YITP Workshop :

Jet & Shock breakouts

"Jet energy distribution inferred from late-time light curves"

(very preliminary)

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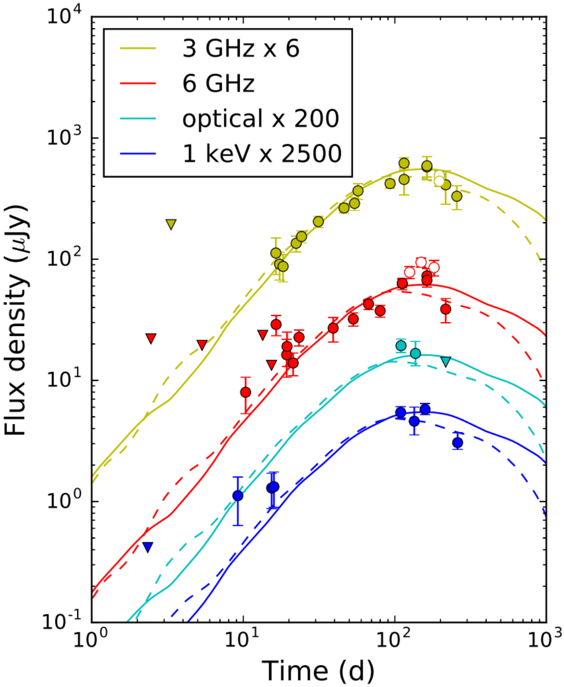
§1. Introduction

Unexpectedly long-rising late-time light curve of GRB170817A

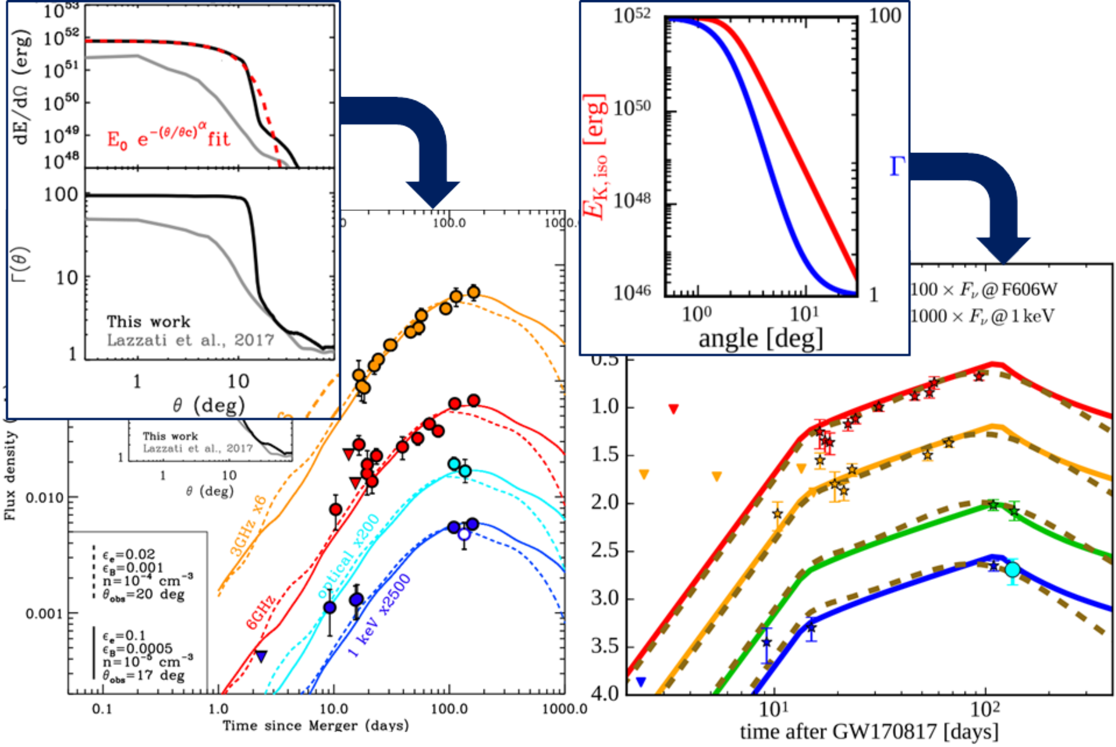


This talk's topic

Structured Jet?

