# CONTENTS

## Elementary Modes of Excitation in Spherical Odd-Mass Nuclei

# Preface

In 1970, A. Kuriyama, T. Marumori and K. Matsuyanagi proposed a systematic microscopic theory to describe the collective excitation in spherical odd-mass nuclei. The theory can be regarded as a natural extension of the conventional quasi-particle-random-phase approximation for the case of spherical even-mass nuclei into the case of spherical odd-mass nuclei. Along this line of thought, investigations of microscopic structures of low-lying collective states in spherical odd-mass nuclei have been made in Japan. Since some interesting results have recently been obtained, it seems opportune to present the systematic survey of these works, though some of them have already been published in the form of separate papers.

Our study has been developed as a part of the research projects organized by the Research Institute for Fundamental Physics from 1969 to 1974. We would like to express our hearty thanks to Prof. K. Takada (Kyushu University) and Prof. M. Yamamura (Kyoto University) and the other members of the research projects for the valuable discussions and intimate collaborations. We are also grateful to Profs. A. Bäcklin, K. Hisatake, T. Inamura, R. A. Meyer, S. Morinobu, Y. Nagai, S. Ohya, T. Shibata, L. Vanneste and W. B. Walters for communicating their experimental results. Our thanks are also due to Miss J. Ito for her advice on English expression of the original manuscript.

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# Part I.

# Introduction

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## §1. Main motive

In the past decade the studies on nuclear structure have found the concept of phonon as an elementary mode of excitation in the nuclear system increasingly significant. On the other hand, the studies have suggested that the simple phonon model (based on the harmonic approximation) cannot give a satisfactory description of rather complicated anharmonicity effects, i.e., deviations from the simple phonon model are quite essential in a finite many-body quantal system such as the nucleus. Furthermore, the recent rapid accumulation of experimental data suggests the existence of certain "hidden regularities" in the complicated anharmonicity effects. Thus, one of the important subjects in current nuclear study is the sublation (aufheben) of the very concept of elementary modes of excitation in connection with the structure of the anharmonicity. Concerning such a subject, several annual research projects have been organized in Japan by the Research Institute for Fundamental Physics since 1969. Some important problems to be attacked at the first stage of the study were set up in the beginning of the research projects. One of them was to investigate the possibility of proposing an algebraic method of pair operators, which starts with the special nucleon-pair operators as the basis operators instead of the "phonon" as an ideal boson.[1] Along this line, the algebraic method is being extensively investigated by Yamamura et al.[2] Another problem was to construct a microscopic theory by which the structure of the complicated anharmonicity effects can be investigated in a simple systematic way. The essential part of our investigation concerning the present paper has been performed as a part of the research along this line.

In order to explain the situation at that time, we start with a brief survey of the results of analyses of the anharmonicity effects in even-even nuclei by Yamamura, Tokunaga and Marumori[3] in 1967 in terms of the boson expansion

method.[4] They first classified the anharmonicity effects into two characteristic types; i) *kinematical effects,* i.e., effects due to the Pauli principle among the quasi-particles belonging to different (bound) quasi-particle pairs which are regarded as ideal bosons (i.e., "phonons") under the quasi-particle-random-phase approximation (RPA), and ii) *dynamical effects,* i.e., effects due to the residual interaction which has been omitted in the RPA. After calculating the kinematical and the dynamical effects (in the pairing-plus-quadrupole-force model) with the use of a perturbation theory based on the boson expansion method, they arrived at the following conclusion[3]: The simple "two-phonon" concept (as a possibility of repeating the excitation of an ideal boson twice) is actually broken in the following sense. i) Both the kinematical and the dynamical effects become unexpectedly large in the absolute values when the "phonon" energy under the RPA comes close to the actual experimental value. ii) When the energy of the "two-phonon" state under the RPA is close to those of the non-collective two-quasi-particle states, the coupling between the "two-phonon" state and the non-collective two-quasi-particle states due to the dynamical effects becomes too significant to be treated by the perturbation theory. In this case, which occurs most often in actual nuclei, we are forced to make a diagonalization of the coupling, which leads to a strong mixing of the two states and breaks the simple "two phonon" concept.

From this conclusion, one may naturally expect that the (quasi-particle-) higher-random-phase approximation (HRPA)[5] is promising in taking these significant anharmonicity effects into account, because it does not use the picture of repeating the "phonon" excitation twice. It is known that in the HRPA (the second RPA) the kinematical effects on the so-called "two-phonon" states due to the Pauli principle among the four quasi-particles are fully taken into account. Furthermore, the dynamical effects, i.e., the coupling between the two-quasi-particle excitation modes and the "two-phonon" modes are properly considered.*) Unfortunately, such a merit of the HRPA is merely one of formal logic. Actually we encounter the well-known formal difficulty which is inherently connected with the non-symmetrical form of the secular matrix coming from the linearized equation of motion for the eigenmode operator of the HRPA. The other rather serious formal difficulty in the HRPA is also known to arise from the spurious-state problem, which originates from the nucleon-number-non-conservation in the quasi-particle basis. As is well-known, it is one of the important advantages of the RPA that both the

*) Since both two-quasi-particle and four-quasi-particle amplitudes (in the sense of the new-Tamm-Dancoff (NTD) approximation with the ground-state correlation) are taken into account in the eigenmode operator of the HRPA, the excitation energies of both the first and the second excited states (which roughly correspond to the "one-phonon" and the "two-phonon" states of the RPA, respectively) are simultaneously obtained through the (linearized) equation of motion for the eigenmode operator in the NTD approximation.

"phonon" states and the (correlated) ground state are orthogonal to the spurious states within the framework of the approximation. However, the HRPA never leads us to either the "physical" excited states or the "physical" ground states which are orthogonal to the spurious states. Thus, we may conclude that, without overcoming these difficulties in essence and not in superficies, we cannot enjoy the above-mentioned essential merit of the HRPA in treating the anharmonicity effects. Nevertheless, any theories or methods overcoming the difficulties had not yet been achieved at that time. This was the reason why the authors' first task in collaboration with Kanesaki, Sakata and Takada[6)~9)] was to construct a new systematic microscopic theory which overcomes the difficulties in the HRPA and to treat both the kinematical and the dynamical anharmonicity effects in a simple systematic way.

## §2. Outline of theory

In contrast with the HRPA, the underlying philosophy of our theory is not to intend a *direct, formal* diagonalization of the Hamiltonian within a subspace characterized by the eigenmode operator of the HRPA, but rather to start with an extraction of the basic physical elements from the subspace.

Our first task is to develop a method which enables us to uniquely separate the spurious components from the quasi-particle states and to precisely keep the one-to-one correspondence between the seniority number and the quasi-particle number. This problem is studied in Chap. 1 of Part II. According to the method developed in Chap. 1, we can regard the space of states described by the quasi-particles as a product space composed of "intrinsic" and "collective" spaces. The "intrinsic" space consists of the states which never involve $J=0$-coupled quasi-particle pairs, while the "collective" space consists of the states which include only $J=0$-coupled quasi-particle pairs and are always orthogonal to the "intrinsic" states. Needless to say, all of the spurious components belong to the "collective" space, and a special one of "collective" vibrations (under the RPA) with zero energy in this subspace is known to be due to the nucleon-number non-conservation.

Secondly, in the "intrinsic" space, we construct the correlated $n$-quasi-particle excitation modes (with $n=2, 4, 6, \cdots$ for even-even nuclei and with $n=1, 3, 5, \cdots$ for odd-mass nuclei) within the framework of the new Tamm-Dancoff (NTD) method with the ground-state correlation. The creation operators of these excitation modes consist of $n$-quasi-particle (creation and annihilation) operators accompanied by the correlation amplitudes involving the ground-state correlation in the NTD sense. The excitation modes are hereafter called the "dressed" $n$-quasi-particle ($n$QP) modes, and their detailed formal structure is studied in Chap. 2 for odd-mass nuclei. In order to specify the dressed $n$QP modes precisely in the "intrinsic" space, as is shown in Chap. 2, it is decisive to use the concept of spherical tensors in the quasi-

spin space which has been introduced through the quasi-spin formalism (for the pairing correlations).[10] The dressed 2QP mode (with the lowest energy eigenvalue), which is the simplest one among the dressed $n$QP modes, is nothing but the "phonon" under the RPA. In this sense, we may say that our theory can be regarded as a natural extension of the RPA. The dressed 4QP states (with the lowest energy eigenvalues) correspond to the "two-phonon" states of the RPA, but the kinematical effects due to the Pauli principle among the four quasi-particles are fully taken into account in these states.

With the aid of the dressed $n$QP modes, we can introduce a set of orthogonal basis vectors consisting of the (correlated) ground state and the dressed $n$QP states. We call the space spanned by the orthogonal set the quasi-particle NTD space. Within the framework of the NTD approximation, this space is, by definition, orthogonal to the "collective" space which involves all of the spurious components. The basic physical idea underlying the introduction of the quasi-particle NTD space is as follows. Let us recall that the use of the quasi-particle basis can be regarded as an attempt to classify both the ground state and the excited states in terms of the seniority number $v$, keeping one-to-one correspondence between the seniority number and the quasi-particle number $n$. Then, the orthogonal basis vectors characterizing the quasi-particle basis are the BCS ground state (with $v=0$) and the $n$-quasi-particle states *with the condition* $n=v$. These orthogonal basis vectors with the definite quasi-particle numbers $n=v$ span the "quasi-particle Tamm-Dancoff (TD) space", which is merely the "intrinsic" space mentioned above. Now it is well known that, in a many-body quantal system such as the nucleus, the ground-state correlation is particularly important as a collective predisposition which allows the correlated excited states to occur from the ground state. We must therefore take account of the importance of both the seniority classification and the ground-state correlation simultaneously, in a way that the essential physical concept obtained in the quasi-particle TD space would still persist in a certain form. The guiding principle to introduce the quasi-particle NTD space lies in the fact that, in the NTD method, the quasi-particle correlations which are asymmetrically attributed to only the excited states in the TD calculations are symmetrically incorporated in the ground state through the ground-state correlation. In contrast with the BCS ground state in the quasi-particle TD space, the ground state in the quasi-particle NTD space is not with a definite seniority number because of the ground-state correlation. In spite of such a breaking of the seniority classification, in the quasi-particle NTD method we can still characterize the excitation modes, i.e., the dressed $n$QP modes by the amount of seniority $\Delta v=n$ which they transfer to the correlated ground state.

Our third task is to find a method of transcription of the physical operators in the quasi-particle TD space into the quasi-particle NTD space. The

transcription should satisfy some self-consistency conditions within the framework of the (employed) NTD approximation under which the quasi-particle NTD space has been introduced. Details of the method of transcription is also discussed in Chap. 2. It is shown that, after the transcription into the quasi-particle NTD space, the residual interaction which has been omitted in constructing the dressed $n$QP modes manifests itself as a coupling between the different excitation modes. In our theory, the dynamical effects are then obtained by diagonalizing the coupling. The eigenmode creation operator, which is obtained by diagonalizing the coupling within the quasi-particle NTD subspace (composed of the dressed 2QP and dressed 4QP states), is formally of the same form as that of the HRPA, when written explicitly in terms of the quasi-particle operators. Nevertheless, in our theory, the difficulties inherent to the HRPA never appear because of our proper choice of the quasi-particle NTD space. From this point of view, the microscopic structure of the so-called "two-phonon" states is being investigated by Iwasaki, Kanesaki, Marumori, Sakata and Takada.[11]

## §3. Dressed 3QP mode as a new type of elementary mode of collective excitation

According to our theory, the simplest of the collective excitation modes in even-even nuclei is the dressed 2QP mode (with the lowest energy eigenvalue) as a "bound" state of two quasi-particles, which is nothing but the "phonon" under the RPA. In the case of odd-mass nuclei, the simplest of the collective excitation modes is the dressed 3QP mode (with the lowest energy eigenvalue). Thus, in the same manner as the RPA for even-mass nuclei leads us to the concept of "phonon" as a boson, the theory may necessarily lead us to the concept of a new kind of fermion-type collective excitation mode, i.e., *the dressed 3QP mode as a "bound" state of three quasi-particles*. So far, the collective excited states in odd-mass nuclei have conventionally been described in terms of the quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen.[12] From this point of view, it is quite interesting to investigate whether or not this new kind of collective mode systematically exists in many spherical odd-mass nuclei, playing an important role in their low-lying collective states.

There was already a positive reason to expect the presence of the new kind of collective mode. In 1967, Bohr and Mottelson[13] emphasized a significant effect of quasi-particle-phonon coupling, which had been completely omitted in the conventional QPC theory of Kisslinger and Sorensen. They have shown the extreme importance of this new effect in terms of the perturbation theory based on the self-consistent particle-vibration-coupling approach[14] (i.e., the "nuclear field theory"), and have suggested that "the

conventional description of collective excited states of almost all spherical odd-mass nuclei is significantly affected by the inclusion of the effect". It has also been demonstrated that the new effect essentially originates from the Pauli principle between the quasi-particles composing the phonon and the odd quasi-particle (i.e., the kinematical effect among the three quasi-particles), and brings about a significant three-quasi-particle correlation. Now, according to the nuclear field theory,[14] the strength of the particle-vibration coupling, $f$, is obtained by dividing a standard coupling matrix element by the phonon energy $\hbar\omega$. In situations where $f \ll 1$ (, the *weak coupling case*), we can safely treat the coupling by the perturbation theory.[15] For $f \gg 1$ (, the *strong coupling case*), the particle produces a static shape deformation, and the coupled system must be treated by a separation between rotational and intrinsic degrees of freedom. The nuclear field theory has clarified that, in contrast with the case of octupole mode where the values of $f_{\lambda=3}$ are typically about 0.1 to 0.3, the coupling strength for the quadrupole mode, $f_{\lambda=2}$, may become larger than unity. This implies that, for the quadrupole mode, the new effect bringing about the significant three-quasi-particle correlation should be taken into account not by the perturbation approximation but by diagonalizing the Hamiltonian in a suitable subspace. The dressed 3QP mode just satisfies this requirement, because it fully takes into account the kinematical effect among the three quasi-particles within the NTD approximation which is not the perturbation approximation. From this point of view, our theory includes the possibility of such an *intermediate coupling case* where the internal structure of the phonon itself is affected to form the dressed 3QP mode as a bound state. (See Fig. 1.)

Along this line of thought, investigations of microscopic structures of low-lying collective states in spherical odd-mass nuclei have been made. We have then obtained the conclusion that the appearance of the low-lying anomalous coupling (AC) states with spin $I=j-1$ can be regarded as a typical phenomena in which the new kind of collective mode (i.e., the dressed 3QP mode as a "bound" state of three quasi-particles) manifests itself as a relatively pure eigenmode.[16] It has also been shown that the physical condition of the enhancement of the three-quasi-particle correlation (characterizing this new collective mode) is not specific to the AC states but more general in odd-mass nuclei.[18] Thus, we have suggested that the new collective mode exists in

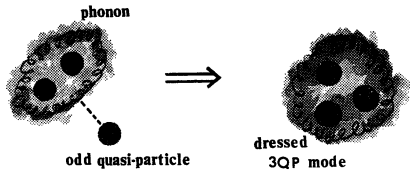


Fig. 1.

many spherical odd-mass nuclei and plays an important role in their low-lying collective states. It seems that recent experiments are revealing the systematic presence of this new kind of collective mode from among complicated spectra of the low-energy excitations in spherical odd-mass nuclei. The detailed review of these investigations[16)~19)] is the main subject in Chaps. 3 and 4 in Part III.

The framework of our theory includes the QPC theory as a special weak coupling case in which the characteristic three-quasi-particle correlation is seriously reduced by some physical conditions in shell structure. Therefore, our theory enables us to investigate the microscopic structure of the breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture". This investigation is the subject of Chap. 5.

The investigations of collective excitations in spherical odd-mass nuclei in Chaps. 3, 4 and 5 have been made with the use of the pairing-plus-quadrupole ($P+QQ$) force.[20)] Since we have widely employed the characteristic properties of the quadrupole force, it is indispensable to examine whether the conclusions obtained from Chap. 3 to Chap. 5 are specific to the $P+QQ$ force or not. This is the problem which is studied in Chap. 6.

## §4. Coupling between pairing mode and dressed *n*QP mode

According to the method developed in Chap. 1, we can regard the space of states in terms of quasi-particles as a product space consisting of the "intrinsic" and "collective" spaces. In this representation, the original quasi-particle interaction is classified into three types: The first represents an interaction causing mixing among the "intrinsic" states, the second among the "collective" states and the last between "collective" and "intrinsic" states. The first-type interaction in the "intrinsic" space can furthermore be divided into two parts: One of them is the so-called *constructive force* which is responsible for constructing the dressed $n$QP modes, and the other the so-called *interactive force* which manifests itself as the coupling among the different $n$QP modes after the transcription into the quasi-particle NTD space. What we have investigated in Part III as the dynamical effect is nothing but the effect originating from this interactive force.

The other new type of dynamical effect may arise from a third-type interaction which causes the mixing between "collective" and "intrinsic" states. Since the "collective" space involves all of the quantum fluctuations of the pairing field, i.e., the excitation modes of $J=0$-coupled quasi-particle pairs, the third-type interaction can be expressed as the coupling between the pairing modes and the dressed $n$QP modes. The formal structure and the physical implication of the coupling are discussed in Chap. 7, although the detailed analysis of its effect in comparison with experiment is in the course of investigation as a next subject.

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# Part II.

# General Formulation of Theory

## Chapter 1. Intrinsic and Collective Degrees of Freedom in Quasi-Spin Space

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## §1. Introduction

It is well known that the use of the quasi-particle basis in the BCS theory can be regarded as an attempt to characterize both the ground state and the excited states in terms of the seniority number $v \equiv \sum_a v_a$*) in such a way that the number of quasi-particles $n \equiv \sum_a n_a$ is equivalent to the seniority $v$. This is one of the most important motives for introducing the quasi-particle basis.

In this approach, there is however a serious difficulty arising from the spurious-state problem due to the nucleon-number non-conservation. Owing to the fact that any quasi-particle basis vectors $|\phi\rangle$ are not eigenstates of the nucleon-number operator $\hat{\mathcal{N}}$, the use of the quasi-particle basis inevitably introduces the spurious states arising from the nucleon-number fluctuations such as $(\hat{\mathcal{N}}-\mathcal{N})|\phi\rangle$, and only the states orthogonal to the spurious states correspond to those of a physical nucleus.

Thus, in the use of the quasi-particle representation, it is decisive to develop a method which can uniquely separate the spurious components from the quasi-particle states $|\phi\rangle$, keeping the one-to-one correspondence between the seniority number $v$ and the quasi-particle number $n$. This is the problem which is studied in this chapter.

*) In this paper we adopt the spherical $j$-$j$ coupling shell model. The single-particle states are then characterized by the set of quantum numbers $\alpha = \{\text{the charge } q, n, l, j, m\}$. In association with the Greek letter $\alpha$, we use the Roman letter $a$ to denote the same set except the magnetic quantum number $m$. We also use the notation $\bar{a}$, which is obtained from $a$ by changing the sign of the magnetic quantum number. We furthermore use the notation $f(\bar{a}) = (-)^{j_a - m_a} f(\bar{a})$ where $f(\alpha)$ is an arbitrary function of $\alpha$.

## §2. Preliminaries

It is well known that the quasi-particle can be regarded as substantiation of the concept of seniority. This is easily seen with the use of the quasi-spin formalism.[1] Since this formalism plays an important role in our theory, we start the discussion with its brief recapitulation.

### 2-1 *Quasi-spin space*

Let us define the quasi-spin operators of the single-particle orbit $a$ as*)

$$\begin{aligned}\hat{S}_+(a)&=\sqrt{\frac{\Omega_a}{2}}\sum_{m_{\alpha_1}m_{\alpha_2}}(j_aj_am_{\alpha_1}m_{\alpha_2}|00)c^\dagger_{\alpha_1}c^\dagger_{\alpha_2},\\ \hat{S}_-(a)&=\sqrt{\frac{\Omega_a}{2}}\sum_{m_{\alpha_1}m_{\alpha_2}}(j_aj_am_{\alpha_1}m_{\alpha_2}|00)c_{\alpha_2}c_{\alpha_1},\\ \hat{S}_0(a)&=\frac{1}{2}(\sum_{m_\alpha}c^\dagger_\alpha c_\alpha-\Omega_a),\quad \Omega_a=j_a+\frac{1}{2},\end{aligned}\tag{2·1}$$

where $c^\dagger_\alpha$ and $c_\alpha$ are the creation and annihilation operators of a nucleon in the single-particle state $\alpha$. These operators then satisfy the same commutation relations as those of the angular-momentum operators:

$$[\hat{S}_+(a),\ \hat{S}_-(a)]=2\hat{S}_0(a),\quad [\hat{S}_0(a),\ \hat{S}_\pm(a)]=\pm\hat{S}_\pm(a).\tag{2·2}$$

The state vectors are specified by the quantum numbers $S(a)$ and $S_0(a)$, which are the eigenvalues of the quasi-spin $\hat{\boldsymbol{S}}(a)^2=\hat{S}_+(a)\hat{S}_-(a)+\hat{S}_0(a)^2-\hat{S}_0(a)$ and its projection $\hat{S}_0(a)$, respectively. They span the quasi-spin subspace of the orbit $a$:**)

$$\{|S(a),\ S_0(a));\ S(a),\ S_0(a)=-S(a),\ -S(a)+1,\ \cdots,\ S(a)\}.\tag{2·3}$$

The physical meaning of the quantum numbers $S(a)$ and $S_0(a)$ is known to be related simply to the seniority number and the nucleon number, respectively, through the relations

$$S(a)=\frac{1}{2}(\Omega_a-v_a)\quad\text{and}\quad S_0(a)=\frac{1}{2}(\mathcal{N}_a-\Omega_a),\tag{2·4}$$

where $v_a$ and $\mathcal{N}_a$ stand for the seniority number and the nucleon number in the orbit $a$, respectively.

With the quasi-spin operators (2·1) we can define irreducible tensor

*) The subscripts $i=1, 2, 3, \cdots$ of $a$ are used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit is necessary.

**) The quasi-spin space for the general many $j$-shell case is simply expressed as the direct product composed of the quasi-spin subspace of each orbit. Therefore, for simplicity, we discuss the case of a single orbit in this section.

operators in the quasi-spin subspace of the orbit $a$, as usual, by the commutation relations

$$\begin{aligned}&[\hat{S}_0(a), \mathcal{T}_{k\kappa}(a)]=\kappa\mathcal{T}_{k\kappa}(a),\\&[\hat{S}_\pm(a), \mathcal{T}_{k\kappa}(a)]=\sqrt{(k\mp\kappa)(k\pm\kappa+1)}\,\mathcal{T}_{k,\kappa\pm1}(a),\end{aligned}\tag{2·5}$$

where $\mathcal{T}_{k\kappa}(a)$ is the $\kappa$-component of an irreducible tensor of rank $k$, and the indicies $k$ and $\kappa$ are analogous to the quantum numbers $S(a)$ and $S_0(a)$ for the corresponding wavefunction multiplet. The index $\kappa$ takes on $2k+1$ values from $-k$ to $k$. The single-particle operators $c_a^\dagger$ and $c_a$ are then regarded as spinors in the quasi-spin subspace:

$$\mathcal{T}_{1/2,1/2}(\alpha)=c_a^\dagger \quad\text{and}\quad \mathcal{T}_{1/2,-1/2}(\alpha)=c_{\bar{a}}\equiv(-)^{j_a-m_a}c_{\bar{a}}.\tag{2·6}$$

The irreducible tensors can be obtained from the products of the spinors by the standard vector-coupling procedures. For example, we have

$$\begin{aligned}&\mathcal{T}_{1,1}(a_1a_2)=c_{a_1}^\dagger c_{a_2}^\dagger,\\&\mathcal{T}_{1,0}(a_1a_2)=\frac{1}{\sqrt{2}}(c_{a_1}^\dagger c_{\bar{a}_2}+c_{\bar{a}_1}c_{a_2}^\dagger),\\&\mathcal{T}_{1,-1}(a_1a_2)=c_{\bar{a}_1}c_{\bar{a}_2}\end{aligned}\tag{2·7a}$$

and

$$\begin{aligned}&\mathcal{T}_{3/2,3/2}(a_1a_2a_3)=c_{a_1}^\dagger c_{a_2}^\dagger c_{a_3}^\dagger,\\&\mathcal{T}_{3/2,1/2}(a_1a_2a_3)=\frac{1}{\sqrt{3}}(c_{a_1}^\dagger c_{a_2}^\dagger c_{\bar{a}_3}+c_{a_1}^\dagger c_{\bar{a}_2}c_{a_3}^\dagger+c_{\bar{a}_1}c_{a_2}^\dagger c_{a_3}^\dagger),\\&\mathcal{T}_{3/2,-1/2}(a_1a_2a_3)=\frac{1}{\sqrt{3}}(c_{a_1}^\dagger c_{\bar{a}_2}c_{\bar{a}_3}+c_{\bar{a}_1}c_{a_2}^\dagger c_{\bar{a}_3}+c_{\bar{a}_1}c_{\bar{a}_2}c_{a_3}^\dagger),\\&\mathcal{T}_{3/2,-3/2}(a_1a_2a_3)=c_{\bar{a}_1}c_{\bar{a}_2}c_{\bar{a}_3}.\end{aligned}\tag{2·7b}$$

Here it should be noted that there is no interference between the coupling of the quasi-spin and that of the ordinary angular momentum, since $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ commute with the angular-momentum operators $\hat{J}_\pm$ and $\hat{J}_0$.

