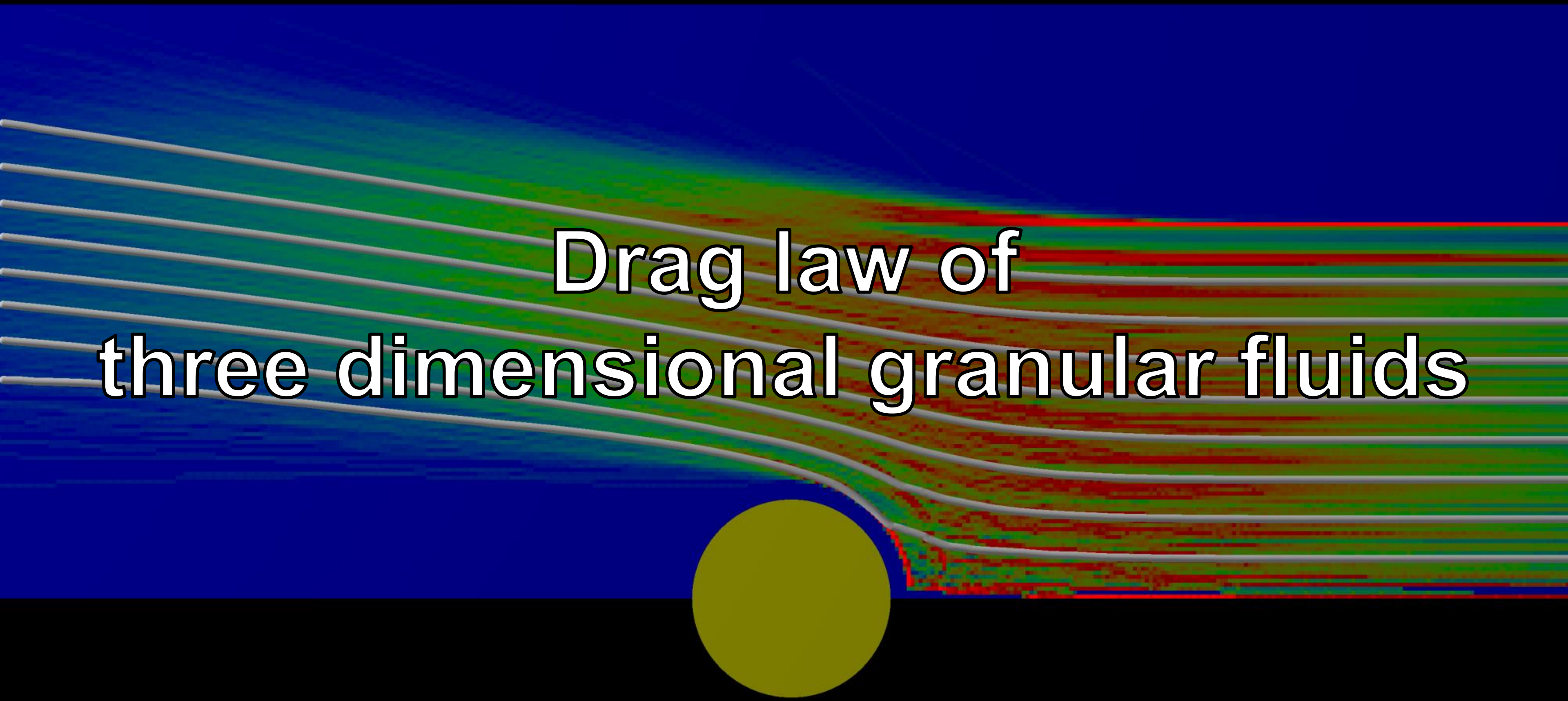


Drag law of three dimensional granular fluids

Satoshi Takada (ERI, Tokyo)
Hisao Hayakawa (YITP, Kyoto)

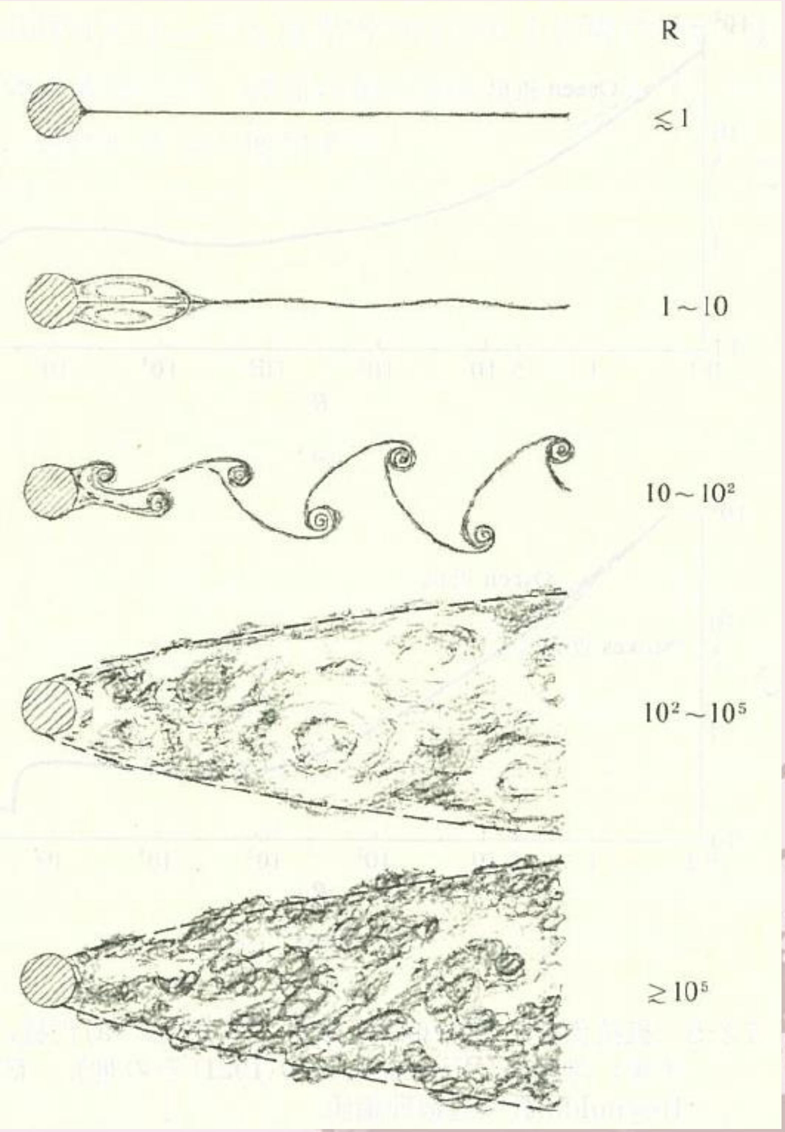
~~2018/6/18~~2018/6/20 Rheology of disordered particles (YITP, Kyoto University)

Introduction

Drag law in fluid

Drag of a tracer in a flow is characterized by Reynolds number

- Slow-speed region
  $F = 6\pi\mu a U$ (3D sphere)
  (Stokes' law)
  - ☛ M. Itami & S. Sasa, J. Stat. Phys. 161, 532 (2015)
- High-speed region $F \propto U^2$
  (Newton's law)
  impulsive force due to collisions



Tomomasa Tatsumi
"Hydrodynamics" (Baifukan)

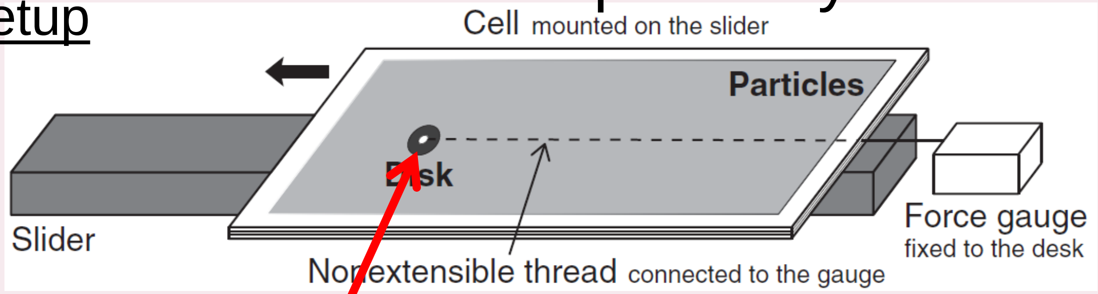
Previous studies (1)

Drag law in a granular media

Y. Takehara & K. Okumura, PRL, **112**, 148001 (2014).

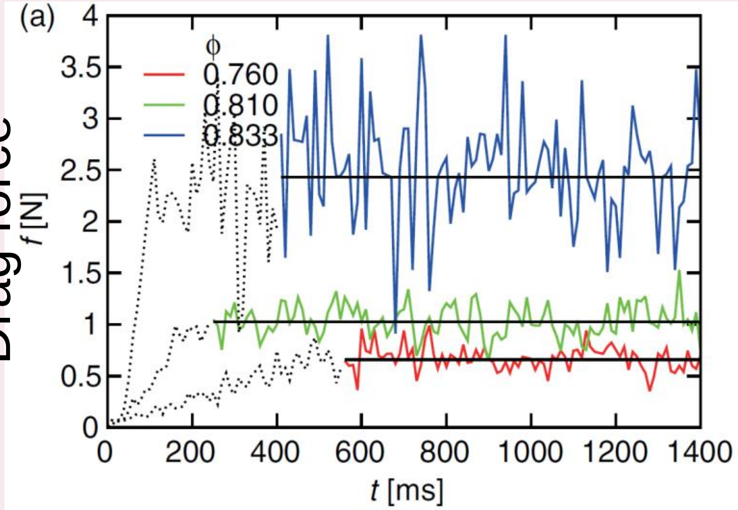
The Cell is pulled by V.

Experimental setup



The Disk is fixed by a wire.

Drag force



time

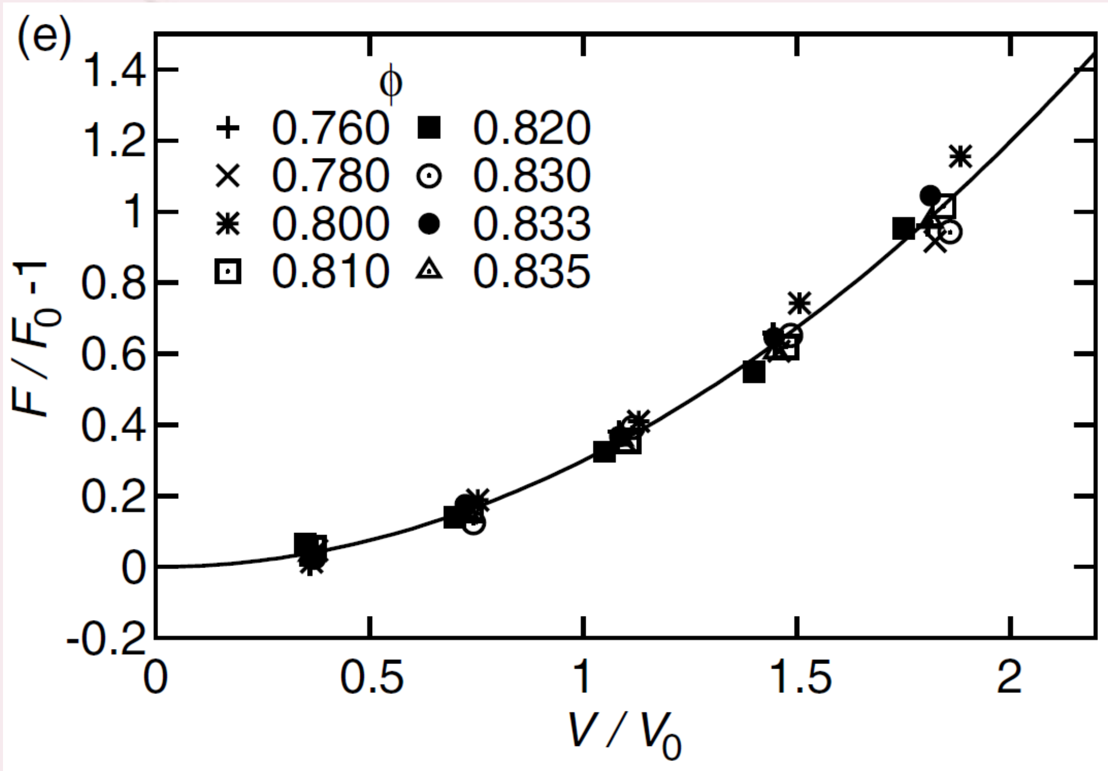
The drag force f is measured.

Time average

Drag force F

Previous studies (2)

Drag law in a granular media



$F = \underline{F_0(\phi)} + \alpha(\phi)V^2$ is a good fitting function.

Yield force

Previous studies (3)

The origin of the term proportional to V^2

Dimensional analysis

- Force $\propto$ $[\mathrm{time}]^{-2}$
- Stiffness k and pulling speed V have the dimension of time.

Unimportant below the jamming

$\Rightarrow$ drag force $\propto V^2$

The origin of the yield force

Another quantity having the dimension of time...

Gravity acceleration g ?

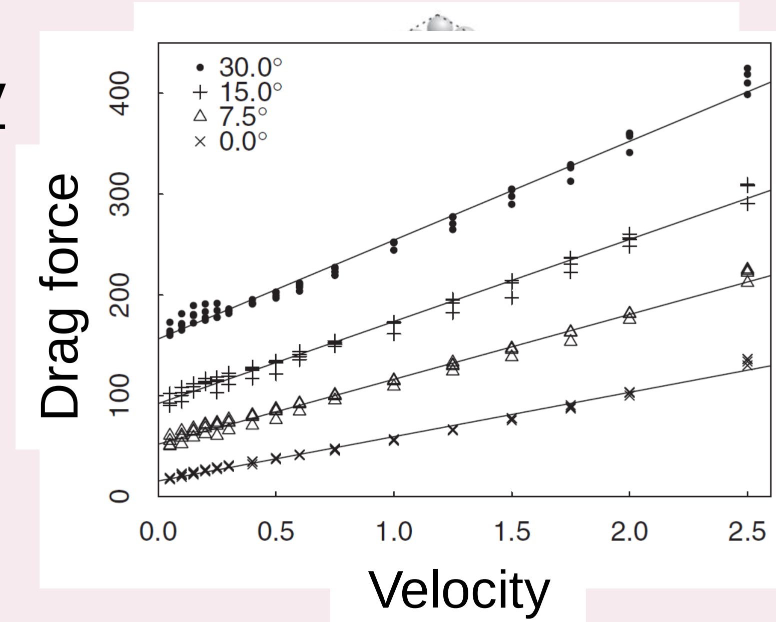
Dry friction between the grains and the bottom plate?

