Centre-sector tunneling, confinement and the quark Fermi surface

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Introduction:

- Yang-Mills *moduli* and *confinement*
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- Yang-Mills moduli and confinement
- Phenomenological impact of centre-sector-tunneling
  → Fermi-Einstein condensation in $SU(2N)$ QCD-like theories
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  → Fermi-Einstein condensation in $SU(2N)$ QCD-like theories

- Does centre-sector-tunneling take place in the hadronic phase?
  → lattice gauge simulations: YM + qHiggs
  → centre-sector-tunneling and the ‘t Hooft loop
  → tunneling coefficient (new!)
**Introduction:**

- Yang-Mills moduli and confinement
- Phenomenological impact of centre-sector-tunneling  
  → Fermi-Einstein condensation in $SU(2N)$ QCD-like theories  
- Does centre-sector-tunneling take place in the hadronic phase?  
  → lattice gauge simulations: YM + qHiggs  
  → centre-sector-tunneling and the 't Hooft loop  
  → tunneling coefficient (new!)
- Does Fermi-Einstein condensation take place in $SU(3)$ with matter?
Yang-Mills moduli

- use lattice gauge theory throughout
Yang-Mills moduli

- Use lattice gauge theory throughout

- Gauge fields: links $U$
- Matter fields: site $q$
- Gauge transformations: site $\Omega$

![Diagram showing lattice structure with nodes labeled link and quark, and transformations indicated.](image)
Yang-Mills moduli

My name is vacuum - the vacuum:
(pert.) vacuum $\leftrightarrow$ all contractable loops are $1$

Example:
$U_\mu(x) = 1$

More vacua?
Yang-Mills moduli

My name is vacuum - the vacuum:

(pert.) vacuum $\leftrightarrow$ all contractable loops are $1$

example:

$U_\mu(x) = 1$

more vacua?

constructing the moduli space

$\Rightarrow$ need to “devide out” the gauge transformations

[Schaden, PRD 71 (2005) 105012]
Yang-Mills moduli

- vacuum $\Rightarrow \text{tr } P(x) = \text{tr } P(y), \quad P: \text{Polyakov line}$
**Yang-Mills moduli**

- vacuum $\Rightarrow \text{tr } P(x) = \text{tr } P(y)$, \text{P: Polyakov line}
- $U^\dagger P(x) U P^\dagger(y) = 1 \Rightarrow P(y) = U^\dagger P(x) U$ q.e.d.

\[ P(x) \quad U \quad P(y) \]

\[ X \quad U \quad Y \]
Yang-Mills moduli

- complete gauge fixing $\rightarrow$ moduli space
  step 1

![Diagram with grid and labels Omega_1, Omega_2, Omega_3, etc., with arrows pointing up and a point labeled P.]
Yang-Mills moduli

- complete gauge fixing $\rightarrow$ moduli space
  step 2
Yang-Mills moduli

- complete gauge fixing $\rightarrow$ moduli space
  step 3

$U P U^\dagger P^\dagger = 1$
$[U, P] = 0$
$U, P \in \text{cartan}$

Centre-sector tunneling, confinement and the quark Fermi surface – p. 8/40
Yang-Mills moduli

- choose $P_1, P_2 \in \text{Cartan}$ such that $\text{tr } P_1 \neq \text{tr } P_2$
- found a variety of \textit{gauge inequivalent} vacua
  (\textit{moduli space}!)
choose $P_1, P_2 \in \text{Cartan}$ such that $\text{tr } P_1 \neq \text{tr } P_2$ found a variety of gauge inequivalent vacua (moduli space!)

centre transformation: $z = \exp\left\{i\frac{2\pi}{N_c}m\right\}$, $m = 1 \ldots N_c$, $SU(N_c)$
choose $P_1, P_2 \in \text{Cartan}$ such that $\text{tr } P_1 \neq \text{tr } P_2$
found a variety of gauge inequivalent vacua (moduli space!)

centre transformation: $z = \exp\{i \frac{2\pi}{N_c} m\}, m = 1 \ldots N_c, SU(N_c)$

- symmetry of the action
- mediates between vacua:
  $\text{tr } P \rightarrow z \text{tr } P$
Centre sector tunneling:

Hypothesis:

- integration over moduli $\Rightarrow$ average of centre sectors
  $\Rightarrow$ confinement!
Centre sector tunneling:

**Hypothesis:**

- integration over moduli $\Rightarrow$ average of centre sectors
  $\Rightarrow$ confinement!

