

# 「現代の核構造論」ミニマム

## Chapter 1 現代的な核構造論への招待

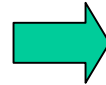
- 1a 間違いだらけの原子核像
- 1b 核構造論の歴史

## Chapter 2 平均場近似とは何か

- 2a 真空とその励起モード
- 2b 対称性の自発的破れとその回復

## Chapter 3 高速回転する原子核

- 3a 超低温核物理学  
高スピンのフロンティア
- 3b 回転座標系での準粒子シェルモデル
- 3c 減衰する回転運動



## Chapter 4 超変形状態の発見

- 4a 変形シェル構造とは何か
- 4b 生成、構造、崩壊
- 4c Wobbling と Precession

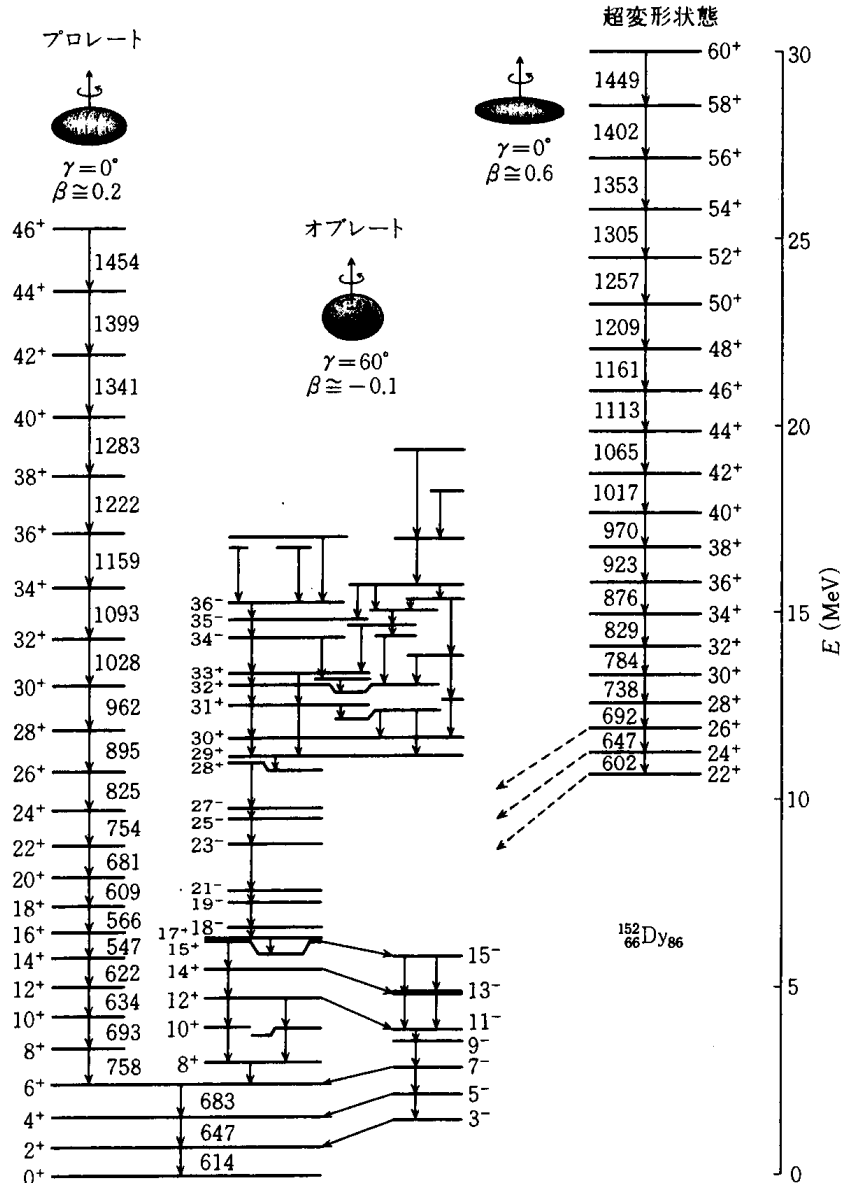
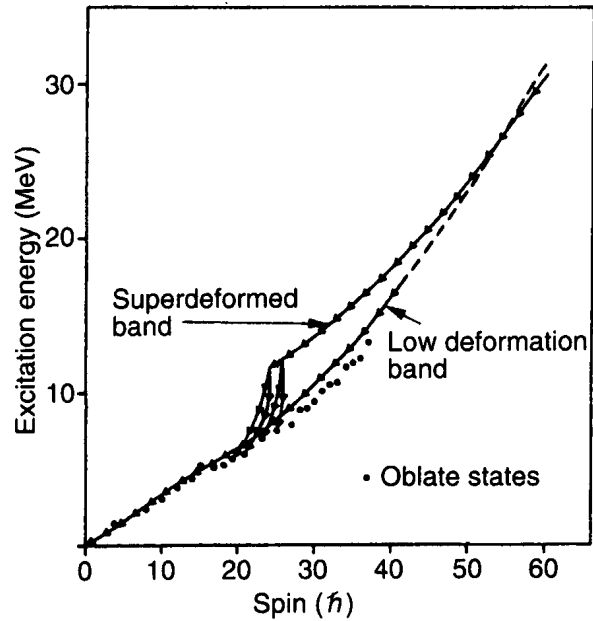
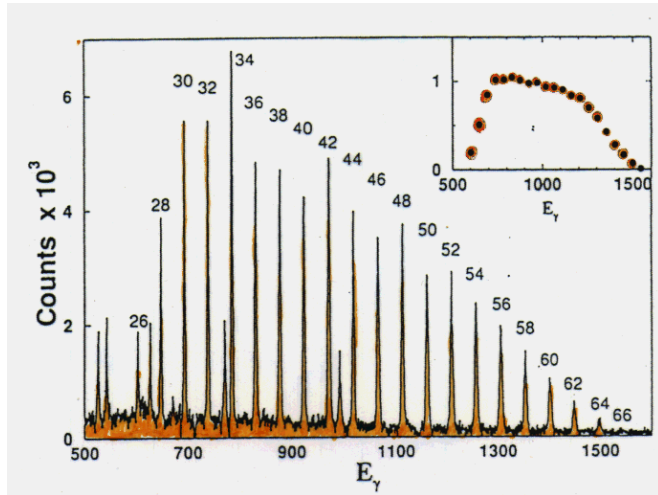
## Chapter 5 大振幅集団運動論

- 5a オブレート・プロレート変形共存現象
- 5b パリティ二重項とカイラル二重項

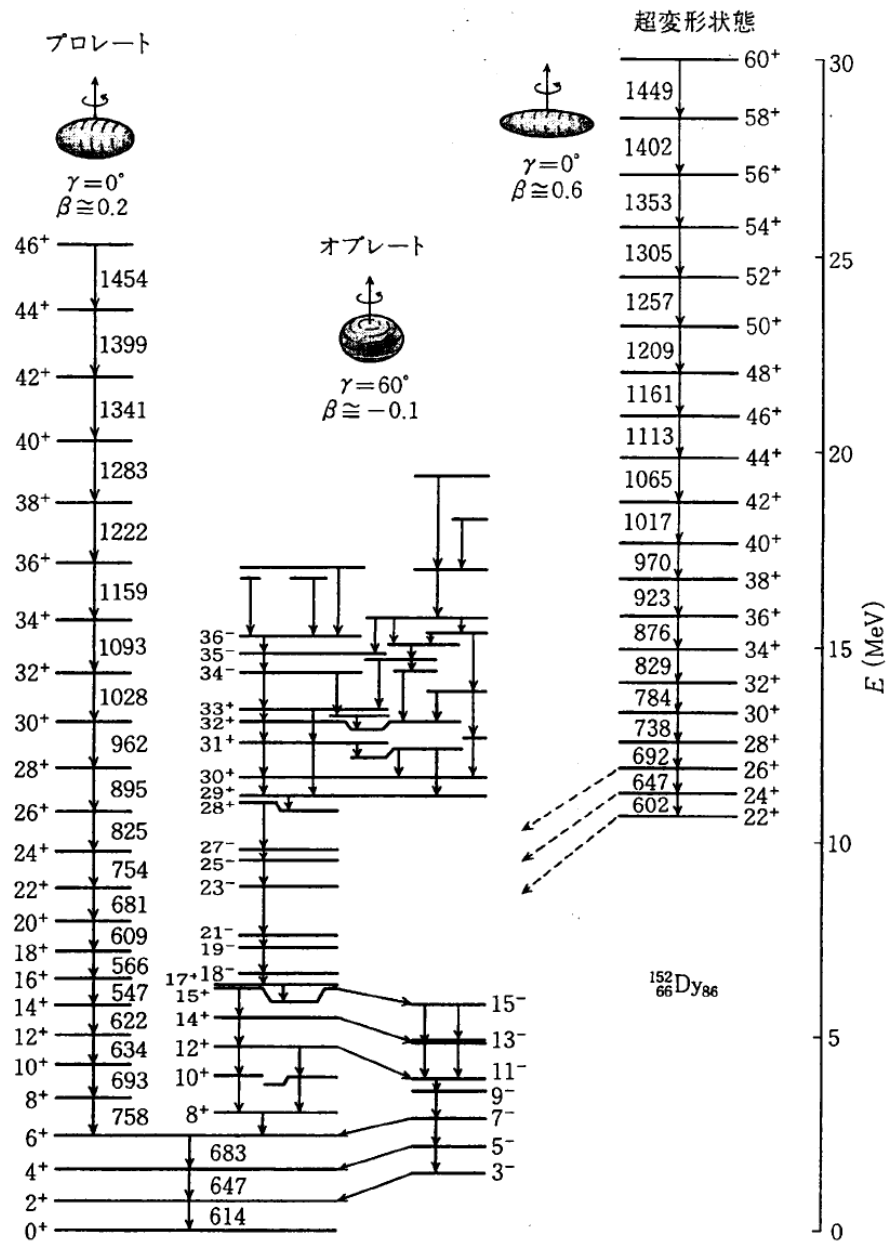
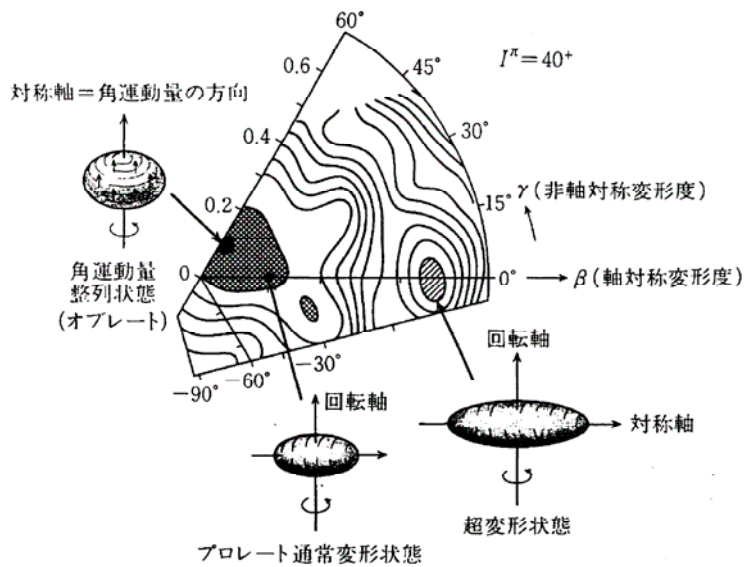
## Chapter 6 不安定核の集団励起モード

- 6a 新しい理論的課題
- 6b 期待される新しい集団現象

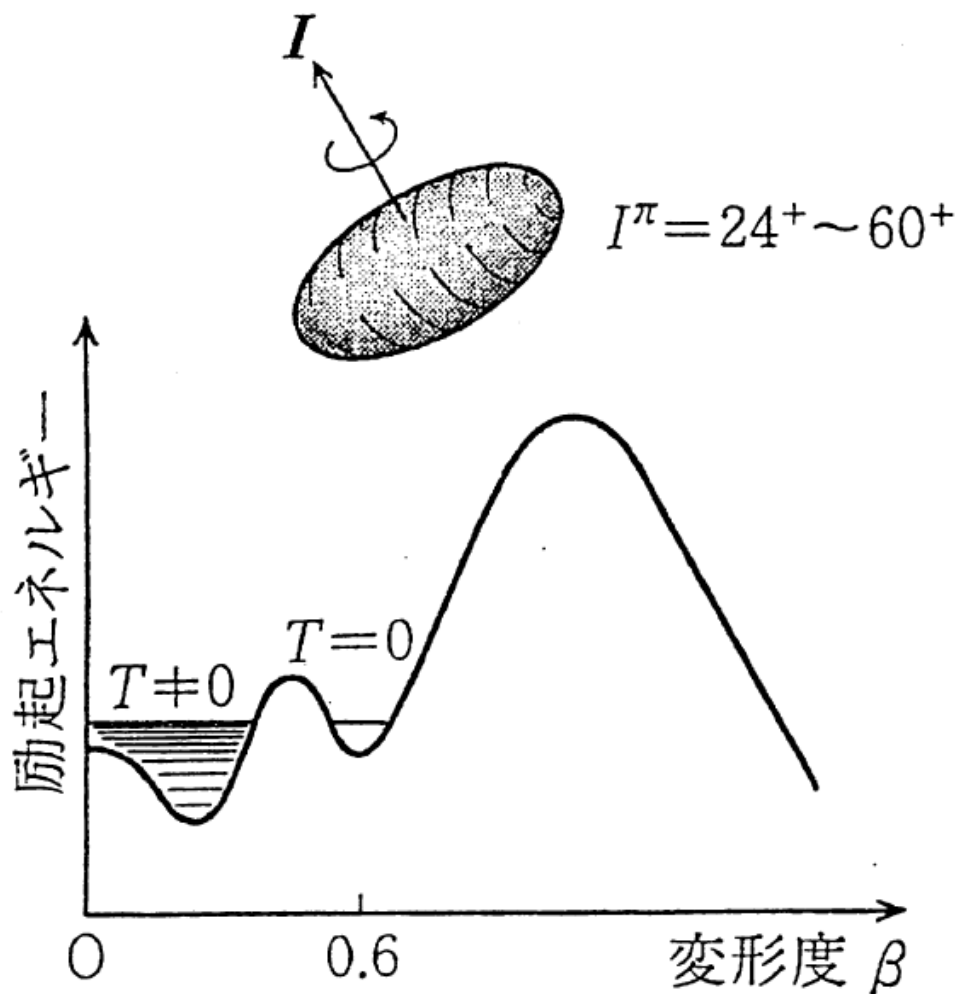
# 超変形回転バンド



# 異なる変形の共存



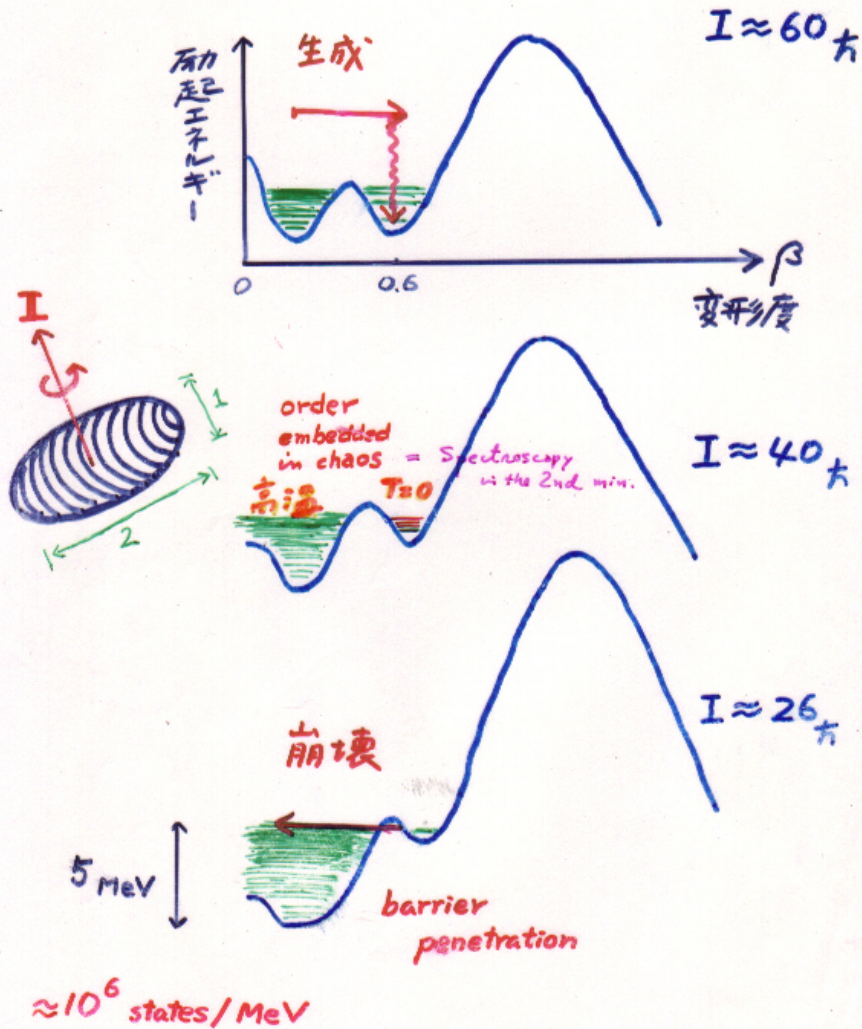
# 高速回転する超変形状態



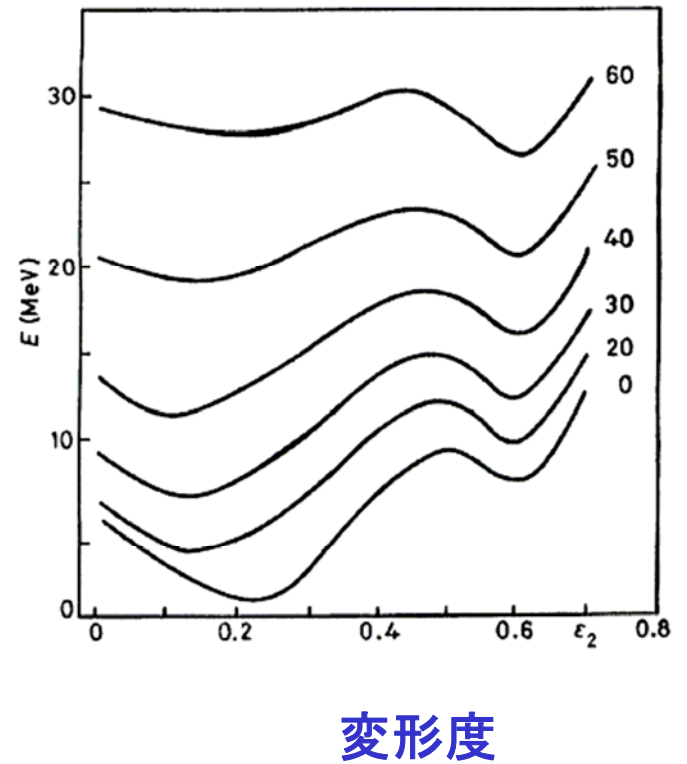
平均場の第2極小点  
(二つの真空)

