Numerical study of the confinement/deconfinement dynamics

X-QCD Japan
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Chiral Symmetry and Confinement in Cold, Dense Quark Matter
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Your Talk contains few Key Words of the Workshop!

Chiral Sym. and Confinement in Cold, Dense Quark Matter

Go Home!
Confinement is Important!
Plan of the Talk

• Magnetic Monopole
• Surface Operators
• Gluon Propagators ( SU(3) )
• Gluon Propagators and Vortex ( SU(2) )
• Transport Coefficient and Vortex ( SU(2) )
Most data are Quench and color SU(2)

Some are SU(3)
Magnetic Degrees of Freedom and the Confinement

Who has seen the Mag. Monopole?

Neither I nor you
Yes, they have seen.
Singular Configuration, or Vortex

Here,
No Monopole!
But it looks like,,,.
Vortices related to the Confinement are a 2-d Object and we need Surface Operator

- S. Gukov and E. Witten, hep-th/0612073.
- A. DiGiacomo and V. I. Zakharov, hep-th/0806.29382

For a Point Charge, Wilson invented Line Operators.
\[ \alpha \int d\sigma_{\mu\nu} F_{\mu\nu}^3 + \beta \int d\sigma_{\mu\nu} \tilde{F}_{\mu\nu}^3 \]

\[ \tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\lambda\rho} F_{\lambda\rho} \]

\[ \alpha \int d\sigma_{\mu\nu} \left( \sqrt{FF} \frac{d\sigma_{\mu\nu}}{d\sigma_{\mu\nu}} \right) + \beta \int d\sigma_{\mu\nu} \left( \frac{FF}{\sqrt{FF}} \frac{d\sigma_{\mu\nu}}{d\sigma_{\mu\nu}} \right) \]

\[ FF = \sum_{\mu\nu} \text{Tr} F_{\mu\nu} F_{\mu\nu} \quad F\tilde{F} = \sum_{\mu\nu} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} \]
Wilson Loops for a charge
Surface Operator
Center Projection

Del Debbio, Faber, Giedt, Greensite, Olejnik

\[
\text{Max} \sum_{x, \mu} \text{Tr} U_{\mu}(x) \rightarrow \text{Max} \sum \text{Tr} \left( U_{\mu} \right)^2
\]

Landau gauge or Coulomb Gauge

\[ Z_{\mu}(x) \equiv \text{sign} \, \text{Tr} U_{\mu}(x) = +1 \text{ or } -1 \]

Gauge Rotation.
Therefore non-local

\[ +1 \in Z_2 \]
\[ -1 \in Z_2 \]
If $Z_\alpha \times Z_\beta \times Z_\gamma \times Z_\delta = -1$, a Vortex pierces the Plaquette.

1-d Object (Charge) \rightarrow Wilson Loop \rightarrow 2-d Object (Vortex Line) \rightarrow Surface Op.
Vortex Removing

\[ U_\mu(x) \Rightarrow Z_\mu(x) \times U_\mu(x) \]

Remember that \( Z_\mu(x) \equiv \text{sign Tr } U_\mu(x) \)

By definition, now \( \text{sign Tr } U_\mu(x) = +1 \) for all links.

All vortices are fading out (by definition).
Surface Operator

\[ \alpha \int d\sigma_{\mu\nu} \left( \sqrt{FF} \frac{d\sigma_{\mu\nu}}{d\sigma_{\mu\nu}} \right) + \beta \int d\sigma_{\mu\nu} \left( \frac{\tilde{FF}}{\sqrt{FF}} \frac{d\sigma_{\mu\nu}}{d\sigma_{\mu\nu}} \right) \]

Operator A

Operator B
Size Dependence of Operator A

A

L
Size Dependence of Operator A with/without the Vortices

After removing Vortices
Fitting

$11.2632 \times L^{2.000}$

$10.3888 \times L^{2.000}$
Size Dependence of Operator B

![Graph showing the size dependence of Operator B with data points and error bars.](image-url)
Size Dependence of Operator B with/without the Vortices
No Vortex Removal
With and Without Vortices
L = 1, 2, 3
Another Possibility

