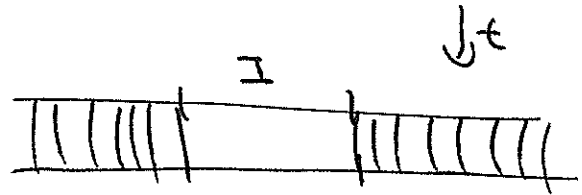
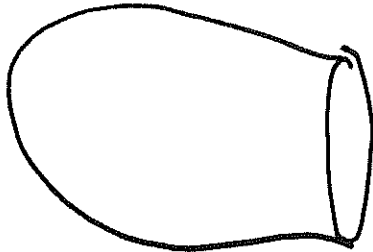


Reduced density matrices

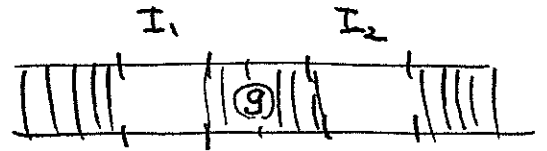
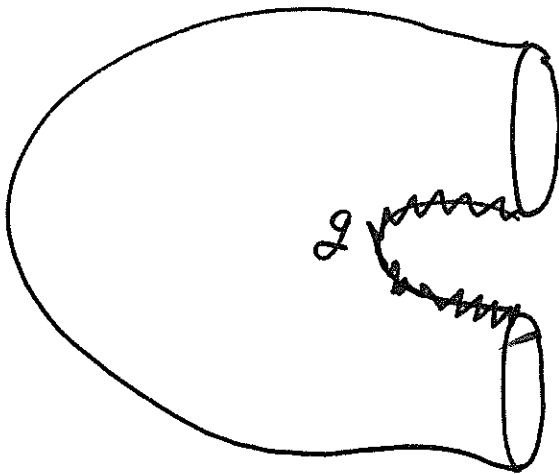
①

• Single interval



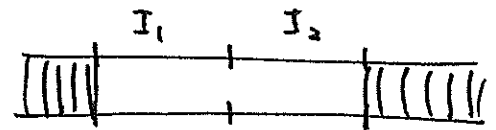
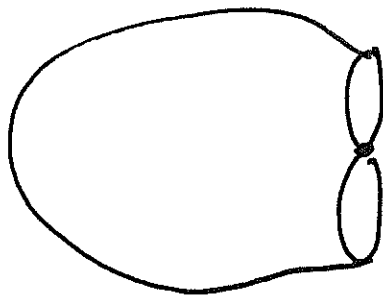
$$= \rho_I$$