Dynamical Ejecta?

Cocoon Emission?



Jet energy distribution ⇒ Light curve

Jet energy distribution ⇐Light curve

Systematic method?

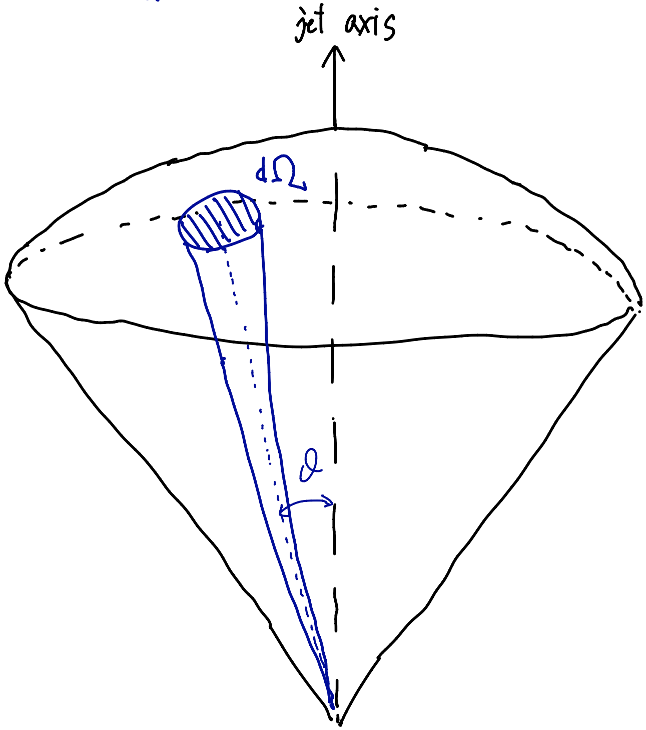
§2. Model

Assumptions:

1. A jet has an axi-symmetric structure where its energy is dependent on the angle.

jet energy contained in a unit solid angle:

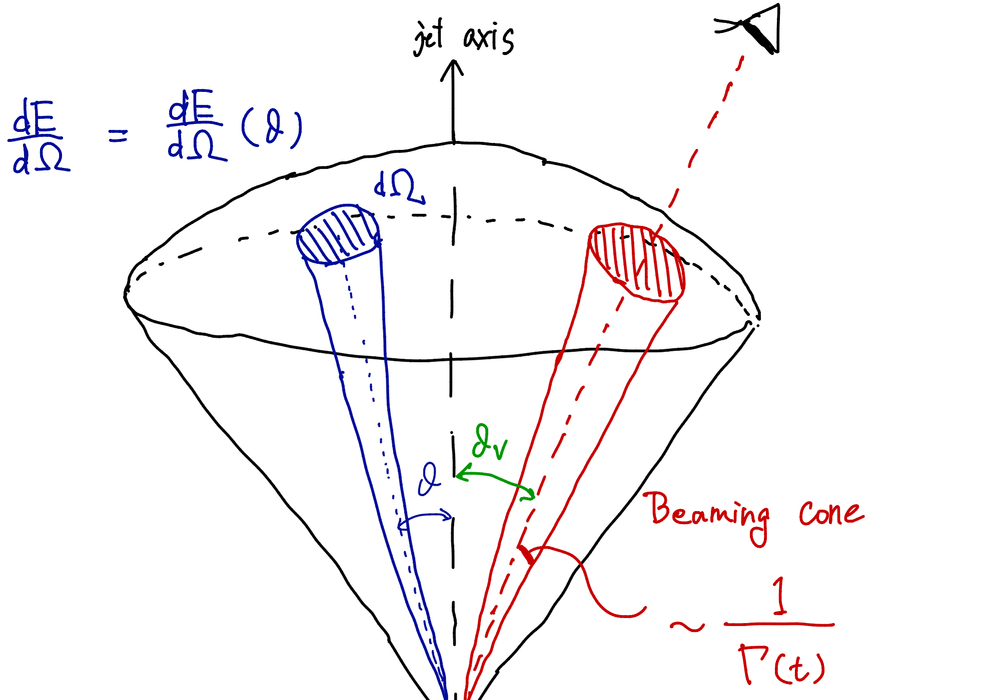
$$\frac{dE}{d\Omega} = \frac{dE}{d\Omega}(\theta)$$



