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U-Plane integrals and integrable systems

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- Correlation functions and contact terms in 4d topological gauge theory
- $G = SU(N)$: Coulomb moduli (u-plane), blowup formula, single-time tau function (after Mariño & Moore)
- multi-time tau function
- other gauge groups

Correlation functions and contact terms in 4d topological gauge theory

L1

Physical Setup

$\mathcal{N}=2$ SUSY
gauge theory

topological
twisting \rightarrow

topological
gauge theory
on X

\downarrow
correlation functions
 \parallel
Donaldson - Witten
invariants of X

generating function of correlation functions:

$$Z_{DW}(xS + yP) = \langle \exp(xI(S) + yO(P)) \rangle$$

$$S \in H_2(X, \mathbb{Z})$$

$$I(S) \sim \int_S G^2 P$$

$$P \in H_0(X, \mathbb{Z})$$

$$O(P) \sim \int C_k \text{Tr } \phi(P)^k$$

x, y : coupling constants

$$b_2^\pm = \dim H^2(X, \mathbb{C})_{\pm} \quad \begin{matrix} \text{self-dual} \\ \text{anti-self-dual} \end{matrix} \quad \sqsubseteq$$

If $b_2^+ > 1$, $Z_{DW} = Z_{SW}$

↑
contribution of moduli space
of Seiberg-Witten monopole eq.
 Z_{DW} is topological invariant.

If $b_2^+ \leq 1$, $Z_{DW} = Z_{SW} + Z_u$

↑
contribution of "u-plane"
(Coulomb moduli)

u-plane integral (Moore-Witten, Losev et al, Mariño-Moore)

$$Z_u = \int_{\mathcal{M}_{\text{Coulomb}}} d\mu A^\chi B^\sigma \exp(U + \chi^2 S^2 T) \Psi$$

U : contribution of $\mathcal{O}(P)$ only

Ψ : partition function of (abelian) gauge fields etc. $\Psi = \sum_{\text{Lattice}} (\dots)$

T : contact term induced by S

$\mathcal{M}_{\text{Coulomb}}$: "u-plane" (quantum moduli space of vacua in S-W effective theory)