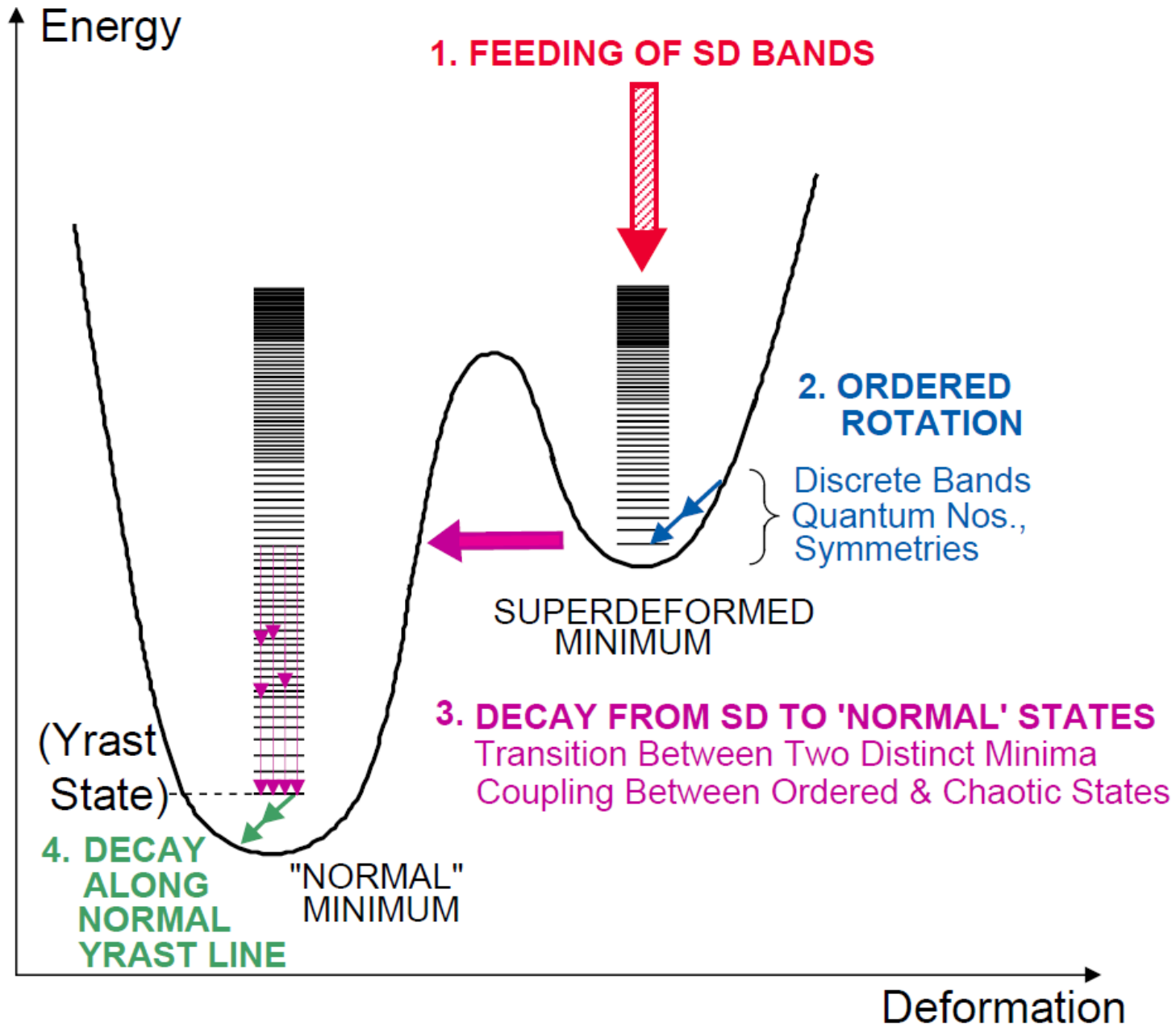
超変形状態から  
常変形状態への  
巨視的トンネル現象

superdeformed band  
超変形回転バンドの生成と崩壊



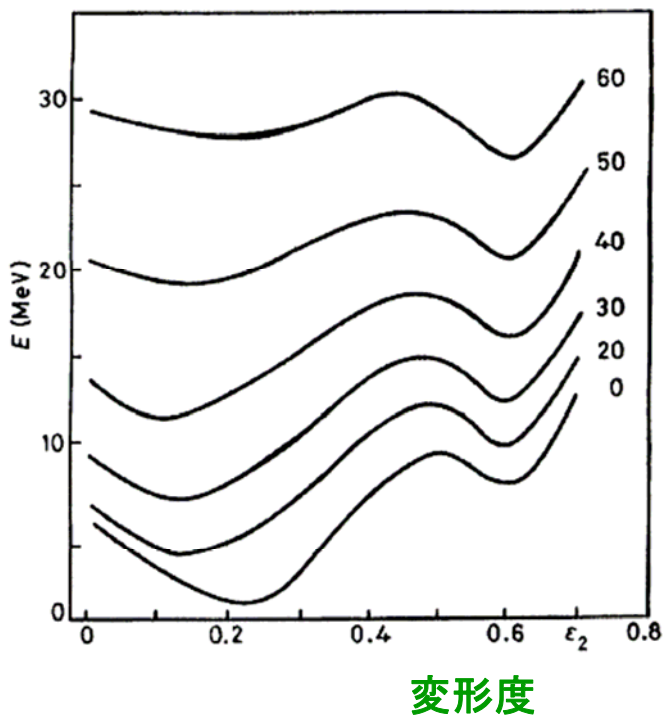
変形ポテンシャルエネルギーの角運動量依存性



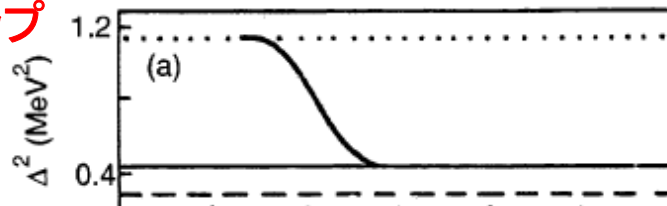


# トンネル崩壊確率 の対相関依存性

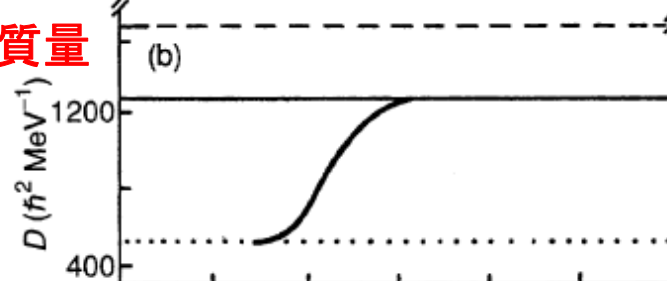
## 変形ポテンシャルの角運動量依存性



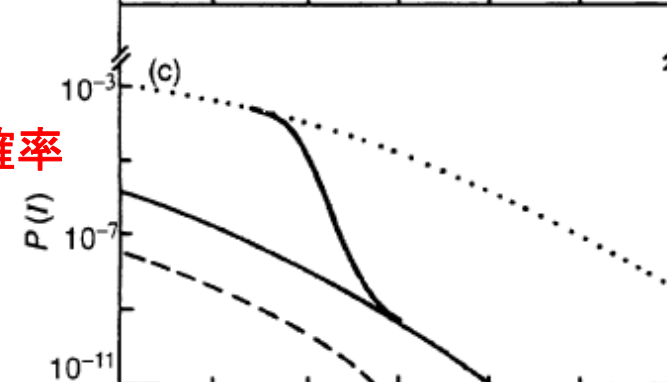
## 対ギャップ



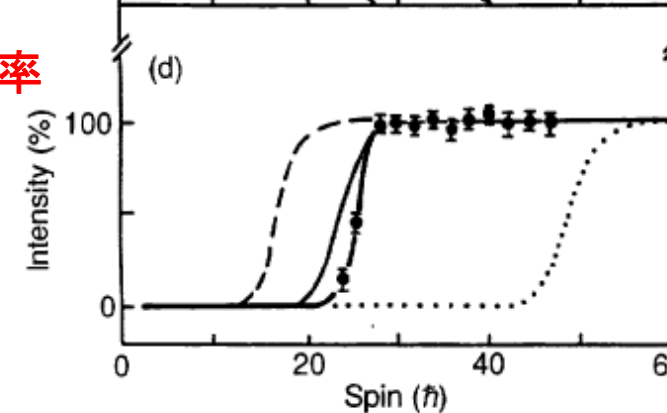
## 集団運動の質量



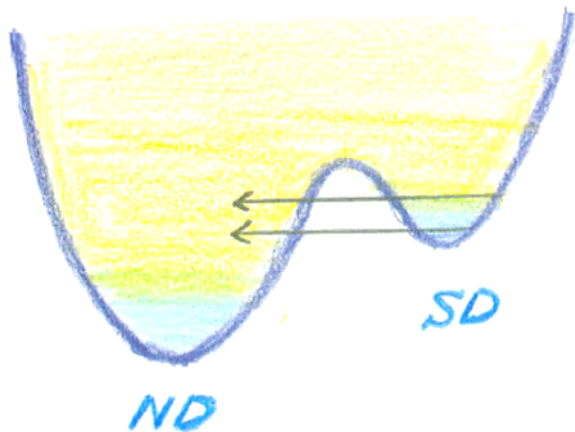
## トンネル確率



## 生存確率



# SD からNDへのトンネル崩壊



$$H = \frac{1}{2} \sum_{ij=1}^2 m_{ij}(q_1, q_2) \dot{q}_i \dot{q}_j + V(q_1, q_2),$$

透過係数

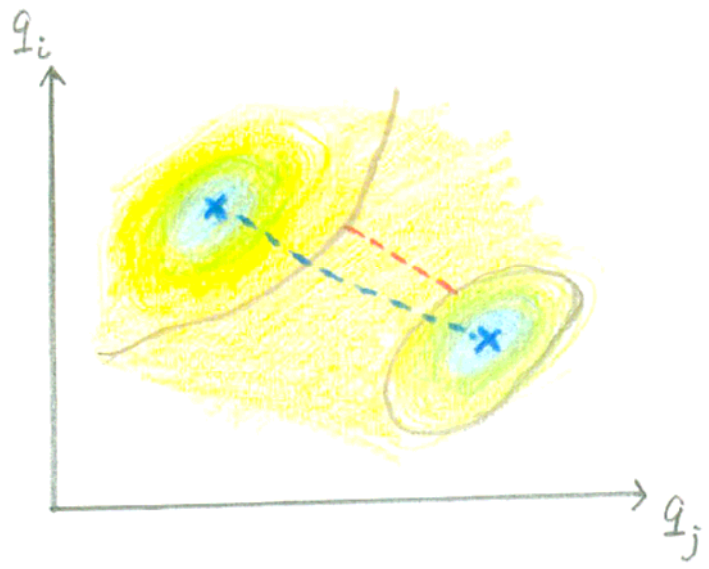
$$T = (1 + \exp 2S)^{-1}$$

作用積分

$$S(E) = \int_{\text{path}} ds \sqrt{2M_0(V(q(s)) - E)},$$

集団質量

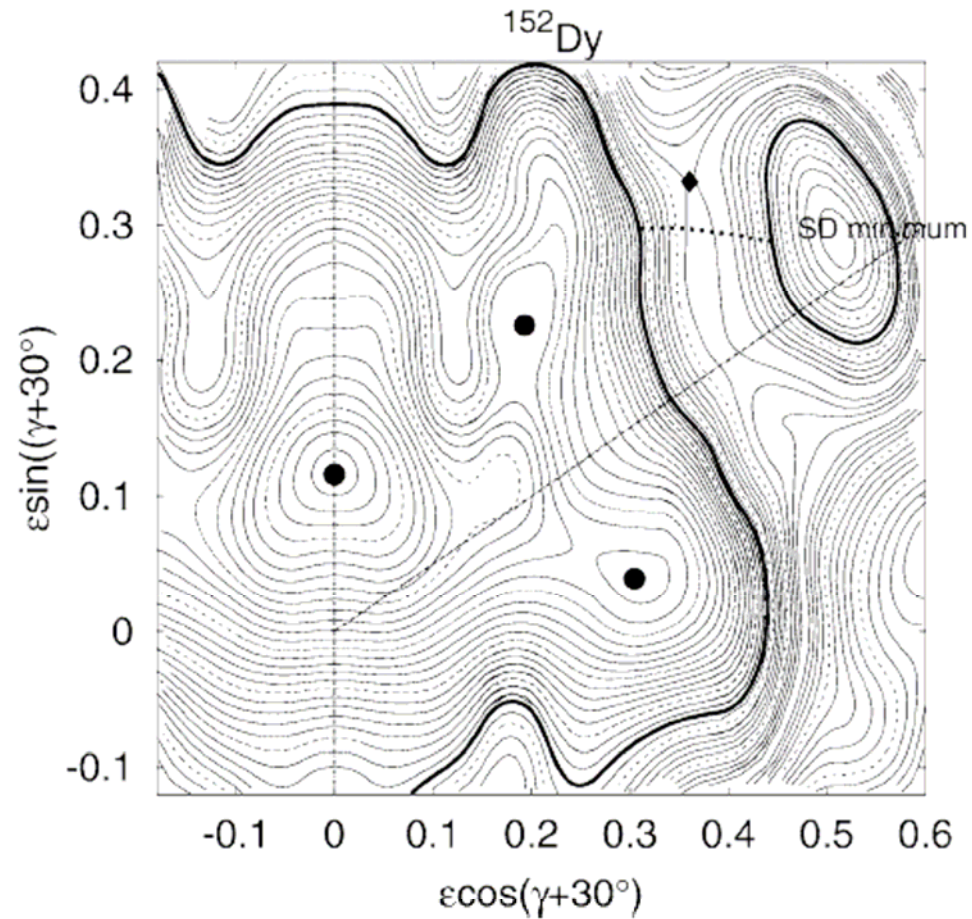
$$M_0 ds^2 = \sum_{ij} m_{ij}(q) dq_i dq_j$$



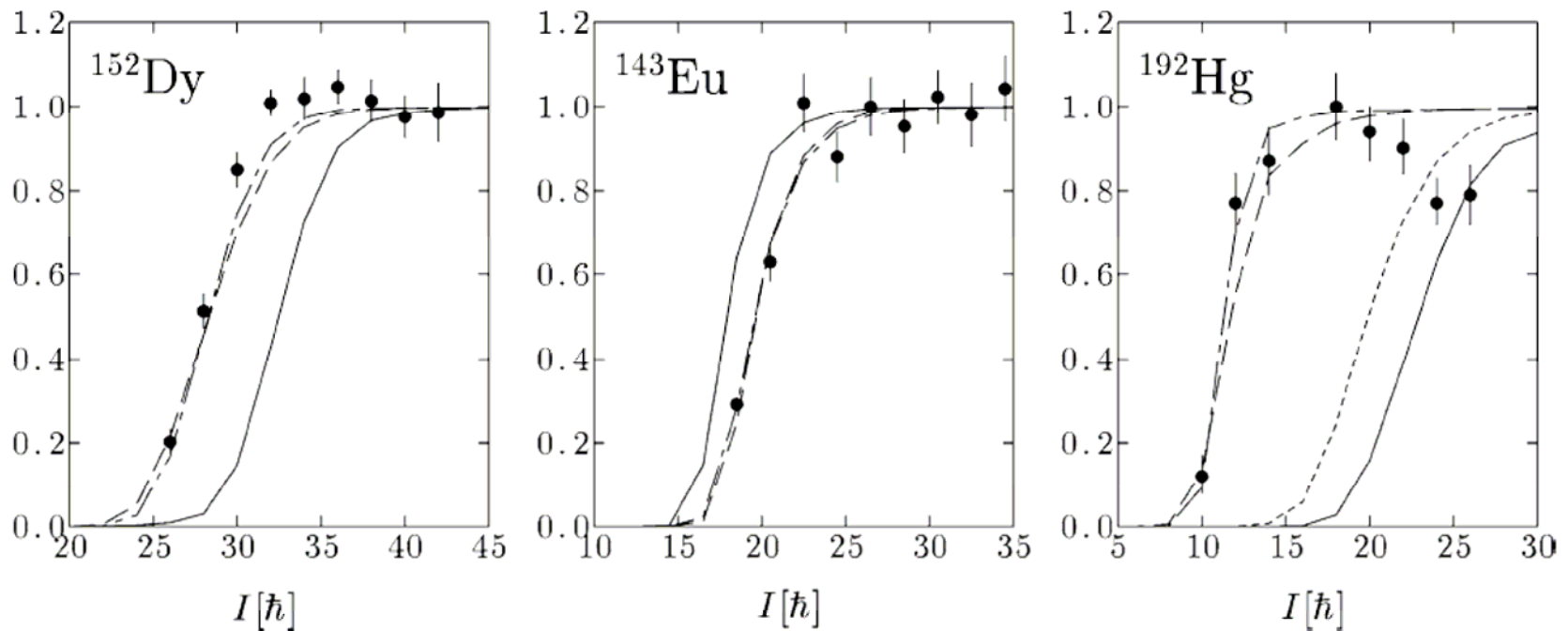
$$q_1 = \epsilon_2 \cos(\gamma + 30^\circ), \quad q_2 = \epsilon_2 \sin(\gamma + 30^\circ).$$



