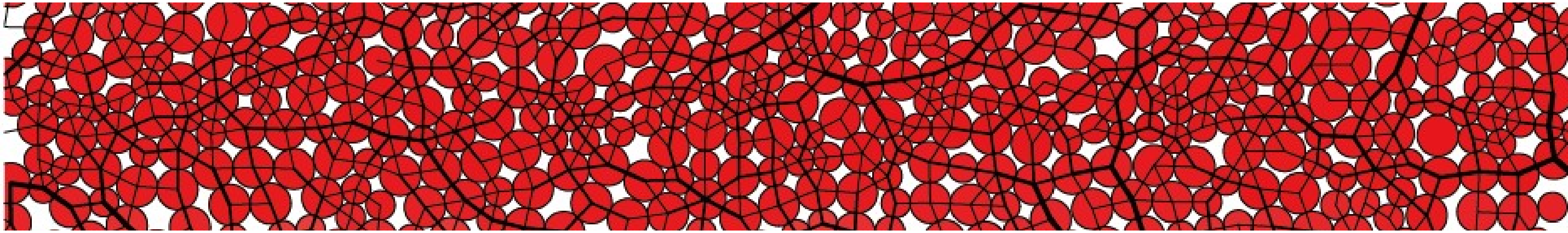
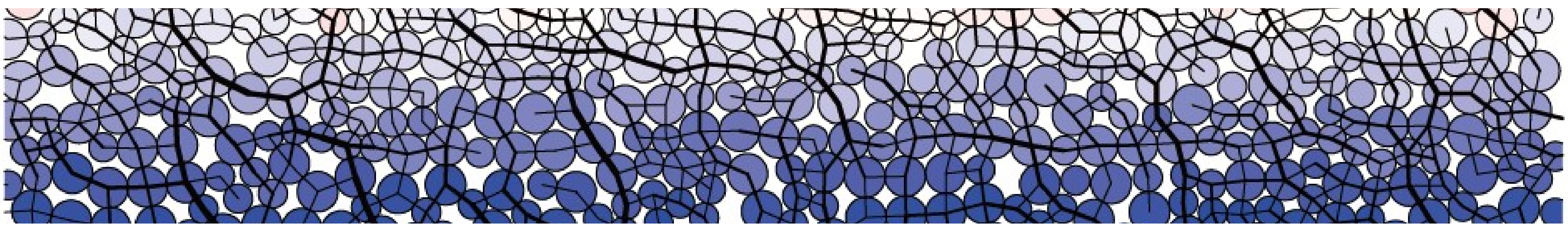
Rheology of disordered particles – suspensions, glassy and granular materials,
YITP, Kyoto University, Kyoto, Japan



Non-local flow behavior of soft athermal particles



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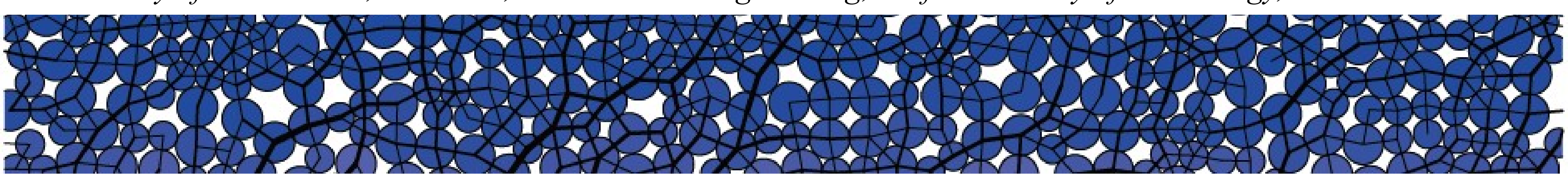


