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# Isomonodromic Deformations and Whitham Equations

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- Int. J. Mod. Phys. A11 (38) (1996), 5505.  
(With T. Nakatsu) hep-th/.....
- solv-int/9704009
- solv-int/9705016

## Outline

- isospectral v.s. isomonodromic problems  
— motivation
- isomonodromic problem as MODULATION  
of isospectral problem — general idea
- examples

# Isospectral Problem

$$\frac{dM(\lambda)}{dt} = [A(\lambda), M(\lambda)] \quad (\text{Lax representation})$$

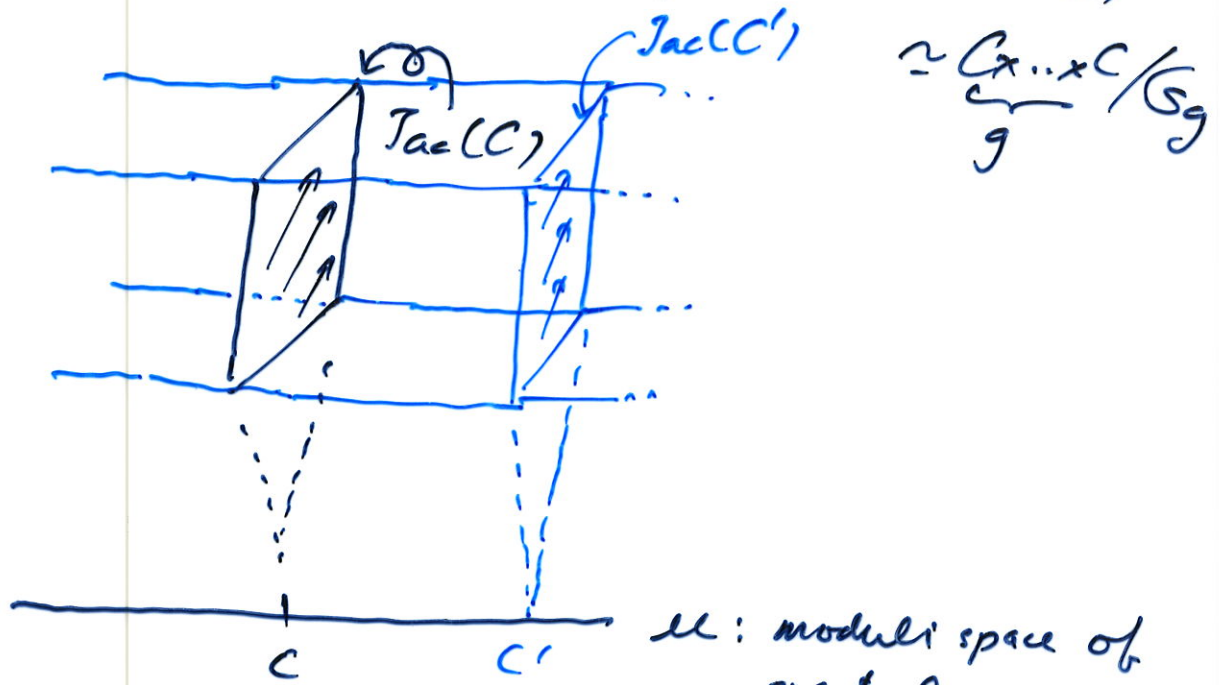
$$\Rightarrow \boxed{\frac{d}{dt} f(\lambda, \mu) = 0} \quad \text{where}$$

$$f(\lambda, \mu) = \det(\mu I - M(\lambda))$$

$$= \mu^n + c_1(\lambda)\mu^{n-1} + \dots + c_n(\lambda)$$

$C: f(\lambda, \mu) = 0$  Spectral curve

Lax equation  $\rightarrow$  linear flow on  $\text{Jac}(C)$



$\mathcal{M}$ : moduli space of spectral curves

$$(\dim \mathcal{M} = g)$$

## Isomonodromic Problem

$$\frac{\partial M(\lambda)}{\partial t} - \frac{\partial A(\lambda)}{\partial \lambda} = [A(\lambda), M(\lambda)]$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t} f(\lambda, \mu) \neq 0}$$

$$\left( \frac{\partial}{\partial t} \log f(\lambda, \mu) = - \text{Tr} \frac{\partial A(\lambda)}{\partial \lambda} (\mu I - M(\lambda))^{-1} \right)$$

Spectral curve depends on  $t$ .

( $\Rightarrow$  usual methods for solving isospectral problems do not work.)

Remark  $\text{Jac}(C) \simeq \underbrace{C \times \dots \times C}_g / \mathbb{S}_g$  is still useful for construction of "Darboux coordinates" ("spectral Darboux coordinates" Harnad et al)

↓

"Hamiltonian structure" of isomonodromic problems  
(K. Okamoto, et al)

cf. Manin's work (P<sub>II</sub> etc) ?

isomonodromic problem  $\neq$  isospectral problem

However, there is much evidence that

isomonodromic problem  $\approx$  modulation of isospectral problem

asymptotically

$x \rightarrow \infty$

$\varepsilon \rightarrow 0$

Examples:

- Boutroux (1913)  $P_I, P_{II}, \dots$
- Garnier (1919) Schlesinger equation

⋮

- Dubrovin, Krichever, Novikov (around '90)

String equations of 2d q-gravity  
( $\approx P_I, \dots$ )

- Vereschagin ('96)  $P_I \sim P_{VI}$