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Strong CP Invariance in Higher Dimensions

井沢 健一

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with T. Watari and T. Yanagida

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with A. Fukunaga

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SYMMETRY GEOMETRY

$$S_{QCD} = \int d^4x \, \frac{1}{2g_s^2} \mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$$

$$+ \frac{\theta}{32\pi^2} \int d^4x \, \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}(F_{\mu\nu}F_{\rho\sigma})$$

$$\theta_{eff} = \theta + \arg(\det Y)$$

$$\theta_{eff} = \langle a(x)/F_a \rangle$$

$$V(a) \simeq -\Lambda_{QCD}^4 \cos(a(x)/F_a)$$

Pecci-Quinn symmetry

$$\mathrm{U}(1)_{PQ} : a(x) \to a(x) + \xi F_a$$

Extra-Quarks

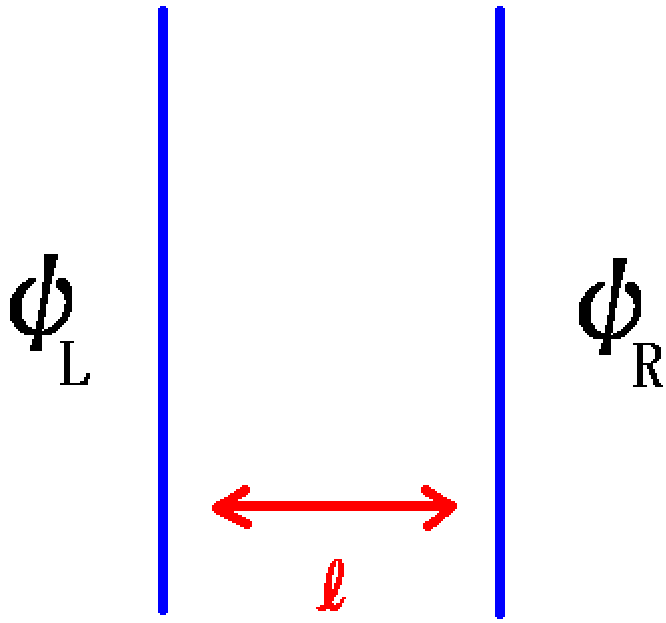
$$\psi_L \to e^{i\xi}\psi_L, \qquad \psi_R \to e^{-i\xi}\psi_R$$

Mass term

$$M_*\psi_L\psi_R^\dagger + \text{h.c.}$$

4-Fermi term

$$\frac{1}{M_*^2}(\psi_L\psi_R^\dagger)^2 + \text{h.c.}$$



$$\frac{M_* \psi_L \psi_R^\dagger}{e^{ml} - e^{-ml}}$$

$M_* \simeq 10^{18}$GeV: $ml \gtrsim 150$

local rigid symmetry **genericity**

Effective Theory Descriptions

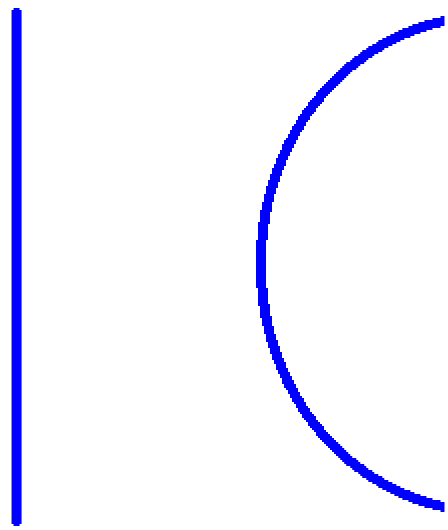
QCD	**Gravity**
strong CP	cosmological constant

changing to variable

chiral anomaly	spacetime inflation

selection by dynamics

axion	quintessence



$$g_{\mu\nu} = e^{2kl}\bar{g}_{\mu\nu}$$

$$R = e^{-2kl}\bar{R}\text{:} \quad kl \gtrsim 140$$

$$S_{YM} = \int d^4x \int_{-l}^{l} dy \; \frac{M_*}{4g^2} \mathrm{tr}(F_{MN}F^{MN})$$

