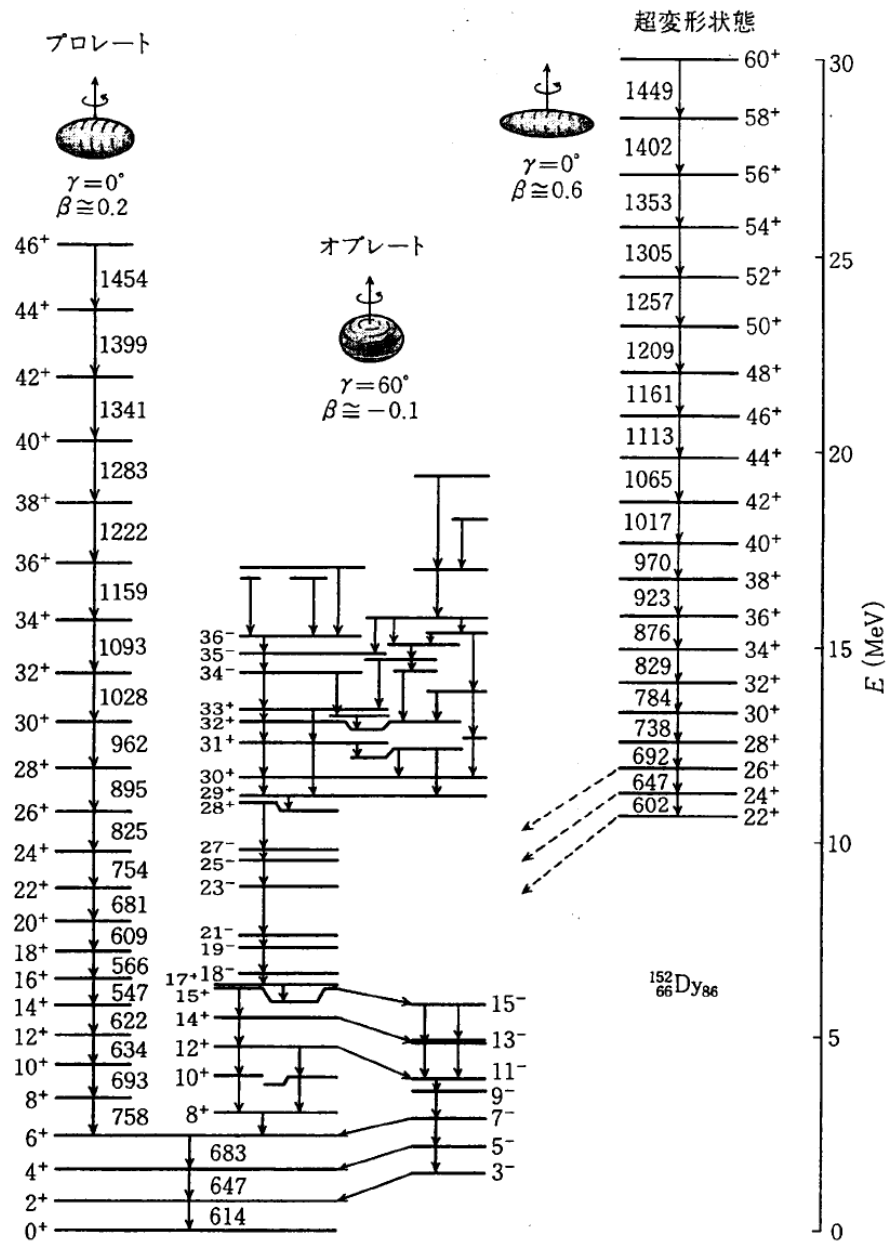
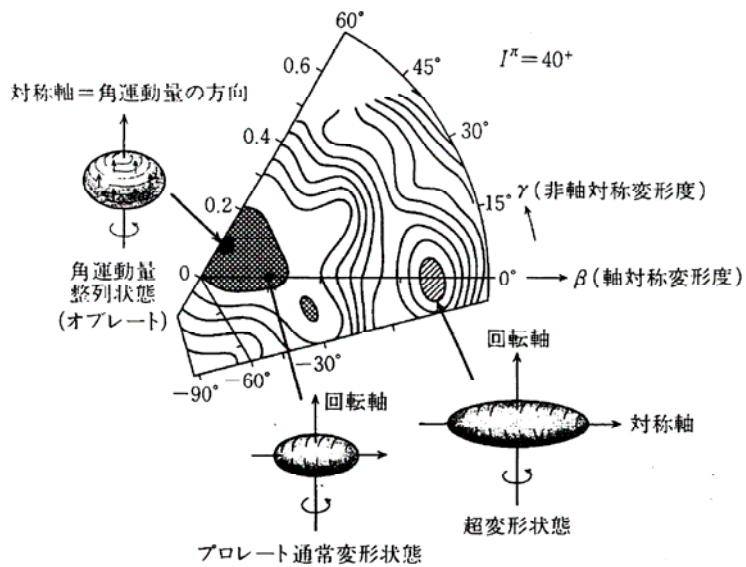
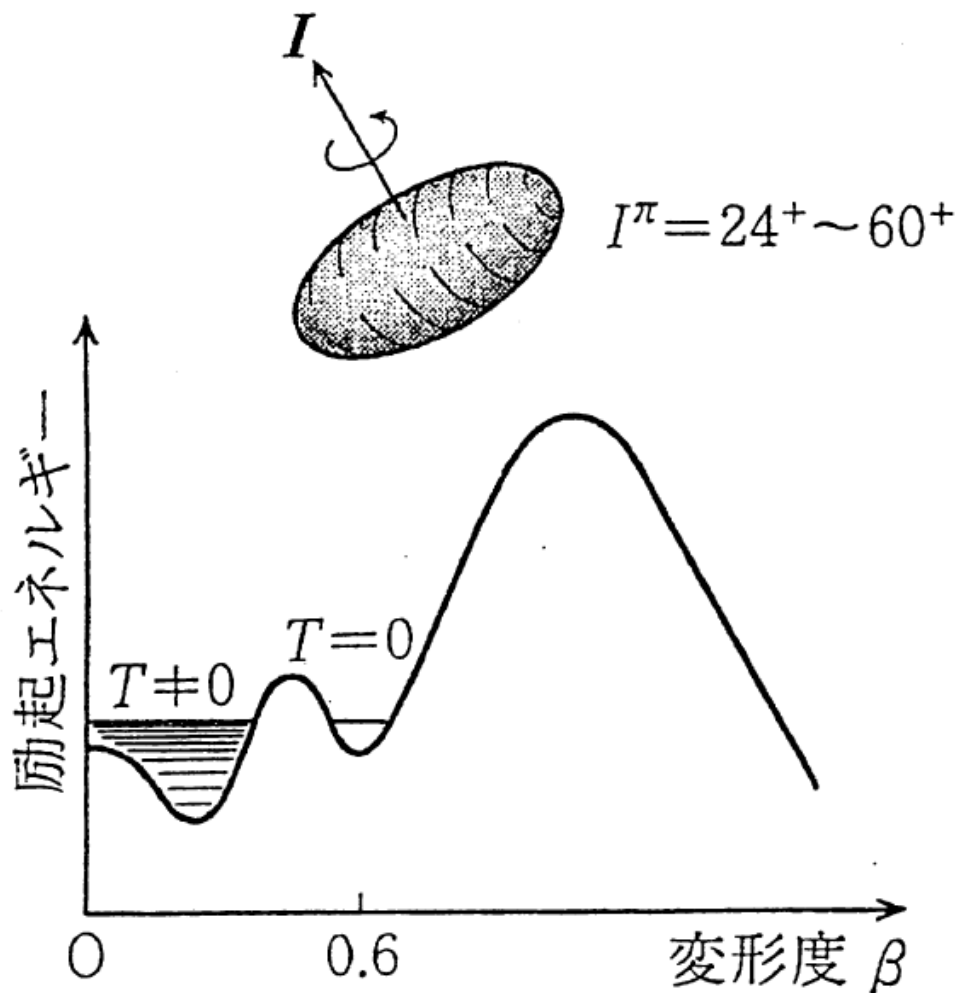


異なる変形の共存



$^{152}_{66}\text{Dy}_{86}$

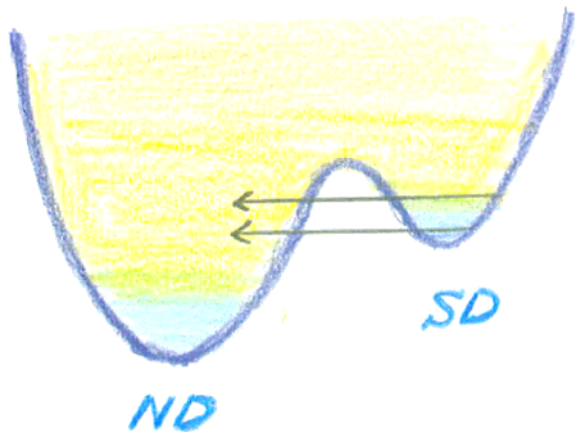
高速回転する超変形状態



平均場の第2極小点
(二つの真空)

超変形状態から
常変形状態への
巨視的トンネル現象

SD からNDへのトンネル崩壊



$$H = \frac{1}{2} \sum_{ij=1}^2 m_{ij}(q_1, q_2) \dot{q}_i \dot{q}_j + V(q_1, q_2),$$