# 超変形状態から通常変形状態へのトンネル崩壊の経路

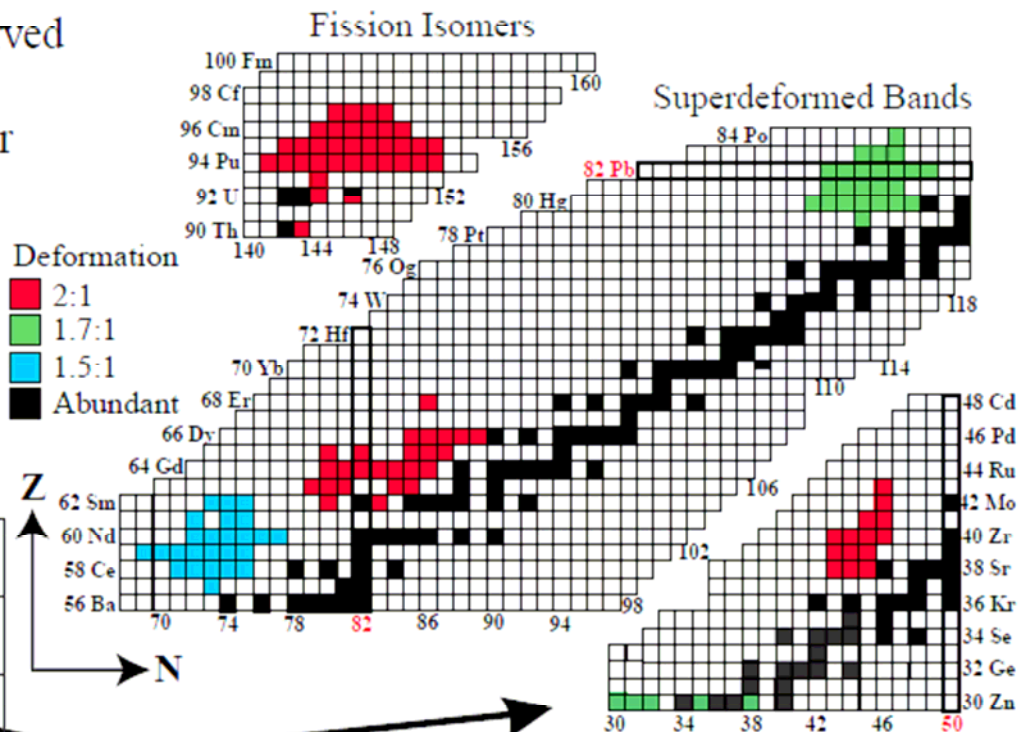
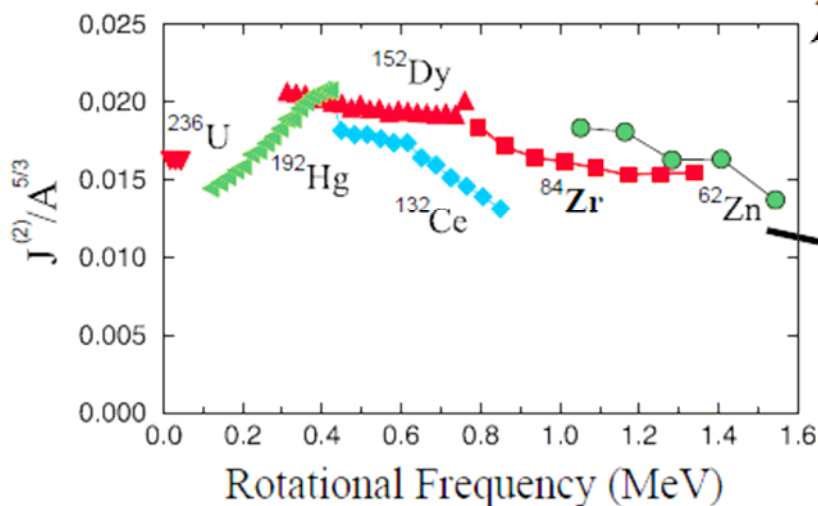


# 超変形状態から通常変形状態へのトンネル崩壊確率



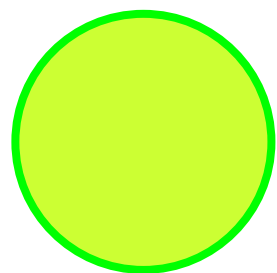
## The Regions of Superdeformation

Superdeformed (SD) bands have been observed throughout the nuclear chart. The moments of inertia, after scaling to  $A^{5/3}$ , are all similar due to the fact that the shape (deformation) of a SD band is largely independent of the mass. The shell gaps, which define the SD shape, depend on the underlying symmetry of the nucleus, and occur at near-integer axis ratios.



The recent observation with GAMMASPHERE of a new region of superdeformation ( $N \sim Z \sim 30$ ) opens up many exciting opportunities, including the possibility to study the most collective states known with a "Shell Model" description.

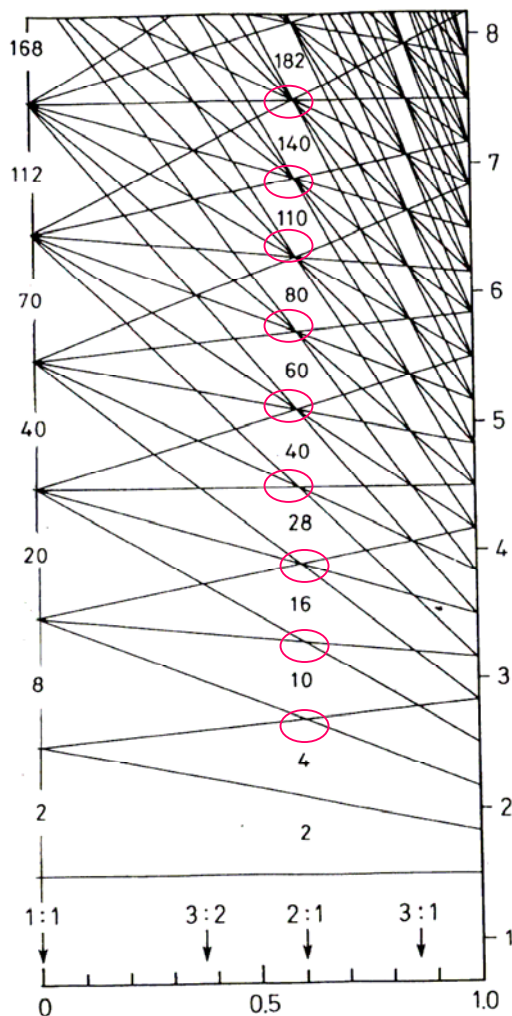
$$e(n_{\perp}, n_z) = \hbar\omega_{\perp}(n_{\perp} + 1) + \hbar\omega_z(n_z + \frac{1}{2})$$



球形

縮退度

- 56 →
- 42 →
- 30 →
- 20 →
- 12 →
- 6 →
- 2 →

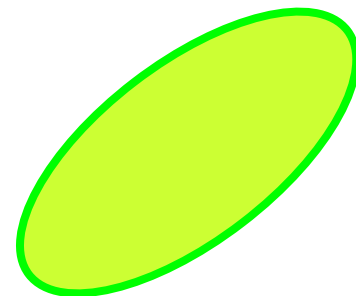


変形度  $\delta_{osc} = (\omega_{\perp} - \omega_z) / \bar{\omega}$

$\omega_{\perp} = \omega_x = \omega_y$

縮退度

- ← 42 } (red bracket)
- ← 42 } (red bracket)
- ← 30 } (red bracket)
- ← 30 } (red bracket)
- ← 20 } (red bracket)
- ← 20 } (red bracket)
- ← 12 } (red bracket)
- ← 12 } (red bracket)
- ← 6 } (red bracket)
- ← 6 } (red bracket)
- ← 2 } (red bracket)
- ← 2 } (red bracket)