Introduction

❖ Local rheology

$$\boxed{\sigma(\mathbf{r}) \ \text{vs.} \ \dot{\gamma}(\mathbf{r})}$$

stress **strain-rate**

❖ Non-local model

Fluidity $f(\mathbf{r}) \equiv \dot{\gamma}(\mathbf{r})/\sigma(\mathbf{r})$

$$f(\mathbf{r}) = f_{\text{bulk}} + \xi^2 \nabla^2 f(\mathbf{r})$$

cooperativity length

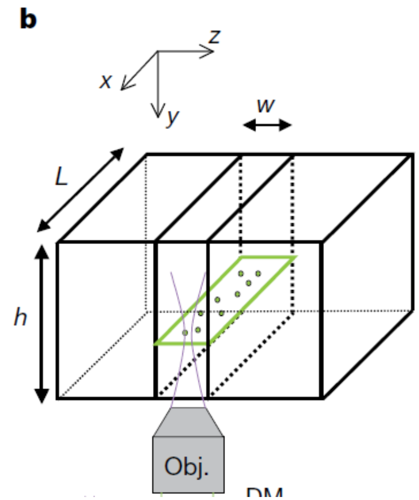
❖ Examples

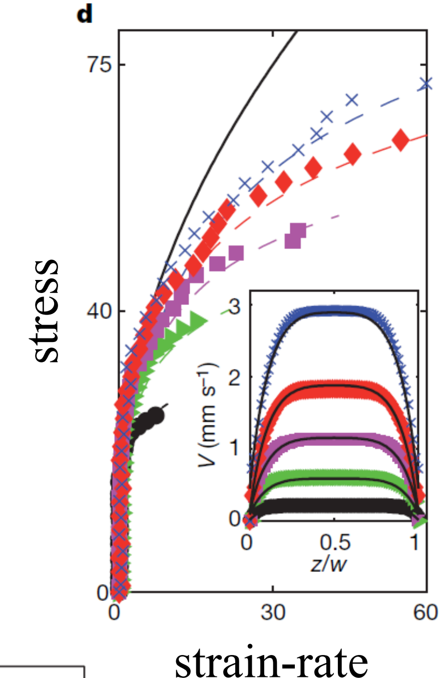
The split-bottom cell, etc.

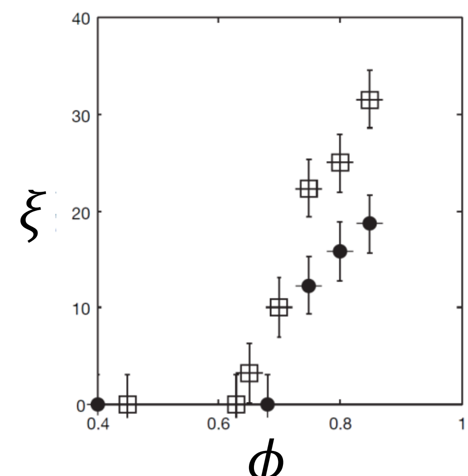
K. Kamrin and G. Koval, *PRL* **108** (2012) 178301.
D.L. Henann and K. Kamrin, PNAS 110 (2013) 6730.

J. Goyon et al., *Nature* **454** (2008) 84.

Flows of emulsion







Motivation

Local

$$\sigma(\mathbf{r}) = \eta(\mathbf{r})\dot{\gamma}(\mathbf{r})$$

viscosity

Non-local

$$\sigma(\mathbf{r}) = \int \eta(\mathbf{r}-\mathbf{r}')\dot{\gamma}(\mathbf{r}')d\mathbf{r}'$$

propagator

The local model is recovered if $\boldsymbol{\eta}(\mathbf{r}-\mathbf{r}') = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')\boldsymbol{\eta}(\mathbf{r}')$.

Does it mean a correlation length diverging at jamming?

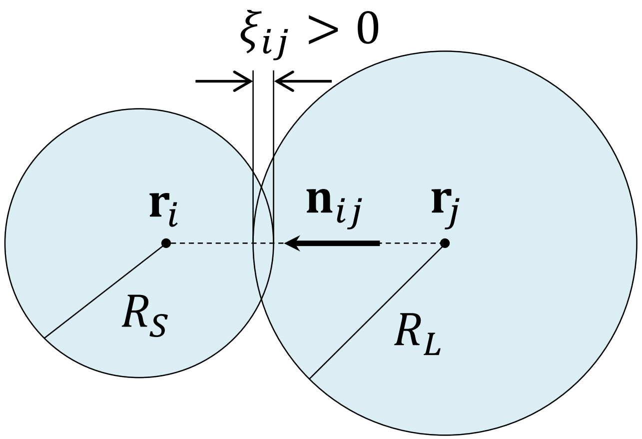
Molecular dynamics simulations

❖ Elastic force and damping

$$\mathbf{f}_i^{(\mathrm{e})} = k \sum_{j \neq i} \xi_{ij} \mathbf{n}_{ij}$$

$$\mathbf{f}_i^{(\mathrm{d})} = -\mu \{ \mathbf{v}_i - \mathbf{u}(\mathbf{r}_i) \}$$