2. The standard model of GRB afterglows is applicable

[the model parameters (e.g., ϵ_B, ϵ_e, p, n, ⋯) are assumed to be constant.]

and the rising of lightcurves originates in the expansion of the beaming cone.

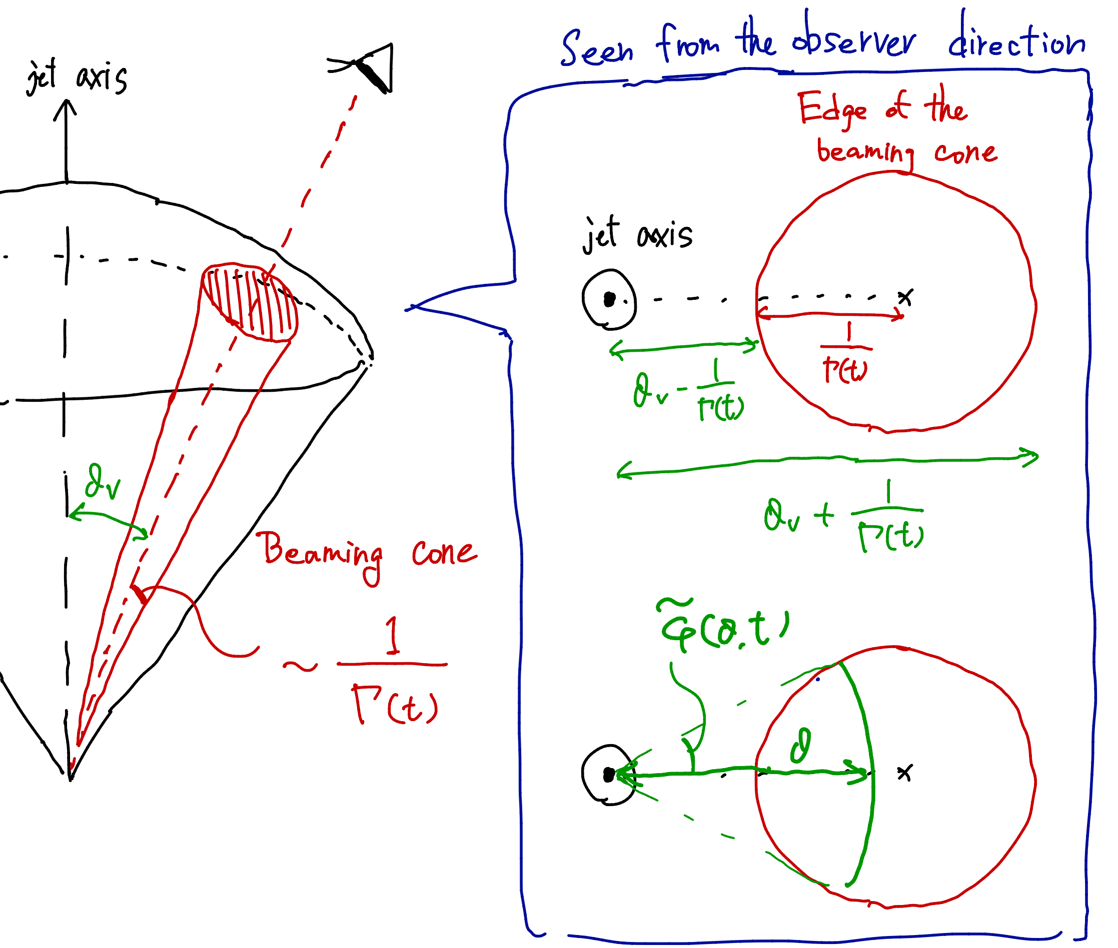


[The angle $\frac{1}{\Gamma(t)}$ increases with time as the jet is decelerated.]

