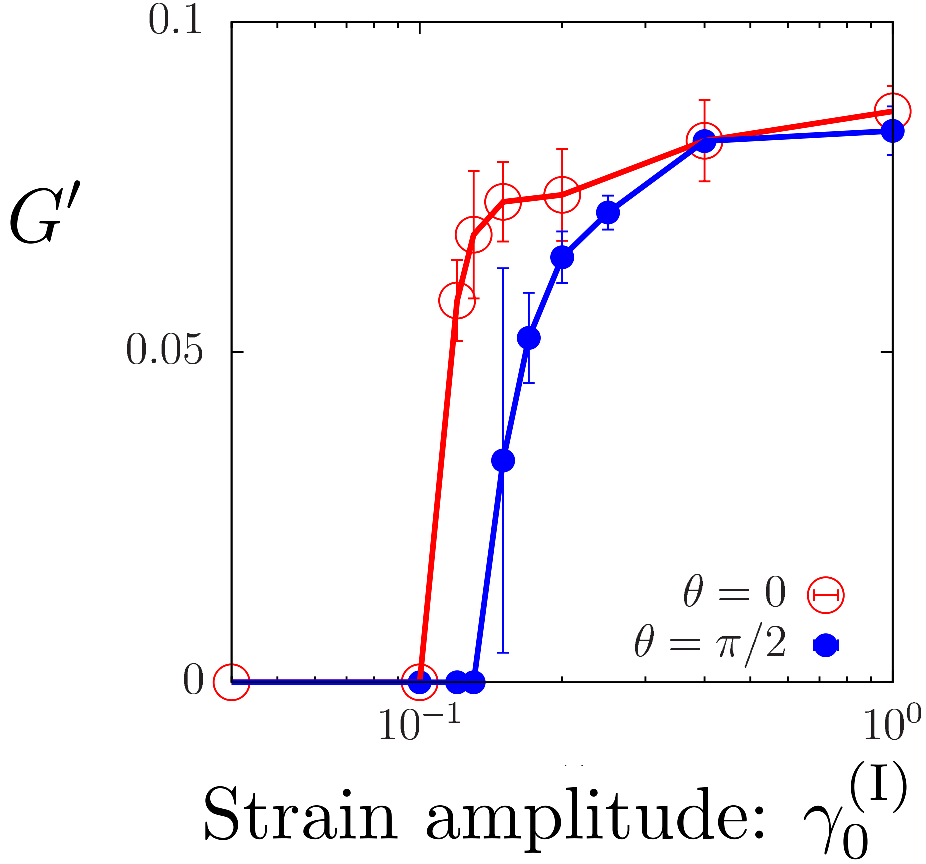
Generalized oscillatory shear:

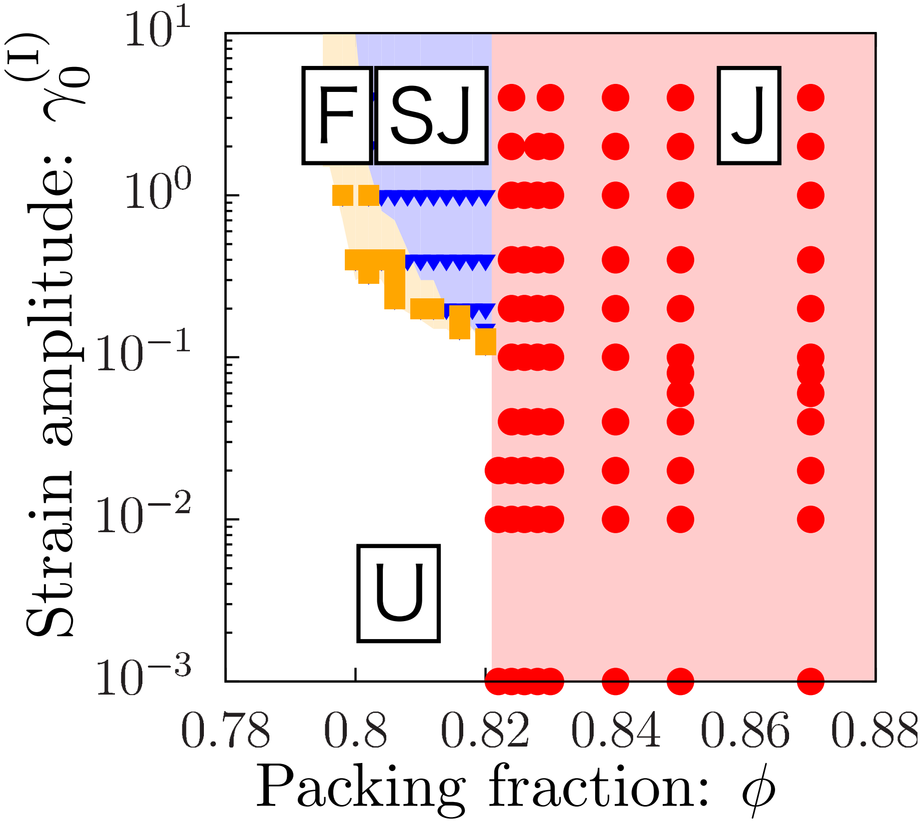
$$\gamma(t) = \gamma_0^{(\mathrm{I})}\{\cos\theta - \cos(\omega t + \theta)\}$$