Flow field



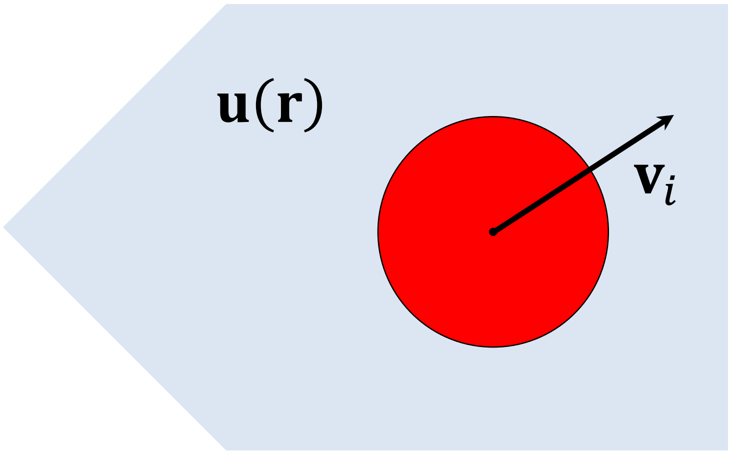
❖ Overdamped dynamics

$$\mathbf{0} = \mathbf{f}_i^{(\mathrm{e})} + \mathbf{f}_i^{(\mathrm{d})}$$

$$\boxed{\therefore \mathbf{v}_i = \mathbf{u}(\mathbf{r}_i) + \mu^{-1} \mathbf{f}_i^{(\mathrm{e})}}$$

Time, $t_0 \equiv \mu / k$

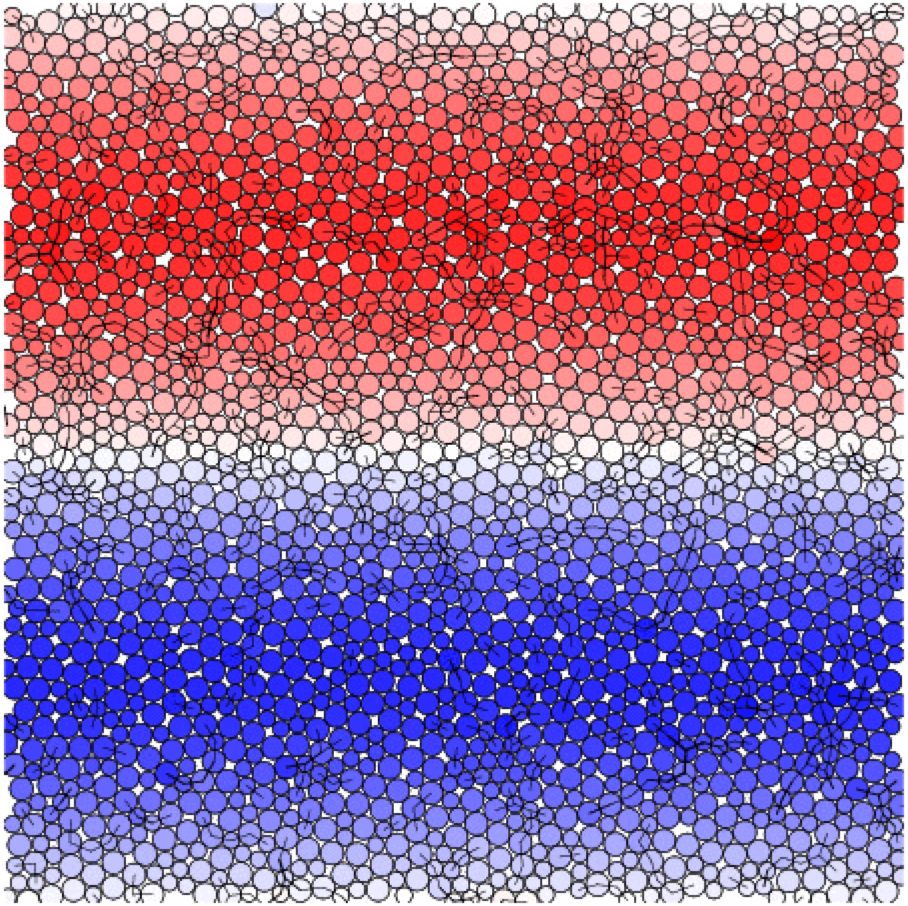
Length, $d_0 \equiv R_S + R_L$



Kolmogorov flows

❖ System

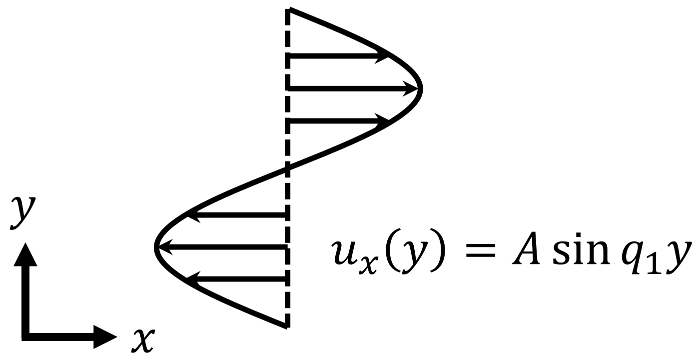
- Bi-dispersed, $R_L/R_S = 1.4$
- The number of disks, $N = 131072$
- $L \times L$ square box, where $L \approx 360 d_0$
- **Periodic boundary conditions**



