

Damping of quantum vibrations revealed in deep sub-barrier fusion

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We demonstrate that when two colliding nuclei approach each other, their quantum vibrations are damped near the touching point. We show that this damping is responsible for the fusion hindrance phenomena measured in the deep sub-barrier fusion reactions. To show those, we, for the first time, apply the random-phase-approximation method to the two-body $^{16}\text{O} + ^{16}\text{O}$ and $^{40}\text{Ca} + ^{40}\text{Ca}$ systems. We calculate the octupole transition strengths for the two nuclei that adiabatically approach each other. The calculated transition strength drastically decreases near the touching point, which strongly suggests the vanishing of the quantum couplings between the relative motion and the vibrational intrinsic degrees of freedom of each nucleus. Based on this picture, we also calculate the fusion cross section for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system by using the coupled-channel method with a damping factor that simulates the vanishing of the couplings. The calculated results reproduce the experimental data well, which indicates that the smooth transition from the sudden to the adiabatic processes indeed occurs in the deep sub-barrier fusion reactions.

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Heavy-ion fusion reactions at low incident energies serve as an important probe for investigating the fundamental properties of the potential tunneling of many-body quantum systems. When two nuclei fuse, a potential barrier, called the Coulomb barrier, is formed because of the strong cancellations between the Coulomb repulsion and the attractive nuclear force. The potential tunneling at incident energies below this Coulomb barrier is called the sub-barrier fusion. One important aspect of the sub-barrier fusion reactions is couplings between the relative motion of the colliding nuclei and the nuclear intrinsic degrees of freedom, such as collective vibrations of the target and/or projectile [1]. Those couplings result in the large enhancement of the fusion cross sections at the sub-barrier incident energies as compared to the estimation of a simple potential model. The coupled-channel (CC) model, which takes the couplings into account, has been successful in explaining this enhancement [2,3].

Recently, it has been possible to measure the fusion cross sections down to extremely deep sub-barrier incident energies [4–7]. The unexpected steep falloff of the fusion cross sections, compared to the standard CC calculations, emerges at the deep sub-barrier incident energies in a wide range of mass systems. These steep falloff phenomena are often called the fusion hindrance. The emergence of the fusion hindrance shows the threshold behavior, which is strongly correlated with the energy at the touching point of the two colliding nuclei [8,9]. In this respect, it has been shown that a key point for understanding this fusion hindrance is the potential tunneling in the density overlap region of the two colliding nuclei (see Fig. 1 in Ref. [8]).

To describe the fusion hindrance phenomena, many theoretical models to extend the standard CC model have been proposed. Based on the sudden picture, Mişicu and Esbensen have proposed that a strong repulsive core exists in the inner part of the Coulomb barrier due to nuclear incompressibility [10]. This model can reproduce the fusion hindrance from the light- to heavy-mass systems well [7,10–12]. Dasgupta

et al. proposed the concept of the quantum decoherence of the channel couplings [5], but there are only simple calculations with this model [13]. Based on the adiabatic picture, which is the opposite limit to the sudden approach, Ichikawa *et al.* introduced the damping factor in the standard CC calculations to smoothly join between the sudden and the adiabatic processes [14]. This model can reproduce the fusion hindrance better than the sudden model. However, the physical origin of the damping factor was still unclear.

In this Rapid Communication, we show the physical origin of the damping factor proposed in Ref. [14]. In the standard CC model, it has been assumed that the properties of the vibrational modes do not change, even when two colliding nuclei touch with each other. However, in fact, the single-particle wave functions drastically change in the two nuclei by approaching each other. This results in the damping of the vibrational excitations, that is, the vanishing of the couplings between the relative motion and the vibrational excitations of each nucleus. To show this, we, for the first time, apply the random-phase-approximation (RPA) method to the two-body $^{16}\text{O} + ^{16}\text{O}$ and $^{40}\text{Ca} + ^{40}\text{Ca}$ systems and calculate the octupole transition strength $B(E3)$ as a function of the distance between the two nuclei. We below show that the obtained $B(E3)$ values for the individual nuclei are indeed damped near the touching point.

