

有理型函数対の空間 の上の可積分系

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1. Integrable system in Atiyah-Hitchin structure (review)

• $\mathcal{M} = \{ (A, B) \mid \text{pair of polynomials} \}$

$$A = \lambda^N + u_2 \lambda^{N-2} + \dots + u_N.$$

$$B = \lambda^{N-1} + v_2 \lambda^{N-2} + \dots + v_N.$$

(with genericity condition)

$$A = \prod_{j=1}^N (\lambda - \alpha_j) \quad , \quad \sum_{j=1}^N \alpha_j = 0.$$

$$B = \prod_{k=1}^{N-1} (\lambda - \lambda_k).$$

$$\dim \mathcal{M} = 2N - 2$$

(u_ℓ, v_ℓ)
 (α_j, λ_j) } coordinates
on \mathcal{M}

- (quasi) symplectic structure

$$\begin{aligned} \Omega &\stackrel{\text{def}}{=} \sum_{j=1}^N d \log B(\alpha_j) \wedge d\alpha_j \\ &= \sum_{k=1}^{N-1} d \log A(\lambda_k) \wedge d\lambda_k \\ &= \sum_{\ell=2}^N du_\ell \wedge d\phi_\ell, \text{ where} \end{aligned}$$

$$\phi_\ell = \sum_{k=1}^{N-1} \int^{\lambda_k} \frac{\lambda^{N-\ell}}{A(\lambda)} d\lambda.$$

$$\therefore \{u_\ell, u_m\} = 0$$

- integrable Hamiltonian system

$\left\{ \begin{array}{l} \text{Hamiltonians } u_2, \dots, u_N \\ \text{angle variables } \phi_2, \dots, \phi_N \end{array} \right.$

- separable in (λ_k, z_k)

$$z_k = A(\lambda_k)$$

- equivalent to open Toda chain

2. Analogous construction on algebraic curve of arbitrary genus

- C_0 : complex (non-singular) algebraic curve of genus g
- $R_1, \dots, R_N \in C_0$: $N > 2g - 2$
(or $N > g$ and in general position)

\Downarrow

$$\dim \mathcal{L}(R_1 + \dots + R_N) = N - g + 1$$

\Uparrow vector space of merom. functions on C_0 with $(f) \geq -R_1 - \dots - R_N$

(poles of at most first order) at R_1, \dots, R_N

$$\therefore \mathbb{P} \mathcal{L}(R_1 + \dots + R_N) \simeq \mathbb{P}^{N-g}$$

$$\Psi$$

$$[f], \quad f \in \mathcal{L}(R_1 + \dots + R_N)$$