## 2-2 *Rotation in quasi-spin space*[2)]

The quasi-spin operators $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ are associated with the transformation of state vectors under the rotation of the coordinate system in the quasi-spin subspace of orbit $a$. Let us take up a new coordinate system $K'$ obtained from the original one $K$ (on which the argument has so far been) by a rotation specified in terms of the Euler angle $\omega_a=(\phi_a, \theta_a, \psi_a)$. The transformed state vectors are then given by

$$|S(a), S_0(a)\rangle = R(\omega_a)|S(a), S_0(a)) = \sum_{S_0(a)'}(S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a))\times|S(a), S_0(a)'), \tag{2·8}$$

where $R(\omega_a)$ is the unitary rotation operator in the quasi-spin subspace of orbit $a$.

$$R(\omega_a)=\exp[-i\phi_a\hat{S}_z(a)]\exp[-i\theta_a\hat{S}_y(a)]\exp[-i\psi_a\hat{S}_z(a)], \qquad \hat{S}_z(a)=\hat{S}_0(a), \quad \hat{S}_y(a)=\frac{1}{2i}(\hat{S}_+(a)-\hat{S}_-(a)), \tag{2·9}$$

and the state vector $|\cdots)$ designates one in the original system $K$ while $|\cdots\rangle$ denotes a state vector in the new coordinate system $K'$. It must be remembered that the quantum numbers $S(a)$ and $S_0(a)$ in the state $|S(a), S_0(a)\rangle$ are the eigenvalues of $\hat{\boldsymbol{S}}(a)^2=R(\omega_a)\hat{\boldsymbol{S}}(a)^2R(\omega_a)^{-1}(=\hat{\boldsymbol{S}}(a)^2)$ and $\hat{S}_0(a)=R(\omega_a)\hat{S}_0(a)R(\omega_a)^{-1}$, respectively. Thus the state vectors defined by (2·8) also span the quasi-spin space:

$$\{|S(a), S_0(a)\rangle;\ S(a), |S_0(a)|\leqq S(a)\}. \tag{2·10}$$

The matrix element of $R(\omega_a)$ defines the conventional $D$-function*) in the quasi-spin subspace:

$$D^{S(a)}_{S_0(a)'S_0(a)}(\omega_a)=(S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a))^* =\langle S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a)\rangle^*. \tag{2·11}$$

With the relation (2·11), the relation (2·8) becomes

$$|S(a), S_0(a)\rangle=\sum_{S_0(a)'}D^{S(a)*}_{S_0(a)'S_0(a)}(\omega_a)|S(a), S_0(a)'). \tag{2·12}$$

Since $R(\omega_a)$ is unitary, this can be rewritten as

$$|S(a), S_0(a))=\sum_{S_0(a)'}D^{S(a)}_{S_0(a)S_0(a)'}(\omega_a)|S(a), S_0(a)'\rangle. \tag{2·13}$$

By definition, the irreducible tensors in the new coordinate system $K'$, $T_{k\kappa}(a)$, are related to those in the original system through

$$T_{k\kappa}(a)=R(\omega_a)\mathcal{T}_{k\kappa}(a)R(\omega_a)^{-1}=\sum_{\kappa'}D^{k*}_{\kappa'\kappa}(\omega_a)\mathcal{T}_{k\kappa'}(a), \qquad \mathcal{T}_{k\kappa}(a)=\sum_{\kappa'}D^{k}_{\kappa\kappa'}(\omega_a)T_{k\kappa'}(a). \tag{2·14}$$

Now let us take up a new coordinate system $K'_{\omega_0}$ specified by the Euler angle $\omega_0\equiv(\phi_a=0,\ -\theta_a,\ \psi_a=0)$. According to Eq. (2·14), we have the quasi-spin spinors $T_{1/2,\kappa}(a)$ in the $K'_{\omega_0}$-system

*) We use a definition of the $D$-function which is adopted by Bohr and Mottelson. Therefore the $D$-function adopted here is the complex conjugate of that of Rose and differs from that employed by Edmonds by the factor $(-)^{S_0(a)-S_0(a)'}$.

$$\begin{bmatrix} T_{1/2,\,1/2}(\alpha) \\ T_{1/2,\,-1/2}(\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\theta_a/2) & -\sin(\theta_a/2) \\ \sin(\theta_a/2) & \cos(\theta_a/2) \end{bmatrix} \begin{bmatrix} \mathcal{T}_{1/2,\,1/2}(\alpha) \\ \mathcal{T}_{1/2,\,-1/2}(\alpha) \end{bmatrix}. \tag{2·15}$$

With the definition

$$T_{1/2,\,1/2}(\alpha)=a_a^\dagger \quad \text{and} \quad T_{1/2,\,-1/2}(\alpha)=a_{\tilde{a}}, \tag{2·16}$$

Eq. (2·15) can be rewritten as

$$\begin{aligned} &a_a^\dagger=u_a c_a^\dagger - v_a c_{\tilde{a}}, \qquad a_a=u_a c_a - v_a c_{\tilde{a}}^\dagger, \\ &u_a=\cos(\theta_a/2), \qquad v_a=\sin(\theta_a/2). \end{aligned} \tag{2·17}$$

This is nothing but the Bogoliubov transformation. We can therefore say that the Bogoliubov transformation merely corresponds to a special rotation $\omega_0$ of the coordinate system in the quasi-spin space.

In this new coordinate system $K'_{\omega_0}$, i.e., in the quasi-particle representation, the quasi-spin operators are given as

$$\begin{aligned} \hat{S}_+(a) &= R(\omega_0)\hat{\mathcal{S}}_+(a)R(\omega_0)^{-1} \\ &= \sqrt{\frac{\Omega_a}{2}}\,\Sigma_{m_{a_1}m_{a_2}}(j_a j_a m_{a_1} m_{a_2}|00)a_{a_1}^\dagger a_{a_2}^\dagger, \\ \hat{S}_-(a) &= \sqrt{\frac{\Omega_a}{2}}\,\Sigma_{m_{a_1}m_{a_2}}(j_a j_a m_{a_1} m_{a_2}|00)a_{a_2}a_{a_1}, \\ \hat{S}_0(a) &= \frac{1}{2}(\Sigma_{m_a}a_a^\dagger a_a - \Omega_a). \end{aligned} \tag{2·18}$$

Since $\hat{\boldsymbol{S}}(a)^2=R(\omega_0)\hat{\boldsymbol{\mathcal{S}}}(a)^2R(\omega_0)^{-1}=\hat{\boldsymbol{\mathcal{S}}}(a)^2$, the quasi-spin quantum number $S(a)$ of the state $|S(a), S_0(a)\rangle$ in the quasi-particle representation $K'_{\omega_0}$ has the same physical meaning as that in the original system:

$$S(a)=\frac{1}{2}(\Omega_a - v_a). \tag{2·19}$$

On the other hand, from relation (2·18) the physical meaning of the quantum number $S_0(a)$ is now related to the number of quasi-particles $n_a$ in the orbit $a$:

$$S_0(a)=\frac{1}{2}(n_a-\Omega_a). \tag{2·20}$$

Needless to say, the BCS ground state $|\phi_0\rangle$ (in the orbit $a$) is given by

$$|\phi_0\rangle = |S(a)=\Omega_a/2,\ S_0(a)=-S(a)\rangle. \tag{2·21}$$

## 2-3 *Definition of collective and intrinsic states*

By the definition of the state $|S(a), S_0(a)=-S(a)\rangle$ of the orbit $a$, we obtain

$$\hat{S}_{-}(a)|S(a),\ S_0(a)=-S(a)\rangle=0, \tag{2·22}$$

which means that there is no $J=0$-coupled quasi-particle pair in this state. In this case, with the aid of Eqs. (2·19) and (2·20), the following relation is obtained:

$$\frac{1}{2}(n_a-\Omega_a)=-\frac{1}{2}(\Omega_a-v_a),\ \text{i.e.,}\ n_a=v_a. \tag{2·23}$$

Thus, for a class of states $|\phi_{\text{intr}}\rangle$ which consists of the direct product of the states satisfying Eq. (2·22), we have

$$\hat{S}_{-}(a)|\phi_{\text{intr}}\rangle=0 \tag{2·24}$$

for all $\hat{S}_{-}(a)$, so that the following well-known statement is satisfied: The use of the quasi-particle basis can be regarded as an attempt to characterize both the ground state and the excited states in terms of the seniority number $v\equiv\sum_a v_a$ in such a way that the number of quasi-particles $n\equiv\sum_a n_a$ is equivalent to the seniority $v$.

The condition (2·24) means that the states $|\phi_{\text{intr}}\rangle$ never contain any $J=0$-coupled quasi-particle pair. In this sense, we call $|\phi_{\text{intr}}\rangle$ "intrinsic states". On the other hand, the states characterized by $v_a=0$ and $n_a\neq 0$ include only $J=0$-coupled quasi-particle pairs and are always orthogonal to the intrinsic states $|\phi_{\text{intr}}\rangle$. Hence we call such a class of states "collective states" $|\phi_{\text{col}}\rangle$. Needless to say, all spurious components due to the nucleon-number non-conservation belong to the "collective states", and a special one of "collective" vibrations (under the RPA) with zero energy in this "collective subspace" is known as due to the nucleon-number non-conservation.

## §3. The Hamiltonian

The Hamiltonian under consideration is that of a spherically symmetric $j$-$j$ coupling shell model with a general two-body interaction which is invariant under rotation, reflection and time reversal:

$$H=\sum_a(\epsilon_a-\lambda_a)c_a^{\dagger}c_a+\sum_{\alpha\beta\gamma\delta}{}^{C}V_{\alpha\beta\gamma\delta}c_\alpha^{\dagger}c_\beta^{\dagger}c_\delta c_\gamma, \tag{3·1}$$

where $\epsilon_a$ and $\lambda_a$ represent the single-particle energy and the chemical potential, respectively. The matrix elements of the interaction satisfy the relations*)

$$\begin{aligned}{}^{C}V_{\alpha\beta\gamma\delta}&=-{}^{C}V_{\beta\alpha\gamma\delta}=-{}^{C}V_{\alpha\beta\delta\gamma}={}^{C}V_{\beta\alpha\delta\gamma}={}^{C}V_{\gamma\delta\alpha\beta}\\&={}^{C}V_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}}.\end{aligned} \tag{3·2}$$

*) It is possible to treat all matrix elements of the Hamiltonian as real quantities if the phase convention is suitably chosen. In this paper we always assume this to be the case.

After the Bogoliubov transformation (2·17) and with the use of notations*),[3]

$$\begin{aligned}
&\Delta_a = -2\sum_\gamma u_c v_c {}^C V_{a\bar{a}\gamma\bar{\gamma}}, \\
&\mu_a = -4\sum_\gamma v_c^2 {}^C V_{a\gamma a\gamma}, \\
&\eta_a = \epsilon_a - \lambda_a - \mu_a,
\end{aligned} \tag{3·3}$$

the Hamiltonian is expressed in terms of the quasi-particle operators as follows:

$$\begin{aligned}
H &= U_0 + H_0 + H_1 + H_{\text{int}}, \qquad (3·4)\\
U_0 &= \sum_a \left[ \left( \eta_a + \frac{1}{2}\mu_a \right) v_a^2 - \frac{1}{2} u_a v_a \Delta_a \right], \\
H_0 &= \sum_a [\eta_a (u_a^2 - v_a^2) + 2 u_a v_a \Delta_a] a_a^\dagger a_a, \\
H_1 &= \sum_a \left[ \eta_a u_a v_a - \frac{1}{2}(u_a^2 - v_a^2)\Delta_a \right] (a_a^\dagger a_{\bar{a}}^\dagger + a_{\bar{a}} a_a), \\
H_{\text{int}} &= H_X + H_V + H_Y, \\
H_X &= \sum_{\alpha\beta\gamma\delta} V_X(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma, \\
H_V &= \sum_{\alpha\beta\gamma\delta} V_V(\alpha\beta\gamma\delta) (a_\alpha^\dagger a_\beta^\dagger a_{\bar{\delta}}^\dagger a_{\bar{\gamma}}^\dagger + a_{\bar{\gamma}} a_{\bar{\delta}} a_\beta a_\alpha), \\
H_Y &= \sum_{\alpha\beta\gamma\delta} V_Y(\alpha\beta\gamma\delta) (a_\alpha^\dagger a_\beta^\dagger a_{\bar{\delta}}^\dagger a_\gamma + a_\gamma^\dagger a_{\bar{\delta}} a_\beta a_\alpha),
\end{aligned}$$

where the abbreviations

$$\begin{aligned}
V_X(\alpha\beta\gamma\delta) &= V_X^{(G)}(\alpha\beta\gamma\delta) + V_X^{(F)}(\alpha\beta\gamma\delta) - V_X^{(F)}(\beta\alpha\gamma\delta) \\
&= (u_a u_b u_c u_{\bar{d}} + v_a v_b v_c v_{\bar{d}}) {}^C V_{a\beta\gamma\delta} + (u_a v_b u_c v_{\bar{d}} + v_a u_b v_c u_{\bar{d}}) {}^C V_{a\bar{\delta}\bar{\beta}\gamma} \\
&\quad - (u_b v_a u_c v_{\bar{d}} + v_b u_a v_c u_{\bar{d}}) {}^C V_{\beta\bar{\delta}\bar{a}\gamma}, \\
V_V(\alpha\beta\gamma\delta) &= u_a u_b v_c v_{\bar{d}} {}^C V_{a\beta\gamma\bar{\delta}}, \\
V_Y(\alpha\beta\gamma\delta) &= 2(u_a u_b u_c v_{\bar{d}} - v_a v_b v_c u_{\bar{d}}) {}^C V_{a\beta\gamma\delta}
\end{aligned}$$

have been used.

The parameters $u$, $v$ and the chemical potentials $\lambda$ are determined as usual by the set of equations

$$\begin{aligned}
&2u_a v_a = \Delta_a / E_a, \qquad u_a^2 - v_a^2 = \eta_a / E_a, \\
&E_a = \sqrt{\eta_a^2 + \Delta_a^2}, \qquad (3·5)\\
&N_n = 2\sum_a \Omega_a v_a^2 = \sum_a \Omega_a (1 - \eta_a / E_a),
\end{aligned}$$

where $N_n$ is the neutron (proton) number and the summation $a$ runs over the neutron (proton) orbits.

*) We assume that, among the single-particle orbits $a$, $b$, ···, a given set of the quantum numbers {charge $q$, parity and $j$-value} occurs at once.

Then $H_0$ takes the form

$$H_0 = \sum_a E_a a_a^\dagger a_a, \tag{3·6}$$

where $E_a$ denotes the quasi-particle energy.

The main part of the pairing correlations is taken into account as the (self-consistent) quasi-particle field. The eigenstates of $H_0$ are given by those of the quasi-spin $\hat{\boldsymbol{S}}(a)^2$ and its projection $\hat{S}_0(a)$, with additional quantum numbers. The term $H_{\text{int}}$ represents the residual interaction among quasi-particles. The role of residual interaction can be classified into three types: The first is the role of mixing among the states in the intrinsic subspace $\{|\phi_{\text{intr}}\rangle\}$, the second among the states in the collective subspace; and the last between collective and intrinsic states. These roles may be expressed as follows:

$$H = H_{\text{col}} + H_{\text{intr}} + H_{\text{coupl}}, \tag{3·7}$$

where $H_{\text{col}}$, $H_{\text{intr}}$ and $H_{\text{coupl}}$ stand for the collective-, intrinsic-, and coupling-Hamiltonians, respectively.

We show, in the remaining part of this chapter, that the original Hamiltonian (3·4) can be transformed unambiguously into the form (3·7).

## §4. Collective variables associated with pairing correlations

### 4-1 *Extension of quasi-spin space and introduction of collective variables*

The procedure of adding pairs of fermion must eventually end if the number of states available is finite, whereas there is nothing to prevent operating again and again on a state with a boson creation operator. In order to define the canonical-conjugate collective variables describing the pairing excitations in terms of boson operators, it is thus necessary to extend the quasi-spin space in such a way that the multiplet in the quasi-spin space (2·10) becomes infinite with allowed values of $S_0(a)$ going to steps of unity from $S(a)+1$ to $+\infty$:

$$\{|S(a), S_0(a)\rangle;\ S(a),\ -S(a) \leqq S_0(a) < +\infty\}. \tag{4·1}$$

With the aid of the extended quasi-spin space we introduce boson operators $b_a^\dagger$ and $b_a$, which satisfy the commutation relations

$$[b_a, b_c^\dagger] = \delta_{ac}, \qquad [b_a, b_c] = [b_a^\dagger, b_c^\dagger] = 0 \tag{4·2}$$

and characterize the state vectors in the extended quasi-spin space by

$$|S(a), S_0(a) = -S(a) + N_a\rangle = \frac{1}{\sqrt{N_a!}} (b_a^\dagger)^{N_a} |S(a), S_0(a) = -S(a)\rangle,$$
$$b_a |S(a), S_0(a) = -S(a)\rangle = 0. \tag{4·3}$$

Then we have

$$\begin{aligned} b_a^\dagger | S(a), S_0(a) = -S(a)+N_a\rangle \\ &= \sqrt{N_a+1}\,| S(a), S_0(a) = -S(a)+N_a+1\rangle, \\ b_a | S(a), S_0(a) = -S(a)+N_a\rangle \\ &= \sqrt{N_a}\,| S(a), S_0(a) = -S(a)+N_a-1\rangle. \end{aligned} \tag{4·4}$$

Explicitly, such an introduction of the boson operators is made in terms of the Holstein-Primakoff transformation[4]

$$\begin{aligned} \mathring{S}_+(a) &= b_a^\dagger \sqrt{2\mathring{S}(a)-\mathring{N}(a)}, \\ \mathring{S}_-(a) &= \sqrt{2\mathring{S}(a)-\mathring{N}(a)}\, b_a, \\ \mathring{S}_0(a) &= \mathring{N}(a)-\mathring{S}(a), \end{aligned} \tag{4·5}$$

where the boson number operator $\mathring{N}(a)$ of the orbit $a$ and the operator $\mathring{S}(a)$ are defined respectively by

$$\mathring{N}(a) = b_a^\dagger b_a, \tag{4·6}$$

$$\mathring{S}(a)\{\mathring{S}(a)+1\} = \mathring{\boldsymbol{S}}(a)^2. \tag{4·7}$$

The operators $\mathring{S}_\pm(a)$, $\mathring{S}_0(a)$ and $\mathring{\boldsymbol{S}}(a)^2$ denote the extensions of the quasi-spin operators $\hat{S}_\pm(a)$, $\hat{S}_0(a)$ and $\hat{\boldsymbol{S}}(a)^2$ into the extended quasi-spin space: In a "physical subspace" which coincides with the quasi-spin space, these extended quasi-spin operators are identical with the original quasi-spin operators. In a "unphysical subspace" which corresponds to the extended part (in the extended quasi-spin space), the operators $\mathring{S}_x(a) = \{\mathring{S}_+(a)+\mathring{S}_-(a)\}/2$ and $\mathring{S}_y(a) = \{\mathring{S}_+(a)-\mathring{S}_-(a)\}/2i$ become anti-hermitian. We may write the extended quasi-spin operators as

$$\begin{aligned} &\mathring{S}_x(a) = i\hat{T}_x(a), \quad \mathring{S}_y(a) = i\hat{T}_y(a), \quad \mathring{S}_0(a) = \hat{T}_0(a), \\ &\hat{T}_+(a) \equiv \hat{T}_x(a)+i\hat{T}_y(a) = b_a^\dagger \sqrt{\mathring{N}(a)-2\mathring{S}(a)}, \\ &\hat{T}_-(a) \equiv \hat{T}_x(a)-i\hat{T}_y(a) = \sqrt{\mathring{N}(a)-2\mathring{S}(a)}\, b_a, \end{aligned} \tag{4·8}$$

so that we have

$$\begin{aligned} \mathring{\boldsymbol{S}}(a)^2 &= \mathring{S}_x(a)^2+\mathring{S}_y(a)^2+\mathring{S}_0(a)^2 \\ &= \hat{T}_0(a)^2-\hat{T}_x(a)^2-\hat{T}_y(a)^2. \end{aligned} \tag{4·9}$$

The algebra of $\hat{T}_x(a)$, $\hat{T}_y(a)$ and $\hat{T}_0(a)$,

$$\begin{aligned} &[\hat{T}_x(a), \hat{T}_y(a)] = -i\hat{T}_0(a), \quad [\hat{T}_y(a), \hat{T}_0(a)] = i\hat{T}_x(a), \\ &[\hat{T}_0(a), \hat{T}_x(a)] = i\hat{T}_y(a), \end{aligned} \tag{4·10}$$

characterizes transformation of the noncompact group with allowed values of $S_0(a)$ from $\{S(a)+1\}$ to $+\infty$: By definitions (4·8) and (4·5) we obtain in the unphysical subspace

$$\hat{T}_{\pm}(a)|S(a),\ S_0(a)\rangle=\sqrt{S_0(a)\{S_0(a)\pm1\}-S(a)\{S(a)+1\}}\,|S(a),\ S_0(a)\pm1\rangle, \tag{4·11}$$

which is consistent with the property $S_0(a)^2\geqq S(a)\{S(a)+1\}$ derived from Eq. (4·9). Then the condition

$$\hat{T}_{-}(a)|S(a),\ S_0(a)_{\mathrm{min}}\rangle=0 \tag{4·12}$$

leads to the relation

$$S_0(a)_{\mathrm{min}}\{S_0(a)_{\mathrm{min}}-1\}=S(a)\{S(a)+1\}, \tag{4·13}$$

which means $S_0(a)_{\mathrm{min}}=S(a)+1$.

The last relation in Eq. (4·5) in the physical space is written as

$$\hat{N}(a)=b_a^{\dagger}b_a=\hat{S}(a)+\hat{S}_0(a), \tag{4·14}$$

which has the eigenvalues

$$N_a=S(a)+S_0(a)=\frac{1}{2}(n_a-v_a). \tag{4·15}$$

This means that the boson number $N_a$ of the orbit $a$ is merely the number of '$J=0$' coupled quasi-particle pairs in the orbit $a$. Therefore, the intrinsic states $|\phi_{\mathrm{intr}}\rangle$, which consist of $|S(a),\ S_0(a)=-S(a)\rangle$ and are defined by Eq. (2·24) always satisfy

$$\mathring{N}(a)|\phi_{\mathrm{intr}}\rangle=0, \tag{4·16}$$

which is consistent with Eq. (4·3). We may thus say that the intrinsic states are not affected at all by the extension of the quasi-spin space and always belong to the physical subspace. This is in marked contrast with the collective states $|\phi_{\mathrm{col}}\rangle$ in which the boson operators play an essential role.

### 4-2 *Canonical coordinates and canonical conjugate momenta*

With the aid of the boson operators $b_a^{\dagger}$ and $b_a$, we can define collective coordinates $\mathring{q}_a$ and their canonical conjugate momenta $\mathring{p}_a$ describing the pairing excitations by

$$\mathring{q}_a=\frac{i}{\sqrt{2}}(b_a-b_a^{\dagger}),\qquad \mathring{p}_a=\frac{1}{\sqrt{2}}(b_a+b_a^{\dagger}). \tag{4·17}$$

Of course, these operators satisfy the canonical commutation relations

$$[\mathring{q}_a,\mathring{p}_b]=i\delta_{ab},\qquad [\mathring{q}_a,\mathring{q}_b]=[\mathring{p}_a,\mathring{p}_b]=0. \tag{4·18}$$

## §5. Canonical transformation into collective representation

### 5-1 *Introduction of auxiliary variables and supplementary condition*

We now apply the canonical transformation method with auxiliary variables[5),6)] to the system under consideration.

