

94-09-05
↓

量子化

Quantization

幾何学

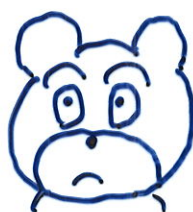
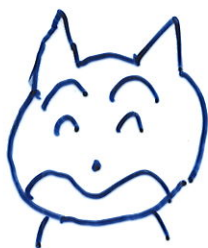
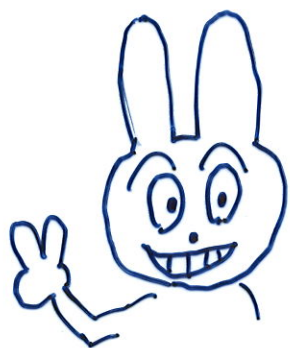
Geometry

可積分系

Integrable Systems

0

2



Exact calculation of path integrals

$$Z = \int \mathcal{D}\Phi e^{A[\Phi]}$$

typical mechanism. trick: -

o Gaussian integral (\leftrightarrow free fields)

$$\Phi: \Sigma \rightarrow M$$

(\rightarrow nontrivial results if Σ, M are nontrivial)

o cancellation

(index th, SUSY, ...)

\rightarrow localization

o symmetries

$$\left(\begin{array}{l} G \curvearrowright \mathcal{D}\Phi, A[\Phi] \\ \int_{\mathcal{F}} \mathcal{D}\Phi e^{A[\Phi]} \cong \int_{\mathcal{F}/G} \dots \end{array} \right)$$

Gaussian integrals

(boson) $\int dx e^{-\sum A_{ij} x_i x_j} = \text{cte} (\det A)^{-1/2}$
↑ symmetric, pos. def.

$\int dz d\bar{z} e^{-\sum A_{ij} z_i \bar{z}_j} = \text{cte} (\det A)^{-1}$
↑ Hermitian, pos. def.

(fermion) $\int d\psi e^{-\sum A_{ij} \psi_i \psi_j} = \text{cte Pfaff } A$
↑ antisymmetric

$\int d\psi d\bar{\psi} e^{-\sum A_{ij} \psi_i \bar{\psi}_j} = \text{cte } \det A$

$\sum A_{ij} x_i x_j \rightarrow -\int \phi \Delta \phi \sim \int (D\phi)^2 + m^2 \phi^2$

$\sum A_{ij} z_i \bar{z}_j \rightarrow \int \partial \phi \cdot \bar{\partial} \phi + \dots$

$\sum A_{ij} \psi_i \bar{\psi}_j \rightarrow \int \psi \not{\partial} \bar{\psi} + \dots$ ($\not{\partial} = \gamma^\mu \partial_\mu$)
Dirac

★ Non-trivial Gaussian integrals — 15

①.1 $\Phi: \Sigma \rightarrow T = \text{torus}$ $\Phi = (\phi_1, \dots, \phi_n)$
 $\phi_i \in S^1$

eg. $\Sigma = \overset{2\text{-dim}}{\text{torus}} \rightarrow T = n\text{-dim torus}$

topological invariants (winding #)

$$\int \mathcal{D}\Phi e^S = \sum_{\text{winding \#}} \int \mathcal{D}\Phi e^S$$

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instanton #
fluctuations

θ-functions
 $\left(\frac{\det'(-\Delta_g)}{\text{vol}_g \Sigma} \right)^{-n/2}$

①.2 $\Phi: \Sigma \rightarrow \mathbb{C}$ $\Sigma = \text{Riemann surface}$

$\psi \sim e^{-i\phi}$, $\bar{\psi} \sim e^{i\phi}$ (free fermion/boson)

$\langle \pi e^{+i\phi(z_i)} \pi e^{-i\phi(\eta_j)} \rangle$

$\sim \frac{\prod E(z_i, z_j) \prod E(\eta_i, \eta_j)}{\prod E(z_i, \eta_j)} \times \frac{\Theta(\sum z_i - \sum \eta_j)}{(\det \bar{\partial}_0)^{2/2}}$

↑
prime forms

↑
theta function
det(Dirac)