透過係数

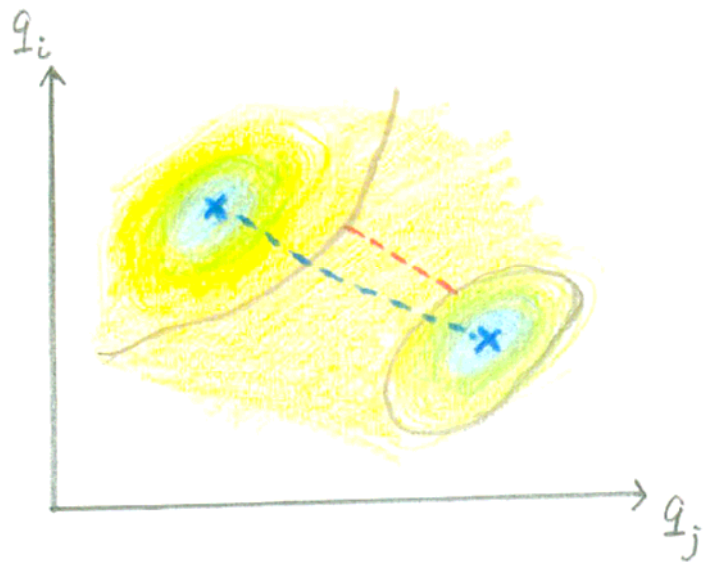
$$T = (1 + \exp 2S)^{-1}$$

作用積分

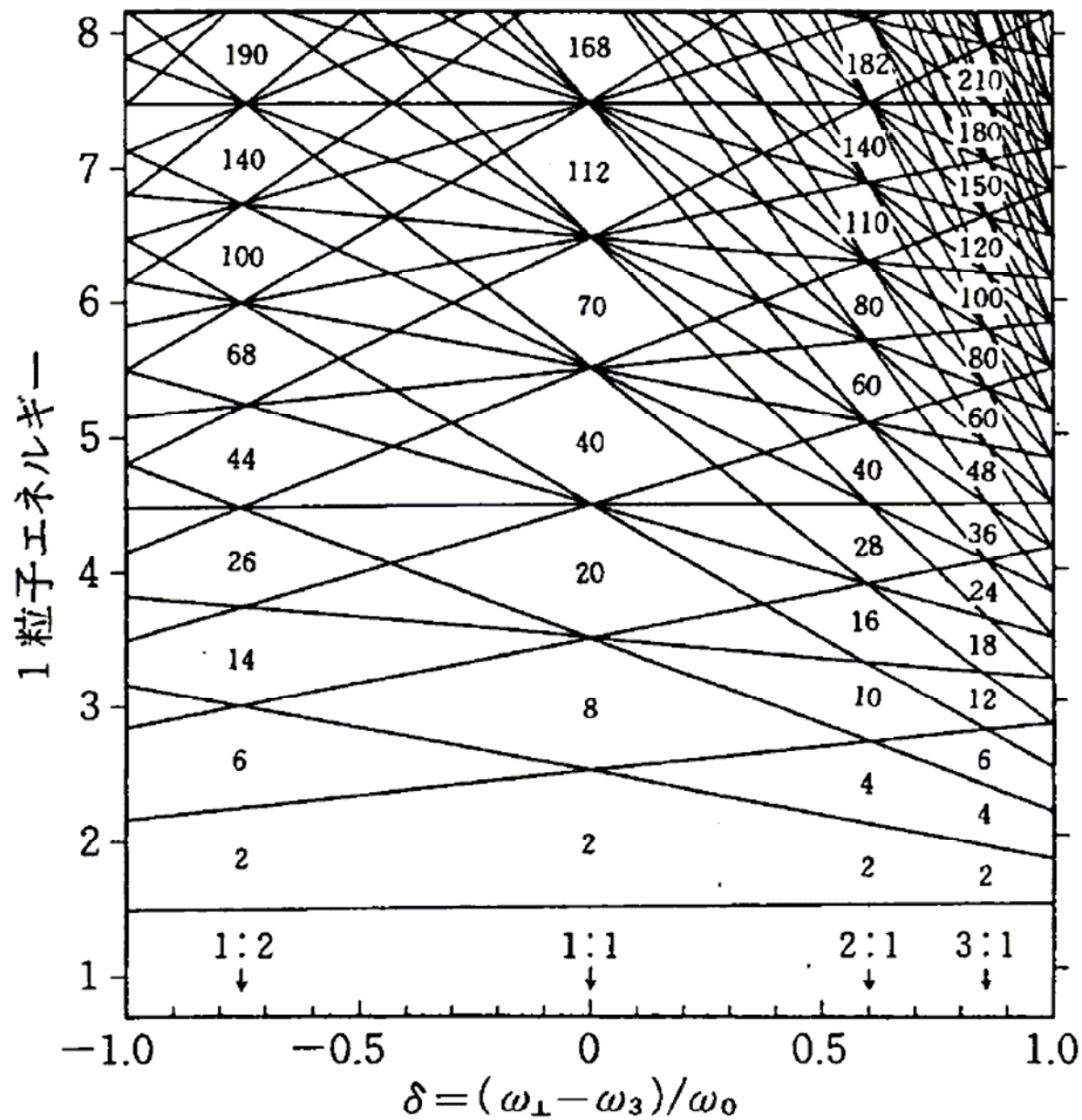
$$S(E) = \int_{\text{path}} ds \sqrt{2M_0(V(q(s)) - E)},$$

集団質量

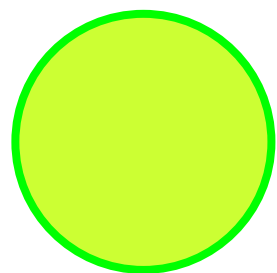
$$M_0 ds^2 = \sum_{ij} m_{ij}(q) dq_i dq_j$$



$$q_1 = \epsilon_2 \cos(\gamma + 30^\circ), \quad q_2 = \epsilon_2 \sin(\gamma + 30^\circ).$$



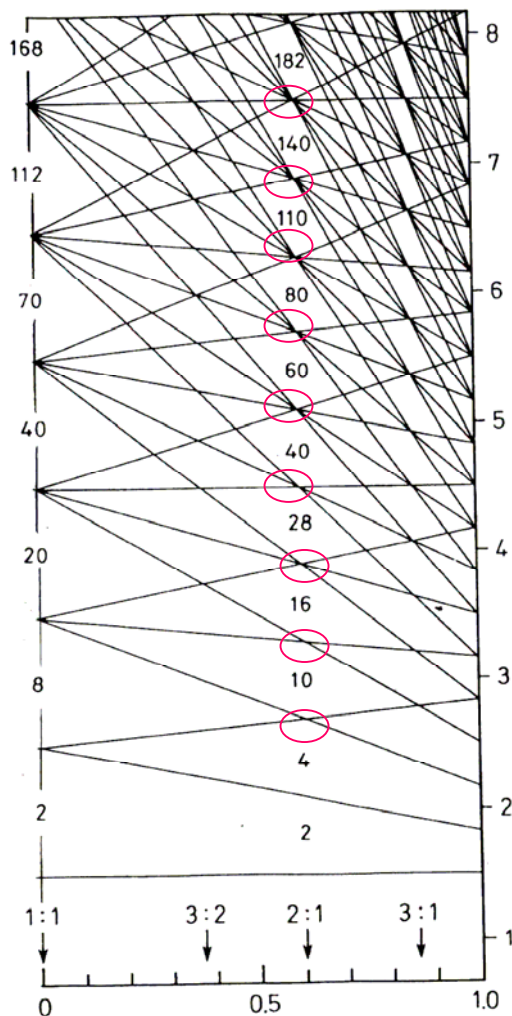
$$e(n_{\perp}, n_z) = \hbar\omega_{\perp}(n_{\perp} + 1) + \hbar\omega_z(n_z + \frac{1}{2})$$



球形

縮退度

- 56 →
- 42 →
- 30 →
- 20 →
- 12 →
- 6 →
- 2 →

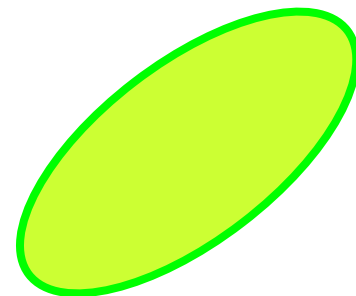


変形度 $\delta_{osc} = (\omega_{\perp} - \omega_z)/\bar{\omega}$

$\omega_{\perp} = \omega_x = \omega_y$

縮退度

- ← 42 } (red bracket)
- ← 42 } (red bracket)
- ← 30 } (red bracket)
- ← 30 } (red bracket)
- ← 20 } (red bracket)
- ← 20 } (red bracket)
- ← 12 } (red bracket)
- ← 12 } (red bracket)
- ← 6 } (red bracket)
- ← 6 } (red bracket)
- ← 2 } (red bracket)
- ← 2 } (red bracket)