First, we introduce redundant canonical variables (i.e., auxiliary variables) $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$, which satisfy the canonical commutation relations

$$[\boldsymbol{q}_a, \boldsymbol{p}_b]=i\delta_{ab}, \qquad [\boldsymbol{q}_a, \boldsymbol{q}_b]=[\boldsymbol{p}_a, \boldsymbol{p}_b]=0, \tag{5·1}$$

and are independent of the quasi-particle operators $(a_a^\dagger, a_a)$ and the boson operators $(b_a^\dagger, b_a)$:

$$\begin{aligned}&[\boldsymbol{q}_a, a_a^\dagger]=[\boldsymbol{q}_a, a_a]=[\boldsymbol{q}_a, b_a^\dagger]=[\boldsymbol{q}_a, b_a]=0,\\ &[\boldsymbol{p}_a, a_a^\dagger]=[\boldsymbol{p}_a, a_a]=[\boldsymbol{p}_a, b_a^\dagger]=[\boldsymbol{p}_a, b_a]=0.\end{aligned} \tag{5·2}$$

With the redundant variables we may define redundant bosons as

$$\boldsymbol{b}_a^\dagger=\frac{1}{\sqrt{2}}(\boldsymbol{p}_a+i\boldsymbol{q}_a), \qquad \boldsymbol{b}_a=\frac{1}{\sqrt{2}}(\boldsymbol{p}_a-i\boldsymbol{q}_a). \tag{5·3}$$

In order to compensate for the over-completeness in the degrees of freedom due to the introduction of the auxiliary variables, we impose on the state vectors a supplementary condition

$$\hat{\boldsymbol{N}}(a)|\Psi\rangle=0, \tag{5·4}$$

$$\hat{\boldsymbol{N}}(a)\equiv\boldsymbol{b}_a^\dagger\boldsymbol{b}_a, \tag{5·5}$$

which physically implies that we are only considering the subspace with no auxiliary bosons. Since the original Hamiltonian $H$ is independent of the auxiliary variables introduced, i.e.,

$$[H, \hat{\boldsymbol{N}}(a)]=0, \tag{5·6}$$

the Schrödinger equation

$$H|\Psi\rangle=E|\Psi\rangle, \tag{5·7}$$

with the supplementary condition (5·4) is exactly equivalent to the starting Schrödinger equation without the auxiliary variables.

### 5-2 *Canonical transformation*

Now, let us define the following canonical transformation:

$$\begin{aligned}&U_{\rm col}=U_1\cdot U_2\cdot U_1,\\ &U_1=\exp[i\textstyle\sum_a \mathring{p}_a\boldsymbol{q}_a], \qquad U_2=\exp[-i\sum_a \mathring{q}_a\boldsymbol{p}_a],\end{aligned} \tag{5·8}$$

where the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ are given by Eq. (4·17). The following relations are then easily derived:

$$\begin{aligned} &U_{\mathrm{col}}\mathring{q}_a U_{\mathrm{col}}^{-1}=\boldsymbol{q}_a, \qquad U_{\mathrm{col}}\mathring{p}_a U_{\mathrm{col}}^{-1}=\boldsymbol{p}_a, \\ &U_{\mathrm{col}}\boldsymbol{q}_a U_{\mathrm{col}}^{-1}=-\mathring{q}_a, \qquad U_{\mathrm{col}}\boldsymbol{p}_a U_{\mathrm{col}}^{-1}=-\mathring{p}_a \end{aligned} \tag{5·9}$$

and thus we have

$$U_{\mathrm{col}}\hat{\boldsymbol{N}}(a)U_{\mathrm{col}}^{-1}=b_a^\dagger b_a. \tag{5·10}$$

This implies that, in the representation after the canonical transformation which we call "collective representation", the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ are completely replaced by the redundant variables $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$, respectively.

The Schrödinger equation in the collective representation is obtained from Eq. (5·7) with the condition (5·4), by regarding both the Hamiltonian $H$ and the state vectors $|\Psi\rangle$ (defined on the physical subspace) as their extensions $\mathring{H}$ and $|\mathring{\Psi}\rangle$ into the extended quasi-spin space:*) It becomes

$$\boldsymbol{H}|\Psi\rangle\!\rangle=E|\Psi\rangle\!\rangle \tag{5·11}$$

with the supplementary condition

$$\mathring{N}(a)|\Psi\rangle\!\rangle=0, \qquad \mathring{N}(a)=b_a^\dagger b_a=\mathring{S}(a)+\mathring{S}_0(a), \tag{5·12}$$

where

$$\boldsymbol{H}=U_{\mathrm{col}}\mathring{H}U_{\mathrm{col}}^{-1}, \qquad |\Psi\rangle\!\rangle=U_{\mathrm{col}}|\mathring{\Psi}\rangle. \tag{5·13}$$

5-3 *Collective representation*

Since the original Hamiltonian $\mathring{H}$ is independent of the auxiliary variables, we have

$$[\mathring{H}, \boldsymbol{q}_a]=[\mathring{H}, \boldsymbol{p}_a]=0, \tag{5·14}$$

which is transformed into the collective representation as

$$[\boldsymbol{H}, \mathring{q}_a]=[\boldsymbol{H}, \mathring{p}_a]=0. \tag{5·15}$$

This implies that, in the collective representation, the collective variables $(\mathring{q}_a, \mathring{p}_a)$ involved implicitly in the original Hamiltonian $\mathring{H}$ are completely replaced by the auxiliary variables $(\boldsymbol{q}_a, \boldsymbol{p}_a)$, and the collective modes of the system are visualized by the explicit appearance of the auxiliary variables in the Hamiltonian $\boldsymbol{H}$. By comparing the supplementary condition (5·12) with

*) Knowledge of the explicit properties of $\mathring{H}$ in the unphysical subspace is actually not necessary at all in our discussions, provided that the commutation properties of $\mathring{H}$ with the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ in the unphysical subspace are the same as those in the physical space.

Eq. (4·16), we can furthermore see that in this representation, the degrees of freedom associated with the quasi-particle operators merely describe the intrinsic motion of the system. Thus, the Hilbert space in this representation may be characterized as the direct product of a boson space (which is associated with the auxiliary bosons $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$) and the intrinsic space composed of the intrinsic states $|\phi_{\text{intr}}\rangle$ (which are defined by Eq. (2·24) and always belong to the physical subspace): The basis vectors (of the orbit $a$) can be represented as

$$|S(a), N_a\rangle\!\rangle \equiv |N_a\rangle_{\text{col}} \cdot |S(a), S_0(a) = -S(a)\rangle, \tag{5·16}$$

where $|N_a\rangle_{\text{col}}$ denotes the collective state vector associated with the boson operators $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$:

$$|N_a\rangle_{\text{col}} \equiv \frac{1}{\sqrt{N_a!}} (\boldsymbol{b}_a^\dagger)^{N_a} |0\rangle_B, \tag{5·17}$$

where $|0\rangle_B$ is the vacuum of the boson, i.e., $\boldsymbol{b}_a|0\rangle_B = 0$, and the states $|S(a), S_0(a) = -S(a)\rangle$ compose the intrinsic states $|\phi_{\text{intr}}\rangle$. It should be noted that, in the collective representation, all unphysical effects as a result of the extension of the quasi-spin space arise only in the collective boson space (associated with $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$) and the intrinsic states remain unchanged in the physical subspace.

## §6. Collective representation of the Hamiltonian

### 6-1 *Perturbative expansion of the Hamiltonian in terms of collective variables*

The collective representation of the Hamiltonian cannot be expected to take a simple and compact form. We here adopt a perturbative expansion in terms of the collective variables. In this expansion, we choose the collective variables $\boldsymbol{X}_\mu^\dagger$ and $\boldsymbol{X}_\mu$ as the basis of the expansion, which are the eigenmode creation and annihilation operators of the pairing vibration under the RPA:

$$\begin{aligned} \boldsymbol{X}_\mu^\dagger &= \sum_a \{\psi_\mu(a)\boldsymbol{b}_a^\dagger + \phi_\mu(a)\boldsymbol{b}_a\}, \\ \boldsymbol{X}_\mu &= \sum_a \{\psi_\mu(a)\boldsymbol{b}_a + \phi_\mu(a)\boldsymbol{b}_a^\dagger\}. \end{aligned} \tag{6·1}$$

It is well known that such pairing vibrations include a special zero-energy solution, which implies that the RPA includes enough of the residual (pairing) interaction to restore the breaking of the nucleon-number conservation by the BCS approximation. Their definitions and the details of the zero-energy solution are given in Appendix 1B.

Thus the auxiliary variables $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$ in the canonical transformation (5·8) are regarded as the functions of these basis operators $\boldsymbol{X}_\mu^\dagger$ and $\boldsymbol{X}_\mu$:

$$\begin{aligned}\boldsymbol{q}_a&=\frac{i}{\sqrt{2}}(\boldsymbol{b}_a-\boldsymbol{b}_a^\dagger)=\frac{i}{\sqrt{2}}\Sigma_\mu\{\psi_\mu(a)+\phi_\mu(a)\}(\boldsymbol{X}_\mu-\boldsymbol{X}_\mu^\dagger),\\ \boldsymbol{p}_a&=\frac{1}{\sqrt{2}}(\boldsymbol{b}_a+\boldsymbol{b}_a^\dagger)=\frac{1}{\sqrt{2}}\Sigma_\mu\{\psi_\mu(a)-\phi_\mu(a)\}(\boldsymbol{X}_\mu^\dagger+\boldsymbol{X}_\mu).\end{aligned}\tag{6·2}$$

By the use of the orthonormality relation

$$\Sigma_a\{\psi_\nu(a)-\phi_\nu(a)\}\{\psi_\mu(a)+\phi_\mu(a)\}=\delta_{\mu\nu},\tag{6·3}$$

the canonical transformation (5·8) is rewritten as

$$\begin{aligned}U_1&=\exp[i\Sigma_a\mathring{p}_a\boldsymbol{q}_a]=\exp[i\Sigma_\mu\mathring{P}_\mu\boldsymbol{Q}_\mu],\\ U_2&=\exp[-i\Sigma_a\mathring{q}_a\boldsymbol{p}_a]=\exp[-i\Sigma_\mu\mathring{Q}_\mu\boldsymbol{P}_\mu],\end{aligned}\tag{6·4}$$

where

$$\begin{aligned}&\mathring{Q}_\mu=\frac{i}{\sqrt{2}}(X_\mu-X_\mu^\dagger),\qquad \mathring{P}_\mu=\frac{1}{\sqrt{2}}(X_\mu+X_\mu^\dagger),\\ &X_\mu^\dagger\equiv\Sigma_a\{\psi_\mu(a)b_a^\dagger+\phi_\mu(a)b_a\},\qquad X_\mu\equiv\Sigma_a\{\psi_\mu(a)b_a+\phi_\mu(a)b_a^\dagger\}.\end{aligned}\tag{6·5}$$

Then, with the aid of the well-known formula

$$\exp(iT)\cdot O\cdot\exp(-iT)=T+i[T,O]-\frac{1}{2}[T,[T,O]]+\cdots,\tag{6·6}$$

we obtain a perturbative expansion of the Hamiltonian in the collective representation in terms of the pairing-vibration modes $(\boldsymbol{X}_\mu^\dagger,\ \boldsymbol{X}_\mu)$:

$$\begin{aligned}\boldsymbol{H}&=U_{\mathrm{col}}\mathring{H}U_{\mathrm{col}}^{-1}\\ &=\hat{h}_{00}+\Sigma_\mu\{\boldsymbol{X}_\mu^\dagger\hat{h}_{10}(\mu)+\boldsymbol{X}_\mu\hat{h}_{01}(\mu)\}\\ &\quad+\frac{1}{2}\Sigma_{\mu\nu}\{\boldsymbol{X}_\mu^\dagger\boldsymbol{X}_\nu^\dagger\hat{h}_{20}(\mu\nu)+\boldsymbol{X}_\nu\boldsymbol{X}_\mu\hat{h}_{02}(\mu\nu)+2\boldsymbol{X}_\mu^\dagger\boldsymbol{X}_\nu\hat{h}_{11}(\mu\nu)\}+\cdots,\end{aligned}\tag{6·7}$$

which is written in a form of the normal ordered product with respect to the creation and annihilation operators $(\boldsymbol{X}_\mu^\dagger,\ \boldsymbol{X}_\mu)$. The operators $h_{ij}$ in Eq. (6·7) are given explicitly as

$$\begin{aligned}\hat{h}_{00}&=\mathring{H}-\Sigma_\mu\{X_\mu^\dagger[X_\mu,\mathring{H}]+[\mathring{H},X_\mu^\dagger]X_\mu\}\\ &\quad+\frac{1}{2}\Sigma_{\mu\nu}\{X_\mu^\dagger X_\nu^\dagger[X_\nu,[X_\mu,\mathring{H}]]+[[\mathring{H},X_\mu^\dagger],X_\nu^\dagger]X_\nu X_\mu\\ &\qquad\qquad+2X_\mu^\dagger[[X_\mu,\mathring{H}],X_\nu^\dagger]X_\nu\}+\cdots,\\ \hat{h}_{10}(\mu)&=\hat{h}_{01}(\mu)^\dagger\\ &=[X_\mu,\mathring{H}]-\Sigma_\nu\{X_\nu^\dagger[X_\nu,[X_\mu,\mathring{H}]]+[[X_\mu,\mathring{H}],X_\nu^\dagger]X_\nu\}+\cdots,\\ \hat{h}_{20}(\mu\nu)&=\hat{h}_{02}(\mu\nu)^\dagger\\ &=[X_\mu,[X_\nu,\mathring{H}]]+\cdots,\\ \hat{h}_{11}(\mu\nu)&=[[X_\mu,\mathring{H}],X_\nu^\dagger]+\cdots.\end{aligned}\tag{6·8}$$

Here we write down explicitly only the terms in $\hat{h}_{ij}$ which include single and double commutators of $\mathring{H}$.

Since $[\boldsymbol{H}, b_a^\dagger]=[\boldsymbol{H}, b_a]=0$ (from Eq. (5·15)), we obtain

$$[\hat{h}_{ij}, b_a^\dagger]=[\hat{h}_{ij}, b_a]=0, \tag{6·9}$$

which means that the operators $\hat{h}_{ij}$ only involve the intrinsic degrees of freedom represented in terms of the quasi-particle basis. Thus, $\hat{h}_{00}$ may be regarded as the intrinsic Hamiltonian, and its eigenstates are always made to satisfy the supplementary condition:

$$b_a|\phi_{\text{intr}}\rangle=0, \quad \text{i.e.,} \quad \hat{S}_-(a)|\phi_{\text{intr}}\rangle=0. \tag{6·10}$$

## 6-2 *Effective Hamiltonian*

Now, let us recall the well-known relation for the pairing-vibration modes:

$$[\mathring{H}, X_\mu^\dagger]=\omega_\mu X_\mu^\dagger+Z_\mu, \tag{6·11}$$

where $Z$ means "interaction" which is neglected under the RPA. With the aid of Eq. (6·11), $\hat{h}_{ij}$ in Eq. (6·8) may be rewritten as

$$\begin{aligned}
&\hat{h}_{00}=\hat{h}_{00}^{(0)}+\hat{h}_{00}^{(1)}+\hat{h}_{00}^{(2)},\\
&\quad \hat{h}_{00}^{(0)}\equiv\mathring{H}-\textstyle\sum_\mu \omega_\mu X_\mu^\dagger X_\mu,\\
&\quad \hat{h}_{00}^{(1)}\equiv-\textstyle\sum_\mu\{X_\mu^\dagger Z_\mu^\dagger+Z_\mu X_\mu\},\\
&\quad \hat{h}_{00}^{(2)}\equiv\frac{1}{2}\textstyle\sum_{\mu\nu}\{X_\mu^\dagger X_\nu^\dagger[X_\nu, Z_\mu^\dagger]+[Z_\mu, X_\nu^\dagger]X_\nu X_\mu+2X_\mu^\dagger[Z_\mu^\dagger, X_\nu^\dagger]X_\nu\}+\cdots,\\
&\hat{h}_{10}(\mu)=\hat{h}_{10}^{(1)}(\mu)+\hat{h}_{10}^{(2)}(\mu),\\
&\quad \hat{h}_{10}^{(1)}(\mu)\equiv Z_\mu^\dagger,\\
&\quad \hat{h}_{10}^{(2)}(\mu)\equiv-\textstyle\sum_\nu\{X_\nu^\dagger[X_\nu, Z_\mu^\dagger]+[Z_\mu^\dagger, X_\nu^\dagger]X_\nu\}+\cdots,\\
&\hat{h}_{11}(\mu\nu)=\hat{h}_{11}^{(0)}(\mu\nu)+\hat{h}_{11}^{(2)}(\mu\nu),\\
&\quad \hat{h}_{11}^{(0)}(\mu\nu)\equiv\omega_\mu\delta_{\mu\nu}, \quad \hat{h}_{11}^{(2)}(\mu\nu)\equiv[Z_\mu^\dagger, X_\nu^\dagger]+\cdots,\\
&\hat{h}_{20}(\mu\nu)=\hat{h}_{20}^{(2)}(\mu\nu)\equiv[X_\nu, Z_\mu^\dagger]+\cdots.
\end{aligned}$$

At this stage, we recall that the space in which all the intrinsic operators $\hat{h}_{ij}$ act must be the intrinsic space, which obeys the supplementary condition (6·10) and consists of the state vectors $|S(a), S_0(a)=-S(a)\rangle$. Therefore, provided that the supplementary condition (6·10) is always kept to be satisfied properly, we may drop all terms in $\hat{h}_{ij}$ which *explicitly* have either the operator $b^\dagger$ on the leftmost side or the operator $b$ on the rightmost side. For instance, we may make such reductions of $\hat{h}_{00}^{(0)}$ and $\hat{h}_{00}^{(1)}$, respectively, as

$$\hat{h}_{00}^{(0)} \Longrightarrow H + \text{const},$$

$$\text{const} \equiv -\langle \phi_0 | \sum_\mu \omega_\mu X_\mu^\dagger X_\mu | \phi_0 \rangle = -\sum_{\mu a} \phi_\mu(a)^2 \omega_\mu, \tag{6·12}$$

$$\begin{aligned}\hat{h}_{00}^{(1)} &= -\sum_\mu [\sum_a \{\psi_\mu(a) b_a^\dagger + \phi_\mu(a) b_a\} Z_\mu^\dagger + Z_\mu \sum_a \{\psi_\mu(a) b_a + \phi_\mu(a) b_a^\dagger\}] \\ &= -\sum_\mu [\{\sum_a \psi_\mu(a) b_a^\dagger Z_\mu^\dagger + \sum_a \phi_\mu(a) [b_a, Z_\mu^\dagger] + \sum_a \phi_\mu(a) Z_\mu^\dagger b_a\} + \text{h.c.}] \\ &\Longrightarrow -\sum_{\mu\nu} [\{\sum_a \phi_\mu(a) \psi_\nu(a) [X_\nu, Z_\mu^\dagger] - \sum_a \phi_\mu(a) \phi_\nu(a) [X_\nu^\dagger, Z_\mu^\dagger]\} + \text{h.c.}]. \end{aligned} \tag{6·13}$$

Thus, the Hamiltonian (6·7) may be effectively written as

$$\boldsymbol{H} = \text{const} + \boldsymbol{H}_{\text{col}} + H_{\text{intr}} + H_{\text{coupl}}, \tag{6·14a}$$

$$\text{const} = -\sum_{\mu a} \phi_\mu(a)^2 \omega_\mu,$$

$$\boldsymbol{H}_{\text{col}} = \sum_\mu \omega_\mu \boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\mu,$$

$$H_{\text{intr}} = H - \sum_{\substack{\mu\nu \\ a}} \phi_\mu(a) \phi_\nu(a) [X_\nu, Z_\mu] - \frac{1}{2} \sum_{\substack{\mu\nu \\ a}} \phi_\mu(a) \psi_\nu(a) \{[X_\mu, Z_\nu^\dagger] + [Z_\mu, X_\nu^\dagger]\},$$

$$\begin{aligned} H_{\text{coupl}} = \sum_\mu (\boldsymbol{X}_\mu^\dagger Z_\mu^\dagger + \boldsymbol{X}_\mu Z_\mu) + \frac{1}{2} \sum_{\mu\nu} \{ & 2 \boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\nu [X_\mu, Z_\nu] \\ & + \boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\nu^\dagger [X_\mu, Z_\nu^\dagger] + \boldsymbol{X}_\nu \boldsymbol{X}_\mu [Z_\mu, X_\nu^\dagger]\}, \end{aligned} \tag{6·14b}$$

where the terms involving commutators higher than double are neglected. The constant term represents the energy of ground-state correlation due to the collective pairing vibration; the terms $H_{\text{intr}}$ and $\boldsymbol{H}_{\text{col}}$ are the intrinsic- and collective-Hamiltonians, respectively, and $H_{\text{coupl}}$ represents the coupling between the collective and intrinsic degrees of freedom.

Now it is clear that we have achieved the aim of unambiguously writing the Hamiltonian in the form (3·7) where the roles of residual interaction are explicitly expressed.

## §7. Concluding remarks

With the explicit use of the quasi-spin formalism, we have defined the collective subspace $\{|\phi_{\text{col}}\rangle\}$, which is associated with the pairing correlations and includes all the spurious components due to the nucleon-number non-conservation in the quasi-particle representation, and the intrinsic subspace $\{|\phi_{\text{intr}}\rangle\}$ which does not include any spurious components. The intrinsic states are characterized by the one-to-one correspondence between the seniority number $v_a$ and the quasi-particle number $n_a$. Furthermore, we have shown that, by an introduction of canonical transformation with auxiliary variables, the collective and intrinsic degrees of freedom are represented with the auxiliary bosons and the quasi-particle operators, respectively. It has been shown that the original Hamiltonian $H$ can be transformed into the effective Hamiltonian which is described in terms of both (collective) boson and fermion degrees of freedom.

In the next chapter we investigate the intrinsic excitation modes in further detail. In Chap. 7 we show that the coupling Hamiltonian $H_{\text{coupl}}$ can be uniquely rewritten in terms of the collective modes of pairing vibration and the elementary modes of intrinsic excitation, within the NTD approximation.

