

Integrable Systems and Branes

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Plan

- Integrable Hamiltonian Systems (C^∞ /analytic/
algebraic)
 - $D=4$ $N=2$ SUSY gauge theories
and integrable systems
 - $D=4$ $N=2$ SUSY gauge theories
and M-theory (M2-branes)
 - Mirror maps and integrable systems
- (• What are branes?)

I. Integrable Hamilton Systems

Ref. Donagi & Markman, alg-geom/9505009
 Donagi, alg-geom/9705010

1. integrable Hamiltonian systems (C^∞),

(C^∞) integrable Hamiltonian system

is a symplectic manifold (X, ω) (or Poisson)
 with Lagrangian fibration $\pi: X \rightarrow U$

$X_u = \pi^{-1}(u)$: Lagrangian submanifold of X

- $H_1, H_2: U \rightarrow \mathbb{R} \Rightarrow \{\pi^*H_1, \pi^*H_2\} = 0$
- $X_u \sim \mathbb{R}^n, T^n$



invariant tori

2. algebraic/analytic integrable Hamiltonian systems

analytic : $X_u = \text{complex torus } (\cong \mathbb{C}^n/L)$

algebraic : $X_u = (\text{polarized}) \text{ abelian variety}$



holomorphic flat structure :

$H_1(X_u, \mathbb{Z}) \cong \mathbb{Z}^{2n}$, skew-symmetric form

↓ smoothly depends on u

flat bundle over U

$\alpha_j, \beta_j (j=1..n)$: symplectic basis

$V_{\mathbb{Z}}^* := \sum \mathbb{Z} \alpha_j \subset H_1(X_u, \mathbb{Z})$

↓ symplectic form

flat structure of U

modelled on $V = \text{Hom}(V_{\mathbb{Z}}^*, \mathbb{C})$

$d z_j = \int_{\alpha_j} \omega$

$\{z_j\}$: flat coordinates

($\sim \{a_i\}$ in SW)

3. Cubic and Prepotential of algebraic IHS I-3

period $\tau: U \rightarrow H/g$ (Siegel upper half plane)

$$(\tau_{ij})$$

$$d\tau_{ij} = \sum \frac{\partial \tau_{ij}}{\partial z_k} dz_k$$

algebraic
integrable
system

cubic :

$$c_{ijk} = \frac{\partial \tau_{ij}}{\partial z_k} \text{ is totally symmetric}$$

potentials :

$$\tau_{ij} = \frac{\partial w_i}{\partial z_j} \quad \{w_i\}: \text{dual coordinates}$$

($\sim \{a_i\}$ in SW)

prepotential \mathcal{F} :

$$w_i = \frac{\partial \mathcal{F}}{\partial z_i}$$

* Similar construction applies to an analytic integrable Hamiltonian system constructed from deformations of Calabi-Yau 3-folds.