To illustrate our main idea, we first discuss a disappearance of the octupole vibration during the $^{16}\text{O} + ^{16}\text{O}$ fusion process. We calculate the mean-field potential with the folding procedure that uses the single Yukawa function to conserve its inner volume [15]. In the two-body system before the touching point, we assume the two sharp-surface spherical nuclei. After the touching point, we describe the nuclear shapes with the lemniscatoids parametrization as shown in Ref. [16]. By using this, we can describe the smooth transition from the two- to the one-body mean-field potentials. The depths of the neutron and proton potentials are taken from Ref. [17]. We use the radius for the proton and neutron potentials R_0 with

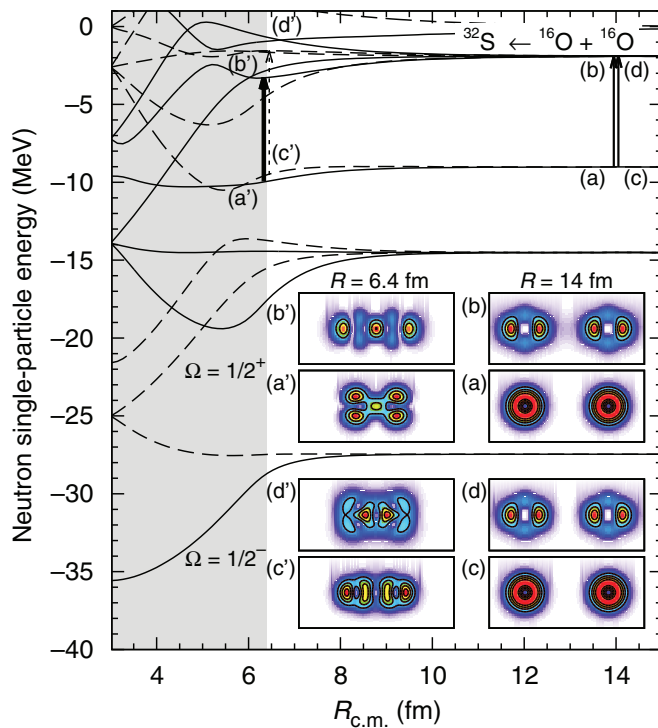


FIG. 1. (Color online) Nilsson diagram for the neutron single-particle states versus the distance between $^{16}\text{O} + ^{16}\text{O}$. The solid and dashed lines denote the positive- and negative-parity states, respectively. The gray area denotes the overlap region of $^{16}\text{O} + ^{16}\text{O}$. The solid and dashed arrows denote the main p - h excitations that generate the octupole vibration. The single-particle density distributions for these p and h states are given in the insets from (a) to (d) and (a') to (d').

$R_0 = 1.26A^{1/3}$ fm, where A is the total nucleon number. In the calculations, the origin is located at the center-of-mass position of the two nuclei. The above procedure works well because Umar and Oberacker found that the double-folding potential with the frozen density agrees almost perfectly with the density-constrained time-dependent Hartree-Fock (TDHF) approach for distances $R \geq 6$ fm [18].

By using the obtained mean-field potentials, we solve the axially symmetric Schrödinger equation with the spin-orbit force. Then, the parity π and the z component of the total angular momentum Ω are the good quantum numbers. The details of the model and the parameters are similar to those in Refs. [15,17]. We calculate the single-particle wave functions of both the projectile and the target by using the one-center single Slater determinant. We expand the total single-particle wave function by many deformed harmonic-oscillator bases in the cylindrical coordinate representation. The deformation parameter of the basis functions is determined so as to cover the target and projectile. The basis functions are taken with their energy lower than $25\hbar\omega$.

Figure 1 shows the Nilsson diagram for the obtained neutron single-particle energies versus the distance between $^{16}\text{O} + ^{16}\text{O}$. The solid and dashed lines denote the positive- and negative-parity states, respectively. The gray area denotes the overlap region of the two nuclei. The distance $R = 6.4$ fm

corresponds to the touching point. Some densities for the obtained single particles at $R = 14$ and 6.4 fm are given in the insets from (a) to (d) and (a') to (d') in Fig. 1, respectively. At $R = 14$ fm, we see that the positive (negative) parity indicates the symmetric (asymmetric) combinations of the single-particle states that refer to the right- and left-sided ^{16}O . Thus, the positive- and negative-parity single-particle states are degenerate for large R . With decreasing R , these single-particle states smoothly change to those for the composite ^{32}S system.