## Appendix 1A. Matrix elements of two-body interaction

According to Eq. (3·1), the general effective two-body interaction which is invariant under rotation, reflection and time reversal is given by

$$H_{\text{int}}=\sum_{\alpha\beta\gamma\delta}\mathcal{V}_{\alpha\beta\gamma\delta}c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma \tag{1A·1}$$

with Eq. (3·2). The invariance properties of $H_{\text{int}}$ under rotations and reflections are explicitly shown when it is rewritten in an invariant tensor product form with respect to the nucleon-pair operators coupled to angular momentum $JM$. Thus, according to Baranger's notations,[3] we are led to write the matrix elements $\mathcal{V}_{\alpha\beta\gamma\delta}$ in the form

$$\begin{aligned}\mathcal{V}_{\alpha\beta\gamma\delta}&=-\frac{1}{2}\sum_{JM}G(abcd;J)(j_aj_bm_am_\beta|JM)(j_cj_dm_\gamma m_\delta|JM)\\&=-\frac{1}{2}\sum_{JM}F(acdb;J)(-)^{j_c-m_\gamma}(j_aj_cm_a\bar{m}_\gamma|JM)\\&\qquad\times(-)^{j_b-m_\beta}(j_dj_bm_\delta\bar{m}_\beta|JM),\end{aligned} \tag{1A·2}$$

upon which the parity and the charge conservations impose the conditions $(-)^{l_a+l_b}=(-)^{l_c+l_d}$ and $q_a+q_b=q_c+q_d$, respectively. When we further impose the isobaric invariance upon $H_{\text{int}}$, Eq. (1A·2) is written, with the use of the isospin formalism, as

$$\begin{aligned}\mathcal{V}_{\alpha\beta\gamma\delta}&=-\frac{1}{2}\sum_{JMTM_T}G(abcd;JT)(j_aj_bm_am_\beta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_a\tau_\beta|\,TM_T\right)\\&\qquad\times(j_cj_dm_\gamma m_\delta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\gamma\tau_\delta|\,TM_T\right)\\&=-\frac{1}{2}\sum_{JMTM_T}F(acdb;JT)s_\gamma(j_aj_cm_a\bar{m}_\gamma|JM)\left(\frac{1}{2}\frac{1}{2}\tau_a\bar{\tau}_\gamma|\,TM_T\right)\\&\qquad\times s_\beta(j_dj_bm_\delta\bar{m}_\beta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\delta\bar{\tau}_\beta|\,TM_T\right),\end{aligned} \tag{1A·3}$$

where $s_\gamma\equiv(-)^{j_c-m_\gamma}(-)^{1/2-\tau_\gamma}$ and $\tau$ denotes the $z$-component of the nucleon isospin. From the hermiticity of $H_{\text{int}}$ and its time reversal invariance, $G$ and $F$ must be real. From Eq. (3·2), we also have the following properties

$$\begin{aligned} G(abcd;JT) &= G(cdab;JT) \\ &= -\theta(abJT)G(bacd;JT) = -\theta(cdJT)G(abdc;JT), \\ F(acdb;JT) &= F(dbac;JT) \\ &= \theta(abJT)\theta(cdJT)F(cabd;JT), \end{aligned} \tag{1A·4}$$

where $\theta(abJT)=(-)^{j_a+j_b-J}(-)^{1-T}$. The $F$ and $G$ type matrix elements are related with each other through the relation

$$F(acdb;JT) = -\sum_{J'T'}(2J'+1)(2T'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix}G(abdc;J'T'). \tag{1A·5}$$

In the text we do not explicitly use the isospin formalism, but use the so-called $m$-scheme in the isospin. By specifying the proton and neutron explicitly by the letters $\pi$ and $\nu$ respectively, the matrix elements in the $m$-scheme of isospin are given in terms of the above $F$ and $G$ type matrix elements as

$$\begin{aligned} &G(a_\pi b_\pi c_\pi d_\pi;J) = G(a_\nu b_\nu c_\nu d_\nu;J) = G(abcd;JT=1), \\ &G(a_\pi b_\nu c_\pi d_\nu;J) = G(a_\nu b_\pi c_\nu d_\pi;J) = \frac{1}{2}[G(abcd;JT=1)+G(abcd;JT=0)], \\ &F(a_\nu c_\pi d_\nu b_\pi;J) = F(a_\pi c_\nu d_\pi b_\nu;J) \\ &\quad = -\frac{1}{2}\sum_{J'}(2J'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}[G(abdc;J'T=1)+G(abdc;J'T=0)], \\ &F(a_\nu c_\nu d_\pi b_\pi;J) = F(a_\pi c_\pi d_\nu b_\nu;J) \\ &\quad = -\frac{1}{2}\sum_{J'}(2J'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}[G(abdc;J'T=1)-G(abdc;J'T=0)], \\ &F(a_\nu c_\pi d_\pi b_\nu;J) = F(a_\pi c_\nu d_\nu b_\pi;J) = 0. \end{aligned} \tag{1A·6}$$

## Appendix 1B. Pairing vibrational modes

In the text we use the pairing vibrational modes as the basis of the perturbative expansion of the Hamiltonian in collective representation. We here give their definitions, and discuss some related problems to the pairing vibrational modes.

### 1B-1 *The pairing Hamiltonian*

We start with the simplest case, i.e., with the pairing Hamiltonian

$$H^{(p)} = \sum_a(\epsilon_a-\lambda)c_a^\dagger c_a - \frac{1}{4}G\cdot\sum_a c_a^\dagger c_{\tilde{a}}^\dagger\cdot\sum_\beta c_{\tilde{\beta}}c_\beta, \tag{1B·1}$$

where $c_{\tilde{a}}^\dagger\equiv(-)^{j_a-m_a}c_{\bar{a}}^\dagger$ and $G$ is the strength of the pairing force. After the

Bogoliubov transformation (2·17), this is written in terms of the quasi-particle operators as

$$H^{(p)}=U_0^{(p)}+H_0^{(p)}+H_1^{(p)}+H_{\rm int}^{(p)}, \tag{1B·2}$$

$$U_0^{(p)}=\sum_a 2\Omega_a\left\{\left(\eta_a+\frac{1}{2}Gv_a^2\right)v_a^2-\frac{1}{2}u_av_a\Delta\right\},$$

$$H_0^{(p)}=\sum_a\{\eta_a(u_a^2-v_a^2)+2u_av_a\Delta\}\hat{n}_a,$$

$$H_1^{(p)}=\sum_a\{2\eta_au_av_a-(u_a^2-v_a^2)\Delta\}\{\hat{S}_+(a)+\hat{S}_-(a)\},$$

$$H_{\rm int}^{(p)}=H_X^{(p)}+H_V^{(p)}+H_Y^{(p)}+H_{\rm exch}^{(p)}, \tag{1B·3}$$

$$H_X^{(p)}=-G\sum_{ac}(u_a^2u_c^2+v_a^2v_c^2)\hat{S}_+(a)\hat{S}_-(c),$$

$$H_V^{(p)}=\frac{1}{2}G\sum_{ac}(u_a^2v_c^2+v_a^2u_c^2)\{\hat{S}_+(a)\hat{S}_+(c)+\hat{S}_-(c)\hat{S}_-(a)\},$$

$$H_Y^{(p)}=G\sum_{ac}(u_a^2-v_a^2)u_cv_c\{\hat{S}_+(a)\hat{n}_c+\hat{n}_c\hat{S}_-(a)\},$$

$$H_{\rm exch}^{(p)}=-G\sum_{ac}u_av_au_cv_c\{\hat{n}_a\hat{n}_c-\hat{n}_a\delta_{ac}\},$$

where

$$\eta_a=\epsilon_a-\lambda-Gv_a^2,\quad \Omega_a=j_a+1/2,\quad \Delta=G\sum_a\Omega_au_av_a,$$

$\hat{n}_a=\sum_{m_a}a_a^\dagger a_a$, and $\hat{S}_\pm(a)$ are defined in Eq. (2·18). As usual, the parameters $u$ and $v$ are determined so as to eliminate the "dangerous term" $H_1^{(p)}$. The quasi-particle energy term $H_0^{(p)}$ is then reduced to

$$H_0^{(p)}=\sum_aE_a\hat{n}_a,$$

$$E_a=\sqrt{\eta_a^2+\Delta^2}. \tag{1B·4}$$

Applying the canonical transformation (5·8) after the extension $H^{(p)}\to\mathring{H}^{(p)}$ (i.e., $\hat{S}_\pm(a)$, $\hat{S}_0(a)\to\mathring{S}_\pm(a)$, $\mathring{S}_0(a)$), we obtain the pairing Hamiltonian in the collective representation:

$$\boldsymbol{H}^{(p)}=U_{\rm col}\cdot\mathring{H}^{(p)}\cdot U_{\rm col}^{-1}=U_0^{(p)}+\boldsymbol{H}_0^{(p)}+\boldsymbol{H}_{\rm int}^{(p)}, \tag{1B·5}$$

$$\boldsymbol{H}_0^{(p)}=\sum_aE_a\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\},$$

$$\boldsymbol{H}_{\rm int}^{(p)}=\boldsymbol{H}_X^{(p)}+\boldsymbol{H}_V^{(p)}+\boldsymbol{H}_Y^{(p)}+\boldsymbol{H}_{\rm exch}^{(p)}, \tag{1B·6}$$

$$\boldsymbol{H}_X^{(p)}=-G\sum_{ac}(u_a^2u_c^2+v_a^2v_c^2)\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\sqrt{\Omega_c-\hat{n}_c-\hat{\boldsymbol{N}}(c)}\,\boldsymbol{b}_c,$$

$$\boldsymbol{H}_V^{(p)}=\frac{1}{2}G\sum_{ac}(u_a^2v_c^2+v_a^2u_c^2)[\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\boldsymbol{b}_c^\dagger\sqrt{\Omega_c-\hat{n}_c-\hat{\boldsymbol{N}}(c)}+\text{h.c.}],$$

$$\boldsymbol{H}_Y^{(p)}=G\sum_{ac}(u_a^2-v_a^2)u_cv_c[\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\{\hat{n}_c+2\hat{\boldsymbol{N}}(c)\}+\text{h.c.}],$$

$$\boldsymbol{H}_{\rm exch}^{(p)}=-G\sum_{ac}u_av_au_cv_c[\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\}\{\hat{n}_c+2\hat{\boldsymbol{N}}(c)\}-\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\}\delta_{ac}].$$

Needless to say, in this collective representation, the boson operators ($\boldsymbol{b}_a^\dagger$, $\boldsymbol{b}_a$) and the quasi-particle operators ($a_a^\dagger$, $a_a$) describe the collective and intrinsic

degrees of freedom respectively, and therefore their mutual interweaving is clearly visualized.

1B-2 *Pairing vibrational modes under RPA*

Now let us make the expansion

$$\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}=\sqrt{\Omega_a}\cdot\left[1-\left\{\frac{\hat{n}_a+\hat{\boldsymbol{N}}(a)}{2\Omega_a}\right\}-\frac{1}{2}\left\{\frac{\hat{n}_a+\hat{\boldsymbol{N}}(a)}{2\Omega_a}\right\}^2+\cdots\right], \quad (1B\cdot7)$$

and then take up the following lowest order terms including only the collective variables from the Hamiltonian (1B·5)

$$\begin{aligned}\boldsymbol{H}_{\rm col}=&2\sum_a E_a\hat{\boldsymbol{N}}(a)-G\sum_{ac}\sqrt{\Omega_a\Omega_c}\,(u_a^2u_c^2+v_a^2v_c^2)\boldsymbol{b}_a^\dagger\boldsymbol{b}_c\\&+\frac{1}{2}G\sum_{ac}\sqrt{\Omega_a\Omega_c}(u_a^2v_c^2+v_a^2u_c^2)(\boldsymbol{b}_a^\dagger\boldsymbol{b}_c^\dagger+\boldsymbol{b}_c\boldsymbol{b}_a),\end{aligned} \quad (1B\cdot8)$$

where we have, as usual, neglected the terms originating from the exchange term $\boldsymbol{H}_{\rm exch}^{(p)}$. This is nothing but the pairing vibrational Hamiltonian within the RPA and can be diagonalized with the pairing vibrational modes:

$$\begin{aligned}&[\boldsymbol{H}_{\rm col},\ \boldsymbol{X}_n^\dagger]=\omega_n\boldsymbol{X}_n^\dagger;\ \omega_n>0,\\&\boldsymbol{X}_n^\dagger=\sum_a\{\psi_n(a)\boldsymbol{b}_a^\dagger+\phi_n(a)\boldsymbol{b}_a\},\end{aligned} \quad (1B\cdot9)$$

which satisfy the commutation relations

$$[\boldsymbol{X}_n,\ \boldsymbol{X}_m^\dagger]=\delta_{nm},\qquad [\boldsymbol{X}_n,\ \boldsymbol{X}_m]=[\boldsymbol{X}_n^\dagger,\ \boldsymbol{X}_m^\dagger]=0. \quad (1B\cdot10)$$

The eigenvalue equation of the amplitudes takes the form

$$\omega_n\begin{bmatrix}\boldsymbol{\psi}_n\\ \boldsymbol{\phi}_n\end{bmatrix}=\begin{bmatrix}\boldsymbol{D}^{(p)} & -\boldsymbol{A}^{(p)}\\ \boldsymbol{A}^{(p)} & -\boldsymbol{D}^{(p)}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_n\\ \boldsymbol{\phi}_n\end{bmatrix}, \quad (1B\cdot11)$$

where the matrix elements of $\boldsymbol{D}^{(p)}$ and $\boldsymbol{A}^{(p)}$ are given by

$$\begin{aligned}&D_{ac}^{(p)}=2E_a\delta_{ac}-G\sqrt{\Omega_a\Omega_c}\,(u_a^2u_c^2+v_a^2v_c^2),\\&A_{ac}^{(p)}=G\sqrt{\Omega_a\Omega_c}(u_a^2v_c^2+v_a^2u_c^2).\end{aligned} \quad (1B\cdot12)$$

The pairing vibrational modes for the general Hamiltonian given by Eq. (3·4) are also given in a similar way. In this case the matrix elements of $\boldsymbol{D}^{(p)}$ and $\boldsymbol{A}^{(p)}$ in Eq. (1B·11) are given as

$$\begin{aligned}&D_{ac}^{(p)}=2E_a\delta_{ac}+\sum_{m_am_\gamma}\frac{1}{\sqrt{\Omega_a\Omega_c}}\,V_X(a\tilde{a}\gamma\tilde{\gamma}),\\&A_{ac}^{(p)}=-2\sum_{m_am_\gamma}\frac{1}{\sqrt{\Omega_a\Omega_c}}\{V_V(a\tilde{a}\gamma\tilde{\gamma})+V_V(\gamma\tilde{\gamma}a\tilde{a})-4V_V(a\gamma a\gamma)\},\end{aligned} \quad (1B\cdot13)$$

where the definitions of $V_X(\alpha\beta\gamma\delta)$ and $V_V(\alpha\beta\gamma\delta)$ are given after Eq. (3·4) in the text.

1B-3 *Mode associated with breaking of nucleon-number conservation*

The eigenvalue equation (1B·11) is known to possess a special zero-energy solution which implies that RPA includes enough of the residual interaction to restore the nucleon-number conservation broken by the BCS solutions: The nucleon-number operator in the RPA is obtained by applying the expansion (1B·7) to the nucleon-number operator in the collective representation and by taking up the lowest order terms:

$$\begin{aligned}&\mathcal{N}_{\mathrm{RPA}}=\sum_a 2\Omega_a v_a^2+\mathcal{N}^{(0)},\\&\mathcal{N}^{(0)}=2\sum_a\sqrt{\Omega_a}\,u_a v_a(\boldsymbol{b}_a^\dagger+\boldsymbol{b}_a).\end{aligned}\tag{1B·14}$$

This operator satisfies

$$[\boldsymbol{H}_{\mathrm{col}},\,\mathcal{N}_{\mathrm{RPA}}]=[\boldsymbol{H}_{\mathrm{col}},\,\mathcal{N}^{(0)}]=0\tag{1B·15}$$

which means that the nucleon-number operator in the RPA itself is a special solution of the RPA equation (1B·9). It is then convenient to define an operator as

$$\boldsymbol{\Phi}^{(0)}=-i\sum_a\Phi(a)(\boldsymbol{b}_a^\dagger-\boldsymbol{b}_a),\tag{1B·16}$$

which satisfies the equation

$$[\boldsymbol{\Phi}^{(0)},\,\mathcal{N}^{(0)}]=i,\qquad[\boldsymbol{H}_{\mathrm{col}},\,\boldsymbol{\Phi}^{(0)}]=-i\frac{1}{I_0}\mathcal{N}^{(0)}.\tag{1B·17}$$

Equation (1B·17) with Eq. (1B·15) is sufficient to determine $\boldsymbol{\Phi}^{(0)}$ and the ($c$-number) inertia parameter $I_0$. The canonical variables ($\mathcal{N}^{(0)}$, $\boldsymbol{\Phi}^{(0)}$) are commutable with pairing vibrational modes ($\boldsymbol{X}_n^\dagger$, $\boldsymbol{X}_n$) with non-zero eigenvalues, and therefore, with the set of operators $\boldsymbol{X}_n^\dagger$, $\boldsymbol{X}_n$, $\mathcal{N}^{(0)}$ and $\boldsymbol{\Phi}^{(0)}$, the boson operators ($\boldsymbol{b}_a^\dagger$, $\boldsymbol{b}_a$) may be expanded as

$$\begin{aligned}\boldsymbol{b}_a^\dagger&=\sum_n\{[\boldsymbol{X}_n,\,\boldsymbol{b}_a^\dagger]\boldsymbol{X}_n^\dagger+[\boldsymbol{b}_a^\dagger,\,\boldsymbol{X}_n^\dagger]\boldsymbol{X}_n\}+i[\boldsymbol{b}_a^\dagger,\,\boldsymbol{\Phi}^{(0)}]\mathcal{N}^{(0)}+i[\mathcal{N}^{(0)},\,\boldsymbol{b}_a^\dagger]\boldsymbol{\Phi}^{(0)}\\&=\sum_n\{\psi_n(a)\boldsymbol{X}_n^\dagger-\phi_n(a)\boldsymbol{X}_n\}+\Phi(a)\mathcal{N}^{(0)}+2i\sqrt{\Omega_a}\,u_a v_a\boldsymbol{\Phi}^{(0)}.\end{aligned}\tag{1B·18}$$

The correlated ground state of $\boldsymbol{H}_{\mathrm{col}}$, denoted by $|0_B)\!)$, is defined in part by the conventional requirement that it be the vacuum of the non-zero modes:

$$\boldsymbol{X}_n|0_B)\!)=0.\tag{1B·19}$$

In addition to this, the zero-energy mode must also be taken into account to complete the definition of $|0_B)\!)$. In order for the ground state to have a definite nucleon number, it is necessary to take on the condition

$$\mathcal{H}^{(0)}|0_B))=0. \tag{1B·20}$$

However, this requirement is known to be too stringent to be compatible with the basic assumptions of the RPA.[7] Therefore, we adopt the following limiting procedure: In so far as the zero-energy mode is concerned, the correlated ground state is assumed to be specified by

$$[\mathcal{H}^{(0)}-i\varepsilon_0 I_0^2\Phi^{(0)}]|0_B))=0, \tag{1B·21}$$

where $\varepsilon_0$ is a real positive parameter. We can then define annihilation and creation operators

$$\begin{aligned} \boldsymbol{X}_0&=\frac{1}{\sqrt{2\varepsilon_0}}\left[\frac{\mathcal{H}^{(0)}}{I_0}-i\varepsilon_0 I_0\Phi^{(0)}\right]=\sum_a\{\psi_0(a)\boldsymbol{b}_a+\phi_0(a)\boldsymbol{b}_a^\dagger\},\\ \boldsymbol{X}_0^\dagger&=\frac{1}{\sqrt{2\varepsilon_0}}\left[\frac{\mathcal{H}^{(0)}}{I_0}+i\varepsilon_0 I_0\Phi^{(0)}\right]=\sum_a\{\psi_0(a)\boldsymbol{b}_a^\dagger+\phi_0(a)\boldsymbol{b}_a\}, \end{aligned} \tag{1B·22}$$

which satisfy $[\boldsymbol{X}_0, \boldsymbol{X}_0^\dagger]=1$. As $\varepsilon_0$ tends toward zero, $\boldsymbol{X}_0$ and $\boldsymbol{X}_0^\dagger$ separately tend toward infinity as $1/\sqrt{\varepsilon_0}$, while the behaviour of the corresponding term of $\boldsymbol{H}_{\rm col}$ in this limit becomes

$$\varepsilon_0 I_0\boldsymbol{X}_0^\dagger\boldsymbol{X}_0\longrightarrow\frac{1}{2I_0}(\mathcal{H}^{(0)})^2, \tag{1B·23}$$

which is finite and is just the one expected from Eq. (1B·17).

With the aid of the operators $(\boldsymbol{X}_0, \boldsymbol{X}_0^\dagger)$, Eq. (1B·18) is now rewritten as

$$\boldsymbol{b}_a^\dagger=\sum_\mu\{\psi_\mu(a)\boldsymbol{X}_\mu^\dagger-\phi_\mu(a)\boldsymbol{X}_\mu\},\qquad \boldsymbol{b}_a=\sum_\mu\{\psi_\mu(a)\boldsymbol{X}_\mu-\phi_\mu(a)\boldsymbol{X}_\mu^\dagger\}. \tag{1B·24}$$

With the use of these modes of pairing vibration we also define the canonical coordinates and canonical conjugate momenta as follows:

$$\boldsymbol{P}_\mu=\frac{1}{\sqrt{2}}(\boldsymbol{X}_\mu+\boldsymbol{X}_\mu^\dagger),\qquad \boldsymbol{Q}_\mu=\frac{i}{\sqrt{2}}(\boldsymbol{X}_\mu-\boldsymbol{X}_\mu^\dagger) \tag{1B·25}$$

which satisfy the canonical commutation relations

$$[\boldsymbol{Q}_\mu, \boldsymbol{P}_\nu]=i\delta_{\mu\nu},\qquad [\boldsymbol{Q}_\mu, \boldsymbol{Q}_\nu]=[\boldsymbol{P}_\mu, \boldsymbol{P}_\nu]=0. \tag{1B·26}$$

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# Chapter 2. Theory of Intrinsic Modes of Excitation in Odd-Mass Nuclei

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## §1. Introduction

In this chapter*) we develop a theory to treat elementary modes of "intrinsic" excitation in odd-mass nuclei, which should be approximate eigenmodes of the intrinsic Hamiltonian $H_{\text{intr}}$ given by the definition (1·6·14)**) in the preceding chapter. We must therefore treat these eigenmodes within the "intrinsic" subspace $\{|\phi_{\text{intr}}\rangle\}$ which obeys the condition $\hat{S}_{-}(a)|\phi_{\text{intr}}\rangle=0$. These eigenmodes are constructed within the framework of the NTD approximation (with the ground-state correlation) and provide the basis vectors for the intrinsic subspace within the NTD approximation. The dressed three-quasi-particle (3QP) mode, on which emphasis is put in this paper, is regarded as one of the simplest modes of intrinsic excitation.

In formulating the theory, for simplicity, we take up only the first term, $H$, of $H_{\text{intr}}$ in (1·6·14) as the intrinsic Hamiltonian. The inclusion of the other terms does not alter the essential ingredients of the discussion in this chapter. Hereafter (with the exception of the last chapter) we concentrate on the study of the intrinsic excitations and drop the symbols expressing "intrinsic" quantities. Furthermore we use the term "collective" in the conventional sense, i.e., for such quantities related to the modes with the ground-state correlation.

## §2. Quasi-particle new-Tamm-Dancoff space

The essence of our theory of the intrinsic modes of excitation is to make an explicit use of the concept of the quasi-particle new-Tamm-Dancoff (NTD)

*) The content of this chapter was already published by the present authors in Prog. Theor. Phys. **52** (1974), 1819.

**) We cite the equations in different chapters by adding the chapter number to the first place of the equation number.

space. To obtain a first understanding of the concept of the quasi-particle NTD space and of the physical operators defined in it, let us start with the conventional quasi-particle Tamm-Dancoff (TD) space characterized by the seniority number.

## 2-1 *Quasi-particle Tamm-Dancoff space*

As shown in §1-Chap. 1, the use of the quasi-particle representation based on the BCS theory can be regarded as an attempt to characterize the intrinsic states in terms of the seniority number $v=\sum_a v_a$, the value of which corresponds to the number of quasi-particles, $n=\sum_a n_a$. Thus the intrinsic-energy spectrum of $H_0$ in odd-mass nuclei has a quite characteristic structure as shown in Fig. 1 and the corresponding states with a fixed odd number of quasi-particles, $n=v(=\sum_a v_a)$, span the $n$-quasi-particle TD space. The quasi-particle TD space for odd-mass nuclei is therefore characterized by the orthonormal state vectors with an odd number of quasi-particles:

$$\begin{aligned}
&|v=1;\alpha\rangle=a_\alpha^\dagger|\phi_0\rangle,\\
&|v=3;\alpha\beta\gamma\rangle=\frac{1}{\sqrt{3!}}a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger|\phi_0\rangle,\\
&|v=5;\alpha\beta\gamma\delta\varepsilon\rangle=\frac{1}{\sqrt{5!}}a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger a_\varepsilon^\dagger|\phi_0\rangle,\\
&\qquad\cdots,
\end{aligned}\tag{2·1}$$

where $|\phi_0\rangle$ is the BCS ground state defined in §§1 and 2 of Chap. 1.

In order to explicitly express the requirement that any state in the quasi-particle TD space must satisfy the supplementary condition (1·6·10), it is convenient to precisely define the quasi-particle TD space by adopting the concept of the quasi-spin tensor, which has been defined in §1-Chap. 1. The quasi-spin tensor operators (in the quasi-particle representation) $T_{k\kappa}(a)$ are defined by the relation (1·2·14), i.e.,

$$T_{k\kappa}(a)=\sum_{\kappa'}D^{k*}_{\kappa'\kappa}(\omega_0)\mathcal{T}_{k\kappa'}(a).$$

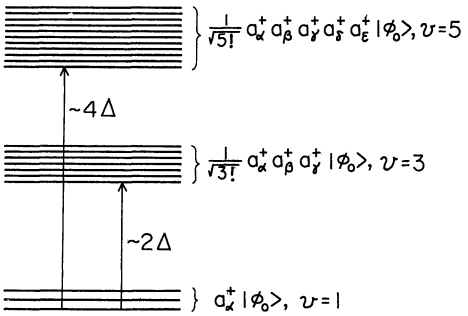


Fig. 1. Schematic energy spectra of $H_0$ in odd-mass nuclei.