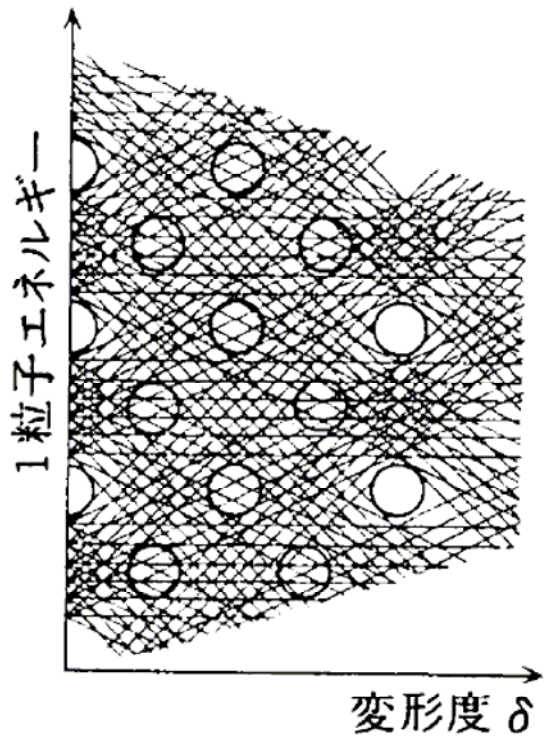
超変形

軸比2:1

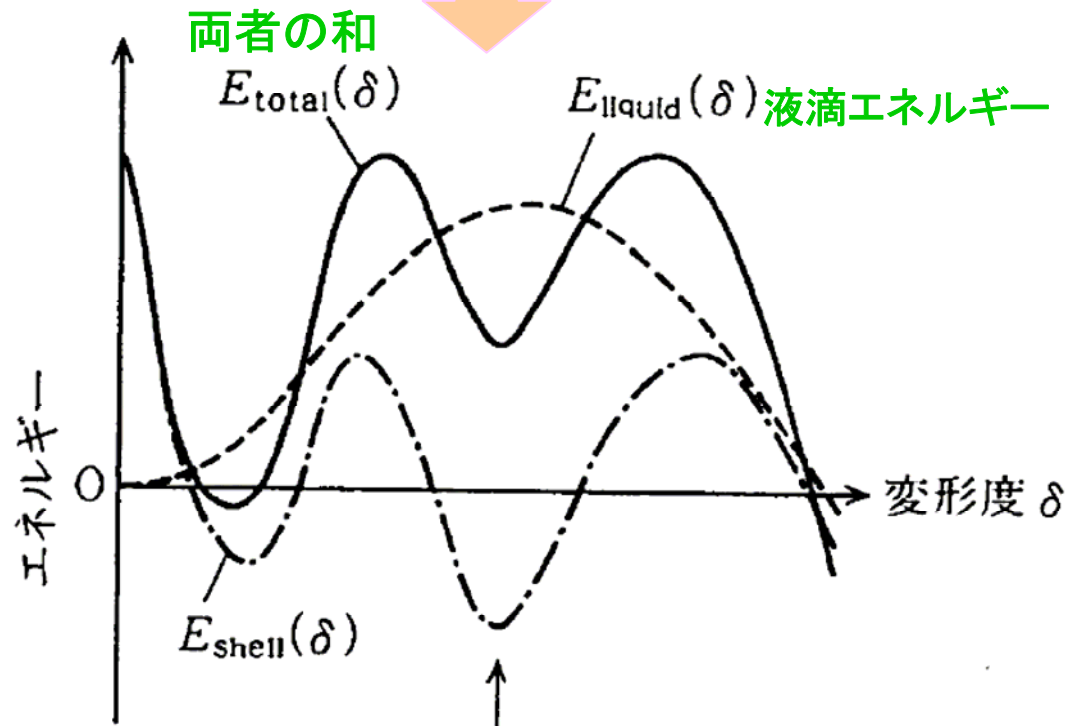
調和振動子ポテンシャルの動力学的対称性

核分裂アイソマー

変形につれてシェル構造は変化する



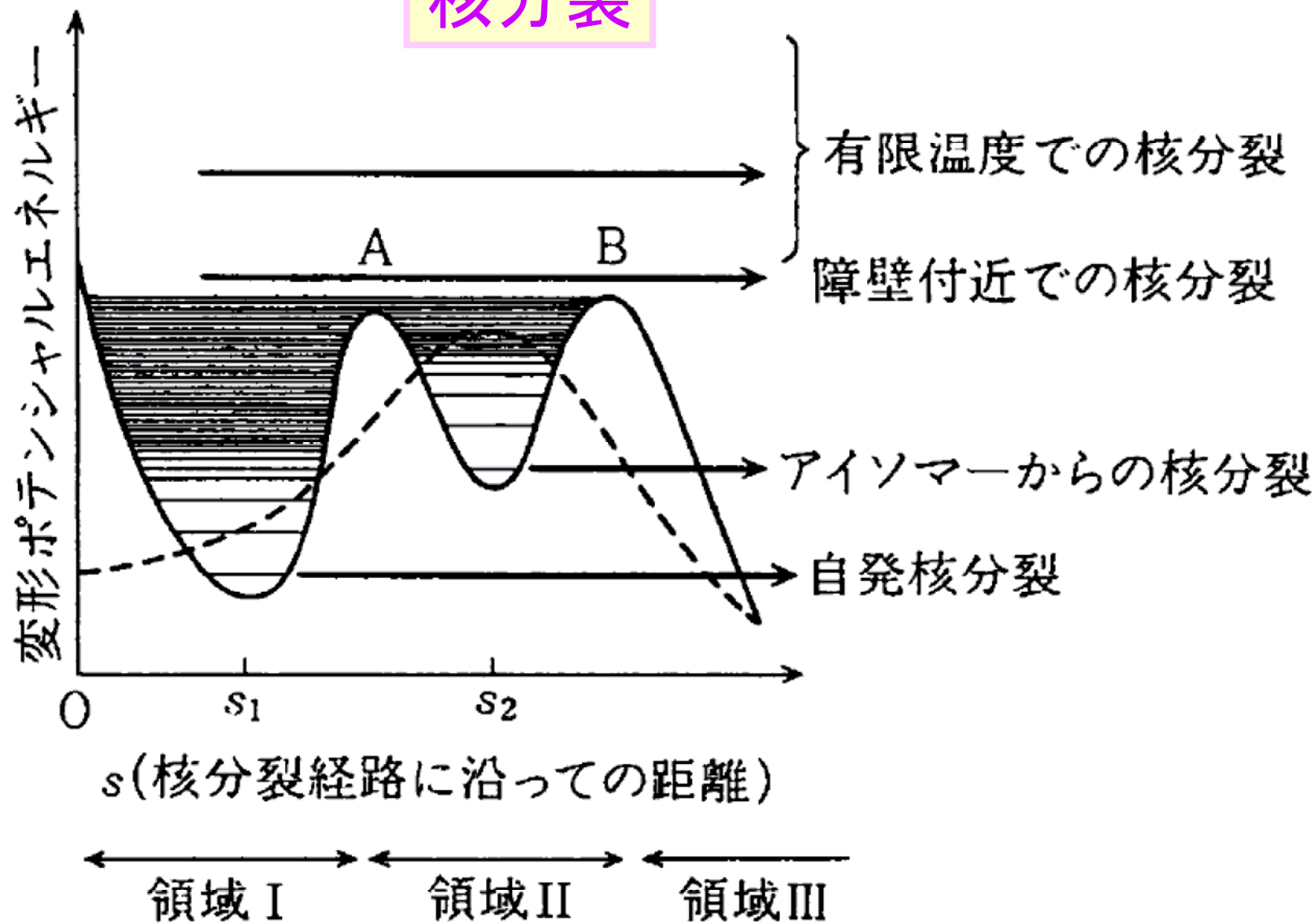
(a)



シェル構造エネルギーの極小点

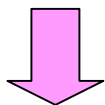
(b)

核分裂



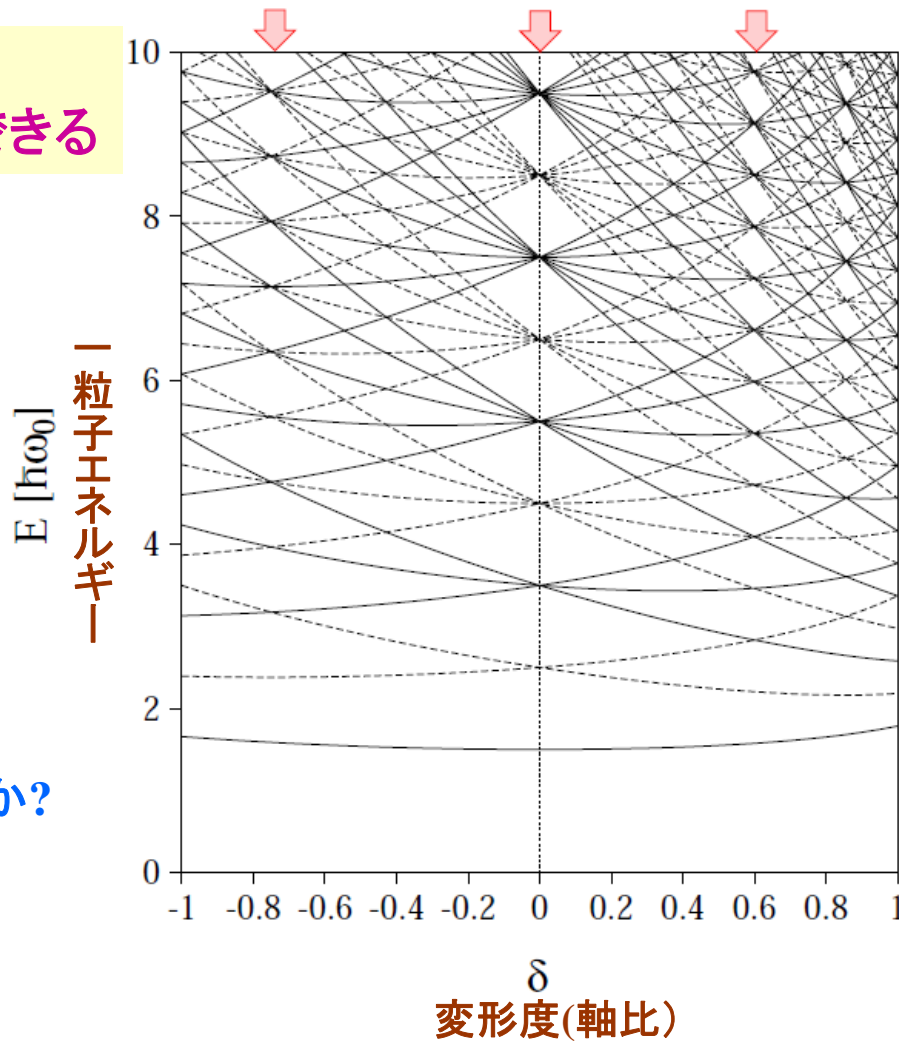
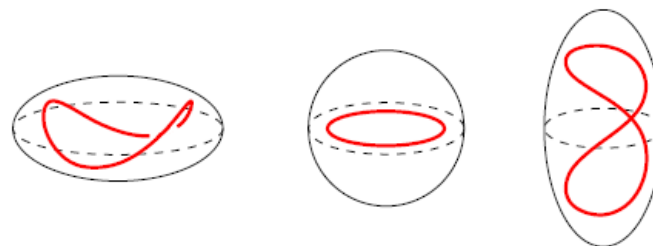
調和振動子ポテンシャル

軸比2:1のとき
8の字型の周期軌道ができる



準位が縮退する
超変形シェル構造の形成

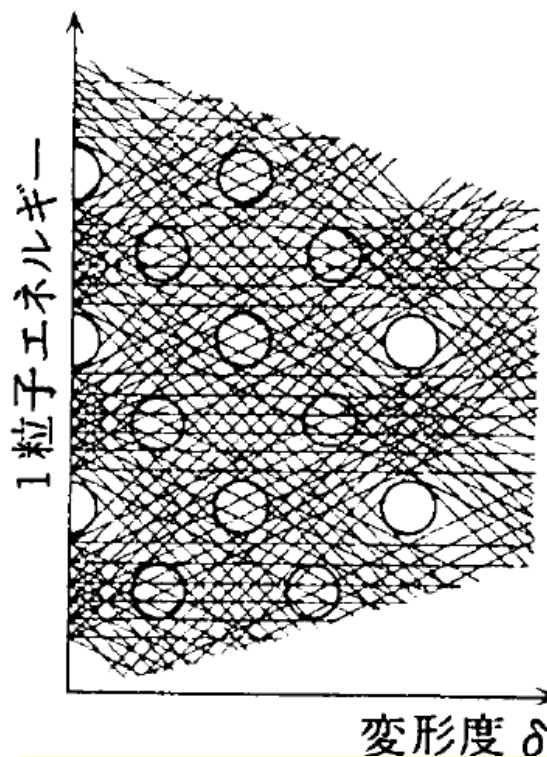
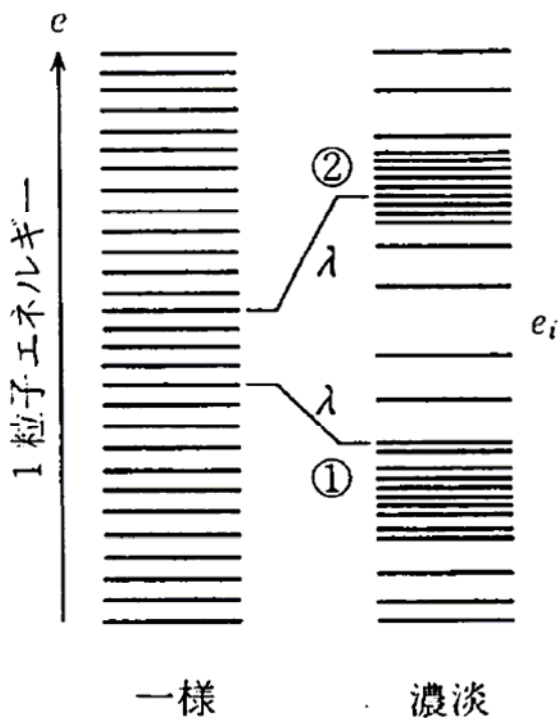
一般のポテンシャルではどうか?