$$\dot{\gamma}(t) = \gamma_0^{(\mathrm{I})}\omega\sin(\omega t + \theta)$$

θ : (Initial or Final) phase of shear

Previous results: $\theta = 0$





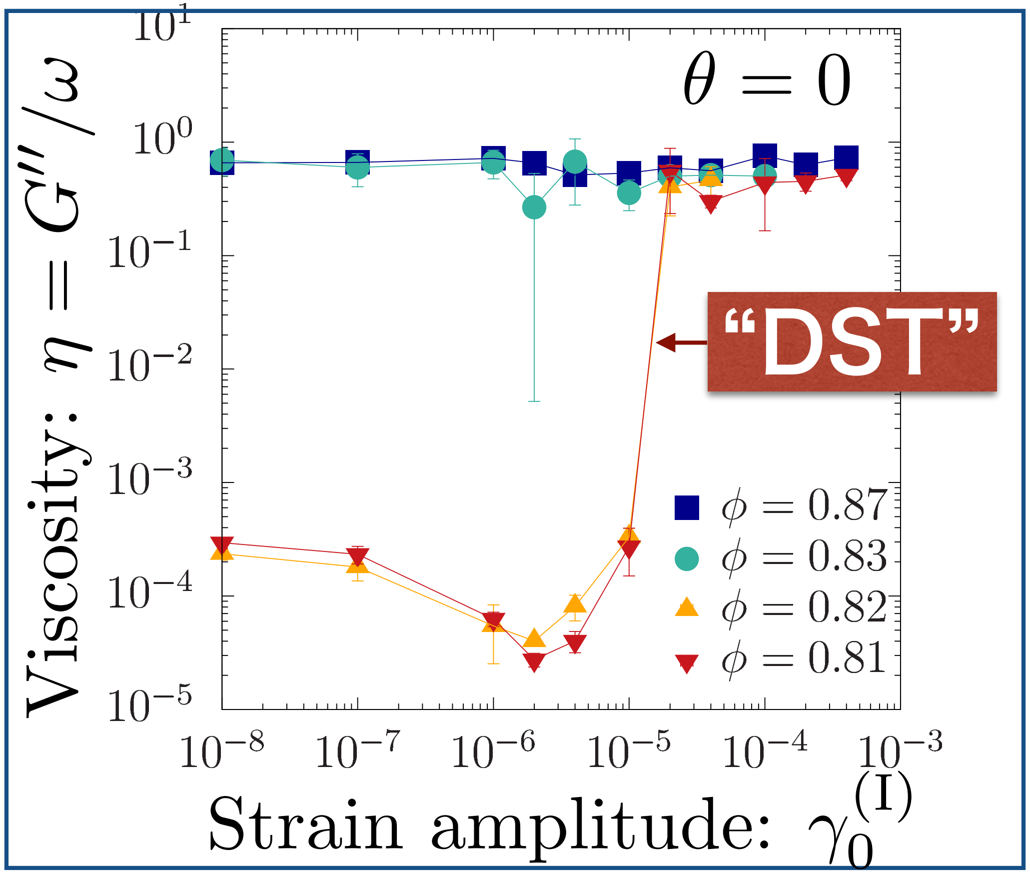
Fragile

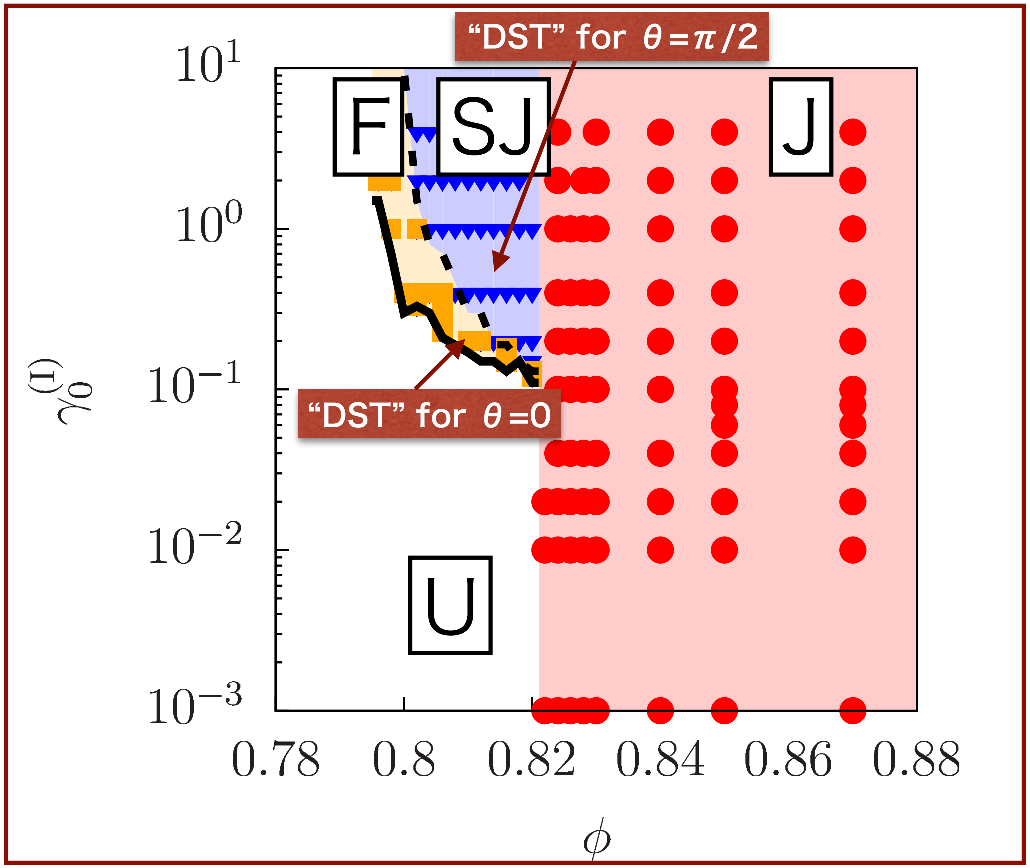
$G' = 0$ for $\theta = 0$

$G' > 0$ for $\theta = \pi/2$

DST-like behavior in loss modulus

Loss modulus : $G'' = \frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\sin(\omega t)}{\gamma_0}$





- η exhibits a rapid increase, which is similar to the discontinuous shear thickening (DST).
- Fragile state exists between "DST" lines for θ=0 and $\pi/2$.

Summary

- Topic : Shear modulus of frictional grains.
- Due to large initial shear, **the shear jamming** occurs and **the shear modulus drastically changes.**
- **The jamming phase diagram** is obtained from mechanical properties.
- There is **“Fragile” phase,** where G’ depends on the phase of the shear.
- **DST-like behavior** is observed.