We can now easily extend the RPA method [19] to the two-body system because the wave functions of both the one- and the two-body systems are described with the one-center Slater determinant. We can directly superpose all combinations of the particle (p) and hole (h) states for the obtained single particles in a unified manner for both the one- and the two-body systems. We solve the RPA equation at each center-of-mass distance between $^{16}\text{O} + ^{16}\text{O}$. At large R values, the RPA solutions with $\Omega^\pi = 0^+$ and 0^- represent the symmetric and asymmetric combinations of the states where the RPA modes are excited in either the right- or the left-sided ^{16}O . When the R values decrease below the touching point, they smoothly change to excitation modes in the composite ^{32}S system. In the calculations, we only take into account the p - h states with the excitation energies below 30 MeV. We use the residual interaction as the density-dependent contact one taken from Ref. [20]. The strength of the residual interaction is determined at each R such that the lowest $\Omega^\pi = 0^-$ solution of the RPA appears at zero energy. We have developed a new scheme based on the Tomonaga theory of collective motion [21,22] that enables us to separate the center of mass, the relative motion, and the intrinsic degrees of freedom. It is a generalization of the known procedure in the RPA [23]. It is also interesting to compare our results with the TDHF dynamics [24].

For the calculated results, the obtained first excited state of the right-sided ^{16}O is the octupole (3^-) one with a large $B(E3)$ value. At $R = 15$ fm, the excitation energy and the $B(E3, 3_1^- \rightarrow 0_1^+)$ value are 5.29 MeV and $102.07 e^2 \text{fm}^6$, respectively. We have checked that those values are consistent with the calculated results of the one-body ^{16}O . Figure 2(a) shows the calculated transition densities and currents [19] for the first excited state with $\Omega^\pi = 0^+$ at $R = 14$ fm. In Fig. 2(a), we can clearly see the octupole vibrations in both ^{16}O 's. At $R = 8.0$ fm, the transition density of the neck part between two ^{16}O 's develops [see Fig. 2(b)]. At $R = 6.4$ fm, the octupole vibrations of each ^{16}O become weak [see Fig. 2(c)]. The degenerating excitation energies of the first excited states with $\Omega^\pi = 0^+$ and 0^- split below $R = 8$ fm. They become 5.82 and 4.83 MeV at $R = 6.4$ fm.

To more clearly see the damping of the octupole vibrations, we calculate the $B(E3)$ value for the right-sided ^{16}O . We can easily calculate it by taking a symmetric linear combination of the octupole transition amplitudes for the positive- and negative-parity RPA solutions. Figure 3(a) shows the calculated $B(E3, 3_1^- \rightarrow 0_1^+)$ values versus the distance between $^{16}\text{O} + ^{16}\text{O}$. In Fig. 3(a), we can see that the $B(E3)$ value for the right-sided ^{16}O (the solid line) falls off at around $R = 8$ fm with decreasing R , which indicates that the octupole vibrations are strongly suppressed near the touching point.