• disjoint interval w/ intermediate flux



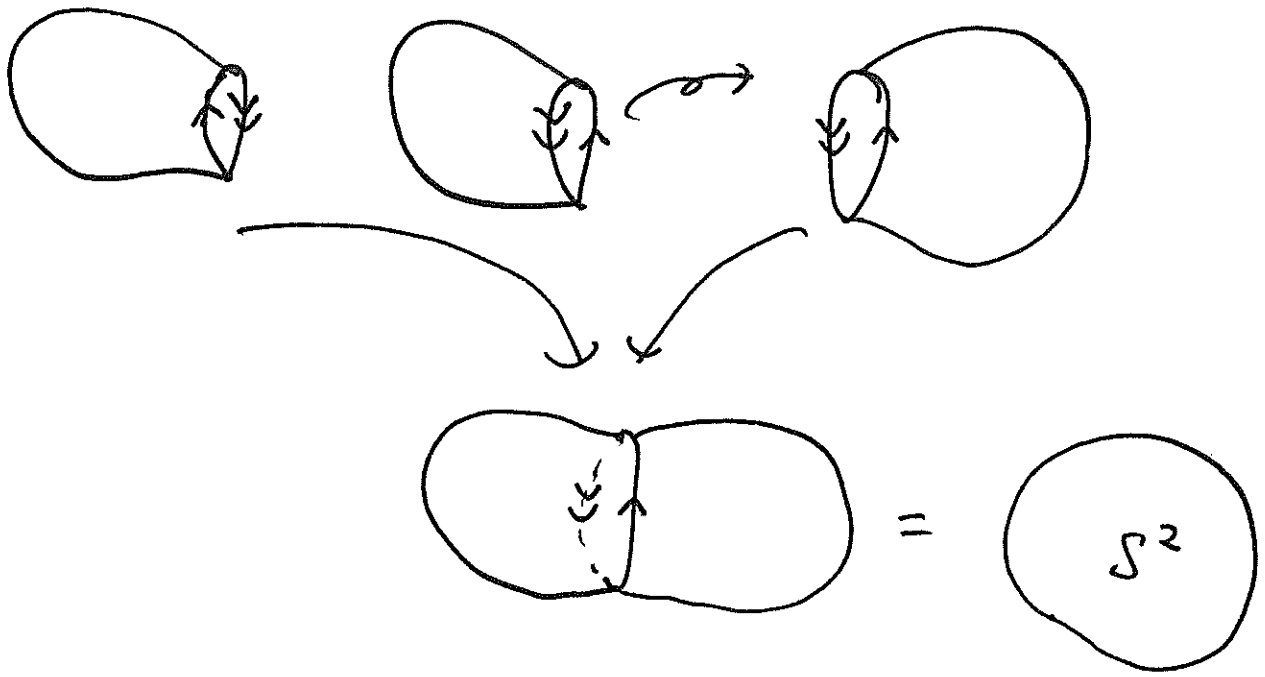
$$= \rho_{I_1, I_2, g}$$

• Adjacent intervals

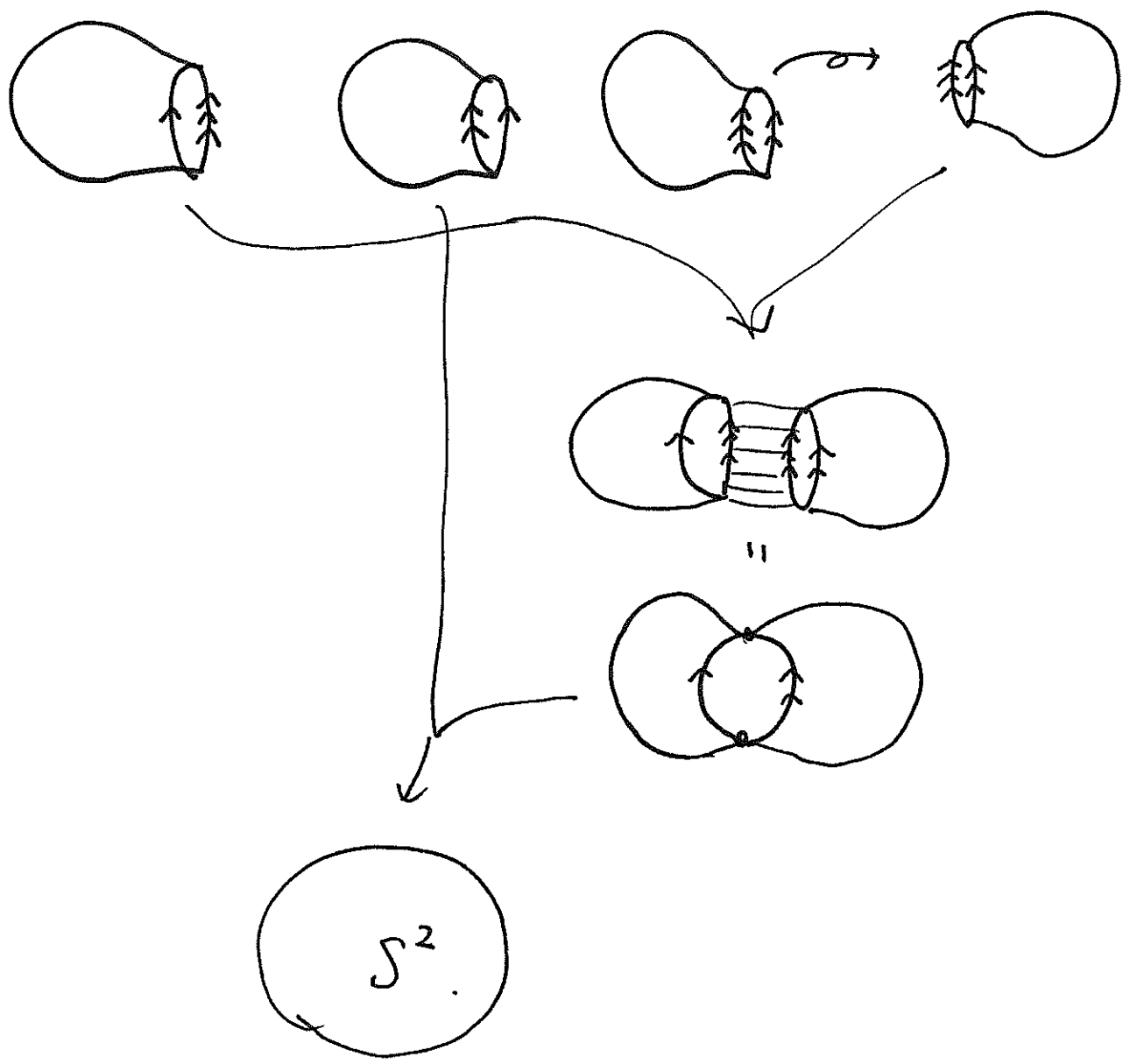


$$= \rho_{I_1, I_2}$$

• $tv P^2$, single interval



tr ρ^2 , single interval

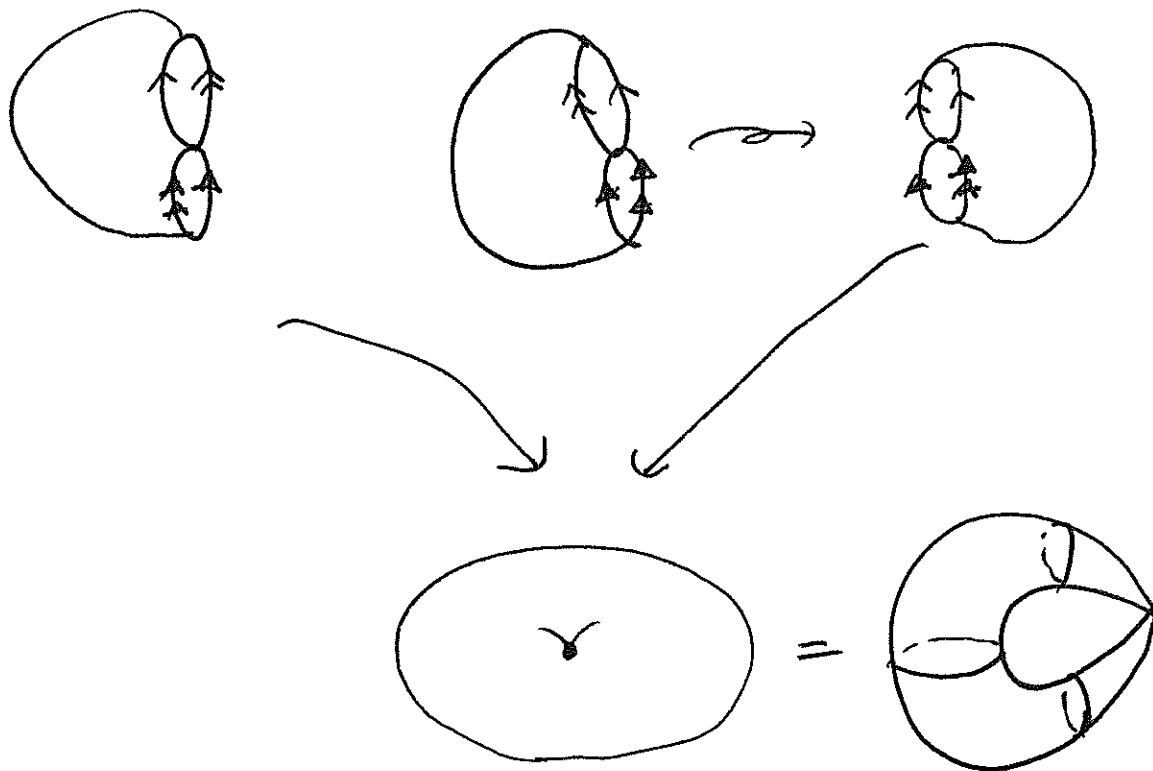


in general

tr $\rho^n = S^2$
 ↑
 single interval

• $tv \rho^2$, adjacent interval

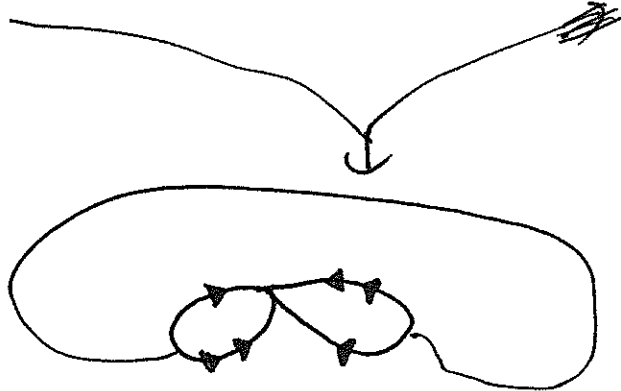
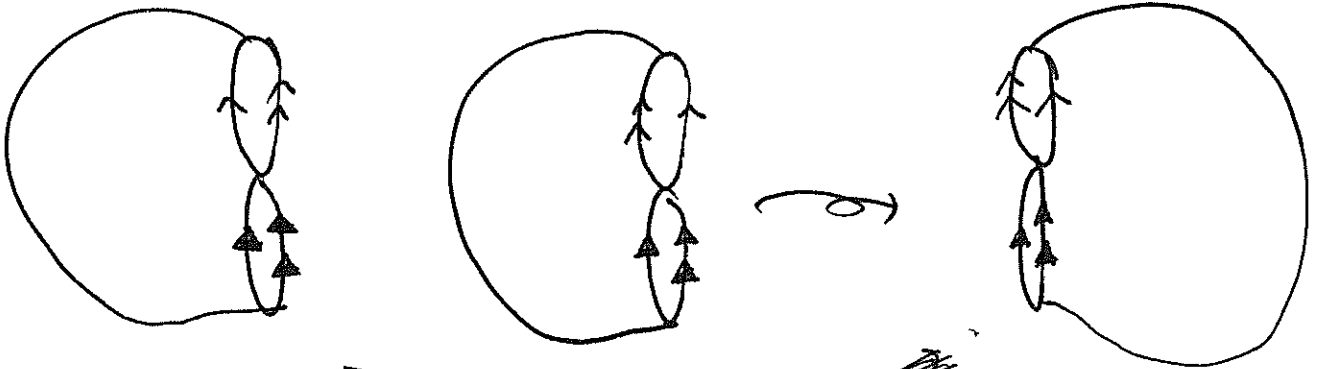
(4)



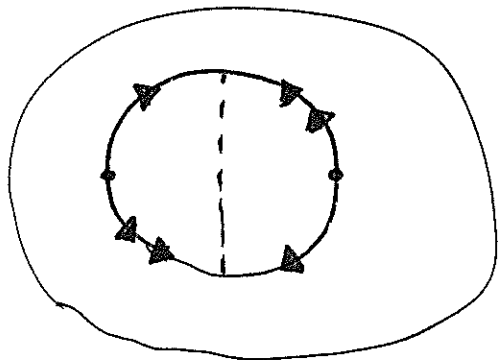
S^2 w/ 2 points identified.

• $tv (\rho^T)^2$, adjacent interval
 \parallel
 $tv \rho^2$.

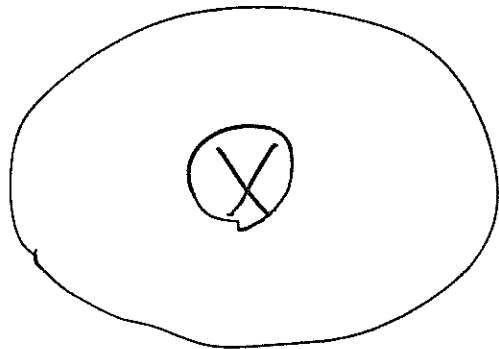
@ $\text{tr} [p U_T^I p^T (U_T^I)^T]$, adjacent interval (5)



"



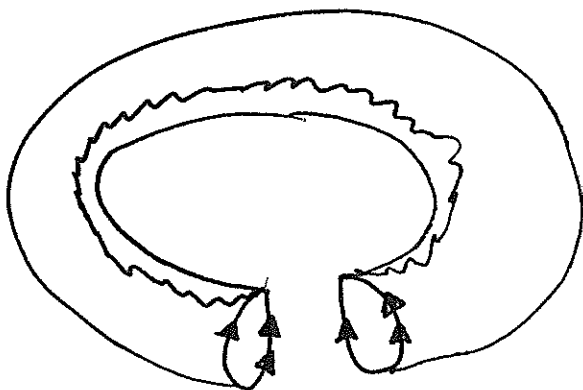
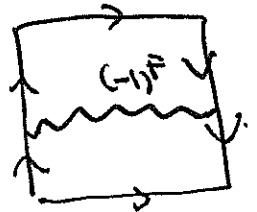
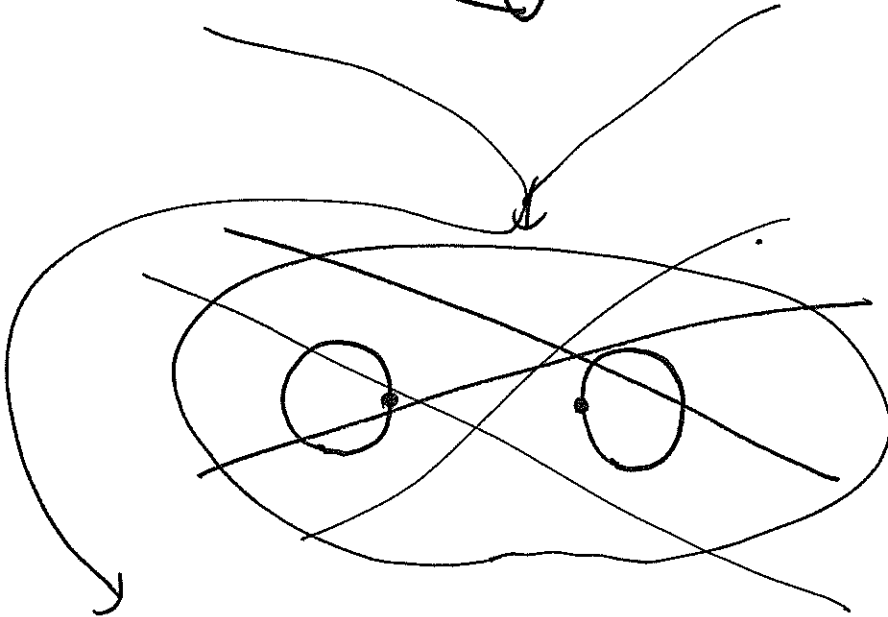
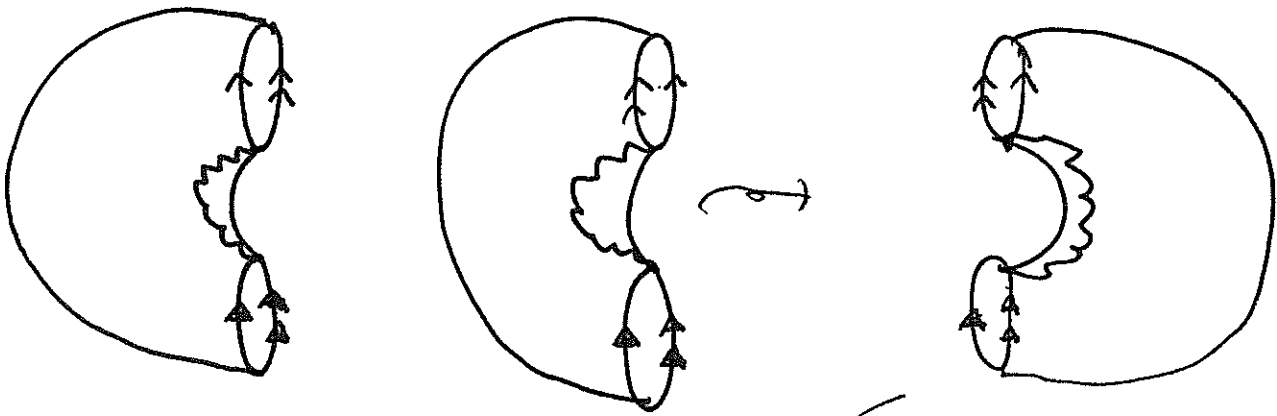
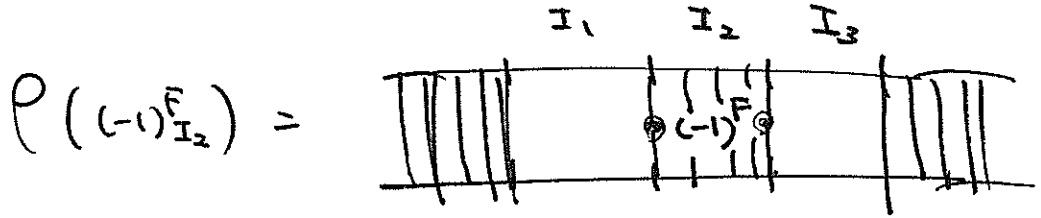
"