Formulation :

$$E_{obs}(t) = \int_{\text{Beaming Cone}} d\Omega \frac{dE}{d\Omega}(\theta)$$

$$= \int_{\theta_v - \frac{1}{\Gamma(t)}}^{\theta_v + \frac{1}{\Gamma(t)}} \sin\theta \, d\theta \int_{-\tilde{\varphi}(\theta,t)}^{\tilde{\varphi}(\theta,t)} d\varphi \frac{dE}{d\Omega}$$



Formulation :

$$E_{obs}(t) = \int_{\text{Beaming Cone}} d\Omega \, \frac{dE}{d\Omega}(\theta)$$

$$= \int_{\theta_v - \frac{1}{\Gamma(t)}}^{\theta_v + \frac{1}{\Gamma(t)}} \sin\theta \, d\theta \int_{-\tilde{\varphi}(\theta,t)}^{\tilde{\varphi}(\theta,t)} d\varphi \, \frac{dE}{d\Omega}(\theta)$$

$$= 2 \int_{\theta_v - \frac{1}{\Gamma(t)}}^{\theta_v + \frac{1}{\Gamma(t)}} \sin\theta \, d\theta \, \tilde{\varphi}(\theta,t) \frac{dE}{d\Omega}(\theta)$$

$$\therefore \; E_{obs}(t) = 2 \int_{\mu_{min}(t)}^{\mu_{max}(t)} d\mu \, \underbrace{\tilde{\varphi}(\mu,t)}_{\text{known}} \, \underbrace{\frac{dE}{d\Omega}(\mu)}_{\text{What we want}}$$

$$\left[\begin{array}{l} \mu_{min}(t) = \cos\left(\theta_v + \frac{1}{\Gamma(t)}\right) \\ \mu_{max}(t) = \cos\left(\theta_v - \frac{1}{\Gamma(t)}\right) \end{array} \right]$$

$E_{obs}(t)$, $\Gamma(t)$: can be given for each epoch if the observed flux and ϵ_p, ϵ_e, n, p, … are specified (Sari+98, Nakar & Piran18) :

$$E_{obs}(t) = 2\times10^{47}\,\text{erg} \left(\frac{F_\nu(t)}{20\,\mu\text{Jy}}\right)^{\frac{3}{3+p}} \left(\frac{t}{10\,\text{d}}\right)^{\frac{3p}{3+p}} \left(\frac{n}{10^{-4}/\text{cc}}\right)^{\frac{p-3}{4(p+3)}} \left(\frac{\epsilon_B}{10^{-2}}\right)^{-\frac{p+1}{4(3+p)}}$$

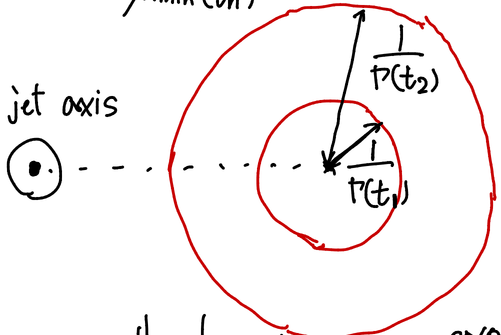
$$\times \left(\frac{\epsilon_e}{10^{-1}}\right)^{-\frac{p-1}{2(3+p)}} \left(\frac{\nu_{obs}}{3\,\text{GHz}}\right)^{\frac{3(p-1)}{2(p+3)}} \left(\frac{D}{40\,\text{Mpc}}\right)^{\frac{6}{3+p}}$$

