Rheology of disordered particles (Part II)
YITP, Kyoto Univ., June 27th

Interaction between two intruders in the granular media

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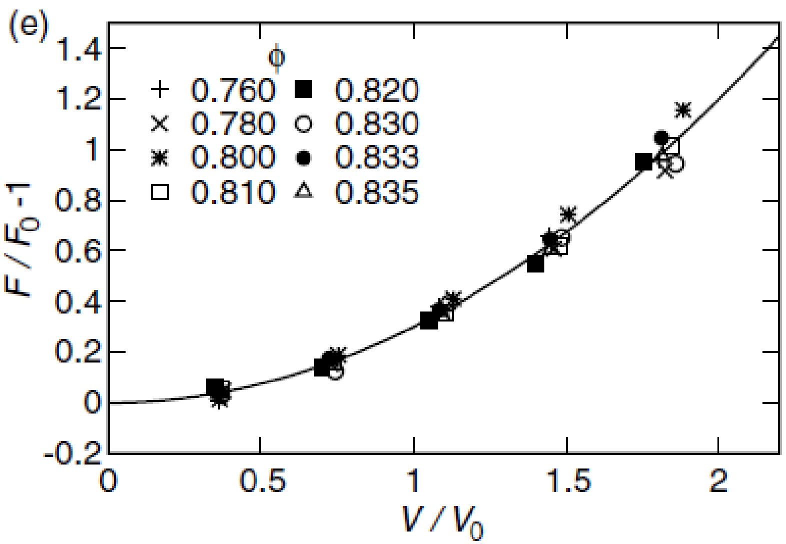
- Interaction in steady granular flow;
  effective repulsive interaction
- Interaction in oscillating granular flow;
  effective attractive interaction
- Discussion: Origin of effective interaction
- Conclusion

Drag low in a granular medium

One body drag

The drag force acting on a tracer
in a 2D single layer was measured.

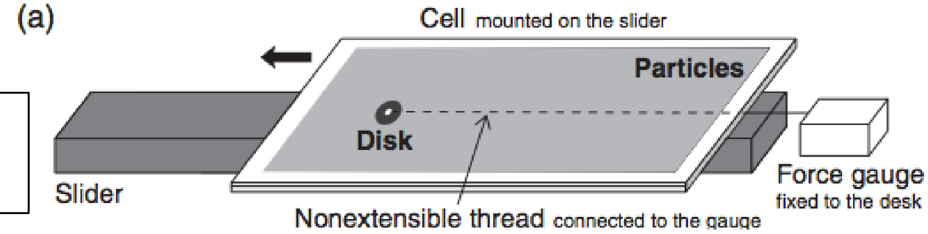
$$\text{Drag force: } F_{\text{drag}} = \alpha V^2 + F_0$$



Y. Takehara & K. Okumura.
PRL, **112**, 148001, (2014).

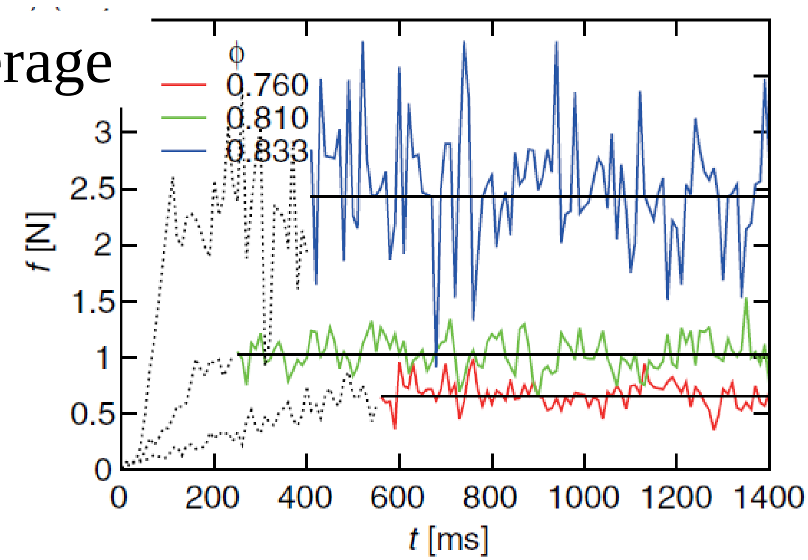
Experimental Setup

Plate moves at a constant speed.



Intruder is fixed.

Time average



Drag low in a granular medium

One body drag

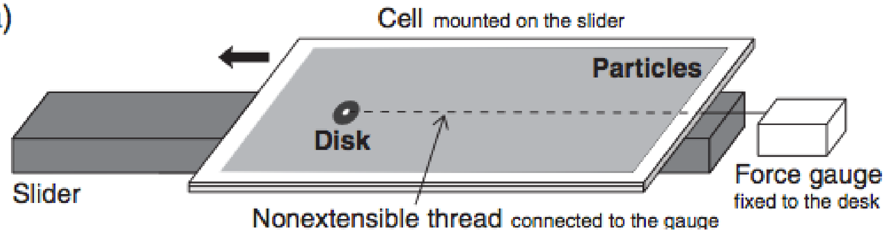
The drag force acting on a tracer in a 2D single layer was measured.

Drag force: $F_{\text{drag}} = \alpha V^2 + F_0$

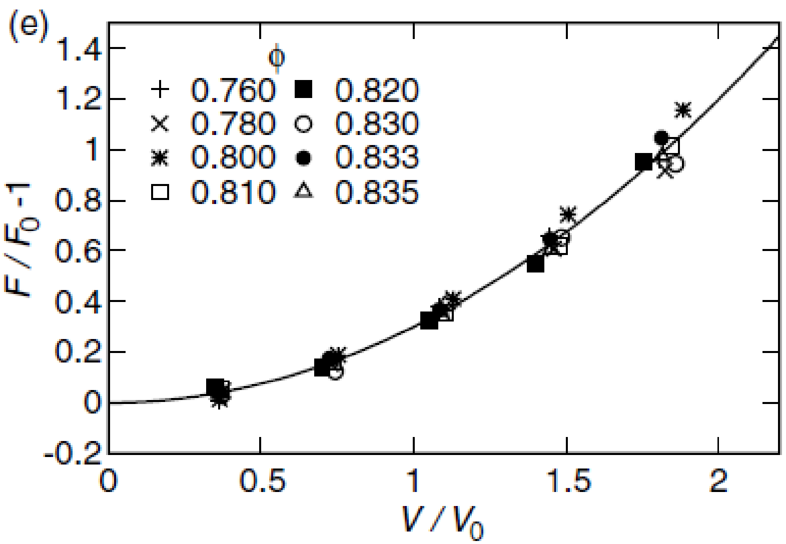
Yield force F_0 was appeared with considering dry friction of bottom plate.

Experimental Setup

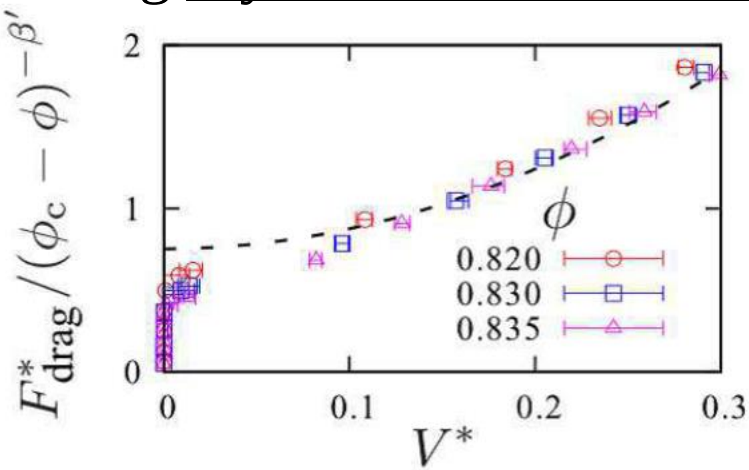
(a)



(e)