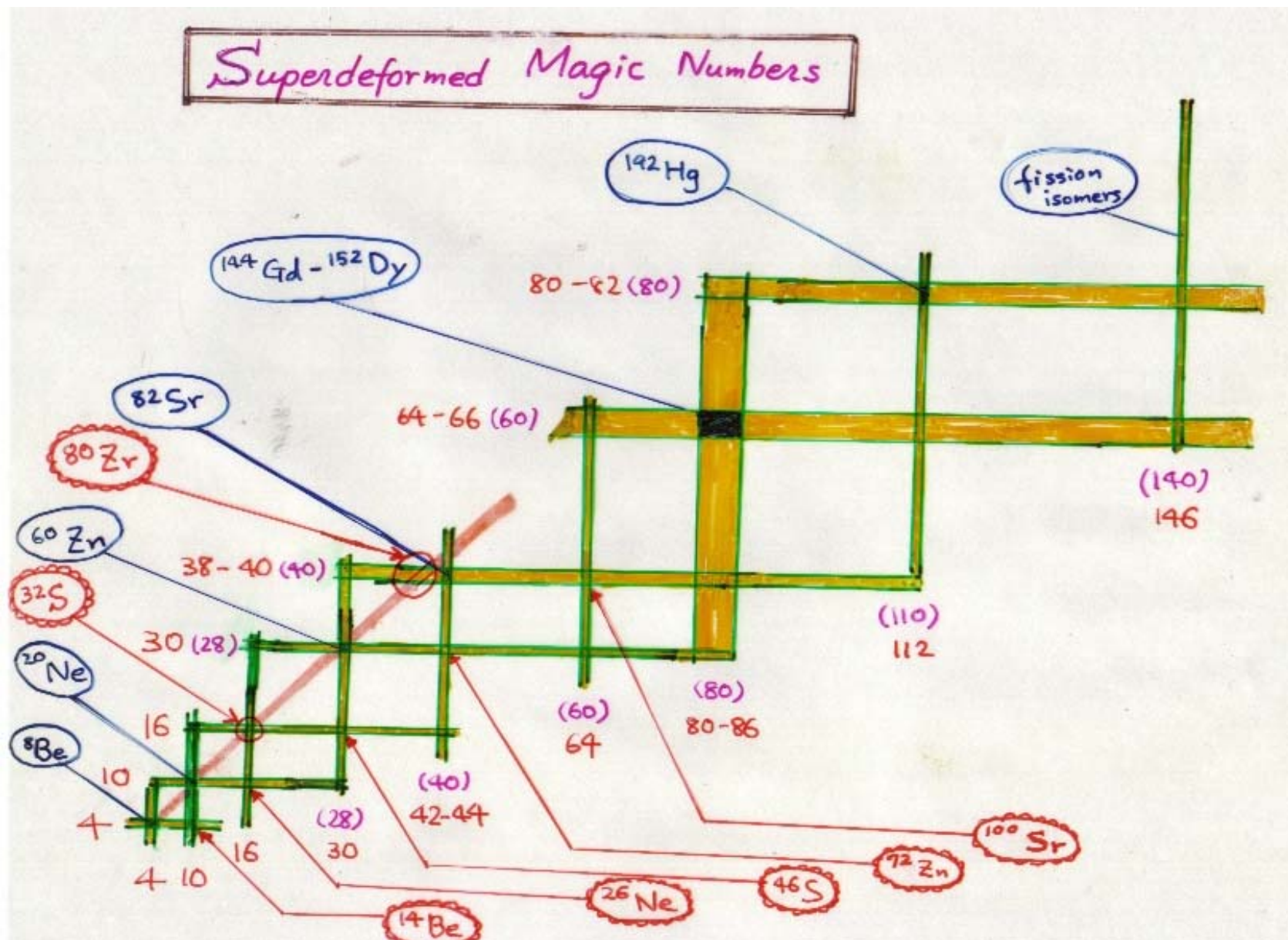


超変形

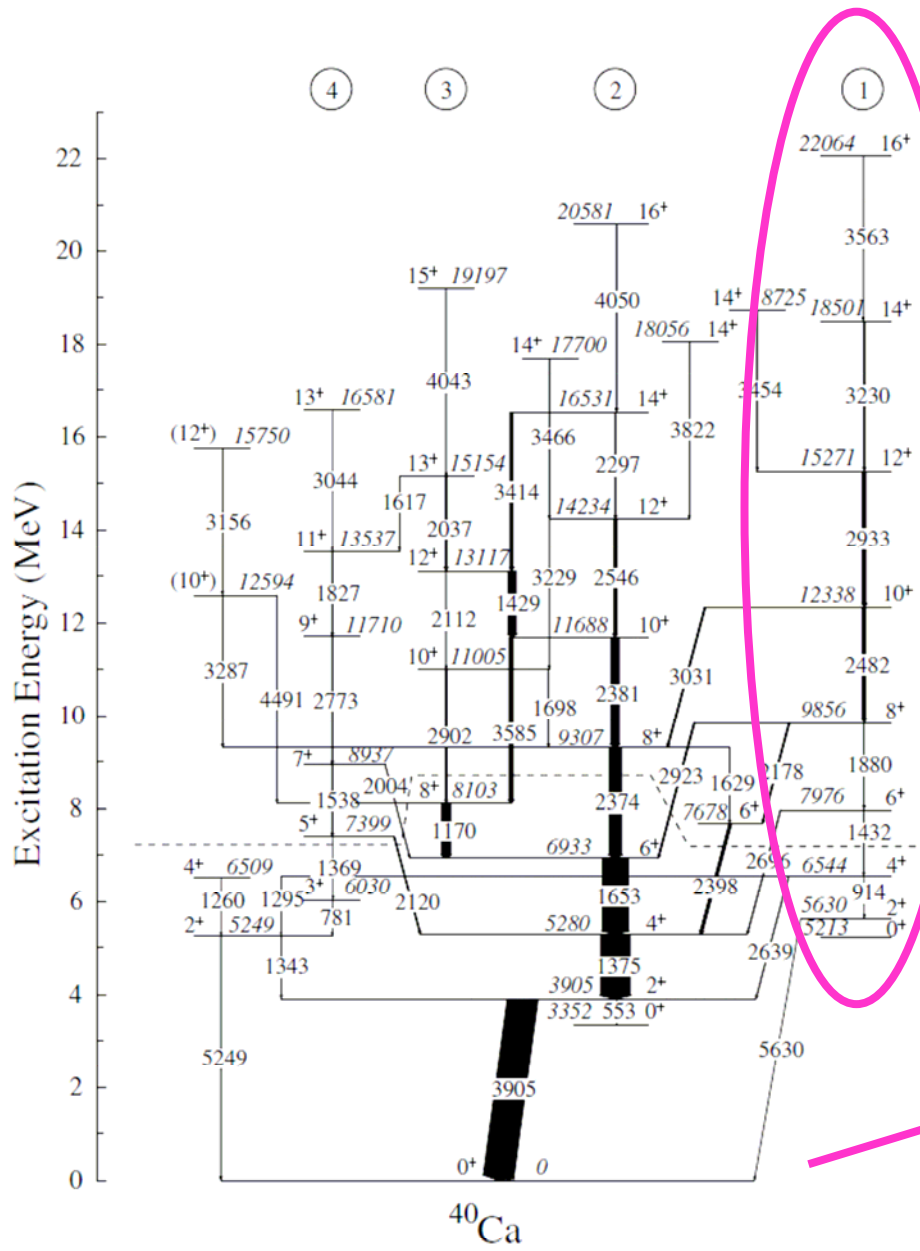
軸比2:1

調和振動子ポテンシャルの動力的対称性

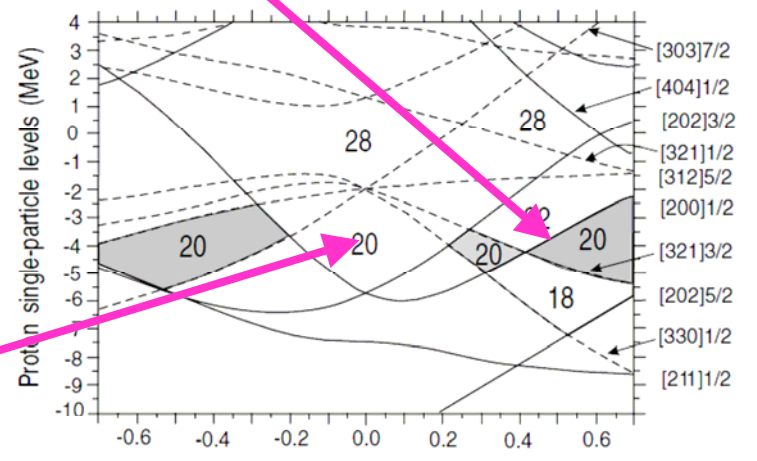
# 超変形の魔法数



# Superdeformation in the Doubly Magic Nucleus $^{40}\text{Ca}_{20}$

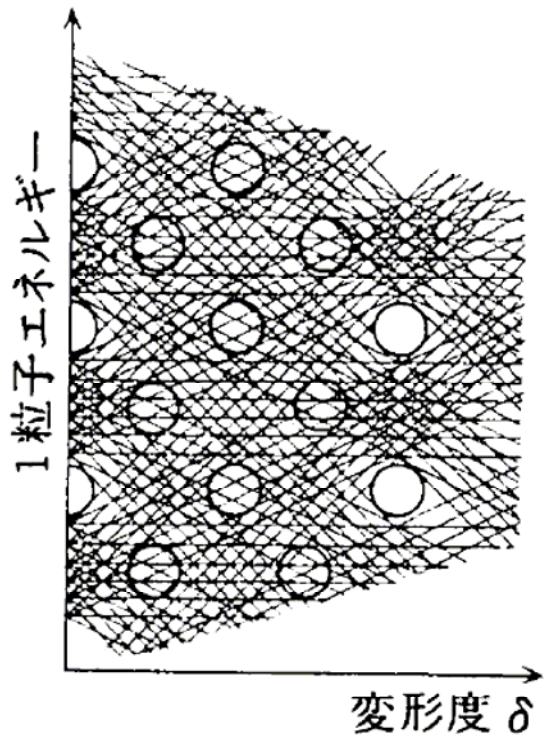


E. Ideguchi, et al.,  
 Phys. Rev. Lett. 87 (2001) 222501

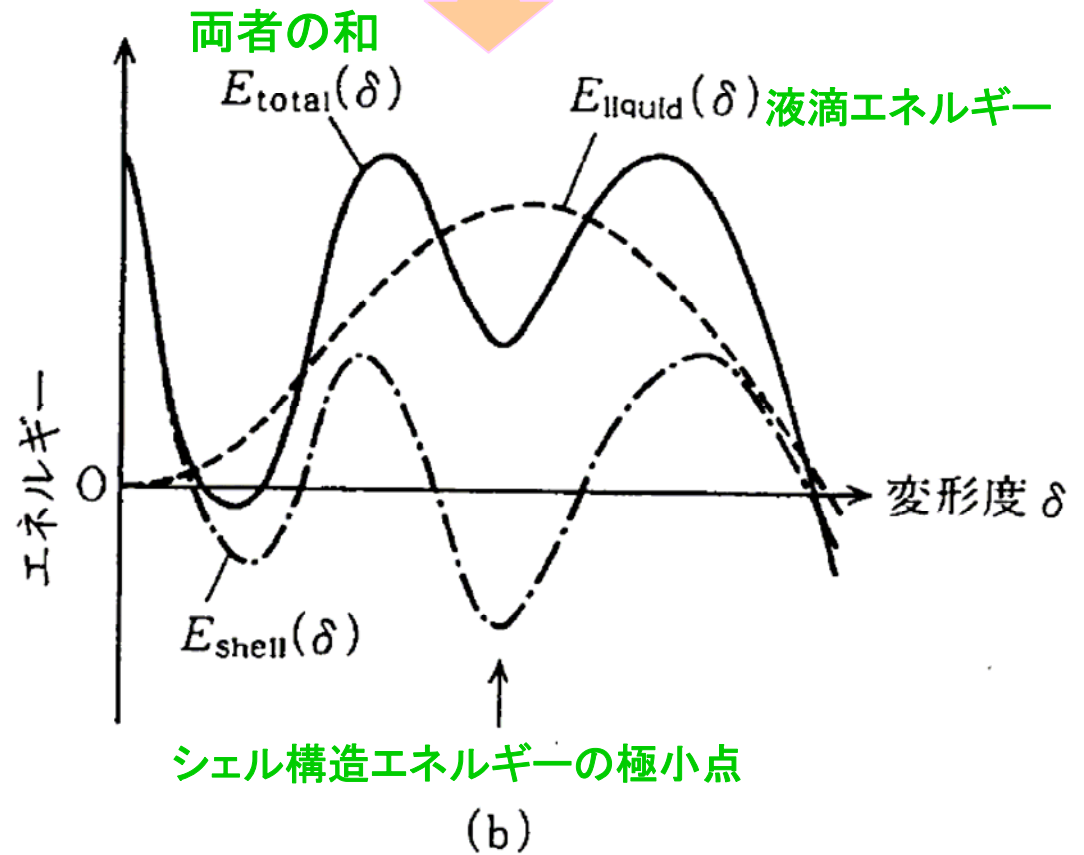


# 核分裂アイソマー

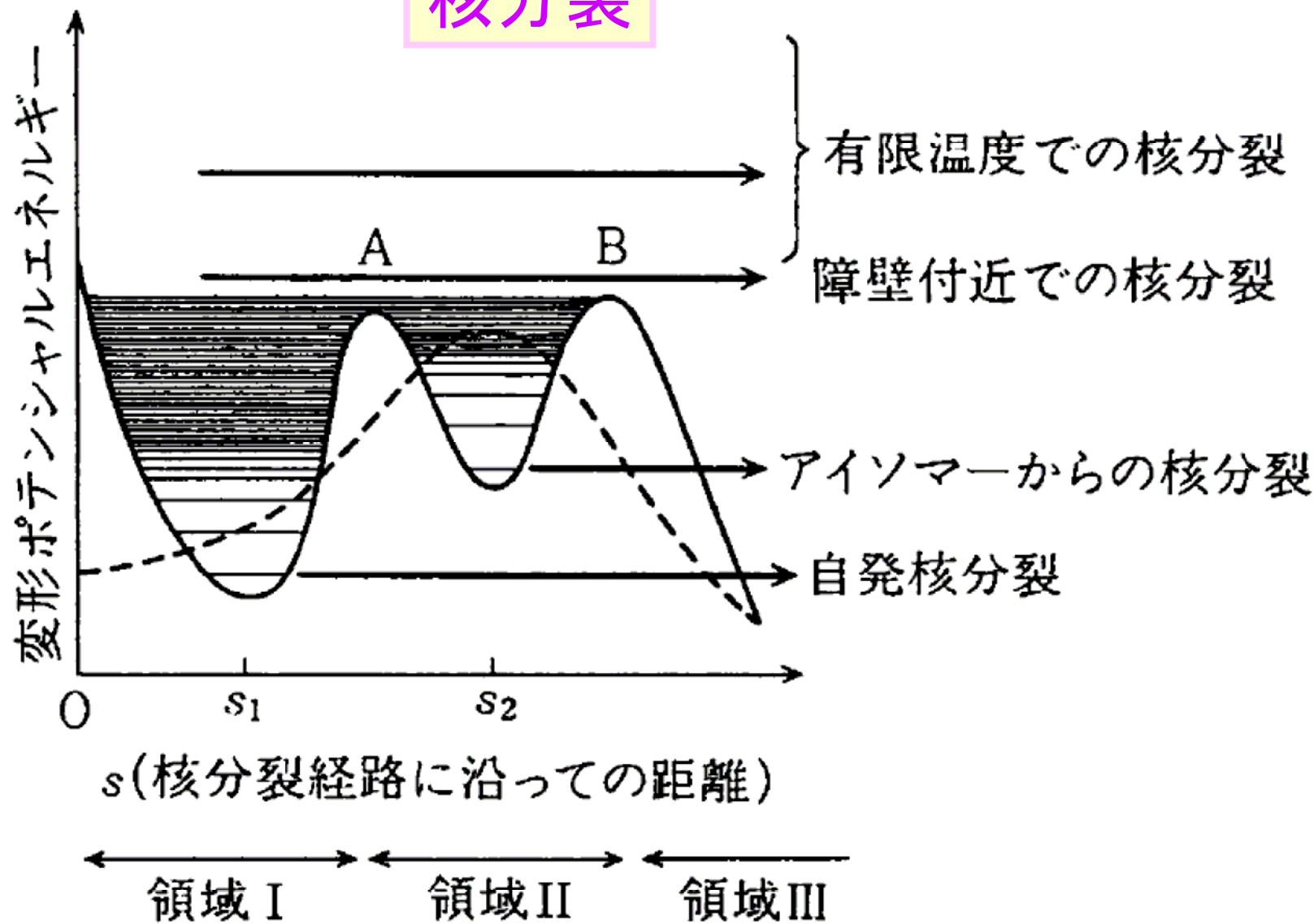
変形につれてシェル構造は変化する



(a)



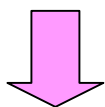
# 核分裂





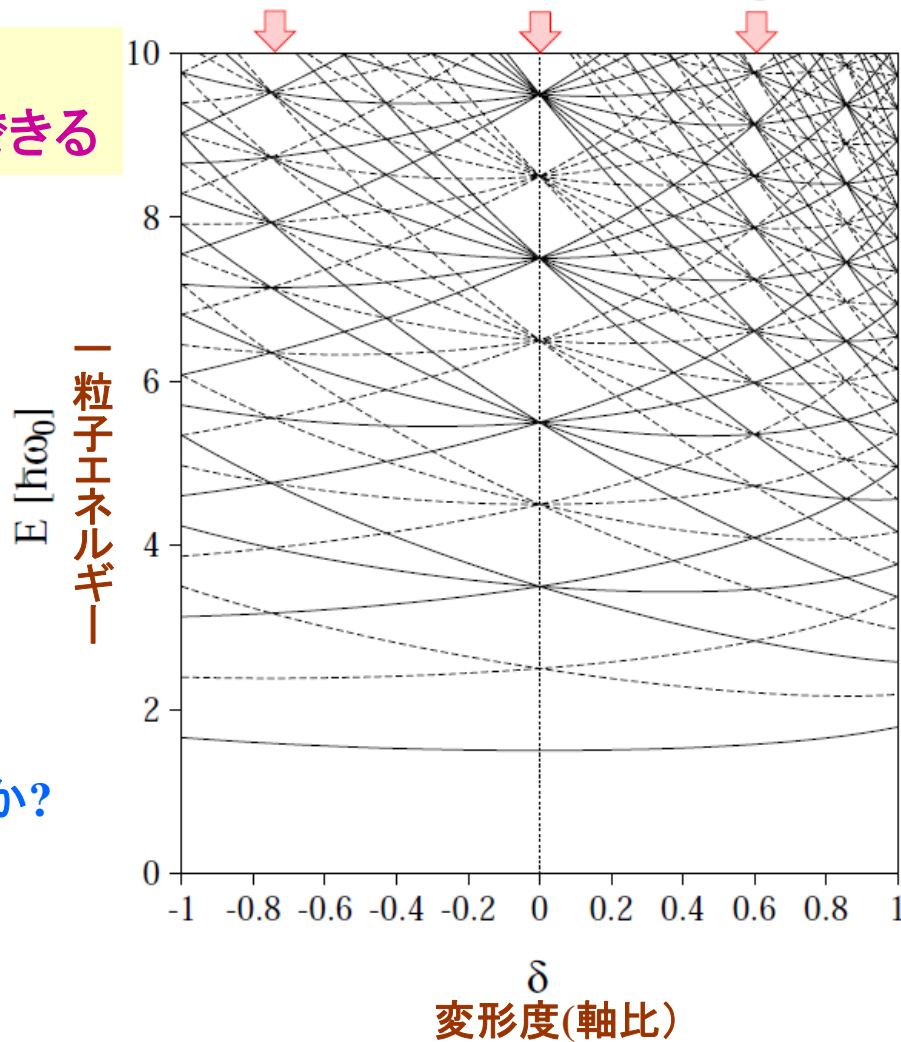
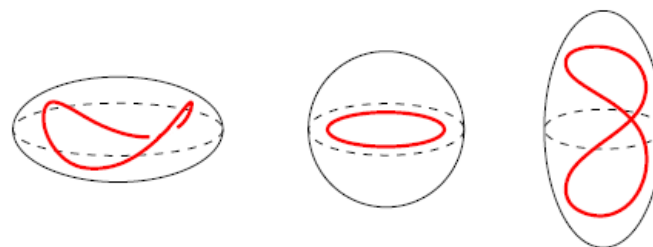
# 調和振動子ポテンシャル

軸比2:1のとき  
8の字型の周期軌道ができる



準位が縮退する  
超変形シェル構造の形成

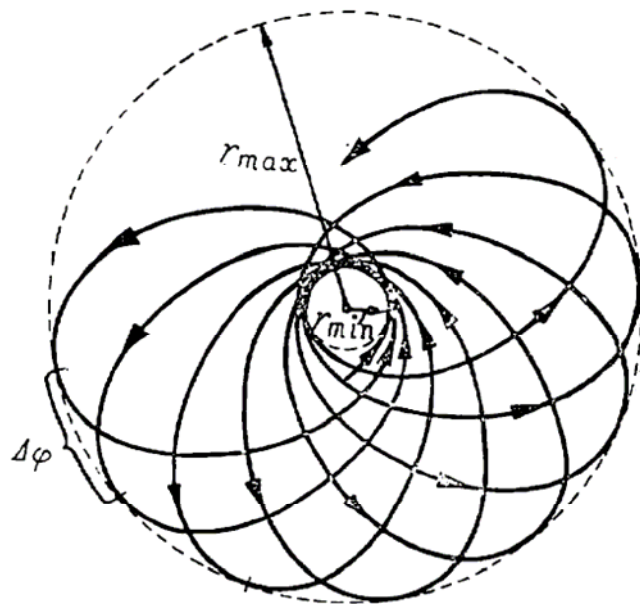
一般のポテンシャルではどうか?



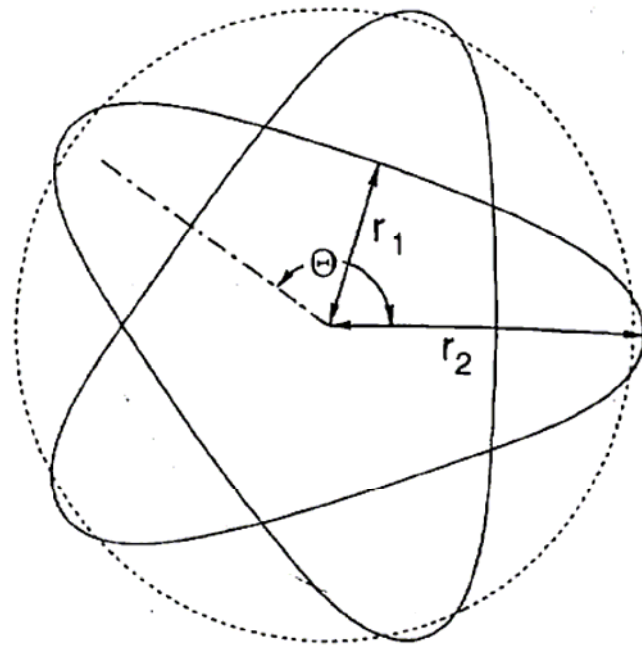
# 球対称ポテンシャルのなかの軌道運動

楕円軌道(クーロン・ポテンシャル、調和振動子)は  
例外中の例外

多重周期運動



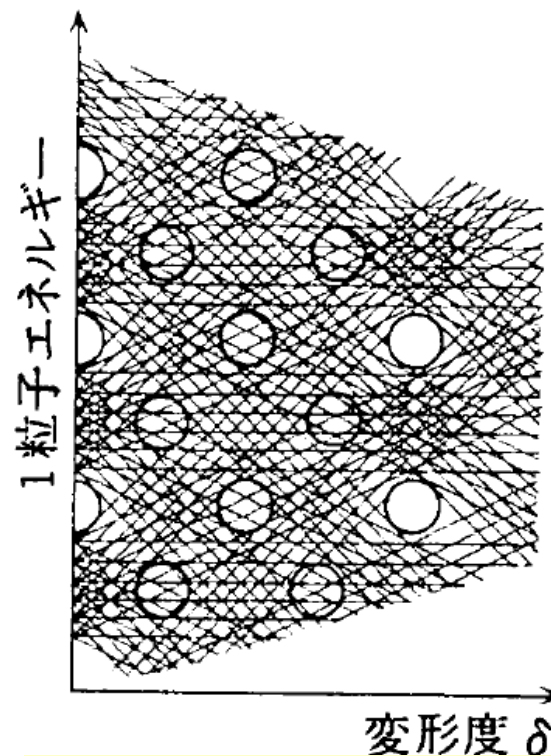
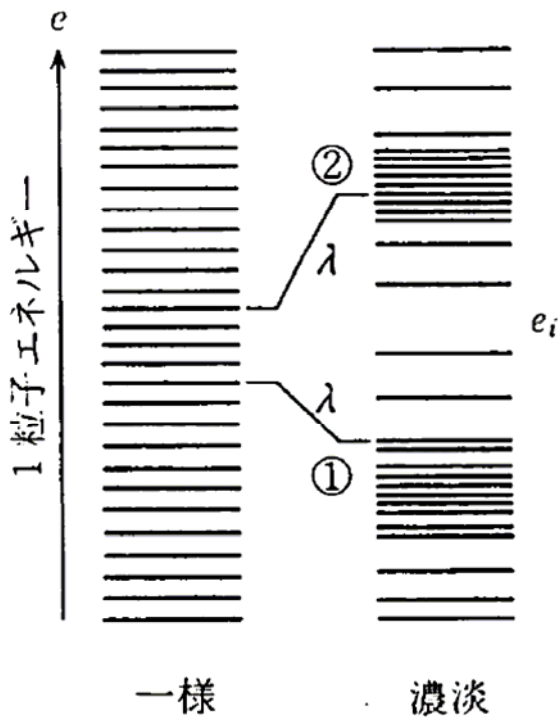
周期運動



# シェル構造とは何か

疎視化して見える1粒子準位スペクトルの規則的な振動パターン

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V(\beta) \right) \psi(\mathbf{r}) = e(\beta) \psi(\mathbf{r})$$



振動パターンは変形とともに変わる

問8 変形した原子核はなぜ存在するのですか。何が「変形」しているのですか。

# 1粒子準位の縮退が起こる条件(可積分系の場合)

作用変数と角変数  $\dot{\theta} = \frac{\partial h}{\partial \mathbf{I}} = \boldsymbol{\omega}(\mathbf{I})$  振動数の組

## EBK(Einstein-Brillouin-Keller)量子条件

$$e(\mathbf{n}) = h(\mathbf{I}) \quad \text{with} \quad \mathbf{I} = \hbar \left( \mathbf{n} + \frac{1}{4} \boldsymbol{\alpha} \right),$$

エネルギー変分

$$\begin{aligned} e(\mathbf{n} + \Delta \mathbf{n}) - e(\mathbf{n}) &= h(\mathbf{I} + \Delta \mathbf{I}) - h(\mathbf{I}) \\ &\simeq \frac{\partial h}{\partial \mathbf{I}} \Delta \mathbf{I} \\ &= \hbar \boldsymbol{\omega} \cdot \Delta \mathbf{n} \\ &= \hbar \omega_1 \Delta n_1 + \hbar \omega_2 \Delta n_2 + \hbar \omega_3 \Delta n_3 \\ &= 0, \end{aligned}$$

量子数の組

振動数が整数比のとき縮退が起こる

# シェル構造とは何か

疎視化して見える1粒子準位スペクトルの規則的な振動パターン

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\beta)\right)\psi(\mathbf{r}) = e(\beta)\psi(\mathbf{r})$$

準位密度に対するトレース公式

$$\begin{aligned}g(e) &= \sum_i \delta(e - e_i) \\ &\simeq \bar{g}(e) + \delta g(e) \\ &= \bar{g}(e) + \sum_{\gamma} A_{\gamma} \cos\left(\frac{1}{\hbar}S_{\gamma}(e) - \frac{\pi}{2}\mu_{\gamma}\right)\end{aligned}$$