- deconfinement: only one state on moduli space is realised

**Remarks:** needs infinitely many dofs (SSB!)

MC simulation better be ergodic

$\langle \text{tr } P \rangle = 0$ also for $T > T_{\text{deconf}}$
Centre sector tunneling:

Hypothesis:

- integration over moduli $\Rightarrow$ average of centre sectors $\Rightarrow$ confinement!
- deconfinement: only one state on moduli space is realised
- remarks: needs infinitely many dofs ($SSB!$)
  MC simulation better be ergodic
  $\langle \text{tr } P \rangle = 0$ also for $T > T_{\text{deconf}}$

- dynamical matter ($QCD!$): flat directions of the vacuum are lifted, but centre sector tunneling still takes place
  in the hadronic phase
- extreme conditions: SSB of centre symmetry on top of explicit breaking
Centre sector tunneling:

Illustration:

- classical action $\Rightarrow$ moduli space

orbifold
Centre sector tunneling:

Illustration:

- quantum effective action, pure YM $\Rightarrow$ centre sectors

SU(3)

centre sectors

barrier!
Centre sector tunneling:

Illustration:

- quantum effective action, $\text{QCD} \Rightarrow \text{centre sectors}$

QCD: hadronic phase

tunneling

centre sectors

vacuum
Centre sector tunneling:

Illustration:

- quantum effective action, $\text{QCD} \Rightarrow \text{centre sectors}$
...before discussing whether centre sector tunnelling in QCD takes place, I discuss the phenomenological impact.
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I discuss the phenomenological impact

consider $SU(2N) +$ dynamical matter (QCD-like theories)
at finite chemical potential $\mu$
Centre sector tunneling:

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- Consider $SU(2N) +$ dynamical matter ($QCD$-like theories) at finite chemical potential $\mu$.

$\Rightarrow$ Fermi-Einstein-Condensation (FEC) $\Leftarrow$
Centre sector tunneling:

- ...before discussing whether centre sector tunnelling in QCD takes place, I discuss the phenomenological impact
- consider $SU(2N) +$ dynamical matter ($QCD$-like theories) at finite chemical potential $\mu$

  $\Rightarrow$ Fermi-Einstein-Condensation (FEC) $\Leftrightarrow$

- ...will talk about $SU(3) +$ matter $= QCD$ later
Fermi-Einstein-Condensation (FEC)

Model consideration:

- $q(x)$: quarks, $m$: mass, $\mu$: chemical potential
- $A_m$: moduli fields $\Rightarrow$ weighted sum over centre sectors
Fermi-Einstein-Condensation (FEC)

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- \( q(x) \): quarks, \( m \): mass, \( \mu \): chemical potential
- \( A_m \): moduli fields \(\Rightarrow\) weighted sum over centre sectors

- partition function: \( \exp\{iA_m\} = Z_m \in Z(N_C) \)

\[
Z = \sum_{m=1}^{N_c} p_m \int Dq D\bar{q} \exp\{\bar{q}(i\partial + (A_m + i\mu)\gamma_0 + im)q\}
\]

- \( p_m \): probability for centre sector \( m \)
  - pure YM-theory: \( p_m = 1/N_c, \forall m \)
  - high \( T \) SSB phase: \( p_{N_c} = 1, p_m = 0 \) for \( m = 1 \ldots N_c - 1 \)
  - hadronic phase: \( p_{N_c} > p_m \neq 0 \) for \( m = 1 \ldots N_c - 1 \)