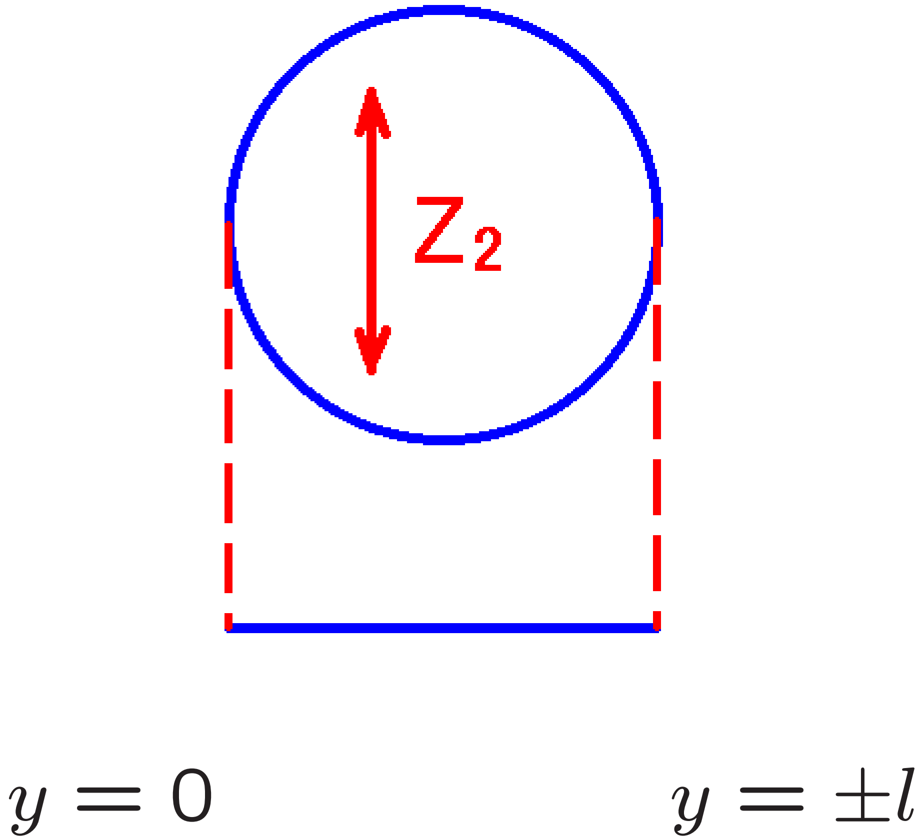
Parity symmetry

$$\mathbf{Z_2} : \begin{cases} A_\mu(x,y) \to A_\mu(x,-y) \\ \\ A_4(x,y) \to -A_4(x,-y) \end{cases}$$

Orbifolding

$$\begin{aligned} A_\mu(x,y) &= A_\mu(x,-y) \\ A_4(x,y) &= -A_4(x,-y) \end{aligned}$$

$$\varepsilon(x,y) = \varepsilon(x,-y)$$



Z_2-fixed planes

$$S_\psi = \int_{y=0} d^4x \; \bar{\psi}_L i \not{D} \psi_L + \int_{y=l} d^4x \; \bar{\psi}_R i \not{D} \psi_R$$

Gauge anomaly

$$\delta S_{eff} \; = \; \frac{i}{24\pi^2} \int_{y=0} \mathrm{tr} \left(\varepsilon d (A dA + \frac{1}{2} A^3) \right)$$

$$- \frac{i}{24\pi^2} \int_{y=l} \mathrm{tr} \left(\varepsilon d (A dA + \frac{1}{2} A^3) \right)$$

$$A = A_M dx^M$$

$$S_{CS} = \int h(y)\,\mathrm{tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right)$$