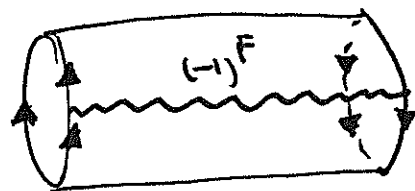
RP^2

$$\textcircled{6} \quad \text{tr} \left\{ \rho((-1)_{I_2}^F) U_T^{I_1} \rho((-1)_{I_2}^F)^{T_1} (U_T^{I_1})^\dagger \right\}$$

disjoint interval w/ intermediate fermion parity flip



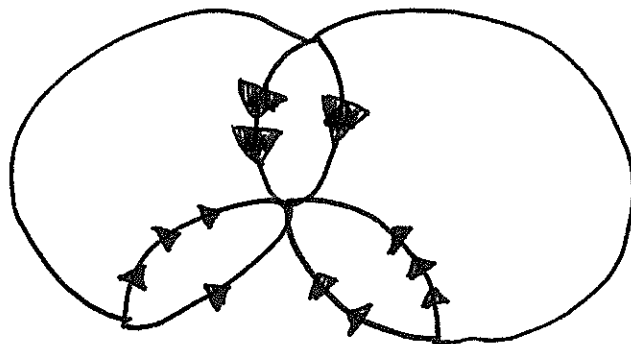
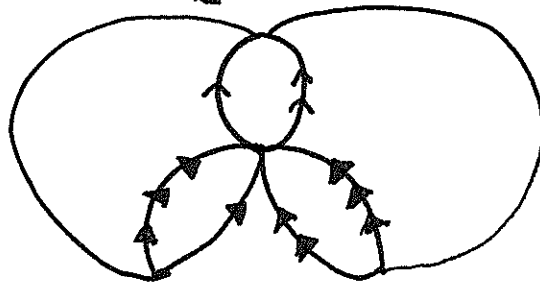
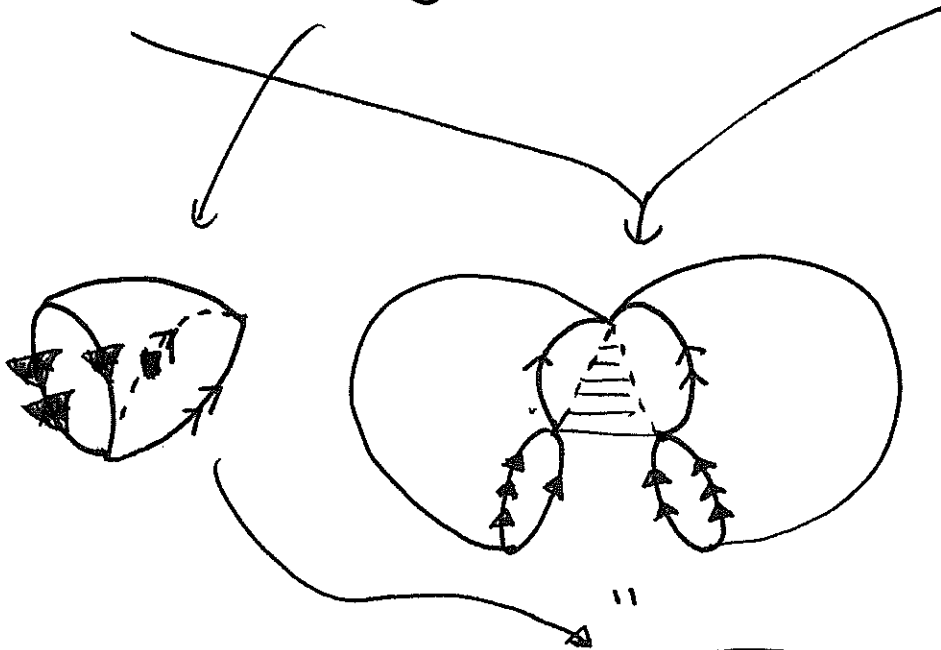
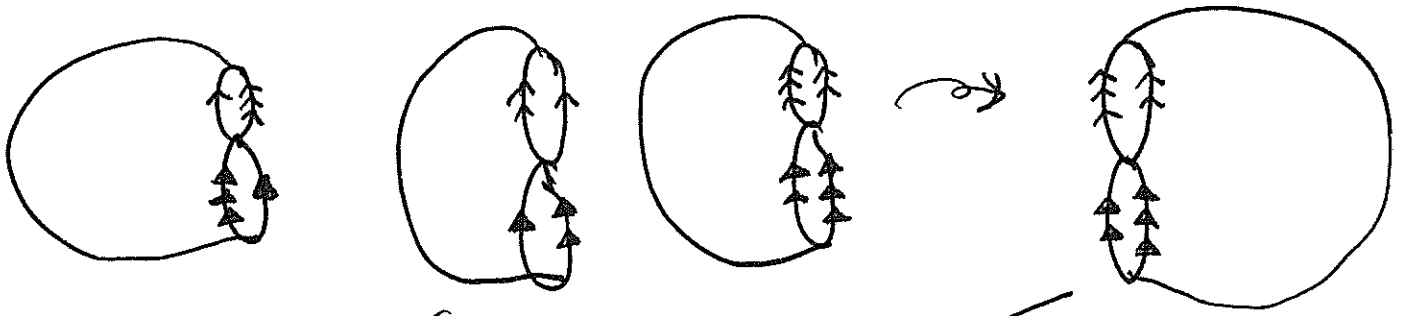
=



Klein bottle, RR sector.

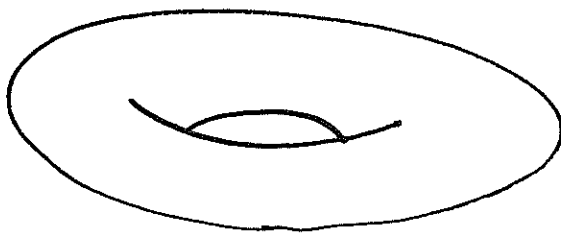
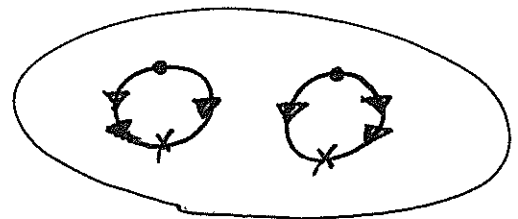
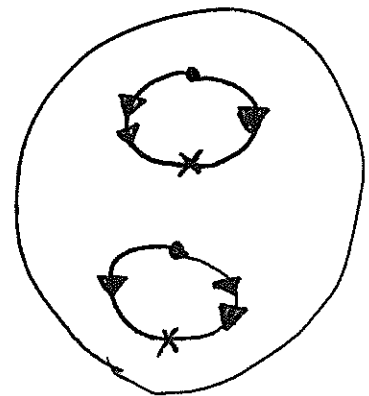
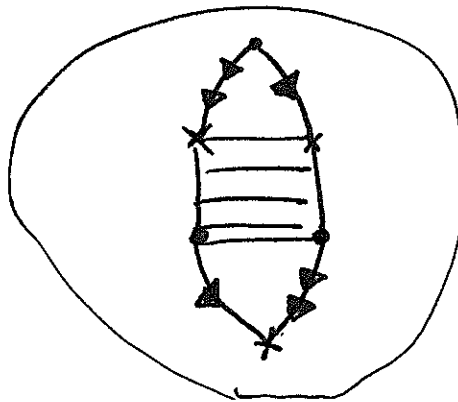
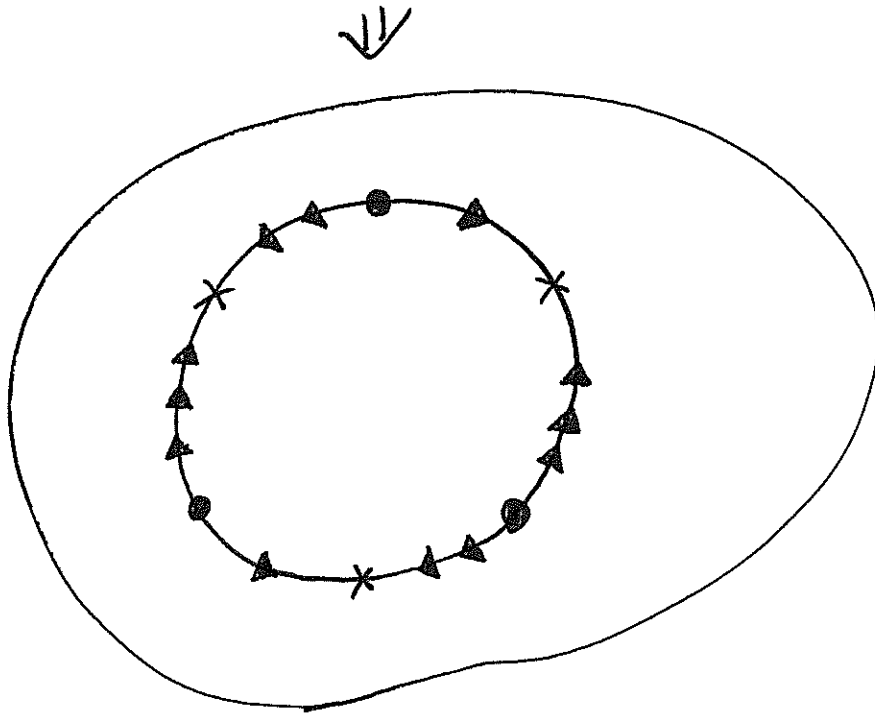
① $tr(PT^3)$, adjacent interval

⑦

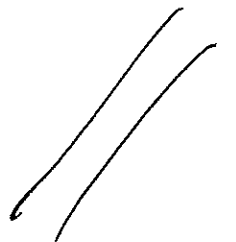


≡

8



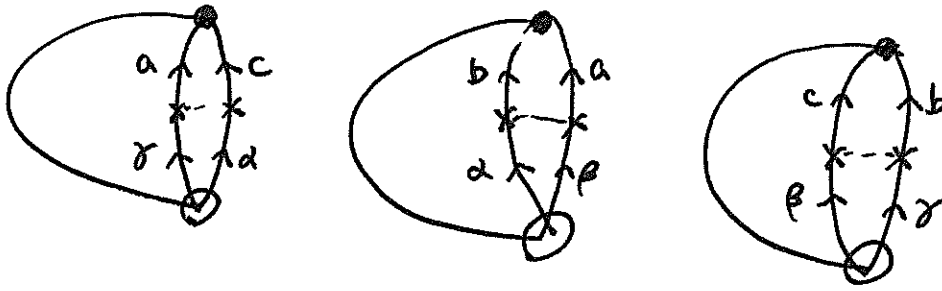
T²



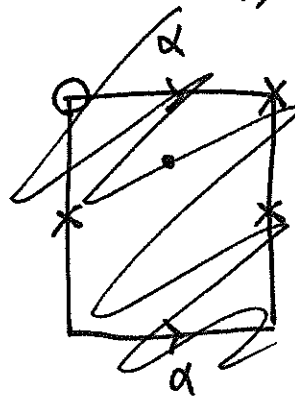
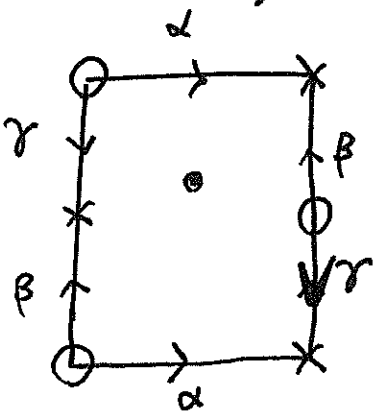
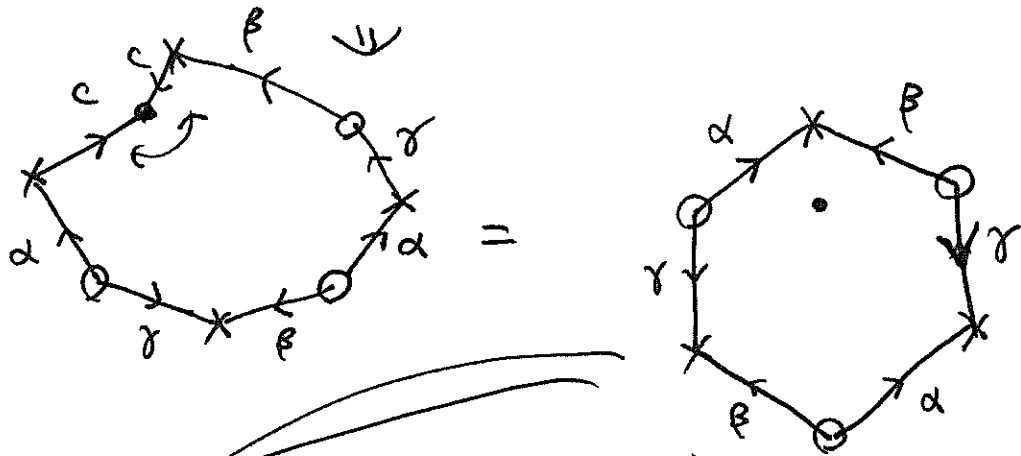
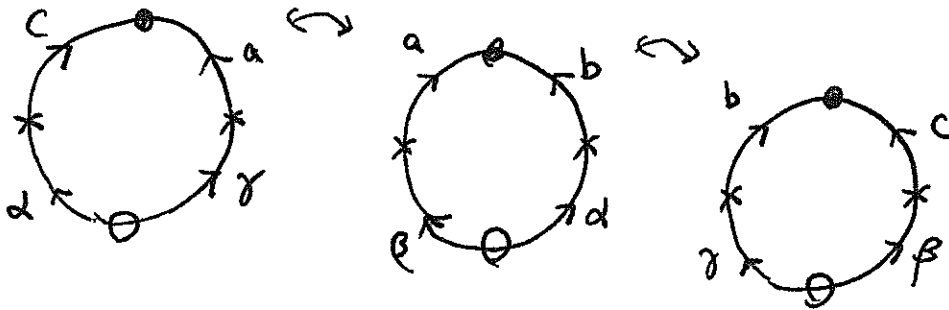
more on