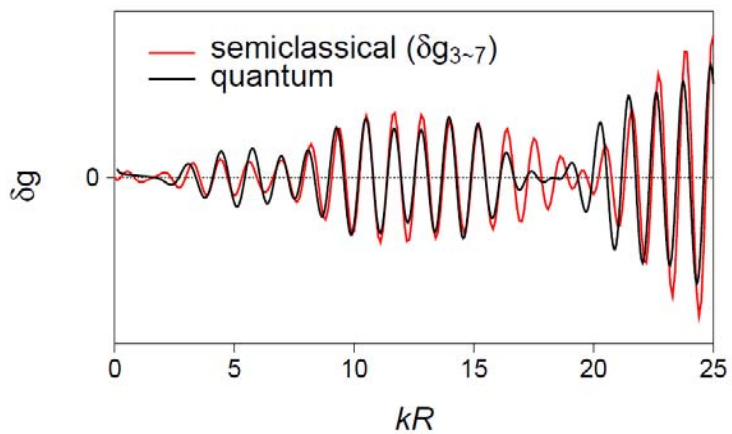
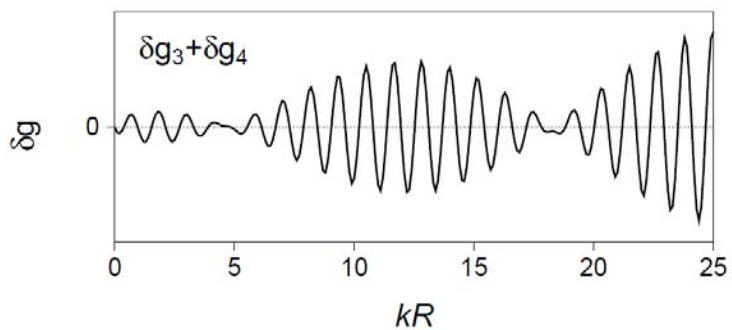
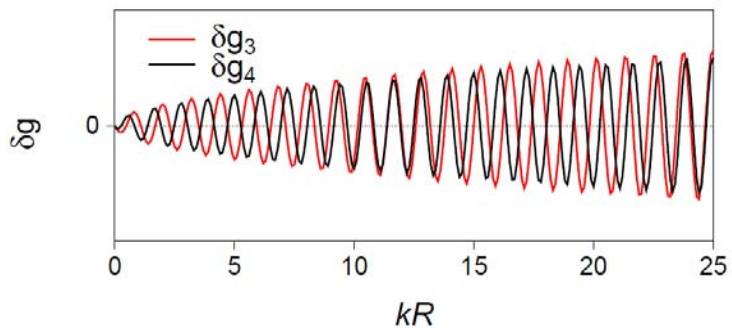
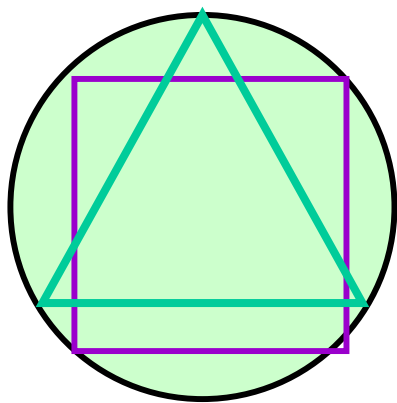
すべての周期軌道に関する総和

作用積分

# Cavity における

## 3角形軌道と4角形軌道の干渉

スーパーシエル構造の形成



# 金属クラスターにおけるスーパーシェル構造の発見

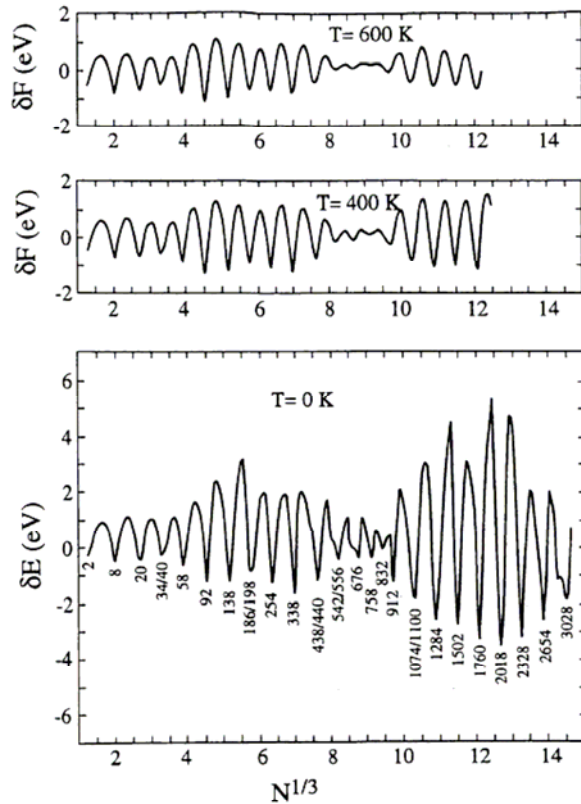


Figure 8.11: Oscillating part of total free energy of spherical sodium clusters, obtained in self-consistent jellium calculations at finite temperatures, plotted versus cube root of the number  $N$  of valence electrons. (After [64].)

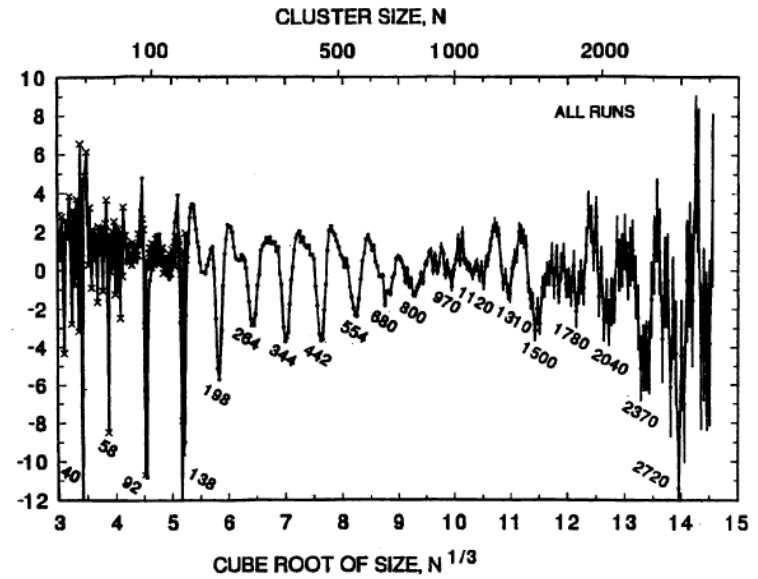
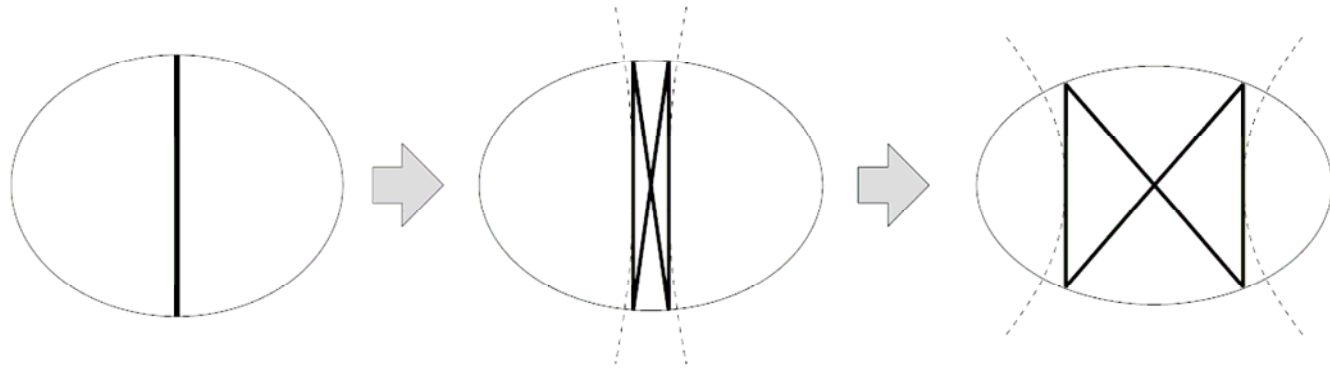


Figure 8.12: Experimental observation of the supershells. Shown are the logarithmically derived relative yields of sodium clusters from an adiabatic expansion source (Fig. 8.10 (below)), but scaled by an extra factor  $N^{1/2} \exp(cN^{1/3})$  (see text for planation). (From [65].)

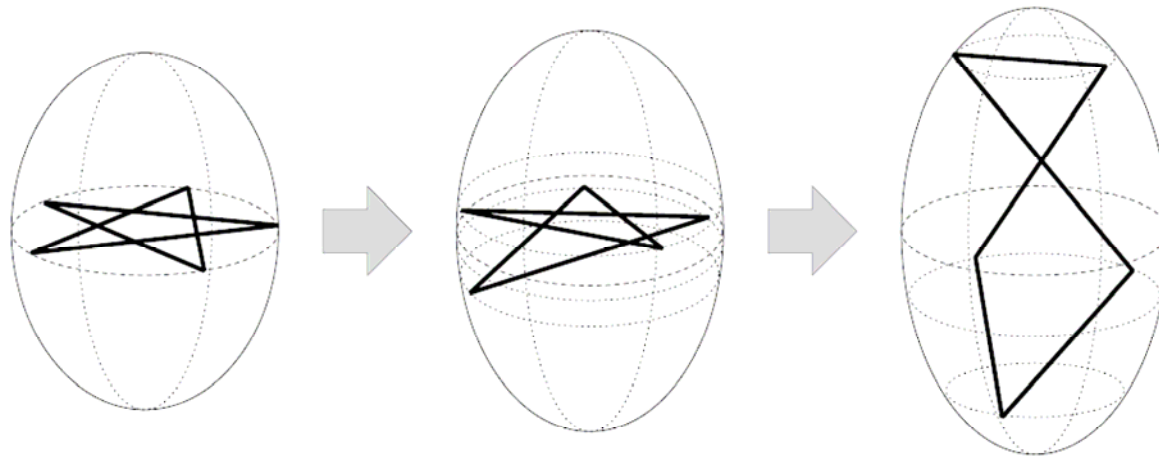
J. Pedersen et al., *Nature* 353 (1991) 733

O. Genzken and M. Brack, *Phys. Rev. Lett.* 67 (1991) 3286

# 周期軌道に分岐現象



楕円ビリヤード: Birth of Butterfly

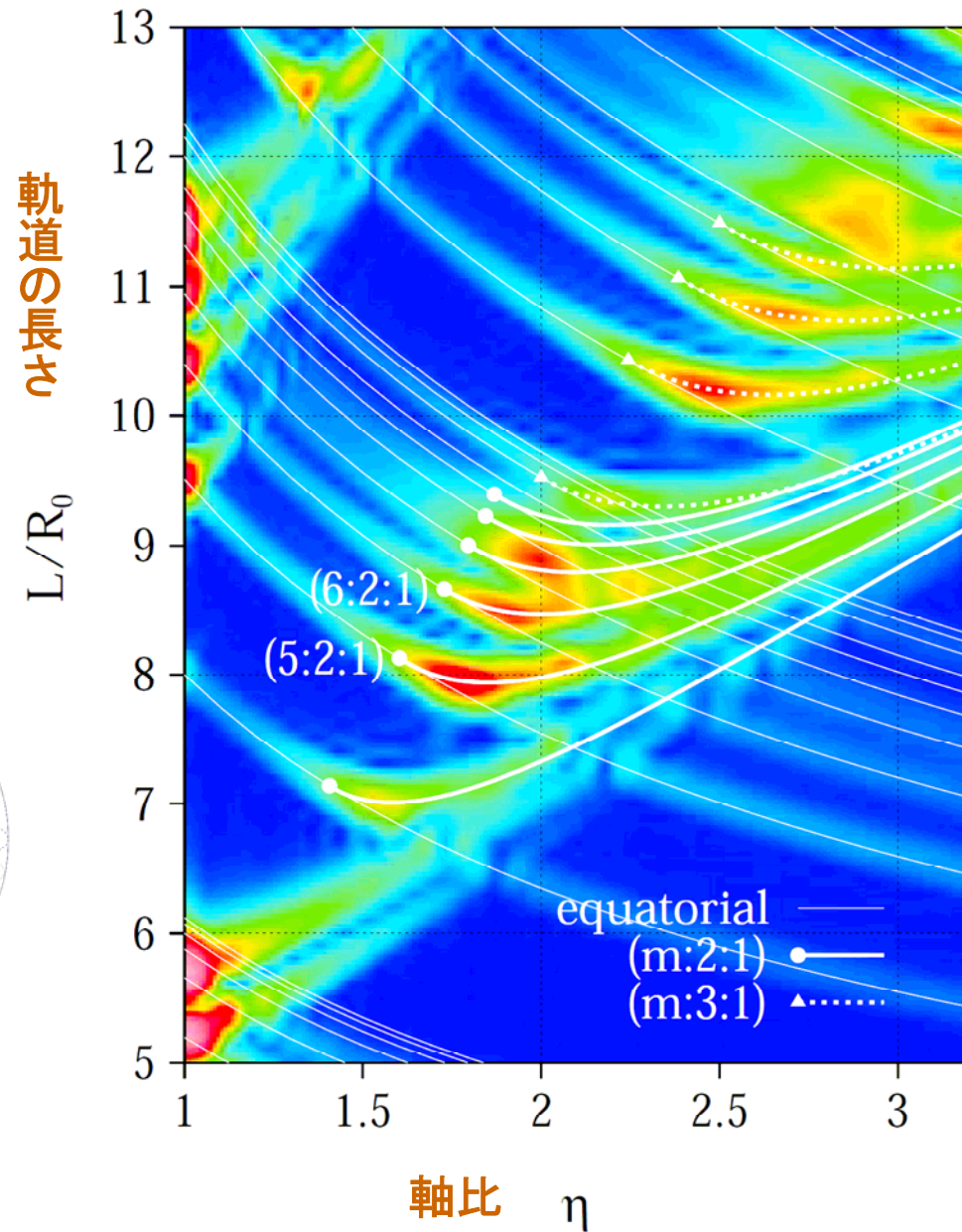
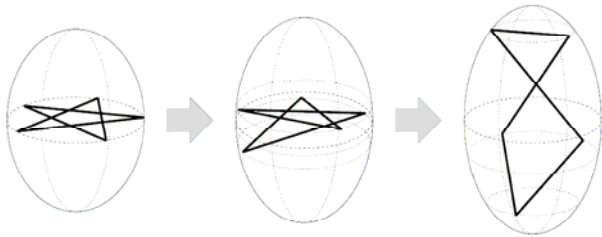


楕円体キャビティ: 3次元軌道の誕生



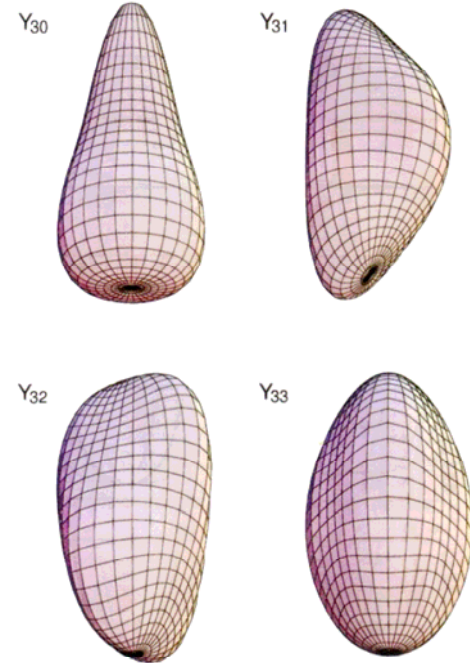
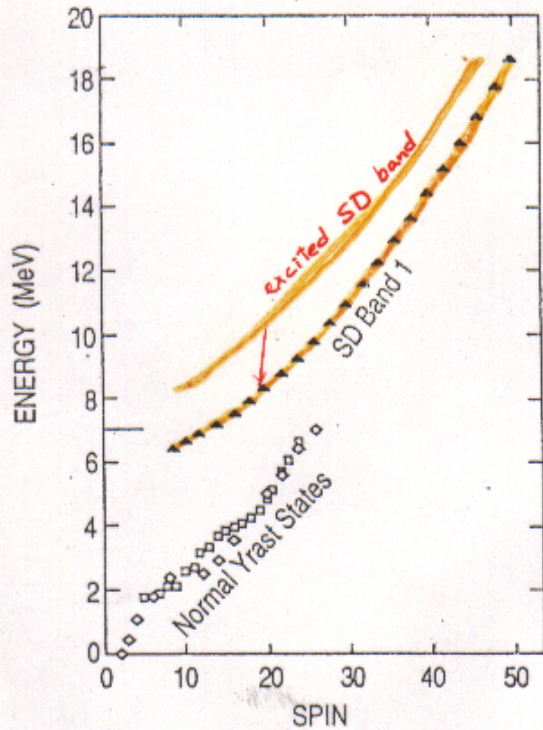
# 軸対称Cavity