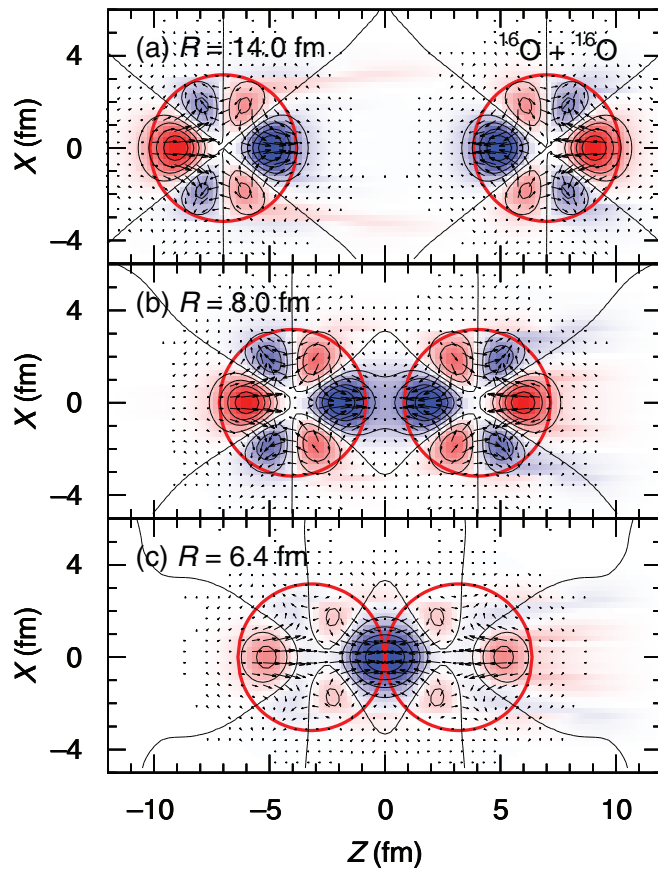


FIG. 2. (Color online) Transition densities and currents for the first excited 3^- state with $\Omega^\pi = 0^+$ at (a) $R = 14.0$ fm, (b) $R = 8.0$ fm, and (c) $R = 6.4$ fm. The contour lines denote the transition density. The arrows denote the current density. These two values are normalized in each plot. The (red) thick solid line denotes the half depth of the mean-field potential.

The damping of the vibrations originates from the change in the single-particle wave functions. At $R = 14$ fm, the major p - h excitations that generate the octupole vibration are those from the $p_{1/2}$ to the $d_{5/2}$ single-particle states in ^{16}O . In these states, the degenerating positive- and negative-parity doublet states contribute equally to generate the octupole vibration. Those can be seen in the density distributions of the $\Omega^\pi = 1/2^+$ and $1/2^-$ states given in the insets from (a) to (d) in Fig. 1. The corresponding p - h excitations are denoted by the solid arrows from (a) to (b) and (c) to (d) in Fig. 1. When the two nuclei approach each other, the features of these single-particle wave functions drastically change. At $R = 6.4$ fm, the neck formation takes place in the positive-parity states, whereas, it is forbidden for the negative-parity states (that have nodes at the touching point). Thus, the density distributions of those parity partners become quite different from each other. [See the insets from (a') to (d').] In the RPA calculation, the contributions from the negative-parity states to the octupole vibration become small at the touching point [see the solid and dotted arrows from (a') to (b') and (c') to (d'), respectively], which results in the decreases in the $B(E3)$ value.

The mechanism for the damping of the quantum vibration is a general one, which is also valid for heavier-mass systems. We

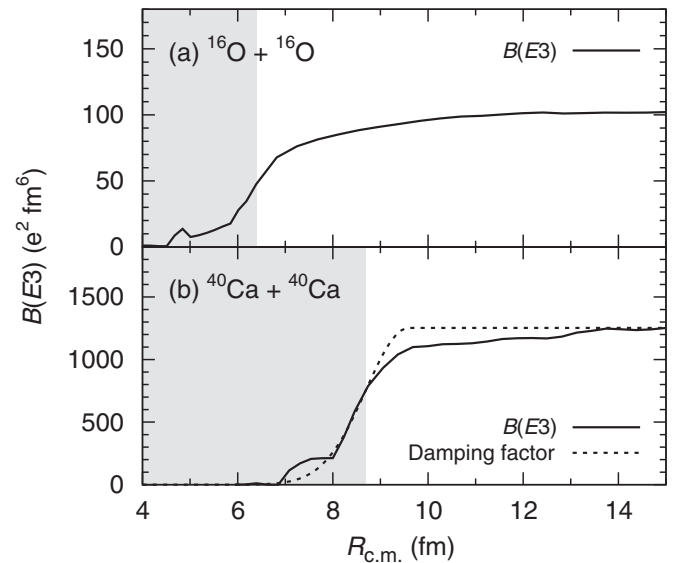


FIG. 3. Transition strengths $B(E3)$ for the first excited state versus the distance between (a) $^{16}\text{O} + ^{16}\text{O}$ and (b) $^{40}\text{Ca} + ^{40}\text{Ca}$. The solid line denotes the calculated results. The gray area denotes the overlap region of the two colliding nuclei. In the bottom panel (b), the dotted line denotes the damping factor of Eq. (2) in Ref. [14], estimated from the experimental data of the fusion cross section.

have also performed the RPA calculations for the $^{40}\text{Ca} + ^{40}\text{Ca}$ and $^{56}\text{Ni} + ^{56}\text{Ni}$ systems. In the calculations, we use $R_0 = 1.27 A^{1/3}$. We obtained the similar damping of the $B(E3)$ values for the both systems. In Fig. 3(b), the solid line denotes the calculated result for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system. The calculated excitation energy and the $B(E3, 3_1^- \rightarrow 0_1^+)$ value for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system are 3.24 MeV and $1253.17 e^2 \text{fm}^6$ at $R = 15.0$ fm, respectively.