Fermi-Einstein-Condensation (FEC)

Results for baryon density:

\[ B = \frac{1}{\pi^2} \int_m^\infty dE \ E \ \sqrt{E^2 - m^2} \ \rho(E, T, \mu) \]

\( \rho(E, T, \mu) \): density of states
Fermi-Einstein-Condensation (FEC)

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\( \rho(E, T, \mu) \): density of states

\[ \rho(E, T, \mu) = \sum_{m} e^{[E - \mu]/T + z_m} w_m, \]

\( z_m \): centre phases, \( w_m \): weights
**Fermi-Einstein-Condensation (FEC)**

Results for baryon density:

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\(\rho(E, T, \mu)\): density of states

\[
\rho(E, T, \mu) = \sum_m \frac{z_m}{e^{[E-\mu]/T} + z_m} w_m,
\]

\(z_m\): centre phases, \(w_m\): weights

\[
w_m = p_m \rho_m / \sum_i p_i \rho_i
\]

\[
\rho_i = \exp\left\{ \frac{V}{\pi^2} \int_{m}^{\infty} dE \frac{E}{\sqrt{E^2 - m^2}} \ln(1 + z_i e^{-\frac{E-\mu}{T}}) \right\}
\]
Fermi-Einstein-Condensation (FEC)

(I) high temperature phase

- remember:
  - sector probability: \( p_{Nc} = 1, \quad p_m = 0 \) for \( m = 1 \ldots N_c - 1 \)
  - centre element: \( z_{Nc} = 1 \)
  - weights: \( w_{Nc} = \rho_{Nc}/\rho_{Nc} = 1, \quad w_m = 0 \) else
Fermi-Einstein-Condensation (FEC)

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\[
\rho(E, T, \mu) = \sum_m z_m \frac{w_m}{e^{(E-\mu)/T} + z_m} = \frac{1}{e^{(E-\mu)/T} + 1}
\]

free quarks with a Fermi surface !!
**Fermi-Einstein-Condensation (FEC)**

(II) hadronic phase \((N_c \text{ even})\)

- sector probability: \(p_{N_c} > p_m \neq 0\) for \(m = 1 \ldots N_c - 1\)
- centre element: \(z_{N_c/2} = \exp\{i \frac{2\pi}{N_c} \frac{N_c}{2}\} = -1\)
- weights:
  \[ w_{Nc/2} \approx \frac{\rho_{Nc/2}^{Nc/2}}{\rho_{Nc/2}^{Nc/2}} = 1, \quad w_m \approx 0 \text{ else} \]

\[
\rho_{Nc/2} = \exp\left\{ \frac{V}{\pi^2} \int_{m}^{\infty} dE \, E \sqrt{E^2 - m^2} \ln\left(1 - e^{-\frac{E - \mu}{T}}\right) \right\} \rightarrow \infty
\]

*Cooper instability familiar from BEC!!*
Fermi-Einstein-Condensation (FEC)

(II) hadronic phase ($N_c$ even)

- sector probability: $p_{N_c} > p_m \neq 0$ for $m = 1 \ldots N_c - 1$
- centre element: $z_{N_c/2} = \exp\{i \frac{2\pi}{N_c} \frac{N_c}{2}\} = -1$

weights:

$$w_{N_c/2} \approx \frac{\rho_{N_c/2}^2 \rho_{N_c/2}}{\rho_{N_c/2}^2 \rho_{N_c/2}} = 1, \ w_m \approx 0 \text{ else}$$

$$\rho_{N_c/2} = \exp\left\{\frac{V}{\pi^2} \int_m^\infty dE \ E \sqrt{E^2 - m^2} \ \ln(1 - e^{-\frac{E - \mu}{T}})\right\} \to \infty$$

Cooper instability familiar from BEC !!

$$\rho(E, T, \mu) = \sum_m \frac{z_m}{e^{(E-\mu)/T} + z_m} \ w_m = \frac{-1}{e^{(E-\mu)/T} - 1}$$

Fermi-Einstein Condensation (FEC) !!