They are constructed from the quasi-spin spinors; for example,

$$
\begin{aligned}
&T_{3/2,\,3/2}(\alpha_1\alpha_2\alpha_3)=a^\dagger_{a_1}a^\dagger_{a_2}a^\dagger_{a_3},\\
&T_{3/2,\,1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{a_1}a^\dagger_{a_2}a_{\bar{a}_3}+a^\dagger_{a_1}a_{\bar{a}_2}a^\dagger_{a_3}+a_{\bar{a}_1}a^\dagger_{a_2}a^\dagger_{a_3}),\\
&T_{3/2,\,-1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{a_1}a_{\bar{a}_2}a_{\bar{a}_3}+a_{\bar{a}_1}a^\dagger_{a_2}a_{\bar{a}_3}+a_{\bar{a}_1}a_{\bar{a}_2}a^\dagger_{a_3}),\\
&T_{3/2,\,-3/2}(\alpha_1\alpha_2\alpha_3)=a_{\bar{a}_1}a_{\bar{a}_2}a_{\bar{a}_3}.
\end{aligned}
\tag{2·2}
$$

Now the quasi-particle TD space for odd-mass nuclei characterized by (2·1) is precisely defined in terms of the set of state vectors

$$
\begin{aligned}
&|v=2s;\ \alpha_1\alpha_2\cdots\alpha_{2s_a},\ \beta_1\beta_2\cdots\beta_{2s_b},\ \cdots\rangle\\
&\quad=O^\dagger_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a=s_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b=s_b;\ \cdots]|\phi_0\rangle,
\end{aligned}
\tag{2·3a}
$$

where

$$
\begin{aligned}
&O^\dagger_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b;\ \cdots]\\
&\quad=\frac{1}{\sqrt{(2s_a)!\,(2s_b)!\cdots}}\mathring{T}_{s_a\kappa_a}(\alpha_1\alpha_2\cdots\alpha_{2s_a})\cdot\mathring{T}_{s_b\kappa_b}(\beta_1\beta_2\cdots\beta_{2s_b})\cdots
\end{aligned}
\tag{2·3b}
$$

with $2s=2(s_a+s_b+\cdots)=v$ in odd numbers. In Eq. (2·3b), we have used the definition

$$
\begin{aligned}
\mathring{T}_{k\kappa}(\alpha_1\alpha_2\cdots\alpha_{2k})\equiv&\sum_{\alpha'_1\alpha'_2\cdots\alpha'_{2k}}P(\alpha_1\alpha_2\cdots\alpha_{2k}|\alpha'_1\alpha'_2\cdots\alpha'_{2k})\\
&\times T_{k\kappa}(\alpha'_1\alpha'_2\cdots\alpha'_{2k}),
\end{aligned}
\tag{2·4}
$$

where the operator $\boldsymbol{P}$ (the matrix elements of which are $P(\alpha_1\alpha_2\cdots\alpha_{2k}|\alpha'_1\alpha'_2\cdots\alpha'_{2k})$) is a projection operator by which the quasi-spin operators $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ are removed from the quasi-spin tensor $T_{k\kappa}(\alpha_1\alpha_2\cdots\alpha_{2k})$ completely. Therefore the eigenvectors of the projection operator $\boldsymbol{P}$ are closely related to the coefficients of fractional parantage (cfp) with the seniority $v_a=2k$ for $(j_a)^{2k}$-configurations, and its explicit form for $k=3/2$ is given in Appendix 2A. By the definition of the operator $O^\dagger_s$ in the relation (2·3), we obtain

$$
\hat{S}_-(a)|v=2s;\ \alpha_1\alpha_2\cdots\alpha_{2s_a},\ \beta_1\beta_2\cdots\beta_{2s_b},\ \cdots\rangle=0. \tag{2·5}
$$

Thus the quasi-particle TD space defined in the above manner satisfies the supplementary condition (1·6·10).

Since the quasi-particle TD space is characterized by the seniority number $v(=n)$ as shown in Fig. 1, a better approximation may be obtained by diagonalizing the quasi-particle interaction $H_{\rm int}$ in the Hamiltonian (1·3·4) in the subspace with a fixed number of quasi-particles. This is well known as the quasi-particle TD approximation. Among the matrix elements of $H_{\rm int}$ in the subspace with a definite number of quasi-particles, the non-zero ones come from only the part $H_X$ which conserves the quasi-particle number.

Therefore the eigenmode-creation operators $Y^{\dagger}_{0s\lambda}$ in the TD approximation are given by the eigenvalue equation

$$[H_0+H_X,\ Y^{\dagger}_{0s\lambda}]\cong\omega_{0s\lambda}Y^{\dagger}_{0s\lambda}\quad(\omega_{0s\lambda}>0) \tag{2·6}$$

with

$$\begin{aligned}Y^{\dagger}_{0s\lambda}=\sum_{\substack{a_i,\beta_j,\cdots\\(s=\sum_a s_a)}}&\Psi_{0s\lambda}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a=s_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b=s_b;\ \cdots]\\&\times O^{\dagger}_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a=s_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b=s_b;\ \cdots].\end{aligned} \tag{2·7}$$

Here $\lambda$ denotes a set of additional quantum numbers to specify the eigenmodes.

The eigenmodes $Y^{\dagger}_{0s\lambda}$ satisfy the anti-commutation relation in the following sense:

$$\begin{aligned}&\{Y_{0s_>\lambda},\ Y^{\dagger}_{0s'_<\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\phi_0\rangle,\\&\{Y^{\dagger}_{0s\lambda},\ Y^{\dagger}_{0s'\lambda'}\}_+=\{Y_{0s\lambda},\ Y_{0s'\lambda'}\}_+=0,\end{aligned} \tag{2·8}$$

where the subscript $>$ (or$<$) of $s_>$ (or $s'_<$) denotes the relation $s\geq s'$. Thus, the set of states $Y^{\dagger}_{0s\lambda}|\phi_0\rangle$ with $2s=n$ in odd numbers provides a complete set of orthonormal bases of the quasi-particle TD space:

$$\langle\phi_0|\{Y_{0s\lambda},\ Y^{\dagger}_{0s'\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}. \tag{2·9}$$

Now it is clear that the quasi-particle TD space for odd-mass nuclei may be characterized with the operators defined by

$$\boldsymbol{Y}^{\dagger}_{0s\lambda}=Y^{\dagger}_{0s\lambda}|\phi_0\rangle\langle\phi_0|,\ \ \boldsymbol{Y}_{0s\lambda}=|\phi_0\rangle\langle\phi_0|Y_{0s\lambda}\ \text{with}\ 2s\ \text{in odd numbers.} \tag{2·10}$$

By definition, the operators $\boldsymbol{Y}^{\dagger}_{0s\lambda}$ satisfy the equations

$$[H_0+H_X,\ \boldsymbol{Y}^{\dagger}_{0s\lambda}]=\omega_{0s\lambda}\boldsymbol{Y}^{\dagger}_{0s\lambda}\quad(\omega_{0s\lambda}>0) \tag{2·11}$$

and

$$\begin{aligned}&\{\boldsymbol{Y}_{0s\lambda},\ \boldsymbol{Y}^{\dagger}_{0s'\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\phi_0\rangle,\\&\{\boldsymbol{Y}^{\dagger}_{0s\lambda},\ \boldsymbol{Y}^{\dagger}_{0s'\lambda'}\}_+=\{\boldsymbol{Y}_{0s\lambda},\ \boldsymbol{Y}_{0s'\lambda'}\}_+=0.\end{aligned} \tag{2·12}$$

The unit operator $\mathbf{1}$ in the quasi-particle TD space for odd-mass nuclei is given by

$$\mathbf{1}=\Sigma'_{s\lambda}\boldsymbol{Y}^{\dagger}_{0s\lambda}\boldsymbol{Y}_{0s\lambda}, \tag{2·13}$$

where $\Sigma'_{s\lambda}$ denotes the summation with respect to $2s$ in odd numbers. With the use of the operators $\boldsymbol{Y}^{\dagger}_{0s\lambda}$, the Hamiltonian $H_0+H_X$ in (1·3·4) is now written as

$$H_0+H_X\longrightarrow\mathbf{1}(H_0+H_X)\mathbf{1}=\Sigma'_{s\lambda}\omega_{0s\lambda}\boldsymbol{Y}^{\dagger}_{0s\lambda}\boldsymbol{Y}_{0s\lambda}. \tag{2·14}$$

Thus, using the elementary excitation operators $Y^{\dagger}_{0s\lambda}$ instead of the quasi-particle operators $a^{\dagger}_{a}$, we obtain another representation of any operator $\hat{F}$ in the quasi-particle TD space for odd-mass nuclei:

$$\hat{F}=\mathbf{1}\hat{F}\mathbf{1}=\sum'_{s\lambda}\sum'_{s'\lambda'}\langle 0s\lambda|\hat{F}|0s'\lambda'\rangle Y^{\dagger}_{0s\lambda}Y_{0s'\lambda'} \tag{2·15}$$

with the definition $|0s\lambda\rangle=Y^{\dagger}_{0s\lambda}|\phi_0\rangle$.

## 2-2 *Quasi-particle NTD space*

Now it is well known that in a finite quantal system such as nucleus, the ground-state correlation is particularly important as a collective predisposition which permits the correlated excited states to occur from the ground state. Actually, we must simultaneously take special account of both the seniority classification and the ground-state correlation, in a way that the essential physical notion obtained in the quasi-particle TD space still persists in a certain form. The guiding principle to introduce the quasi-particle NTD space lies in the fact that, in the new-Tamm-Dancoff (NTD) method, the quasi-particle correlations attributed asymmetrically to only the excited states in the TD calculations are symmetrically incorporated in the ground state through the ground-state correlation. The quasi-particle NTD space for odd-mass nuclei is thus defined with a set of basis vectors,

$$Y^{\dagger}_{s\lambda}|\Phi_0\rangle \tag{2·16}$$

with $2s$ in odd numbers, where $Y^{\dagger}_{s\lambda}$ are creation operators of "*dressed*" $n$ $(=2s)$-quasi-particle modes constructed within the framework of the NTD method with the ground-state correlation, and $|\Phi_0\rangle$ is the corresponding ground state. Contrary to the BCS ground state $|\phi_0\rangle$, the state $|\Phi_0\rangle$ does not have a definite seniority number because of the ground-state correlation. In spite of the breakdown of the seniority number, we can still characterize the excitation modes in the quasi-particle NTD space by the amount of seniority $\Delta v(=2s=n)$ which they transfer to the ground state $|\Phi_0\rangle$.

In exactly the same way as the conventional spherical tensor operator is characterized by the amount of angular momentum it transfers to the state on which it acts, the quasi-spin-tensor operator $T_{s\kappa}$ is characterized by the amount of the transferred seniority $\Delta v=2s$ to the state on which it operates. Therefore, we can define the dressed $n(=2s)$-quasi-particle modes $Y^{\dagger}_{s\lambda}$ in terms of the direct product of the quasi-spin-tensor operators defined in each orbit with the total transferred seniority $\Delta v=2s=\sum_a 2s_a$:

$$\begin{aligned} Y^{\dagger}_{s\lambda}=\sum_{\substack{a_i,\beta_j,\cdots \\ (s=\sum_a s_a)}}\sum_{\kappa_a,\kappa_b,\cdots}\Psi_{s\lambda}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\kappa_a;\beta_1\beta_2\cdots\beta_{2s_b},\kappa_b;\cdots] \\ \times O^{\dagger}_{s}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\kappa_a;\beta_1\beta_2\cdots\beta_{2s_b},\kappa_b;\cdots], \end{aligned} \tag{2·17}$$

where $O^{\dagger}_{s}[\alpha_i,\kappa_a;\beta_j,\kappa_b;\cdots]$ is defined by (2·3b). Within the framework of

the NTD approximation, the eigenvalue equation which the amplitude $\Psi_{s\lambda}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \cdots]$ must satisfy is as usual given by

$$[H_0+H_X+H_V,\ Y^\dagger_{s\lambda}]\cong\omega_{s\lambda}Y^\dagger_{s\lambda},\quad(\omega_{s\lambda}>0) \tag{2·18}$$

where the part $H_V$ of the quasi-particle interaction $H_{\text{int}}$ in (1·3·4) introduces the ground-state correlation.

The part $H_V$ together with $H_X$ is known to be essential in constructing the collective excitation modes within the framework of the NTD method. Hence we call the parts $H_X$ and $H_V$ the *constructive force* (of the collective modes). The part $H_Y$ in (1·3·4) changes the number of quasi-particles, and has no contribution in the TD calculation with the definite number of quasi-particles. In so far as the NTD method is adopted (in describing the dressed $n$-quasi-particle mode) as an improvement of the TD method (for $n$-quasi-particles) the part $H_Y$ does not play any important role. The part $H_Y$ plays a decisive role as an essential coupling between the various dressed $n$-quasi-particle modes. Hence we call it the *interactive force.**)

The dressed $n$-quasi-particle modes $Y^\dagger_{s\lambda}$ (with $2s=n$) must satisfy the fermion-type anti-commutation relation in the quasi-particle NTD space,

$$\{Y_{s>\lambda},\ Y^\dagger_{s'<\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\Phi_0\rangle, \tag{2·19}$$

just as the $n$-quasi-particle modes $Y^\dagger_{0s\lambda}$ (with $2s=n$) in the quasi-particle TD space satisfy (2·8). This requirement is necessary, together with the eigenvalue equation (2·18), for prescribing the elementary excitation modes in terms of the concept of transferred seniority. When (2·19) is satisfied within the framework of the NTD approximation, the set of states $Y^\dagger_{s\lambda}|\Phi_0\rangle$ with $2s=n$ in odd numbers becomes a complete set of orthonormal bases in the quasi-particle NTD space for odd-mass nuclei:

$$\langle\Phi_0|\{Y_{s\lambda},\ Y^\dagger_{s'\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}, \tag{2·20}$$

and, in the same way as (2·13), the unit operator in the quasi-particle NTD space for odd-mass nuclei is given by

$$\mathbf{1}=\sum'_{s\lambda}\boldsymbol{Y}^\dagger_{s\lambda}\boldsymbol{Y}_{s\lambda}, \tag{2·21}$$

where

$$\boldsymbol{Y}^\dagger_{s\lambda}=Y^\dagger_{s\lambda}|\Phi_0\rangle\langle\Phi_0|,\qquad \boldsymbol{Y}_{s\lambda}=|\Phi_0\rangle\langle\Phi_0|\,Y_{s\lambda}. \tag{2·22}$$

In terms of the elementary excitation operators $\boldsymbol{Y}^\dagger_{s\lambda}$, any physical operator $\hat{F}$ is easily transcribed into the quasi-particle NTD sapce:

$$\hat{F}\longrightarrow\hat{\boldsymbol{F}}=\mathbf{1}\hat{F}\mathbf{1}=\sum'_{s\lambda}\sum'_{s'\lambda'}\langle\Phi_0|\,Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle\cdot\boldsymbol{Y}^\dagger_{s\lambda}\boldsymbol{Y}_{s'\lambda'}. \tag{2·23}$$

*) It should be also noted that the matrix elements of $H_Y$ contain the reduction $(u, v)$-factors which can be quite small in the middle of the shell, while the matrix elements of $H_X$ and $H_V$ involve the enhancement $(u, v)$-factors which are close to unity for low-lying states in the middle of the shell.

Thus, the actual problem is how to estimate the matrix elements $\langle\Phi_0|Y_{s\lambda}\times\hat{F}Y^{\dagger}_{s'\lambda'}|\Phi_0\rangle$. As shown in §5, however, a simple rule will be found when the anti-commutation relation (2·19) is satisfied.

In the following sections we concretely study the quasi-particle NTD subspace which consists of the dressed quasi-particle modes with the transferred seniority $\Delta\nu(=2s)=1$ and 3, because we are considering the low-lying collective excited states in odd-mass nuclei.

## §3. Structure of dressed three-quasi-particle modes

According to the definition (2·17), the eigenmode operators of the dressed three-quasi-particle modes (which satisfy Eq. (2·18) with $2s=3$ within the NTD approximation) are written in the explicit form:

$$\begin{aligned}C^{\dagger}_{\lambda}=&\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\psi_{\lambda}(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)a^{\dagger}_{\alpha}a^{\dagger}_{\beta}a^{\dagger}_{\gamma}\\&+\frac{1}{\sqrt{3!}}\sum_{a_1a_2a_3}\phi^{(1)}_{\lambda}(a_1a_2a_3)\mathring{T}_{3/2,-1/2}(a_1a_2a_3)\\&+\frac{1}{\sqrt{2!}}\sum_{\substack{a_1a_2\gamma\\(a\neq c)}}\phi^{(2)}_{\lambda}(a_1a_2;\gamma)\mathring{T}_{10}(a_1a_2)a_{\tilde{\gamma}}\\&+\sum_{\substack{(\alpha\beta)\gamma\\(a,b\neq c)}}(1+\delta_{ab})^{-1/2}\phi^{(3)}_{\lambda}(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)a^{\dagger}_{\gamma}a_{\tilde{\alpha}}a_{\tilde{\beta}}.\end{aligned}\tag{3·1}$$

Here the symbol $\sum_{(\alpha\beta)\gamma}$ denotes the summation over the orbit-pair $(ab)$, $m_{\alpha}$, $m_{\beta}$ and $\gamma$, and

$$\begin{aligned}&\boldsymbol{P}(\alpha\beta\gamma)a^{\dagger}_{\alpha}a^{\dagger}_{\beta}a^{\dagger}_{\gamma}\equiv\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')a^{\dagger}_{\alpha'}a^{\dagger}_{\beta'}a^{\dagger}_{\gamma'},\\&\boldsymbol{P}(\alpha\beta)a_{\tilde{\alpha}}a_{\tilde{\beta}}\equiv\sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')a_{\tilde{\alpha}'}a_{\tilde{\beta}'},\end{aligned}\tag{3·2}$$

where the operators $\boldsymbol{P}$ stand for the projection operators by which any quasi-spin operator is removed from the products of quasi-particles $(a^{\dagger}_{\alpha}, a_{\alpha})$. Their explicit forms are given in Appendix 2A. A direct calculation of Eq. (2·18) with (3·1) leads to the following eigenvalue equation which the correlation amplitudes should satisfy:

$$\omega_{\lambda}\begin{bmatrix}\boldsymbol{\psi}_{\lambda}\\ \boldsymbol{\phi}_{\lambda}\end{bmatrix}=\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A}\\ \boldsymbol{A}^{T} & -\boldsymbol{d}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_{\lambda}\\ \boldsymbol{\phi}_{\lambda}\end{bmatrix},\tag{3·3}$$

where $\boldsymbol{\psi}_{\lambda}$ and $\boldsymbol{\phi}_{\lambda}$ are the vector notations symbolizing the sets of amplitudes $\psi_{\lambda}(\alpha\beta\gamma)$ and $\{\phi^{(1)}_{\lambda}(a_1a_2a_3), \phi^{(2)}_{\lambda}(a_1a_2;\gamma), \phi^{(3)}_{\lambda}(\alpha\beta;\gamma)\}$, respectively, and the explicit forms of matrices $\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ are given in Appendix 2B. The projection operators $\boldsymbol{P}$ involved in these matrices guarantee that the correlation amplitudes automatically satisfy the relations

$$\begin{aligned}
&\psi_\lambda(\alpha\beta\gamma)=\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma\,|\,\alpha'\beta'\gamma')\psi_\lambda(\alpha'\beta'\gamma'),\\
&\phi_\lambda^{(1)}(\alpha_1\alpha_2\alpha_3)=\sum_{\alpha_1'\alpha_2'\alpha_3'}P(\alpha_1\alpha_2\alpha_3\,|\,\alpha_1'\alpha_2'\alpha_3')\phi_\lambda^{(1)}(\alpha_1'\alpha_2'\alpha_3'),\\
&\phi_\lambda^{(2)}(\alpha_1\alpha_2\,;\,\gamma)=\sum_{\alpha_1'\alpha_2'}P(\alpha_1\alpha_2\,|\,\alpha_1'\alpha_2')\phi_\lambda^{(2)}(\alpha_1'\alpha_2'\,;\,\gamma),\\
&\phi_\lambda^{(3)}(\alpha\beta\,;\,\gamma)=\sum_{\alpha'\beta'}P(\alpha\beta\,|\,\alpha'\beta')\phi_\lambda^{(3)}(\alpha'\beta'\,;\,\gamma)
\end{aligned}\tag{3·4}$$

which mean that the correlation amplitudes never contain any component due to the nucleon-number fluctuations (i.e., due to the quasi-spin operators).

Equation (3·3) tells us that with the use of the definition of the metric matrix

$$\tau=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\tag{3·5}$$

the correlation amplitudes satisfy the orthonormality relation in the sense

$$(\boldsymbol{\varphi}_{\lambda'}^T,\ \boldsymbol{\phi}_{\lambda'}^T)\tau\begin{bmatrix}\boldsymbol{\varphi}_\lambda\\ \boldsymbol{\phi}_\lambda\end{bmatrix}=\varepsilon_\lambda\delta_{\lambda\lambda'},\tag{3·6}$$

where $\varepsilon_\lambda$ is the sign function with $|\varepsilon_\lambda|=1$ and $\boldsymbol{\varphi}_\lambda^T$ denotes the transposed vector of $\boldsymbol{\varphi}_\lambda$. Due to the introduction of the backward-going components, the eigenvalue equation (3·3) has "extra" unphysical solutions which have the large amplitudes $\boldsymbol{\phi}_{\lambda_0}$ and the small amplitudes $\boldsymbol{\varphi}_{\lambda_0}$.*) As long as the eigenvalues $\omega_\lambda$ are real, the physical solutions have the large amplitudes $\boldsymbol{\varphi}_\lambda$ and the small amplitudes $\boldsymbol{\phi}_\lambda$. Thus the positive $\varepsilon_\lambda$ corresponds to the physical solutions, and we can classify the eigenmode operators $C_\lambda^\dagger$ in (3·1) as follows:

$$C_\lambda^\dagger=\begin{cases}Y_\lambda^\dagger & \text{for}\quad \varepsilon_\lambda=1,\\ A_{\lambda_0} & \text{for}\quad \varepsilon_{\lambda_0}=-1.\end{cases}\tag{3·7}$$

The physical dressed 3-quasi-particle states are given as

$$|\lambda\rangle=Y_\lambda^\dagger|\Phi_0\rangle,\tag{3·8}$$

where $|\Phi_0\rangle$ is the correlated ground state (within the framework of the NTD approximation). The existence of the extra eigenmodes $A_{\lambda_0}^\dagger$, which have no physical meaning, imposes an important condition upon the state vectors in the quasi-particle NTD space: Any state vector $|\Phi\rangle$ which actually has physical meaning must satisfy the supplementary condition

$$A_{\lambda_0}|\Phi\rangle=0.\tag{3·9}$$

## §4. Structure of ground-state correlation

It is now quite important to examine the compatibility of Eqs. (3·3) and

*) Hereafter the unphysical solutions are specified by the subscript $\lambda_0$.

(2·19). In this section, we show that the requirement (2·19) is satisfied within the NTD approximation when we properly take into account the characteristics of the introduced ground-state correlation.