Previous studies on 3D drag

- 3D drag simulation under gravity with friction

  J. E. Hilton & A. Tordesillas, PRE, **88**, 062203 (2013)

$$F = F_0 + \alpha V$$

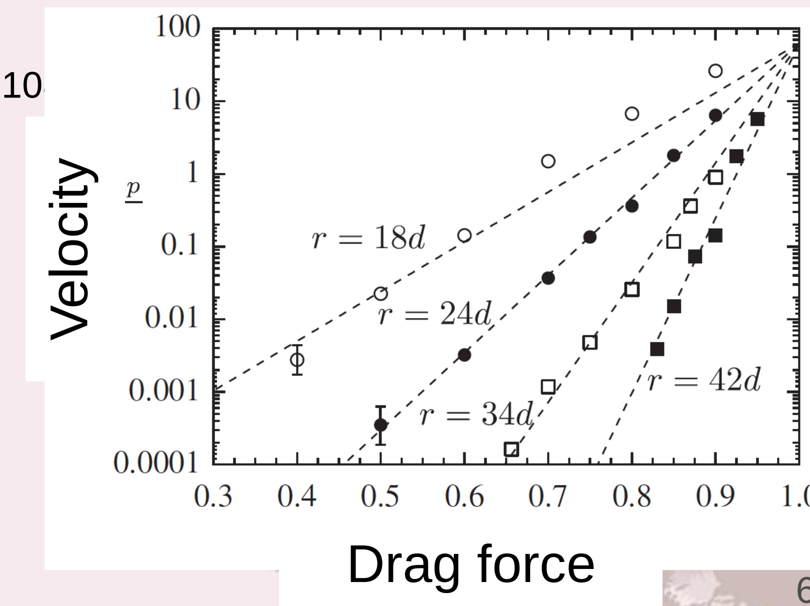


- Drag experiment of rod

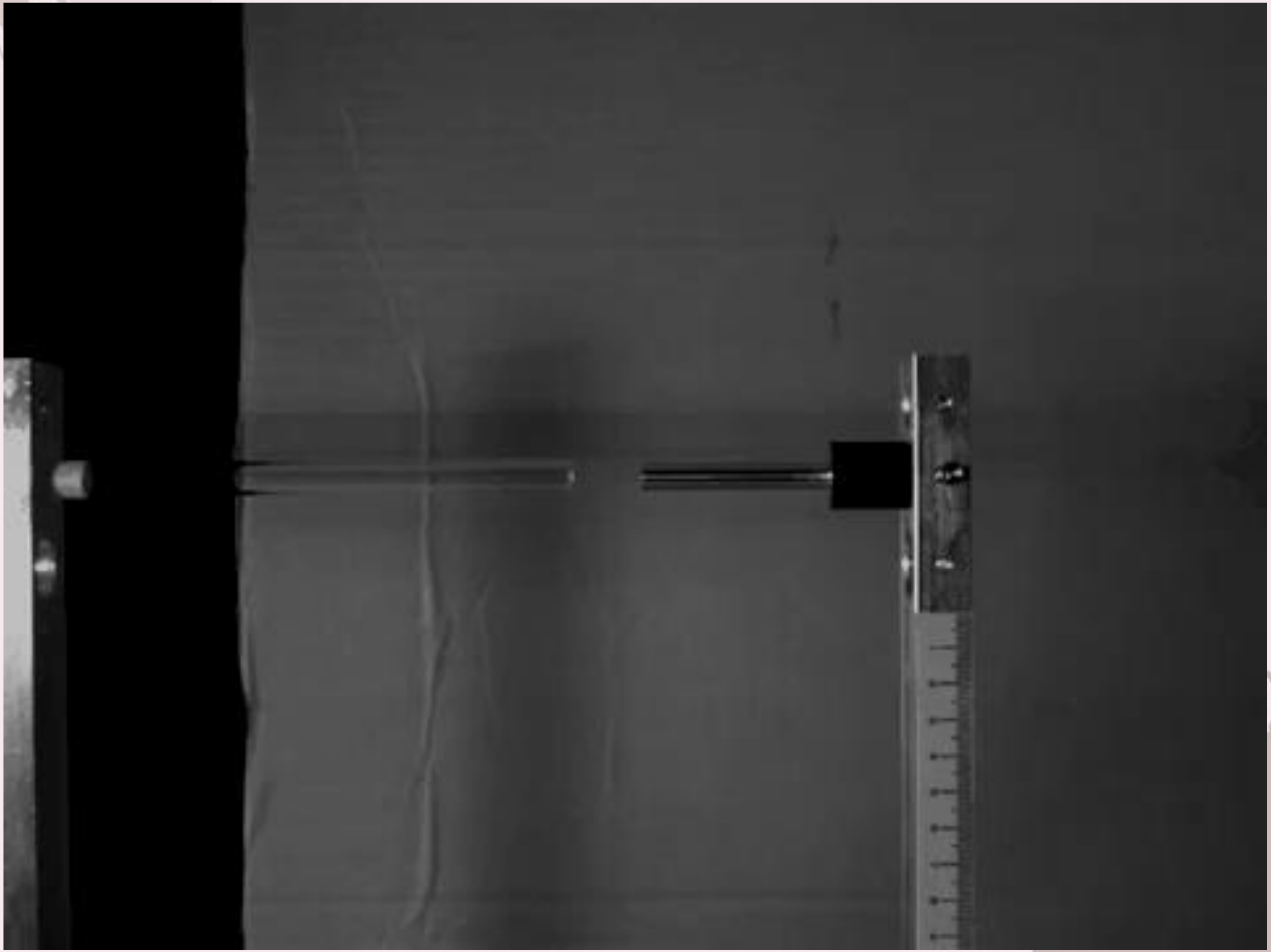
  K.A. Reddy, Y. Forterre, and O. Pouliquen, PRL, **106**, 10

$$V = \exp\left(\frac{F - F_c}{F_0}\right)$$

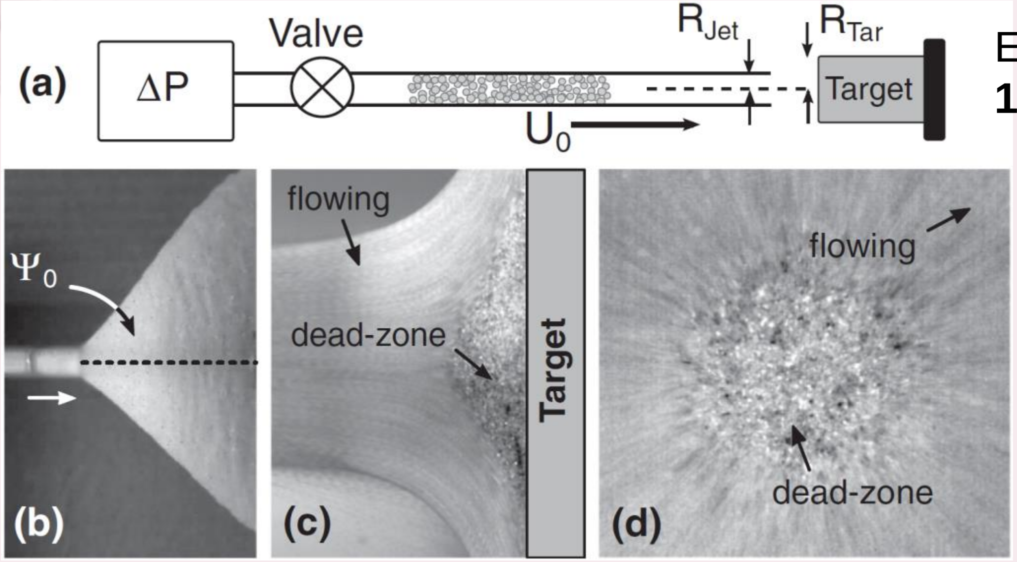
$$\Leftrightarrow F = a + b\log(V)$$



Perfect fluidity in granular jet (1)



Perfect fluidity in granular jet (2)



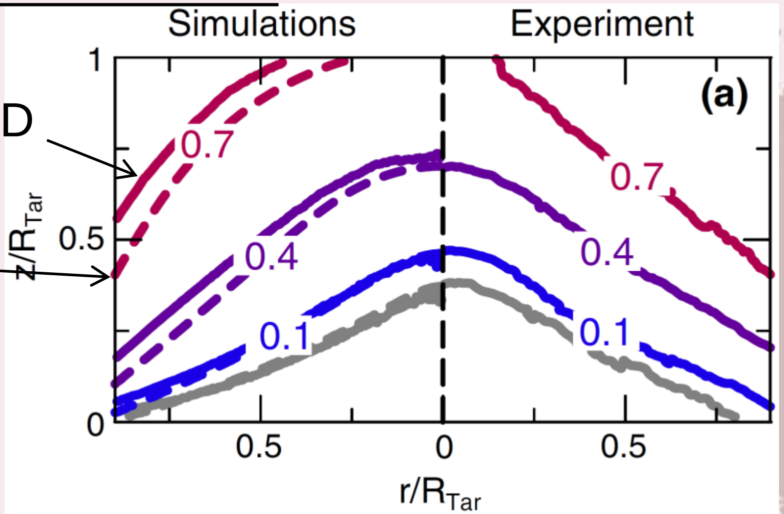
Ellowitz et al. PRL **111**, 168001 (2013)

Chicago group suggested the perfect fluidity in granular jet problem.

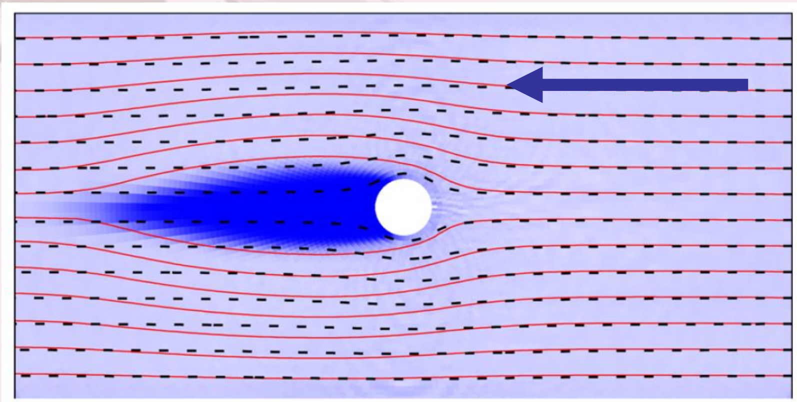
$$\nabla \cdot \boldsymbol{u} = 0,$$
$$\rho(\partial_t + u \cdot \nabla)\boldsymbol{u} = \nabla \cdot \boldsymbol{\sigma}$$
$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \underline{\mu p \dot{\gamma}/|\dot{\gamma}|}$$

Coulombic friction

Speed contours



Our previous study: Drag law in 2D granular media

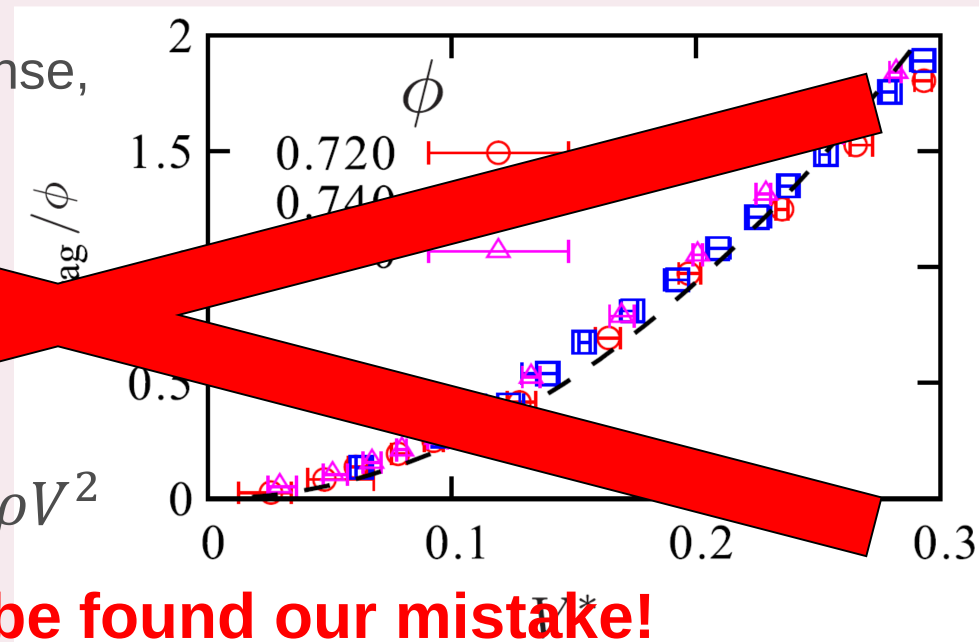


S. Takada and H. Hayakawa,
J. Eng. Mech. **143**, C4016004 (2017)

- Active microrheology
- frictionless system

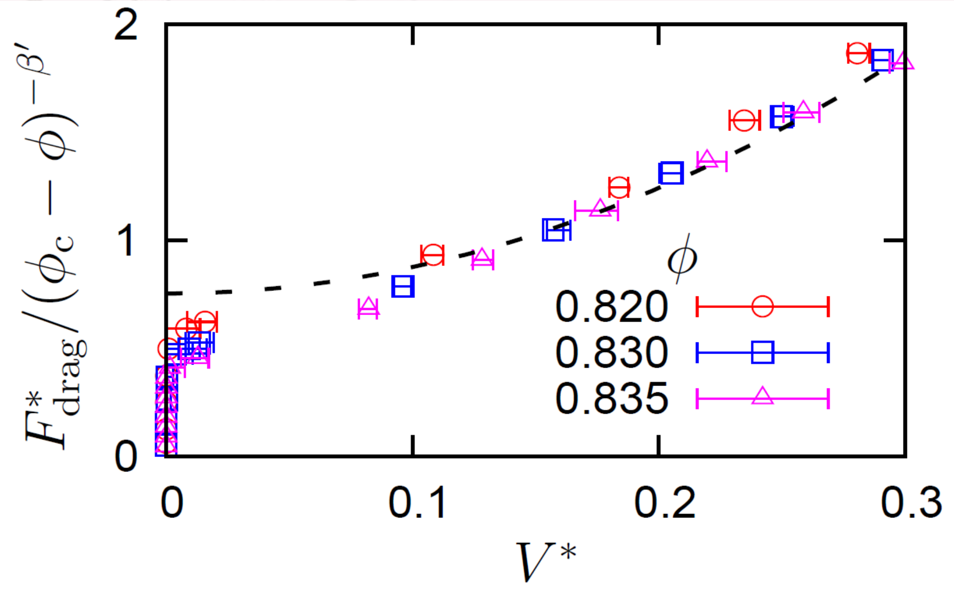
When the system is moderately dense,
the dr... law is explained by
- the perfect flu...
- vacancy.

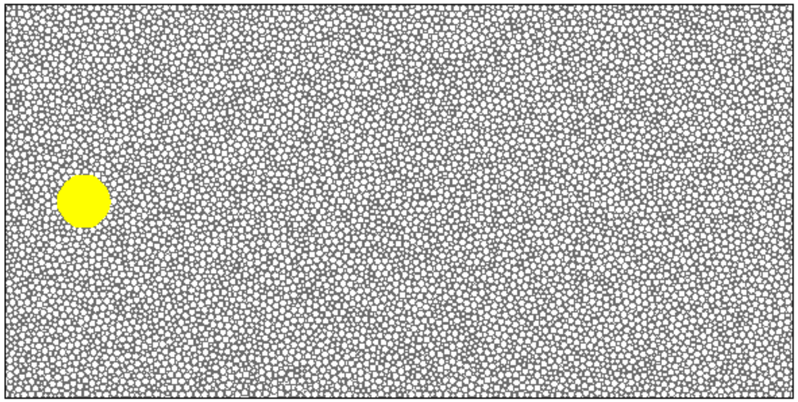
F_{drag}
$= \left(\frac{3 + 2 \cdots}{2} \cdots - \frac{\cdots}{3} \sin^2\theta_0 \right) \sin\theta_0 D \rho V^2$



Recently, Dr. Tanabe found our mistake!