❖ Flow field

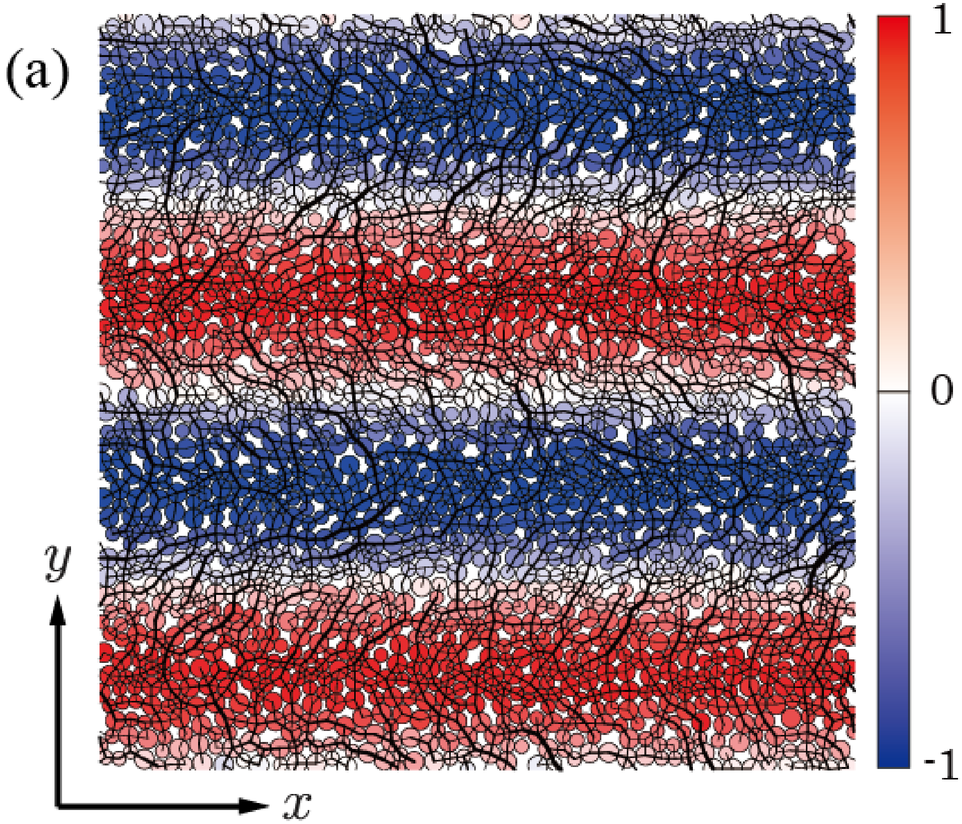
$$\boxed{\mathbf{u}(\mathbf{r}) = (A \sin q_n y\, , 0)}$$

- ***Wave-number***, $q_n = 2n\pi/L$
- ***Amplitude***, $A = 10^{-4} \sim 10^{-1}\, d_0/t_0$
- ***Area fraction***, $\phi = 0.80 \sim 0.85$
- **Steady states**, $20 \le At/d_0 \le 50$

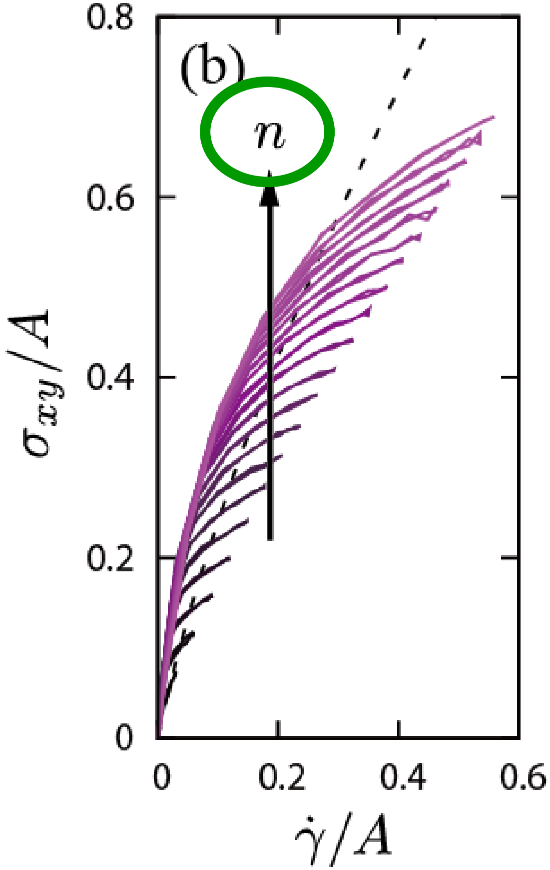


Flow-curves

Wave number, $n = 2$



Flow-curves



- $\boldsymbol{\sigma_{xy}(y)}$ **and** $\boldsymbol{\dot{\gamma}(y) \equiv \nabla_y v_x(y)}$**, are local quantities.**