We fix $L' = 2$
After removing Vortices
Surface Operators - Summary

- The surface operators surely feel the Vortices.
- But the we are struggling to understand them more clearly.
I ♥ Gluon Propagators
Gluon Propagator in the confinement (Quench, SU(3), Old Days Calculation)

\[ e^{-|\vec{p}|t} \]

48^3 \times 64

\[ \beta = 6.8 \]

\[ \vec{p} = \left( \frac{2\pi}{N_x}, 0, 0 \right) \]

Landau Gauge

Nakamura, 1995
Coulomb Gauge QCD a la Zwanziger

\[
H = \frac{1}{2} \int d^3 x \left( E_i^{tr} (x) + B_i^2 \right) + \frac{1}{2} \int d^3 x d^3 y (\rho(x) V(x, y) \rho(y))
\]

\[
g^2 \left\langle A_0 (x) A_0 (y) \right\rangle = g^2 D_{00} (x - y)
\]

\[
= V(x - y) + P(x - y)
\]

\[
V(x, y) = \int d^3 z \frac{1}{M(x, z)} \left( -\partial^2 \right) \frac{1}{M(z, y)} M \equiv -\vec{D} \vec{\partial}
\]

\[
= g^2 \left\langle \vec{V}(\vec{x}, \vec{y}) \right\rangle \delta(x_4 - y_4)
\]

\[
V_{phys} (R) \leq V_{coul} (R)
\]
Color Coulomb Potential

\[ V(x, y) = \int d^3z \frac{1}{M(x, z)} \left( -\partial^2_{(z)} \right) \frac{1}{M(z, y)} \]

\[ g^2 \left\langle A_0(x) A_0(y) \right\rangle = V(x-y) + P(x-y) \]

\[ g^2 \left\langle \mathbf{V}(\mathbf{x}, \mathbf{y}) \right\rangle \delta(x_4 - y_4) \]

\[ V_{phys}(R) \leq V_{coul}(R) \]
Eigen-Values of FP Operator accumulate near zero, i.e., Gribov boundary.

Ghost Dressing Function. Scaling Test

\[ J(p^2) \equiv \bar{p}^2 \left\langle M^{-1}\right\rangle(\bar{p}) \]
Transverse Gluon Propagators

Momentum Space

Co-ordinate Space

Color Coulomb Potential at $T>T_c$

$T/T_c \approx 1.5$

$V(r) \text{ [GeV]}$

$r \text{ [fm]}$

- color-Coulomb potential
- static potential

Transverse Gluon Propagators at $T > T_c$

$D^{tr} (p) T$

- $56^3 \times 4$, $T / T_c \approx 5.6$
- $56^3 \times 6$, $T / T_c \approx 3.7$
- $56^3 \times 4$, $T / T_c \approx 3.8$
- $56^3 \times 6$, $T / T_c \approx 2.5$
- $56^3 \times 8$, $T / T_c \approx 1.9$

Y. Nakagawa, A. Nakamura, T. Saito, H. Toki (in preparation)
Transverse Gluon Propagators at $T>T_c$

Co-ordinate Space

$\xi = 4$

\begin{align*}
24^3 \times 6, \beta=6.10, \quad &T / T_c \approx 4.7 \\
24^3 \times 6, \beta=5.95, \quad &T / T_c \approx 3.7 \\
40^3 \times 6, \beta=5.95, \quad &T / T_c \approx 3.7 \\
40^3 \times 6, \beta=5.75, \quad &T / T_c \approx 2.5
\end{align*}

Y. Nakagawa, A. Nakamura, T. Saito, H. Toki (in preparation)
Gluon Propagators - Summary

• Gribov-Zwanziger Scenario works also at $T>T_c$!
• Is this a good news or bad news?
Gluon Behavior
with/without Vortex
◆ Lattice simulations

- Removing center vortices eliminates confinement and restores chiral symmetry ([de Forcrand, D’Elia, PRL82, 4582(1999)](de Forcrand, D’Elia, PRL82, 4582(1999))
Maximal center projection

◆ Numerical technique
  Direct Maximal Center Projection (MCP) by Debbio, et. al, PRDv58,094501

◆ We apply the MCP to all configurations of the SU(2) gauge field

\[
\text{Maximize } R = \frac{1}{VT} \sum_{x,t} \text{Tr}[U_\mu(x,t)]^2
\]
\[
Z_\mu(x) = \text{sgn} \text{Tr}[U_\mu(x)]
\]

