

~~Three~~ ^{Four} types of isomonodromic systems on $\Sigma = \mathbb{C}/\mathbb{Z} + \pi\tau$

0. scalar type (Okamoto, Iwasaki, Kawai)
1. Gaudin type (new!)
2. Calogero-Gaudin type
(Konopnik - Samtleben, Levin - Olshanetsky)
3. Calogero-Moser type
(Levin - Olshanetsky)

Result: Explicit construction

based on r -matrix

and Hamiltonian structure

Concerns:

relation to CFT, KZ equations,
spin systems, integrable systems,
Whitham equations, Seiberg-
Witten theory, ...

Isomonodromic System of Gaudin Type

(Lett. Math. Phys. 44 (1998) 147-156)

$$L(z) = \sum_{j=1}^N \sum_{(ab)} w_{ab}(z-t_j) J_{ab} A_j^{ab}$$

$$A_j(z) = \operatorname{res}_{w=t_j} \operatorname{tr}_2 (r(z-w) I \otimes L(w))$$

$$= \sum_{(ab)} w_{ab}(z-t_j) J_{ab} A_j^{ab}$$

$$A_z(z) = -\frac{1}{2\pi\sqrt{-1}} \sum_{j=1}^N \sum_{ab} w_{ab}(z) \times$$

$$\times \left(\frac{\partial \sigma_{ab}(z)}{\partial \sigma_{ab}(z)} - \frac{\partial \sigma_{ab}(0)}{\partial \sigma_{ab}(0)} \right) J_{ab} A_j^{ab}$$

$$A_j^{ab} = A_j^{ab}(t_1, \dots, \tau)$$

$$r(z-w) = \sum_{(ab)} w_{ab}(z-w) J_{ab} \otimes J^{ab}$$

(Belavin's \mathbb{Z}_n -symmetric r-matrix)

- $SU(n)$ only
- Related to CFT & Spin system
 - twisted WZW model (Kuroki & Takebe)
 - elliptic KZ equation (Etingof)
 - elliptic $SU(n)$ Gaudin model (K&T)

Isomonodromic System of Calogero - Gaudin type 3.

$$L(z) = \sum_{j=1}^N \sum_{a \neq b} \sigma_{q_a - q_b} (z - t_j) E_{ab} A_j^{ab} \\ + \sum_{j=1}^N \sum_a p(z - t_j) E_{aa} A_j^{aa} \\ + \sum_a p_a E_{aa} \quad \left(\sum_{j=1}^N A_j^{aa} = 0 \right).$$

$$A_j(z) = \operatorname{res}_{w=t_j} \operatorname{tr}_2 (r(z-w) I \otimes L(w)) \\ = \sum_{a \neq b} \sigma_{q_a - q_b} (z - t_j) E_{ab} A_j^{ba} + \sum_a p(z - t_j) E_{aa} A_j^{aa}.$$

$$A_z(z) = -\frac{1}{2\pi f_1} \sum_{j=1}^N \sum_{a \neq b} \sigma_{q_a - q_b} (z - t_j) \times$$

$$\times (p(z - t_j - q_a + q_b) + p(q_a - q_b)) E_{ab} A_j^{ab} \\ - \frac{1}{4\pi f_1} \sum_{j=1}^N \sum_a (p(z - t_j)^2 - p(z - t_j)) E_{aa} A_j^{aa}.$$

$$r(z-w) = \sum_{a \neq b} \sigma_{q_a - q_b} (z-w) E_{ab} \otimes E_{ba} \\ + \sum_a p(z-w) E_{aa} \otimes E_{aa}$$

(Felder and Wieczorkowski's dynamical R-matrix)

q_a, p_a : Calogero-Moser particles

$$A_j^{ab} = A_j^{ab}(t_1, \dots, z) \\ q_j = q_j(t_1, \dots, z) \\ p_j = p_j(t_1, \dots, z)$$