Y. Takehara & K. Okumura. PRL, **112**, 148001, (2014).

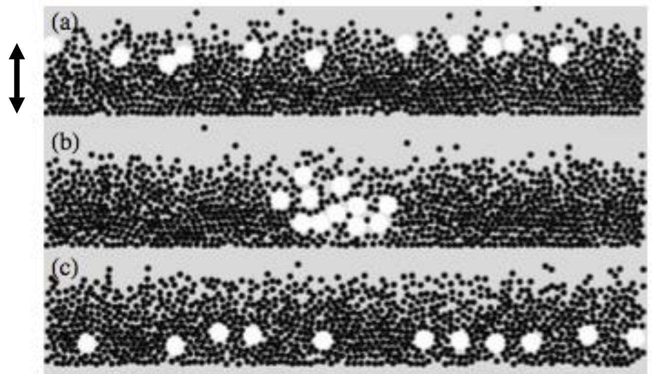


S. Takada & H. Hayakawa. J. Eng. Mech. **143** (2017).

Interaction of intruders in a granular medium

Brazil Nuts Effect (ED)

D. A. Sanders, et al, PRL, **93**, 20, (2004)



$\rho_L/\rho_S < 1$

$\rho_L/\rho_S = 1$

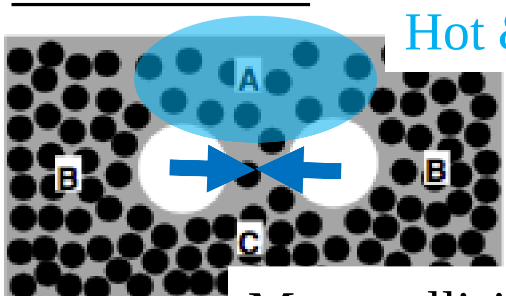
$\rho_L/\rho_S > 1$

$\boldsymbol{\rho_L/\rho_S \approx 1.0} \Longrightarrow$ **Attraction**

ρ_L, ρ_S: Large/Small granular density

Mechanism

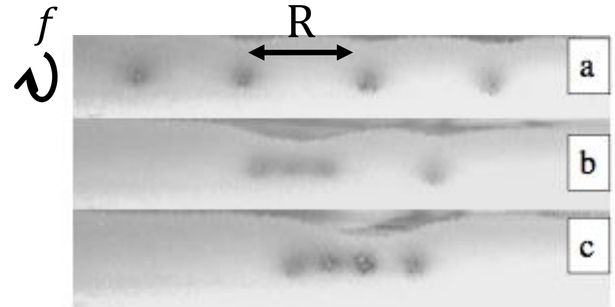
Hot & dilute region



More collisions at region B than region A cause attraction (going to lower density)

Rolling Cylinder (Exp.)

I. Zuriguel, et al, PRL, **95**, 258002, (2005)



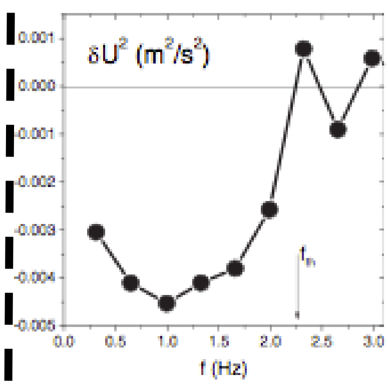
(a) $f > f_{th}$ $\Longrightarrow$ Separation

(b),(c) $f < f_{th}$ $\Longrightarrow$ Attraction

$\boldsymbol{f < f_{th}} \Longrightarrow$ **Attraction**

$\boldsymbol{f > f_{th}} \Longrightarrow$ **Separation**

Mechanism



$f < f_{th} \Longrightarrow \delta U^2 < 0$

(large fluctuation outside)

$\Longrightarrow \delta P = \rho\, \delta U^2/2 < 0$

Pressure difference makes attraction.

$$\delta U^2 = -[U_{rms}(out)^2 + V_{rms}(out)^2 - (U_{rms}(in)^2 + V_{rms}(in)^2)]$$

Interaction of intruders in a granular medium

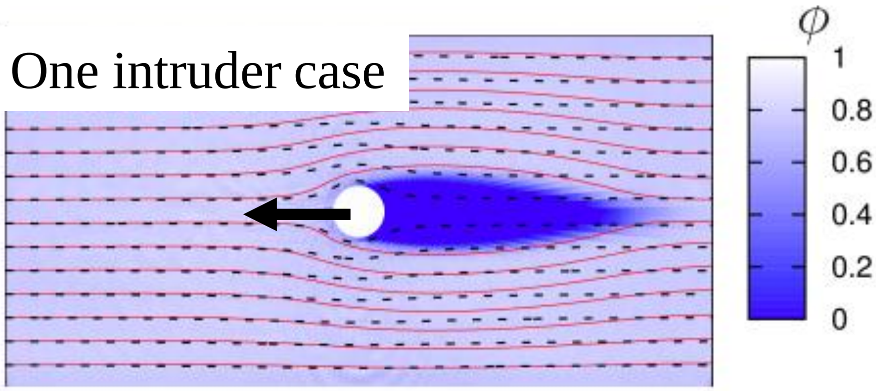
Question

How do the two intruders interact each other under steady / oscillatory granular flow ?

Which interaction are observed in the case of two intruders?

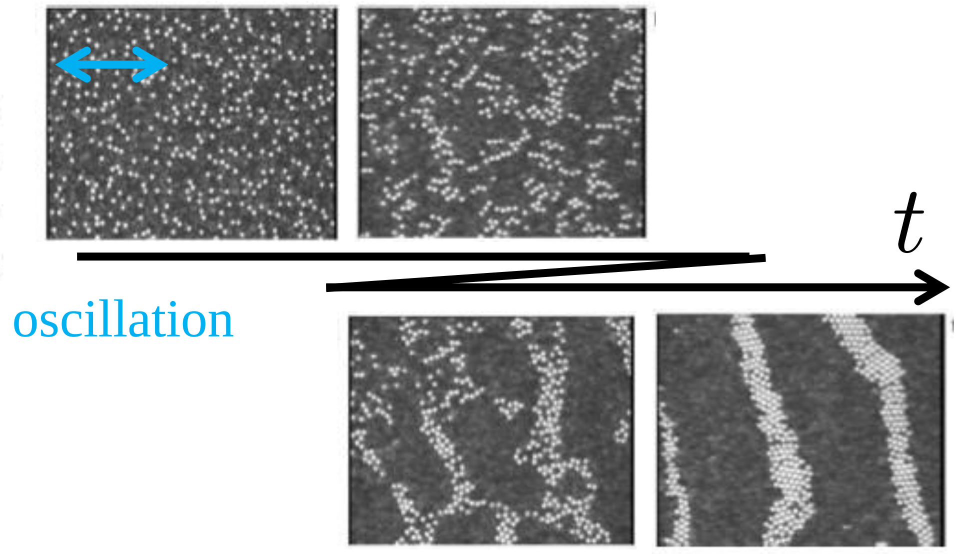
What cause the attractive interaction between intruders?

Density profile & streamline of grains



S. Takada & H. Hayakawa.
J. Eng. Mech. **143**, (2017).

Granular segregation



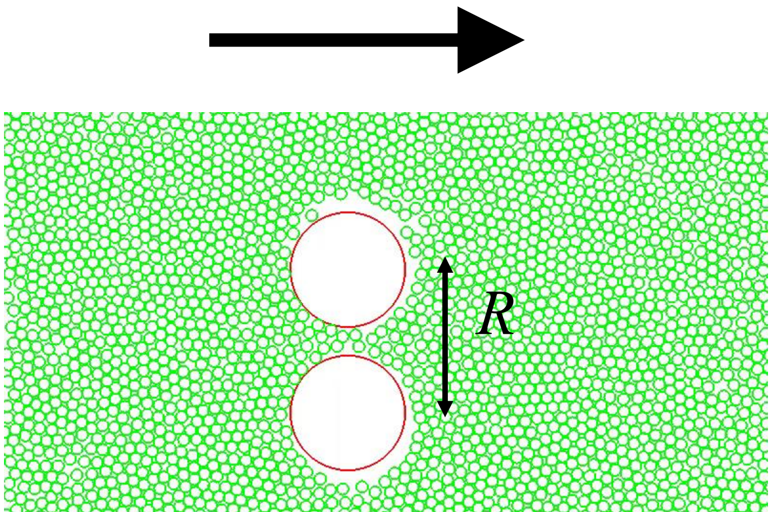
P. M. Reis, et al.,Phys. Rev. E **74**, 051306 (2006)

Model (DEM)

$$m_i \ddot{r}_i = \sum_j F_c^{i,j} + F_{\mathrm{ex}}$$

i	: Disk index
$\boldsymbol{r}_i$: Position	m_i : Mass

$$V_{\mathrm{ex}}^x = V_{\mathrm{ex}}\ (d\sqrt{k_n/m})$$



Intruders are fixed on system

1. Repulsive force (only normal force)

$$F_c^{i,j} = k_n \delta^{i,j} n^{i,j} - \gamma_n \boldsymbol{v}_n^{i,j}$$

2. Driving force

$$F_{\mathrm{ex}} = -\mu(\dot{\boldsymbol{r}}_i - \boldsymbol{V}_{\mathrm{ex}})$$

- Diameter of disks (common density)

$d_i = d(1 \pm 0.1r)$: Small disks

(r : Uniform distr.)