周期軌道分岐による  
シェル構造の形成

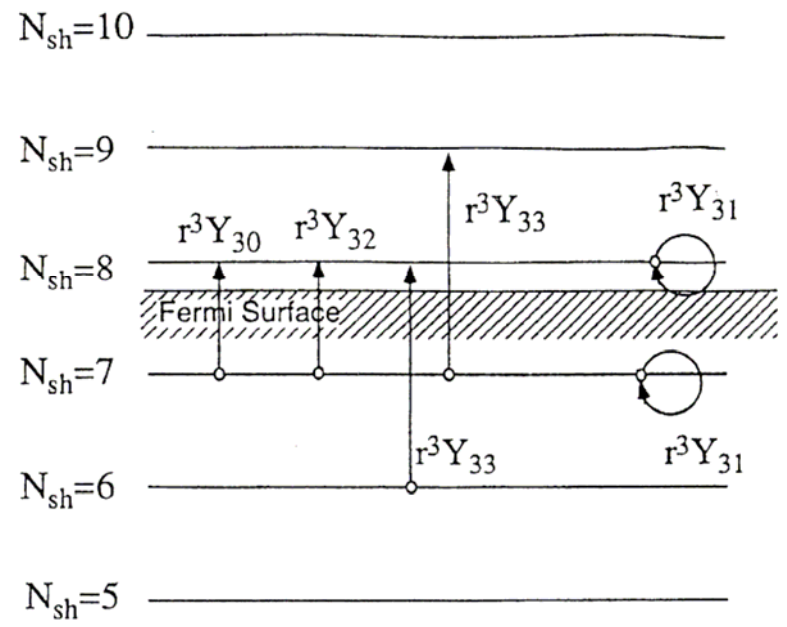
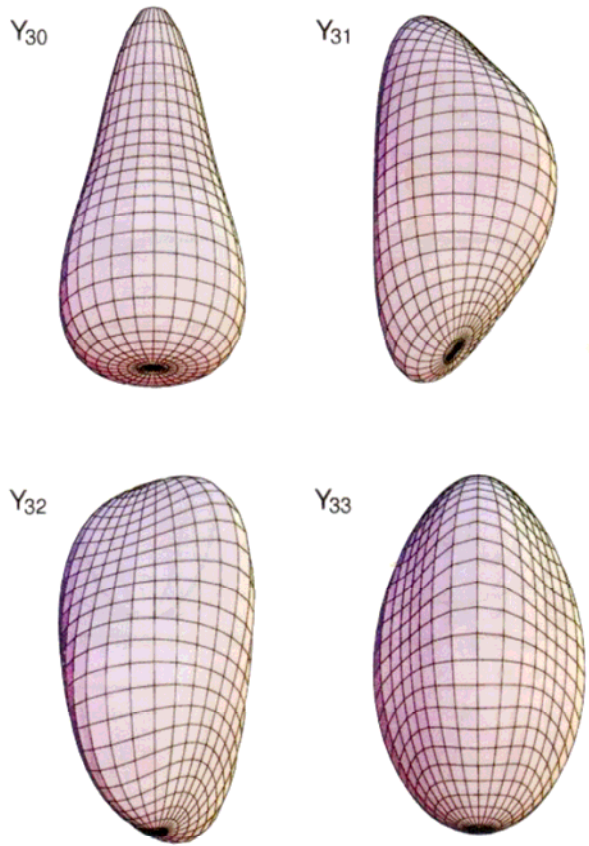


# 超変形イラスト状態の上に形成される新しい集団励起モードの探求

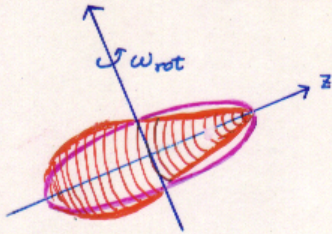
Search for New Collective Modes  
built on SD High-Spin States



# バナナ変形(Y31)モードは超変形のソフトモード

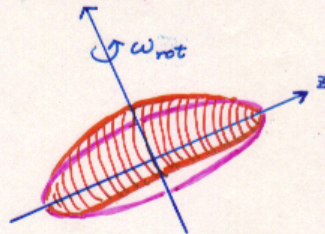


# Octupole Vibrations at the 2:1 shape



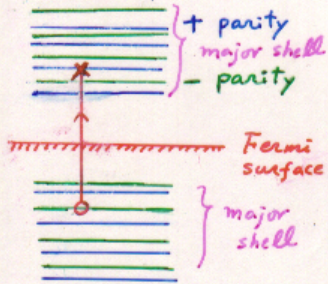
"pear"

$K=0$



"banana"

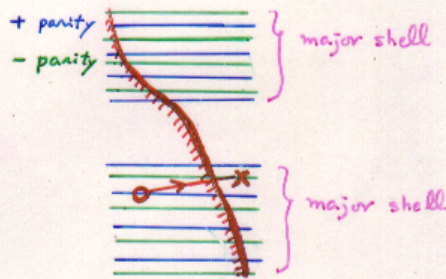
$K=1$



$\Delta=0$

$r^3 Y_{30}$

$\Delta N_{shell} = 1$



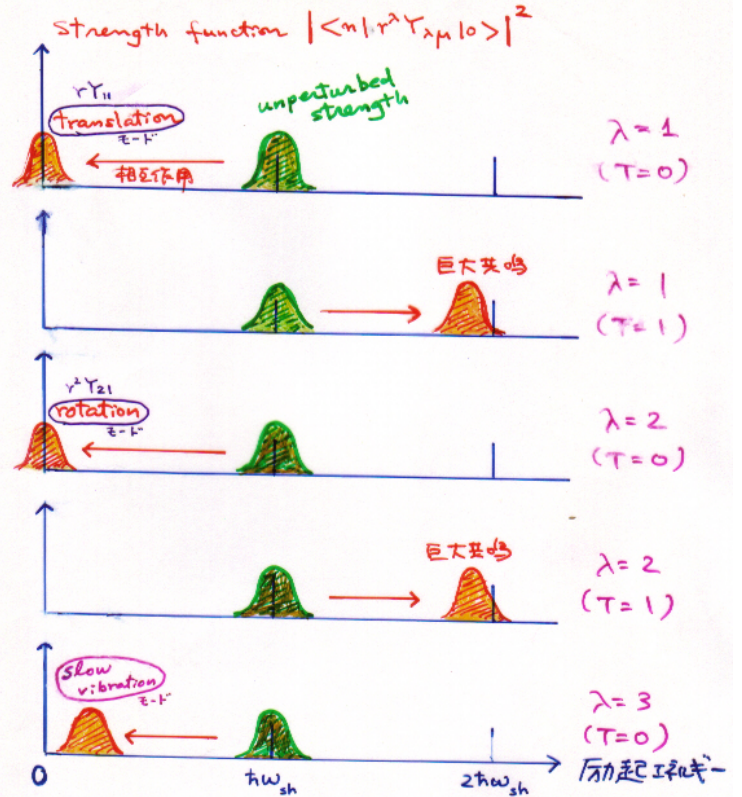
"Superconducting,  
Superdeformed Nuclei"

$r^3 Y_{31}$

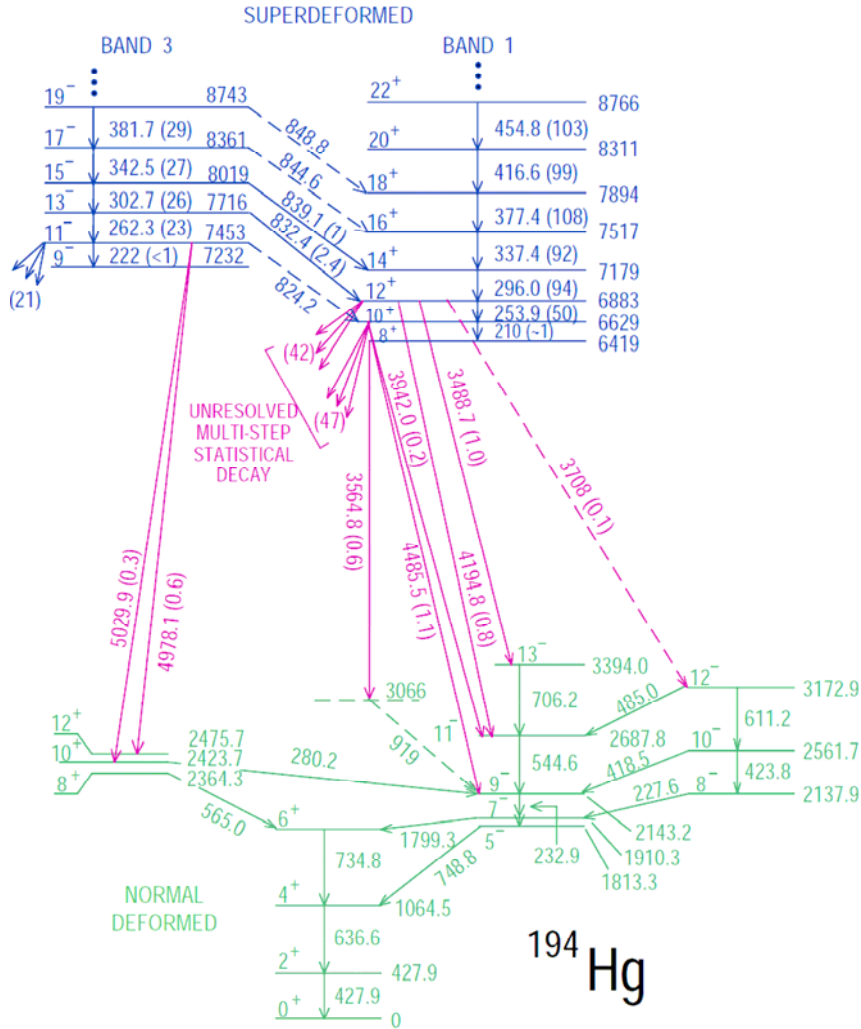
$\Delta \neq 0$

$\Delta N_{shell} = 0$

## 2:1 H.O. model



# 超変形イラストからの励起モード

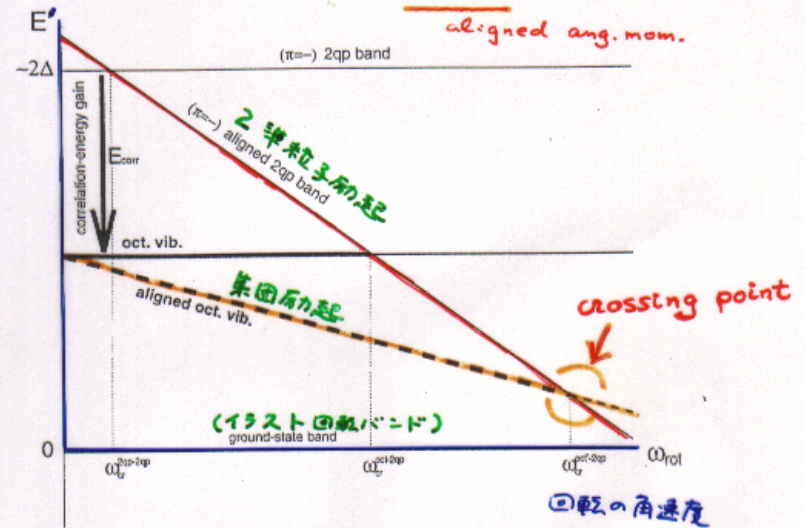


(根元念図)



イラスト系から測った

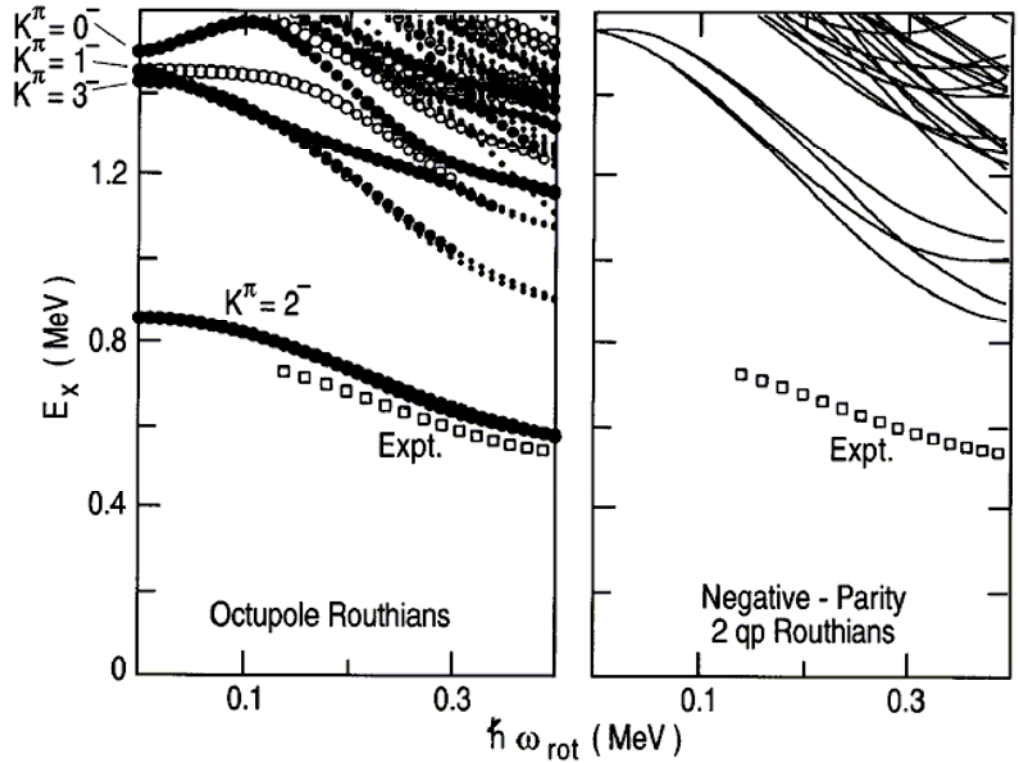
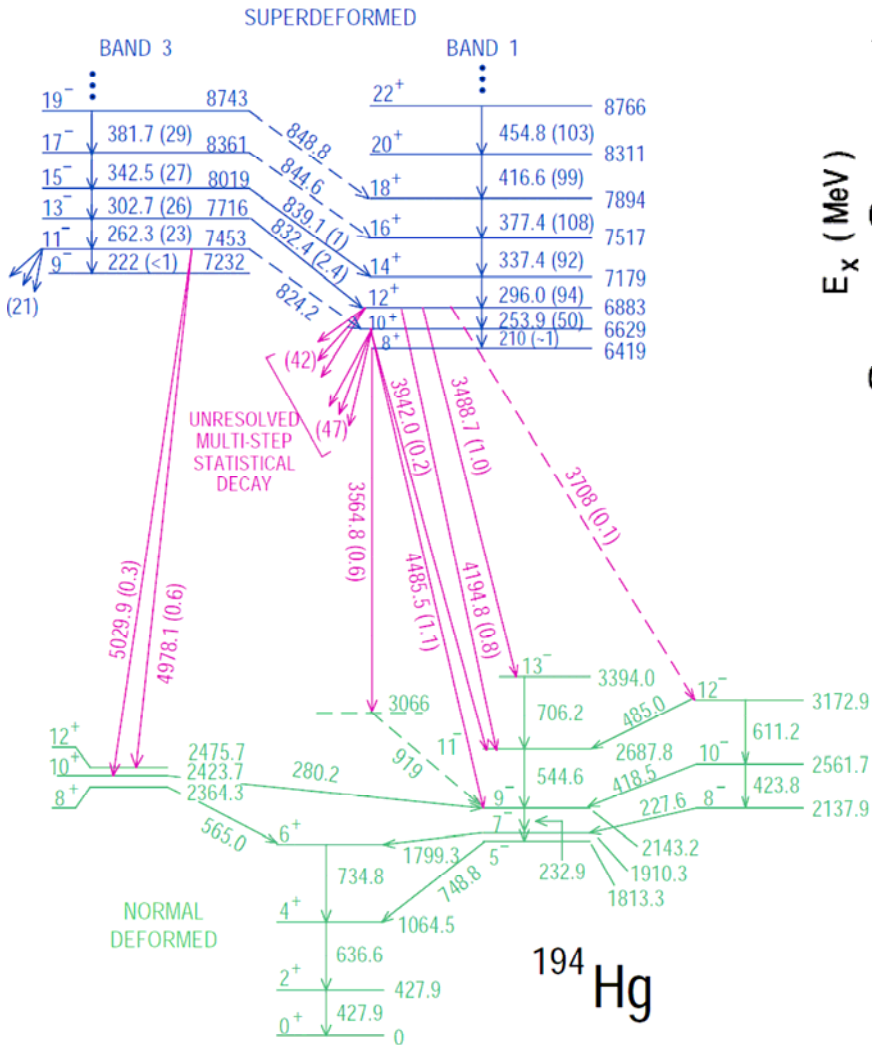
回転座標系でのエネルギー  $E' = E - \omega_{rot} J_x$



aligned oct. vib. と aligned 2qp の競争

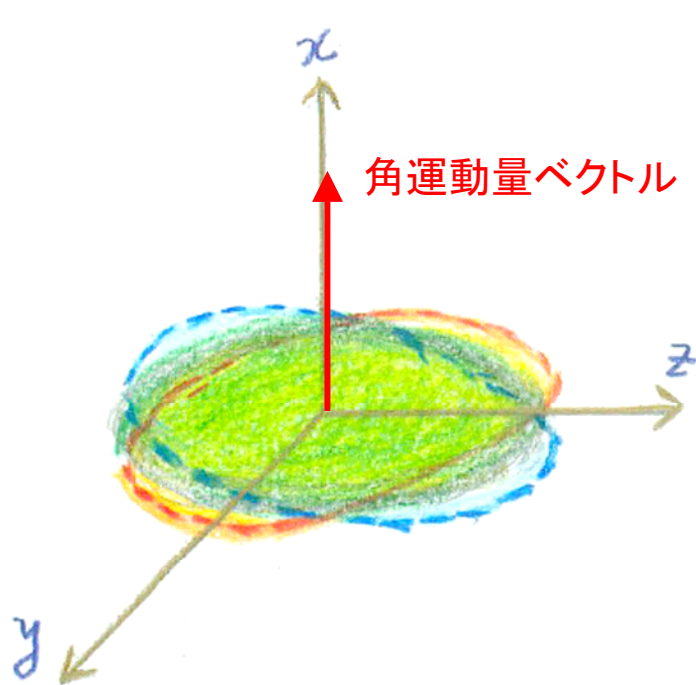
T. Nakatsukasa, K. Matsuyanagi, S. Mizutori, Y.R. Shimizu, Argonne Conf. (1996); Phys. Rev. C 53 (1996) 2213.