シェル構造とは何か

疎視化して見える1粒子準位スペクトルの規則的な振動パターン

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\beta) \right) \psi(\mathbf{r}) = e(\beta) \psi(\mathbf{r})$$



振動パターンは変形とともに変わる

問8 変形した原子核はなぜ存在するのですか。何が「変形」しているのですか。

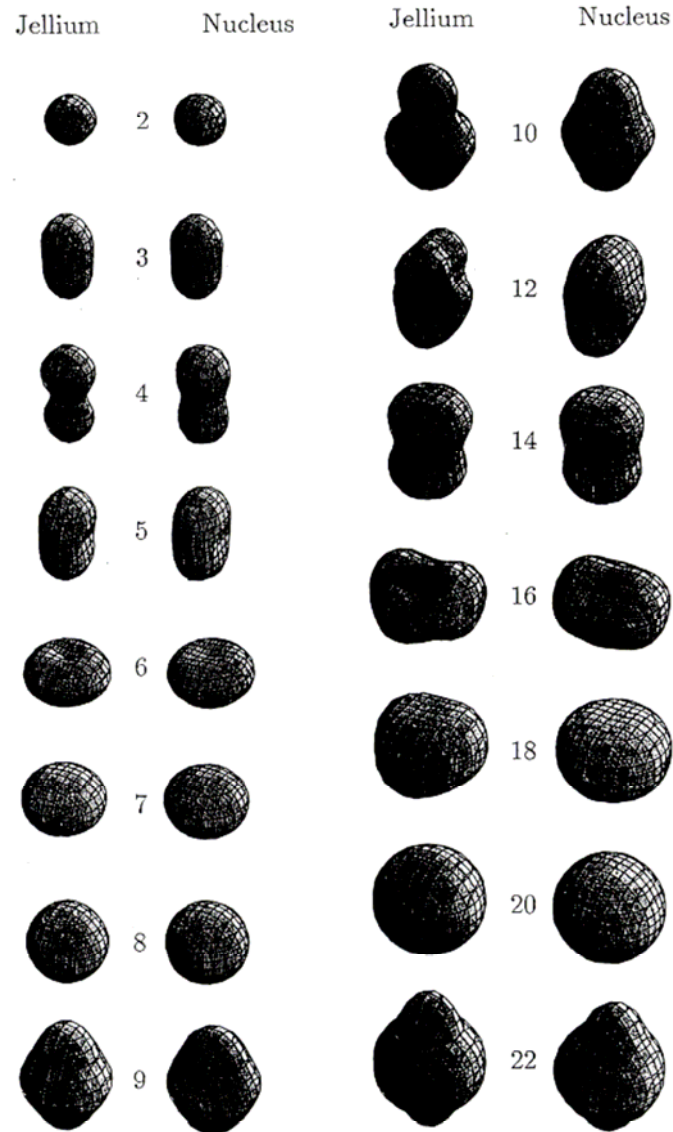


Fig. 4. Constant-density surfaces for electron clusters and nuclei. The density of the surface is 0.00125 atomic units (or scaled units for nuclei), which corresponds to 38% of the bulk density.

1粒子準位の縮退が起こる条件(可積分系の場合)

作用変数と角変数 $\dot{\theta} = \frac{\partial h}{\partial \mathbf{I}} = \boldsymbol{\omega}(\mathbf{I})$ 振動数の組

EBK(Einstein-Brillouin-Keller)量子条件

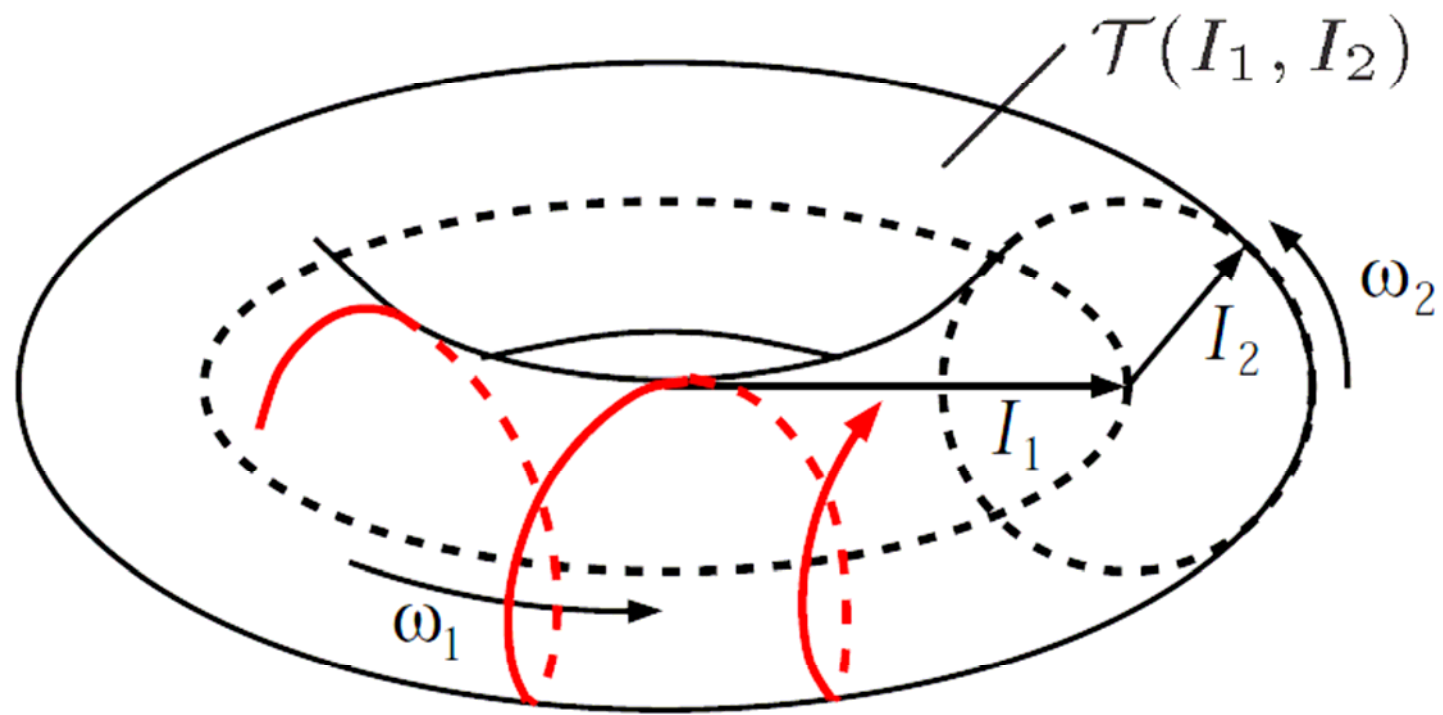
$$e(\mathbf{n}) = h(\mathbf{I}) \quad \text{with} \quad \mathbf{I} = \hbar \left(\mathbf{n} + \frac{1}{4} \boldsymbol{\alpha} \right),$$

エネルギー変分

$$\begin{aligned} e(\mathbf{n} + \Delta \mathbf{n}) - e(\mathbf{n}) &= h(\mathbf{I} + \Delta \mathbf{I}) - h(\mathbf{I}) \\ &\simeq \frac{\partial h}{\partial \mathbf{I}} \Delta \mathbf{I} \\ &= \hbar \boldsymbol{\omega} \cdot \Delta \mathbf{n} \\ &= \hbar \omega_1 \Delta n_1 + \hbar \omega_2 \Delta n_2 + \hbar \omega_3 \Delta n_3 \\ &= 0, \end{aligned}$$

量子数の組

振動数が整数比のとき縮退が起こる



シェル構造とは何か

疎視化して見える1粒子準位スペクトルの規則的な振動パターン

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\beta)\right)\psi(\mathbf{r}) = e(\beta)\psi(\mathbf{r})$$

準位密度に対するトレース公式

$$\begin{aligned}g(e) &= \sum_i \delta(e - e_i) \\&\simeq \bar{g}(e) + \delta g(e) \\&= \bar{g}(e) + \sum_{\gamma} A_{\gamma} \cos\left(\frac{1}{\hbar}S_{\gamma}(e) - \frac{\pi}{2}\mu_{\gamma}\right)\end{aligned}$$