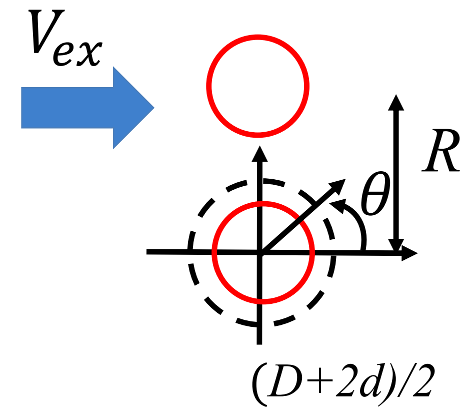
$D = 10d$: Intruder

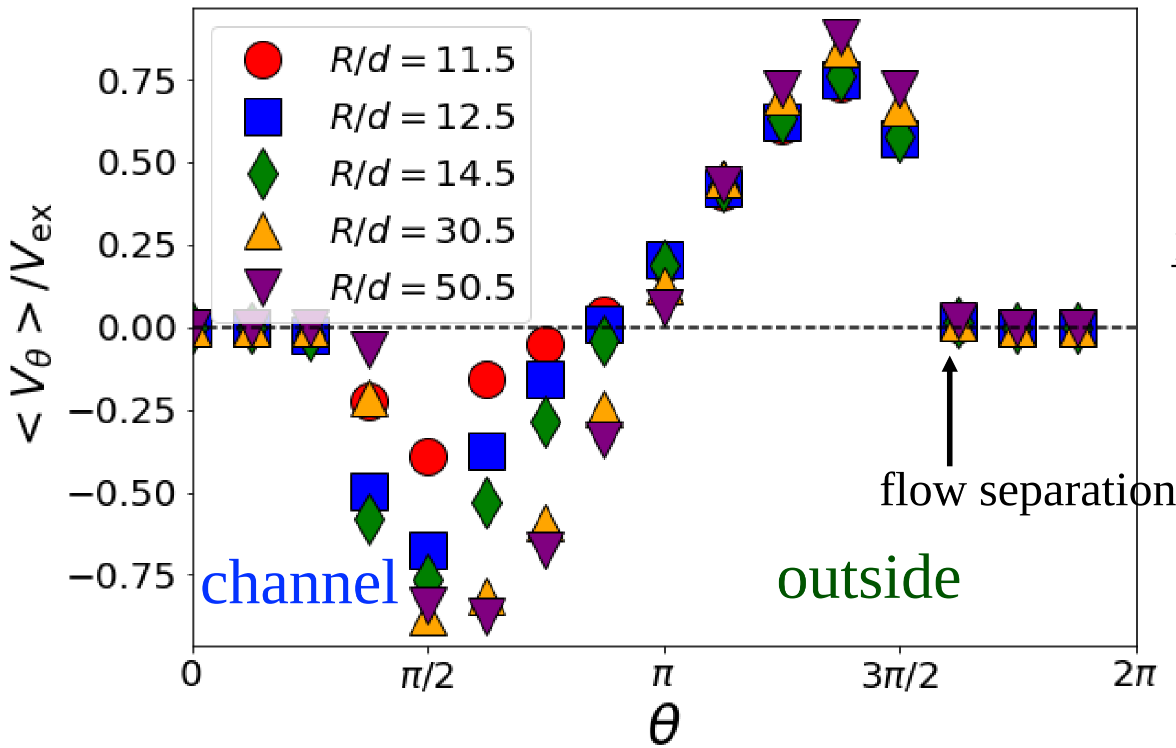
Boundary condition

Fixed flat wall $L_y = 150d$

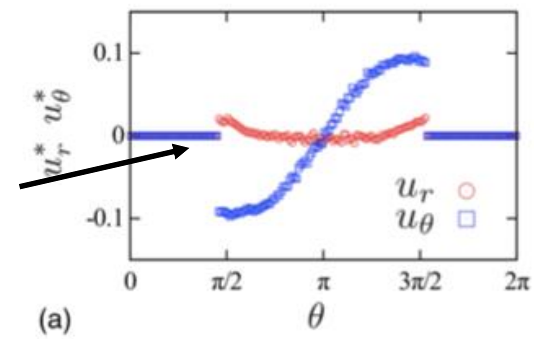
Periodic boundary $L_x = 200d$

Velocity field around intruder





1-intruder case



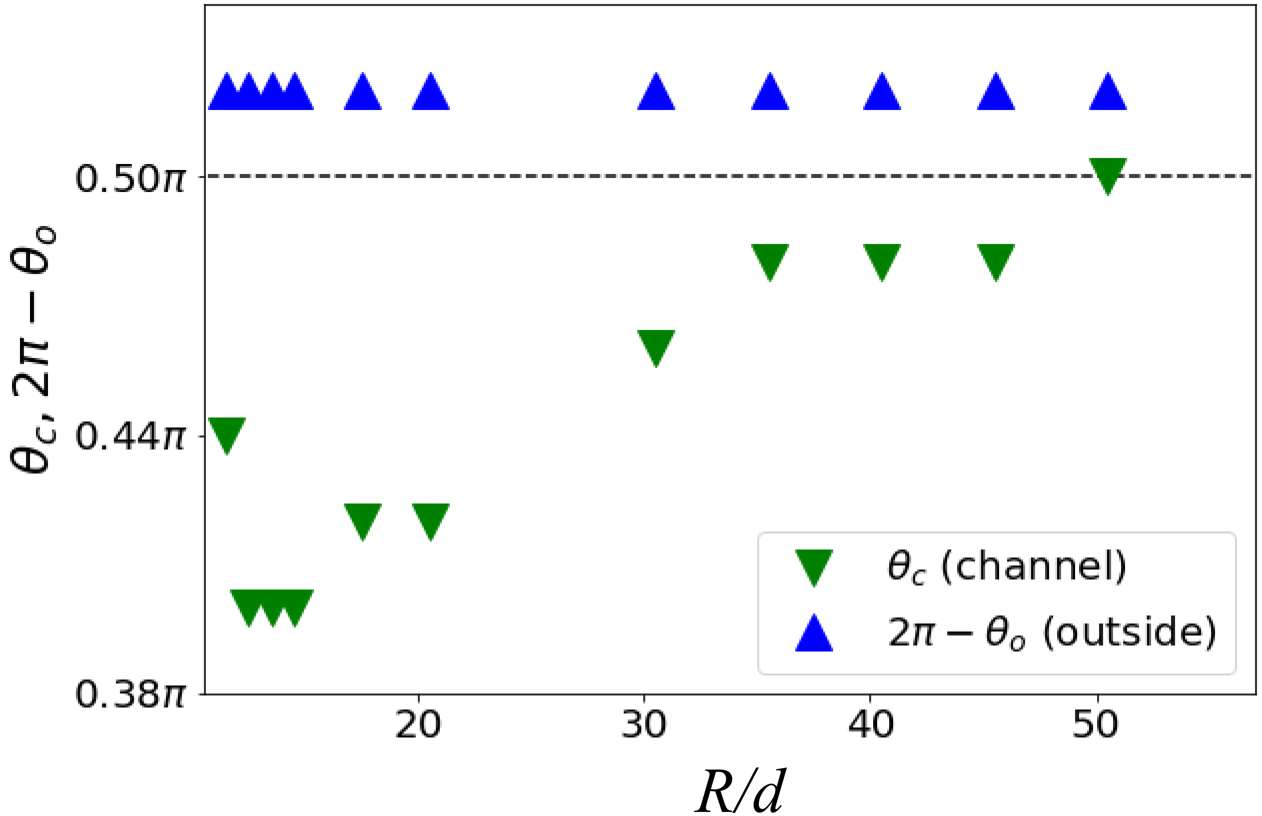
J. Eng. Mech. **143**, (2017).