$$F = dA + A^2$$

Parity symmetry: $h(y) = -h(-y)$

$$\begin{aligned}
\delta S_{CS} &= \int h(y)\,\mathrm{tr}\left((d\varepsilon)d(AdA + \frac{1}{2}A^3)\right) \\
&= -\int (dh(y))\,\mathrm{tr}\left(\varepsilon d(AdA + \frac{1}{2}A^3)\right)
\end{aligned}$$

$$S = S_{YM} + S_{CS} + S_\psi, \qquad \delta S = 0$$

$$h(y) = \frac{i}{48\pi^2}\frac{y}{|y|}$$

anomaly inflow

Color $\times$ Hypercolor: $\mathsf{SU(3)}_C \times \mathsf{SU(3)}_H$

$$\psi_L(\mathbf{3},\mathbf{3}^*), \qquad \psi_R(\mathbf{3},\mathbf{3}^*)$$

$$\left\langle \psi_L \psi_R^\dagger \right\rangle \simeq F_a^3 \qquad\qquad \mathbf{3}\times\mathbf{3}^* = \mathbf{8}+\mathbf{1}$$

$$\chi_L(\mathbf{1},\mathbf{3}^*), \qquad \chi_R(\mathbf{1},\mathbf{3}^*)$$

$$\left\langle \psi_L \psi_R^\dagger \right\rangle \simeq F_a^3, \qquad \left\langle \chi_L \chi_R^\dagger \right\rangle \simeq F_a^3$$

$$(\mathbf{3}+\mathbf{1})\times(\mathbf{3}^*+\mathbf{1}) = \mathbf{8}+\mathbf{3}+\mathbf{3}^*+\mathbf{1}+\mathbf{1}$$

$$\int_{y=0} d^4x \frac{1}{M_*^5}(\psi_L)^3(\psi_L)^3 + \int_{y=l} d^4x \frac{1}{M_*^5}(\psi_R^\dagger)^3(\psi_R^\dagger)^3 + \text{h.c.}$$

$$\tilde{V}(a) \simeq \frac{F_a^{14}}{M_*^{10}} f(a/F_a)$$

$$\theta_{eff} \simeq \frac{F_a^{14}}{M_*^{10}{\Lambda_{QCD}}^4}$$

$M_* \simeq 10^{18}$GeV, $\theta_{eff} < 10^{-9}$:

$F_a \lesssim 10^{12}$GeV

- Supersymmetry

- Gravity

$(V, \Phi) \qquad \Phi \supset iA_4$

$$K = \frac{1}{4\pi}\ \mathrm{Im}\left(\frac{\partial \mathcal{F}(\Phi)}{\partial \Phi}\Phi^\dagger\right)$$

$$W = \frac{-i}{16\pi}\left(\frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^2}\right)\mathcal{W}^\alpha \mathcal{W}_\alpha$$

$$\mathcal{F} = i\tfrac{2\pi}{g^2}\Phi^2 + ih\Phi^3$$

$$\Rightarrow hA_4 F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}$$

rotation (11) or (9) is a local quantity like the anomaly itself. Thanks to the gauge covariance, the evaluation of the last expression in plane wave basis is again simple as is shown in Appendix B. We have

$$\begin{aligned}&\delta_1\left\langle\delta_2 S\right\rangle-\delta_2\left\langle\delta_1 S\right\rangle\\&\stackrel{M\to\infty}{=}\frac{1}{64\pi^2}\int d^8z\ \mathrm{tr}\,\Delta_1\Big(\big[\mathcal{D}^\alpha\Delta_2,W_\alpha\big]+\big[\overline{D}_{\dot\alpha}\Delta_2,\overline{W}'^{\dot\alpha}\big]+\big\{\Delta_2,\mathcal{D}^\alpha W_\alpha\big\}\Big).\end{aligned}\tag{14}$$

