

Landau-Lifshitz equation,
elliptic AKNS hierarchy
and Sato Grassmannian

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Lax equation
(zero-curvature eq) } on algebraic curve

$$\left. \begin{aligned} \partial_t A(P) &= [B(P), A(P)] \\ [\partial_x - A(P), \partial_t - B(P)] &= 0 \end{aligned} \right\} (r \times r)$$

$P \in \Gamma$: algebraic curve

$A(P), B(P)$: meromorphic on Γ

fixed poles P_1, \dots, P_k
order m_1, \dots, m_k

$g = \text{genus}(\Gamma)$

$g = 0 \rightarrow \# \text{ equation} = \# \text{ variables}$

$g > 0 \rightarrow$ overdetermined if A
and B take a general form

(Zakharov - Mikhailov)

Krichever (2002) —

- add extra "movable" poles to A and B $\gamma_1, \dots, \gamma_{rg} \in \Gamma$
- special structure of A, B at these poles
 → extra parameters $\alpha_1, \dots, \alpha_{rg} \in \mathbb{P}^{r-1}$

γ_s, α_s ($s = 1, \dots, rg$): Tyurin parameters

Krichever's result:

Introducing these new parameters as dynamical variables leads to a consistent Lax/zero-curvature equation.