Flow profiles

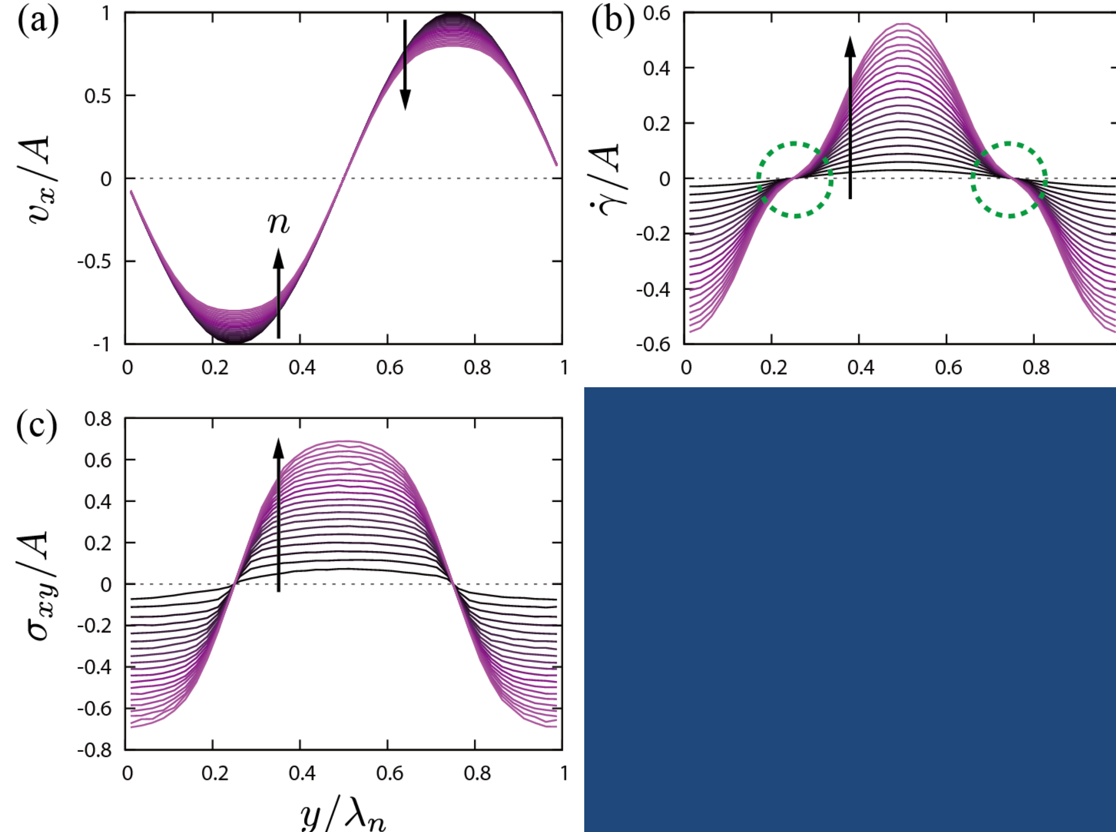
❖ Shear-localization

Increase of n

$v_x(y)$ is flattened

$\dot{\gamma}(y)$ decreases

$\phi(y)$ increases



- **Local rheology is mixing the data of different densities.**
- **Are the *non-local effects* caused by density gradients*?**