Our previous study: Drag law in 2D granular media

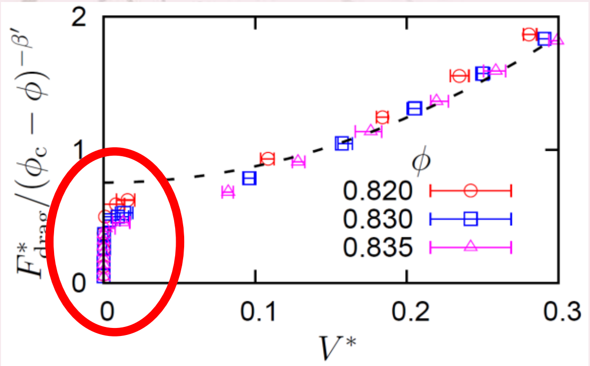




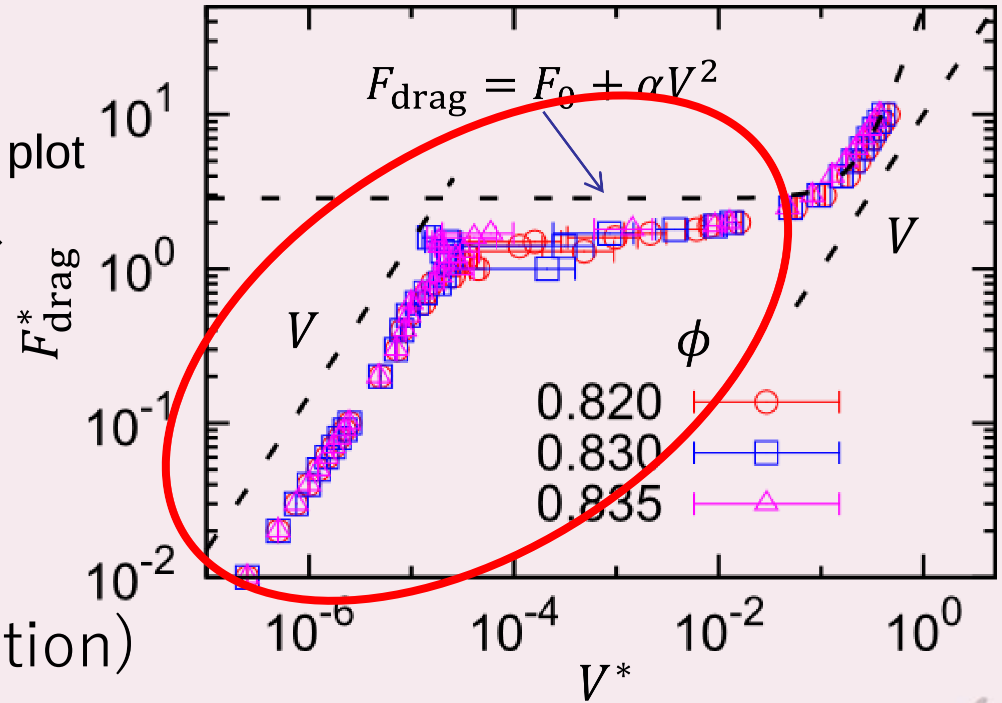
Percolation of force chains!

Introduction of the dry friction of the bottom plate
⇒ Existence of the yield force
⇒ Friction is the origin of the constant force.

Our previous study & Motivation



log-log plot



Various V dependence

- Linear regime (creep)
- Plateau regime (activation)

Motivation

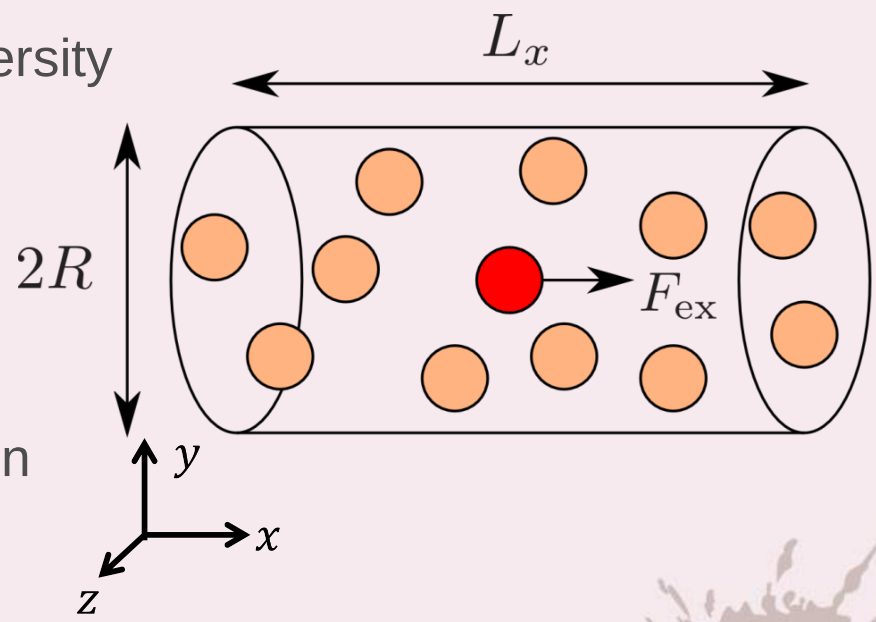
What determines the velocity dependence?

⇒ We clarify the relationship between $F_{\rm drag}$ and V based on the 3D DEM simulation.

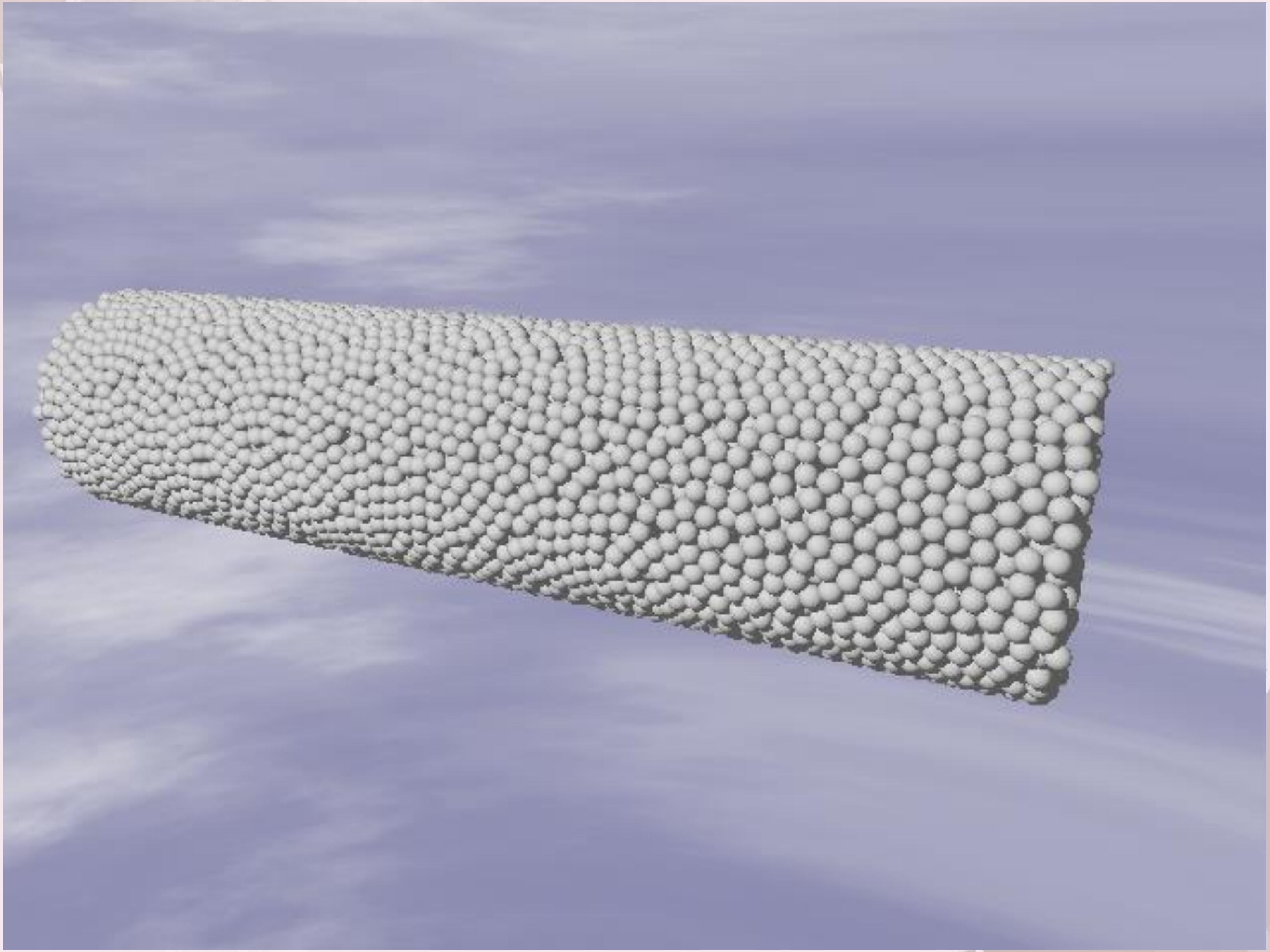
Setup

3D DEM simulation for frictionless systems

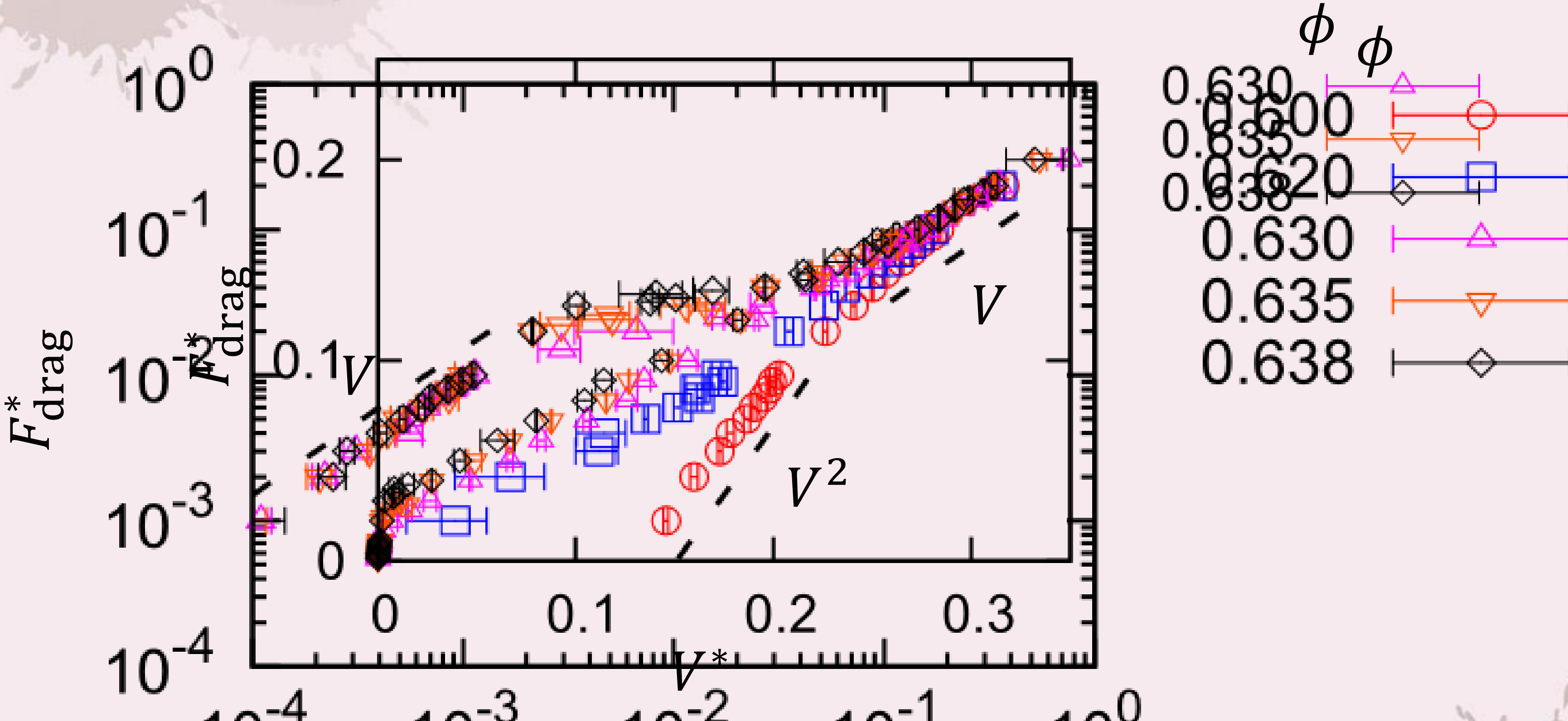
- Particles
  - mass m, diameter d: monodispersity
  - Number of grains $\sim 10^4$
  - Restitution coefficient: $e = 0.8$
- Intruder
  - Diameter D
  - Pulling with $F = F_{\rm ex}$ in x-direction
- System
  - Cylinder $L_x = 60d, R = 7.5d$
  - Boundary condition:
    x-direction ⇒ periodic boundary
    y, z-direction ⇒ flat physical boundary (curvature R)



Movie



Drag law ($D = d$)



F_{drag} = (Yield force) + (term ∝ V) ?

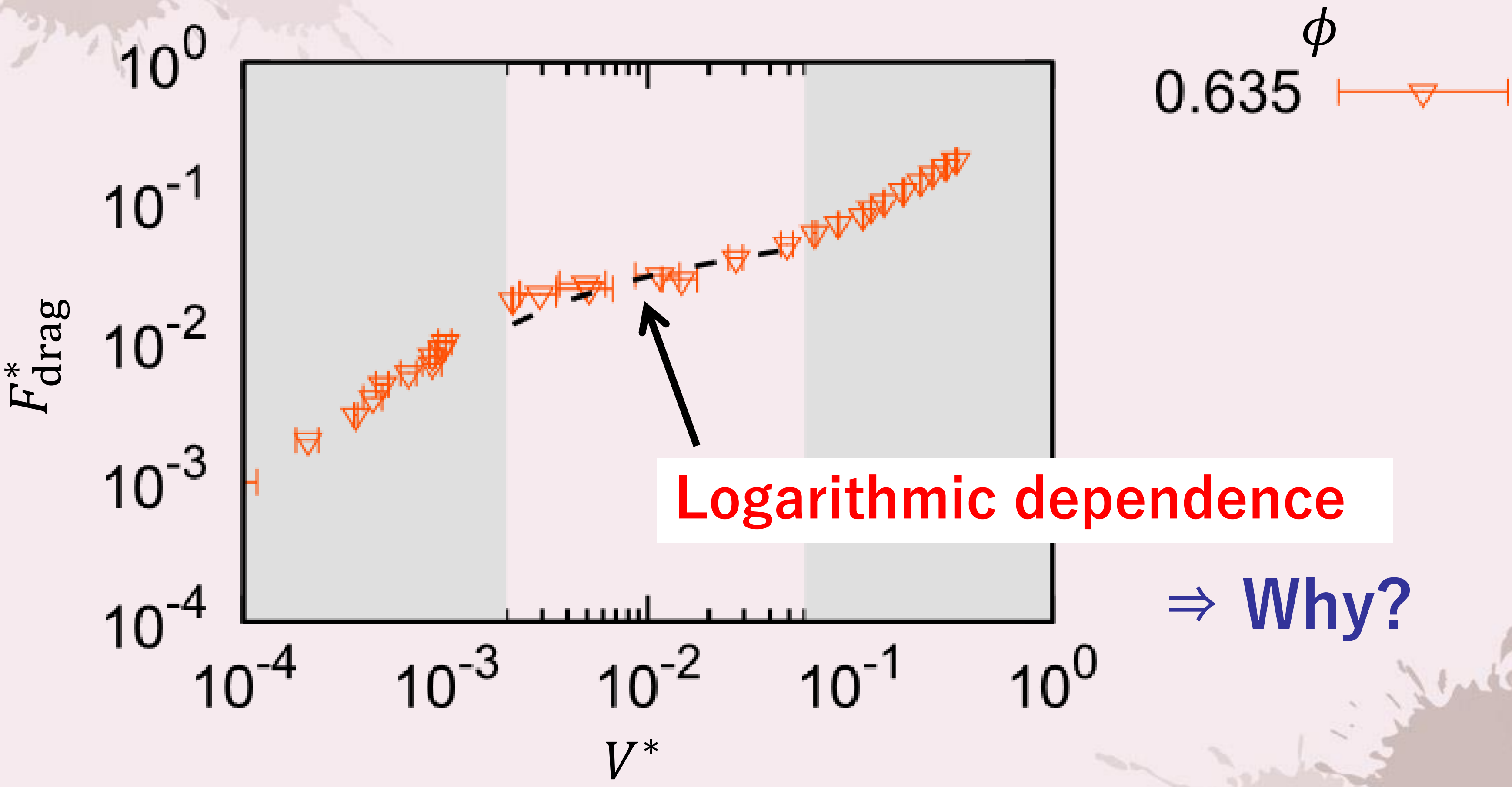
V^*

Various V dependence

- V regimes
- V^2 regime (perfect fluidity)
- Plateau regime

Similar to PRE, **88**, 062203 (2013)