In spite of the asymmetric appearances of 1 and 2 in this expression, one can confirm by using the reality constraint (A.2) that this is actually odd under the exchange $1\leftrightarrow 2$. From Eq. (14), we can read off the left hand side of Eq. (10):

$$\begin{aligned}&\int d^8z\,\delta_\Lambda V^a(z)\Big[\frac{\delta}{\delta V^b(z')}\Big\langle\frac{\delta S}{\delta V^a(z)}\Big\rangle-\frac{\delta}{\delta V^a(z)}\Big\langle\frac{\delta S}{\delta V^b(z')}\Big\rangle\Big]\\&\stackrel{M\to\infty}{=}\frac{1}{64\pi^2}\int d^8z\int_0^1 d\beta\\&\qquad\times\mathrm{tr}\,e^{-\beta V}T^b\delta(z-z')e^{\beta V}\Big(\mathcal{D}^\alpha\big\{i\Lambda,W_\alpha\big\}-\overline{D}_{\dot\alpha}\big\{e^{-V}i\Lambda^\dagger e^V,\overline{W}'^{\dot\alpha}\big\}\Big),\end{aligned}\tag{15}$$

which satisfies Eq. (10) in conjunction with Eq. (8); this fact provides the consistency check of Eq. (14). Finally, from Eqs. (7), (8) and (14), we obtain the consistent gauge anomaly

$$\begin{aligned}&\delta_\Lambda\varGamma[V]\\&\stackrel{M\to\infty}{=}-\frac{1}{64\pi^2}\int d^6z\ \mathrm{tr}\,i\Lambda W^\alpha W_\alpha+\frac{1}{64\pi^2}\int d^6\overline{z}\ \mathrm{tr}\,e^{-V}i\Lambda^\dagger e^V\overline{W}'_{\dot\alpha}\overline{W}'^{\dot\alpha}\\&\qquad+\frac{1}{64\pi^2}\int d^8z\int_0^1 dg\int_0^1 d\beta\ \mathrm{tr}\,e^{-\beta gV}\delta_\Lambda V e^{\beta gV}\\&\qquad\qquad\times\Big(\big[\mathcal{D}^\alpha V,W_\alpha\big]+\big[\overline{D}_{\dot\alpha}V,\overline{W}'^{\dot\alpha}\big]+\big\{V,\mathcal{D}^\alpha W_\alpha\big\}\Big)_{V\to gV}.\end{aligned}\tag{16}$$

Here, as indicated, the quantities inside the round bracket are defined by substituting the gauge superfield V involved by gV. On the other hand, the gauge variation $\delta_\Lambda V$ is given by Eq. (5) as it stands *without* setting $V\to gV$. It is obvious that our consistent anomaly is proportional to the anomaly d^{abc}, as expected.

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by Y. Ohshima, K. Okuyama, H. Suzuki, and H. Yasuta

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by T. Kugo and K. Ohashi

tant action is characterized solely by this cubic polynomial $\mathcal{N}(M)$, and we find the vector multiplet action

$$
\begin{aligned}
&e^{-1}\mathcal{L}_{\rm VL} \\
&\quad = +\tfrac{1}{2}\mathcal{N}\Big(4C+16t\cdot t+\tfrac{1}{2\alpha}F_{ab}(A)(4v^{ab}+i\bar\psi_c\gamma^{abcd}\psi_d)-8i\bar\psi\cdot\gamma\chi-4i\bar\psi_a\gamma^{ab}t\psi_b\Big) \\
&\qquad -\mathcal{N}_I\left(\begin{array}{c} 2t\cdot Y^I-8i\bar\Omega^I\chi-4i\bar\psi\cdot\gamma t\Omega^I+ig[\bar\Omega,\,\Omega]^I \\ -G^I_{ab}(W)(v^{ab}+\tfrac{i}{4}\bar\psi_c\gamma^{abcd}\psi_d) \end{array}\right) \\
&\qquad -\tfrac{1}{2}\mathcal{N}_{IJ}\left(\begin{array}{c} -\tfrac{1}{4}G^I(W)\cdot G^J(W)+\tfrac{1}{2}\mathcal{D}_aM^I\mathcal{D}^aM^J-Y^I\cdot Y^J \\ +2i\bar\Omega^I(\not{\mathcal{D}}-\tfrac{1}{2}\gamma\cdot v+t)\Omega^J+i\bar\psi_a(\gamma\cdot G(W)-2\not{\mathcal{D}}M)^I\gamma^a\Omega^J \\ -2(\bar\Omega^I\gamma^a\gamma^{bc}\psi_a)(\bar\psi_b\gamma_c\Omega^J)+2(\bar\Omega^I\gamma^a\gamma^b\psi_a)(\bar\psi_b\Omega^J) \end{array}\right) \\
&\qquad -\mathcal{N}_{IJK}\left(\begin{array}{c} -i\bar\Omega^I(\tfrac{1}{4}\gamma\cdot G(W)+Y)^J\Omega^K \\ +\tfrac{2}{3}(\bar\Omega^I\gamma^{ab}\Omega^J)(\bar\psi_a\gamma_b\Omega^K)+\tfrac{2}{3}(\bar\psi^i\cdot\gamma\Omega^{Ij})(\bar\Omega^J_{(i}\Omega^K_{j)}) \end{array}\right) \\
&\qquad + e^{-1}\mathcal{L}_{\text{C-S}}\ , \qquad\qquad (2\cdot 11)
\end{aligned}
$$