Interpretation:

Centre-sector tunneling, confinement and the quark Fermi surface
Fermi-Einstein-Condensation (FEC)

Interpretation:

- Centre dressed quarks acquire Bose statistic and condense because of a Cooper instability
Fermi-Einstein-Condensation (FEC)

Interpretation:

- **centre dressed quarks** acquire Bose statistic and **condense** because of a Cooper instability.
- quarks are still represented by Grassmann fields but the **spin-statistic theorem** does not apply as long as **colour** is confined.
Centre sector tunneling:

Does centre sector tunneling take place in the hadronic phase of QCD?
Model considerations:
The SU(2) - qHiggs model

- Degrees of freedom:
  - gluons $\rightarrow U_\mu(x)$
  - Higgs (scalar, fundamental rep.) $\rightarrow \phi(x)$
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The SU(2) - qHiggs model

- Degrees of freedom:
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- Coupling constants:
  - bare gauge coupling $\rightarrow g \leftrightarrow \beta$
  - Higgs mass $\rightarrow \kappa : \text{mass} \propto 1/\kappa$
  - NO Higgs quartic coupling (quadratic Higgs)
Model considerations:

The SU(2) - qHiggs model

- Degrees of freedom:
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- need good ergodcity:
  - 1HMC for the gluon sector
  - HMC for the Higgs sector
  - simulations at the HPCC, Plymouth
Model consideration:

String breaking

- Pure Yang-Mills theory *(no dynamical quarks)*

![Diagram of electric flux tube with electric field E ~ σr between charges Q and Q̅.](image-url)
Model consideration:

String breaking

- Pure Yang-Mills theory (no dynamical quarks)

\[ E \sim \sigma r \]

- QCD (with dynamical quarks)

\[ E \sim \sigma r \sim 2m \]
Model consideration:

SU(2) + qHiggs: carefully tune the bare mass (\(\kappa\))
Model consideration:

SU(2) + qHiggs:
carefully tune the bare mass ($\kappa$)

shows string breaking
breaks center symmetry
abundance of configs
Centre sector tunneling: YM + qHiggs

- Polyakov line: \[ P = \left\langle \sum \vec{x} \prod_t U_0(\vec{x}, t) \right\rangle \]
Centre sector tunneling: YM + qHiggs

- Polyakov line: \( P = \langle \sum \bar{x} \prod_t U_0(\bar{x}, t) \rangle \)

\[<P>/V^3\]

\[T_c\] shift of \( T_c \):

\[300 \text{MeV} \rightarrow 170 \text{MeV}\]
A lesson from the Ising model:
A lesson from the Ising model:

\[
\begin{array}{c}
\text{vacuum 1} \\
+1
\end{array} \quad \xrightarrow{\text{centre transf.}} \quad \begin{array}{c}
\text{vacuum 2} \\
-1
\end{array}
\]

magnetisation: \( \langle M \rangle = 0 \) (Swendsen-Wang cluster alg)
Centre sector tunneling: YM + qHiggs

A lesson from the Ising model:

- magnetisation: $\langle M \rangle = 0$ (Swendsen-Wang cluster alg)

- use: $\left\langle M(0) \sum_x M(x) \right\rangle \propto \begin{cases} \xi^3 = \text{finite, symm. phase} \\ V_3 \text{ SSB phase} \end{cases}$
Centre sector tunneling: YM + qHiggs

- Polyakov line: \( \langle p(0) \sum_x p(x) \rangle \propto \left\{ \begin{array}{l} \sigma^{-3} = \text{confinement phase} \\ V_3 \text{ high } T \text{ phase} \end{array} \right\} \)
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  $V_3$ high $T$ phase