① $tr[(\rho^T)^3]$. adjacent

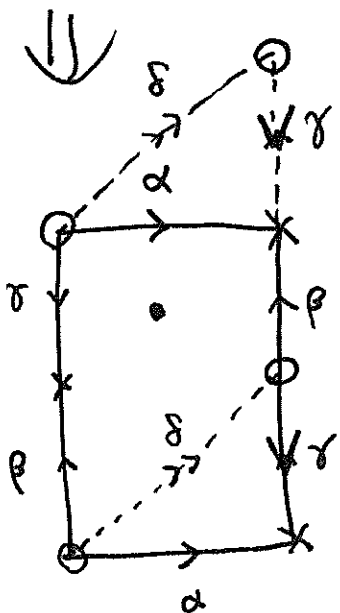
•, x, ⊙ : identified points (9)



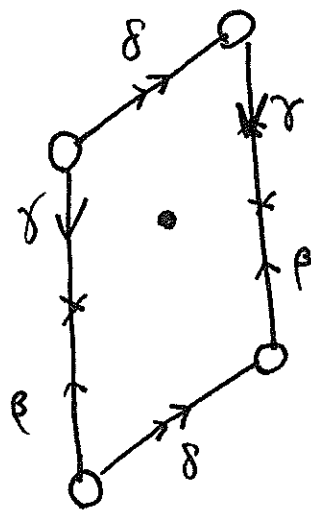
ii)



⇓



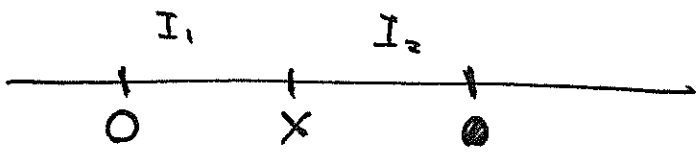
=



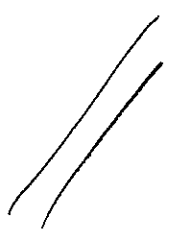
//



boundary conditions

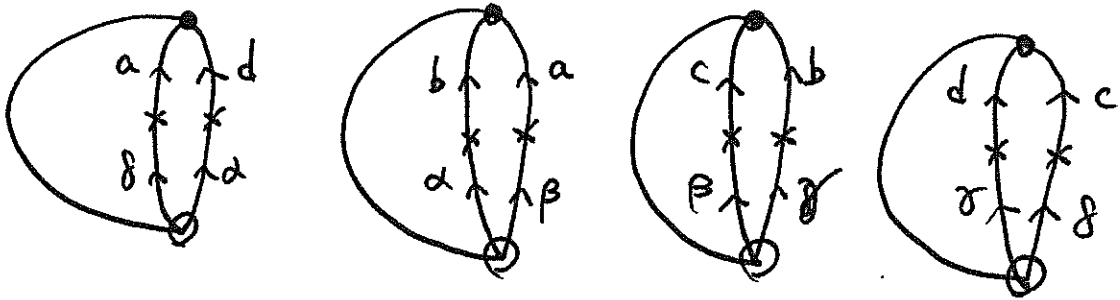


(~~Ohm~~ Ohmori - Tachikawa)

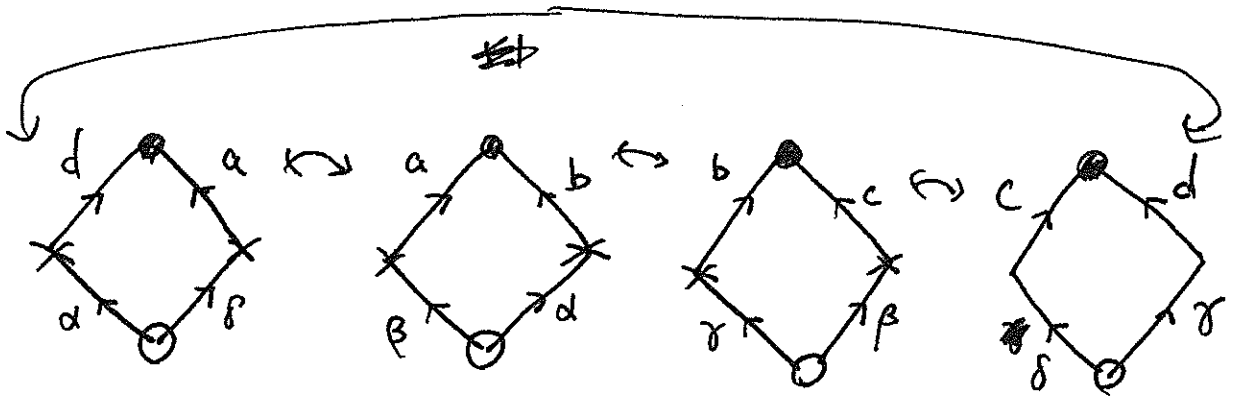


• $\text{tr}\{(P^T)^4\}$ adjacent

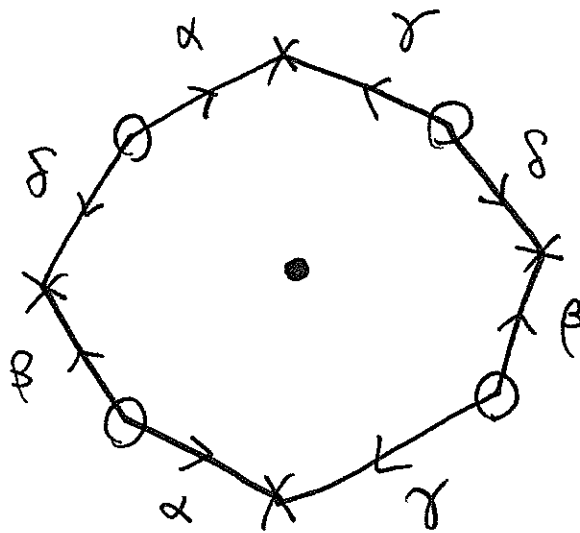
(11)



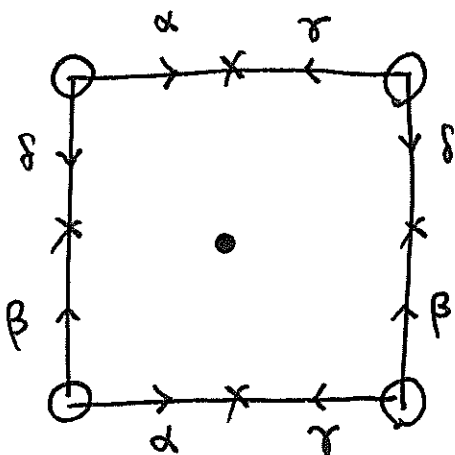
117



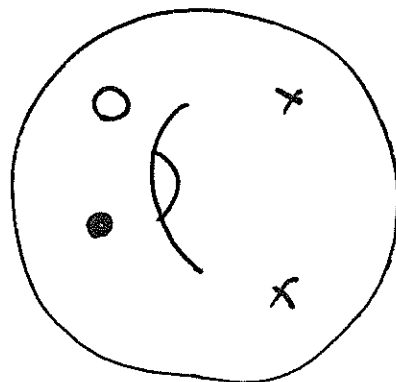
11



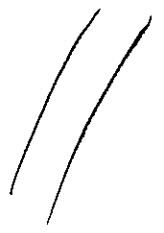
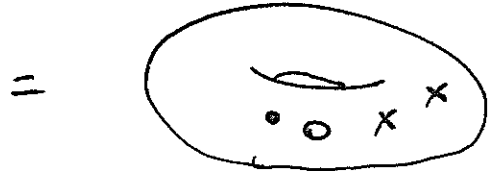
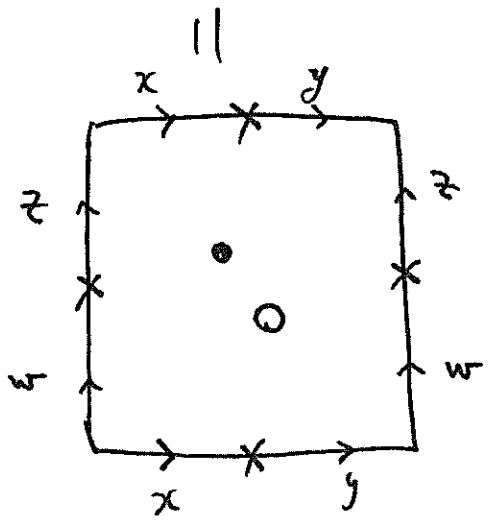
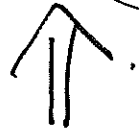
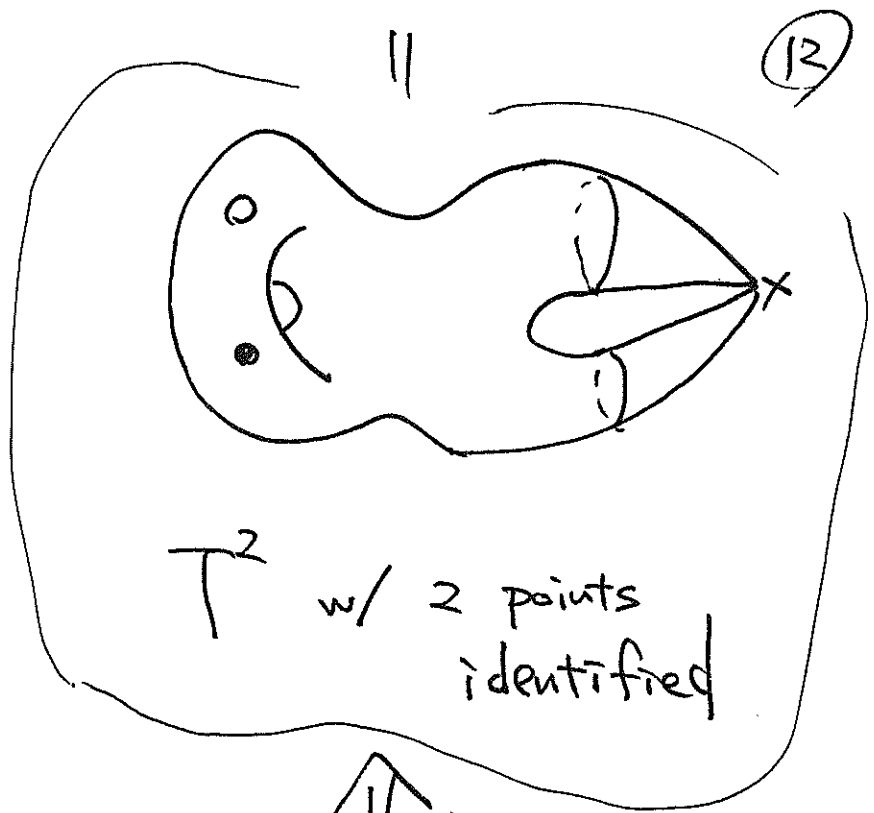
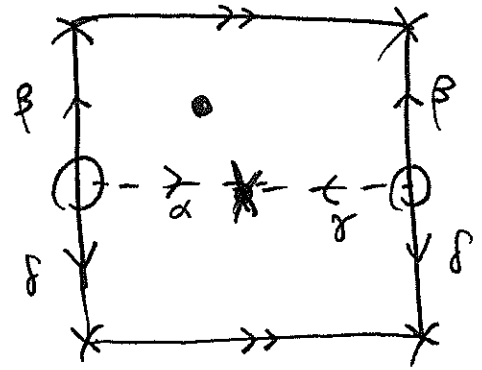
11