As shown above, the octupole vibrations are damped near the touching point, which results in the vanishing of the couplings between the relative motion and the vibrational intrinsic degrees of freedom. This vanishing would lead to the smooth transition from the sudden to the adiabatic process as shown in Ref. [14]. It is clear that such an effect has not been taken into account in the standard CC model. One candidate to include is the introduction of the damping factor proposed in Ref. [14]. To clearly see the effect of the vanishing of the couplings, we calculate the fusion cross section for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system by using the computer code CCFULL [25] coupled with the damping factor based on the model of Ref. [14].

In the calculations, we include the couplings to only the low-lying 3^- and 2^+ states and to single-phonon and all mutual excitations of these states. We take the energies and the deformations of each state taken from Ref. [7] to reproduce the experimental data well. We use the same deformation parameters for the Coulomb and nuclear couplings. For the parameters of the Yukawa-plus-exponential (YPE) model, we use $r_0 = 1.191$ and $a = 0.68$ fm.

It is remarkable that the damping factor strongly correlates with the calculated $B(E3)$ value for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system. To show this, we take $\lambda_\alpha = 0$ in the damping factor of Eq. (2)

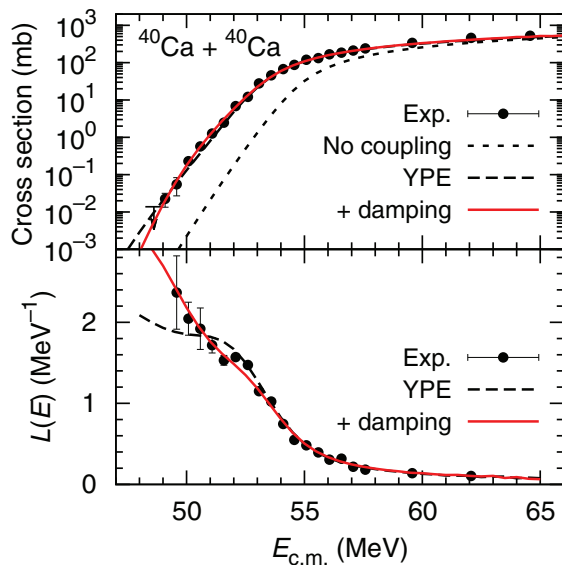


FIG. 4. (Color online) Fusion cross section (upper panel) and its logarithmic derivative (lower panel) for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system versus the incident energies. The solid circles denote the experimental data taken from Ref. [7]. The solid and dashed lines denote the calculated results of the coupled-channel method by using the YPE potential with and without the damping factor, respectively. The dotted line denotes the calculated result without the couplings.

in Ref. [14] for simplicity. We tune the parameters R_d and a_d in the damping factor so as to reproduce the experimental data of the fusion cross section. We obtain $R_d = 9.6$ and $a_d = 0.9$ fm as the best fits to the data. In Fig. 3(b), the dotted line denotes the obtained damping factor normalized at $R = 15$ fm.

Figure 4 shows the calculated fusion cross section (upper panel) and its logarithmic derivative $d \ln(E_{c.m.} \sigma_{\text{fus}})/dE_{c.m.}$ (lower panel) for the $^{40}\text{Ca} + ^{40}\text{Ca}$ system. The solid and dashed lines denote the calculation with and without the damping factor, respectively. The dotted line denotes the calculation without the couplings. In Fig. 4, we can see that the calculated results with the damping factor reproduce the experimental data well, which is better than the sudden model [7,12]. In our model, the calculated astrophysical S factor has a peak structure. We also performed the CC calculation for the $^{48}\text{Ca} + ^{48}\text{Ca}$ system, and the calculated result reproduces the experimental data well. The CC calculations with the damping factor also already reproduced the experimental data for the $^{64}\text{Ni} + ^{64}\text{Ni}$, $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{16}\text{O} + ^{208}\text{Pb}$ systems well [14].

