

torus上の2つのhyperbola $x-y$ と $x+y$ の交点を λ, t として変換
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$$P_{\text{II}}: \frac{d^2 \lambda}{dt^2} = \frac{1}{2} \left(\frac{1}{\lambda} + \frac{1}{\lambda-1} + \frac{1}{\lambda-t} \right) \left(\frac{d\lambda}{dt} \right)^2$$

$$- \left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{\lambda-t} \right) \frac{d\lambda}{dt}$$

$$+ \frac{\lambda(\lambda-1)(\lambda-t)}{t^2(t-1)^2} \left(\alpha + \beta \frac{t}{\lambda^2} + \gamma \frac{t-1}{(\lambda-1)^2} + \delta \frac{t(t-1)}{(\lambda-t)^2} \right)$$

$$\left. \begin{array}{l} \lambda = \frac{\wp(q) - e_1}{e_2 - e_1} \\ t = \frac{e_3 - e_1}{e_2 - e_1} \end{array} \right\} \begin{array}{l} \wp(\omega_j) = e_j \\ \omega_1 = \frac{1}{2}, \omega_2 = \frac{1+\tau}{2} \\ \omega_3 = \frac{\tau}{2} \end{array}$$

$$(2\pi i)^2 \frac{d^2 q}{d\tau^2} = \sum_{j=0}^3 \alpha_j \wp'(q + \omega_j) \quad (\omega_0 = 0)$$

(Fuchs; Manin)

• Picardの解: $(\alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = 0)$

$$q = c_1 + c_2 \tau$$

$(c_1, c_2: \text{constant})$

• Hamiltonian $H = \frac{1}{2} p^2 - \sum_{j=0}^3 \alpha_j \wp(q + \omega_j)$

Inozemtsev系 (ell. CM系の一様)

$$H = \frac{1}{2} \sum_{j=1}^{\ell} p_j^2 + \frac{g^2}{2} \sum_{\varepsilon, \varepsilon' = \pm 1} \sum_{j \neq k} \rho(\varepsilon q_j + \varepsilon' q_k) \\ + \sum_{a=0}^3 g_a^2 \sum_{j=1}^{\ell} \rho(q_j + \omega_a)$$

自励系 $\frac{dq_j}{dt} = \{q_j, H\}, \quad \frac{dp_j}{dt} = \{p_j, H\}$



$\frac{d}{dt} \rightarrow 2\pi i \frac{d}{d\tau}$

$$f(u) = f(u|1, \tau)$$

非自励系

$$2\pi i \frac{dq_j}{d\tau} = \{q_j, H\}, \quad 2\pi i \frac{dp_j}{d\tau} = \{p_j, H\}$$

$\ell = 1$: Maninの方程式 $(\sum_{j \neq k} \rightarrow 0)$

Levin & Olshanetsky ('97)

"Painlevé - Calogero 対応"

Hitchin系 \rightarrow 等モイド02-変形

楕円型 CM 系 (ルート系に付随してきまる)

- untwisted model ($p, q \in \mathbb{R}^d \supset \Delta$)

$$H = \frac{1}{2} p^2 + \frac{1}{2} \sum_{\alpha \in \Delta} g_{\nu(\alpha)} \rho(q \cdot \alpha)$$

- twisted model (D'Hoker & Phong)

$$H = \frac{1}{2} p^2 + \frac{1}{2} \sum_{\alpha \in \Delta} g_{\nu(\alpha)} \rho_{\nu(\alpha)}(q \cdot \alpha)$$

- extended twisted model (Bordner & Sasaki)

$B_e, C_e, \boxed{BC_e}$

ρ, ρ_{ν} 両方含む

↓
Inozemtsev 系

$$\rho_{\nu}(u) = \rho(u / \frac{1}{\nu(\alpha)}, \tau)$$

Lax 対 (\Rightarrow 可積分性)

- Lie 代数の表現空間 (minimal) における構成 (---, D'Hoker & Phong)

- Weyl 群の表現空間 (実際には Δ に対しての軌道を基底とするものを便利) における構成 (Bordner, et al) "root-type Lax pair"

(\rightarrow 特に Inozemtsev 系の Lax 対が"と"まる)