=

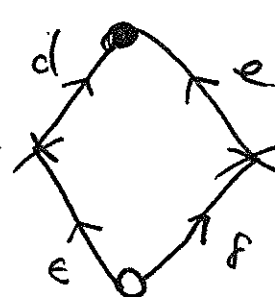
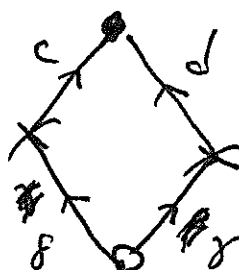
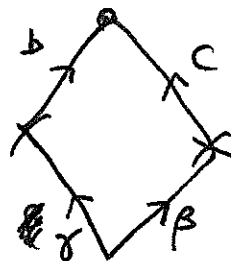
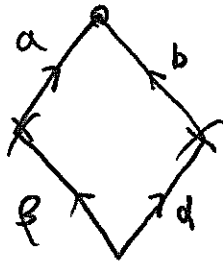
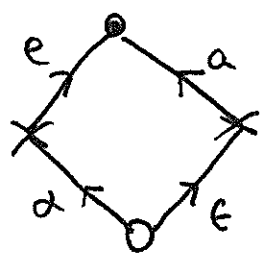


11

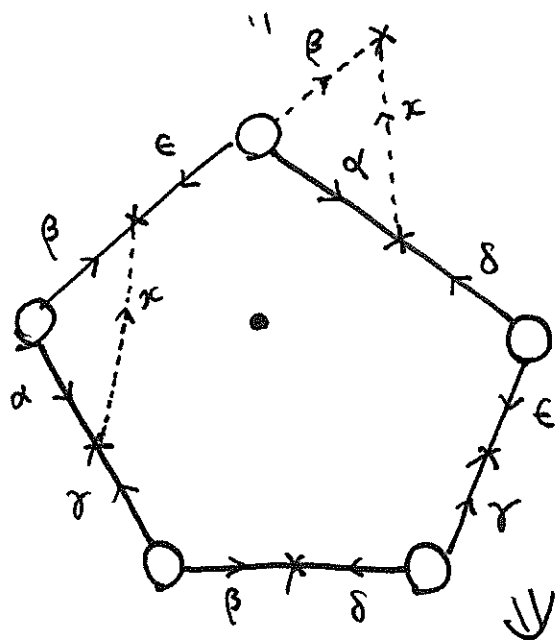
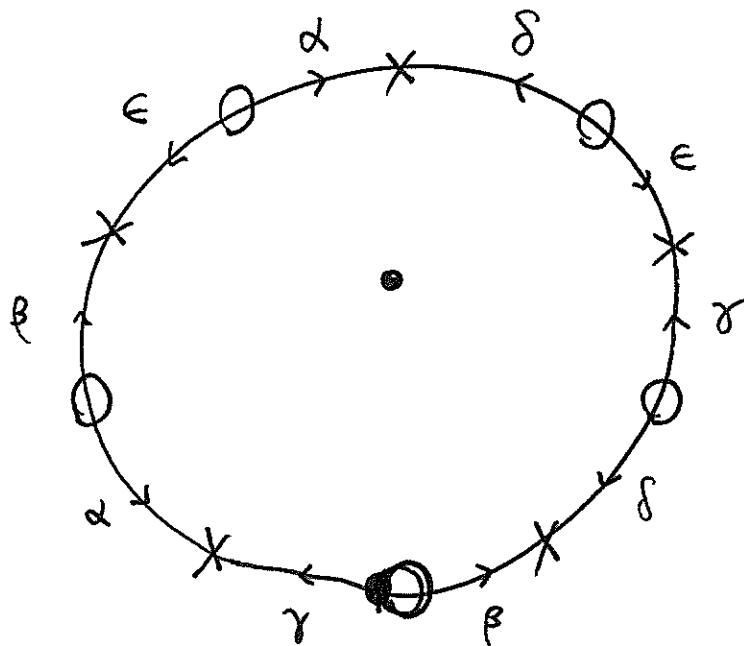


② $\text{tr}[(\rho^T)^5]$ adjacent

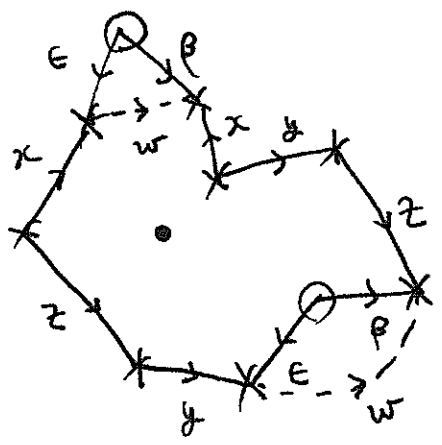
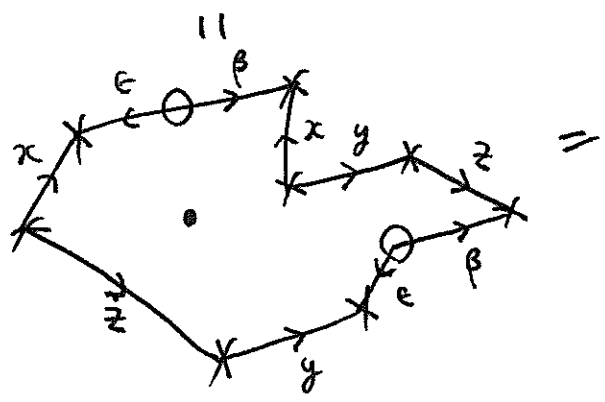
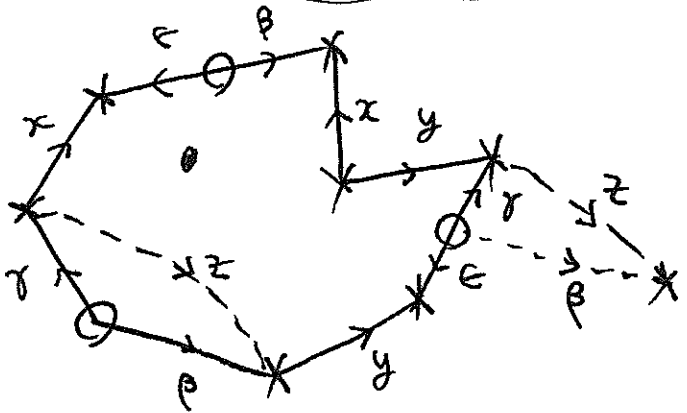
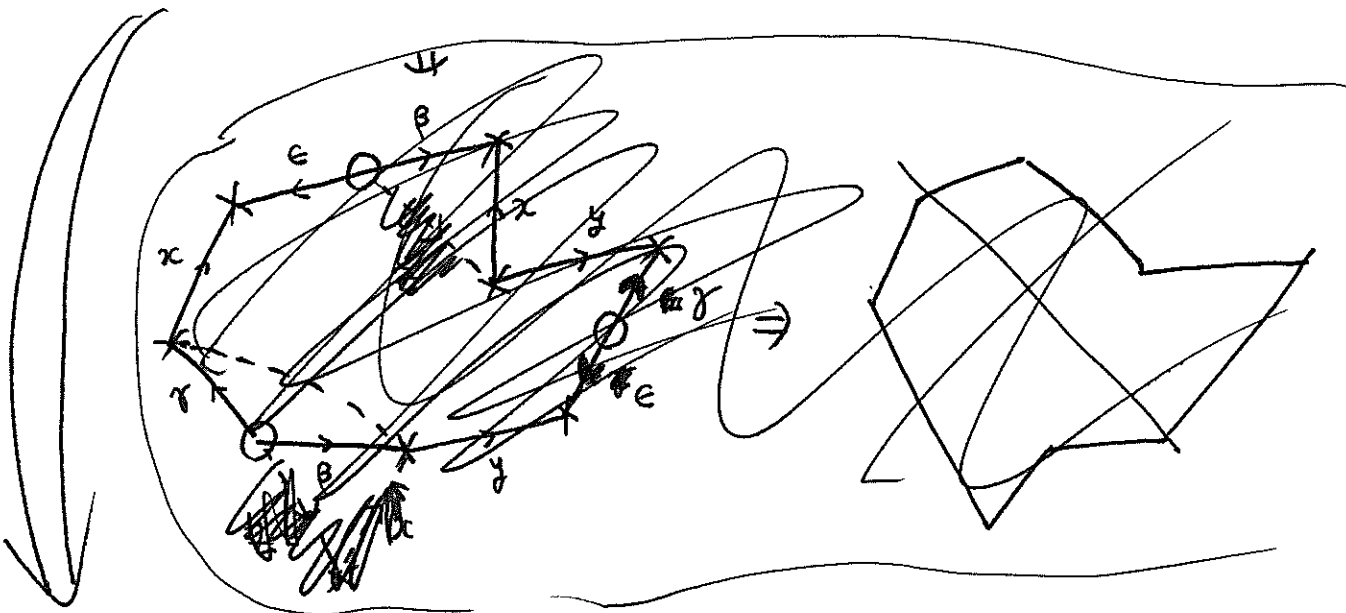
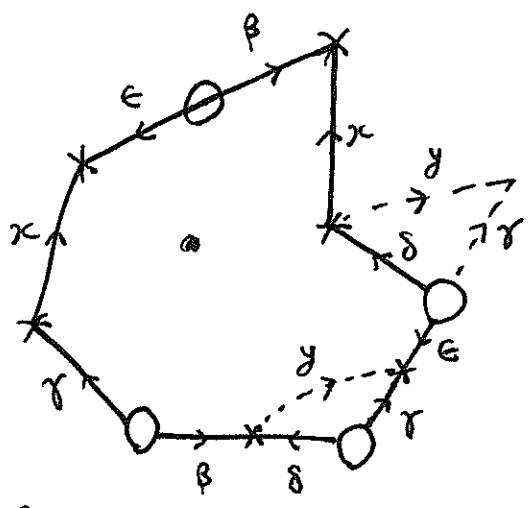
(13)