◆ Removing center vortex (via de Forcrand – D’Elia procedure, PRL82,4582(1999)):

\[
U_\mu(x) \rightarrow U'_\mu(x) = Z_\mu(x)U_\mu(x)
\]

→ Color confinement disappears and chiral symmetry restores.
Center removal for quark potential

- Removing vortices eliminates confinement.

\[ V(r)/\sigma \]

Confinement region (T=0)

- Removing center vortices eliminates confinement and restores chiral symmetry (de Forcrand, D’Elia, PRL82, 4582(1999))
Gluon propagators in the Landau gauge

SU(2)

Landau gauge

$T/T_c = 1.40$
\[ T / T_c = 3.0 \quad \text{SU}(2) \]

\[ T / T_c = 6.0 \]
Gluon propagators in the Coulomb gauge

\[ D_{\text{Electric}} \quad D_{\text{Magnetic}} \]

- Time-time (electric) correlator diverges in the infrared limit.
  - Instantaneous linearly rising potential and non-zero thermal string tension that depends on magnetic scaling
- Spatial-Spatial (magnetic) correlator is suppressed in the infrared limit.

SU(2)
Gluon Propagators with and without Vortex

Summary

• Gluons at $T>T_c$ have contribution of Vortex.
One of X-QCD Japan projects is to calculate Transport Coefficients at RHIC and LHC temperature regions and to see if they are different. What does “Perfect Fluid” mean?
\[ \eta = -\int d^3 x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x}',t') >_{ret} \]

\[ \frac{4}{3} \eta + \varsigma = -\int d^3 x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x},t) T_{11}(\vec{x}',t') >_{ret} \]

\[ \chi = -\frac{1}{T} \int d^3 x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x},t) T_{01}(\vec{x}',t') >_{ret} \]

\( \eta \) : Shear Viscosity

\( \varsigma \) : Bulk Viscosity

\( \chi \) : Heat Conductivity

we do not consider in Quench simulations.

\[ T_{\mu\nu}(\vec{x}',t') \]

\[ T_{\mu\nu}(\vec{x},t) \]

\[ -\infty < t' < t_1 < t \]
History

1995

U(1)
Coulomb and Confinement Phases

SU(2)
Two Definitions:
F = \log U
F = U - 1

1995

SU(2): \beta = 3.0
Dia\& Pert

1998

SU(3)
Improved Action

2005

The first calculation of \( \eta/s \) on the lattice, which is consistent with KSS bound.

\[ \eta \]
\[ s \]
Viscosity by Lattice, 2007

Nakamura and Sakai

\[ \frac{\eta}{s} \quad (2007) \]

\( \eta \): shear viscosity
\( s \): Entropy density

\( 16^3 \times 8 \), \( 24^3 \times 8 \)

Meyer

KSS bound

\[ \frac{T}{T_c} \]
Fluctuations in MC sweeps

SU(3), T=2

$\langle G(T=2) \rangle_{10000}$

Stand, $\beta=6.25$: $\langle G(2) \rangle_{all} = 1.7 \times 10^{-6}$

Imp, $\beta=3.3$: $\langle G(2) \rangle_{all} = 6.4 \times 10^{-7}$

Sweep $x 10^4$

Standard Action

Improved Action
$16^3 \times 8$, $\beta = 6.539$ (Wilson), $\beta = 3.05$ (Iwasaki)
中川君のデータ

$4 / \approx \quad cTT \quad 5 / \approx \quad cTT$

$G_{12}(\tau)$

$T / T_c \approx 4$

$T / T_c \approx 5$
\[
\langle T_{12}(0)T_{12}(t) \rangle
\]

\[\beta = 2.66 \quad (T / T_c \approx 3.24)\]

\[16^3 \times 8\]
\[ \langle T_{12}(0)T_{12}(t) \rangle \]

\[ \beta = 3.0 \ (T / T_c \approx 5.1) \]

\[ 16^3 \times 8 \]

\[ SU(2) \]

60 × 1000 Sweeps

With Multi-Hit
+ Vortex Removal

With Multi-Hit

Standard Plaq. Action
\( \langle T_{12}(0)T_{12}(t) \rangle \)