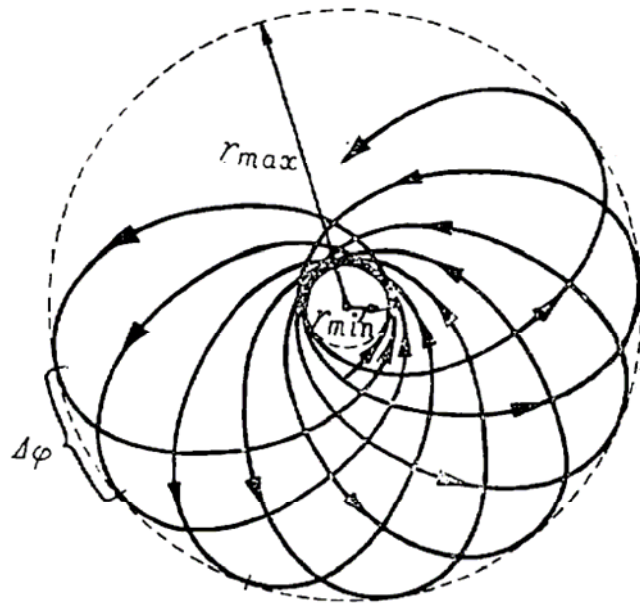
すべての周期軌道に関する総和

作用積分

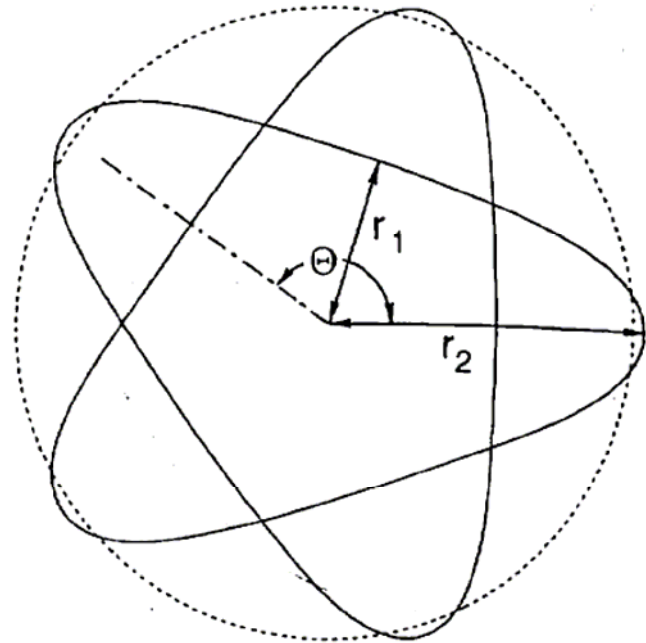
球対称ポテンシャルのなかの軌道運動

楕円軌道(クーロン・ポテンシャル、調和振動子)は
例外中の例外

多重周期運動



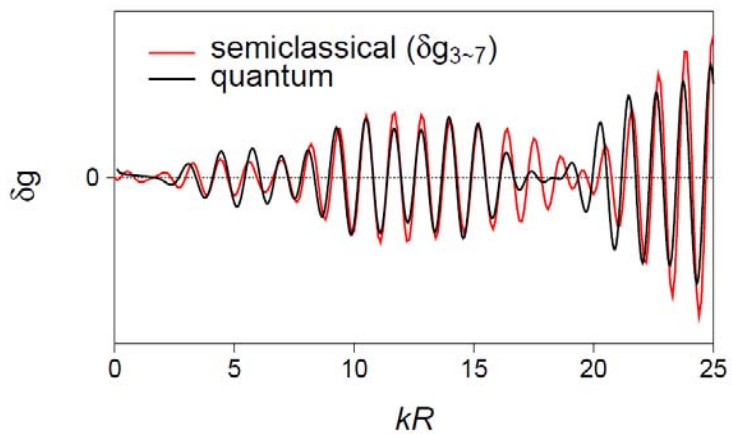
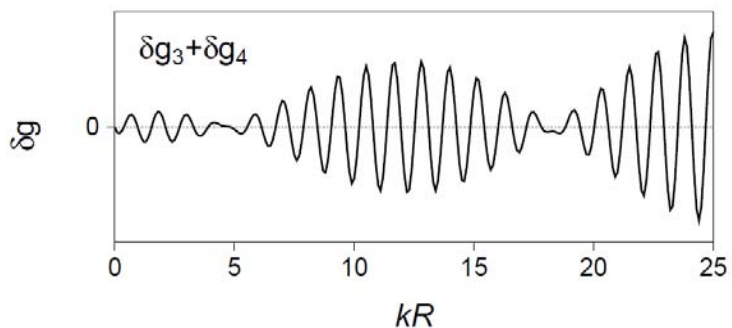
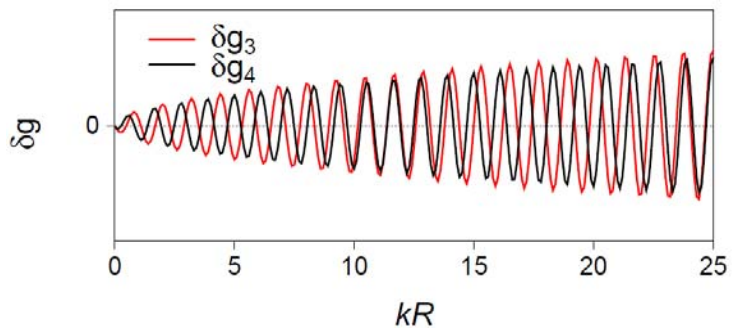
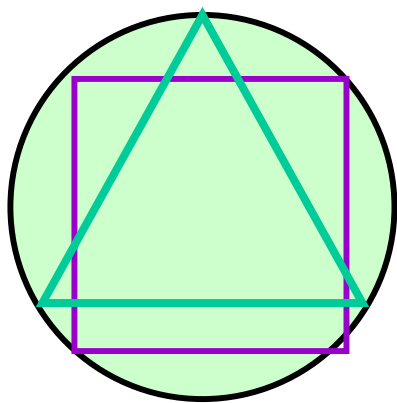
周期運動



Cavity における

3角形軌道と4角形軌道の干渉

スーパーシエル構造の形成



金属クラスターにおけるスーパーシェル構造の発見

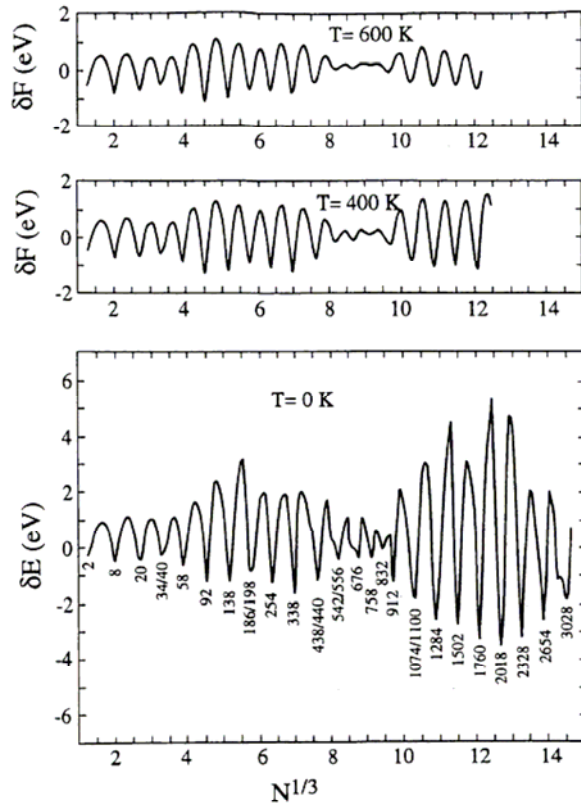


Figure 8.11: Oscillating part of total free energy of spherical sodium clusters, obtained in self-consistent jellium calculations at finite temperatures, plotted versus cube root of the number N of valence electrons. (After [64].)

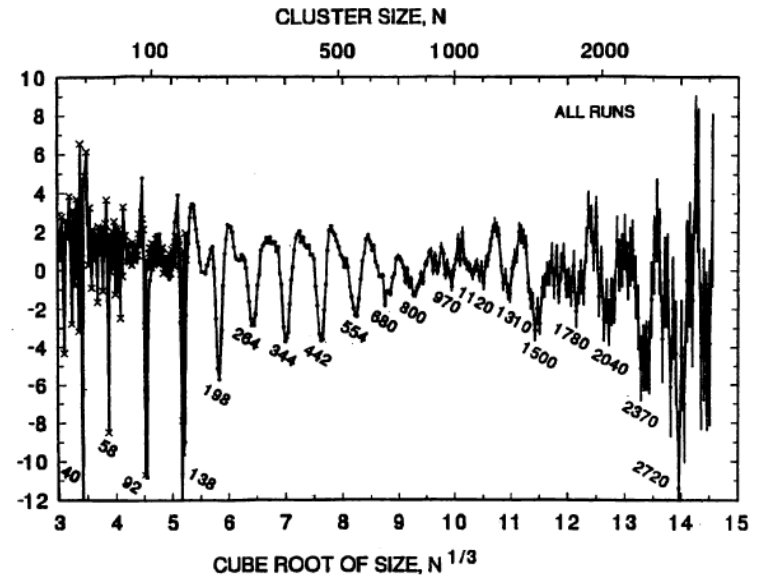
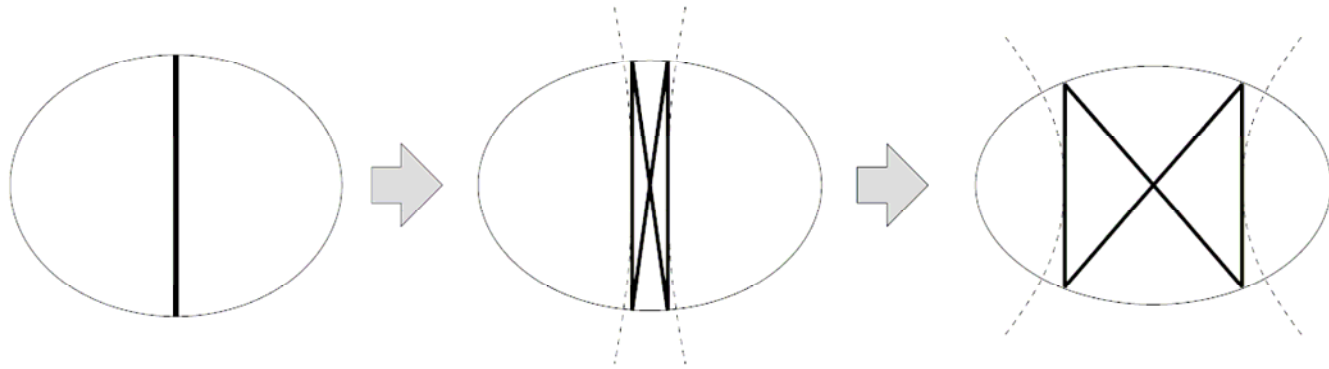


Figure 8.12: Experimental observation of the supershells. Shown are the logarithmically derived relative yields of sodium clusters from an adiabatic expansion source Fig. 8.10 (below), but scaled by an extra factor $N^{1/2} \exp(cN^{1/3})$ (see text for planation). (From [65].)

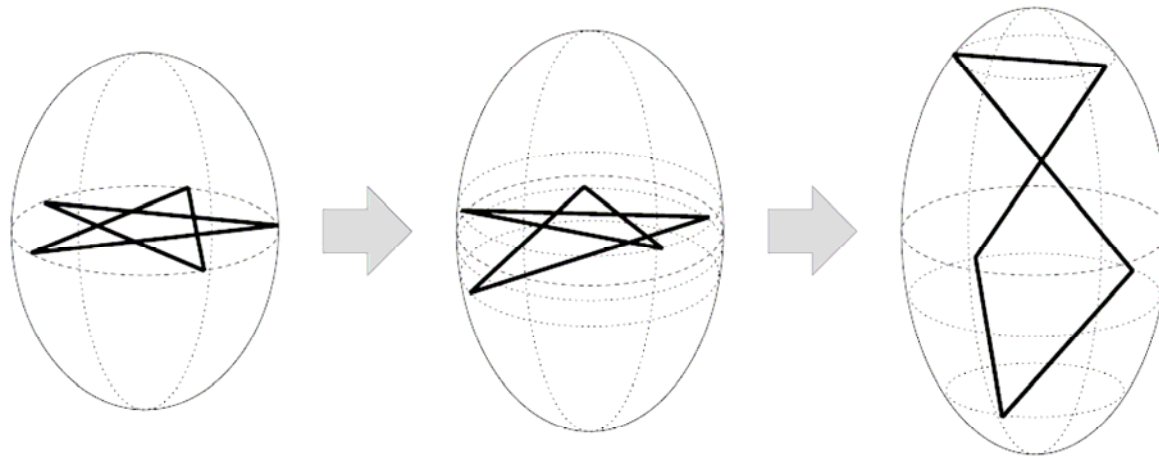
J. Pedersen et al., Nature 353 (1991) 733