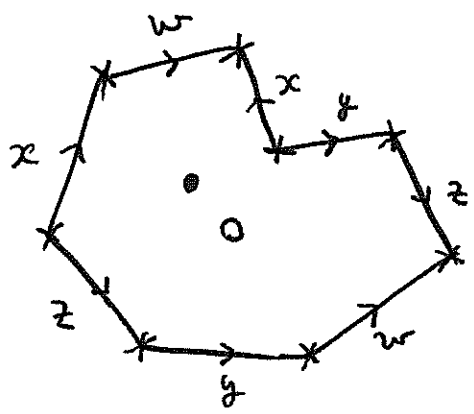
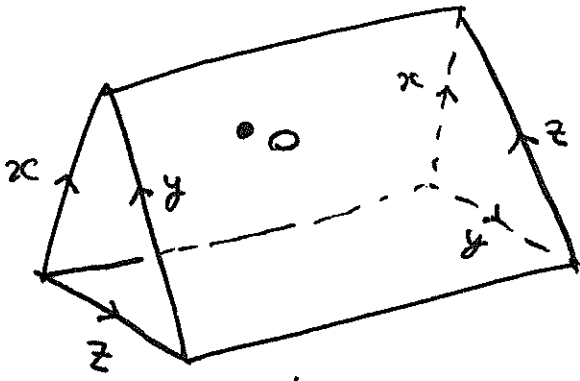
||



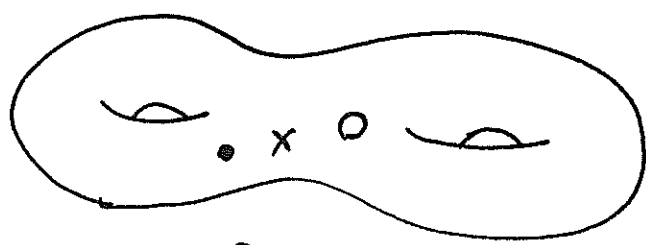
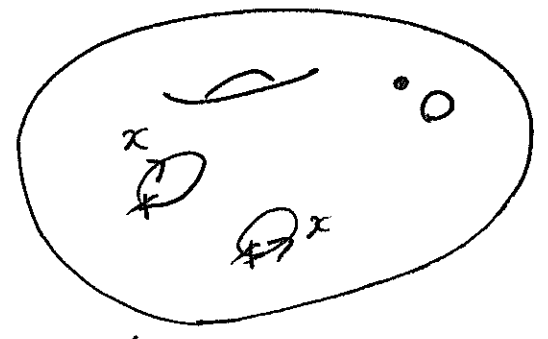
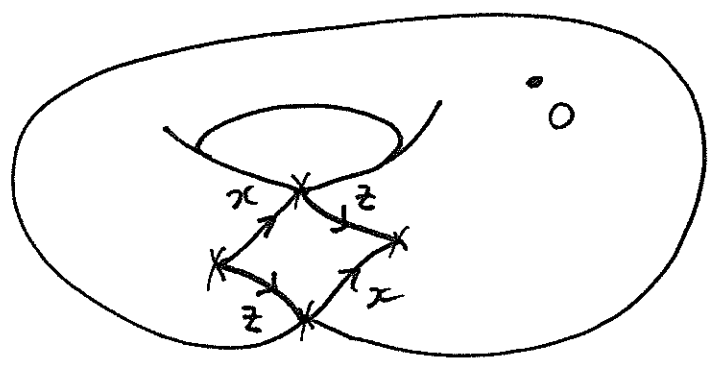
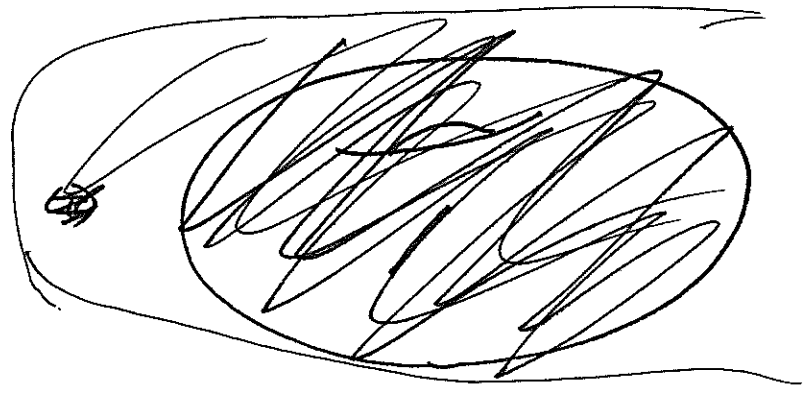
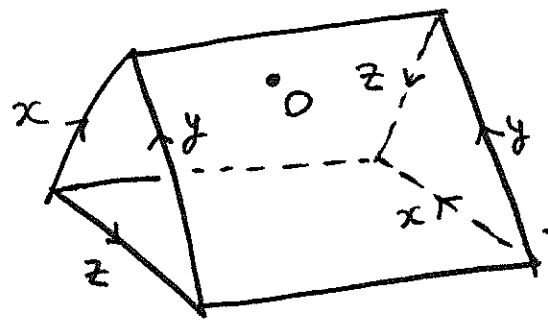
⇓



⇓



manifold?

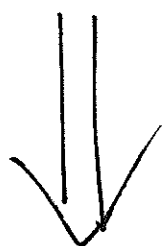
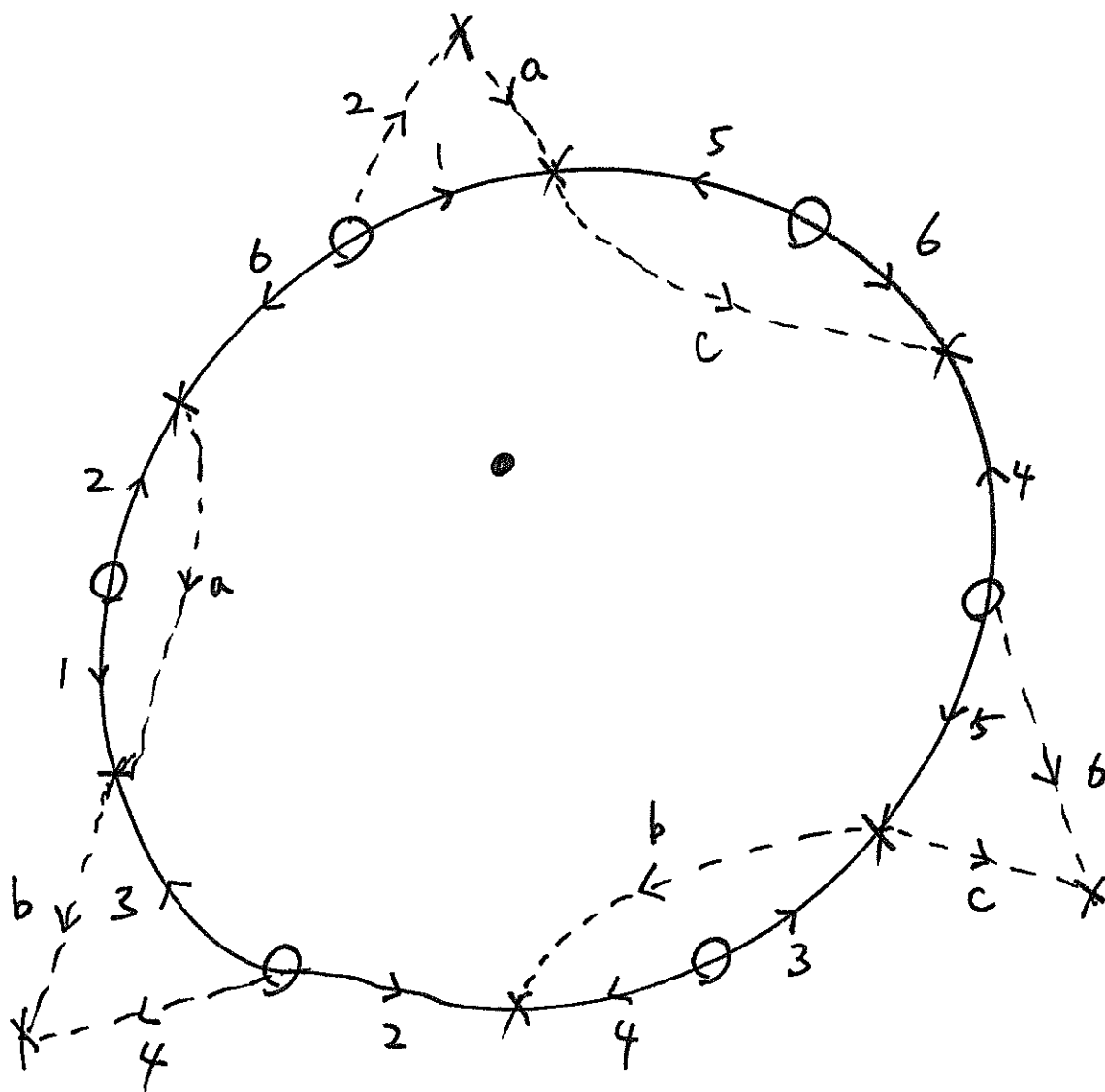