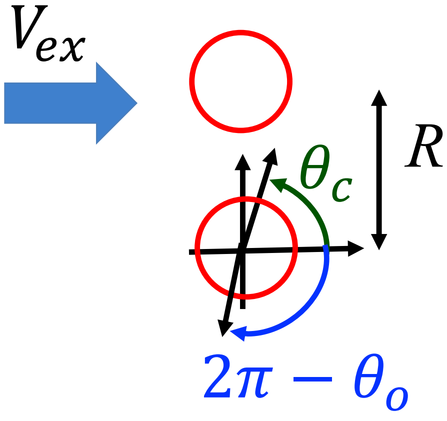
Velocity profiles of grains within $(D+2d)/2$ seem similar at outside.

Flow stagnates at narrow channel and the profile shows asymmetric form.

Little dependency on flow speed V_{ex}.

Separation angle of granular flow





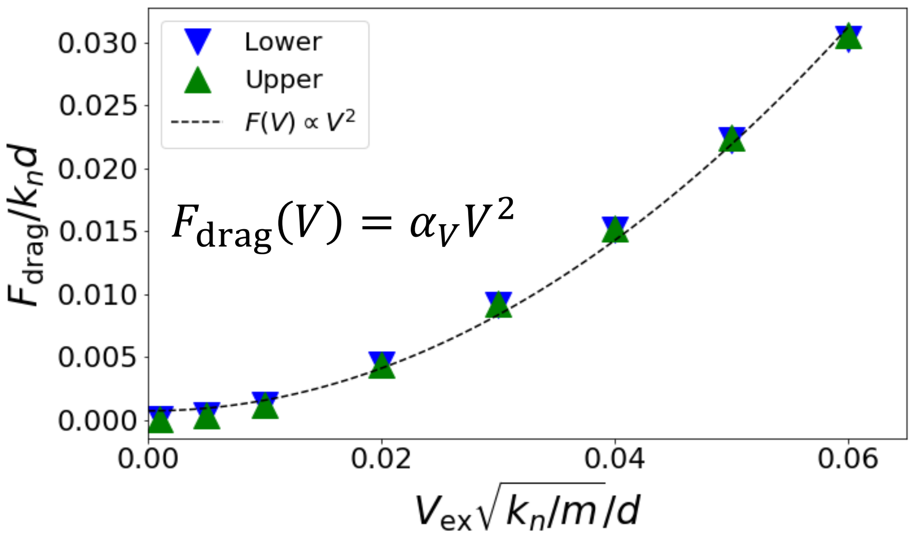
Granular flow peel off in same point at outside.

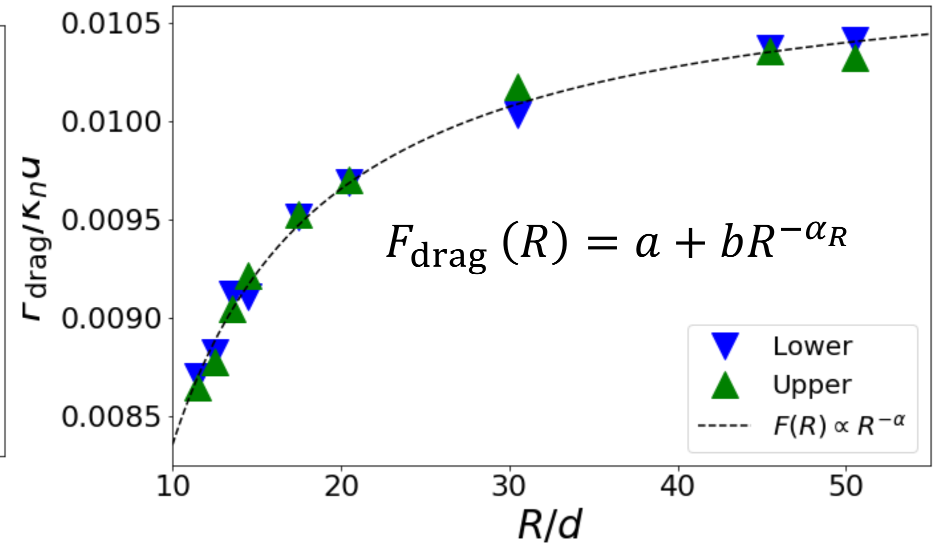
Flow separation is suppressed in narrow channel.

Separation angle become to symmetrical with increasing *R*.

Drag force F_{drag} for V_{ex} & R

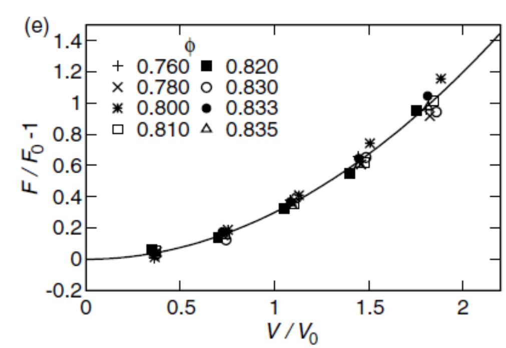
$$F_{\mathrm{drag}} = < \sum_j F_{c,x}^{\mathrm{intruder},j} >$$





$F_{\mathrm{drag}}(V)$ can be expressed with V^2 with no yield term.

<u>1-body drag exp.</u>



PRL, **112**, 148001, (2014)

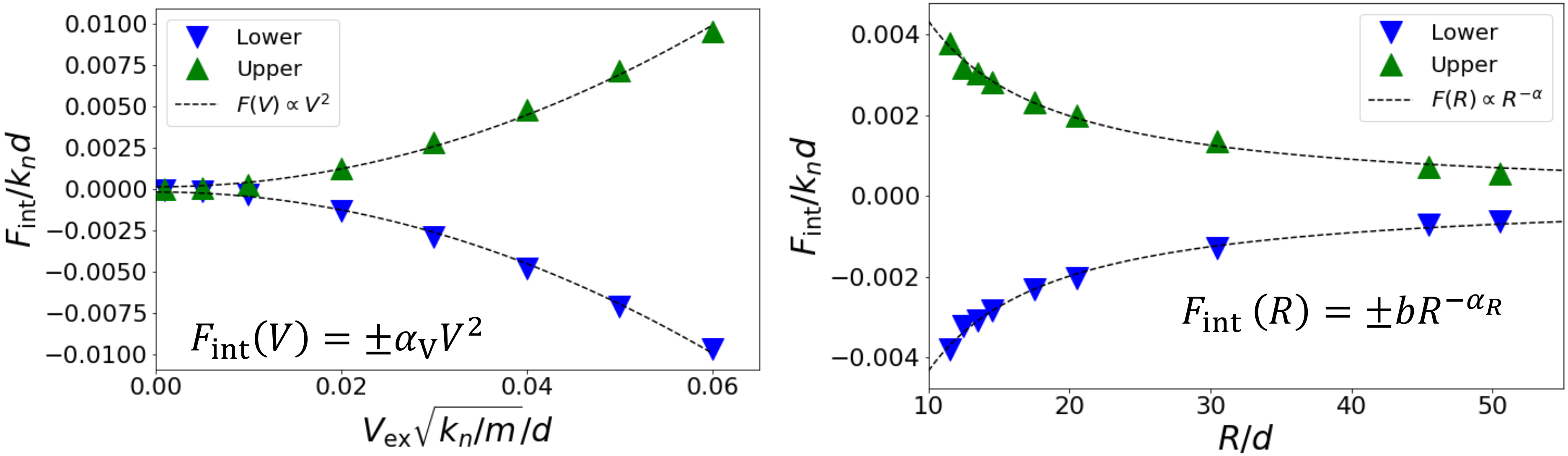
$F_{\mathrm{drag}}(R)$ can be expressed as $F(R) \propto R^{-\alpha_R}$.