where $\mathcal{N}_I=\partial\mathcal{N}/\partial M^I$, $\mathcal{N}_{IJ}=\partial^2\mathcal{N}/\partial M^I\partial M^J$, etc., and $\mathcal{L}_{\text{C-S}}$ is the Chern-Simons term:

$$
\begin{aligned}
\mathcal{L}_{\text{C-S}} = \tfrac{1}{8}c_{IJK}\epsilon^{\lambda\mu\nu\rho\sigma}W^I_\lambda\Big(&F^J_{\mu\nu}(W)F^K_{\rho\sigma}(W)+\tfrac{1}{2}g[W_\mu,W_\nu]^JF^K_{\rho\sigma}(W) \\
&+\tfrac{1}{10}g^2[W_\mu,W_\nu]^J[W_\rho,W_\sigma]^K\Big)\,. \qquad (2\cdot 12)
\end{aligned}
$$

We have checked the supersymmetry invariance of this action for general non-Abelian cases as follows. When the gauge coupling g is set equal to zero, the action reduces to one with the same form as that for the Abelian case, and thus the invariance is guaranteed by the above derivation. When g is switched on, the covariant derivative $\mathcal{D}_\mu$ comes to include the G-covariantization term $-g\delta_G(W_\mu)$, and the field strength $F_{\mu\nu}(W)$ comes to include the non-Abelian term $-g[W_\mu,W_\nu]$. We, however, can use the variables $\mathcal{D}_\mu\phi$ ($\phi=M^I$, Ω^I) and $F_{\mu\nu}(W)$ as they stand in the action and in the supersymmetry transformation laws, keeping these g-dependent terms implicit inside of them. Then, we have only to keep track of *explicitly* g-dependent terms and make sure that these terms vanish in the supersymmetry transformation of the action. The explicitly g-dependent terms in the action are only the term $-ig\mathcal{N}_I[\bar\Omega,\Omega]^I$, aside from those in the Chern-Simons term. The Chern-Simons term is special because it contains the gauge field W^I_μ explicitly, and its supersymmetry transformation as a whole yields no explicit g-dependent terms, as we show below. In the supersymmetry transformations $\delta\phi$, explicitly g-dependent terms do not appear for $\phi=M^I$, Ω^I, $G^I_{\mu\nu}(W)$ or $F^I_{\mu\nu}(W)$, but appear only in δY^{Iij}, $\delta(\mathcal{D}_\mu M^I)$ and $\delta(\mathcal{D}_\mu\Omega^I)$. (For the latter

M/CY$_3$

$$C^{(3)} dC^{(3)} dC^{(3)} \qquad\qquad A = \int_{Q_2} C^{(3)}$$

Dp-brane

$$\int G_C d^{-1} \,\mathrm{tr}\left(\exp \frac{iF}{2\pi}\right)$$

Orbifolding

$$V(x,y) \;=\; V(x,-y)$$
$$\Phi(x,y) \;=\; -\Phi(x,-y)$$

Color $\times$ Hypercolor: SU$(3)_C\times$SU$(5)_H$

$$Q = ({Q^i}_A, {Q^4}_A)\text{:} \quad (\mathbf{3}+\mathbf{1},\mathbf{5}^*)$$
$$\bar{Q} = ({\bar{Q}_i}{}^A, {\bar{Q}_4}{}^A)\text{:} \quad (\mathbf{3}^*+\mathbf{1},\mathbf{5})$$