# 超変形状態の上のソフトOctupole振動モード



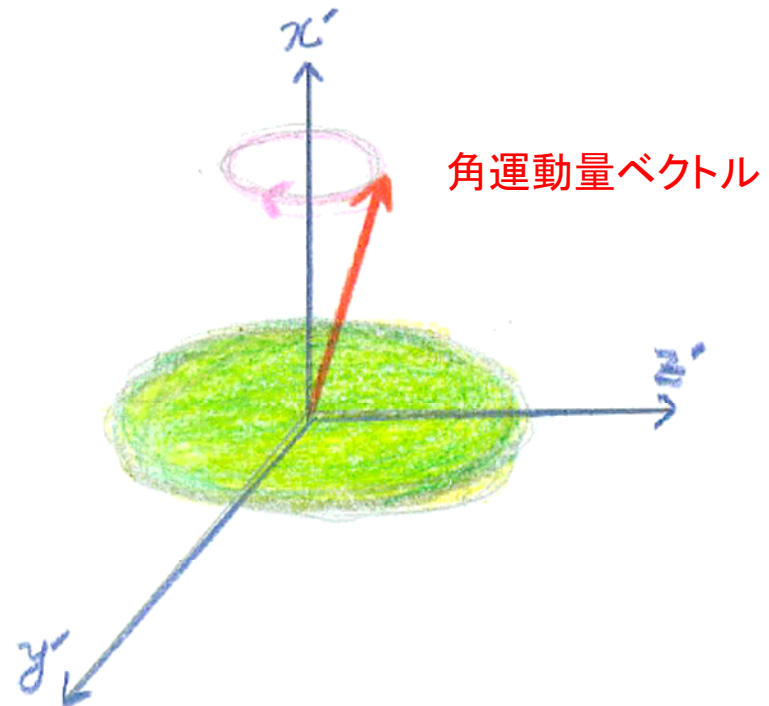
G. Hackman et al.,  
 Phys. Rev. Lett. 79 (1997) 4100

# Wobbling Motion の2つの描像



一様回転する座標系からみると  
形が振動しているように見える

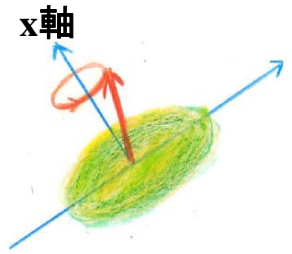
(角運動量ベクトルの方向をx軸と定義)



原子核の  
主軸に固定された座標系から見ると  
角運動量ベクトルが周期運動  
しているように見える

非軸対称な原子核

# Wobbling Motion のパラドックス



Wobblingモード

$$H = \frac{I_x^2}{2\mathcal{J}_x} + \frac{I_y^2}{2\mathcal{J}_y} + \frac{I_z^2}{2\mathcal{J}_z}$$

$$B^\dagger = xb^\dagger - yb$$

$$I^2 = I_x^2 + I_y^2 + I_z^2$$

固有座標系

$$I_\pm = I_y \pm iI_z$$

$$[I_-, I_+] = 2I_x \approx 2I$$

for  $I_x \gg I_y, I_z$

ボソン演算子

$$b = \frac{I_-}{\sqrt{2I}}, \quad b^\dagger = \frac{I_+}{\sqrt{2I}}$$

$$[b, b^\dagger] = 1$$

$$H = \frac{I^2}{2\mathcal{J}_x} + \hbar\omega(B^\dagger B + \frac{1}{2})$$

Wobbling振動数

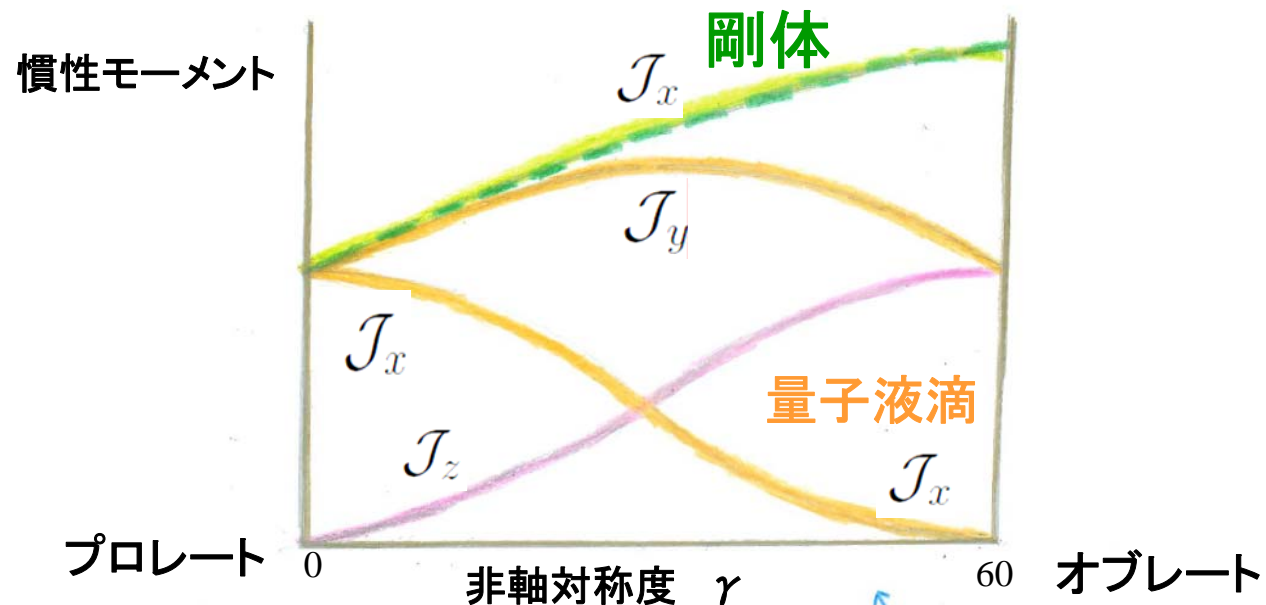
$$\hbar\omega = I \sqrt{\left(\frac{1}{\mathcal{J}_y} - \frac{1}{\mathcal{J}_x}\right)\left(\frac{1}{\mathcal{J}_z} - \frac{1}{\mathcal{J}_x}\right)}$$

$$E(n, I) = \frac{I(I+1)}{2\mathcal{J}_x} + \hbar\omega\left(n + \frac{1}{2}\right)$$

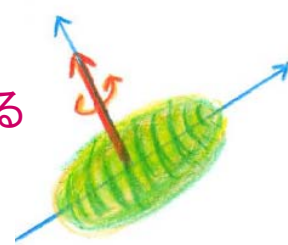
このモードが存在するためには  $\mathcal{J}_x \geq \mathcal{J}_y, \mathcal{J}_x$  が必要



# Wobbling Motion のパラドックス



破られた対称性を回復する  
量子力学的回転運動



対称軸と回転軸は直交

核子の角運動量整列  
(回転運動の消滅)



対称軸と回転軸は平行

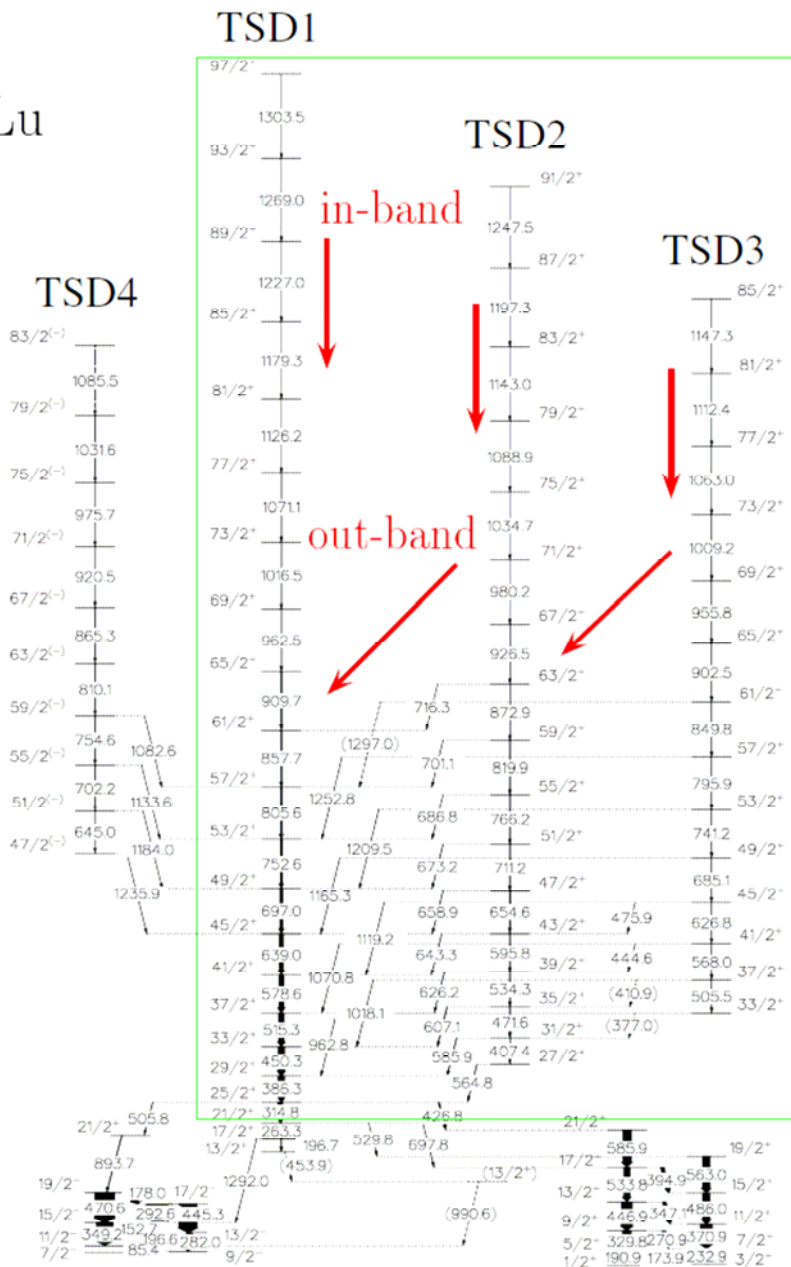
**Wobbling振動数**

$$\hbar\omega = I \sqrt{\left(\frac{1}{\mathcal{J}_y} - \frac{1}{\mathcal{J}_x}\right)\left(\frac{1}{\mathcal{J}_z} - \frac{1}{\mathcal{J}_x}\right)}$$

量子液滴では虚数になる！

$\mathcal{J}_x \geq \mathcal{J}_y, \mathcal{J}_x$  が必要

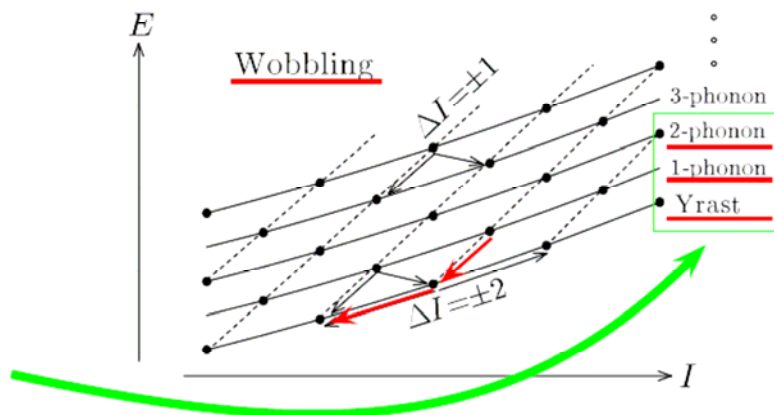
$^{163}\text{Lu}$



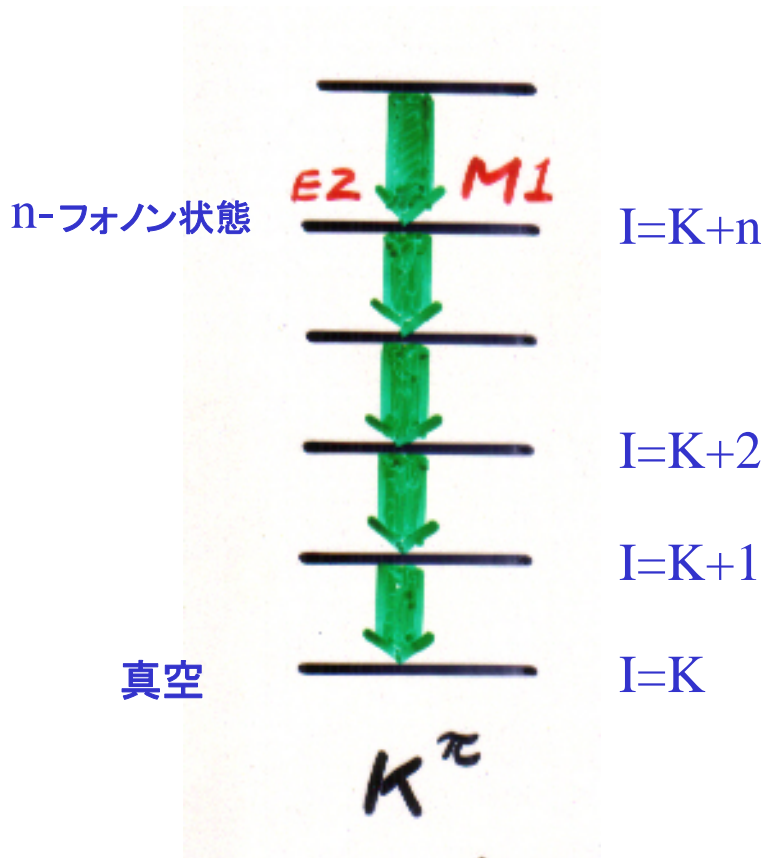
# Wobbling Spectra

D. R. Jensen et al., Eur. Phys. J. **A19** (2004), 173

First identified by  
Ødegård et al. (2001)



# High-K アイソマーの上立つ回転バンド



$$B(E2: I+1 \rightarrow I)$$

$$\approx \frac{5}{16\pi} e^2 Q_0^2 \underbrace{\frac{3}{K}}_{\text{振幅}} \cdot \underbrace{(I-K+1)}_{\text{量子の数}}$$

$$B(M1: I+1 \rightarrow I)$$

$$\approx \frac{3}{4\pi} ((g_K - g_R) K)^2 \frac{1}{K} (I-K+1)$$

# Precession as a phonon (vibrational mode)

High- $K$  rotor formula

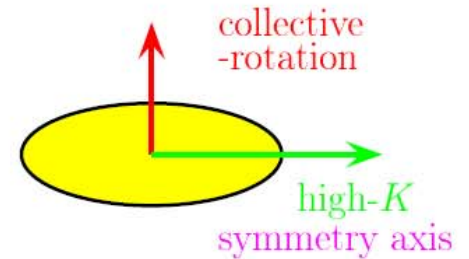
$$E_{\text{high-}K}(I) = \frac{1}{2\mathcal{J}_\perp} [I(I+1) - K^2], \quad (I \geq K)$$

putting  $n = I - K$  and  $n \ll K$  (*large !*), one obtains

Harmonic Spectra:

$$E_{\text{high-}K}(n) = \omega_{\text{prec}} \left( n + \frac{1}{2} + \frac{n(n+1)}{K} \right), \quad \boxed{\omega_{\text{prec}} \equiv \frac{K}{\mathcal{J}_\perp}}$$

anharmonicity      precession phonon energy



RPA method

$E2$  transitions

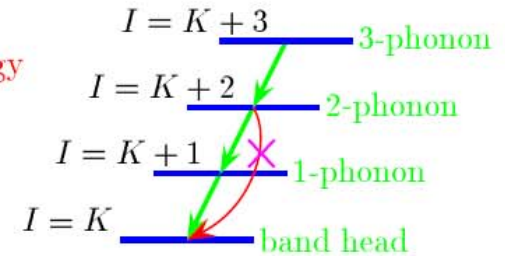
$$B(E2) \propto Q_0^2 \langle I_f K 20 | I_i K \rangle^2$$

then

$$\begin{cases} B(E2; n+1 \rightarrow n) \propto 3[(n+1)/K] & \cdots (\frac{1}{K}) \\ B(E2; n+2 \rightarrow n) \propto (3/2)[(n+2)(n+1)/K^2] & \cdots (\frac{1}{K})^2 \end{cases}$$