$$\Gamma(t) = 4\left(\frac{F_\nu(t)}{20\,\mu\mathrm{Jy}}\right)^{\frac{1}{2(3+p)}}\left(\frac{t}{10\,\mathrm{d}}\right)^{-\frac{3}{2(3+p)}}\left(\frac{n}{10^{-4}/\mathrm{cc}}\right)^{-\frac{p+5}{8(3+p)}}\left(\frac{\epsilon_B}{10^{-4}}\right)^{-\frac{p+1}{8(3+p)}}$$
$$\times\left(\frac{\epsilon_e}{10^{-1}}\right)^{-\frac{p-1}{2(3+p)}}\left(\frac{\nu_{obs}}{3\,\mathrm{GHz}}\right)^{\frac{p-1}{4(p+3)}}\left(\frac{D}{40\,\mathrm{Mpc}}\right)^{\frac{1}{3+p}}$$

$$\boxed{\begin{gathered}\underbrace{E_{obs}(t)}_{\text{known}} = 2\int_{\mu_{min}(t)}^{\mu_{max}(t)} d\mu\, \underbrace{\tilde{\varphi}(\mu,t)}_{\text{known}}\, \underbrace{\frac{dE}{d\Omega}(\mu)}_{\text{What we want}} \\ \left[\begin{aligned}\mu_{min}(t) &= \cos\left(\theta_v + \frac{1}{\Gamma(t)}\right) \\ \mu_{max}(t) &= \cos\left(\theta_v - \frac{1}{\underbrace{\Gamma(t)}_{\text{known}}}\right)\end{aligned}\right]\end{gathered}}$$

How to get $\frac{dE}{d\Omega}(\mu)$: Discretize the time : t_n $(n=1,2,3,\cdots)$

$$E_{obs}(t_n) = 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \tilde{\varphi}(\mu,t_n)\, \frac{dE}{d\Omega}(\mu)$$



As time passes, the beaming cone expands and more information on $\frac{dE}{d\Omega}(\mu)$ is contained in $E_{obs}(t)$.

The integral $\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \tilde{\varphi}(\mu, t_n) \frac{dE}{d\Omega}(\mu)$ can be also discretized : $\int d\mu \to \sum_i \Delta\mu_i$

$\Rightarrow$ Algebraic equation for $\frac{dE}{d\Omega}(\mu_i)$,

$$\mu_i \in [\mu_{min}(t_n), \mu_{max}(t_n)]$$

$\Rightarrow$ Since the jet energy may be more dominant near the jet axis rather than jet outer part, the contribution from $\frac{dE}{d\Omega}(\mu_{min}(t_n))$ can be neglected.

$\Rightarrow$ Solving the algebraic equation, we obtain the jet energy $\frac{dE}{d\Omega}(\mu_{max}(t_n))$ for each time as

$$\frac{dE}{d\Omega}(\mu_{max}(t_n)) = \text{Function of } E_{obs}(t_n),\ \frac{dE}{d\Omega}(\mu_{max}(t_{n-1})),\ \frac{dE}{d\Omega}(\mu_{max}(t_{n-2})),\ \cdots\cdots$$

This is our basic strategy!

... but the application is not so straightforward ...

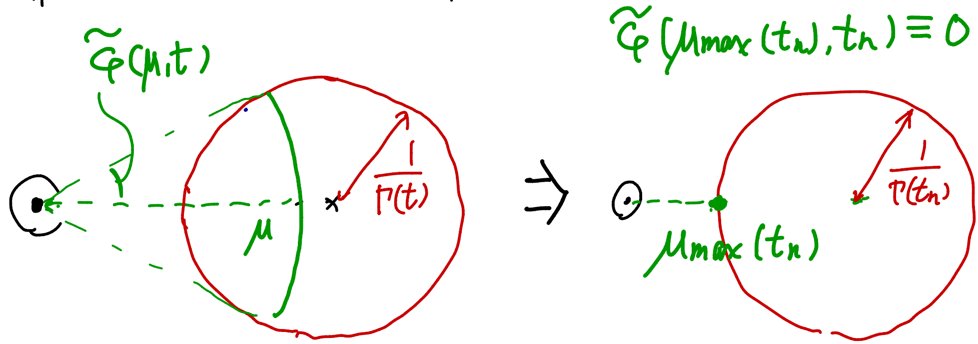
$$E_{obs}(t_n) = 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu \, \tilde{\varphi}(\mu, t_n) \frac{dE}{d\Omega}(\mu)$$

$$\Big\Downarrow \quad \text{some discretization method} \quad \int d\mu \longrightarrow \sum_i \Delta\mu_i$$