$(\alpha_R \approx 1)$

$F_{\mathrm{drag}}(R)$ is suppressed when R is small because the flow speed decreases.

Interaction F_{int} for V_{ex} & R

$$F_{\text{int}} = \langle \sum_j F_{c,y}^{\text{intruder},j} \rangle$$

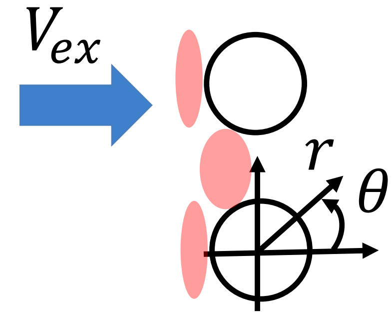


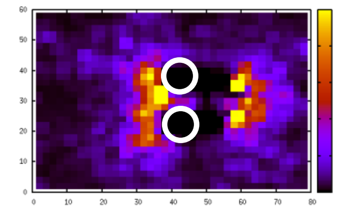
$F_{\text{int}}(V)$ can be expressed with V^2 & effective interaction is **repulsive**.

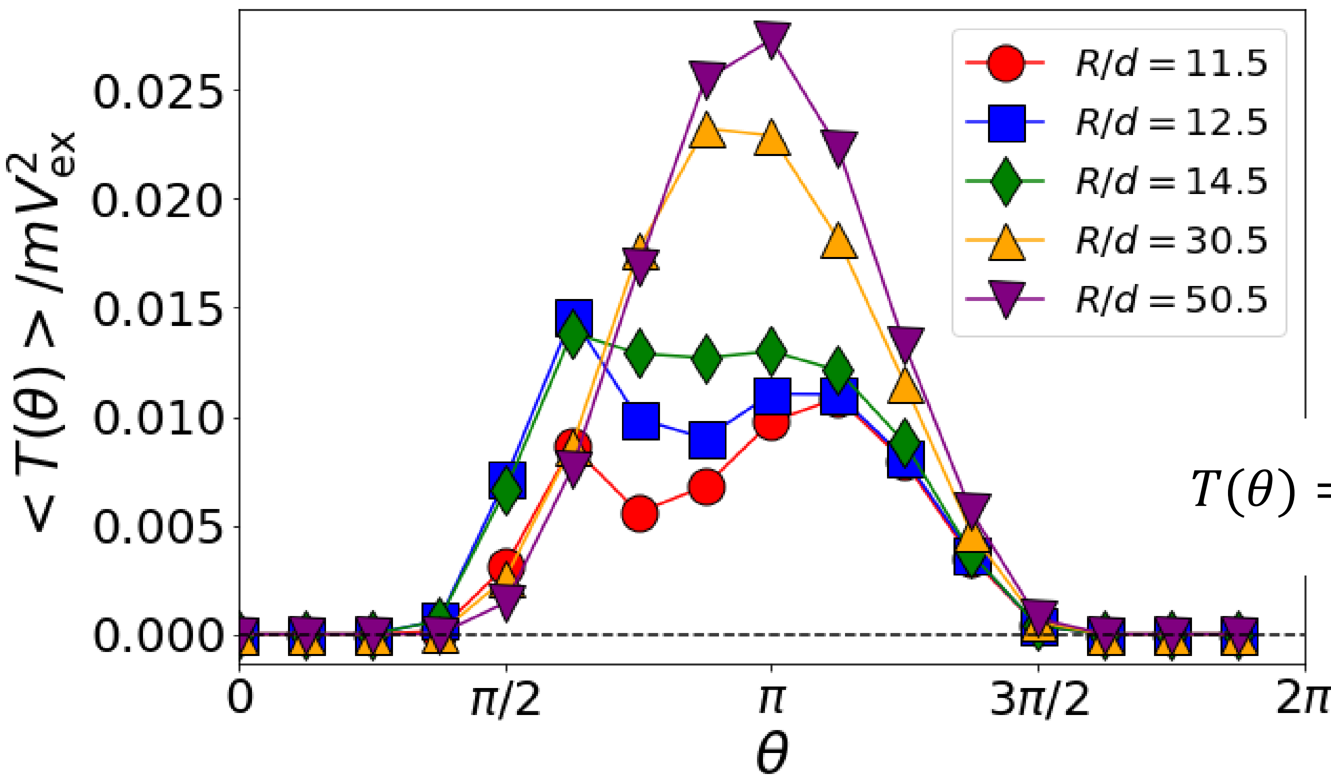
=> contrast to perfect fluidity (Bernoulli's principle)

$F_{\text{int}}(R)$ can be expressed as $F(R) \propto \boldsymbol{R^{-\alpha_R}}$ $(\alpha_R \approx 1)$
& asymptotically approaches 0 as increasing R.

Origin of repulsive interaction







$$T(\theta) = \sum_i m_i(\overline{\boldsymbol{v}_A} - \boldsymbol{v}_i)^2/N_A$$

When R is small, there are two peaks in the profile of granular temperature at the channel & front face ($\theta \approx \pi/2, \pi$).

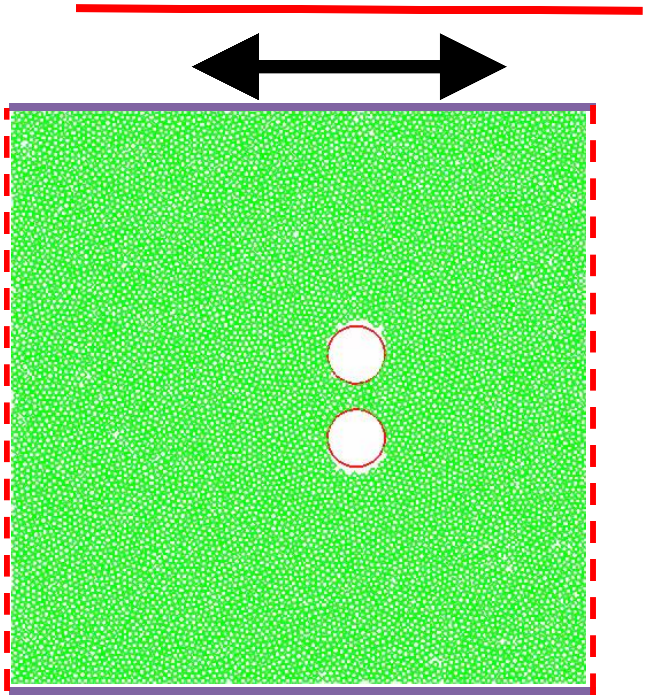
Increasing R, these two peaks merge into the peak at $\theta = \pi$ and symmetric profile, then the interaction F_{int} becomes weak.

