

EXTENSIONS OF "SOLITON EQUATIONS" ^①

① • SELF-DUALITY EQUATION
IN YANG-MILLS THEORY

② • SELF-DUALITY EQUATION
IN KÄHLER GEOMETRY

3 • SUPER KP HIERARCHY

CONCERNS: —

1 AND 2 — "HIGHER DIMENSIONAL"
(OR "MULTI-DIMENSIONAL")
EXTENSION OF "NONLINEAR
INTEGRABLE SYSTEMS"

1, 2, 3 ...
SYMMETRIES
???

3 — EXTENSION OF METHOD OF
"D-MODULE" (M. SATO) ON
KP HIERARCHY

(2)

GENERALIZED SELF-DUALITY EQUATION IN YANG-MILLS THEORY

$$(x, p) = (x^1, \dots, x^{2N}, p^1, \dots, p^{2N})$$

4N-DIMENSIONAL SPACE-TIME

I. $K = K(x, p) \in \text{Lie } G$

$$\frac{\partial^2 K}{\partial x^i \partial p^j} - \frac{\partial^2 K}{\partial x^j \partial p^i} + \left[\frac{\partial K}{\partial x^i}, \frac{\partial K}{\partial x^j} \right] = 0$$

$$i, j = 1, \dots, 2N$$

II. $J = J(x, p) \in G$

$$\frac{\partial}{\partial x^i} \left(\frac{\partial J}{\partial p^j} \cdot J^{-1} \right) - \frac{\partial}{\partial x^j} \left(\frac{\partial J}{\partial p^i} \cdot J^{-1} \right) = 0$$

- I AND II ARE EQUIVALENT.

(3)

AUXILIARY VARIABLES

$$W = 1 + \sum_{n=-\infty}^{-1} W_n(x, p) \lambda^n$$

$$\hat{W} = \sum_{n=0}^{\infty} \hat{W}_n(x, p) \lambda^n \quad \text{THAT SATISFY:}$$

$$\left(\frac{\partial}{\partial p_i} - \lambda \frac{\partial}{\partial x_i} + A_i \right) \Psi = 0,$$

$$\Psi = W, \hat{W}$$

$$A_i = -\frac{\partial K}{\partial x_i} = -\frac{\partial J}{\partial p_i} \cdot J^{-1}$$

$$W_{-1} = -K$$

$$\hat{W}_0 = J$$

→ NON LINEAR SYSTEM
ON $\{W_n, \hat{W}_n\}$

↻
SYMMETRIES

((W, V) - SYSTEM)