4-1 *Prescription of structure of ground-state correlation*

First of all, let us investigate the characteristics of the ground-state correlation (due to the dressed 3-quasi-particle modes). The structure of the ground-state correlation should be determined *in principle* through the properties of the fundamental eigenvalue equation (3·3). As is seen from Eq. (2·18), the fundamental equation contains only the matrix elements of the constructive force, $H_X$ and $H_V$. The diagrams considered in the correlated ground state $|\Phi_0\rangle$ are therefore closed diagrams composed by combining only the matrix elements of $H_X$ and $H_V$, so that $|\Phi_0\rangle$ may be written as a superposition of 0-, 4-, 8-quasi-particle states:

$$|\Phi_0\rangle = C_0|\phi_0\rangle + \sum_{\alpha\beta\gamma\delta} C_1(\alpha\beta\gamma\delta)a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger|\phi_0\rangle + \sum_{\alpha\beta\gamma\delta\varepsilon\xi\mu\nu} C_2(\alpha\beta\gamma\delta\varepsilon\xi\mu\nu)a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger a_\varepsilon^\dagger a_\xi^\dagger a_\mu^\dagger a_\nu^\dagger|\phi_0\rangle + \cdots, \quad (4\cdot1)$$

where $C_0$ is the constant related to the normalization of $|\Phi_0\rangle$. The coefficients $C$ in (4·1) should be determined by the conditions $Y_\lambda|\Phi_0\rangle=0$ and $A_{\lambda_0}|\Phi_0\rangle=0$, within the framework of the NTD approximation (which is used to obtain the fundamental equation (3·3) ). This procedure suggests that, with the basic approximation in the NTD method (i.e., $O(n_0/2\Omega)\approx 0$ ),*) the correlated ground state $|\Phi_0\rangle$ may be written in a symbolized form

$$|\Phi_0\rangle = C_0 \exp\left[\frac{1}{\sqrt{4!}} k \sum_{\alpha\beta\gamma\delta} \chi_{J=0}(\alpha\beta\gamma\delta)a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger\right]|\phi_0\rangle \equiv C_0 \exp[W]|\phi_0\rangle, \quad (4\cdot2)$$

where the constant $k$ and $\chi_{J=0}(\alpha\beta\gamma\delta)$ are defined through the relations

$$\begin{aligned} &\frac{1}{\sqrt{4!}} k\cdot\chi_{J=0}(\alpha\beta\gamma\delta) \equiv C_1(\alpha\beta\gamma\delta)/C_0, \\ &\sum_{\alpha\beta\gamma\delta}\chi_{J=0}(\alpha\beta\gamma\delta)^2 = 1. \end{aligned} \quad (4\cdot3)$$

Needless to say, $\chi_{J=0}(\alpha\beta\gamma\delta)$ never contain any component of the $J=0$-coupled quasi-particle pair, so that it should satisfy

$$\chi_{J=0}(\alpha\beta\gamma\delta) = \sum_{\alpha'\beta'\gamma'\delta'} P(\alpha\beta\gamma\delta|\alpha'\beta'\gamma'\delta')\chi_{J=0}(\alpha'\beta'\gamma'\delta'). \quad (4\cdot4)$$

The ground-state correlation written in the symbolized form (4·2) should be interpreted as to be characterized by the following prescriptions:

(1) For an arbitrary operator $\hat{O}$, we have

*) Here $n_0$ and $2\Omega$ are defined through $\langle\Phi_0|a_\alpha^\dagger a_\beta|\Phi_0\rangle \cong \delta_{\alpha\beta}\cdot n_0/2\Omega$, so that $n_0$ denotes the average number of quasi-particles in the ground state and $2\Omega$ the total number of single-particle states under consideration.

$$\begin{aligned}\hat{O}|\Phi_0\rangle &= C_0\hat{O}\exp[W]|\phi_0\rangle \\ &= C_0\exp[W]\{\hat{O}+[\hat{O},W]+\frac{1}{2}[[\hat{O},W],W]+\cdots\}|\phi_0\rangle \\ &\Rightarrow C_0\exp[W]\{\hat{O}+[\hat{O},W]\}|\phi_0\rangle. \quad \text{(the NTD approximation)}\end{aligned} \tag{4·5}$$

(2) Since the basis operators characterizing the ground-state correlation are $O^{\dagger}_{3/2}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \gamma_k, \kappa_c]$ (Eq. (2·3b) with $s=3/2$) which construct the dressed 3-quasi-particle modes and since *the operators* $O^{\dagger}_{3/2}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \gamma_k, \kappa_c]$ *are antisymmetric with respect to the indices belonging to the same single-particle orbit*, all quantities which appear in the last expression of Eq. (4·5) must maintain the same property.

According to prescription (1), the supplementary condition (3·9) with (4·2) leads to the relation

$$\boldsymbol{\phi}_{\lambda_0}-k\boldsymbol{C}\boldsymbol{\phi}_{\lambda_0}=0 \tag{4·6}$$

with

$$\begin{aligned}&C_{\alpha\beta\gamma,\alpha'_1\alpha'_2\alpha'_3}\equiv 3\sqrt{2}\,\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}'_1\tilde{\alpha}'_2)\delta_{\gamma\alpha'_3}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),\\ &C_{\alpha\beta\gamma,\alpha'_1\alpha'_2\gamma'}\equiv 6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}'_2\tilde{\gamma}')\delta_{\gamma\alpha'_1}\boldsymbol{P}^T(\alpha'_1\alpha'_2),\\ &C_{\alpha\beta\gamma,\alpha'\beta'\gamma'}\equiv 6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\delta_{\gamma\gamma'}(1+\delta_{\alpha'\beta'})^{-1/2}\boldsymbol{P}^T(\alpha'\beta').\\ &\quad(\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\equiv(-)^{j_{\alpha'}-m_{\alpha'}}(-)^{j_{\beta'}-m_{\beta'}}\chi_{J=0}(\alpha\beta\bar{\alpha}'\bar{\beta}'))\end{aligned}$$

For simplicity, the following abbreviations are used:

$$\begin{aligned}&\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma,\alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta'\gamma')\\ &\quad\equiv\sum_{\sigma\mu\nu}\sum_{\sigma'\mu'\nu'}P(\alpha\beta\gamma|\sigma\mu\nu)f(\sigma\mu\nu,\sigma'\mu'\nu')P(\sigma'\mu'\nu'|\alpha'\beta'\gamma'),\\ &\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma,\alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta')\\ &\quad\equiv\sum_{\sigma\mu\nu}\sum_{\mu'\nu'}P(\alpha\beta\gamma|\sigma\mu\nu)f(\sigma\mu\nu,\mu'\nu'\gamma')P(\mu'\nu'|\alpha'\beta'),\end{aligned} \tag{4·7}$$

where $f(\alpha\beta\gamma, \alpha'\beta'\gamma')$ is an arbitrary function with respect to $(\alpha\beta\gamma, \alpha'\beta'\gamma')$. Combining Eqs. (4·6) and (3·6) and using the symmetry property of $\chi_{J=0}(\alpha\beta\gamma\delta)$ with respect to the permutation of $(\alpha\beta\gamma\delta)$, we obtain an equation to determine $\chi_{J=0}(\alpha\beta\gamma\delta)$ in terms of the physical amplitudes:

$$\boldsymbol{\phi}_{\lambda}-k\boldsymbol{C}^T\boldsymbol{\phi}_{\lambda}=0. \tag{4·8}$$

### 4-2 *Orthonormality relations of intrinsic modes of excitation*

The particular importance of prescription (2) manifests itself when we evaluate the following expression:

$$\{Y_\lambda, a^\dagger_\gamma\}_+|\Phi_0\rangle=\Big\{\frac{\sqrt{6}}{2}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}$$
$$+\frac{1}{\sqrt{2}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi^{(1)}_\lambda(\gamma_1\gamma_2\gamma_3)a^\dagger_{\gamma_2}a^\dagger_{\gamma_1}$$
$$+\sum_{\substack{\gamma_1\gamma_2\alpha\\(\alpha\neq c)}}\delta_{\gamma\gamma_1}\phi^{(2)}_\lambda(\gamma_1\gamma_2;\alpha)a^\dagger_{\tilde{\alpha}}a^\dagger_{\gamma_2}$$
$$+\sum_{\substack{(\alpha'\beta')\gamma'\\(a,b\neq c)}}\delta_{\gamma\gamma'}(1+\delta_{a'b'})^{-1/2}\phi^{(3)}_\lambda(\alpha'\beta';\gamma')a^\dagger_{\beta'}a^\dagger_{\tilde{\alpha}'}\Big\}|\Phi_0\rangle. \tag{4·9}$$

In this case we must evaluate the first term. With the aid of prescription (1), we first obtain

$$\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$
$$=\sqrt{6}\,k\sum_{\alpha'\beta'\gamma'}\sum_{\alpha\beta}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a^\dagger_{\tilde{\alpha}}a^\dagger_{\tilde{\beta}}|\Phi_0\rangle.$$

Prescription (2) then leads the right-hand side to

$$6k\sum_{\alpha'\beta'\gamma'}\sum_{\alpha\beta}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a^\dagger_{\tilde{\alpha}}a^\dagger_{\tilde{\beta}}|\Phi_0\rangle$$
$$\Rightarrow\sqrt{6}\,k\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\boldsymbol{P}(\gamma_1\gamma_2\gamma_3)\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_3}\chi_{J=0}(\gamma_1\gamma_2\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a^\dagger_{\gamma_1}a^\dagger_{\gamma_2}|\Phi_0\rangle$$
$$+2\sqrt{6}\,k\sum_{\substack{\gamma_1\gamma_2\alpha\\(\alpha\neq c)}}\delta_{\gamma\gamma_1}\boldsymbol{P}(\gamma_1\gamma_2)\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_1}\chi_{J=0}(\gamma_2\alpha\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a^\dagger_{\gamma_2}a^\dagger_{\tilde{\alpha}}|\Phi_0\rangle$$
$$+2\sqrt{6}\,k\sum_{\substack{(\alpha\beta)\\(a,b\neq c)}}\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a^\dagger_{\tilde{\alpha}'}a^\dagger_{\tilde{\beta}'}|\Phi_0\rangle$$
$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi^{(1)}_\lambda(\gamma_1\gamma_2\gamma_3)a^\dagger_{\gamma_1}a^\dagger_{\gamma_2}|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(\alpha\neq c)}}\delta_{\gamma\gamma_1}\phi^{(2)}_\lambda(\gamma_1\gamma_2;\alpha)a^\dagger_{\gamma_2}a^\dagger_{\tilde{\alpha}}|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi^{(3)}_\lambda(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a^\dagger_{\tilde{\alpha}'}a^\dagger_{\tilde{\beta}'}|\Phi_0\rangle,$$

where Eq. (4·8) is used in the last expression. Thus, we finally obtain

$$\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$
$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi^{(1)}_\lambda(\gamma_1\gamma_2\gamma_3)a^\dagger_{\gamma_1}a^\dagger_{\gamma_2}|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(\alpha\neq c)}}\delta_{\gamma\gamma_1}\phi^{(2)}_\lambda(\gamma_1\gamma_2;\alpha)a^\dagger_{\gamma_2}a^\dagger_{\tilde{\alpha}}|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi^{(3)}_\lambda(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a^\dagger_{\tilde{\alpha}'}a^\dagger_{\tilde{\beta}'}|\Phi_0\rangle, \tag{4·10}$$

so that Eq. (4·9) simply becomes

$$\{Y_\lambda, a_\nu^\dagger\}_+|\Phi_0\rangle=0. \tag{4·11}$$

We are now in a position to show that requirement (2·19) is satisfied. A direct calculation with the aid of Eq. (4·10) leads us to the relation

$$\begin{aligned}
&\{Y_{\lambda'}, Y_\lambda^\dagger\}_+|\Phi_0\rangle \\
&=\Big[\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\psi_{\lambda'}(\alpha\beta\gamma)+\frac{3}{2}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'} \\
&\quad+\frac{\sqrt{3}}{2}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\Big\{\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_{\lambda'}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\gamma_2}^\dagger a_{\gamma_1}^\dagger \\
&\quad+\sqrt{2}\sum_{\substack{\gamma_1\gamma_2\alpha'\\(\alpha'\neq c)}}\delta_{\gamma\gamma_1}\phi_{\lambda'}^{(2)}(\gamma_1\gamma_2;\alpha')a_{\tilde{\alpha}}^\dagger a_{\tilde{\gamma}_2}^\dagger \\
&\quad+\sqrt{2}\sum_{\substack{(\alpha'\beta')\gamma'\\(\alpha',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_{\lambda'}^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\tilde{\beta}}^\dagger a_{\tilde{\alpha}}^\dagger\Big\} \\
&\quad+\frac{\sqrt{3}}{2}\sum_\gamma\Big\{\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}a_{\tilde{\gamma}_2}+\sqrt{2}\sum_{\substack{\gamma_1\gamma_2\alpha\\(\alpha\neq c)}}\delta_{\gamma\gamma_1}\phi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}a_{\tilde{\alpha}} \\
&\quad+\sqrt{2}\sum_{\substack{(\alpha\beta)\gamma'\\(a,b\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_\lambda^{(3)}(\alpha\beta;\gamma')}{\sqrt{1+\delta_{ab}}}a_{\tilde{\alpha}}a_{\tilde{\beta}}\Big\}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}\Big]|\Phi_0\rangle \\
&=(\boldsymbol{\phi}_{\lambda'}^T\boldsymbol{\phi}_\lambda\ -\boldsymbol{\phi}_{\lambda'}^T\boldsymbol{\phi}_\lambda)|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle,
\end{aligned} \tag{4·12}$$

where all the terms with $O(n_0/2\Omega)\approx O(k^2/2\Omega)\approx 0$ have been dropped according to the basic approximation in the NTD method, and Eq. (3·6) has been used in the last relation.

4-3 *Orthogonality to collective degrees of freedom*

The ground-state correlation function $\chi_{J=0}(\alpha\beta\gamma\delta)$ should satisfy Eq. (4·4). Hence, with the aid of (4·5), we obtain

$$\hat{S}_-(a)|\Phi_0\rangle=0, \tag{4·13}$$

where $\hat{S}_-(a)$ is defined in Eq. (1·2·18). Equation (4·13) shows that the correlated ground state has no zero-coupled quasi-particle pairs. With the aid of Eq. (4·13), we have

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=[\hat{S}_-(a), Y_\lambda^\dagger]|\Phi_0\rangle. \tag{4·14}$$

Since the inner product of the state vectors on the right-hand side of Eq. (4·14) is of the order $O(n_0/2\Omega)\approx 0$, we can also see that the dressed 3-quasi-particle states have no zero-coupled pairs under the basic approximation $O(n_0/2\Omega)\approx 0$, i.e.,

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=0. \tag{4·15}$$

Furthermore, we can see that the "one-quasi-particle" states $Y_{s=1/2,\alpha}^\dagger|\Phi_0\rangle \equiv a_\alpha^\dagger|\Phi_0\rangle$ (with $\Delta_v=1$) also have no zero-coupled pairs, i.e.,

$$\hat{S}_{-}(a)a_{\alpha}^{\dagger}|\Phi_0\rangle=0, \tag{4·16}$$

because we have

$$\hat{S}_{-}(a)a_{\beta}^{\dagger}|\Phi_0\rangle=[\hat{S}_{-}(a),\ a_{\beta}^{\dagger}]|\Phi_0\rangle=\delta_{ab}a_{\bar{\beta}}|\Phi_0\rangle,$$

the inner product of which is of the order $O(n_0/2\Omega)\approx 0$ by the definition $\langle\Phi_0|a_{\alpha}^{\dagger}a_{\beta}|\Phi_0\rangle\cong\delta_{\alpha\beta}n_0/2\Omega$. Therefore, our quasi-particle NTD subspace, consisting of the modes with the transferred seniority $\Delta v(=2s)=1$ and 3, does not include any zero-coupled quasi-particle pairs within the basic approximation $O(n_0/2\Omega)\approx 0$. Thus, the subspace is orthogonal to any pairing-vibrational "collective" state.

## §5. Transcription of Hamiltonian and electromagnetic multipole operators into quasi-particle NTD subspace

### 5-1 *Quasi-particle NTD subspace*

The basis vectors of the quasi-particle NTD subspace under consideration are

$$\{Y_{s=1/2,\alpha}^{\dagger}|\Phi_0\rangle\equiv a_{\alpha}^{\dagger}|\Phi_0\rangle,\quad Y_{s=3/2,\lambda}^{\dagger}|\Phi_0\rangle\equiv Y_{\lambda}^{\dagger}|\Phi_0\rangle\}, \tag{5·1}$$

the orthonormality of which is satisfied (under the basic approximation $O(n_0/2\Omega)\approx 0$) because of Eqs. (4·11) and (4·12). The unit operator in this subspace is defined by

$$\mathbf{1}=\sum_{\alpha}\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\alpha}+\sum_{\lambda}\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda}, \tag{5·2}$$

where

$$\boldsymbol{a}_{\alpha}^{\dagger}=a_{\alpha}^{\dagger}|\Phi_0\rangle\langle\Phi_0|,\qquad \boldsymbol{Y}_{\lambda}^{\dagger}=Y_{\lambda}^{\dagger}|\Phi_0\rangle\langle\Phi_0|. \tag{5·3}$$

The elementary excitation operators $(\boldsymbol{a}_{\alpha}^{\dagger},\ \boldsymbol{Y}_{\lambda}^{\dagger})$ in the quasi-particle NTD subspace satisfy the relations

$$\boldsymbol{a}_{\alpha}|\Phi_0\rangle=\boldsymbol{Y}_{\lambda}|\Phi_0\rangle=0, \tag{5·4}$$

and

$$\begin{aligned}&\{\boldsymbol{Y}_{\lambda},\ \boldsymbol{Y}_{\lambda'}^{\dagger}\}_{+}|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle,\\ &\{\boldsymbol{a}_{\alpha},\ \boldsymbol{a}_{\beta}^{\dagger}\}_{+}|\Phi_0\rangle=\delta_{\alpha\beta}|\Phi_0\rangle,\\ &\{\boldsymbol{Y}_{\lambda},\ \boldsymbol{a}_{\alpha}^{\dagger}\}_{+}|\Phi_0\rangle=0.\end{aligned} \tag{5·5}$$

The non-repeatability of excitations is a trivial result and is expressed as

$$\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda'}^{\dagger}|\Phi_0\rangle=\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\beta}^{\dagger}|\Phi_0\rangle=\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{Y}_{\lambda}^{\dagger}|\Phi_0\rangle=0. \tag{5·6}$$

### 5-2 *Transcription rule into quasi-particle NTD subspace*

Now let us consider the transcription of a physical operator $\hat{F}$ (such

as the Hamiltonian and the electromagnetic multipole operators) into the NTD subspace. According to Eq. (2·23), it is necessary to evaluate the matrix elements $\langle\Phi_0|\, Y_{s\lambda}\hat{F}Y^{\dagger}_{s'\lambda'}|\Phi_0\rangle$ within the framework of the NTD approximation. For this purpose we must make full use of the properties of the eigenmode operators, such as relations (4·11) and (4·12). Hence, we rewrite the matrix element in the following two forms:

$$\langle\Phi_0|\, Y_{s_>\lambda}\hat{F}Y^{\dagger}_{s'_<\lambda'}|\Phi_0\rangle = \begin{cases} \langle\Phi_0|\{[Y_{s_>\lambda},\hat{F}],\ Y^{\dagger}_{s'_<\lambda'}\}_+|\Phi_0\rangle+\langle\Phi_0|\hat{F}\{Y_{s_>\lambda},\ Y^{\dagger}_{s'_<\lambda'}\}_+|\Phi_0\rangle, & (5\cdot7a) \\ \langle\Phi_0|\{Y_{s_>\lambda},[\hat{F},\ Y^{\dagger}_{s'_<\lambda'}]\}_+|\Phi_0\rangle+\langle\Phi_0|\{Y_{s_>\lambda},\ Y^{\dagger}_{s'_<\lambda'}\}_+\hat{F}|\Phi_0\rangle. & (5\cdot7b) \end{cases}$$

The evaluation of the first terms, which include a double commutator, is easily made. For the second terms, it is convenient to use the form (5·7a), because in general we obtain

$$\{Y_{s_<\lambda},\ Y^{\dagger}_{s'_>\lambda'}\}_+|\Phi_0\rangle\neq0 \quad \text{for} \quad s\neq s',$$

i.e.,

$$\langle\Phi_0|\{Y_{s_>\lambda},\ Y^{\dagger}_{s'_<\lambda'}\}_+\neq0 \quad \text{for} \quad s\neq s',$$

which is in contrast with the simple relation (2·19), i.e., relation (4·11). Therefore, we adopt the form (5·7a) and easily obtain

$$\langle\Phi_0|\, Y_{s_>\lambda}\hat{F}Y^{\dagger}_{s'_<\lambda'}|\Phi_0\rangle=\langle\Phi_0|\{[Y_{s_>\lambda},\hat{F}],\ Y^{\dagger}_{s'_<\lambda'}\}_+|\Phi_0\rangle +\delta_{ss'}\delta_{\lambda\lambda'}\langle\Phi_0|\hat{F}|\Phi_0\rangle. \tag{5·8}$$

This means the following transcription rule for evaluating the matrix elements $\langle\Phi_0|\, Y_{s\lambda}\hat{F}Y^{\dagger}_{s'\lambda'}|\Phi_0\rangle$: *First, we calculate the commutation relation between the physical operator $\hat{F}$ and the eigenmode operator of the higher transferred seniority number, and afterwards take the anti-commutation relation with the eigenmode operator of the lower transferred seniority number.*

## 5-3 *Transcribed operators*

Using the transcription rule, we obtain

$$\begin{aligned} &\langle\Phi_0|\, Y_{\lambda'}HY^{\dagger}_{\lambda}|\Phi_0\rangle=\{\omega_\lambda+\langle\Phi_0|H|\Phi_0\rangle\}\delta_{\lambda\lambda'}, \\ &\langle\Phi_0|\, a_\alpha Ha^{\dagger}_{\beta}|\Phi_0\rangle=\{E_\alpha+\langle\Phi_0|H|\Phi_0\rangle\}\delta_{\alpha\beta} \end{aligned} \tag{5·9}$$

and

$$\begin{aligned} &\langle\Phi_0|\, Y_\lambda Ha^{\dagger}_{\alpha}|\Phi_0\rangle\,(=\langle\Phi_0|\, Y_\lambda H_Y a^{\dagger}_{\alpha}|\Phi_0\rangle) \\ &\quad=-\sqrt{6}\,\textstyle\sum_{\alpha'\beta'\gamma'}V_Y(\alpha'\beta'\alpha\tilde{\gamma}')\psi_\lambda(\alpha'\beta'\gamma') \\ &\qquad+\sqrt{2}\,\textstyle\sum_{\alpha'_1\alpha'_2\alpha'_3}\{V_Y(\alpha'_1\alpha'_2\tilde{\alpha}'_3\alpha)-2V_Y(\alpha'_1\alpha'_2\alpha\tilde{\alpha}'_3)\}\phi^{(1)}_\lambda(\alpha'_1\alpha'_2\alpha'_3) \end{aligned}$$

$$-2\sum_{\substack{a_1'a_2'\gamma' \\ (a'\neq c')}}\{V_Y(a\tilde{\gamma}'a_1'a_2')+V_Y(\tilde{a}_2'aa_1'\gamma')+V_Y(a_1'\gamma'\tilde{a}_2'a)\}\phi_\lambda^{(2)}(a_1'a_2';\gamma')$$
$$+2\sum_{\substack{(a'\beta')\gamma' \\ (a',b'\neq c')}}\{V_Y(a'\beta'\tilde{\gamma}'a)+2V_Y(a\tilde{a}'\gamma'\beta')\}\frac{\phi_\lambda^{(3)}(a'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}. \qquad (5\cdot10)$$

According to Eq. (2·23), we can obtain the explicit form of the transcribed Hamiltonian in the quasi-particle NTD subspace:

$$\begin{aligned}\boldsymbol{H}&=\boldsymbol{1}H\boldsymbol{1}=U\cdot\boldsymbol{1}+\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}\\&\equiv U\cdot\boldsymbol{1}+\sum_a E_a\boldsymbol{a}_a^\dagger\boldsymbol{a}_a+\sum_\lambda\omega_\lambda\boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_\lambda+\sum_{a\lambda}V_{\mathrm{int}}(a,\lambda)\cdot(\boldsymbol{Y}_\lambda^\dagger\boldsymbol{a}_a+\boldsymbol{a}_a^\dagger\boldsymbol{Y}_\lambda),\end{aligned} \qquad (5\cdot11)$$

where $V_{\mathrm{int}}(a,\lambda)\equiv\langle\Phi_0|\,Y_\lambda Ha_a^\dagger|\Phi_0\rangle$, and $U$ is a constant related to the correlation energy of the ground state due to the dressed 3-quasi-particle modes. As seen from the matrix elements $V_{\mathrm{int}}(a,\lambda)$ given in Eq. (5·10), the *effective interaction* $\boldsymbol{H}^{(\mathrm{int})}$ between the different types of modes results from only the interactive force $H_Y$ of the original interaction.