**Improve Action (Symanzik)**

16^3 \times 8, Symanzik, \( \beta = 2.2 \) (\( T \sim 2.5T_c \)), \( \sim 6k \) confs.

\[ G_{12} = [\langle T_{11}, T_{12} \rangle T_{12}^2] \]

\( \beta = 2.2 \) (\( T / T_c \approx 2.5 \))

16^3 \times 8

SU(2)

60000 \times 100\) Sweeps
Transport Coefficient
Summary

• Transport Coefficient knows if there are Vortices or not!
• It is interesting to see if Vortices change the Viscosity or not.
Conclusion

• QCD is a Quantum Field Theory.
• QCD has a very special feature, the Confinement.
• If a singular structure of the quantum field (gluon field) explains the confinement, it is very interesting (at least to me).
• The surface operators feel Vortex.
• Transport Coefficients change if we remove the Vortex.
• We want to understand QGP natures in terms of the Surface operators in future.
In other words,

QCD/QGP physics provides an opportunity to see a new quantum field mechanism, the singularities on surfaces.

K.R.
Backup Slides
Gluon propagators at finite temperature

◆ Self-energy and propagators

$$\Pi^{\mu\nu} = G P_T^{\mu\nu} + F P_L^{\mu\nu} \quad D^{\mu\nu} = \frac{1}{G + k^2} P_T^{\mu\nu} + \frac{1}{F + k^2} P_L^{\mu\nu} + \frac{\rho}{k^2} \frac{k^\mu k^\nu}{k^2} \quad \rho = 0 : \text{Landau gauge}$$

◆ Projection operators

$$P_T^{00} = P_T^{0i} = P_T^{i0} \quad P_T^{ij} = \delta^{ij} - \frac{k^i k^j}{k^2}, \quad P_L^{\mu\nu} = \delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - P_T^{\mu\nu}, \quad (P_T) = P_T, \quad (P_L) = P_L, \quad P_T P_L = 0$$

◆ Electric and magnetic gluon propagators

$$D_E(k, k_0 = 0) = D^{00} = \frac{1}{F + k^2}, \quad F(0, 0) = m_E \sim g(T)T$$

$$D_M(k, k_0 = 0) = D^{ii} = \frac{1}{G + k^2}, \quad G(0, 0) = m_M \sim g^2(T)T$$

◆ Gauge field, correlator and unequal time propagators

$$A^a_\mu(x, t) = \text{Tr} \sigma^a U_\mu(x, t) \quad D_{\mu\nu}(\vec{q}, t) = \frac{1}{V(N_c^2 - 1)} \sum_x A^a_\mu(x, t') A^a_\nu(y, t'') e^{i\vec{q}(x - y)}$$

◆ After taking a sum of $t$ with $q_0 = 0$,

$$D_{\mu\nu}(\vec{q}, q_0 = 0) = \frac{1}{N_t} \sum_t D_{\mu\nu}(\vec{q}, t)$$

◆ Note that for Coulomb gauge case we use equal-time propagators here.
Lattice setup

- SU(2) lattice calculation with quenched Wilson-gauge action
- Landau (Coulomb) gauge on the lattice in the path-integral formula satisfies the following condition:

\[ \partial_\mu A_\mu(x,t) = 0 \Rightarrow \text{Maximize } R = \frac{1}{VT} \sum_{x,t} \text{Re} \text{Tr} U_\mu(x,t) \left| \sum_\mu \text{Tr} \sigma^a \left( U_\mu(x) - U_\mu(x - \hat{\mu}) \right) \right|^2 \leq 10^{-\text{eps}} \]

Wilson-Mandula Method (PLB185,127(1987))

- Parameters:
  - Lattice size: 24x24x24x4
  - beta: 2.2-2.6, corresponding to the temperatures \( T/T_c \) are approx. 1.40, 3.00 and 6.00.
  - Configurations: 10k discarded and about 20-30 confs. are used to measure.
  - Convergence criteria: \( \text{eps} = 10^{-8} \) for gauge fixing and \( \text{eps} = 10^{-16} \) for maximal center projection.

- Procedure:

  \textit{Gauge updated} --> \textit{Maximal center projection} --> \textit{Gauge fixing}
Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$\langle T_{\mu\nu}(0)T_{\mu\nu}(\tau) \rangle$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)