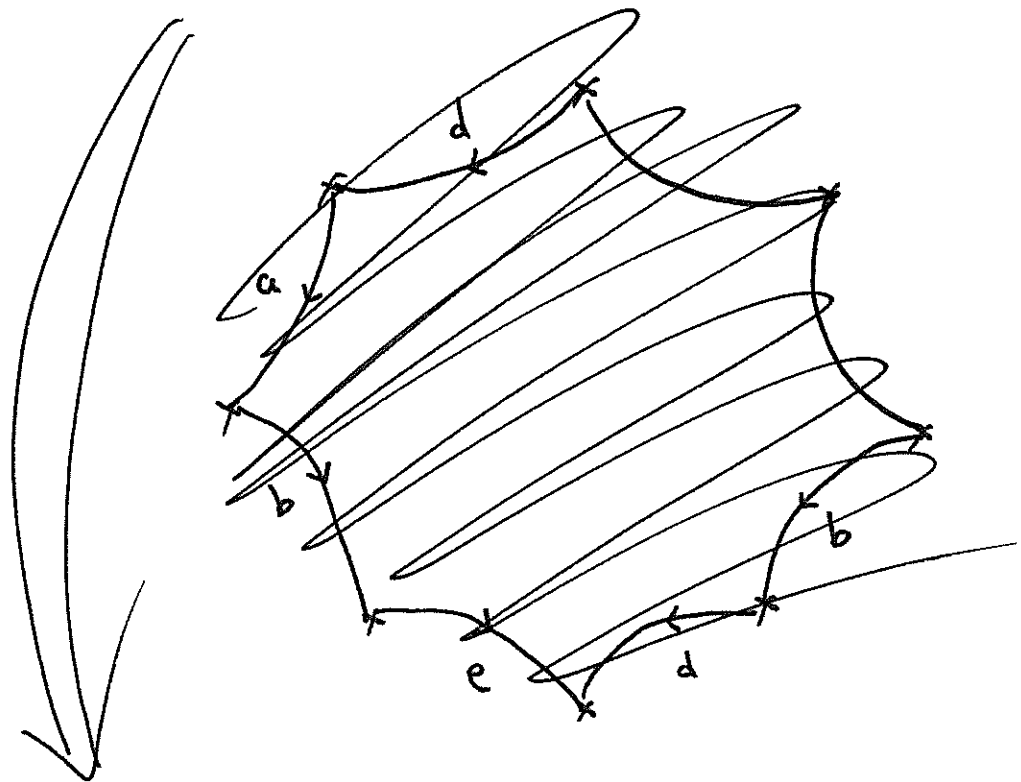
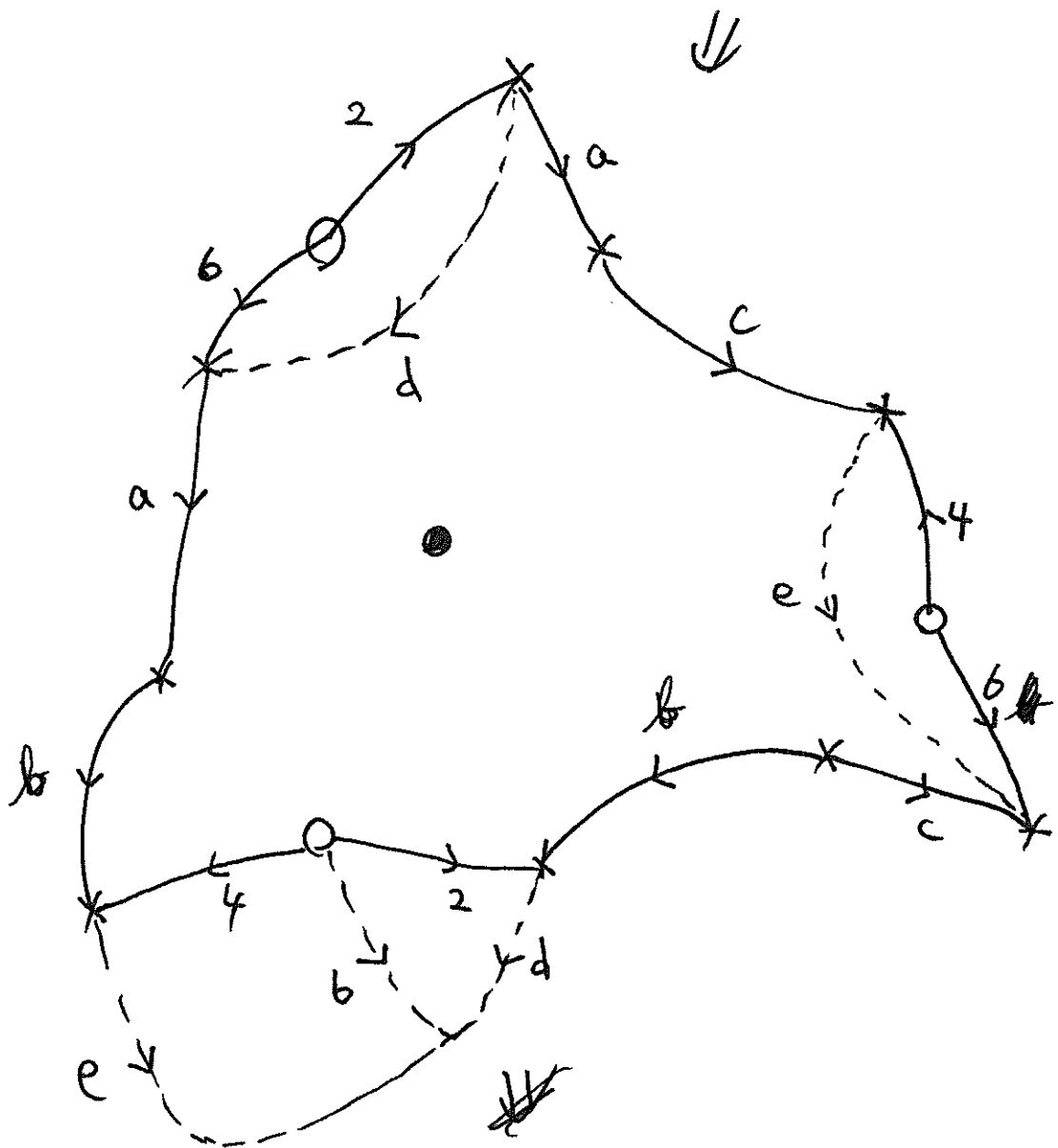
T^2

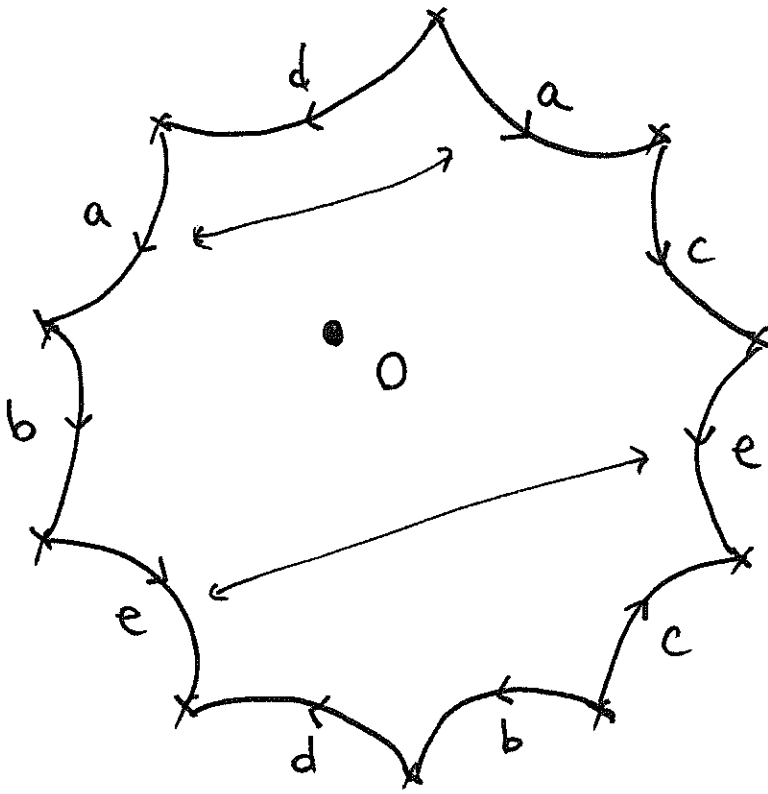


• $\text{tr}[(PT)^6]$, adjacent

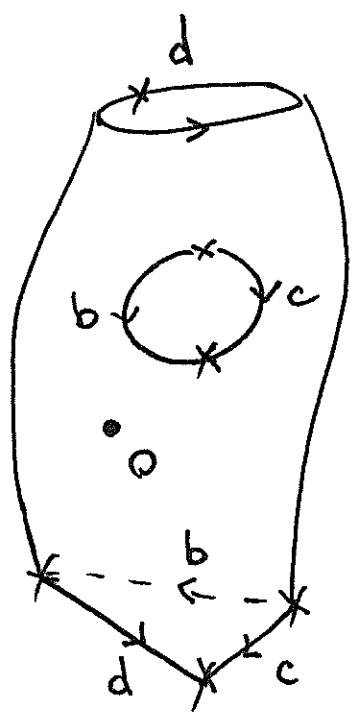
(16)



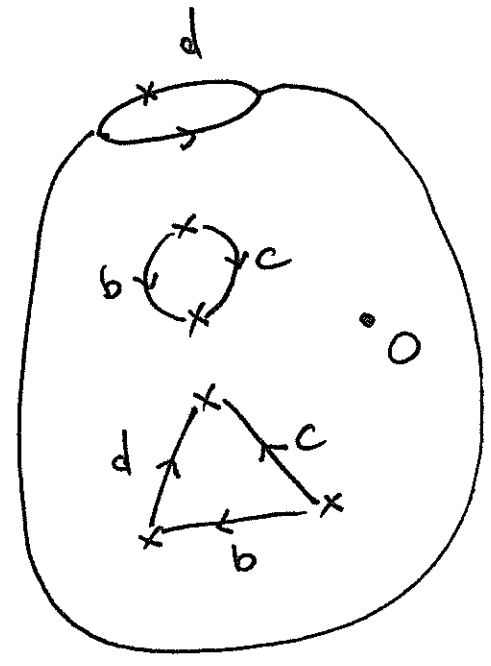




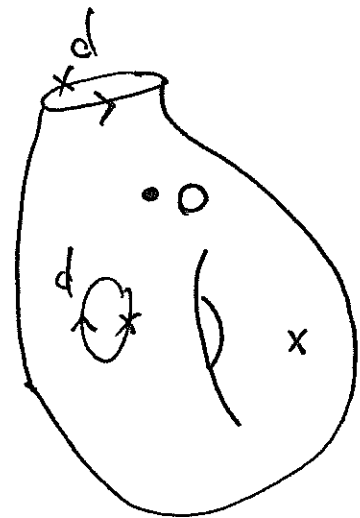
||

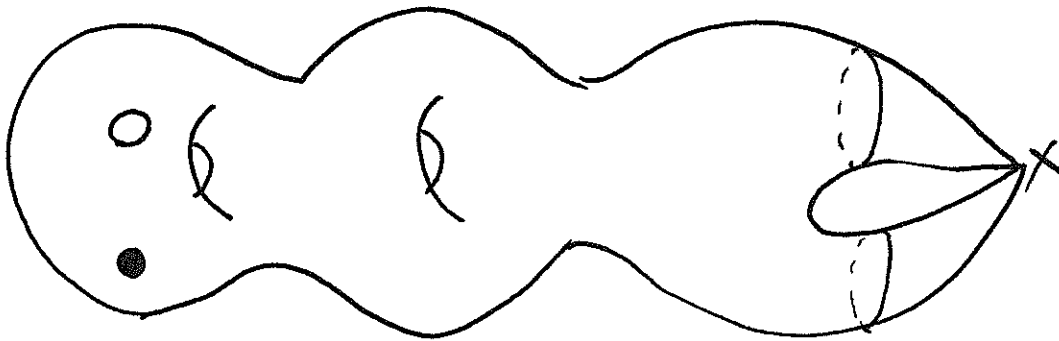
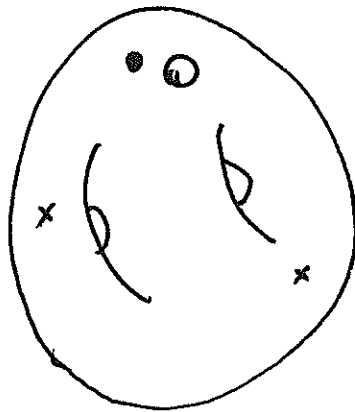
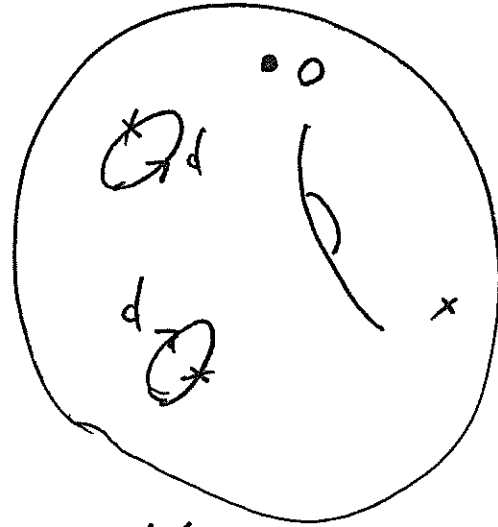


||

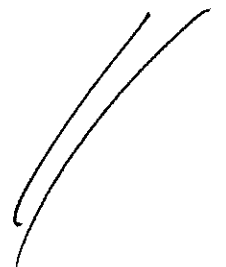


||





Σ_2 w/ 2 points identified.