The electromagnetic multipole operators are the one-body operators written in general as

$$\begin{aligned}\hat{O}_{LM}^{(\pm)}&=\sum_{a\beta}(a|\,O_{LM}^{(\pm)}|\beta)c_a^\dagger c_\beta\\&\equiv\sum_{a\beta}\{\mathring{O}_{LM}^{(\pm)}(a\beta)(a_a^\dagger a_\beta^\dagger\pm a_{\tilde{\beta}}a_{\tilde{a}})+\bar{O}_{LM}^{(\pm)}(a\beta)a_a^\dagger a_\beta\}\\&\quad+\sum_a(a|\,O_{LM}^{(\pm)}|a)v_a^2\cdot\frac{1\pm1}{2},\end{aligned} \qquad (5\cdot12)$$

where the double symbol $(\pm)$ is related to the conventional transformation property*) of the multipole operators with respect to the time reversal, and $\mathring{O}_{LM}^{(\pm)}$ and $\bar{O}_{LM}^{(\pm)}$ are defined respectively by

$$\begin{aligned}\mathring{O}_{LM}^{(\pm)}(a\beta)&=-\frac{1}{2}(a|\,O_{LM}^{(\pm)}|\tilde{\beta})(u_av_b\pm v_au_b),\\\bar{O}_{LM}^{(\pm)}(a\beta)&=(a|\,O_{LM}^{(\pm)}|\beta)(u_au_b\mp v_av_b).\end{aligned} \qquad (5\cdot13)$$

By definition, $\mathring{O}_{LM}^{(\pm)}(a\beta)$ satisfies the relation $\mathring{O}_{LM}^{(\pm)}(a\beta)=-\mathring{O}_{LM}^{(\pm)}(\beta a)$, and $\bar{O}_{LM}^{(\pm)}(a\beta)$ the relation $\bar{O}_{LM}^{(\pm)}(a\beta)=\pm\bar{O}_{LM}^{(\pm)}(\tilde{\beta}\tilde{a})$. With the aid of the transcription rule (5·8), we now obtain the transcribed electromagnetic multipole operators in the quasi-particle NTD subspace:

$$\begin{aligned}\hat{O}_{LM}^{(\pm)}\longrightarrow\hat{\boldsymbol{O}}_{LM}^{(\pm)}&=\boldsymbol{1}\hat{O}_{LM}^{(\pm)}\boldsymbol{1}\\&=C_{LM}^{(\pm)}\cdot\boldsymbol{1}+\sum_{a\beta}O_{LM}^{(\pm)}(a\beta)\boldsymbol{a}_a^\dagger\boldsymbol{a}_\beta+\sum_{\lambda\lambda'}O_{LM}^{(\pm)}(\lambda\lambda')\boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_{\lambda'}\\&\quad+\sum_{a\lambda}\{O_{LM}^{(\pm)}(a\lambda)\boldsymbol{a}_a^\dagger\boldsymbol{Y}_\lambda+O_{LM}^{(\pm)}(\lambda a)\boldsymbol{Y}_\lambda^\dagger\boldsymbol{a}_a\}.\end{aligned} \qquad (5\cdot14)$$

*) The time reversal property of the electromagnetic multipole operator $\hat{O}_{LM}$ is characterized by $T\hat{O}_{LM}T^\dagger=\tau(-)^M\hat{O}_{L\bar{M}}$, where $\tau=\pm1$. The operators $\hat{O}_{ML}^{(+)}$ and $\hat{O}_{LM}^{(-)}$ denote those with $\tau=+1$ and $\tau=-1$, respectively.

Full expressions of the coefficients $C^{(\pm)}_{LM}$, $O^{(\pm)}_{LM}(\alpha\beta)$, $O^{(\pm)}_{LM}(\lambda\lambda')$ and $O^{(\pm)}_{LM}(\alpha\lambda)$ are given in Appendix 2D.

Any physical operator can be transcribed in the same way into the quasi-particle NTD subspace.

## §6. Concluding remarks

On the basis of the quasi-particle NTD method, we have developed a systematic microscopic theory describing the collective excitations in spherical odd-mass nuclei. The theory has led us to a concept of a new kind of fermion-type collective excitation modes, in exactly the same manner as the RPA for even-even nuclei leads us to the concept of "phonon" as a boson. Needless to say, the framework of our theory includes that of the quasi-particle-phonon-coupling theory as a specially approximated version. The dressed 3-quasi-particle mode involves the phonon-like collective correlation and the three-quasi-particle correlation in a unified manner.

In Part III we apply this theory to describe the low-lying collective excited states in spherical odd-mass nuclei, and show that this unified picture plays an important role in clarifying the structure of low-lying states.

## Appendix 2A. Projection operators

In Eq. (3·2) we have used the projection operators, $\boldsymbol{P}(\alpha\beta\gamma)$ and $\boldsymbol{P}(\alpha\beta)$, defined by

$$\begin{aligned}&\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma)\equiv\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')f(\alpha'\beta'\gamma'),\\&\boldsymbol{P}(\alpha\beta)g(\alpha\beta)\equiv\sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')g(\alpha'\beta'),\end{aligned}\tag{2A·1}$$

by which arbitrary functions $f(\alpha\beta\gamma)$ and $g(\alpha\beta)$ are antisymmetrized with respect to $(\alpha, \beta, \gamma)$ and $(\alpha, \beta)$ respectively, and any angular-momentum-zero-coupled-pair component is removed from the antisymmetrized functions $f^{A}(\alpha\beta\gamma)$ and $g^{A}(\alpha\beta)$. Here we give their explicit definitions.

The antisymmetrization operator of three-body system is given by

$$P^{A}(\alpha\beta\gamma|\alpha'\beta'\gamma')=\frac{1}{3!}\sum_{\mathcal{P}(\alpha'\beta'\gamma')}\delta_{\mathcal{P}}\cdot\mathcal{P}(\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}),\tag{2A·2}$$

where $\sum_{\mathcal{P}(\alpha'\beta'\gamma')}$ denotes the summation over all the permutation with respect to $(\alpha', \beta', \gamma')$ and $\delta_{\mathcal{P}}$ takes the value $+1$ for even permutations and the value $-1$ for odd permutations. As is easily seen, this operator satisfies the relation of projection operator:

$$\sum_{\alpha''\beta''\gamma''}P^{A}(\alpha\beta\gamma|\alpha''\beta''\gamma'')P^{A}(\alpha''\beta''\gamma''|\alpha'\beta'\gamma')=P^{A}(\alpha\beta\gamma|\alpha'\beta'\gamma').\tag{2A·3}$$

In the coupled-angular-momentum representation, the antisymmetrization operator (2A·2) is represented by

$$\begin{aligned}P_I^A(ab(J)c|a'b'(J')c') &= \sum_{m_\alpha m_\beta m_\gamma}\sum_{m_{\alpha'}m_{\beta'}m_{\gamma'}}\sum_{MM'}(j_a j_b m_\alpha m_\beta|JM)\\ &\times(Jj_cMm_\gamma|IK)(j_{a'}j_{b'}m_{\alpha'}m_{\beta'}|J'M')(J'j_{c'}M'm_{\gamma'}|IK)P^A(\alpha\beta\gamma|\alpha'\beta'\gamma')\\ &= \frac{1}{3!}(1+\hat{\mathcal{P}}_{abJ})\Big[\delta_{aa'}\delta_{bb'}\delta_{cc'}\delta_{JJ}\\ &\quad+\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} j_c & j_b & J' \\ j_a & I & J\end{Bmatrix}(1+\hat{\mathcal{P}}_{bcJ'})\delta_{ac'}\delta_{bb'}\delta_{ca'}\Big],\end{aligned} \tag{2A·4}$$

where the operation of $\hat{\mathcal{P}}$ on any function $f$ is defined by

$$\hat{\mathcal{P}}_{abJ}f_J(ab) = -(-)^{j_a+j_b-J}f_J(ba). \tag{2A·5}$$

With the aid of Eq. (2A·4), the projection operator $P_I(ab(J)c|a'b'(J')c')$, which removes any angular-momentum-zero-coupled-pair component (from the functions on which it operates), is easily obtained as follows:

$$P_I(ab(J)c|a'b'(J')c') = P_I^A(ab(J)c|a'b'(J')c')$$

$$\begin{cases} +0 & \text{for } a\neq b\neq c\neq a,\\ -P_I^A(ab(J)c|a'b'(J')c')\delta_{J0} & \text{for } a=b\neq c,\\ -\dfrac{P_I^A(ab(0)c|a'b'(J')c')P_I^A(ab(J)c|a'b'(0)c')}{P_I^A(ab(0)c|a'b'(0)c')} & \text{for } a=b=c,\\ +(-)^{j_a+j_c+J}(2J+1)^{1/2}\begin{Bmatrix} j_a & j_b & J\\ I & j_c & 0\end{Bmatrix}P_I^A(ac(0)b|a'b'(J')c') & \text{for } a=c\neq b,\\ +(-)^{j_b+j_c}(2J+1)^{1/2}\begin{Bmatrix} j_b & j_a & J\\ I & j_c & 0\end{Bmatrix}P_I^A(bc(0)a|a'b'(J')c') & \text{for } a\neq b=c. \end{cases} \tag{2A·6}$$

The projection operators $P_I$ thus defined satisfy the following properties:

$$\text{i)}\quad \sum_{a''b''c''J''}P_I(ab(J)c|a''b''(J'')c'')P_I(a''b''(J'')c''|a'b'(J')c') = P_I(ab(J)c|a'b'(J')c'), \tag{2A·7a}$$

$$\text{ii)}\quad P_I(ab(J)c|a'b'(J')c') = P_I(a'b'(J')c'|ab(J)c), \tag{2A·7b}$$

$$\text{iii)}\quad P_I(ab(0)c|a'b'(J')c') = P_I(ab(J)c|a'b'(0)c') = 0. \tag{2A·7c}$$

With the expression (2A·6), the projection operator $P(\alpha\beta\gamma|\alpha'\beta'\gamma')$ in (2A·1) (in the $m$-scheme) is given through the relation

$$\begin{aligned}P_I(ab(J)c|a'b'(J')c') &= \sum_{m_\alpha m_\beta m_\gamma}\sum_{m_{\alpha'}m_{\beta'}m_{\gamma'}}\sum_{MM'}(j_a j_b m_\alpha m_\beta|JM)\\ &\times(Jj_cMm_\gamma|IK)(j_{a'}j_{b'}m_{\alpha'}m_{\beta'}|J'M')(J'j_{c'}M'm_{\gamma'}|IK)P(\alpha\beta\gamma|\alpha'\beta'\gamma').\end{aligned} \tag{2A·8}$$

The projection operator $P(\alpha\beta|\alpha'\beta')$ of two-body system is defined in a similar way and its explicit form in the coupled-angular-momentum representation is trivially given by

$$P_J(ab|a'b')=\frac{1}{2}(\delta_{aa'}\delta_{bb'}-(-)^{j_a+j_b-J}\delta_{ab'}\delta_{ba'})(1-\delta_{J0}). \tag{2A·9}$$

## Appendix 2B. Matrix elements of the secular equation for the dressed 3-quasi-particle modes

Here, we give the explicit forms for the matrix elements of $\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ in the eigenvalue equation (3·3).

With the definitions

$$V^{(fi)}_{\alpha\beta\alpha'\beta'}\equiv 2[V^{(G)}_{Xi}(\alpha\beta\alpha'\beta')+V^{(F)}_{Xi}(\alpha\beta\alpha'\beta')-V^{(F)}_{Xi}(\beta\alpha\alpha'\beta')], \tag{2B·1}$$

$$\begin{aligned}V^{(bi)}_{\alpha\beta\alpha'\beta'}\equiv 2[&V_{Vi}(\alpha\beta\alpha'\beta')+V_{Vi}(\alpha'\beta'\alpha\beta)-V_{Vi}(\alpha\tilde{\beta}'\tilde{\beta}\alpha')-V_{Vi}(\tilde{\beta}\alpha'\alpha\tilde{\beta}')\\&+V_{Vi}(\beta\tilde{\beta}'\tilde{\alpha}\alpha')+V_{Vi}(\tilde{\alpha}\alpha'\beta\tilde{\beta}')],\end{aligned} \tag{2B·2}$$

we first introduce the matrices $3\boldsymbol{D}^i$, $\boldsymbol{d}^i$ and $\boldsymbol{A}^i$, the elements of which are given as follows:

$$3D^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\boldsymbol{P}(\alpha\beta\gamma)[(E^i_a+E^i_b+E^i_c)\delta_{aa'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}+3V^{(fi)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}]\boldsymbol{P}^T(\alpha'\beta'\gamma'), \tag{2B·3a}$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha'_1\alpha'_2\alpha'_3}=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)[E^i_a\delta_{\alpha_1\alpha'_1}\delta_{\alpha_2\alpha'_2}\delta_{\alpha_3\alpha'_3}+V^{(fi)}_{\alpha'_1\alpha'_2\alpha_1\alpha_2}\delta_{\alpha'_3\alpha_3}]\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),$$

$$d^i_{\alpha_1\alpha_2\gamma,\alpha'_1\alpha'_2\gamma'}=\boldsymbol{P}(\alpha_1\alpha_2)[E^i_c\delta_{\alpha_1\alpha'_1}\delta_{\alpha_2\alpha'_2}\delta_{\gamma\gamma'}+2V^{(fi)}_{\alpha'_2\gamma'\alpha_2\gamma}\delta_{\alpha'_1\alpha_1}]\boldsymbol{P}^T(\alpha'_1\alpha'_2),$$

$$d^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}[(E^i_a+E^i_b-E^i_c)\delta_{aa'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}+2V^{(fi)}_{\alpha'\beta'\alpha\beta}\delta_{\gamma'\gamma}]\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}}, \tag{2B·3b}$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha'_1\alpha'_2\gamma'}=\sqrt{2}\,\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(fi)}_{\alpha'_2\gamma'\alpha_1\alpha_2}\delta_{\alpha'_1\alpha_3}\boldsymbol{P}^T(\alpha'_1\alpha'_2),$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha'\beta'\gamma'}=\sqrt{2}\,\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(fi)}_{\alpha'\beta'\alpha_1\alpha_2}\delta_{\gamma'\alpha_3}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

$$d^i_{\alpha_1\alpha_2\gamma,\alpha'\beta'\gamma'}=2\boldsymbol{P}(\alpha_1\alpha_2)V^{(fi)}_{\alpha'\beta'\alpha_2\gamma}\delta_{\gamma'\alpha_1}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

$$A^i_{\alpha\beta\gamma,\alpha'_1\alpha'_2\alpha'_3}=\sqrt{3}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha'_1\alpha'_2}\delta_{\gamma\alpha'_3}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),$$

$$A^i_{\alpha\beta\gamma,\alpha'_1\alpha'_2\gamma'}=\sqrt{6}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha'_2\gamma'}\delta_{\gamma\alpha'_1}\boldsymbol{P}^T(\alpha'_1\alpha'_2), \tag{2B·3c}$$

$$A^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\sqrt{6}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

where we have used the abbreviations for the projection operators in (3·2),

for simplicity. The matrix elements of $\mathbf{3D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ are then given by the following replacements in (2B·3):

$$V_{Xi}^{(G)}(\alpha\beta\gamma\delta) \Rightarrow V_X^{(G)}(\alpha\beta\gamma\delta), \qquad V_{Xi}^{(F)}(\alpha\beta\gamma\delta) \Rightarrow V_X^{(F)}(\alpha\beta\gamma\delta),$$
$$V_{Yi}(\alpha\beta\gamma\delta) \Rightarrow V_Y(\alpha\beta\gamma\delta), \qquad E_a^i \Rightarrow E_a, \tag{2B·4}$$

where $V_X^{(G)}(\alpha\beta\gamma\delta)$, $V_X^{(F)}(\alpha\beta\gamma\delta)$ and $V_Y(\alpha\beta\gamma\delta)$ are given after Eq. (3·4) in Chap. 1.

It is obvious that the suffix $i$ of $V$ and $E$ is completely superfluous in the above equations. The suffix $i$ has been used here merely from formal point of view and its usefulness will become clear in Appendix 7A.

## Appendix 2C. Interaction between the dressed 3-quasi-particle mode and the single-quasi-particle mode

Here, we give the explicit form of the matrix element $V_{\text{int}}(\alpha, \lambda)$ in Eq. (5·11) in a formally convenient way.

Let us first evaluate the matrix element of the following operators:

$$K_Y = \sum_{\alpha\beta\gamma\delta} \{V_{Y1}(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_{\tilde{\delta}}^\dagger a_\gamma + V_{Y2}(\alpha\beta\gamma\delta) a_\gamma^\dagger a_{\tilde{\delta}} a_\beta a_\alpha\}. \tag{2C·1}$$

With the aid of the transcription rule given in §5-2, we obtain

$$\begin{aligned}\langle\Phi_0 | Y_\lambda K_Y a_\alpha^\dagger | \Phi_0\rangle &= \langle\Phi_0 | \{[Y_\lambda, K_Y], a_\alpha^\dagger\}_+ | \Phi_0\rangle \\ &= (\boldsymbol{\psi}_\lambda^T, \boldsymbol{\phi}_\lambda^T)\cdot\boldsymbol{B}(\alpha),\end{aligned} \tag{2C·2}$$

where

$$\boldsymbol{B}(\alpha') = \begin{pmatrix} \boldsymbol{B}^1(\alpha') \\ \boldsymbol{B}^2(\alpha') \end{pmatrix} \tag{2C·3}$$

the elements of which are defined by

$$\begin{aligned}
B^1_{\alpha\beta\gamma}(\alpha') &= -\boldsymbol{P}(\alpha\beta\gamma)\sqrt{6}\, V_{Y1}(\alpha\beta\alpha'\tilde{\gamma}), \\
B^2_{\alpha_1\alpha_2\alpha_3}(\alpha') &= -\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\sqrt{2}\, \{V_{Y2}(\tilde{\alpha}_1\tilde{\alpha}_2\alpha_3\tilde{\alpha}') - 2V_{Y2}(\alpha'\tilde{\alpha}_1\alpha_3\alpha_2)\}, \\
B^2_{\alpha_1\alpha_2\gamma}(\alpha') &= -\boldsymbol{P}(\alpha_1\alpha_2) 2\{V_{Y2}(\tilde{\alpha}_2\tilde{\gamma}\alpha_1\tilde{\alpha}') - V_{Y2}(\alpha'\tilde{\alpha}_2\alpha_1\gamma) + V_{Y2}(\alpha'\tilde{\gamma}\alpha_1\alpha_2)\}, \\
B^2_{\alpha\beta\gamma}(\alpha') &= -\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}} 2\{V_{Y2}(\tilde{\alpha}\tilde{\beta}\gamma\tilde{\alpha}') - V_{Y2}(\alpha'\tilde{\alpha}\gamma\beta) + V_{Y2}(\alpha'\tilde{\beta}\gamma\alpha)\}.
\end{aligned} \tag{2C·4}$$

The matrix element of the interaction, $V_{\text{int}}(\alpha, \lambda)$ in Eq. (5·11), is then compactly given by

$$V_{\text{int}}(\alpha, \lambda) = (\boldsymbol{\psi}_\lambda^T, \boldsymbol{\phi}_\lambda^T)\cdot\boldsymbol{B}(\alpha) \tag{2C·5}$$

with the following replacements in (2C·4):

$$V_{Y1}(\alpha\beta\gamma\delta)=V_{Y2}(\alpha\beta\gamma\delta)\Rightarrow V_Y(\alpha\beta\gamma\delta), \tag{2C·6}$$

where $V_Y(\alpha\beta\gamma\delta)$ is given after Eq. (3·4) in Chap. 1.

The operator $K_Y$ has been used here merely from formal point of view and its usefulness will become clear in Appendix 7A.

## Appendix 2D. Matrix elements of electromagnetic multipole operators

Here, we give the explicit forms of the coefficients in Eq. (5·14).

Let us first evaluate the matrix elements of the following operators:

$$\mathring{F}^{(\pm)}_{LM}=\sum_{\alpha\beta}\{\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)a^\dagger_\alpha a^\dagger_\beta\pm\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)a_{\tilde\beta}a_{\tilde\alpha}\}, \tag{2D·1}$$

$$\bar F^{(\pm)}_{LM}=\sum_{\alpha\beta}\bar F^{(\pm)}_{LM}(\alpha\beta)a^\dagger_\alpha a_\beta. \tag{2D·2}$$

With the aid of the transcription rule given in §5-2, we obtain

$$\begin{aligned}\langle\Phi_0|\,Y_\lambda\mathring{F}^{(\pm)}_{LM}a^\dagger_{\alpha'}|\Phi_0\rangle&=\sqrt{6}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)\\&\pm\{\sqrt{2}\sum_{\alpha_1\alpha_2\alpha_3}\phi^{(1)}_\lambda(\alpha_1\alpha_2\alpha_3)\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\delta_{\alpha_3\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha_1\alpha_2)\\&+2\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\phi^{(2)}_\lambda(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\delta_{\alpha_1\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha_2\gamma)\\&+2\sum_{\substack{(\alpha\beta)\gamma\\(a\neq c,b\neq c)}}\phi^{(3)}_\lambda(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)/\sqrt{1+\delta_{ab}}\,\},\end{aligned} \tag{2D·3a}$$

$$\begin{aligned}\langle\Phi_0|\,a_{\alpha'}\mathring{F}^{(\pm)}_{LM}Y^\dagger_\lambda|\Phi_0\rangle&=\pm\sqrt{6}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{2LM}(\tilde\alpha\tilde\beta)\\&+\sqrt{2}\sum_{\alpha_1\alpha_2\alpha_3}\phi^{(1)}_\lambda(\alpha_1\alpha_2\alpha_3)\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\delta_{\alpha_3\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha_1\tilde\alpha_2)\\&+2\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\phi^{(2)}_\lambda(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\delta_{\alpha_1\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha_2\tilde\gamma)\\&+2\sum_{\substack{(\alpha\beta)\gamma\\(a\neq c,b\neq c)}}\phi^{(3)}_\lambda(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha\tilde\beta)/\sqrt{1+\delta_{ab}}\end{aligned} \tag{2D·3b}$$

and

$$\langle\Phi_0|\,Y_\lambda\bar F^{(\pm)}_{LM}Y^\dagger_{\lambda'}|\Phi_0\rangle=(\boldsymbol{\phi}^T_\lambda,\boldsymbol{\phi}^T_\lambda)\begin{bmatrix}\boldsymbol{F}&0\\0&\boldsymbol{f}\end{bmatrix}\begin{bmatrix}\boldsymbol{\phi}_{\lambda'}\\\boldsymbol{\phi}_{\lambda'}\end{bmatrix}, \tag{2D·4}$$

where the matrix elements of $\boldsymbol{F}$ and $\boldsymbol{f}$ are respectively defined by

$$F_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=3\boldsymbol{P}(\alpha\beta\gamma)\bar F^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}\boldsymbol{P}^T(\alpha'\beta'\gamma'), \tag{2D·5a}$$

$$f_{a_1a_2a_3,a'_1a'_2a'_3}=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\{2\bar F^{(\pm)}_{LM}(\tilde\alpha'_3\tilde\alpha_3)-\bar F^{(\pm)}_{LM}(\alpha_3\alpha'_3)\}\delta_{a_1a'_1}\delta_{a_2a'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),$$

$$f_{a_1a_2a_3,a'_1a'_2\gamma'}=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\sqrt{2}\,\bar F^{(\pm)}_{LM}(\tilde\gamma'\tilde\alpha_3)\delta_{a_1a'_1}\delta_{a_2a'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2),$$

$$f_{a_1a_2\gamma,a'_1a'_2a'_3}=\boldsymbol{P}(\alpha_1\alpha_2)\sqrt{2}\,\bar F^{(\pm)}_{LM}(\tilde\alpha'_3\tilde\gamma)\delta_{a_1a'_1}\delta_{a_2a'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),$$

$$f_{a_1a_2a_3,\alpha'\beta'\gamma'}=-\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\sqrt{2}\,\bar F^{(\pm)}_{LM}(\alpha_3\gamma')\delta_{a_1\alpha'}\delta_{a_2\beta'}\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{a'b'}},$$

$$f_{\alpha\beta\gamma,a'_1a'_2a'_3}=-\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\sqrt{2}\,\bar F^{(\pm)}_{LM}(\gamma\alpha'_3)\delta_{\alpha a'_1}\delta_{\beta a'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),$$

$$
\begin{aligned}
f_{\alpha_1\alpha_2\gamma,\alpha_1'\alpha_2'\gamma'} &= \boldsymbol{P}(\alpha_1\alpha_2)\,[\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}_1'\tilde{\alpha}_1)-\bar{F}^{(\pm)}_{LM}(\alpha_1\alpha_1')\}\delta_{\alpha_2\alpha_2'}\delta_{\gamma\gamma'} \\
&\quad +\bar{F}^{(\pm)}_{LM}(\tilde{\gamma}'\tilde{\gamma})\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}+\bar{F}^{(\pm)}_{LM}(\alpha_1\alpha_1')\delta_{\alpha_2\gamma'}\delta_{\gamma\alpha_2'}]\boldsymbol{P}^T(\alpha_1'\alpha_2'), \\
f_{\alpha_1\alpha_2\gamma,\alpha'\beta'\gamma'} &= \boldsymbol{P}(\alpha_1\alpha_2)2\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}'\tilde{\alpha}_2)\delta_{\alpha_1\gamma'}\delta_{\gamma\beta'} \\
&\quad -\bar{F}^{(\pm)}_{LM}(\alpha_1\gamma')\delta_{\alpha_2\alpha'}\delta_{\gamma\beta'}\}\,\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{\alpha'b'}}, \\
f_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'} &= \frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}2\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}_2'\tilde{\alpha})\delta_{\gamma\alpha_1'}\delta_{\beta\gamma'}-\bar{F}^{(\pm)}_{LM}(\gamma\alpha_1')\delta_{\alpha\alpha_2'}\delta_{\beta\gamma'}\}\,\boldsymbol{P}^T(\alpha_1'\alpha_2'), \\
f_{\alpha\beta\gamma,\alpha'\beta'\gamma'} &= \frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}2\{2\bar{F}^{(\pm)}_{LM}(\tilde{\beta}'\tilde{\beta})\delta_{\alpha\alpha'}\delta_{\gamma\gamma'} \\
&\quad -\bar{F}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}\}\,\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{a'b'}}\,. \qquad \text{(2D·5b)}
\end{aligned}
$$

The explicit forms of the coefficients in Eq. (5·14) are then given by

$$
\begin{aligned}
&C^{(\pm)}_{LM}\equiv\textstyle\sum_\alpha(\alpha|O^{(\pm)}_{LM}|\alpha)v_\alpha^2\cdot\dfrac{1\pm1}{2}, \\
&O^{(\pm)}_{LM}(\alpha\beta)\equiv\langle\Phi_0|a_\alpha\hat{O}^{(\pm)}_{LM}a^\dagger_\beta|\Phi_0\rangle-C^{(\pm)}_{LM}\delta_{\alpha\beta}=\bar{O}^{(\pm)}_{LM}(\alpha\beta), \\
&O^{(\pm)}_{LM}(\lambda\alpha)\equiv\langle\Phi_0|Y_\lambda\hat{O}^{(\pm)}_{LM}a^\dagger_\alpha|\Phi_0\rangle=\langle\Phi_0|Y_\lambda\mathring{F}^{(\pm)}_{LM}a^\dagger_\alpha|\Phi_0\rangle, \\
&O^{(\pm)}_{LM}(\alpha\lambda)\equiv\langle\Phi_0|a_\alpha\hat{O}^{(\pm)}_{LM}Y^\dagger_\lambda|\Phi_0\rangle=\langle\Phi_0|a_\alpha\mathring{F}^{(\pm)}_{LM}Y^\dagger_\lambda|\Phi_0\rangle, \\
&O^{(\pm)}_{LM}(\lambda\lambda')\equiv\langle\Phi_0|Y_\lambda\hat{O}^{(\pm)}_{LM}Y^\dagger_{\lambda'}|\Phi_0\rangle-C^{(\pm)}_{LM}\delta_{\lambda\lambda'}=\langle\Phi_0|Y_\lambda\bar{F}^{(\pm)}_{LM}Y^\dagger_{\lambda'}|\Phi_0\rangle \qquad \text{(2D·6)}
\end{aligned}
$$

with the following replacements in (2D·3) and (2D·4):

$$
\begin{aligned}
&\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)=\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)\Rightarrow\mathring{O}^{(\pm)}_{LM}(\alpha\beta), \\
&\bar{F}^{(\pm)}_{LM}(\alpha\beta)\Rightarrow\bar{O}^{(\pm)}_{LM}(\alpha\beta),
\end{aligned}
\qquad \text{(2D·7)}
$$

where $\mathring{O}^{(\pm)}_{LM}(\alpha\beta)$ and $\bar{O}^{(\pm)}_{LM}(\alpha\beta)$ are defined by (5·13).