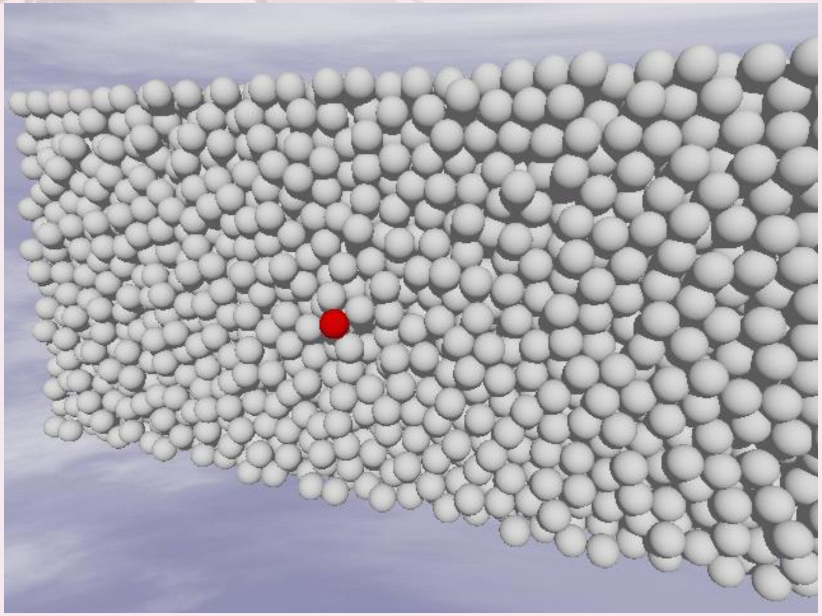
Drag law



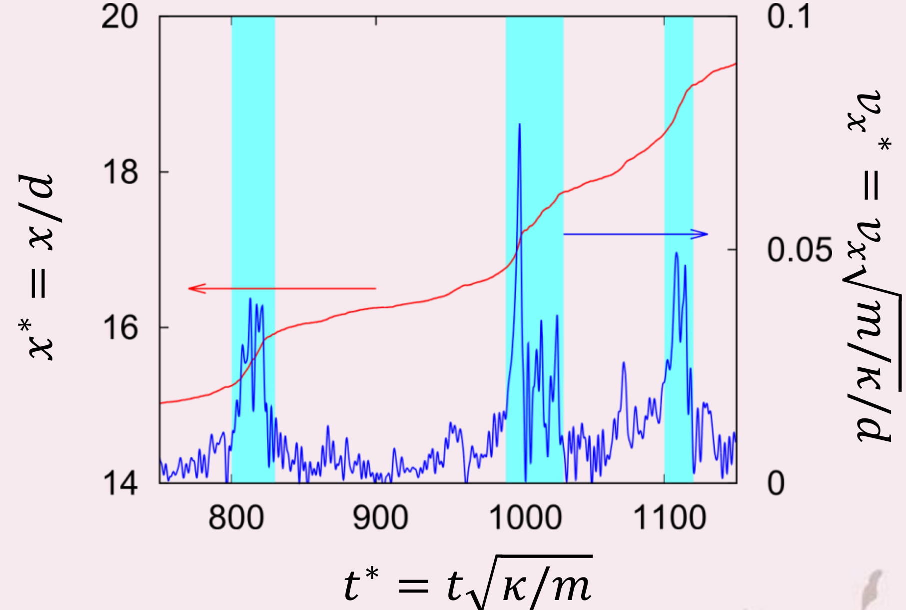
Logarithmic dependence

⇒ **Why?**

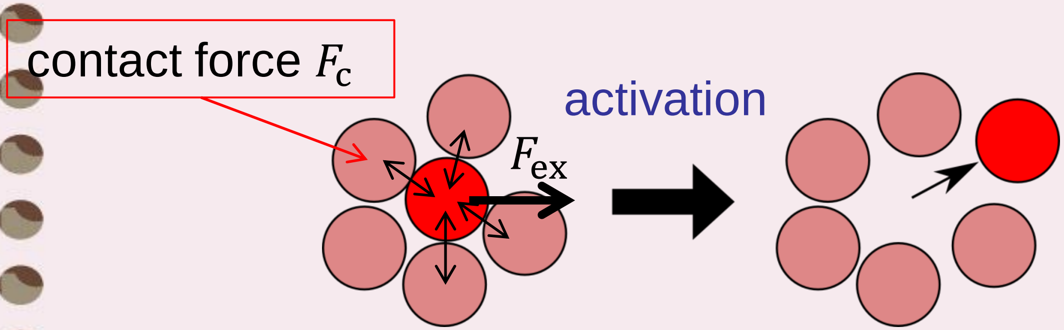
Log regime



Time evolution of
the position and velocity of the tracer



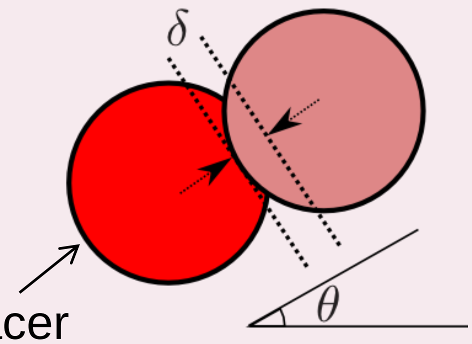
Activated processes occur ?



Average contact force $\langle F \rangle_c$
~ External force F_{ex} ?

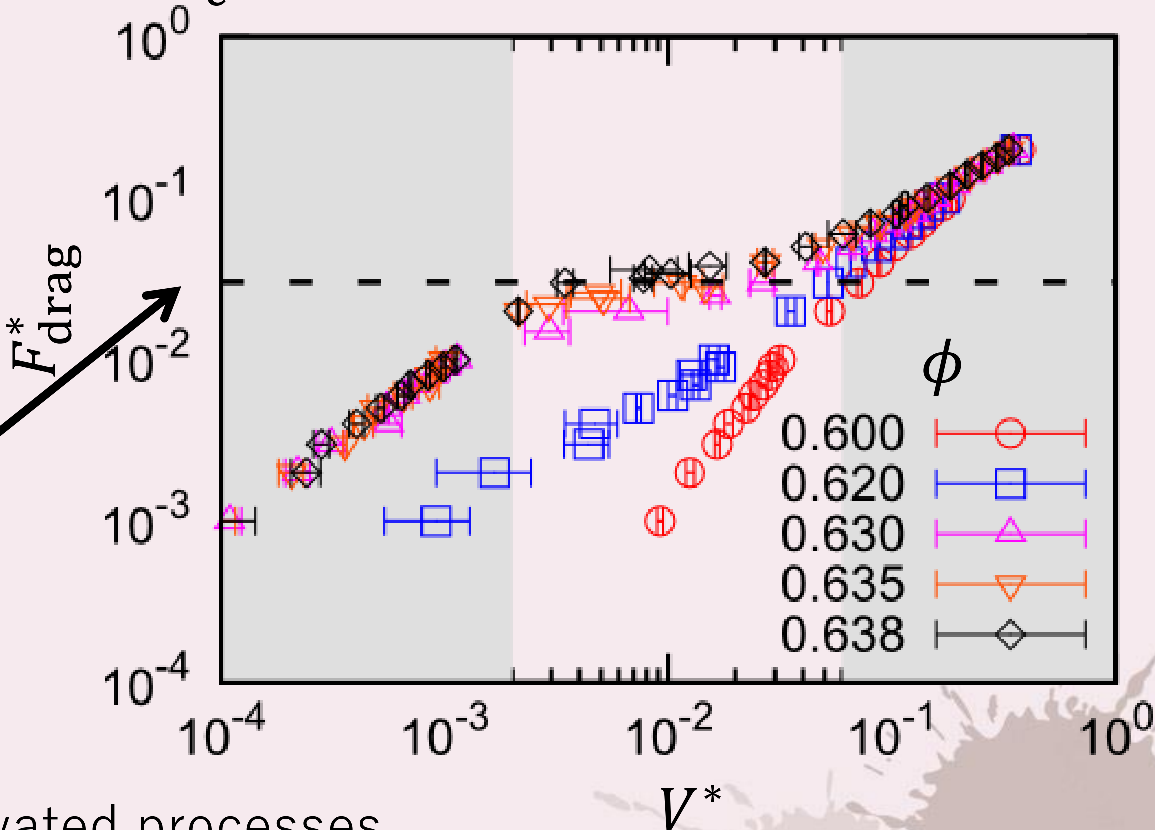
Activated process

Time-averaged contact force $\bar{F}_c$ from the simulation



$$\bar{F}_c = \overline{\sum_{\mathrm{contact}} \kappa\delta\cos\theta}$$

κ: spring constant



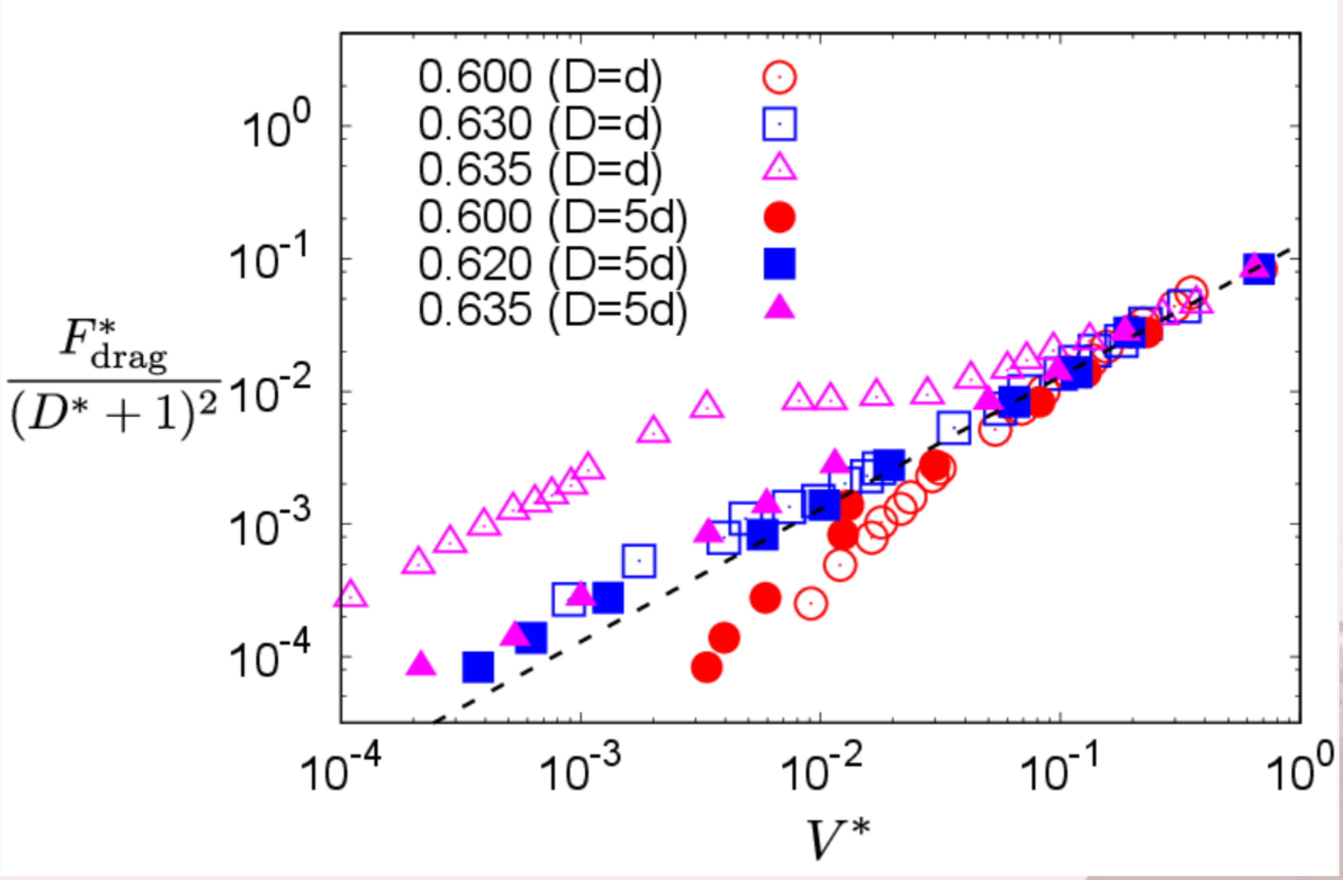
Good agreement ⇒ Activated processes

Logarithmic dependence!

cf). PRL, **106**, 108301 (2011)

Drag law ($D > d$)

When the intruder is much larger than the surrounding particles, the plateau regime vanishes.
⇒ This fact seems to validate our conjecture (activation process).



Short summary of Part I

We have performed
the three-dimensional drag simulation.
This system is force-controlled (active microrheology).
Many characteristic regimes

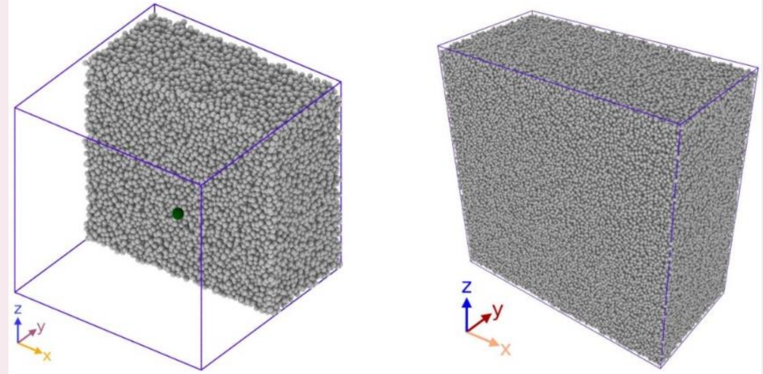
- Quadratic regime ⇒ Newtonian
  (Similar to 2D system)
- Log regime ⇒ Activated process
  This regime vanishes for larger D.

Next question:
What is the proper drag law?

Especially, the drag law for frictional grains under gravity.
⇒ We numerically study this problem.