Data suggest tunneling for $\beta < \beta_c$

Despite explicit centre symm. breaking
centre sector transformation: \( |\psi\rangle \rightarrow |Z \psi\rangle \)

Wigner Weyl: \( \langle \psi | Z \psi \rangle = 1 \)

SSB: \( \langle \psi | Z \psi \rangle = 0, \quad (V_3 \rightarrow \infty) \)
Centre sector tunneling: ’t Hooft loop

- centre sector transformation: $|\psi\rangle \rightarrow |Z\psi\rangle$
  
  Wigner Weyl: $\langle \psi | Z \psi \rangle = 1$
  
  SSB: $\langle \psi | Z \psi \rangle = 0$, $(V_3 \rightarrow \infty)$

- consider: $\frac{\sum_\psi \langle \psi | e^{-H/T} | Z \psi \rangle}{\sum_\psi \langle \psi | e^{-H/T} | \psi \rangle} = \begin{cases} 1 & \text{for Wigner Weyl} \\ 0 & \text{for SSB} \end{cases}$
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variable subs.

‘t Hooft loop!
pure SU(2) YM-theory:
[ de Forcrand, v. Smekal,
PRD 66 (2002) 011504.]
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also: ‘t Hooft loop ↔ vortex free energy
Centre sector tunneling: ‘t Hooft loop

- pure SU(2) YM-theory:
  [ de Forcrand, v. Smekal,
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- ‘t Hooft loop with dynamical matter ???

here: $\langle \psi | Z | \psi \rangle$ well defined ↔ centre sector tunneling
Centre sector tunneling: 't Hooft loop

- pure SU(2) YM-theory:
  

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- difficult to calculate with dynamical matter
  
  [100,000 configs]
Centre sector tunneling: ‘t Hooft loop

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  here: $\langle \psi |Z |\psi \rangle$ well defined ↔ centre sector tunneling

- difficult to calculate with dynamical matter
  [100,000 configs]

- need something else....
Interface tension: YM + qHiggs

A lesson from the Ising model:

- vacuum 1
- vacuum 2

frustration!

periodicity

+1  bond: −1  −1
Interface tension: YM + qHiggs

A lesson from the Ising model:

Interface energy: $F$

\[ \exp\left\{-\frac{F}{T}\right\} = \frac{\text{partition function}}{\text{partition function}} \]
Interface tension:

- Define:

\[ P_\prec = \sum_{V_\prec} \text{tr} \prod_t U_0(x) \]
\[ P_\succ = \sum_{V_\succ} \text{tr} \prod_t U_0(x) \]
**Interface tension:**

- Define:
  \[ P_\prec = \sum_{V_\prec} \text{tr} \prod_t U_0(x) \]
  \[ P_\succ = \sum_{V_\succ} \text{tr} \prod_t U_0(x) \]

- Center map: \( C(P) \)
  \[ C = n : P \in \mathbb{C} \rightarrow \mathbb{Z}_n \]
Interface tension:

- Define:
  \[ P_\leq = \sum_{V_\leq} \text{tr} \prod_t U_0(x) \]
  \[ P_\geq = \sum_{V_\geq} \text{tr} \prod_t U_0(x) \]

- Center map: \( C(P) \)
  \[ C = n : P \in \mathbb{C} \rightarrow \mathbb{Z}_n \]

- Centre interface energy: \( F_{nm} \)
  \[ \exp\{-F_{nm}/T\} = \frac{1}{N} \int \mathcal{D}U \mathcal{D}\phi \delta\left(m, C(P_\leq)\right) \delta\left(n, C(P_\geq)\right) e^S \]
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  \]

- Tunneling coefficient:
  probability \((C(P_\prec) \text{ and } C(P_\succ))\) are different
**Interface tension: YM + qHiggs**

- tunneling coefficient for SU(2):

<table>
<thead>
<tr>
<th></th>
<th>$C(P^-)$</th>
<th>$C(P^+)$</th>
</tr>
</thead>
<tbody>
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- tunneling $\rightarrow \frac{1}{2}$
- SSB $\rightarrow 0$
Interface tension: YM + qHiggs

