超幾何系ワークショップ in 神戸 '97

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Gaudin Model, KZ Equation and Isomonodromic Problem on Torus

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Ver. 5 will soon appear

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Goal : Construct an <u>elliptic</u> analogue of the Schlesinger equation

Schlesinger equation — Isomonodromic Problem of matrix system

$$\frac{dY}{d\lambda} = M(\lambda)\, Y, \qquad n \times n$$

$$M(\lambda) = \sum_{i=1}^{N} \frac{A_i}{\lambda - t_i} \; : \text{ rational } (X = \mathbb{CP}^1)$$

Elliptic analogue — Isomonodromic Problem with elliptic coefficients

$$M(\lambda) = \, ? \qquad (X = \mathbb{C}/\mathbb{Z} + \tau\mathbb{Z})$$

Cf. Scalar equation

$$\frac{d^2y}{d\lambda^2} = q(\lambda)\, y, \quad \text{higher order}, \ldots$$

— Okamoto, Iwasaki, Kawai

(Complex analytic and geometric approach)

Approaches.

Complex analytic and geometric approach

Okamoto, Iwasaki, Kawai

twisted de Rham cohomology

CFT, r-matrix approach

① Reshetikhin, Harnad) $g = 0$ (Schlesinger eq)

KZ (Knizhnik-Zamolodchikov) equation

rational r-matrix

(rational Gaudin model)

② Korotkin, Samtleben, Lavin, Olshanetsky) $g = 1$

KZB (K-Z-Bernard) equation

dynamical r-matrix

Calogero-Gaudin model

My approach

elliptic KZ equation of Etingof

Belavin's r-matrix

elliptic SU(n) Gaudin model of Kuroki and Takebe

Road Map

Elliptic Gaudin model

$J(\lambda)$, $\underbrace{H_0, H_1, \cdots, H_N}_{\text{quantum Hamiltonians}}$

↓ classical limit

Non-autonomous multi-time Hamiltonian system ← **geometric interpretation of isomonodromic deformations**

symplectic form Ω on moduli space $\mathcal{M}$ of mero. connections with given <u>constant</u> local monodromy data

$M(\lambda)$, $\underbrace{H_0, H_1, \cdots, H_N}_{\text{classical}}$

$\tau, t_1, \cdots, t_N$

↓

Lax representation

$\left[\frac{\partial}{\partial\lambda} - M(\lambda), \cdots\right] = 0$

↓

Isomonodromic linear system

$\frac{\partial Y}{\partial\lambda} = M(\lambda)Y, \cdots$

↑ classical limit + picture changing (Schrödinger → Heisenberg)

Elliptic KZ equation