Setup

In collaboration with
S. Kumar and K. A. Reddy
(IITC, India)



System I
$40d \times 40d \times 38d$
70,001 particles

System II
$80d \times 40d \times 80d$
300,001 particles

- 3D DEM simulation by LAMMPS
- With gravity and friction
- Polydisperse particles
  ($0.9d \sim 1.1d$: uniformly distributed)
- We control V, and measure F_{drag}.
- We introduce the Froude number (dimensionless speed)

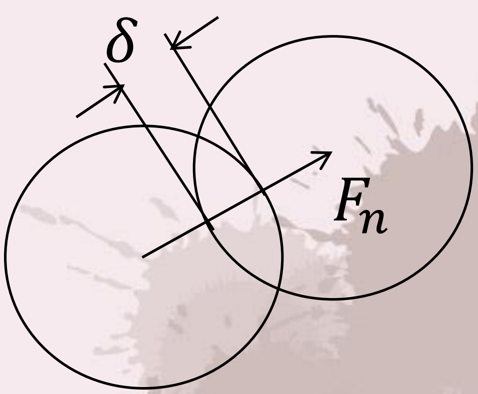
$$\boxed{\mathrm{Fr} = \frac{V}{\sqrt{gD}}}$$

= ratio of two characteristic time scales:
$t_1 = D/V$: forward motion in x-direction
$t_2 = \sqrt{D/g}$: falling in z-direction

Interaction model

- Hertzian model + dashpot (proportional to the relative velocity)

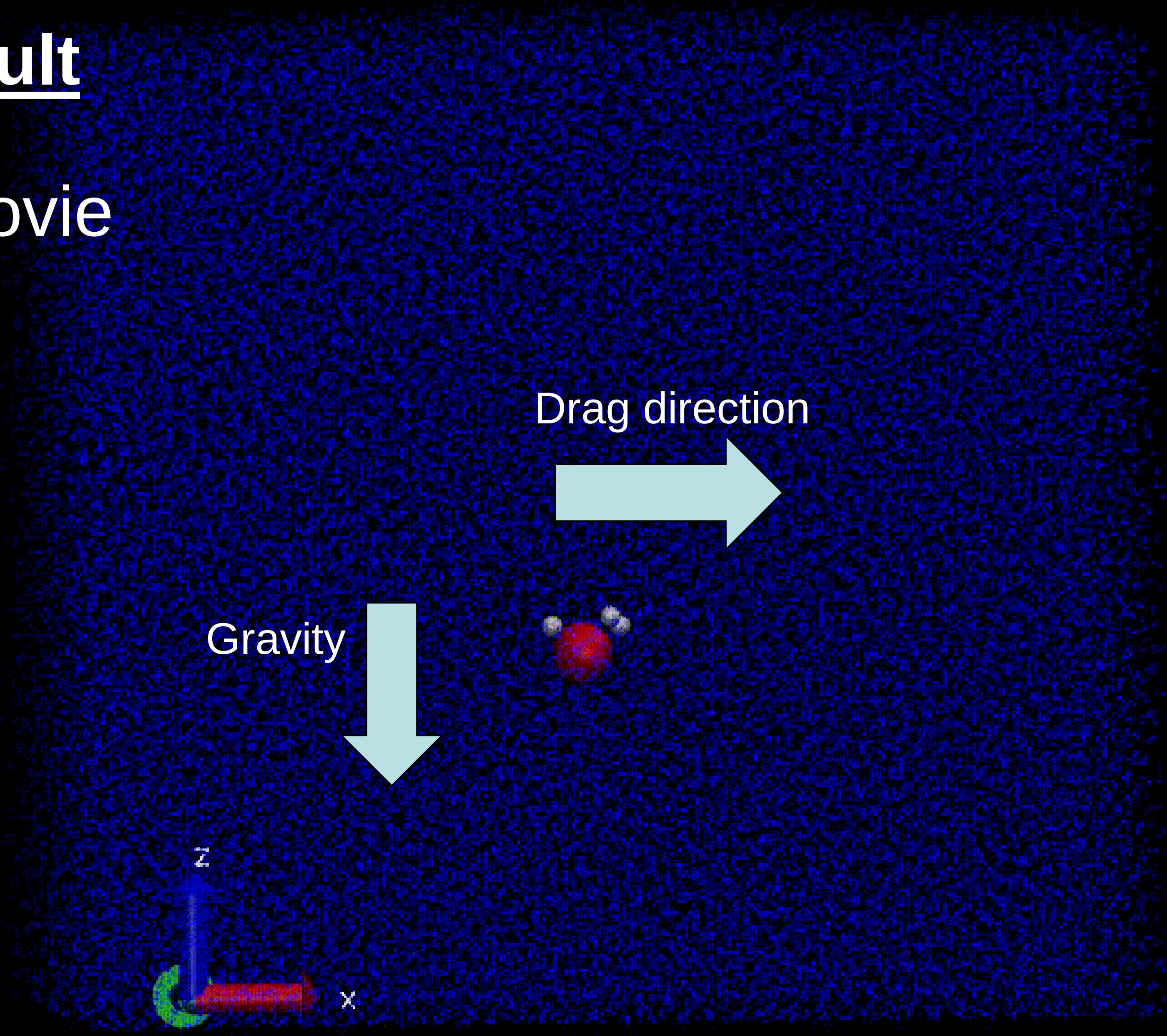
$$F_n = \sqrt{R_{\mathrm{eff}}\delta}(K_n\delta - m_{\mathrm{eff}}\gamma_n v_n)$$
$$F_t = -\min\left(\mu F_n, \sqrt{R_{\mathrm{eff}}\delta}(K_t \Delta s_t + m_{\mathrm{eff}}\gamma_t v_t)\right)$$
$$K_n = 2 \times 10^8 \rho d g, K_t = 2.45 \times 10^8 \rho d g$$



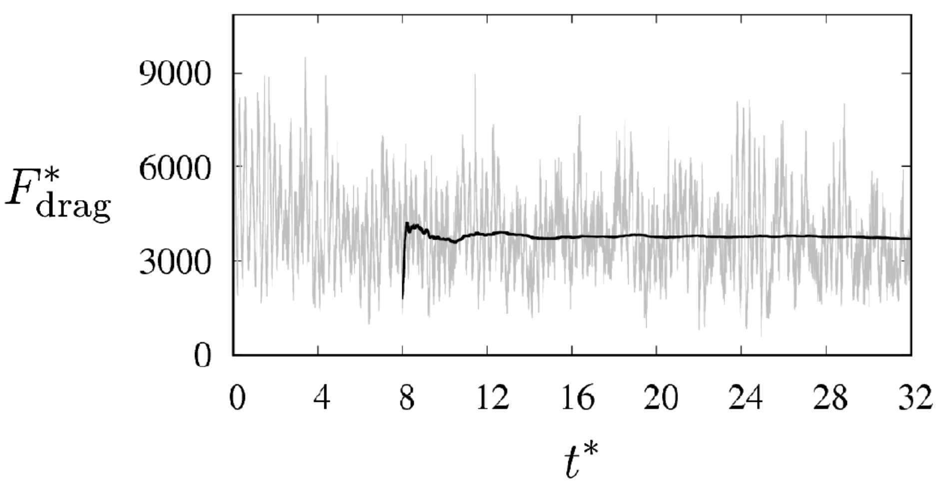
※All quantities are nondimensionalized in terms of ρ, d, g.

Result

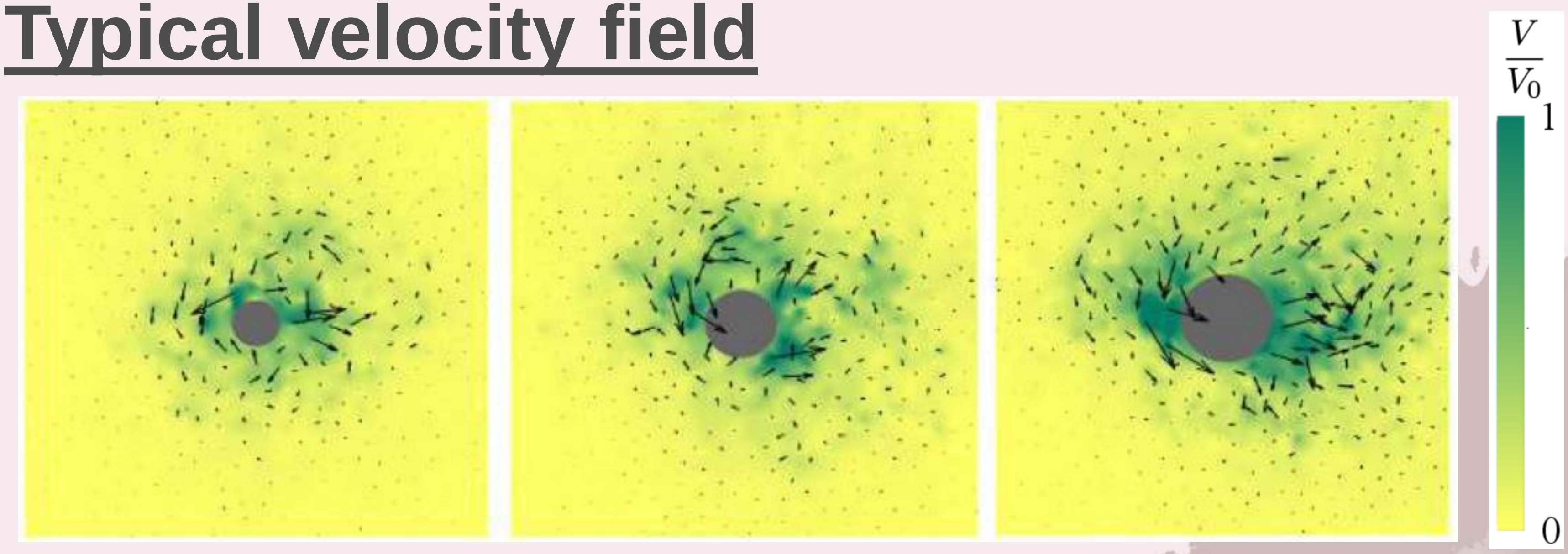
- movie



Typical time evolution of the drag force

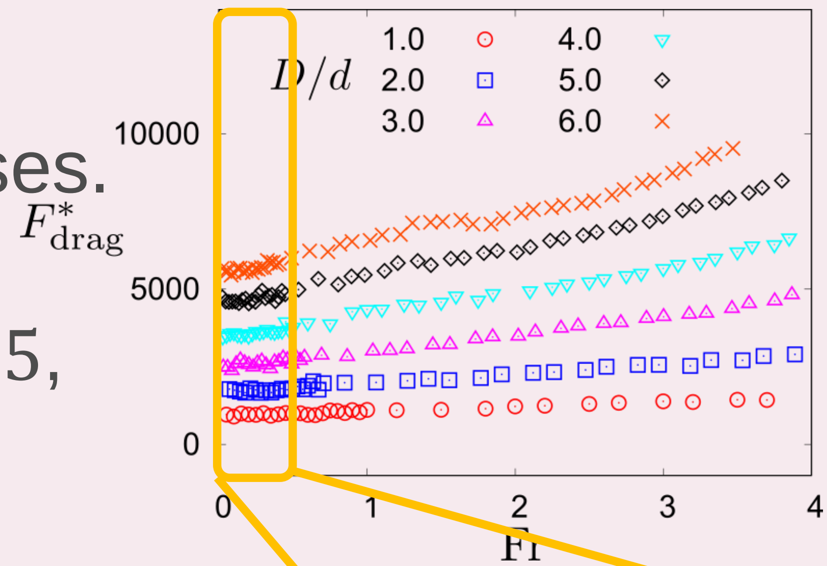


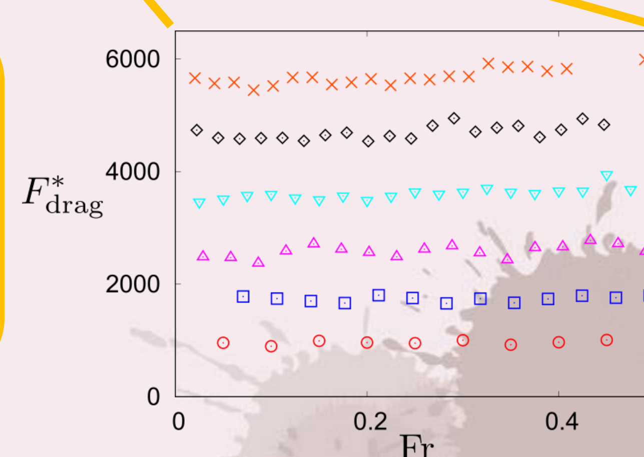
Typical velocity field



Drag force vs. Froude number (constant depth)

- F_{drag} increases as the diameter D increases. $F_{\mathrm{drag}} = F_{\mathrm{drag}}(\mathrm{Fr})$.
- Especially, for $0 \le \mathrm{Fr} \le 0.5$, $F_{\mathrm{drag}} \simeq F_{\mathrm{Y}} \equiv \mathrm{const.}$

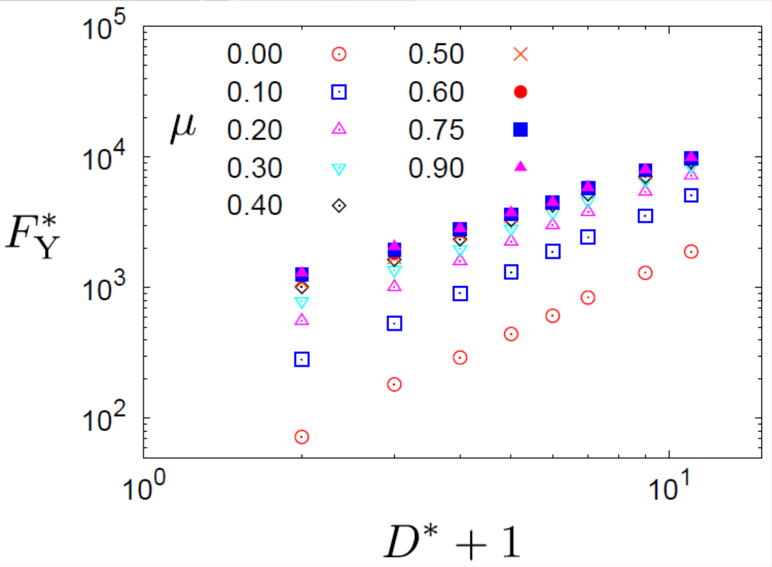




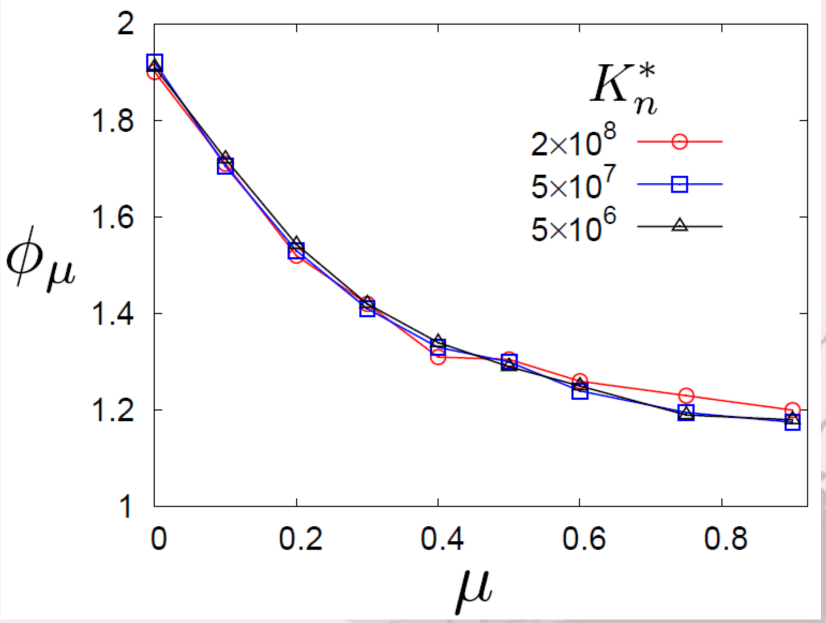
Question:
How can we scale these drag laws depending on the intruder diameter D or the depth h?

$\Rightarrow$ We focus on the static (constant) regime $(0 < \mathrm{Fr} \le 0.5)$.

Diameter D dependence in the static regime



- F_Y increases as μ increases.
- F_Y depends the power of $(D^* + 1)$.
  ⇒ We define the exponent ϕ_μ.



Yield force is scaled by $D^* + 1$ with

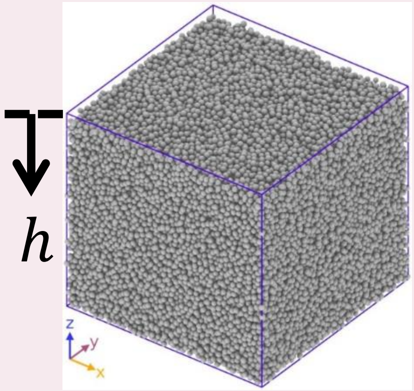
$$\frac{F_Y^*}{f(\mu)(D^* + 1)^{\phi_\mu}} = \text{const.}$$

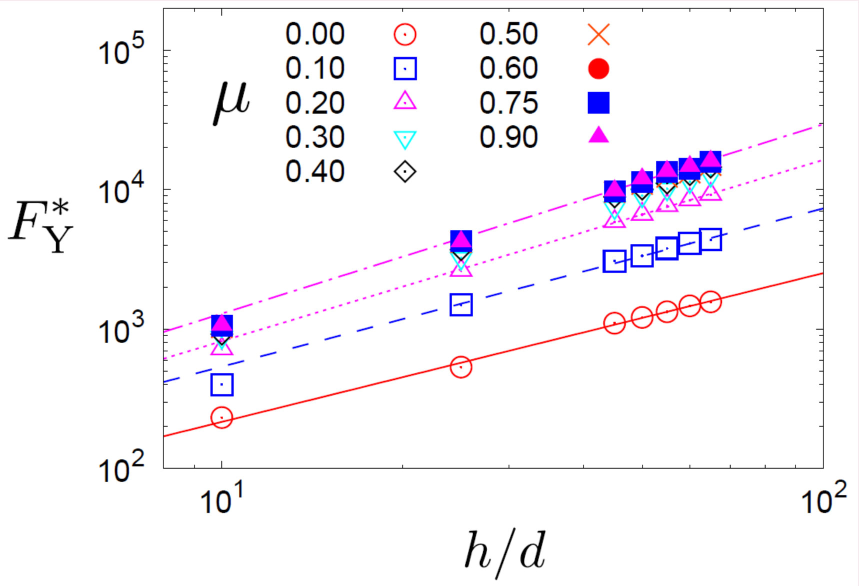
※$f(\mu)$ depends only on μ.