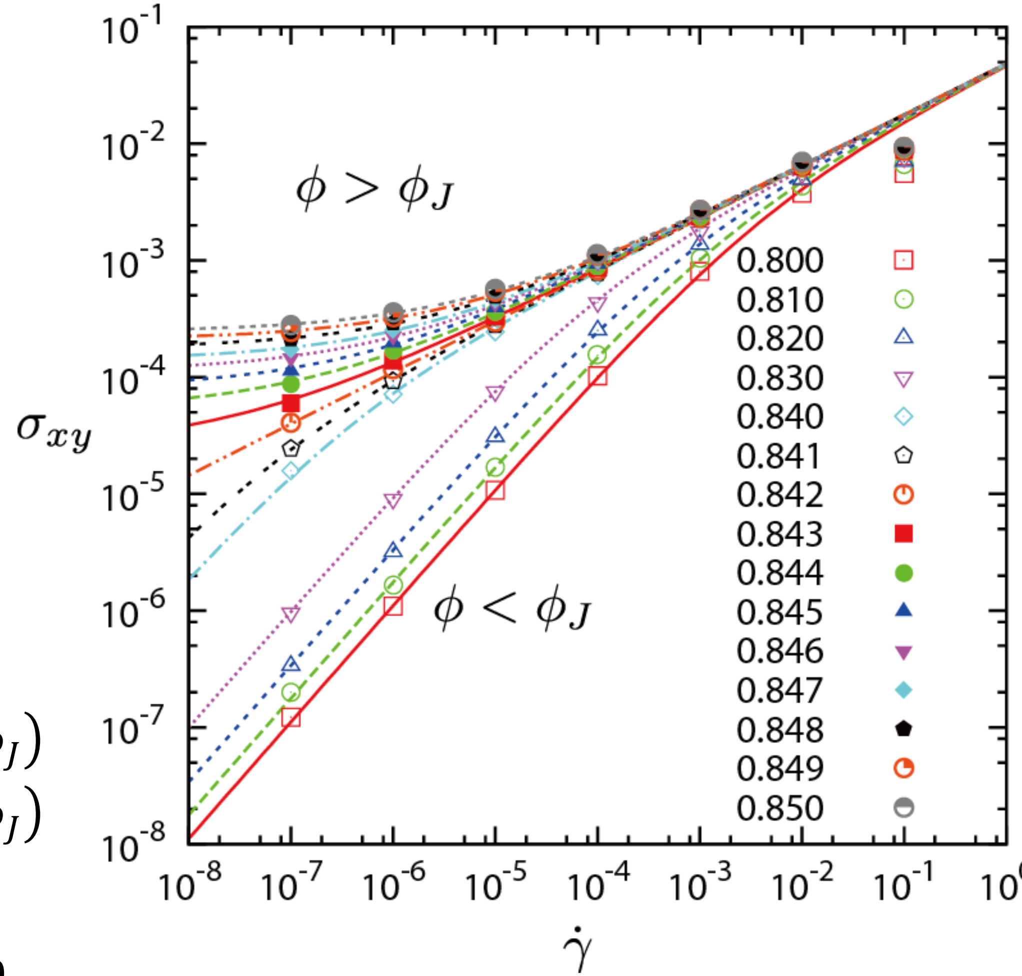
*D. Bonn et al., *Rev. Mod. Phys.* **89** (2017) 035005.

ϕ-dependence

❖ Homogeneous flows

Symbols

MD simulations of
simple shear flows



The lines

$$\boxed{\sigma_{xy} = \eta_s(\phi, \dot{\gamma})\dot{\gamma}}$$

$$\eta_s(\phi, \dot{\gamma}) = \begin{cases} \eta_0\left(\dot{\gamma}^a + c\Delta\phi^b\right)^{-1} & (\phi < \phi_J) \\ \sigma_Y(\phi)\dot{\gamma}^{-1} + \eta_0\dot{\gamma}^{-a} & (\phi > \phi_J) \end{cases}$$

- Fitting constants, a, b, c, and η_0
- The yield stress, $\sigma_Y(\phi)$

Local model

❖ Constitutive laws

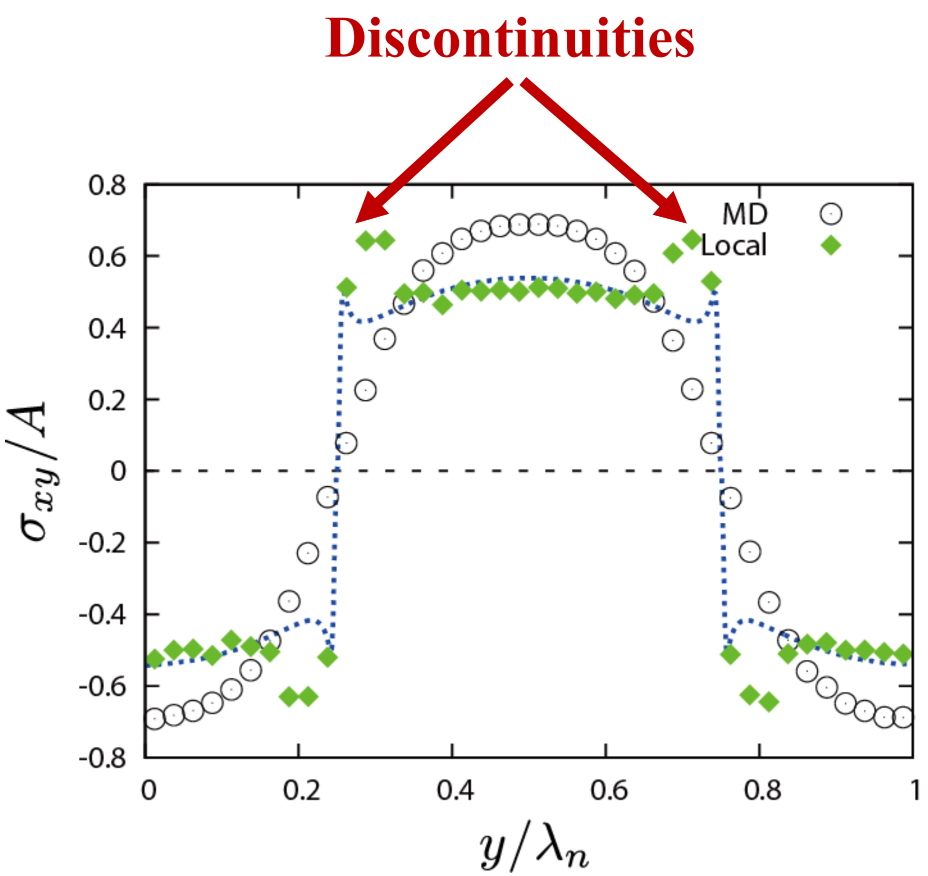
$$\sigma_{xy}^{L}(y) = \eta_s[\phi(y), \dot{\gamma}(y)]\dot{\gamma}(y)$$

