標準的な断熱ポンプ：1D class D

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複素フェルミオン$a_j^\dagger/a_j$に対して，マヨラナフェルミオンを

$$c_{\sigma,2j-1}=a_{\sigma,j}+a^\dagger_{\sigma,j},\quad c_{\sigma,2j}=\frac{a_{\sigma,j}-a^\dagger_{\sigma,j}}{i} \tag{1}$$

と導入する．

模型は以下：

$$H(\theta)=\begin{cases} H_I(\theta), & (\theta\in[0,\pi]),\\ H_{II}(\theta), & (\theta\in[\pi,2\pi]),\end{cases} \tag{2}$$

$$H_I(\theta)=\frac{i}{2}\sum_j\left[\cos\theta(c_{\uparrow,2j-1}c_{\uparrow,2j}+c_{\downarrow,2j-1}c_{\downarrow,2j})+\sin\theta(c_{\uparrow,2j-1}c_{\downarrow,2j-1}+c_{\downarrow,2j}c_{\uparrow,2j})\right], \tag{3}$$

$$H_{II}(\theta)=\frac{i}{2}\sum_j\left[\cos\theta(c_{\uparrow,2j-1}c_{\uparrow,2j}+c_{\downarrow,2j-1}c_{\downarrow,2j})+\sin\theta(c_{\uparrow,2j-1}c_{\downarrow,2j+1}+c_{\downarrow,2j+2}c_{\uparrow,2j})\right]. \tag{4}$$

- $\theta=0\to2\pi$なるtextureをひとつ導入した基底状態のフェルミオン・パリティが(-1)であることが確認できる．

BdGハミルトニアンを計算しよう．南部表示$\psi_{\sigma,j}=(a_{\sigma,j},a^\dagger_{\sigma,j})^T$を導入する．

$$\begin{aligned}\frac{i}{2}\sum_j c_{\sigma,2j-1}c_{\sigma,2j}&=\frac{1}{2}\sum_j(a_{\sigma,j}+a^\dagger_{\sigma,j})(a_{\sigma,j}-a^\dagger_{\sigma,j}) && (5)\\ &=\frac{1}{2}\sum_j(a^\dagger_{\sigma,j}a_{\sigma,j}-a_{\sigma,j}a^\dagger_{\sigma,j}) && (6)\\ &=\frac{1}{2}\sum_j\psi^\dagger_{\sigma,j}\tau_z\psi_{\sigma,j}, && (7)\end{aligned}$$

$$\begin{aligned}\frac{i}{2}\sum_j(c_{\uparrow,2j-1}c_{\downarrow,2j-1}+c_{\downarrow,2j}c_{\uparrow,2j})&=\frac{i}{2}\sum_j\{(a_{\uparrow,j}+a^\dagger_{\uparrow,j})(a_{\downarrow,j}+a^\dagger_{\downarrow,j})-(a_{\downarrow,j}-a^\dagger_{\downarrow,j})(a_{\uparrow,j}-a^\dagger_{\uparrow,j})\} && (8)\\ &=\frac{1}{2}\sum_j(ia_{\uparrow,j}a_{\downarrow,j}-ia_{\downarrow,j}a_{\uparrow,j}+ia^\dagger_{\uparrow,j}a^\dagger_{\downarrow,j}-ia^\dagger_{\downarrow,j}a^\dagger_{\uparrow,j}) && (9)\\ &=\frac{1}{2}\sum_j(a^\dagger_{\uparrow,j},a^\dagger_{\downarrow,j},a_{\uparrow,j},a_{\downarrow,j})\begin{pmatrix} & & & i\\ & & -i & \\ & i & & \\ -i & & & \end{pmatrix}\begin{pmatrix}a_{\uparrow,j}\\ a_{\downarrow,j}\\ a^\dagger_{\uparrow,j}\\ a^\dagger_{\downarrow,j}\end{pmatrix} && (10)\\ &=\frac{1}{2}\sum_j\psi^\dagger_j(-\tau_x\sigma_y)\psi_j, && (11)\end{aligned}$$

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\begin{aligned}
\frac{i}{2}\sum_j (c_{\uparrow,2j-1}c_{\downarrow,2j+1}+c_{\downarrow,2j+2}c_{\uparrow,2j}) &= \frac{i}{2}\sum_j \{(a_{\uparrow,j}+a^\dagger_{\uparrow,j})(a_{\downarrow,j+1}+a^\dagger_{\downarrow,j+1})-(a_{\downarrow,j+1}-a^\dagger_{\downarrow,j+1})(a_{\uparrow,j}-a^\dagger_{\uparrow,j})\} \qquad (12)\\
&= \frac{1}{2}\sum_j (ia_{\uparrow,j}a_{\downarrow,j+1}-ia_{\downarrow,j+1}a_{\uparrow,j}+ia^\dagger_{\uparrow,j}a^\dagger_{\downarrow,j+1}-ia^\dagger_{\downarrow,j+1}a^\dagger_{\uparrow,j}) \qquad (13)\\
&= \frac{1}{2}\sum_j (ie^{ik}a_{\uparrow,-k}a_{\downarrow,k}-ie^{-ik}a_{\downarrow,-k}a_{\uparrow,k}+ie^{ik}a^\dagger_{\uparrow,k}a^\dagger_{\downarrow,-k}-ie^{-ik}a^\dagger_{\downarrow,k}a^\dagger_{\uparrow,-k}) \qquad (14)\\
&= \frac{1}{2}\sum_k (a^\dagger_{\uparrow,k},a^\dagger_{\downarrow,k},a_{\uparrow,-k},a_{\downarrow,-k})\begin{pmatrix} & & & ie^{ik}\\ & & -ie^{-ik} & \\ & ie^{ik} & & \\ -ie^{-ik} & & & \end{pmatrix}\begin{pmatrix} a_{\uparrow,k}\\ a_{\downarrow,k}\\ a^\dagger_{\uparrow,-k}\\ a^\dagger_{\downarrow,-k}\end{pmatrix} \qquad (15)\\
&= \frac{1}{2}\sum_k \psi^\dagger_k(-\cos k\tau_x\sigma_y-\sin k\tau_x\sigma_x)\psi_k. \qquad (16)
\end{aligned}
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ここでフーリエ変換を

$$
\psi_j=\sum_k \psi_k e^{ikj}=(a_k,a^\dagger_{-k}) \qquad (17)
$$

で導入した．よってBdGハミルトニアンは

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\begin{aligned}
\mathcal{H}_I(k,\theta) &= \cos\theta\tau_z-\sin\theta\tau_x\sigma_y, \qquad (18)\\
\mathcal{H}_{II}(k,\theta) &= \cos\theta\tau_z-\sin\theta(\cos k\tau_x\sigma_y+\sin k\tau_x\sigma_x). \qquad (19)
\end{aligned}
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- $(k.\theta)$空間で定義される$\mathbb{Z}_2$数は非自明？?