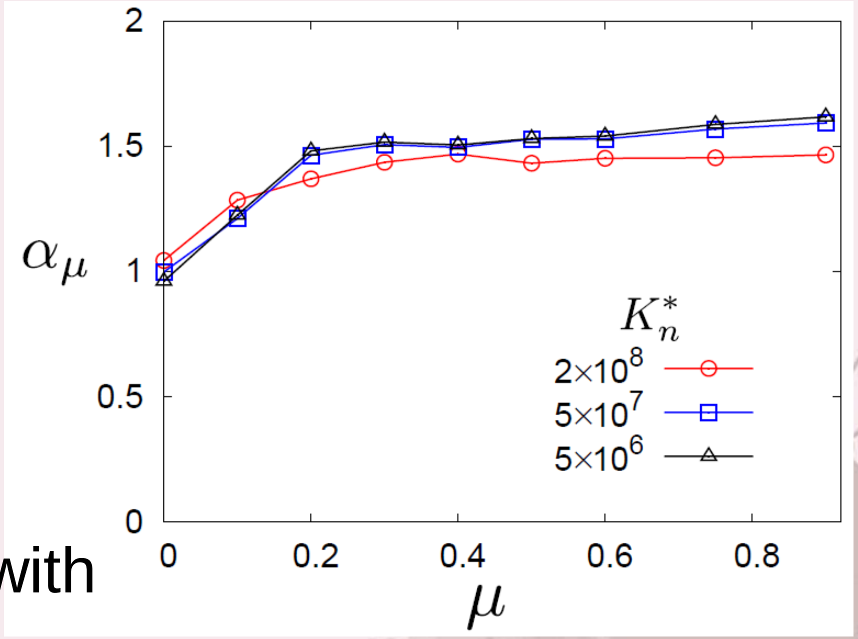
Next, how is the depth dependence?

Depth h dependence (constant diameter D)

- As well as $(D^* + 1)$, F_Y also depends on the power of h.
  ⇒ We define the exponent α_μ.
  α_μ is the increasing function of μ.







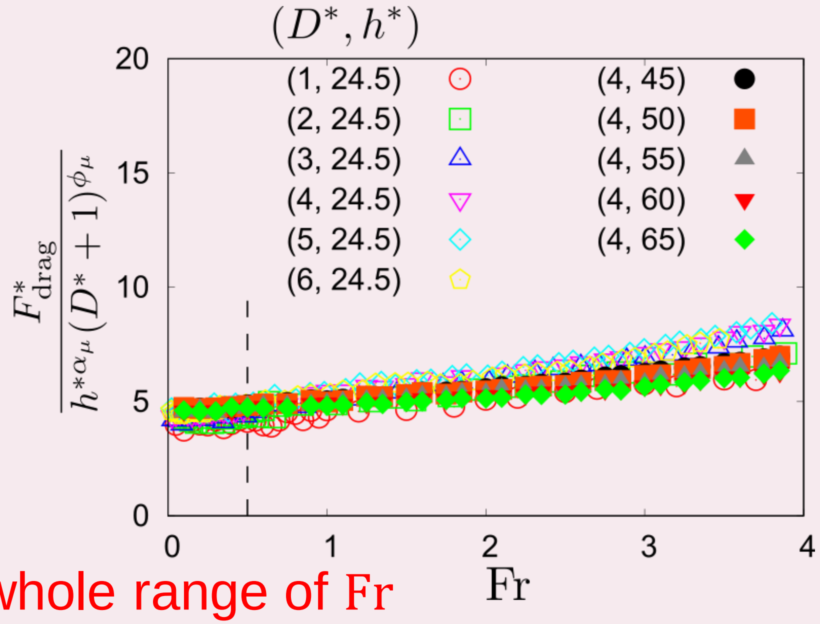
- Yield force is also scaled by h with

$$\boxed{\frac{F^*_\mathrm{Y}}{g(\mu)h^{*\alpha_\mu}} = \mathrm{const.}}$$

Scaling law

From the above discussions,
the yield force can be scaled as

$$F_{\mathrm{Y}} \propto \rho (D^* + 1)^{\phi_\mu} h^{\alpha_\mu} g$$



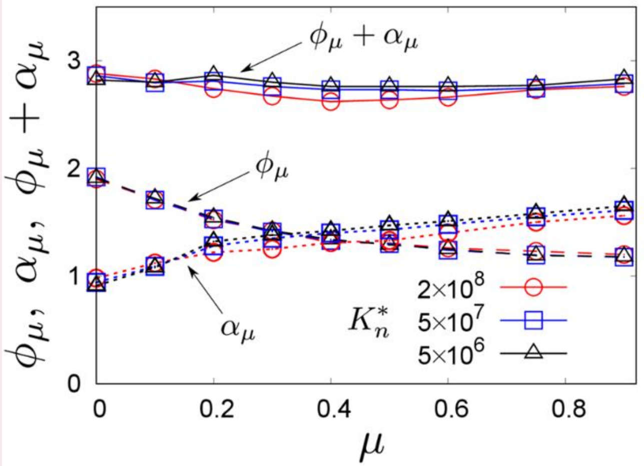
And this scaling can be applied for the whole range of Fr except for the shallow region.

$$\frac{F_{\mathrm{drag}}}{F_{\mathrm{Y}}} = F^*_{\mathrm{dynamic}}(\mathrm{Fr})$$

Sum rule

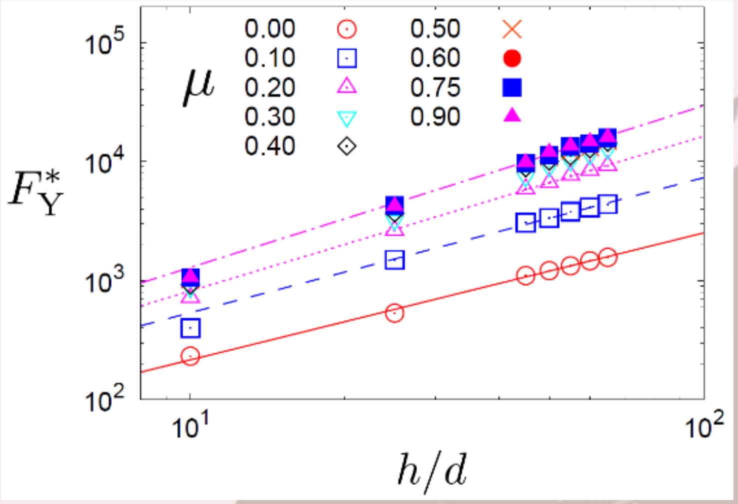
Two exponents ϕ_μ and α_μ satisfy an approximate sum rule

Sum rule
$\phi_\mu + \alpha_\mu \approx 3$



Discussion

- Why is the sum rule $\phi_\mu + \alpha_\mu \approx 3$ satisfied?
  From the dimensional analysis, $F_{\text{drag}} = [\text{length}]^3$
  Which do other quantities have the dimension of length?
  $\Rightarrow D, h \Rightarrow (D+d)^{\phi_\mu} h^{\alpha_\mu}, \phi_\mu + \alpha_\mu = 3$
- Why $\phi_{\mu=0} \approx 2$?
  Collision cross section is given by $\pi(D+d)^2/4$. $\Rightarrow \phi_{\mu=0} \approx 2$
- Why does ϕ_μ (α_μ) decrease (increase)?
  Or really power dependence?
  We do not still have any answer.
  $\Rightarrow$ Larger (in height) simulation should be done.



Short summary of Part II

- We have performed DEM simulation to study the drag law in 3D granular media.
- There exist two regimes depending on the Froude number (static and dynamic parts).
- The drag law for the whole Fr regime can be scaled in terms of $D + d$ and h.
- There exists an approximately sum rule ($\phi_\mu + \alpha_\mu \approx 3$) between two exponents.

Future work

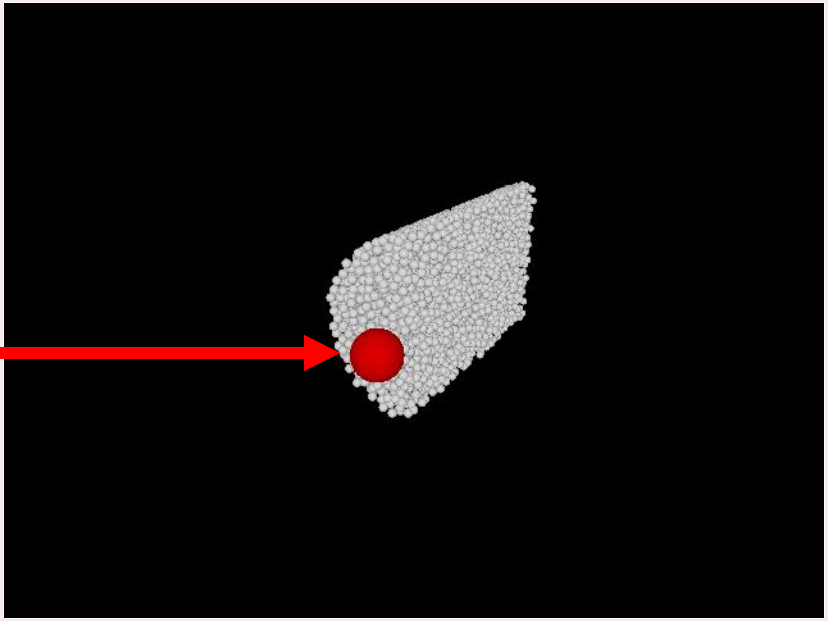
- Larger simulations are needed.
- Force chain network

Question

- Can we use periodic boundary condition in the pulling direction?
  ⇒ It may affect the results
  (especially for high velocity regime).
- When we watch the movie (in Part I), the surrounding particles seem to have a finite temperature?
  ⇒ What happens when the particles have the finite temperature?

⇒ We study passive microrheology.

Setup



Velocity control simulation

Soft core simulation (e=1)

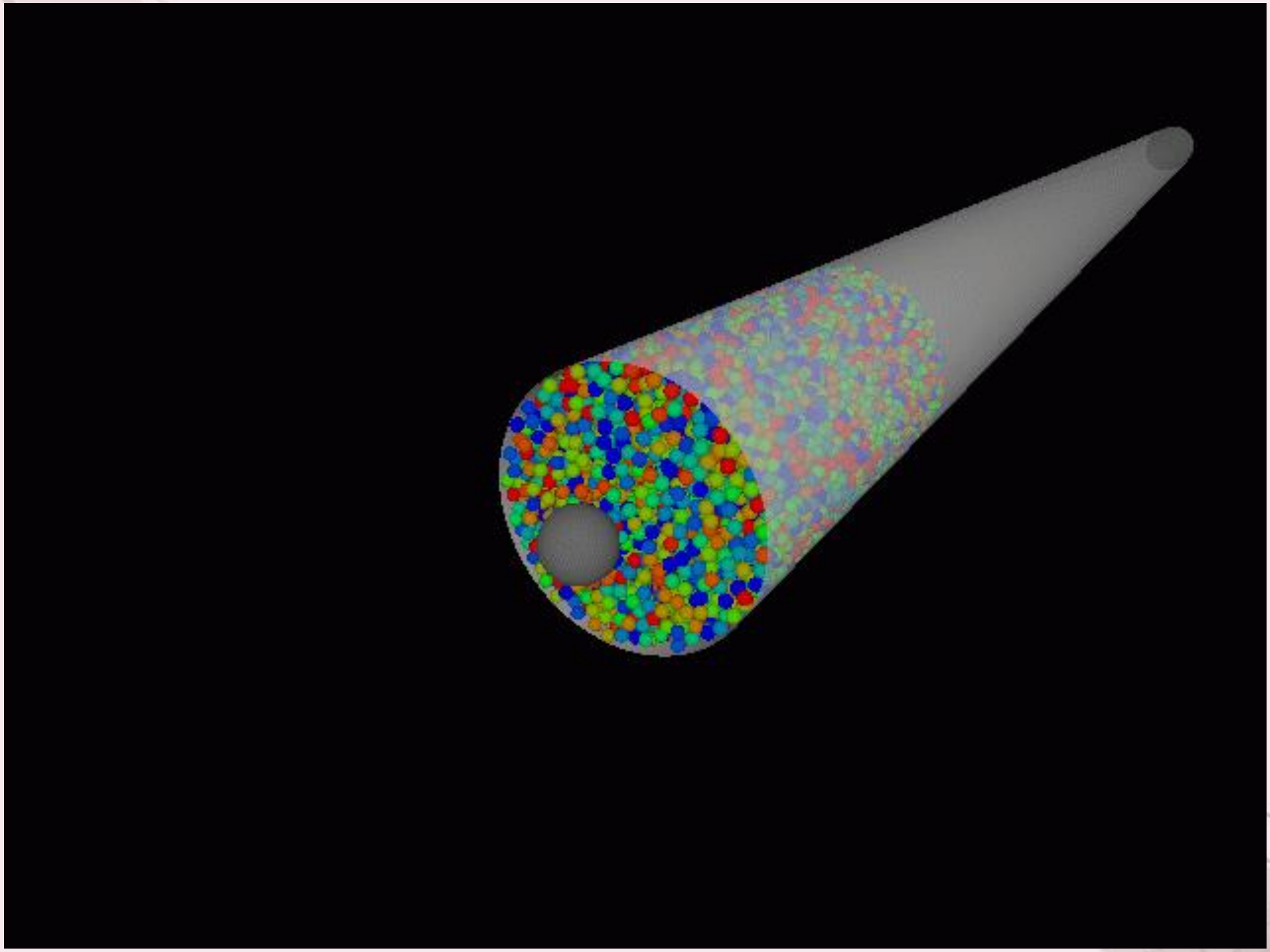
N=20,000, monodisperse

Initial packing fraction: $\phi = 0.4$

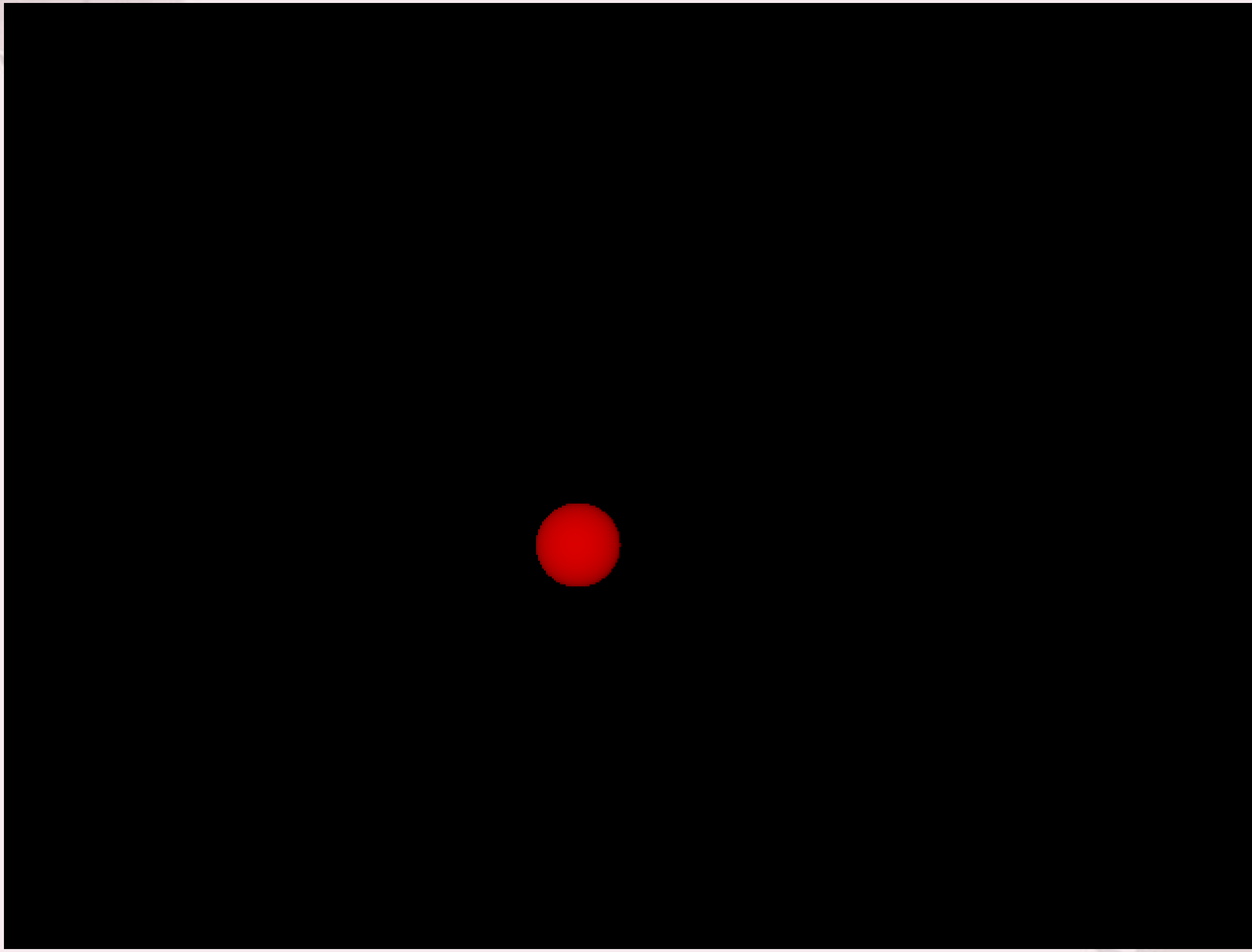
- Intruder
  - Intruder is fixed.
  - Diameter $D = 5d$
  - Mass $M = \infty$
- Surrounding particles
  - Monodisperse
  - At $t = 0$, the velocity V is added.
  - No overlaps at first.