O. Genzken and M. Brack, Phys. Rev. Lett. 67 (1991) 3286

周期軌道に分岐現象



楕円ビリヤード: Birth of Butterfly



楕円体キャビティ: 3次元軌道の誕生

トレース積分

$$g = \int e^{iS(x)} dx$$

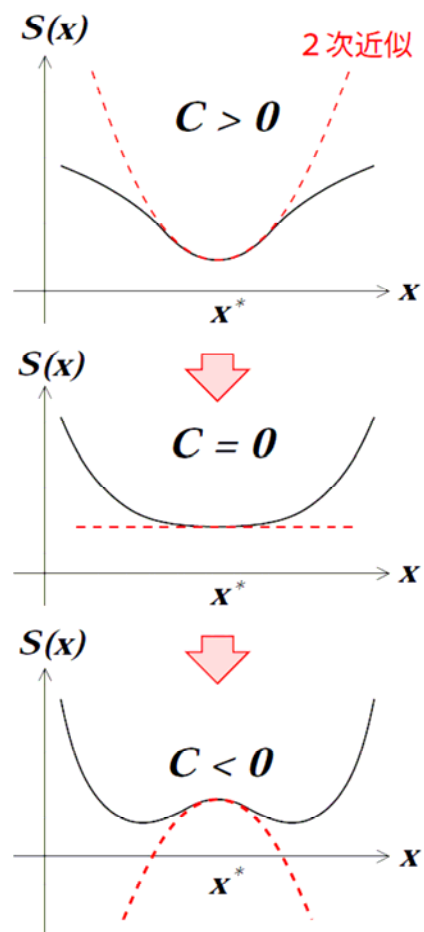
↓

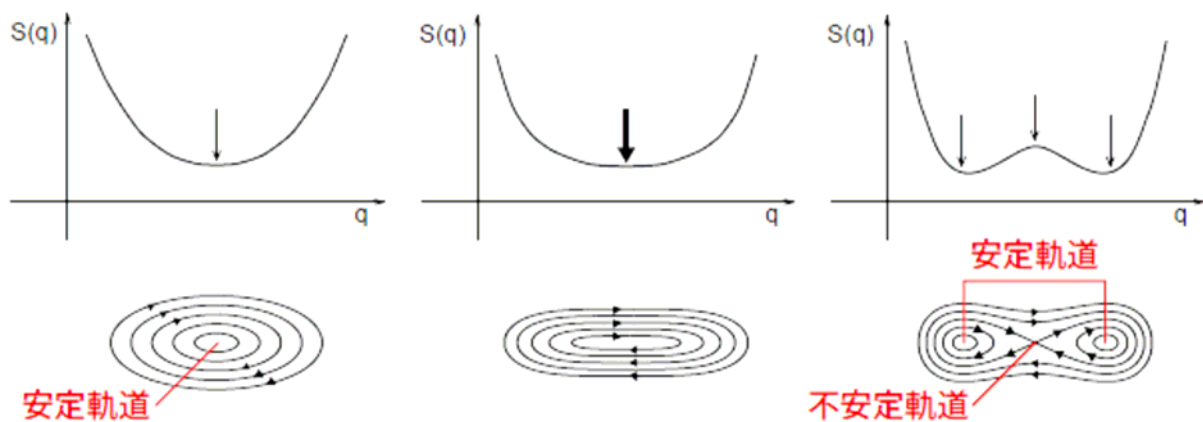
停留位相近似（鞍点近似）

$$g \sim e^{iS(x^*)} \int e^{i\frac{C}{2}(x-x^*)^2} dx$$

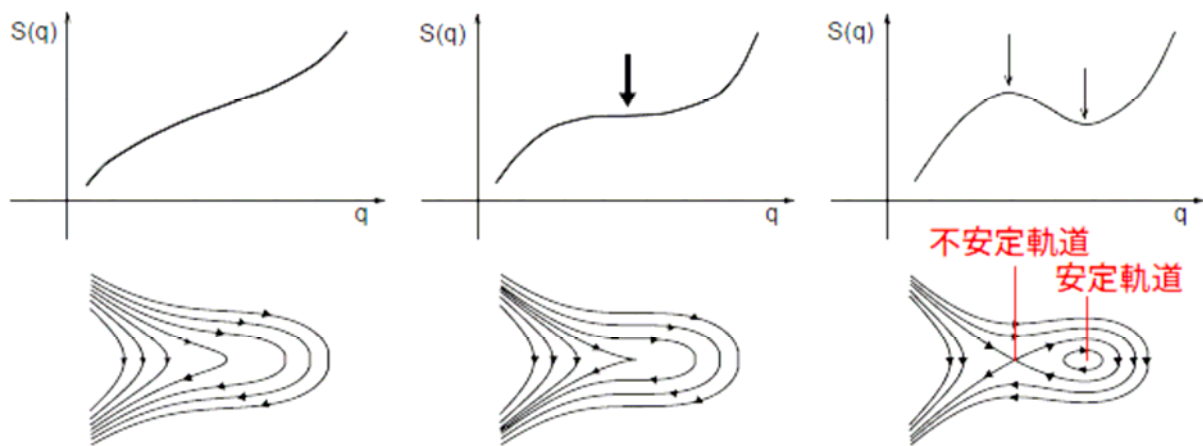
$$\propto \frac{e^{iS^*}}{\sqrt{C}} \rightarrow \text{発散}$$

周期軌道分岐点: $C = 0$

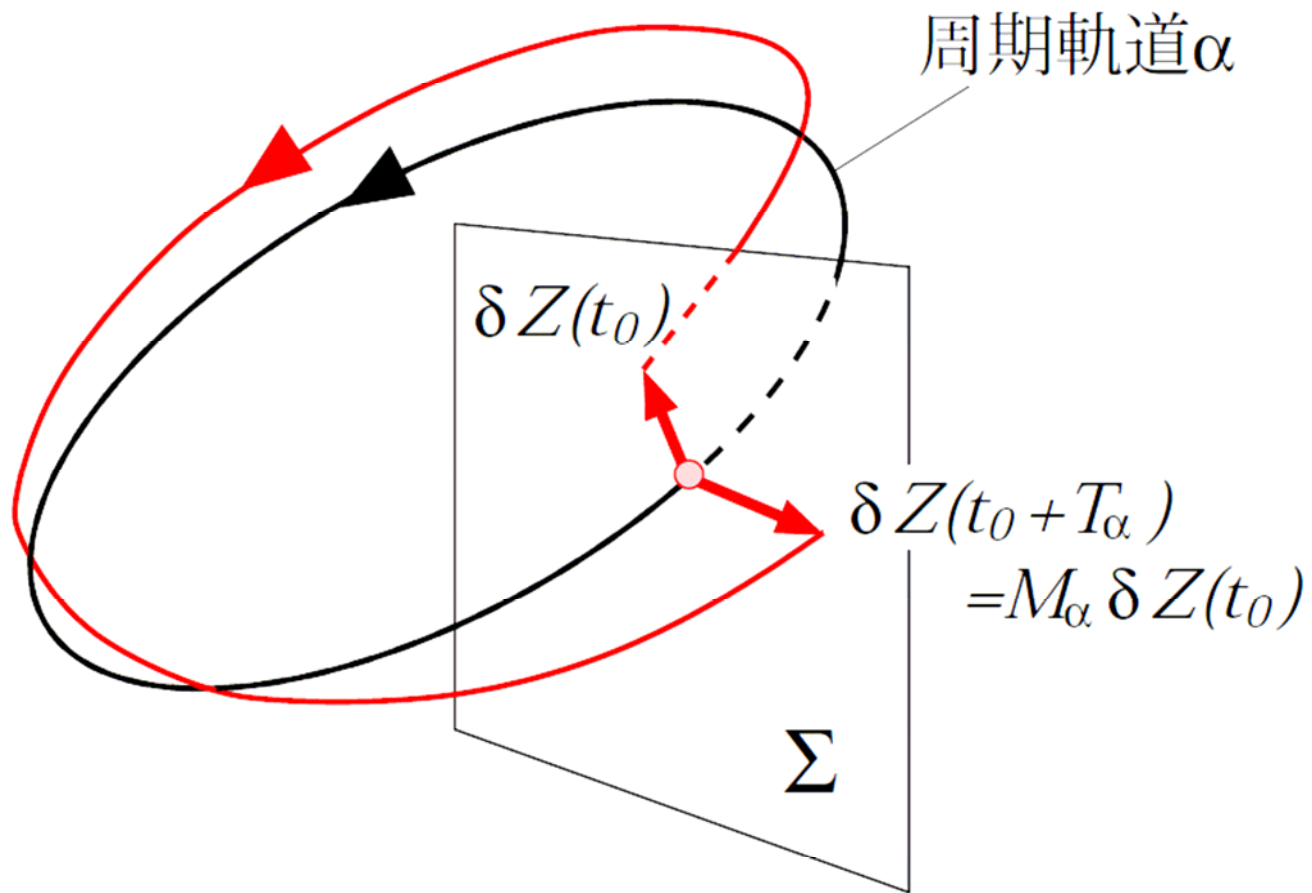




a) 熊手型分岐



b) 鞍点-結節点分岐



$$g(E) = \bar{g}(E) + \sum_{\alpha} A_{\alpha}(E) \cos\left(\frac{1}{\hbar} S_{\alpha}(E) - \frac{\pi}{2} \mu_{\alpha}\right)$$

$$\delta E = \frac{2\pi\hbar}{\partial S_{\alpha}(E)/\partial E} = \frac{2\pi\hbar}{T_{\alpha}}$$