Model (DEM)

$$m_i \ddot{r}_i = \sum_j F_c^{i,j} + F_{\text{ex}}$$

i : Disk index	
$\boldsymbol{r}_i$: Position	m_i : Mass

$$V_{\text{ex}}^{x}(t) = A\nu \sin(\nu t)$$



Boundary condition

Fixed flat wall $L_y = 100d$

Periodic boundary $L_x = 100d$

1. Repulsive force (only normal force)

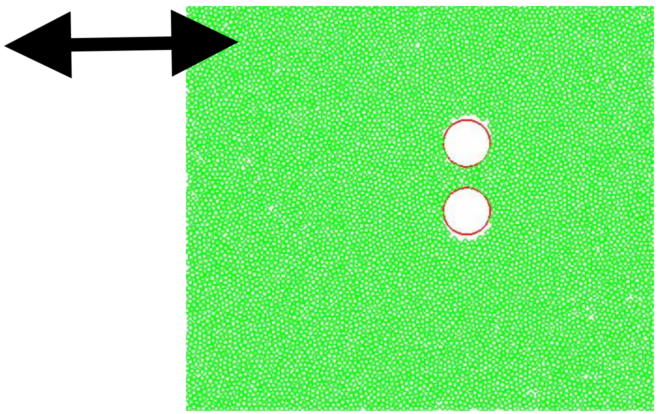
$$F_c^{i,j} = k_n \delta^{i,j} n^{i,j} - \gamma_n \boldsymbol{v}_n^{i,j}$$

2. Driving force $F_{\text{ex}} = -\mu(\dot{\boldsymbol{r}}_i - \boldsymbol{V}_{\text{ex}})$

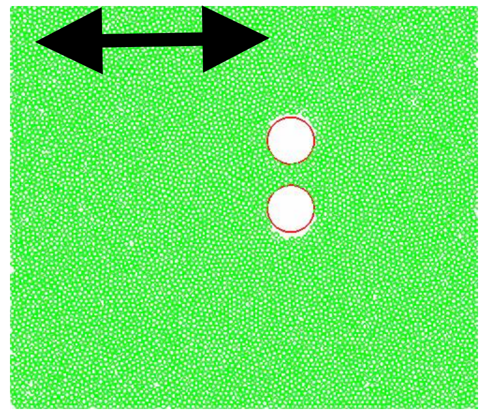
- All disks including intruders can move
- Parameters: (A, ν)
  Amplitude & Frequency
  Initial distance $R_0 = 14.5d$ is fixed

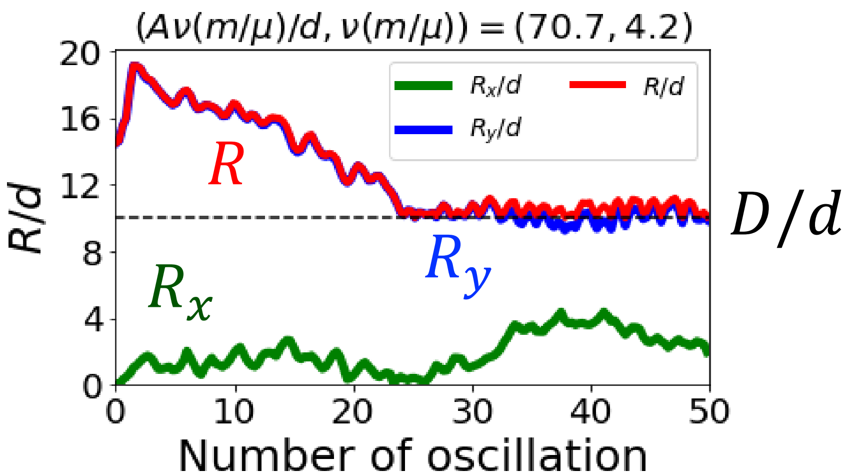
Oscillational flow: intruders dynamics

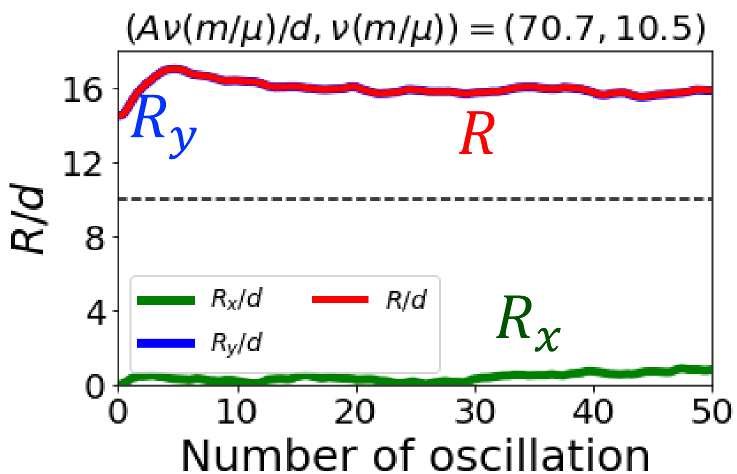
Small ν: attractive



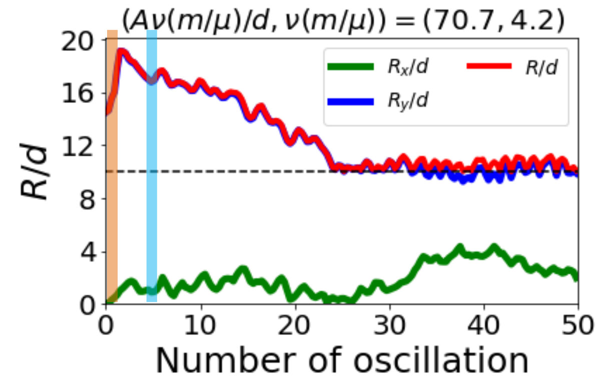
Large ν : non-attractive



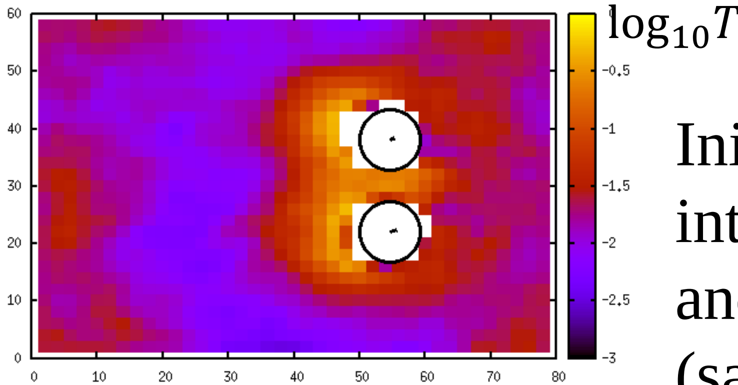




Granular temperature at repulsive & attractive period

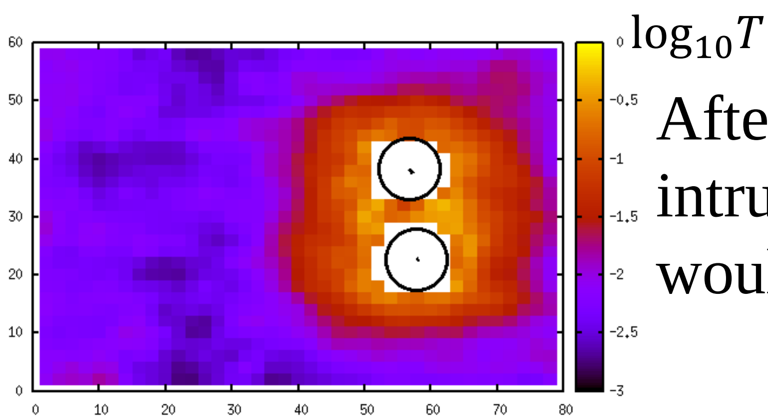


$1/\nu < t < 2/\nu$: Repulsive period



Initially, left side region and region between intruders become high temperature, and they would cause initial separation (same reason with steady flow).