3D simulation ($T = 0$)

$\phi = 0.4$
$e = 1$
$V^* = 10^{-0.5}$



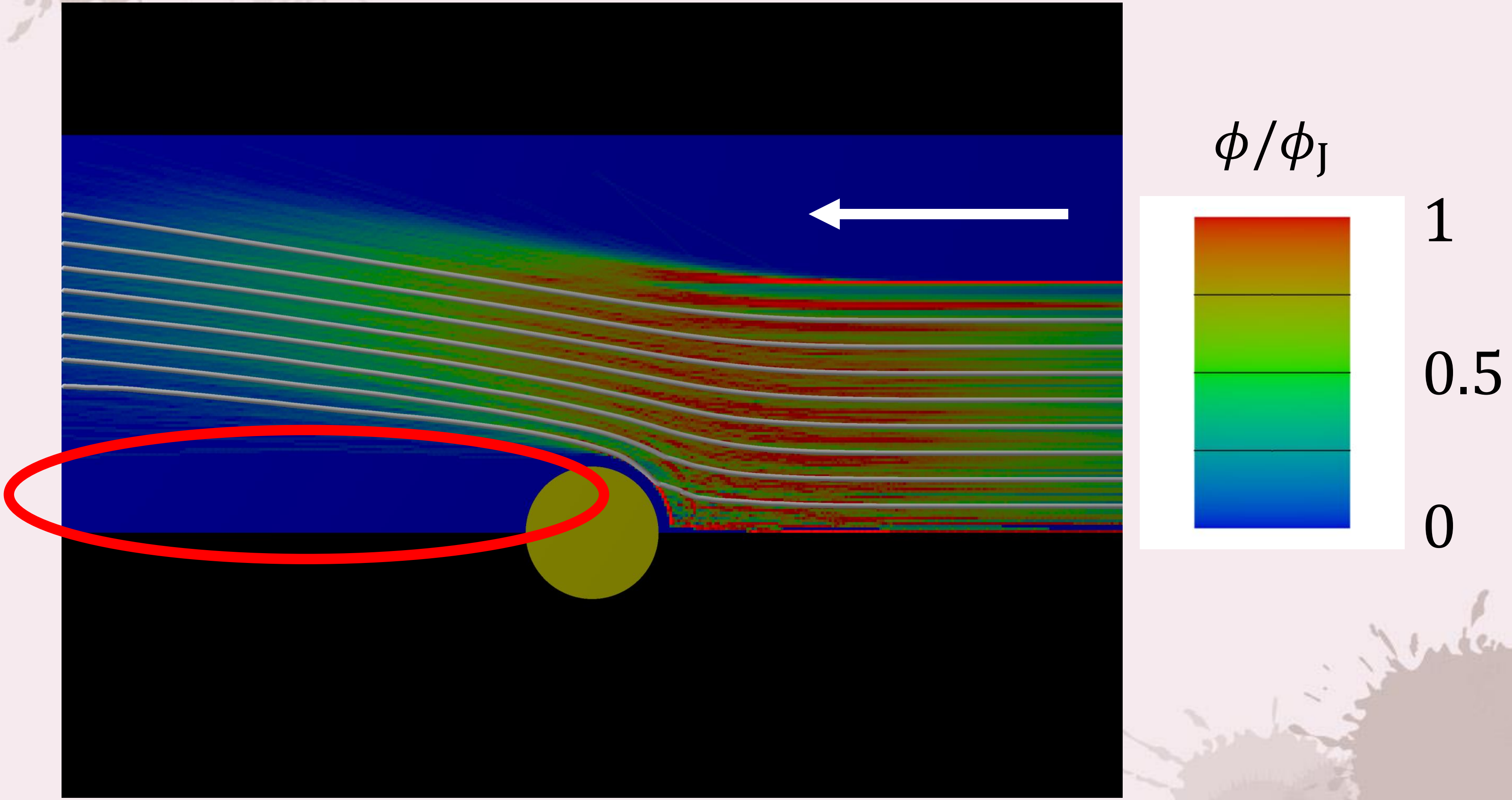
Force chain



Force chains only exist near the intruder.

Density profile

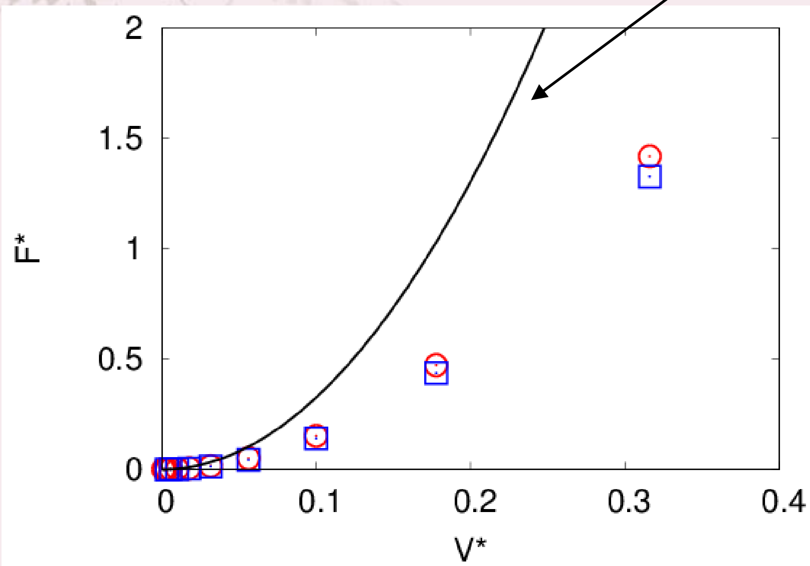
White lines… stream lines

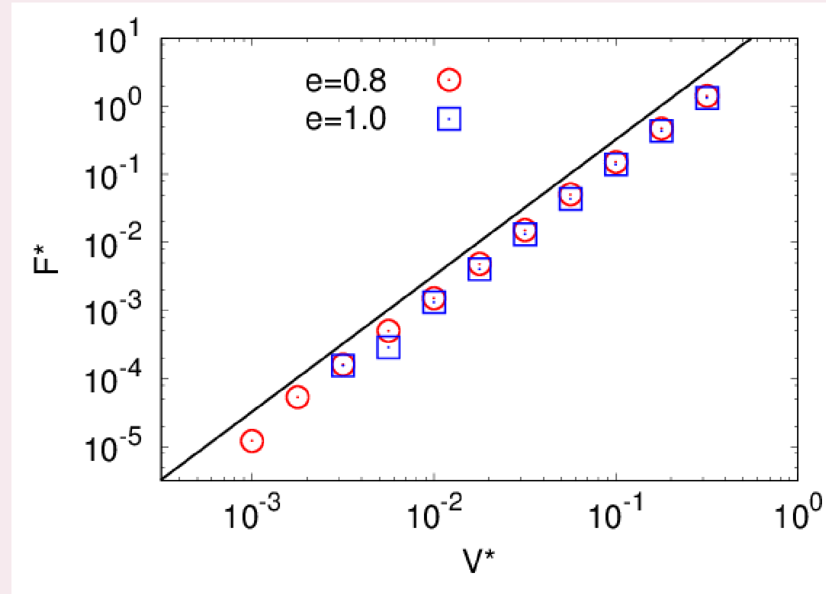


Large vacant region behind the intruder.

Drag law ($T = 0$)

$$F_{\mathrm{drag}} = \left(\frac{3+2e}{8} - \frac{9}{64}\sin^2\theta_0\right)\sin^2\theta_0 \rho D^2 V^2$$



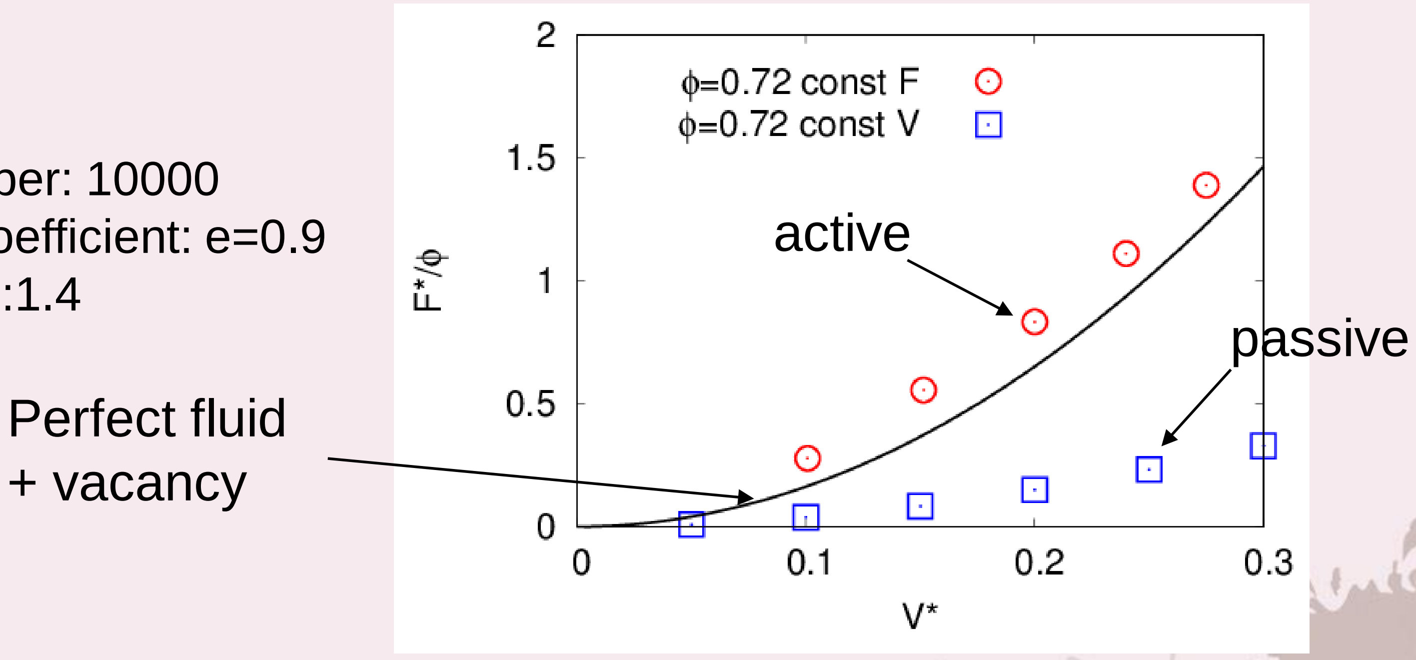


- The drag law is insensitive to the restitution coefficient.
- The drag force is proportional to V^2.
- The drag force is two times smaller than that obtained by the perfect fluid + vacancy model.

Comparison with active microrheology

Difference between active (force control) and passive (velocity control) microrheology?

Setup:
2D system
Particle number: 10000
Restitution coefficient: e=0.9
Bidisperse: 1:1.4



Tendency is opposite!
Why? We still have no idea.

Finite temperature system

We consider the case that
the particles have a finite temperature $T(> 0)$.

Introduction of dimensionless parameters

- Dimensionless drag force

$$C_D = \frac{F}{\frac{1}{2}\rho V^2 S}$$

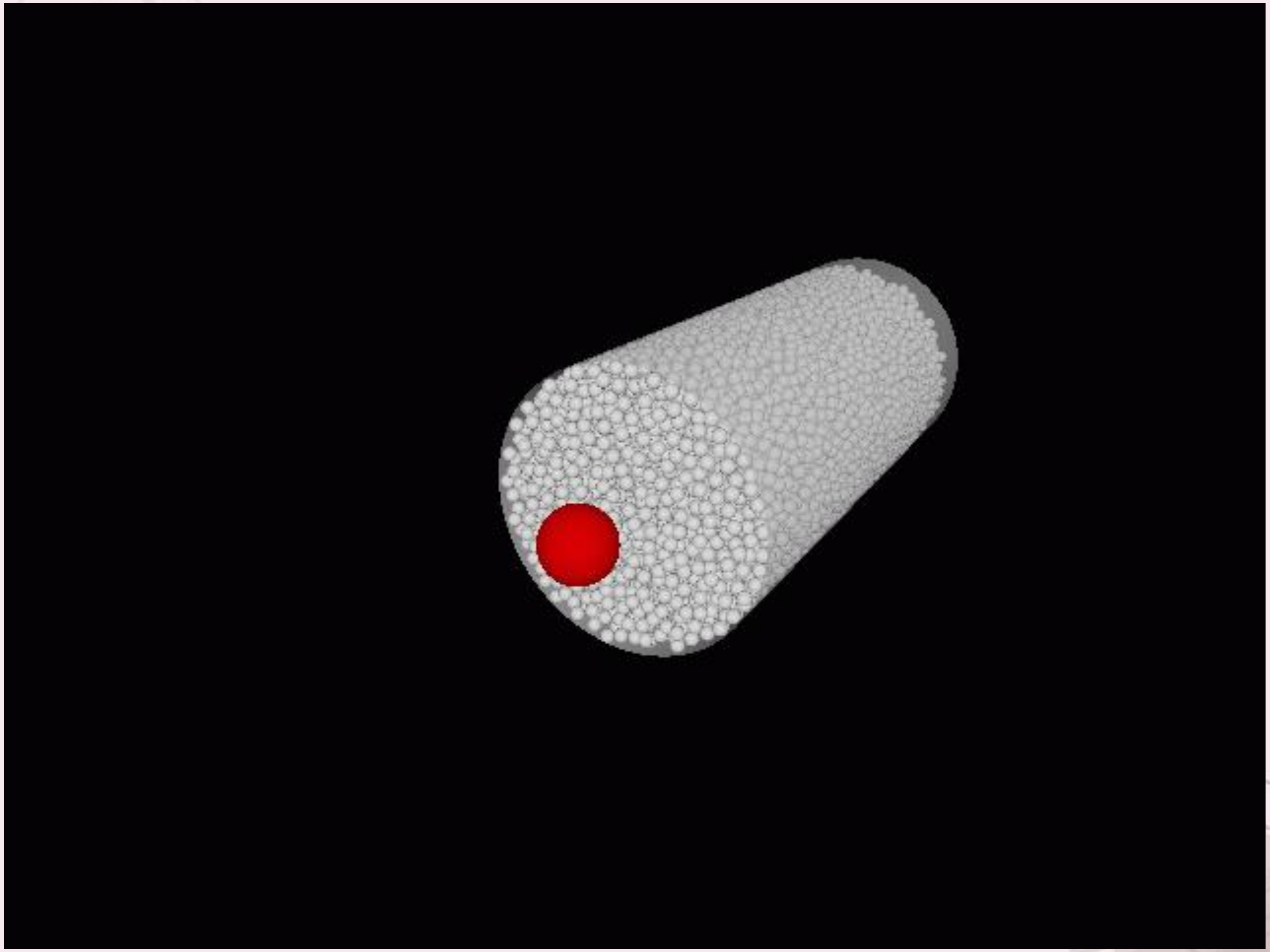
(We measure F at $V^* = 0.1$.)