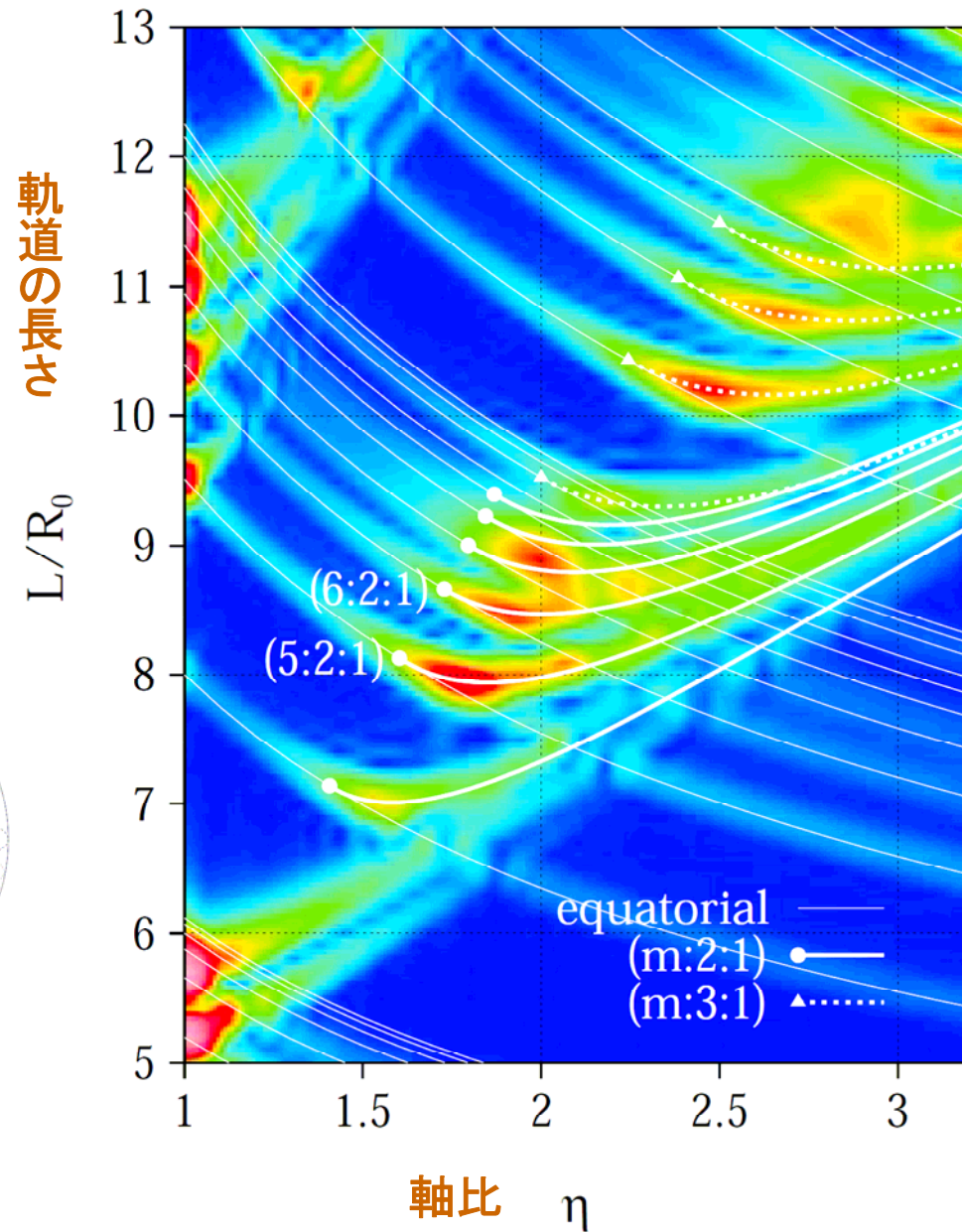
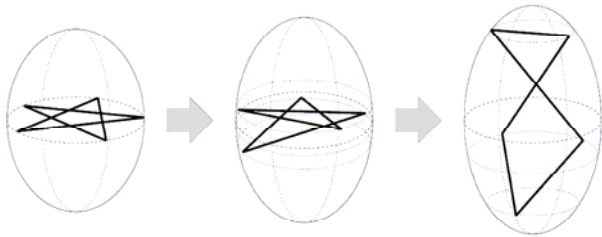
$$\eta = R_z / R_{\perp}$$

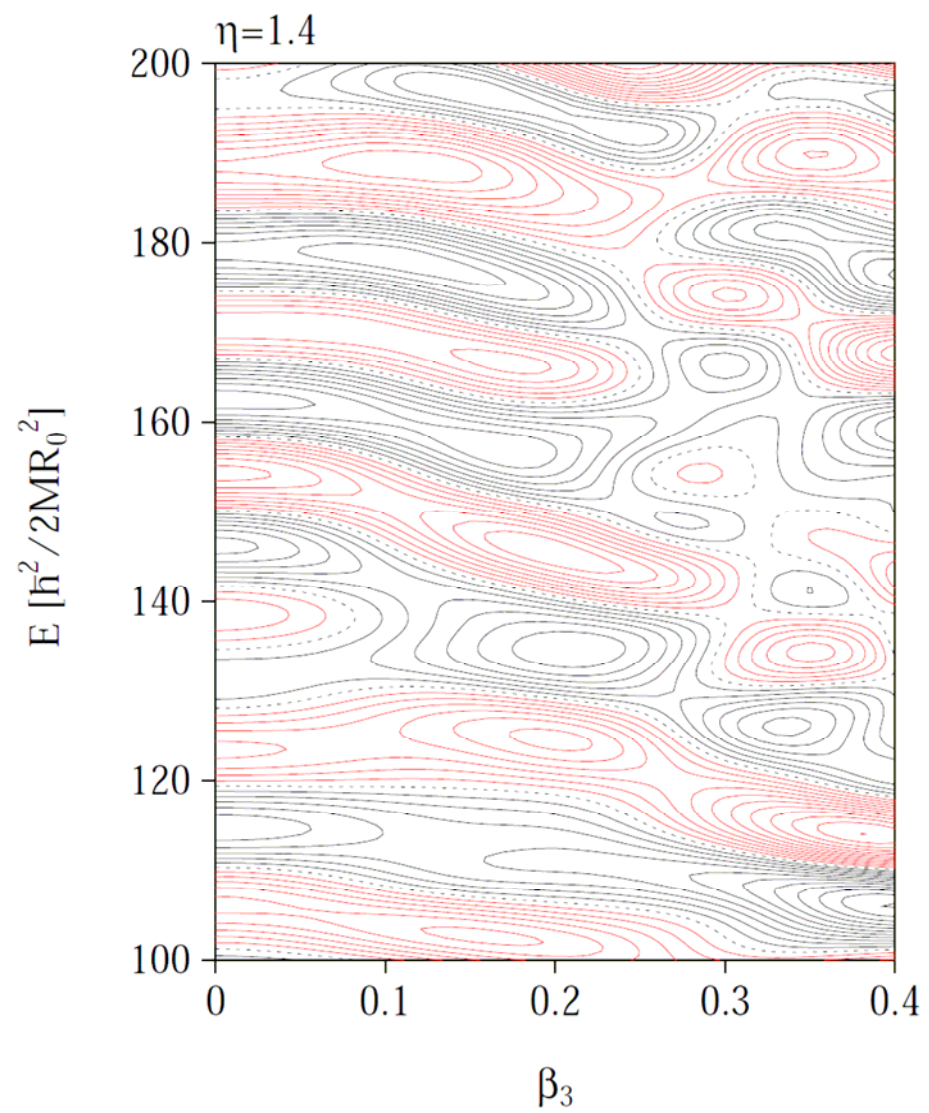
$$\delta g(E = \hbar^2 k^2 / 2m) = \sum_{\alpha} A_{\alpha} \cos(kL_{\alpha} - \pi\mu_{\alpha}/2)$$

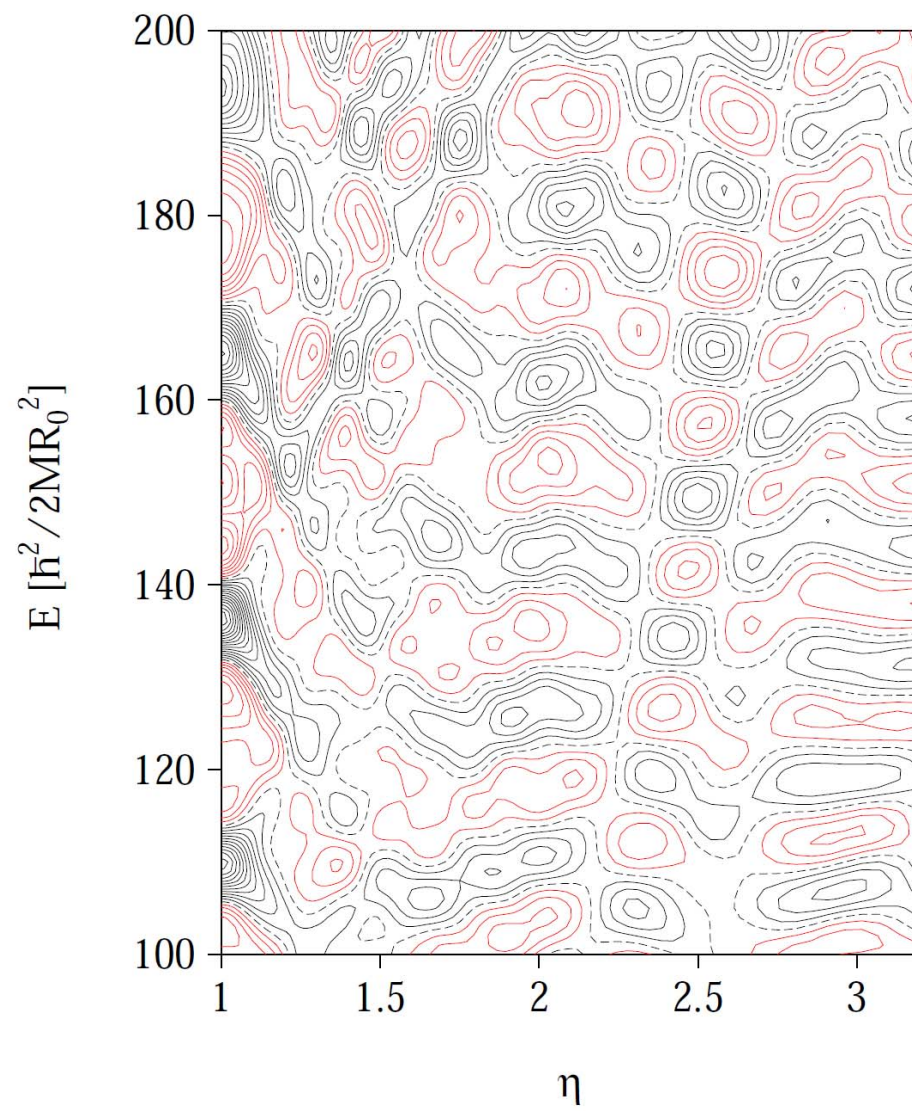
$$F(L) = \int e^{ikL} \delta g(k) dk = \sum_{\alpha} A'_{\alpha} \delta(L - L_{\alpha})$$