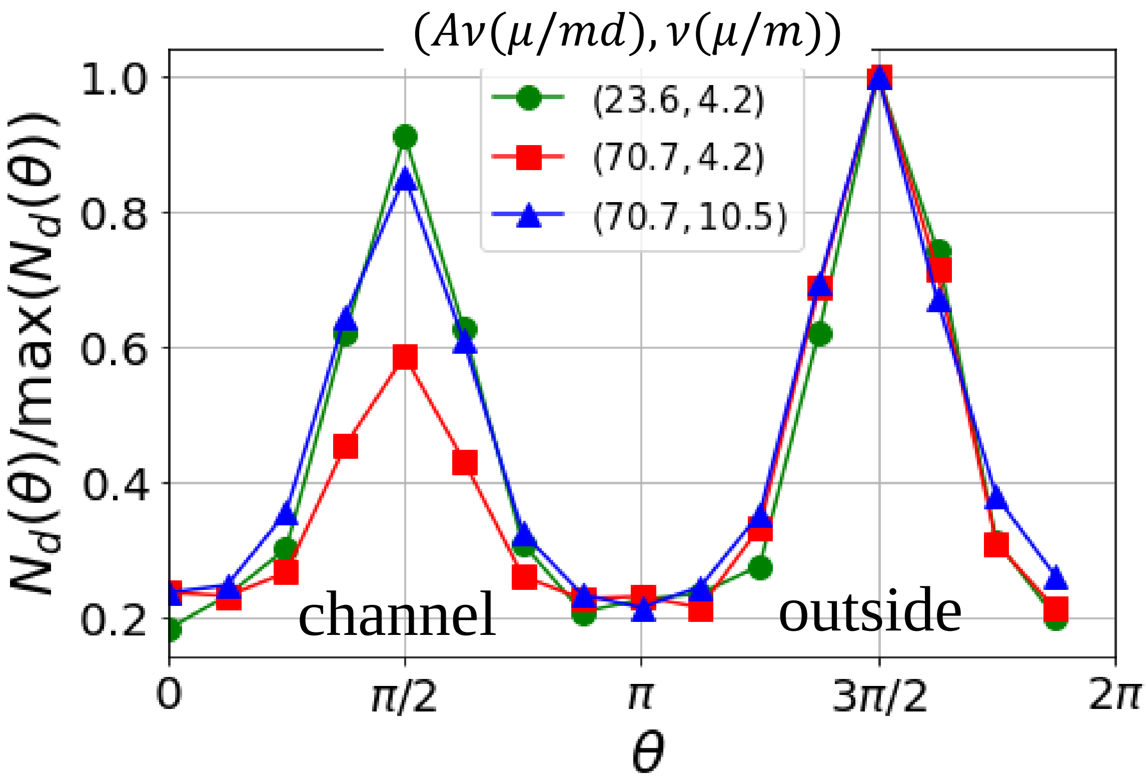
$5/\nu < t < 6/\nu$: Attractive period

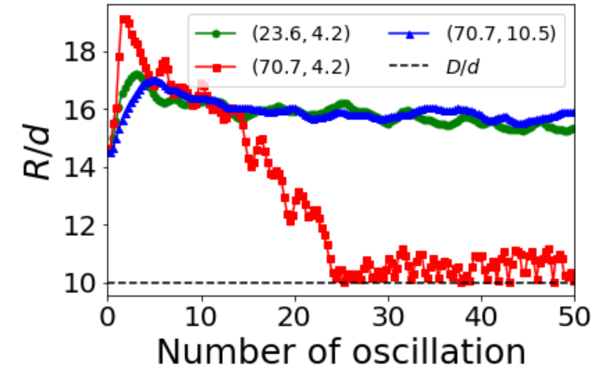


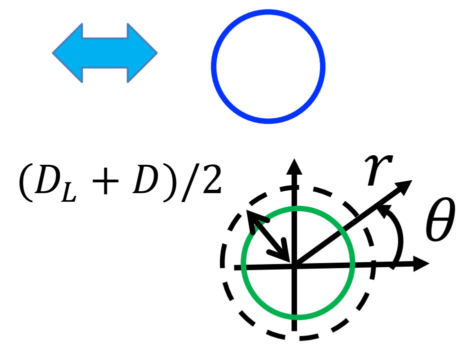
After several oscillations, the region around intruders become high temperature and this would prevent the intruders from separation.

Summation of granular temperature during an oscillation at each point

Number of disks around intruders





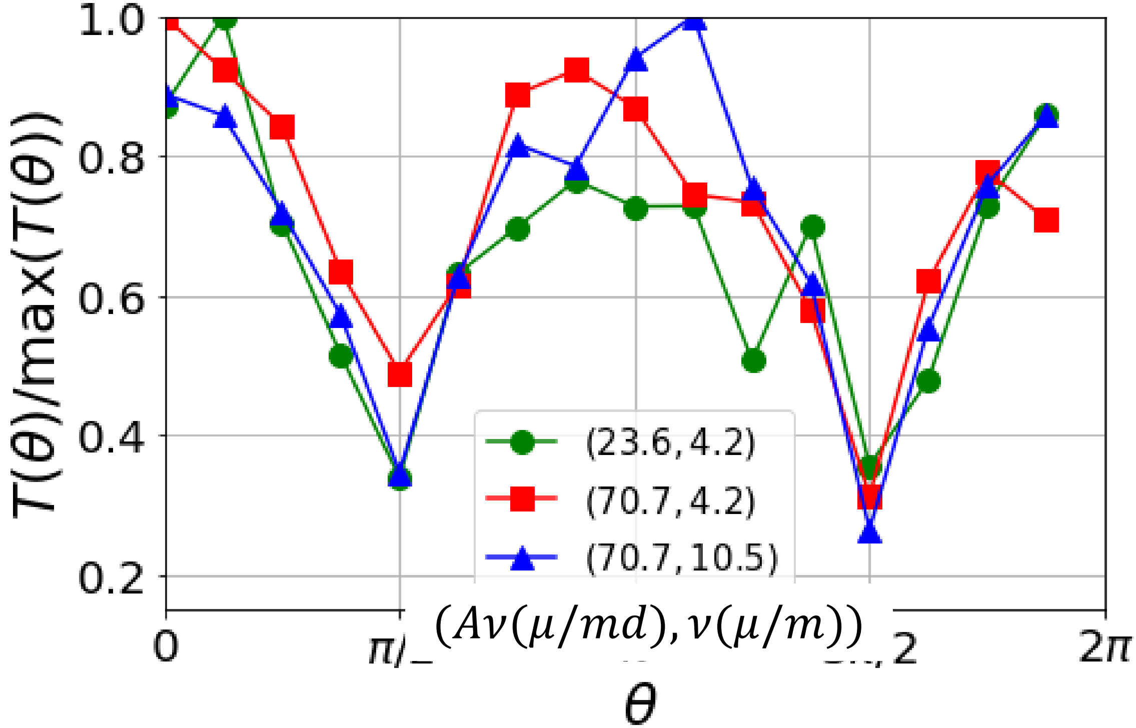


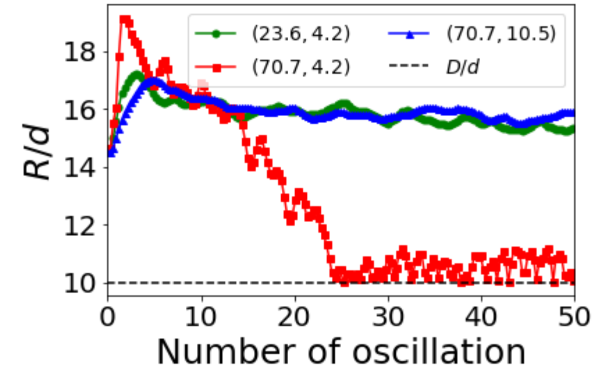
NA) There are little differences between channel and outside.

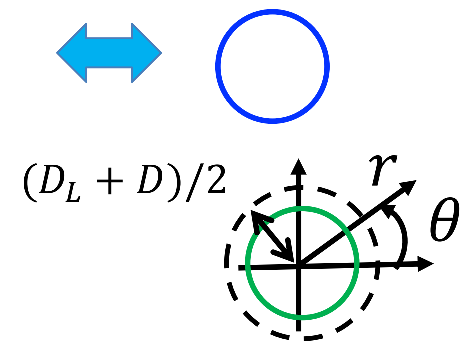
A) **Channel region has less number of disks** than outside. Moreover, the number of disks at the channel is small.

NA): abbreviation of non-attractive cases and A): abbreviation of attractive cases.

Granular temperature around intruders





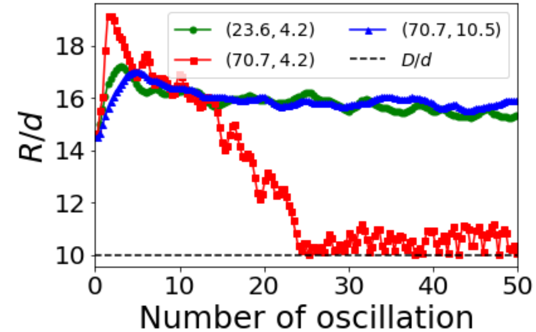


A) **Channel region has higher temperature** than that of outer region.