SU(5)$_H$ dynamics

$$W_{eff} = \frac{\Lambda_H^{11}}{\det Q_A \bar{Q}^A}$$

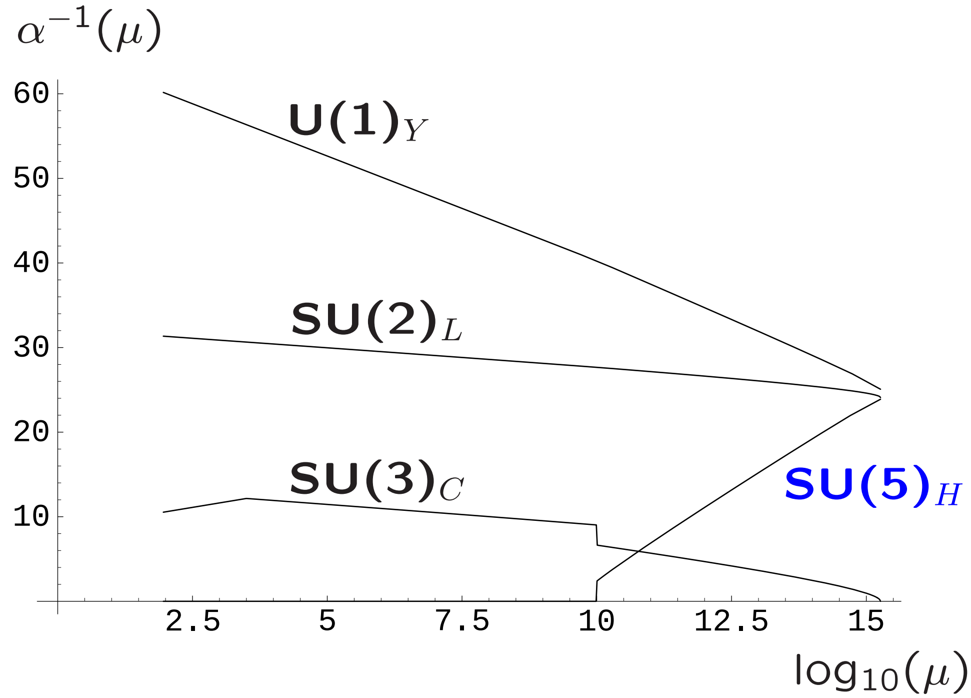
soft masses $V = m^2|Q|^2 + m^2|\bar{Q}|$

$$\left\langle Q_A \bar{Q}^A \right\rangle \sim \left(\frac{\Lambda_H^{11}}{m} \right)^{\frac{1}{5}}$$

$$(\mathbf{3}+\mathbf{1}) \times (\mathbf{3}^* + \mathbf{1}) = \mathbf{8} + \mathbf{3} + \mathbf{3}^* + \mathbf{1} + \mathbf{1}$$

$$\int_{y=0} d^4x d^2\theta d^2\bar{\theta} \frac{1}{M_*} Q^\dagger Q \bar{d}$$

$$\int_{y=0} d^4x d^2\theta d^2\bar{\theta} \frac{1}{M_*^4} (Q)^3 Q D^2 Q$$



$\alpha^{-1}(\mu)$
60
50
40
30
20
10
U(1)$_Y$
SU(2)$_L$
SU(3)$_C$
SU(5)$_H$
2.5
5
7.5
10
12.5
15
$\log_{10}(\mu)$

String/M Theory Realizations?

QCD	Gravity
strong CP	cosmological constant

background configurations

intersecting branes	flux vacua

objects inside

chiral fermions	anti-D branes