THE GEOMETRIC REPRESENTATION OF U–PROJECTIVE RESOLUTION OF MODULE OVER PATH ALGEBRA OF TYPE A_n AND $\widetilde{A}_n$

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ABSTRACT

Projective resolutions of modules play an important role in homological algebra. In 2002, Davvaz and Shabani-Solt introduced the notions of U–complex, U–homology, etc. to generalize certain concepts in homological algebra. Inspired by this, the first and the third authors introduced the notion of U–projective resolutions and U–extension modules in 2016.

We also found that every short exact sequence of modules and module homomorphisms over hereditary algebra can always be extended into a long exact sequence of U–homologies (the modified homologies) consisting of U–extension modules. This encouraged us to further research on U–extension modules over hereditary algebras. The algebra of our interest is a path algebra generated by a finite acyclic quiver. In this paper we will discuss U–projective resolutions of modules over such algebra of type A_n and $\widetilde{A}_n$.

A sequence $\ldots \xrightarrow{d_{k+2}} M_{k+1} \xrightarrow{d_{k+1}} M_k \xrightarrow{d_k} M_{k-1} \xrightarrow{d_{k-1}} \ldots$ of modules and module homomorphisms is said to be **exact** if $Imd_{k+1} = \ker d_k$ for every k. Davvaz and Parnian-Garamaleky introduced the notion of U–exact sequence, which is a generalization of an exact sequence. The idea is to replace $\ker d_k$ with $d_k^{-1}(U_{k-1})$, for every k, where U_k is a submodule of M_k for each k. Davvaz and Shabani-Solt then redefined this concept using the so-called U–complex approach which further assumes that Imd_{k+1} must contain U_k for every k. Thus, an U–exact sequence is a sequence $\ldots \xrightarrow{d_{k+2}} M_{k+1} \xrightarrow{d_{k+1}} M_k \xrightarrow{d_k} M_{k-1} \xrightarrow{d_{k-1}} \ldots$ of modules and module homomorphisms such that $Imd_{k+1} = d_k^{-1}(U_{k-1}) \supseteq U_k$ where U_k is a submodule of M_k, for each integer k.

Mahatma and Muchtadi-Alamsyah extended these result by proposing a way to define U–projective resolutions and U–extension modules. The aim of this paper is to state a formula for U–projective resolutions of kQ–modules where Q is quiver of type A_n and $\widetilde{A}_n$ and to give their geometric representations based on result by Baur and Torkildsen in 2016.

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