To summarize, we have demonstrated the damping of the quantum vibrations when two nuclei adiabatically approach each other. To show this, we, for the first time, applied the RPA method to the two-body $^{16}\text{O} + ^{16}\text{O}$ and $^{40}\text{Ca} + ^{40}\text{Ca}$ systems and calculated the $B(E3)$ values of each nucleus. We have shown that the calculated $B(E3)$ value is indeed damped near the touching point. We have also shown that the damping factor proposed in Ref. [14] strongly correlates with the calculated $B(E3)$ values and the calculations of the fusion cross section coupled with the damping factor reproduce the experimental data well. This indicates that the fusion hindrance originates from the damping of the quantum couplings and strongly suggests that the smooth transition from the sudden to the adiabatic processes occurs near the touching point.

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- [1] M. Dasgupta *et al.*, *Annu. Rev. Nucl. Part. Sci.* **48**, 401 (1998).
- [2] A. B. Balantekin and N. Takigawa, *Rev. Mod. Phys.* **70**, 77 (1998).
- [3] K. Hagino and N. Takigawa, *Prog. Theor. Phys.* **128**, 1061 (2012).
- [4] C. L. Jiang *et al.*, *Phys. Rev. Lett.* **89**, 052701 (2002); **93**, 012701 (2004); C. L. Jiang, B. B. Back, H. Esbensen, R. V. F. Janssens, and K. E. Rehm, *Phys. Rev. C* **73**, 014613 (2006); C. L. Jiang, B. B. Back, R. V. F. Janssens, and K. E. Rehm, *ibid.* **75**, 057604 (2007); C. L. Jiang, K. E. Rehm, B. B. Back, and R. V. F. Janssens, *ibid.* **79**, 044601 (2009).
- [5] M. Dasgupta *et al.*, *Phys. Rev. Lett.* **99**, 192701 (2007).
- [6] A. M. Stefanini *et al.*, *Phys. Rev. C* **78**, 044607 (2008); *Phys. Lett. B* **679**, 95 (2009).
- [7] G. Montagnoli *et al.*, *Phys. Rev. C* **85**, 024607 (2012).
- [8] T. Ichikawa, K. Hagino, and A. Iwamoto, *Phys. Rev. C* **75**, 064612 (2007).
- [9] E. S. Zin Thein, N. W. Lwin, and K. Hagino, *Phys. Rev. C* **85**, 057602 (2012).
- [10] Ş Mişicu and H. Esbensen, *Phys. Rev. Lett.* **96**, 112701 (2006); *Phys. Rev. C* **75**, 034606 (2007).
- [11] H. Esbensen, C. L. Jiang, and A. M. Stefanini, *Phys. Rev. C* **82**, 054621 (2010).
- [12] Ş Mişicu and F. Carstoiu, *Phys. Rev. C* **84**, 051601(R) (2011).
- [13] A. Diaz-Torres, D. J. Hinde, M. Dasgupta, G. J. Milburn, and J. A. Tostevin, *Phys. Rev. C* **78**, 064604 (2008).
- [14] T. Ichikawa, K. Hagino, and A. Iwamoto, *Phys. Rev. Lett.* **103**, 202701 (2009); *EPJ Web Conf.* **17**, 07001 (2011).
- [15] M. Bolsterli *et al.*, *Phys. Rev. C* **5**, 1050 (1972).
- [16] T. Ichikawa, K. Hagino, and A. Iwamoto, *Phys. Rev. C* **75**, 057603 (2007).
- [17] P. Möller *et al.*, *At. Data Nucl. Data Tables* **59**, 185 (1995).
- [18] A. S. Umar and V. E. Oberacker, *Phys. Rev. C* **74**, 021601(R) (2006).
- [19] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
- [20] S. Shlomo and G. Bertsch, *Nucl. Phys. A* **243**, 507 (1975).
- [21] S. Tomonaga, *Prog. Theor. Phys.* **13**, 468 (1955).
- [22] T. Ichikawa and K. Matsuyanagi (to be published).
- [23] S. Kamenzhiev, R. J. Liotta, E. Litvinova, and V. Tselyaev, *Phys. Rev. C* **58**, 172 (1998).
- [24] R. Keser, A. S. Umar, and V. E. Oberacker, *Phys. Rev. C* **85**, 044606 (2012).
- [25] K. Hagino, N. Rowley, and A. T. Kruppa, *Comput. Phys. Commun.* **123**, 143 (1999).