Symbols

$\phi(y)$ and $\dot{\gamma}(y)$ are taken from MD

The dotted line

The solution of **force-balance eq.**



The local model fails and generates discontinuities in the shear localized regions (shear-bands).

Non-local model

❖ Constitutive laws

Normalized propagator

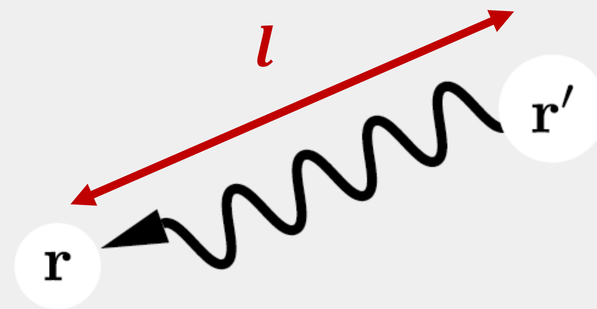
$$\eta(y-y') \equiv \alpha(y-y')\eta_s[\phi(y'),\dot{\gamma}(y')]$$

The Taylor expansion, $\boldsymbol{l \equiv y - y'}$

$$\sigma_{xy}(y) = \int \alpha(l)\sigma_{xy}^{L}(y-l)dl$$

$$= \sigma_{xy}^{L}(y) + \xi^2\nabla_y^2\sigma_{xy}^{L} + \cdots$$

$$\boxed{\therefore (1-\xi^2\nabla_y^2)\sigma_{xy}(y) \approx \sigma_{xy}^{L}(y)}$$



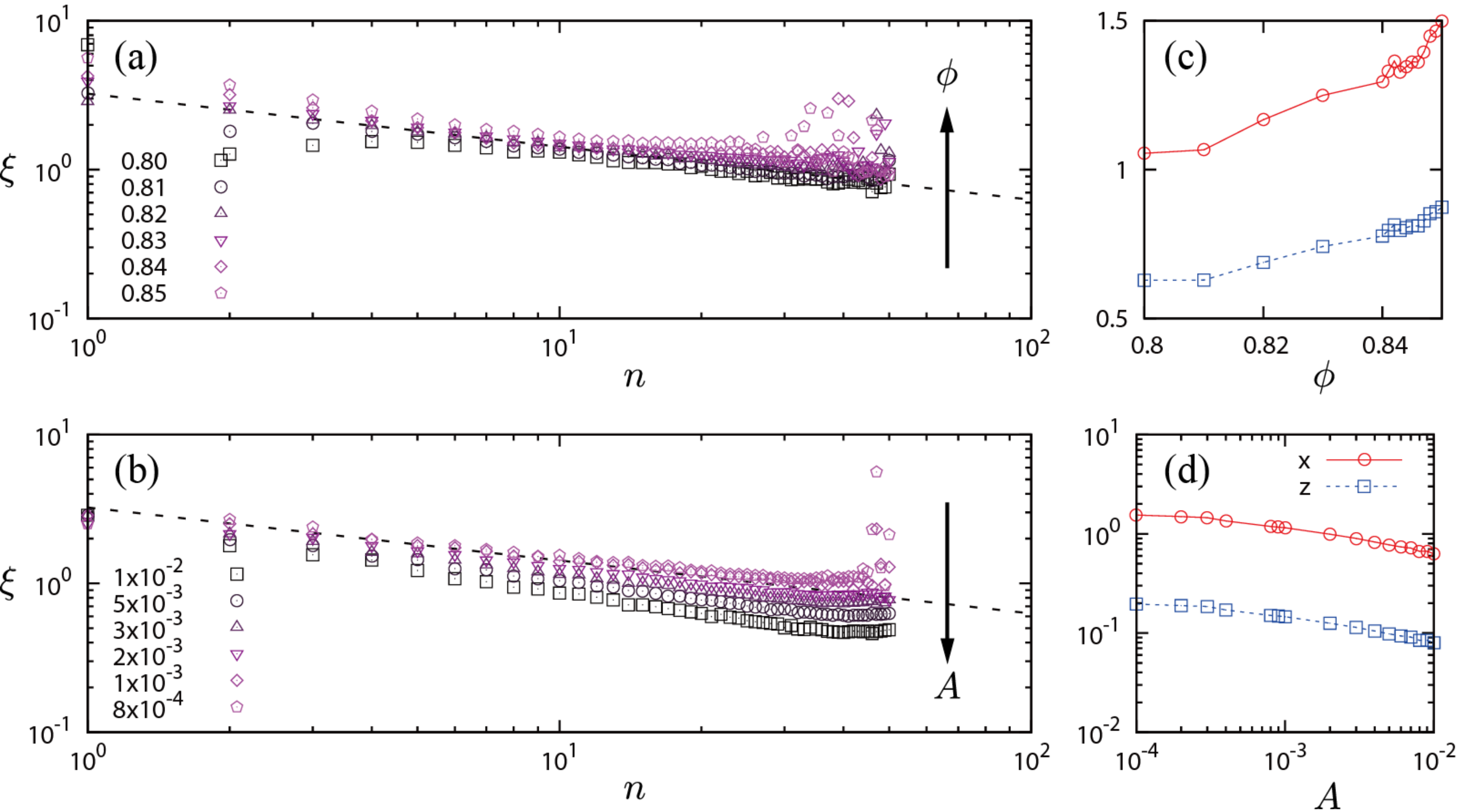
$$\sigma(\mathbf{r}) = \int \eta(\mathbf{r}-\mathbf{r}')\dot{\gamma}(\mathbf{r}')d\mathbf{r}'$$

