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変形KP階層のq-類似と その準古典極限

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アブストラクト訂正

(第1式) $\oint_{\lambda=\infty} \tau(s', t' - [\lambda^2]) \tau(s, t + [\lambda^2]) \underbrace{\lambda^{s'-s}}_{\text{ゆけずい3}} e^{\int (t', \lambda) - \int (t, \lambda)} d\lambda = 0$

(第3式) $[\alpha]_q^{(n)} = \left(\dots, \frac{\underbrace{(1-q)^2}_{\leftarrow (1-q^2)} \alpha^2}{2(1-q^2)}, \dots, \frac{(1-q)^{\underbrace{k}_{n \rightarrow k}} \alpha^{\underbrace{k}_{n \rightarrow k}}}{\underbrace{k(1-q^k)}_{n \rightarrow k}}, \dots \right)$

(2nd-item, 第1式)

$$[qx]_q^{(n)} = [\lambda]_q^{(n)} - \sum_{m=0}^{n-1} [e^{2\pi i m/n} \underbrace{(1-q)^{\frac{1}{n}} x^{\frac{1}{n}}}_{x \rightarrow \lambda}]$$

(2nd-item, 第5式)

$$(1-q^n) D_{q^n} (x_n) \Psi_q = B_n \Psi_q, \quad B_n = \dots$$

(2nd-item, 下段) $L_2 \rightarrow \underbrace{(Li_2)}_{\text{ゆけずい3}}$

(2nd-item, 下から3行目) $\partial S / \partial s = p \rightarrow \underbrace{e^{\partial S / \partial s}}_{\text{ゆけずい3}} = p$

Modified KP hierarchy

- tau function: $\tau(s, t)$
 $s \in \mathbb{Z}, t = (t_1, t_2, \dots)$

- bilinear equations: For $s' \geq s$,

$$\oint_{\lambda=0} \tau(s', t' - [\lambda^{-1}]) \tau(s, t + [\lambda^{-1}]) \times \lambda^{s'-s} e^{\xi(t', \lambda) - \xi(t, \lambda)} d\lambda = 0.$$

$$\oint_{\lambda=0} : \textcircled{\infty}$$

$$[\alpha] = \left(\alpha, \frac{\alpha^2}{2}, \dots, \frac{\alpha^k}{k}, \dots \right)$$

$$\xi(t, \lambda) = \sum_{k=1}^{\infty} t_k \lambda^k$$

- wave function:

$$\Psi(s, t, \lambda) = \frac{\tau(s, t - [\lambda^{-1}])}{\tau(s, t)} \lambda^s e^{\xi(t, \lambda)}$$

$$\Psi^*(s, t, \lambda) = \frac{\tau(s, t + [\lambda^{-1}])}{\tau(s, t)} \lambda^{-s} e^{-\xi(t, \lambda)}$$

$$\partial_{t_n} \Psi(s, t, \lambda) = \underbrace{(e^{n\lambda s} + a_{n,1} e^{(n-1)\lambda s} + \dots + a_{n,n})}_{\text{difference operator}} \Psi(s, t, \lambda)$$

- yet another "modification" (2nd modification) $\angle 2$

$$\oint_{\lambda=\infty} \tau(s', t' - [\lambda_1^{-1}] - \dots - [\lambda_n^{-1}] - [\lambda^{-1}]) \\ \times \tau(s, t + [\lambda^{-1}]) \lambda^{s'-s} e^{\xi(t', \lambda) - \xi(t, \lambda)} \\ \times \prod_{j=1}^n (\lambda - \lambda_j) d\lambda = 0$$



(\therefore) Substitute

$$t' \rightarrow t' - [\lambda_1^{-1}] - \dots - [\lambda_n^{-1}]$$

in the previous bilinear equation.

The exponential factor $e^{\xi(t', \lambda) - \xi(t, \lambda)}$

gets multiplied by

$$\exp\left(-\sum_{j=1}^n \xi([\lambda_j^{-1}], \lambda)\right) \\ = \prod_{j=1}^n \exp\left(-\sum_{k=1}^{\infty} \frac{\lambda^k}{k \lambda_j^k}\right) \\ = \prod_{j=1}^n \left(1 - \frac{\lambda}{\lambda_j}\right). \quad \square$$