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Interface tension: YM + qHiggs

**tunneling coefficient for SU(2):**

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$tunneling \quad \rightarrow \quad \frac{1}{2}$

$SSB \quad \rightarrow \quad 0$
### Interface tension: SU(3) + quarks

#### Tunneling coefficient for SU(3):

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- Tunneling → \( \frac{2}{3} \)
- SSB → 0
Interface tension: SU(3) + quarks

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- Tunneling \( \rightarrow \frac{2}{3} \)
- SSB \( \rightarrow 0 \)

Using MILC configs

\( N_f = 2, m a = 0.0125 \)

\( \beta_c \approx 5.26 \)
Centre sector tunneling:

evidence that centre sector tunneling take places in in the hadronic phase of QCD!

QCD: hadronic phase

tunneling

centre sectors

vacuum
Centre sector tunneling:

- evidence that centre sector tunneling take places in the hadronic phase of QCD!

QCD: hadronic phase

- is there FEC for $SU(N)$, $N$ odd, such as $SU(3)$?

$$
Z = -\frac{1}{2} + i \frac{\sqrt{3}}{2}
$$

$$
Z = 1
$$

$$
Z = -\frac{1}{2} - i \frac{\sqrt{3}}{2}
$$
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Fermi-Einstein-Condensation (FEC)

Model consideration:

- \( q(x) \): quarks, \( m \): mass, \( \mu \): chemical potential
- \( A_m \): moduli fields \( \Rightarrow \) weighted sum over \textit{centre sectors}

- partition function:
  \[
  \exp\{iA_m\} = Z_m \in Z(N_C)
  \]

\[
Z = \sum_{m=1}^{N_c} p_m \int Dq D\bar{q} \exp\{\bar{q}(i\partial + (A_m + i\mu)\gamma_0 + im)q\}
\]

- \( p_m \): probability for centre sector \( m \)
- hadronic phase: \( p_m \approx 1/N_c, \forall m \)
- high \( T \) SSB phase: \( p_{N_c} = 1, p_m = 0 \) for \( m = 1 \ldots N_c - 1 \)
Fermi-Einstein-Condensation (FEC)

high-$T$ SSB phase
Fermi-Einstein-Condensation (FEC)

- high-$T$ SSB phase
- quarkyonic phase

$\rho(E)$ vs $E/m$ graph with two curves:
- high $T$ (SSB)
- hadronic

Fermi sphere of quarks
Fermi-Einstein-Condensation (FEC)

high-$T$ SSB phase
quarkyonic phase
hadronic phase
Fermi-Einstein-Condensation (FEC)

high-$T$ SSB phase
quarkyonic phase
hadronic phase

- origin of the **Cooper instability**:

$$\exp\left\{-\frac{F}{T}\right\} = Z = \sum_{m=1}^{N_c} p_m \int \mathcal{D}q \, \mathcal{D}\bar{q} \ldots \rightarrow 0$$
Conclusions:

- Sum over Yang-Mills moduli $\Rightarrow$ confinement

Yes, there is a QCD perturbation theory with confinement!

Quantum level: centre-sector tunneling $\Rightarrow$ confinement
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  Fermi-Einstein condensation in $SU(N)$ QCD-like theories

  $N$ even [analytical] ✔, $N$ odd [numerical] ✔
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  **Fermi-Einstein condensation** in $SU(N)$ QCD-like theories

  $N$ even [analytical] ✔, $N$ odd [numerical] ✔

- Evidence that centre-sector-tunneling takes place
  in the hadronic phase:
  lattice gauge simulations: $SU(2) + \text{qHiggs}$, $SU(3) + N_f = 2$
  $\rightarrow$ tunneling coefficient

  **generalised ’t Hooft loop** ($\langle \psi | Z \psi \rangle$) with dynamical matter...