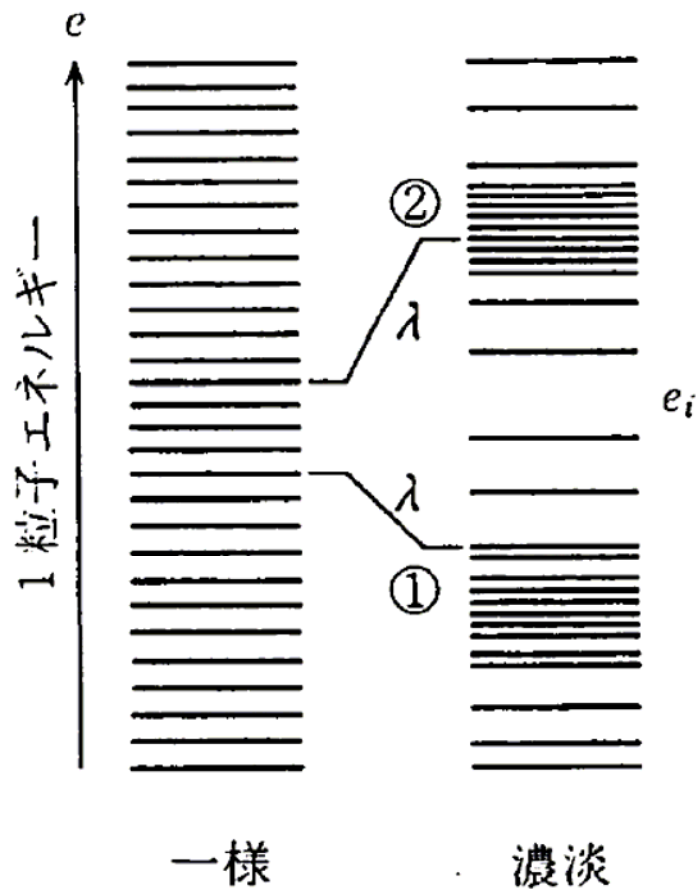
軸対称Cavity

周期軌道分岐による
シェル構造の形成

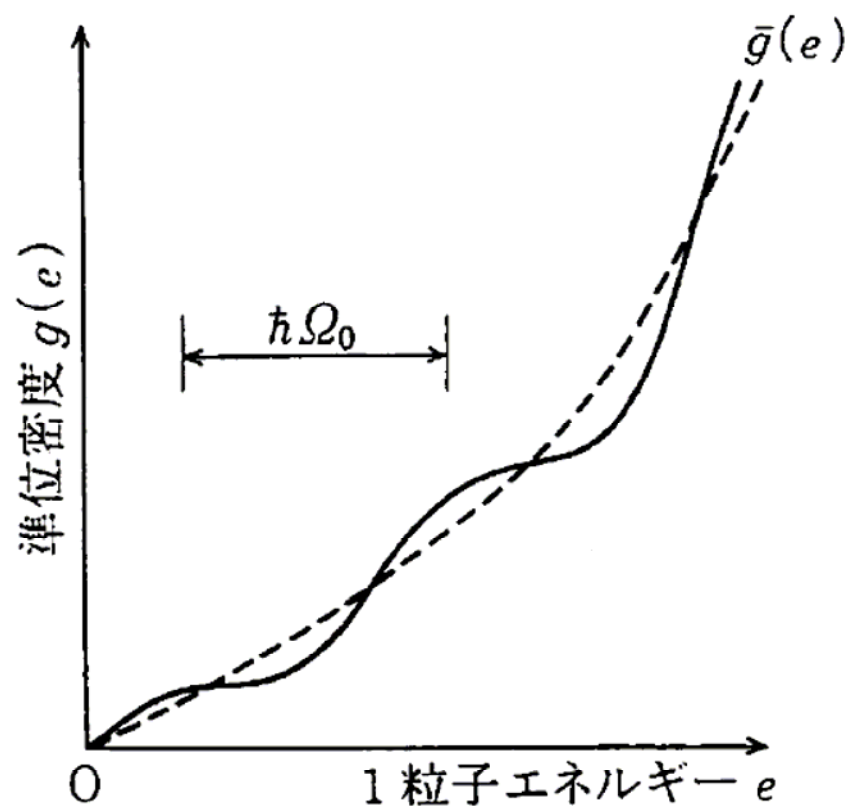








(a)



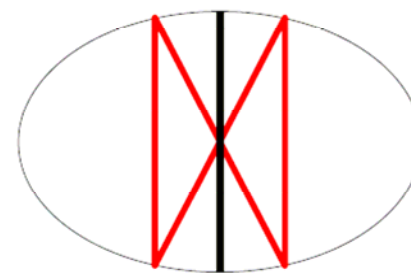
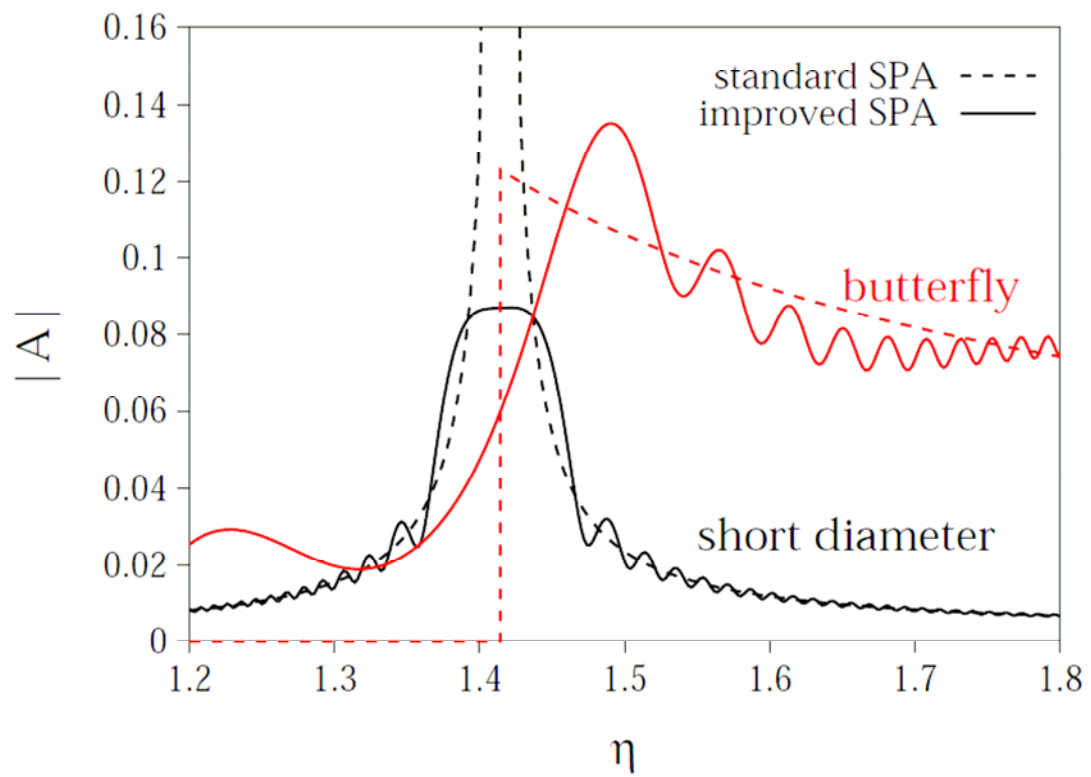
(b)

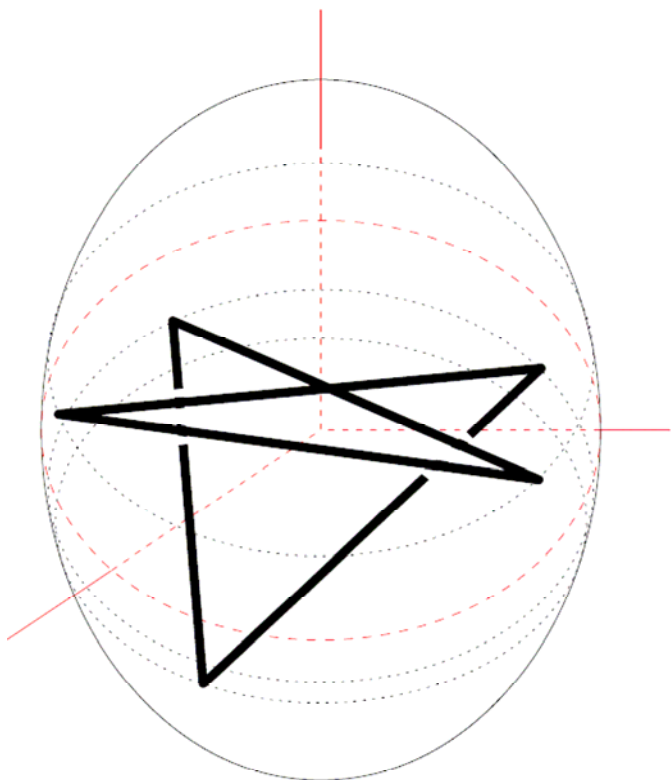
$$g(E) = \frac{dN(E)}{dE} = \sum_n \delta(E - E_n)$$

$$I_i \equiv \oint_{\gamma_i} \mathbf{p} \cdot d\mathbf{r} = \hbar (n_i + \alpha_i/4), \quad (n_i = 0, 1, 2, \dots)$$

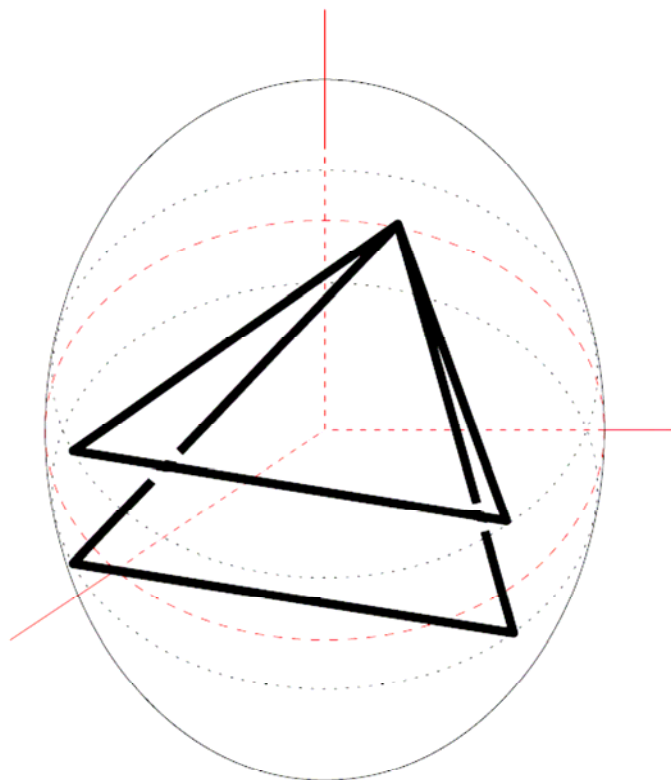
$$\Delta E \simeq \frac{\partial H(\mathbf{I})}{\partial \mathbf{I}} \cdot \Delta \mathbf{I} = \hbar \boldsymbol{\omega}(\mathbf{I}) \cdot \Delta \mathbf{n}$$

$$\mathbf{n} \equiv \{n_i\}$$





(5:2:1)



(6:2:1)