$\frac{dE}{d\Omega}(\mu_{max}(t_n))$ appears as

$$\underbrace{\tilde{\varphi}(\mu_{max}(t_n))}_{=\,0} \frac{dE}{d\Omega}(\mu_{max}(t_n))$$

Recall: The definition of $\tilde{\varphi}(\mu, t)$



$\Rightarrow \frac{dE}{d\Omega}(\mu_{max}(t_n))$ is not obtained

The difficulty can be overcome by performing a partial integral before the discretization:

$$E_{obs}(t_n) = 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \tilde{\varphi}(\mu,t_n)\, \frac{dE}{d\Omega}(\mu)$$

$$= 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \frac{d}{d\mu}\int_{\mu_{min}(t_n)}^{\mu} \tilde{\varphi}(\mu',t_n)\, d\mu'\, \frac{dE}{d\Omega}(\mu)$$

$$= 2\left[\frac{dE}{d\Omega}\int_{\mu_{min}(t_n)}^{\mu} d\mu'\, \tilde{\varphi}(\mu',t_n)\right]_{\mu_{min}(t_n)}^{\mu_{max}(t_n)}$$

$$- 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu) \int_{\mu_{min}(t_n)}^{\mu} \tilde{\varphi}(\mu',t_n)\, d\mu'$$

$$\boxed{\begin{aligned} \therefore E_{obs}(t_n) &= 2\,\frac{dE}{d\Omega}(\mu_{max}(t_n))\, I(\mu_{max}(t_n), t_n) \\ &\quad - 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu\, \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu)\, I(\mu, t_n), \\ I(\mu,t_n) &\equiv \int_{\mu_{min}(t_n)}^{\mu} d\mu'\, \tilde{\varphi}(\mu',t_n) \neq 0 \quad \text{unless } \mu = \mu_{min}(t_n) \end{aligned}}$$

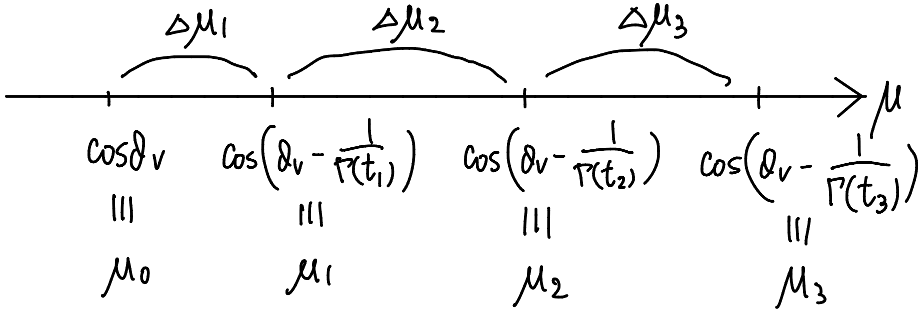
$$E_{obs}(t_n) = 2\frac{dE}{d\Omega}(\mu_{max}(t_n))\, I(\mu_{max}(t_n), t_n) - 2\int_{\mu_{min}(t_n)}^{\mu_{max}(t_n)} d\mu \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu)\, I(\mu, t_n)$$

$$\simeq 2\frac{dE}{d\Omega}(\mu_{max}(t_n))\, I(\mu_{max}(t_n), t_n) - 2\int_{\mu_{min}(t_1)}^{\mu_{max}(t_n)} d\mu \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu)\, I(\mu, t_n)$$

($\because$ The jet outer part is sub-dominant for the observed energy)