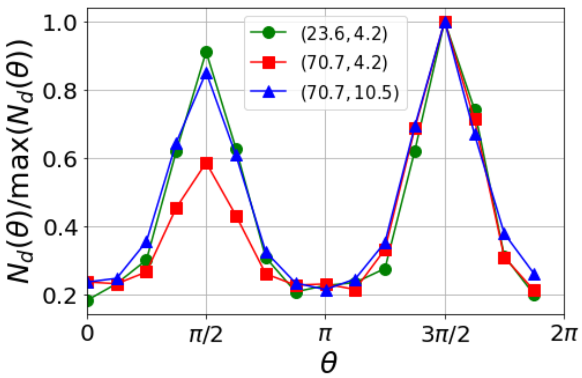
The granular temperature for (NA) is smaller than that of (A) for all θ.

x-direction ($\theta = 0, \pi, 2\pi$) shows higher temperature than the others.

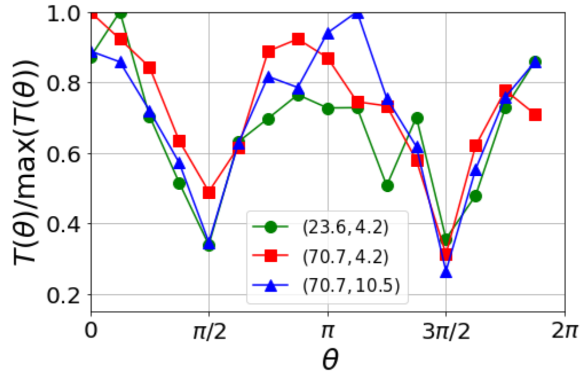
Origin of attractive interaction



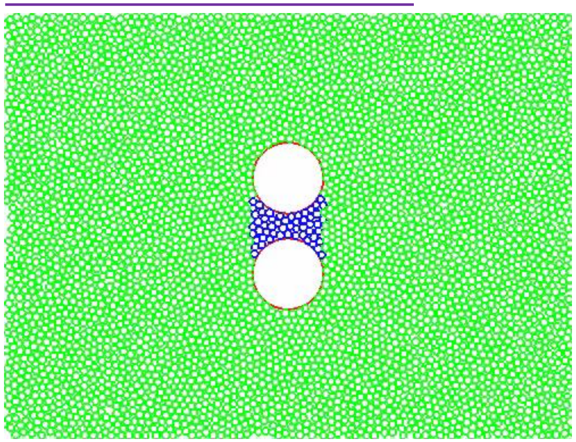
Number of disks



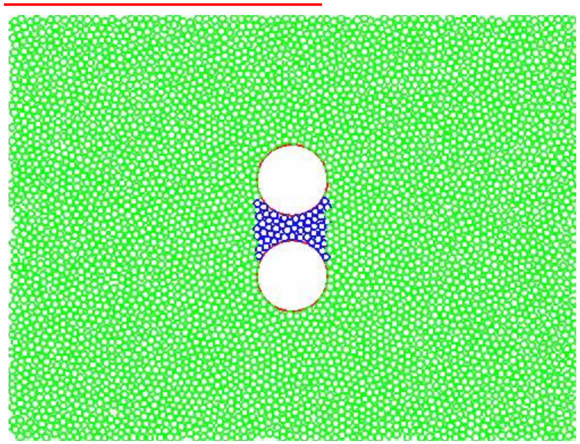
Granular temperature



Non attractive case



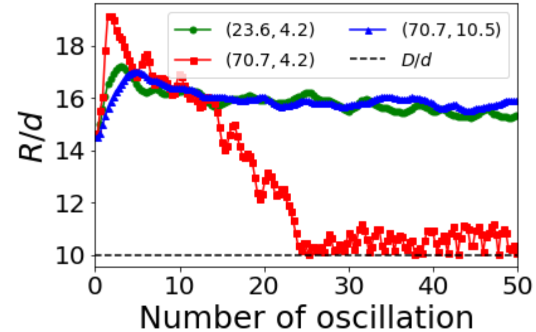
Attractive case



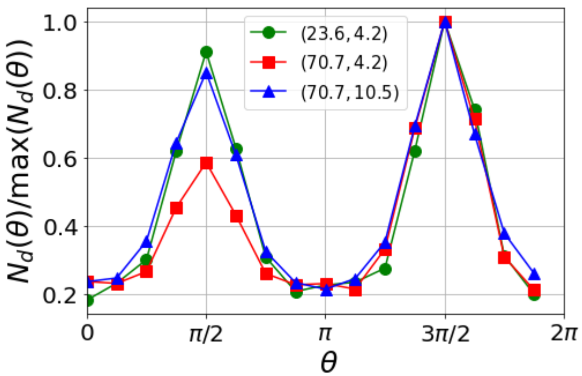
If the **frequency is enough small and amplitude is large**, small disks between intruders move out and the other disks are rarely able to invade to channel area because of higher temperature there.

Initially, the disks locate between two intruders are colored with blue.

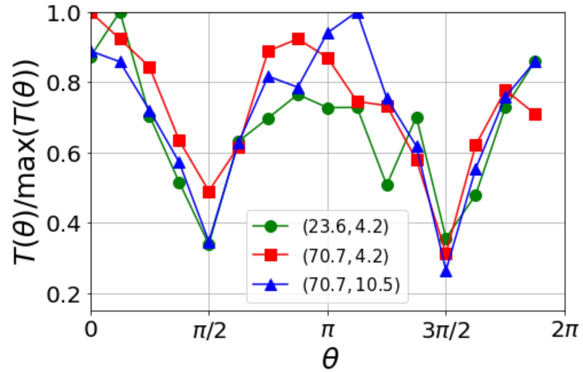
Origin of attractive interaction



Number of disks

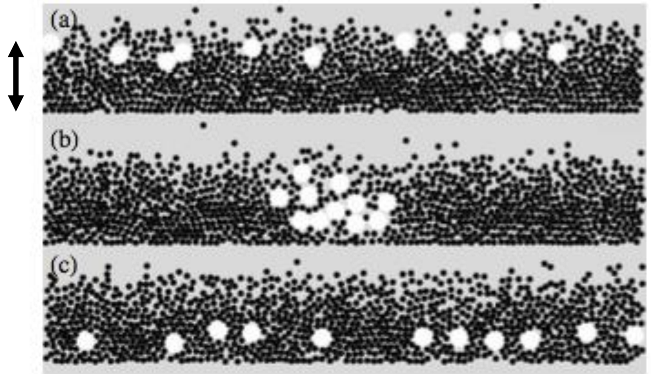


Granular temperature



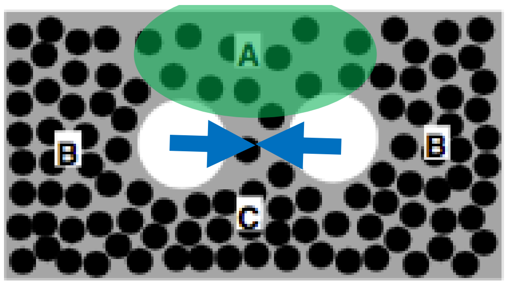
Mechanism of attractive interaction is similar to that of vertical oscillation. Dilute region of granular medium between intruders introduce attractive interaction in both case.

Brazil Nuts Effect (ED)



D. A. Sanders, et al, PRL, **93**, 20, (2004)

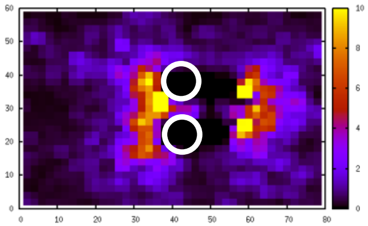
Lower density region



More collisions at region B than region A cause attraction (going to lower density)

We numerically obtained the effective interaction within two kinds of granular flow

Interaction in steady granular flow



- High granular temperature at channel region makes **repulsive interaction** of intruders
- Drag force can be expressed similar as one-body problem ($\propto V^2$)

Interaction in oscillatory granular flow

- **Dilute hot region** at channel makes **attractive interaction**
- The granular temperature of parallel direction to the flow is agitated and this prevents the intruders from separation