The width of propagator

$$\xi^2 \equiv \langle l^2\rangle/2 = (1/2)\int l^2\alpha(l)dl$$

A.C. Eringen, "*Nonlocal Continuum Field Theories*"

Length scales



The length scale of non-locality does not show any critical divergences and is comparable with the disk diameter.

Summary

- *Kolmogorov flows* of soft athermal disks are simulated by MD simulations to examine the non-locality below jamming.
- *Local model* fails even if the density gradients are taken into account, where the discontinuities are generated in the shear-localized regions.
- *Non-local model* well explains the profiles and wave-number dependent flow curves.
- We conclude that the range of non-locality (the width of propagator, ξ) may not diverge at jamming transition.

Bubble (Katgert, et al. 2010)

$$\xi \sim 3d_0$$

Granular, fixed pressure (Bouzid, et al. 2013)

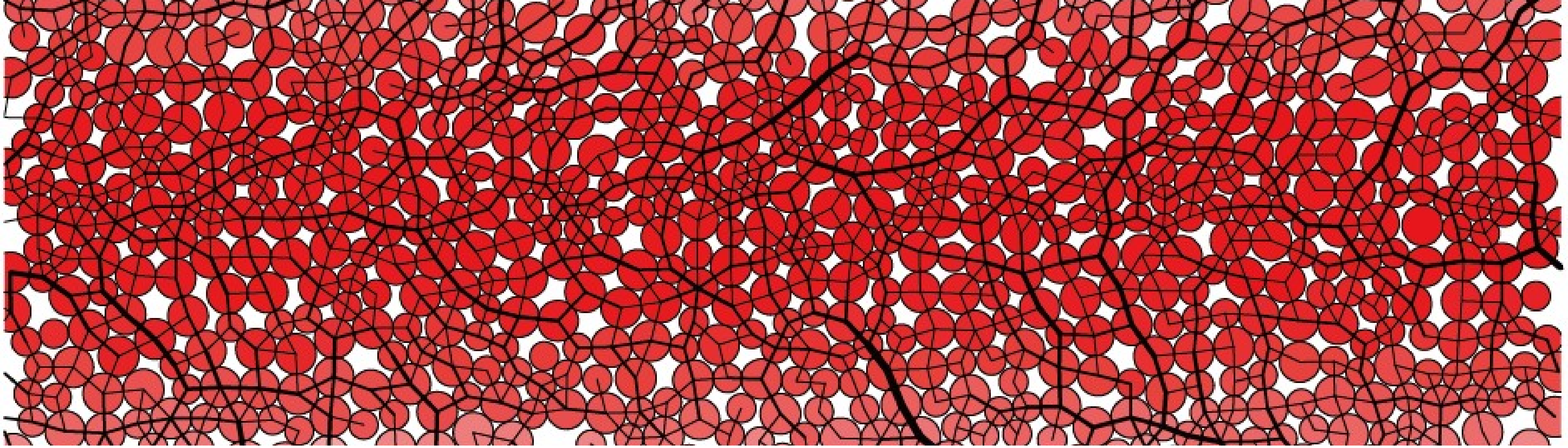
$$\mu \equiv \sigma_{xy}/p \qquad \xi \sim |\mu - \mu_c|^{-1/2}$$

Emulsion (Jop, et al. 2012)

$$\xi \sim 3d_0$$

Non-local elasticity (Baumgarten, et al. 2012)

$$\xi \sim 2d_0$$



Thank you very much!