$\Downarrow$ discretization with trapezoidal method [Accuracy $\sim \mathcal{O}((\Delta\mu)^2)$]

$$E_{obs}(t_n) = 2\frac{dE}{d\Omega}(\mu_n)\, I(\mu_n, t_n) - 2\sum_{i=1}^{n} \frac{\Delta\mu_i}{2}\left[\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_i)\, I(\mu_i, t_n) + \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{i-1})\, I(\mu_{i-1}, t_n)\right]$$



$$E_{obs}(t_n) = 2\frac{dE}{d\Omega}(\mu_n)\, I(\mu_n, t_n)$$
$$- 2\sum_{i=1}^{n-1} \frac{\Delta\mu_i}{2}\left[\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_i)\, I(\mu_i, t_n) + \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{i-1})\, I(\mu_{i-1}, t_n)\right]$$
$$- 2\cdot\frac{\Delta\mu_n}{2}\left[\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_n)\, I(\mu_n, t_n) + \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{n-1})\, I(\mu_{n-1}, t_n)\right]$$

$$\Rightarrow \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_n)$$
$$= -\frac{E_{obs}(t_n)}{\Delta\mu_n I(\mu_n, t_n)} + \frac{2}{\Delta\mu_n}\frac{dE}{d\Omega}(\mu_n)$$
$$- \sum_{i=1}^{n-1}\frac{\Delta\mu_i}{\Delta\mu_n}\left[\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_i)\frac{I(\mu_i, t_n)}{I(\mu_n, t_n)} + \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{i-1})\frac{I(\mu_{i-1}, t_n)}{I(\mu_n, t_n)}\right]$$
$$- \frac{I(\mu_{n-1}, t_n)}{I(\mu_n, t_n)}\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{n-1})$$

with

$$\frac{dE}{d\Omega}(\mu_n) = \frac{dE}{d\Omega}(\mu_{n-1}) + \frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_{n-1})\,\Delta\mu_n$$

Basic equations

§ Boundary values

The equations are solved for $n \geq 2$ ($\Leftrightarrow t \geq t_2 \Leftrightarrow \mu \geq \mu_2$)

We need boundary values $\frac{dE}{d\Omega}(\mu_1)$, $\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_1)$, $\frac{d}{d\mu}\left(\frac{dE}{d\Omega}\right)(\mu_0)$

i.e., we need information of the energy distribution for $\mu < \mu_2$

This should be assumed, while it satisfies the observational constraint at $t = t_1$:

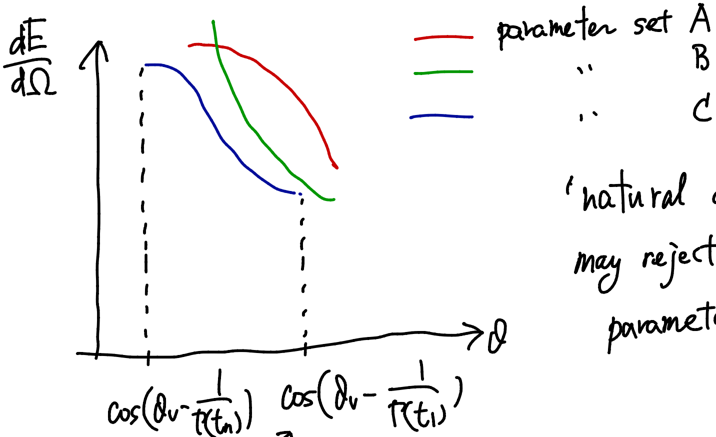
$$E_{obs}(t_1) = 2\int_{\mu_{min}(t_1)}^{\mu_{max}(t_1)} d\mu\, \tilde{\varphi}(\mu, t_1)\, \frac{dE}{d\Omega}(\mu)$$

☆ For an assumed $\frac{dE}{d\Omega}(\mu)$ for $\mu < \mu_2$ $\Rightarrow$ An energy distribution $\frac{dE}{d\Omega}(\mu)$ for $\mu \geq \mu_2$

☆ The assumed distribution $\frac{dE}{d\Omega}(\mu)$ for $\mu < \mu_2$ can be posteriori checked and modified to give smooth overall distribution $\frac{dE}{d\Omega}(\mu)$ in the entire range.

Now, we are finding a good way to give and modify the boundary values.

Expected:



'natural outcome' may reject some parameter range!

model parameter dependent
$(\epsilon_e, \epsilon_B, p, n, \cdots)$

Thank you!!!