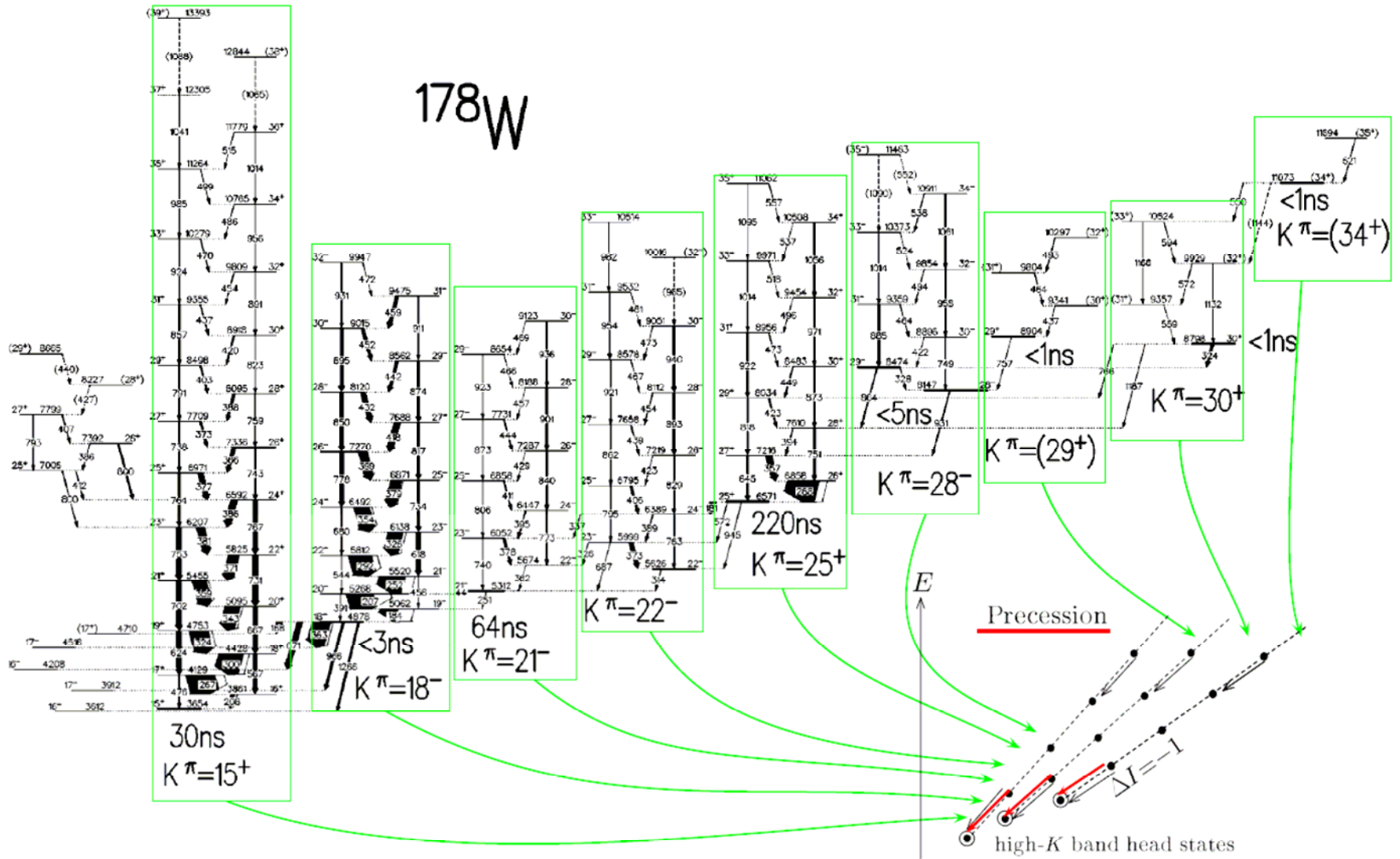
i.e. much smaller transition from two-phonon state ( $K \gg 1$ ):

$$B(E2; n+2 \rightarrow n) \ll B(E2; n+1 \rightarrow n) \quad (n \ll K)$$

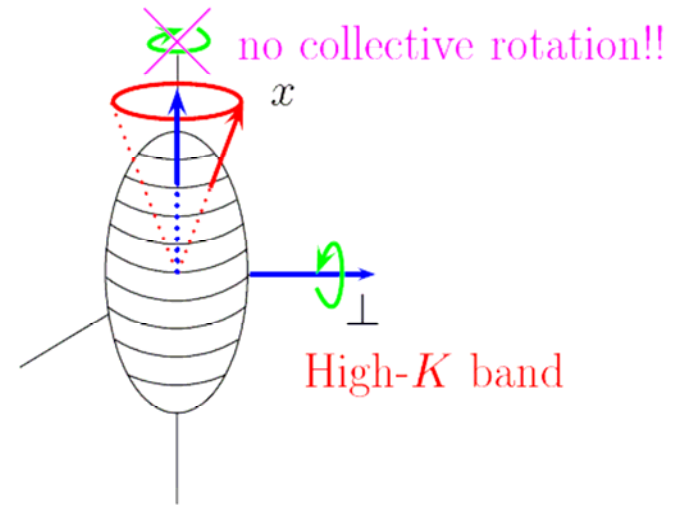
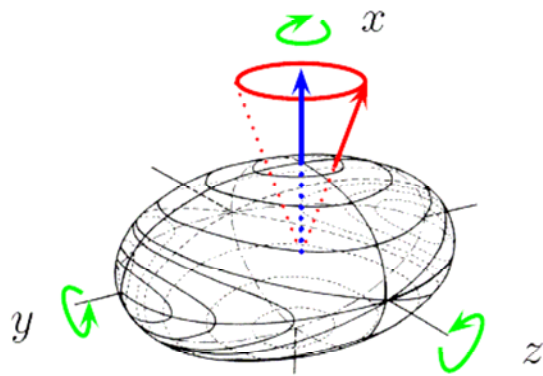


# Precession Spectra

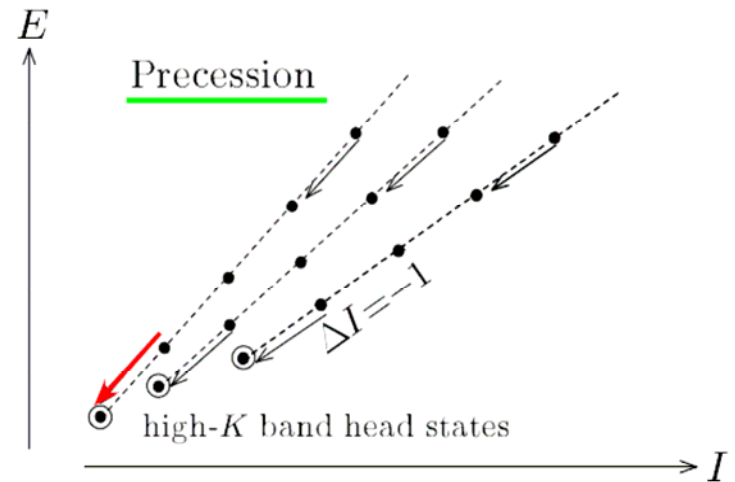
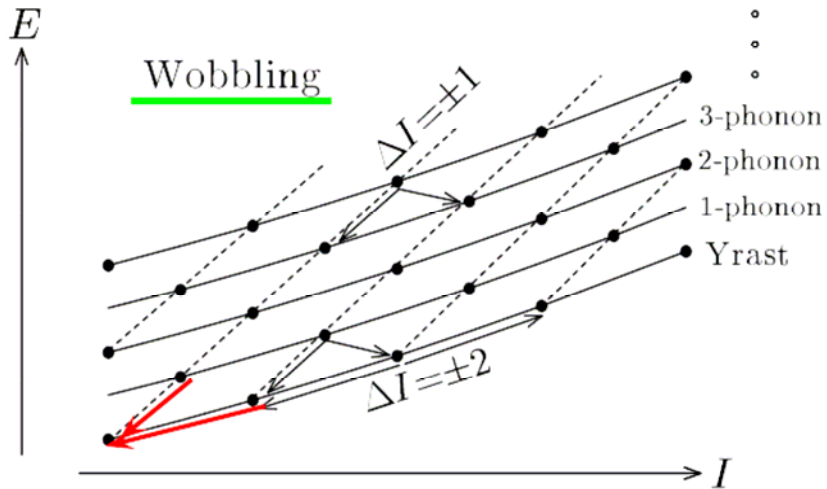
D. M. Cullen et al., Phys. Rev. **C60** (1990), 064301.



high- $K$  rotational bands: known for many years

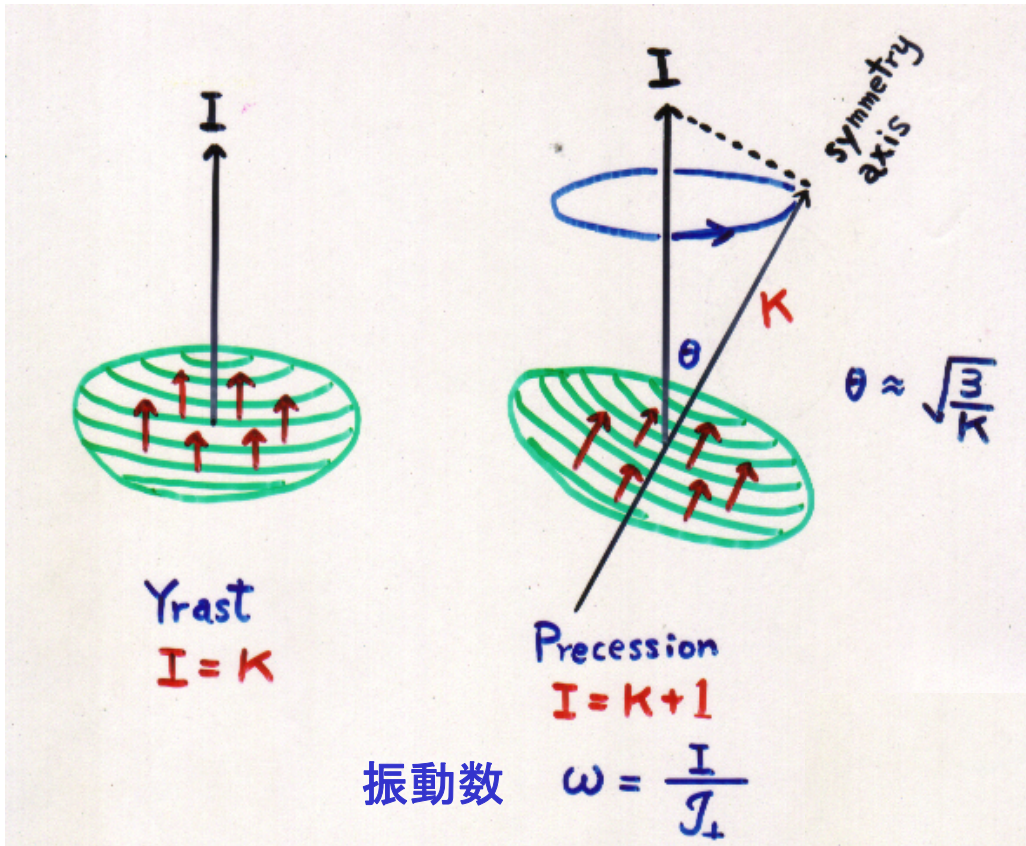


## Quantum Mechanical Spectra

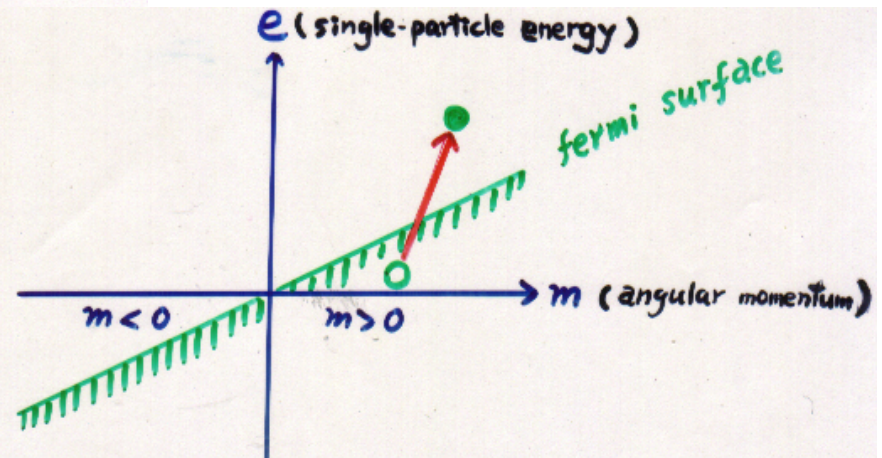


(Y.R. Shimizu, talk at Lund conf. 2005)

# 歳差(Precession)運動のRPA

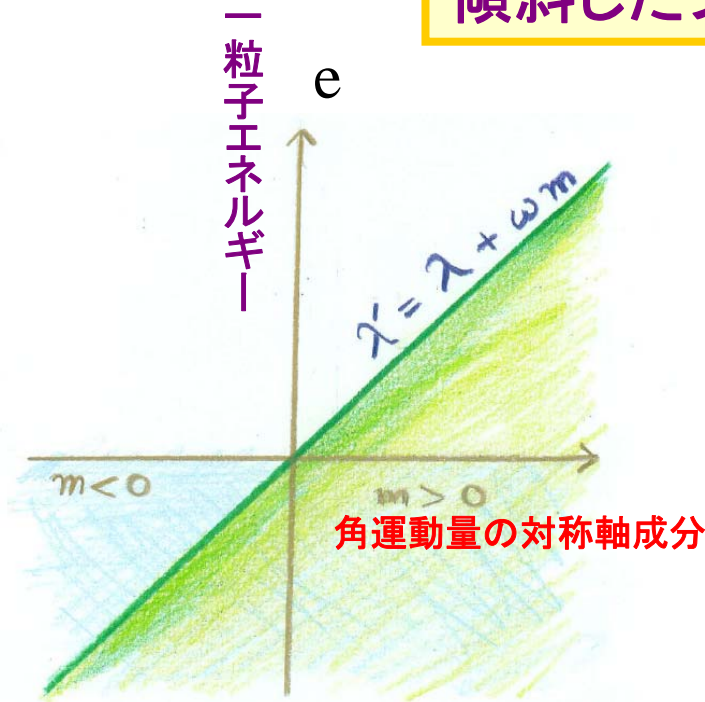


微視的構造



傾斜したフェルミ面

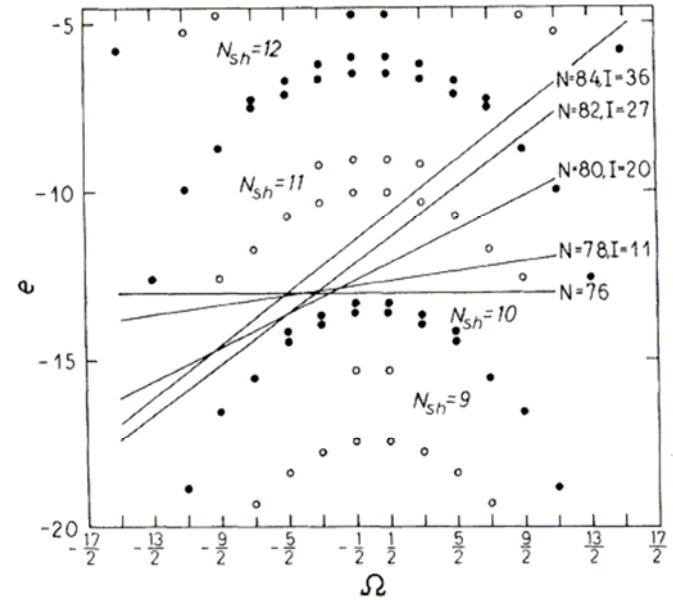
# 傾斜したフェルミ面



$$\begin{aligned}
 E &= \sum_i (e_i - \lambda) - \omega \sum_i m_i \\
 &= \sum_i (e_i - \lambda - \omega m_i) \\
 &= \sum_i (e_i - \lambda'_i)
 \end{aligned}$$

$$I = \sum_i m_i$$

— 粒子エネルギー —

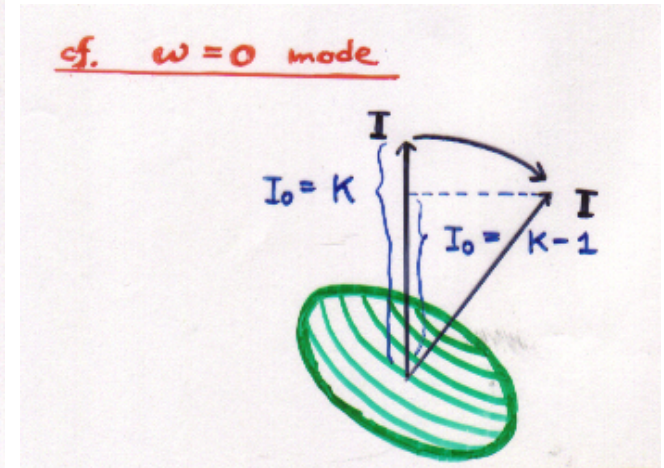
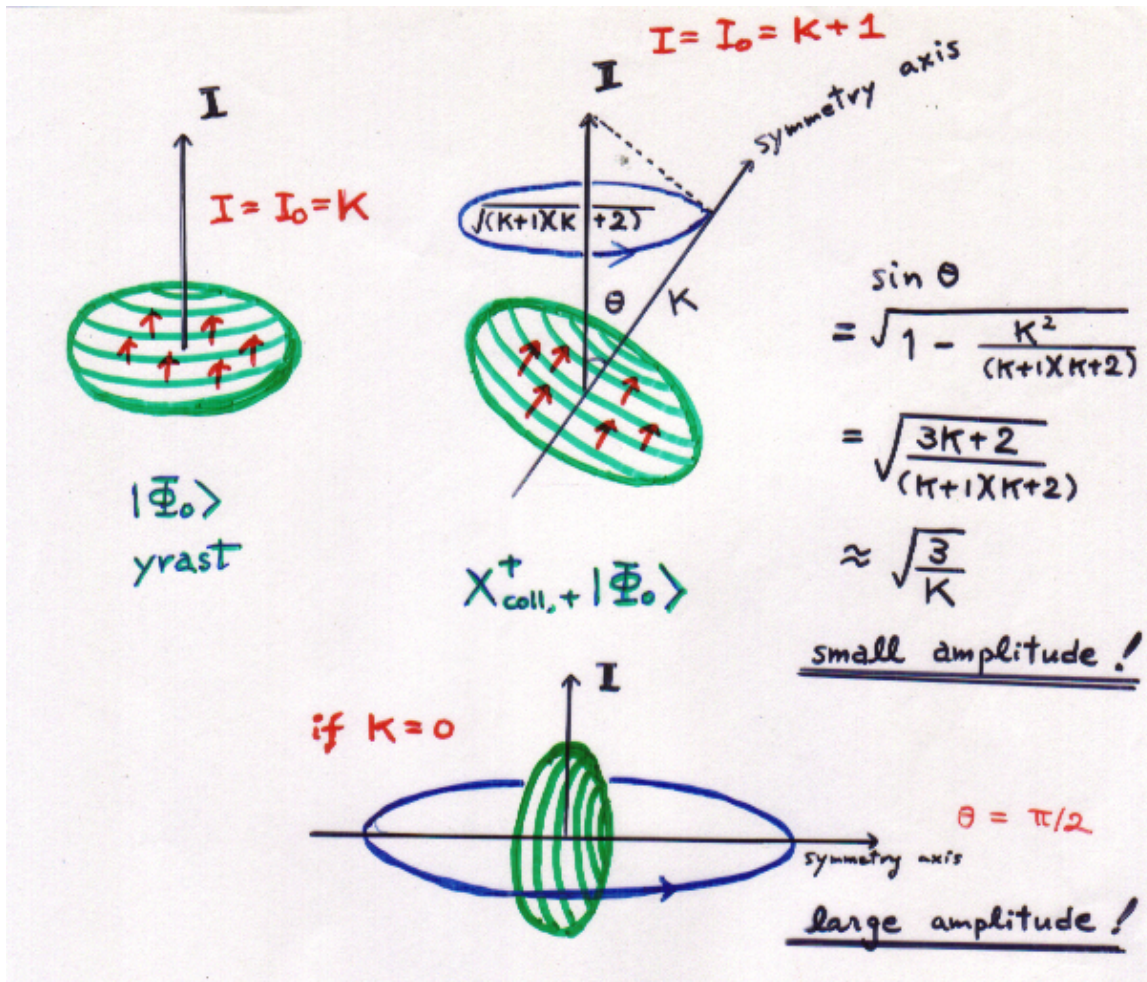


$m$

角運動量の対称軸成分



# Precession モードとNambu-Goldstoneモード



角運動量の  
方向だけを変える