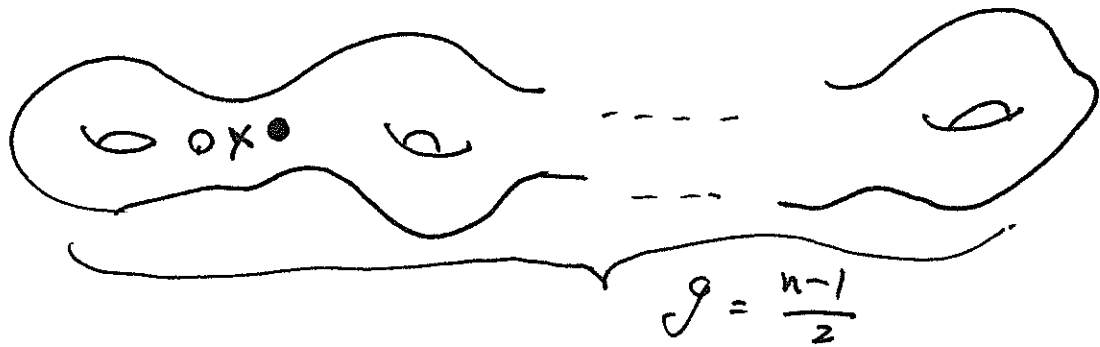


Conjecture

(20)

odd n $tv((P^T)^n)$, adjacent.

\Rightarrow genus $\frac{n-1}{2}$ surface $\sum \frac{n-1}{2}$

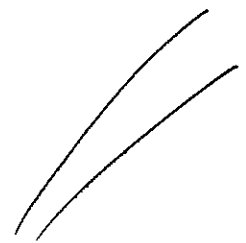


even n $tv((P^T)^n)$, adjacent

$\Rightarrow g = \frac{n}{2} - 1$ surface w/ 2 points identified.



$$g = \frac{n}{2} - 1$$

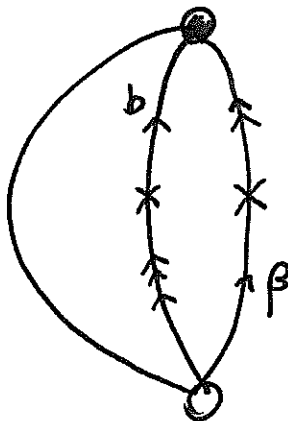
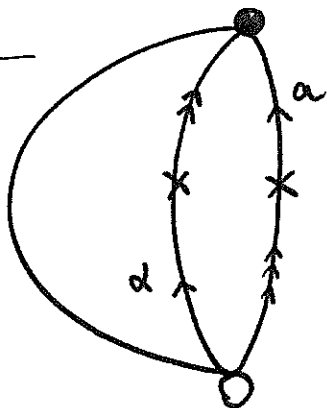


Proof

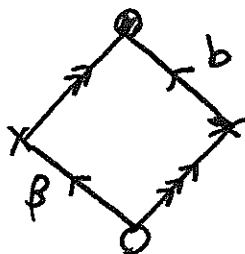
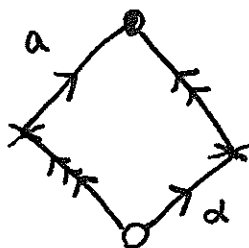
(2)

1st step

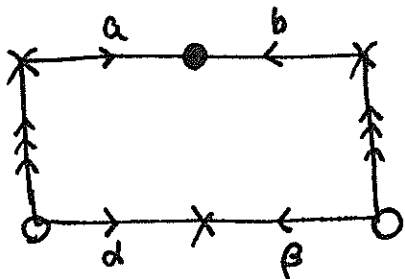
$P^T P^T =$



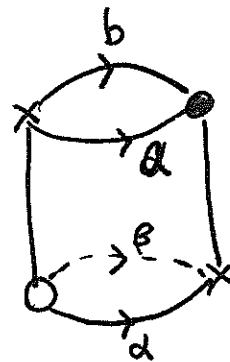
=



=

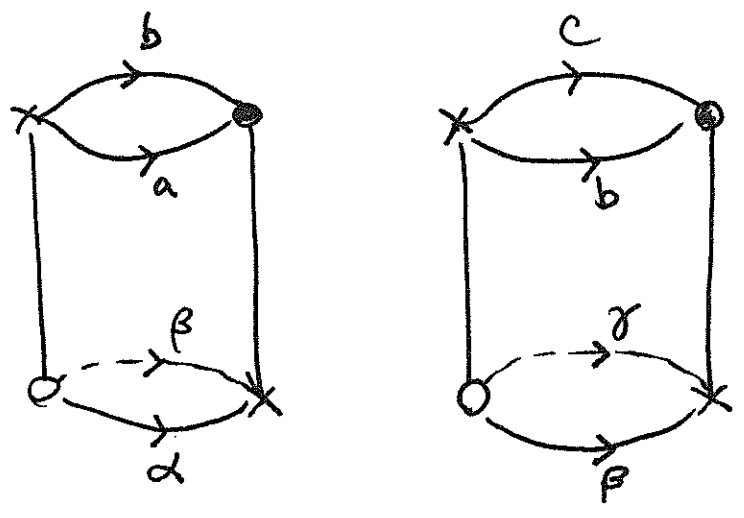


=

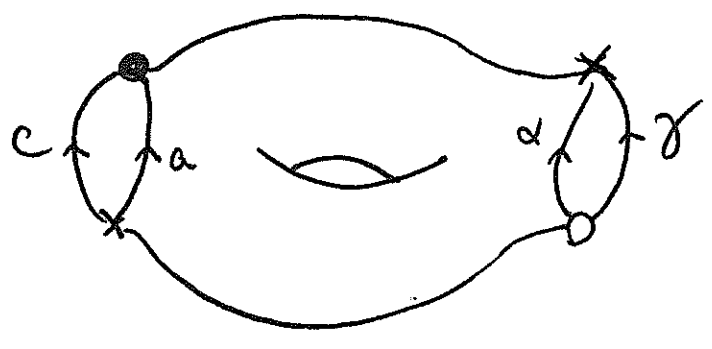


2nd step

$(P^T)^f =$

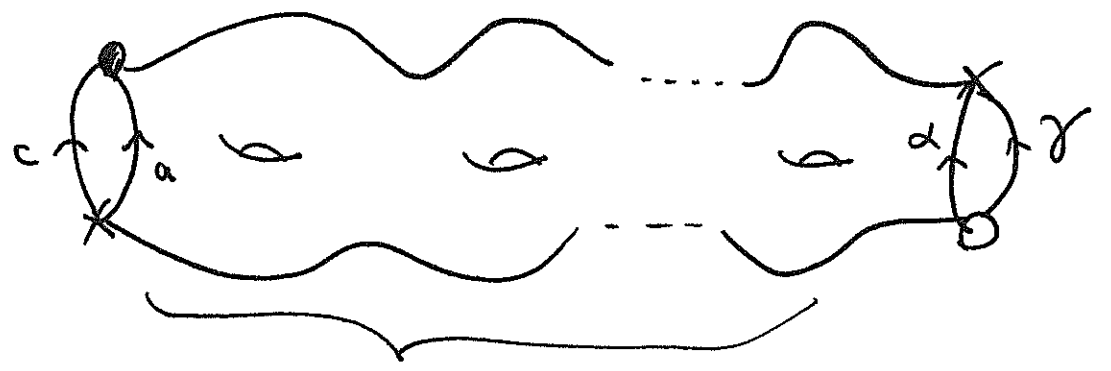


$=$



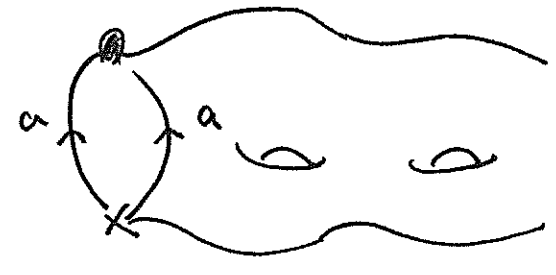
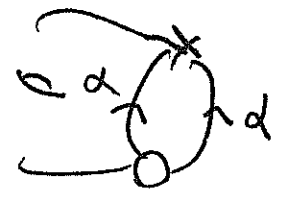
In the same way,



$(P^T)^{2n} =$

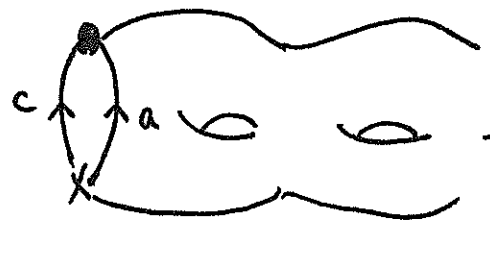
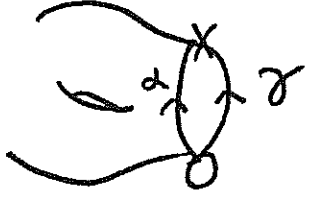
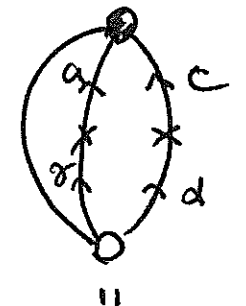


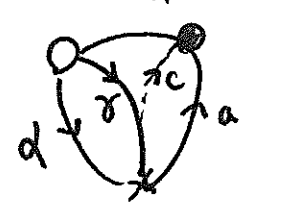
$(n-1)$ genus

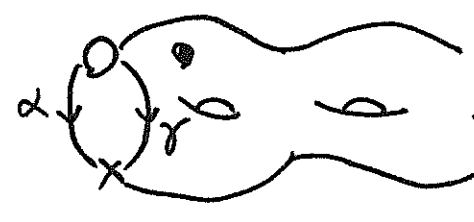
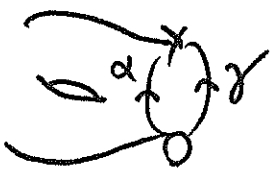
3rd step

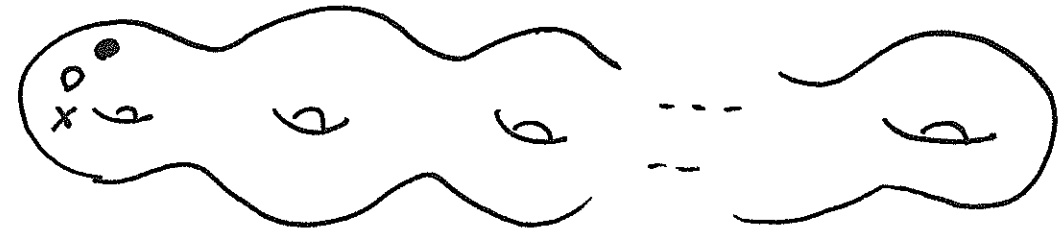
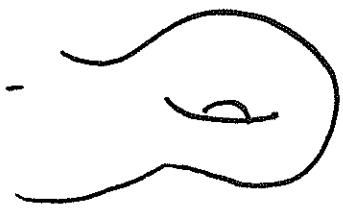
• $\text{tr}(P^T)^{2n} =$  \dots 

$=$  \dots 

• $\text{tr}(P^T)^{2n+1} (P^T)^{2n} =$  \dots  

$=$ 

$=$  \dots 

$=$  \dots 

n genus.