- We also define the dimensionless velocity as

$$R = \frac{V}{v_T} = V\sqrt{\frac{m}{2T}}.$$

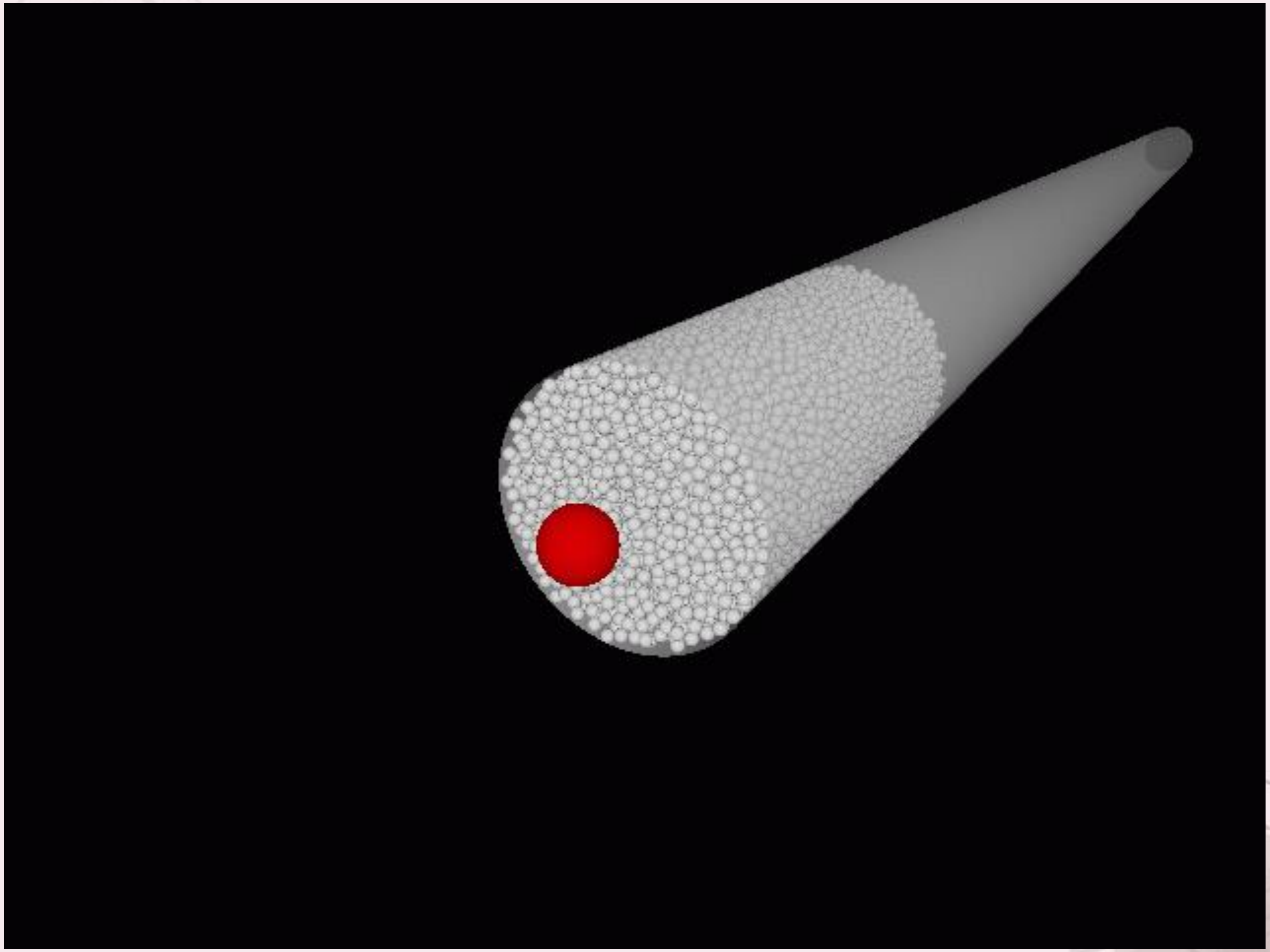
Movie ($R = 1$)

$N = 20{,}000$
$\phi = 0.4$
$e = 1$
$V^{*} = 0.1$

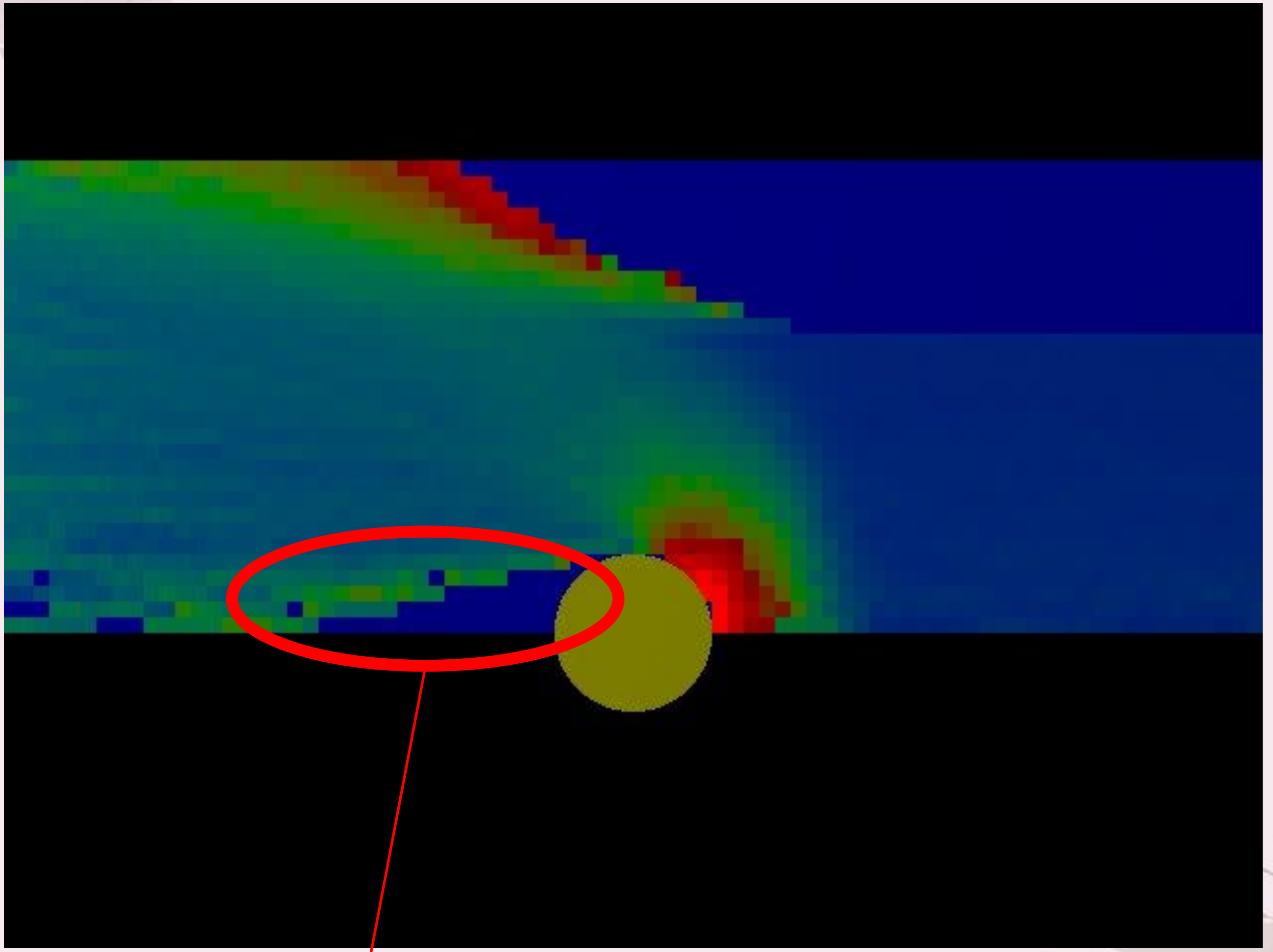


Movie ($R = 0.1$)

$N = 20{,}000$
$\phi = 0.4$
$e = 1$
$V^{*} = 0.1$

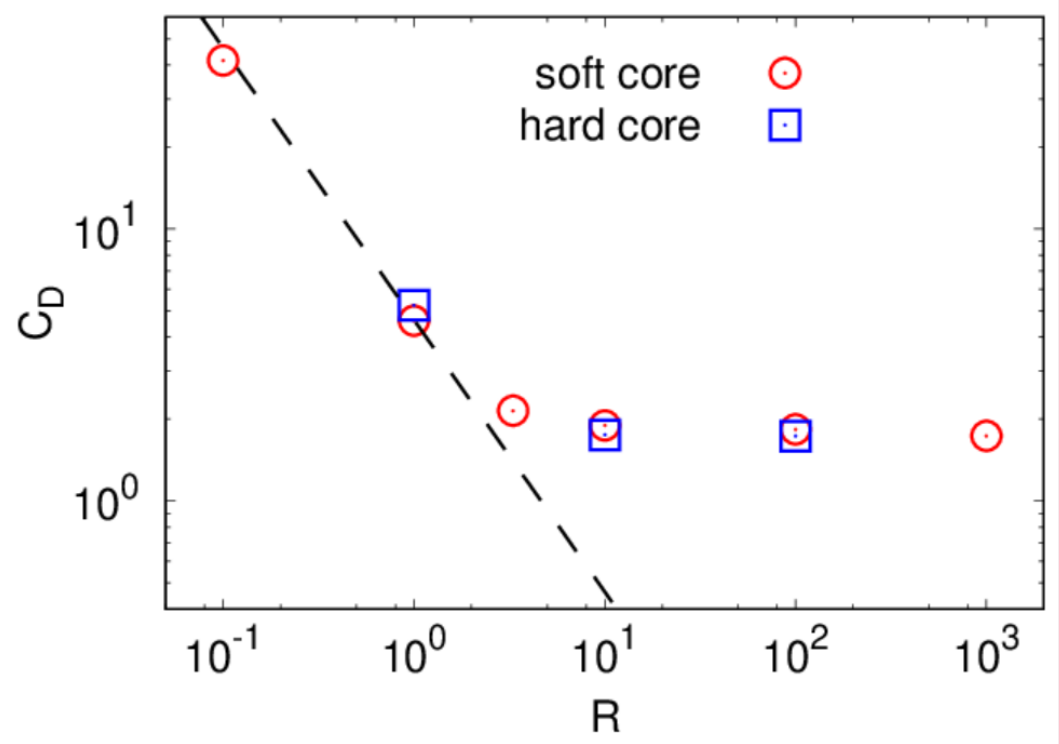


Density field ($R = 0.1$)



Vacant regime becomes smaller.
(Particles can go around the intruder.)

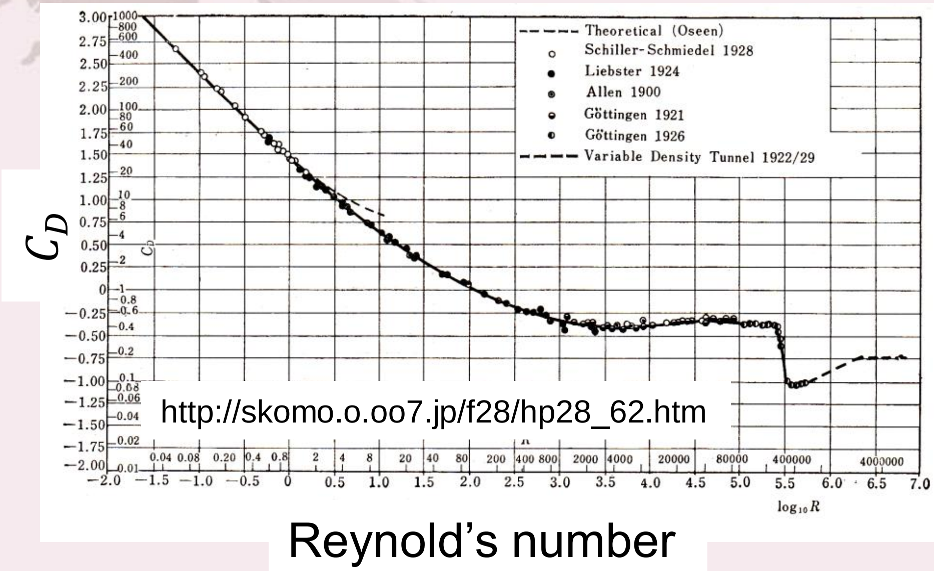
Drag law between C_D vs. R



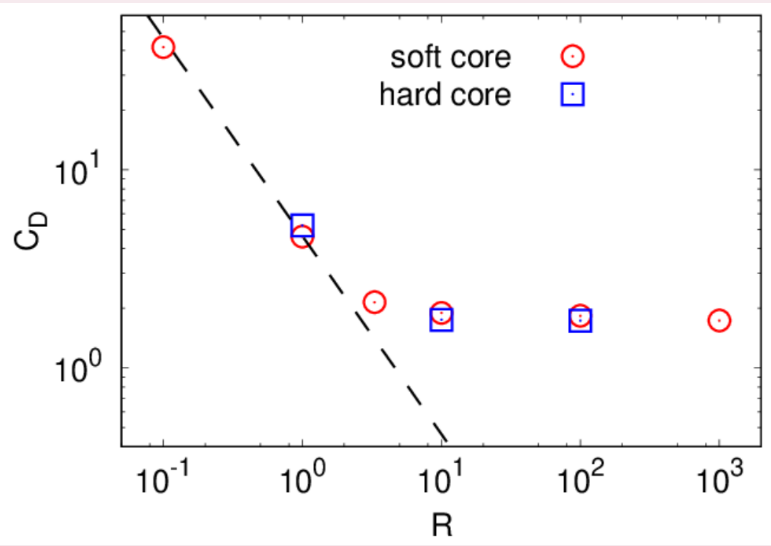
- Hard-core limit is realized for soft-core simulations.
- R dependence
  R: large ⇒ $C_D = const.$ ⇒ $F \propto V^2$ (Newtonian)
  R: small ⇒ $C_D \propto 1/V$ ⇒ $F \propto V$ (Stokes' law)
  Consistent with Stokesian drag $F = 4\pi\eta \frac{D}{2} V$ (slip surface)
  (η: viscosity for the surrounding particles ← Garzo & Dufty, PRE (1999))

Discussion

Fluid



Our system



- Transient regime between $C_D \propto \frac{1}{R}$ and $C_D \propto \mathrm{const.}$
  Fluid: ○ ⇔ Our system: ✖
- High R limit
  Fluid: turbulence (Karman vortex and separation vortex)
  ⇔ Our system: converge to constant
  What causes these?
  → roughness of the surface, thermal wall?

Summary of Part III

- We have performed simulations.
- Velocity control system.
- $T = 0$
  Drag force $\propto V^2$ (Newtonian)
- $T > 0$
  $R \gg 1$: Newtonian
  $R \ll 1$: Stokesian
  $\rightarrow$ consistent with Stokes drag $F = 4\pi\eta \frac{D}{2} V$

Future work

- Transient behavior
- Introduction of surface roughness or thermal wall