The operators, $\mathring{F}$ and $\bar{F}$, have been used here merely from formal point of view and its usefulness will become clear in Appendix 7B.

# Part III.
# Analysis of Low-Lying States in Spherical Odd-Mass Nuclei
## Chapter 3. Structure of the Anomalous Coupling States with Spin $I=(j-1)$

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## §1. Introduction

### 1–1 *Outline*

According to the $j$-$j$ coupling shell model, a high-$j$ orbit having parity opposite to that of the other orbits appears systematically in each major shell. (See Fig. 1.) When this unique-parity orbit, such as $1f^{-}_{7/2}$ and $1g^{+}_{9/2}$, is filled with nucleons in odd numbers, a competition between a spin $j$- and a spin $(j-1)$-state for the ground state occurs quite regularly. Such extra low-lying states with spin $I=(j-1)$ and with unique parity have been called the anomalous coupling (AC) states.

The AC states are well known as the typical phenomena which cannot be interpreted within the framework of the conventional quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen.[1)]

The main purpose of this chapter*) is to introduce a new microscopic model of the AC states in the light of recent experimental developments illuminating the structure of the AC states. In the microscopic model proposed here, the AC states are regarded as *typical manifestations* of the dressed three-quasi-particle (3QP) modes which have

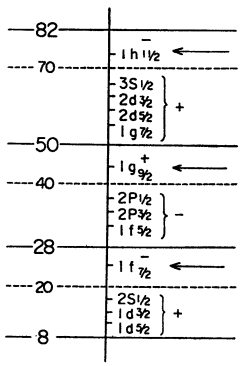



Fig. 1. Schematic representation of shell structure. The arrow denotes the high-spin, unique-parity orbit in each major shell.

*) The content of this chapter was already published by the present authors in Prog. Theor. Phys. **47** (1972), 498; **51** (1974), 779.

been introduced in Chap. 2. It is shown that, under the special condition of shell structure for the appearance of the AC states, the dressed 3QP modes manifest themselves as relatively pure eigenmodes without coupling to the single-quasi-particle (1QP) modes. Then, in the same manner as the $2^+$ phonon modes (the dressed two-quasi-particle modes) are regarded as elementary excitations in spherical even-even nuclei, the AC states are regarded as typical phenomena which exhibit the elementary modes of low-energy collective excitations in spherical odd-mass nuclei. From this point of view, the mechanism of appearance of the collective 3QP correlation, which is responsible for the particular favouring of the spin $(j-1)$ states, and the process of its growth are clarified.

After a short summary of the recent experimental evidences showing collective character of the AC states, the motive for introducing the new microscopic model is discussed in §2 in connection with the picture of phonon-quasi-particle coupling. In §3, starting with the $j$-$j$ coupling shell model with the pairing-plus-quadrupole (P+QQ) force,[3] we formulate the microscopic model of the AC states in a concrete form by using the general theory developed in Chap. 2.*) It is shown that the model introduced involves two essential characteristics of the AC states in a unified manner: One characteristic aspect represented by Kisslinger's 3QP "intruder state"[4] and the other characteristic aspect of strong collectiveness underlying the quasi-particle-phonon-coupling state.[1),2)] Furthermore, by investigating the stability of the spherical BCS vacuum against the collective 3QP correlation, we point out an interesting relation between our new viewpoint and the Bohr-Mottelson's old suggestion[5] concerning the possible connection between the appearance of the spin $(j-1)$ state as the ground state and the onset of quadrupole deformation. In the course of these, the relations between our microscopic model and the recent works based on the semi-microscopic models[6),7)] (which start from the particle-vibration coupling Hamiltonian[8]) are also discussed by putting special emphasis on their underlying picture for the AC states.

It is shown in §4 that the model introduced can give us an intuitive and perspective understanding of the characteristics of electromagnetic properties of the AC states. The theoretical predictions given there are examined in §5 by comparing results of numerical calculations with available experimental data. Here, special attention is paid to the systematical agreement with the common properties of the AC states observed in the experiments over a wide range of spherical odd-mass nuclei rather than numerical agreement with the experimental value at a specific nucleus. In the theoretical calculation, the coupling effect coming from the 1QP mode in the next upper

*) Throughout Part III, we take up the first term, $H$, of $H_{\mathrm{intr}}$ (1·6·14) as the intrinsic Hamiltonian, in the same way as in Chap. 2.

major shell, such as $1g^{+}_{7/2}$ and $1h^{-}_{9/2}$, on the dressed 3QP mode with $I=(j-1)$ is also taken into account, and, therewith, the smallness of its mixing effect on the properties of the AC states is examined. The results clearly show how we can understand the various properties of the AC states in a unified manner within the framework of the microscopic model proposed on the basis of the theory developed in Chap. 2. Thus we conclude in §6 that, in the first order approximation, the AC states with $I=(j-1)$ can very well be recognized as the typical manifestations of the dressed 3QP modes.

### 1-2 *Finding of collective nature of AC states*

Since the special lowering of the spin $(j-1)$ state is in clear contrast to the simple pairing-coupling scheme which favours the spin $j$ state characterized by the seniority $v=1$, the phenomena showing the competition between the spin $j$ and $(j-1)$ states have been discussed with special interest from the viewpoint of the nuclear coupling scheme. It has been known from the very beginning of the proposal of the $j$-$j$ coupling shell model, that one of the possibilities of reproducing the extremely low-lying $(j-1)$ state is to introduce a sufficiently long-range effective force in multi-nucleon configurations $j^n$.[9)~15)] It was shown that, within the $j^3$ configurations, the spin $(j-1)$ state characterized by the seniority $v=3$ is especially lowered in energy as the range of the effective force becomes larger.[10)~15)] Kisslinger's interpretation of the $(j-1)$ state as the 3QP "intruder" state[4)] may be regarded as a model in terms of the P+QQ force, elaborated along this line of development.

Another possibility of explaining the AC states is to introduce a possibility of quadrupole deformations in nuclei: In view of the fact that the lowest state of $j^3$ configurations in the oblately deformed potential has the spin $I=K=j-1$ (the aligned coupling scheme), Bohr and Mottelson suggested[5)] a possible connection between the appearance of the $(j-1)$ state as the ground state and the onset of the quadrupole deformation. From this point of view, the competition between the $j$ and $(j-1)$ states is considered as a phenomenon reflecting directly the growth of quadrupole instability.

Now let us first make a systematics of the excitation energies of the $(j-1)$ states on the basis of recent accumulation of experimental data. In Fig. 2 are presented the excitation energies of the $7/2^+$ states measured from those of the 1QP $9/2^+$ states, as a function of the neutron number $N$. These $7/2^+$ states in the odd-proton Tc, Rh and Ag isotopes are the most well-known examples of the AC states with spin $I=(j-1)$. We can then notice a striking similarity between the behaviour of the excitation energies of the $2^+$ phonon states in the sequence of even-even nuclei and that of the $(j-1)$ states in the sequence of odd-mass nuclei: As a function of $N$, the excitation energies of the $7/2^+$ states change in quite a parallel way to those of the $2^+$

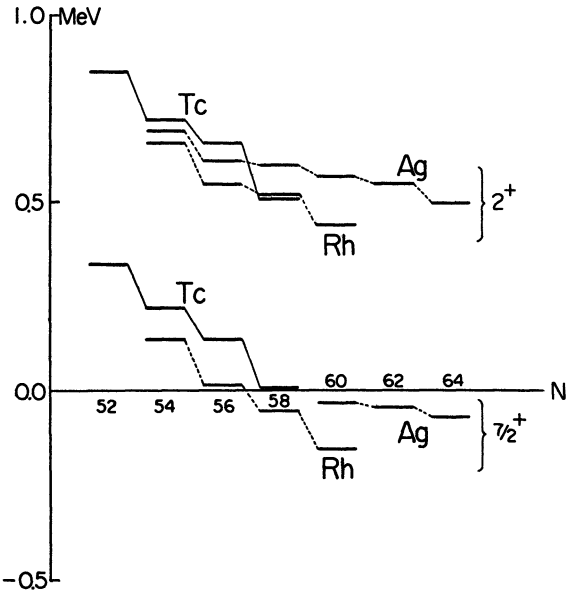



Fig. 2. Comparison between the excitation-energy systematics of the $7/2^+$ states and those of $2^+$ phonon states. The phonon-energies presented are the average values between the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(N, Z) = 1/2\{\omega_{2^+}(N, Z-1)+\omega_{2^+}(N, Z+1)\}$. The energies of the $7/2^+$ states are those measured from the 1QP $9/2^+$ states.

phonon states, aside from the fact that they are shifted down about 0.5 MeV compared to those of $2^+$ phonons. This similarity indicates a collective character of the $7/2^+$ states which is difficult to understand with either of the two interpretations mentioned above.

In addition to the well-known $7/2^+$ states in nuclei belonging to the $1g^+_{9/2}$-region, recent experimental works have revealed a number of new examples of the AC states with spin $9/2^-$ in odd-neutron Cd, Te and Xe isotopes belonging to the $1h^-_{11/2}$-region. The excitation energies of the $9/2^-$ states measured from the 1QP $11/2^-$ states show also the trend similar to that of the $2^+$ phonon states in the neighbouring even-even nuclei.

Furthermore, recent measurements on various electromagnetic properties of the AC states have been providing us important information, directly showing their collective character. One of the most important findings is that the $E2$ transitions from the $(j-1)$ states to the 1QP states with spin $j$ are strongly enhanced while the corresponding $M1$ transitions are moderately hindered. The amount of enhancement of the $E2$ transitions is comparable (or somewhat larger) to that of the $E2$ transitions from the $2^+$ phonon states to the ground states in the adjacent even-even nuclei. Thus, the strongly collective nature of the AC states has been clearly exhibited.

A possible origin of the striking $E2$ enhancements from the $(j-1)$ states may be ascribable to the quasi-particle-phonon-coupling nature based on the theory of Kisslinger and Sorensen.[1] However, the special energy-lowering of the AC states with spin $I=(j-1)$ has not at all been accounted for within the framework of the conventional QPC theory of Kisslinger and Sorensen. Considering the striking $E2$ enhancement as an essential characteristic of the AC states, Sano and Ikegami[16),17)] carried out the calculation based on the conventional QPC theory by enlarging the shell model space to several major shells. An extension of the conventional QPC theory to another direction has also been attempted by different authors.[18)~20)] However, it turnd out later,[21),22)] that it is difficult to ensure the conditions for eliminating the spurious states and for satisfying the Pauli principle between the quasi-particles composing the phonon and the odd quasi-particle, within a mere formal extension of the framework of the conventional QPC theory.

The three different kinds of approaches mentioned above have succeeded in explaining partial aspects of the $(j-1)$ state. That is, for the special favouring of the $I=(j-1)$ coupling in $j^3$ configuration, the spherical shell model with a long-range effective force, for the possible appearance of the $(j-1)$ state as the ground state, the aligned coupling scheme in the deformed model, and for the strong collectiveness exhibited by the enhancement of $B(E2;\ j-1\to j)$, the quasi-particle-phonon-coupling model. However, their mutual relationships have not yet been clarified and, therewith, the essential understanding of the structure of the AC states has not been achieved.

In the following, we first investigate the missing effect of the conventional QPC theory of Kisslinger and Sorensen, which is the main cause for the special favouring of the $(j-1)$ state.

## §2. A new type of quasi-particle-phonon-coupling giving rise to collective 3QP correlations

An important effect of the quasi-particle-phonon coupling, which has been neglected for a long time, was emphasized by Bohr and Mottelson[23)] in 1967: "In the *phenomenological* phonon-quasi-particle coupling model, the lowest-order-perturbation effects which contribute to all the energies of the different states of the multiplet composed of the odd quasi-particle and the one-phonon, are shown in Figs. 3A and 3B. The diagrams of the type A are nothing but the conventional ones which have so far been treated as 'phonon-quasi-particle coupling' in the QPC theory of Kisslinger and Sorensen, while the diagrams of the type B never appear in this QPC theory. The physical effect underlying the diagrams B is that *the phonon disassociates into a pair of quasi-particles, one of which reassociates with the odd quasi-*

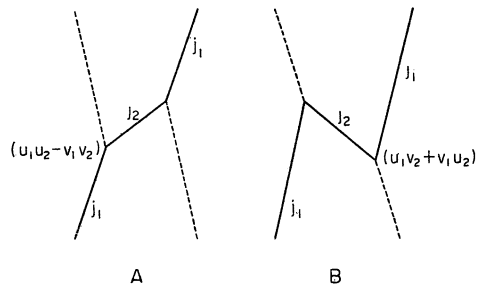



Fig. 3A. Contribution of lowest order. The solid and broken lines represent the quasi-particle and the phonon, respectively. This type of particle-vibration coupling is accompanied by the reduction factor $(u_1u_2-v_1v_2)$.
Fig. 3B. Contribution of lowest order. This type of particle-vibration coupling is accompanied by the enhancement factor $(u_1v_2+v_1u_2)$. Both (A) and (B) are taken from reference 23).

*particle while the remaining quasi-particle is now the odd quasi-particle.* This effect is essentially based on the Pauli principle between the quasi-particle composing the phonon and the odd quasi-particle. The extreme importance of the diagrams of type B can be recognized as follows. The diagrams of type A consist of the factor $(u_1u_2-v_1v_2)$ which can be quite small, while the diagrams of type B involve the coupling with the factor $(u_1v_2+v_1u_2)$ which is close to unity for low-lying states in the middle of the shell. Thus it is likely that the description of collective excited states of almost all spherical odd-mass nuclei is significantly effected by the inclusion of the effect."

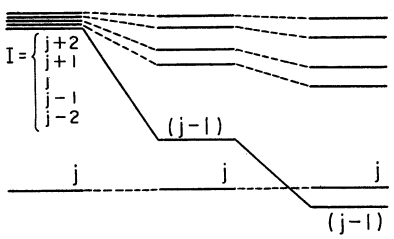
In the conventional QPC theory, the phonon is regarded as the ideal boson described by the random-phase approximation (RPA) and is commutable with the odd quasi-particle. Therefore, the effect which underlies the diagrams of type B and is based on the Pauli principle between odd quasi-particle and quasi-particles composing the phonon is *in principle* not taken into account within the framework of the theory. From the viewpoint of boson expansion methods in odd-mass nuclei, such effect is called "kinematical anharmonicity effect" based on a new type of quasi-particle-phonon coupling which is derived from two types of the original interaction, $H_X$ and $H_Y$ in Eq. (1·3·4), represented in Fig. 7 in §3. The new type of coupling is in clear contrast to the coupling in the conventional QPC theory, which is derived from the original interaction, $H_Y$ in Eq. (1·3·4), represented in Fig. 7 and causes "dynamical anharmonicity effect." The significance of the new type of coupling has also been emphasized in the course of investigating the "anharmonicity effects" in terms of the boson expansion method.

Of course, as the kinematical anharmonicity effect becomes more significant, the higher order diagrams of the type B must be taken into account.

Therefore, in such a situation, it is required to take the essential effect into account not by the perturbation approximation but by diagonalizing the Hamiltonian in a "certain subspace," in such a way that we adopt the new Tamm-Dancoff approximation (i.e., the RPA) when constructing the phonon modes in even-even nuclei.

It is now easy to recognize that the dominant cause which brings about the special favouring of the spin $(j-1)$ state is nothing but the effect of type B: Let us operate the effect on the degenerate multiplet composed of the odd quasi-particle and the one-phonon. Then, as is shown in §3, the $(j-1)$ state in the multiplet is affected most strongly and its excitation energy is extremely lowered as the effect becomes stronger. The AC states with $I=(j-1)$ are regarded as the phenomena in which the effect grows extremely. In fact we see the experimental data clearly exhibiting this process, for example, in the cases of Nb-Tc-Rh isotopes in §5.

In order to investigate the effect of type B on the basis of the microscopic theory, let us replace the phonon line in the diagrams B in Fig. 3 with the conventional correlated-two-quasi-particle line shown in Fig. 5, by taking account of the composite structure of the phonon. Then the diagrams B can be decomposed into the corresponding microscopic diagrams in Fig. 6.




Eig. 4.

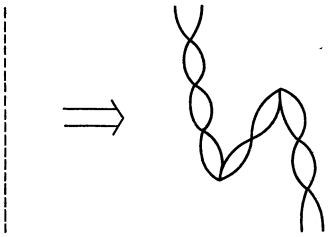


Fig. 5.

Fig. 4. Schematic illustration showing the relation between the process of growth of the 3QP correlation and the increase of the splitting of the multiplet composed of phonon plus odd quasi-particle.

Fig. 5. Representation of the phonon as a correlated two quasi-particles.

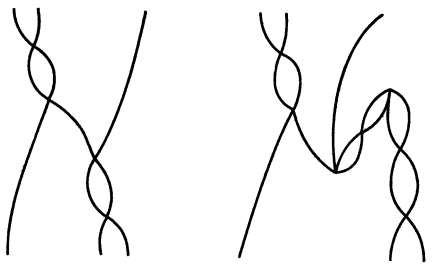


Fig. 6. Microscopic structure of diagram 3B.

The structure of the diagrams in Fig. 6 shows that they are composed of only two types of the quasi-particle interaction, $H_X$ and $H_V$, which are also well known to be responsible for the phonon mode in even-even nuclei. The situation never changes even when we take account of any higher order diagram of type B, and the excited state corresponding to any such diagram is always represented as a superposition of the particle states with 3, 7, 11, 15, ... quasi-particles. These considerations lead us to the conclusion that if we succeed in constructing the correlated three-quasi-particle mode, including the corresponding ground-state correlation in the framework of the new-Tamm-Dancoff (NTD) approximation (by using the two types of the quasi-particle interaction $H_X$ and $H_V$), the above mentioned requirement that the effect of type B should be taken into account not by the perturbation but by diagonalizing the Hamiltonian in a "certain subspace" is satisfied in a very suitable way. Thus we are now at a position to introduce a new microscopic model of the AC states with spin $I=(j-1)$ as typical manifestations of the dressed 3QP modes formulated in Chap. 2.

## §3. Microscopic model of AC states as dressed 3QP modes

### 3–1 *The Hamiltonian*

Let us start with the spherically symmetric $j$-$j$ coupling shell-model Hamiltonian with the pairing-plus-quadrupole (P+QQ) force in the quasi-particle representation:

$$\begin{aligned} H &= H_0 + :H_{QQ}: \\ &= \sum_{\alpha} E_a a_\alpha^\dagger a_\alpha - \frac{1}{2}\chi \sum_M :\hat{Q}_{2M}^\dagger \hat{Q}_{2M}:, \end{aligned} \tag{3·1}$$

where $\chi$ is the strength of the quadrupole force, and $E_a$ is the quasi-particle energy, determined as usual together with the parameters $u_a$ and $v_a$ of the Bogoliubov transformation. The symbol : : denotes the normal product with respect to the quasi-particle operators $a_\alpha^\dagger$ and $a_\alpha$, and the quantity $\hat{Q}_{2M}$ is the mass-quadrupole-moment operator in terms of quasi-particles,

$$\begin{aligned} \hat{Q}_{2M} = \frac{1}{2}\sum_{ab} q(ab)[\xi(ab)\{A_{2M}^\dagger(ab) + A_{2\widetilde{M}}(ab)\} \\ + \eta(ab)\{B_{2M}^\dagger(ab) + B_{2\widetilde{M}}(ab)\}], \end{aligned} \tag{3·2}$$

where

$$q(ab) \equiv \frac{1}{\sqrt{5}}(a\|r^2 Y_2\|b) \tag{3·3}$$

and

$$\left.\begin{aligned}\xi(ab)&\equiv(u_a v_b+v_a u_b),\\ \eta(ab)&\equiv(u_a u_b-v_a v_b).\end{aligned}\right\} \tag{3·4}$$

The operators $A^{\dagger}_{JM}(ab)$, $A_{J\widetilde{M}}(ab)$, $B^{\dagger}_{JM}(ab)$ and $B_{J\widetilde{M}}(ab)$ are the conventional pair operators defined by

$$\left.\begin{aligned}A^{\dagger}_{JM}(ab)&\equiv\sum_{m_\alpha m_\beta}(j_a j_b m_\alpha m_\beta|JM)a^{\dagger}_\alpha a^{\dagger}_\beta,\\ B^{\dagger}_{JM}(ab)&\equiv-\sum_{m_\alpha m_\beta}(j_a j_b m_\alpha m_\beta|JM)a^{\dagger}_\alpha a_{\tilde\beta},\\ A_{J\widetilde{M}}(ab)&\equiv(-)^{J-M}A_{J,\bar M}(ab),\\ B_{J\widetilde{M}}(ab)&\equiv(-)^{J-M}B_{J,\bar M}(ab),\end{aligned}\right\} \tag{3·5}$$

where

$$a_{\tilde\beta}\equiv s_\beta a_{\bar\beta}\equiv(-)^{j_b-m_\beta}a_{\bar\beta}. \tag{3·6}$$

The quadrupole force $:H_{QQ}:$ acting among quasi-particles can be divided into following parts according to the roles they play in constructing the elementary excitation modes:

$$\left.\begin{aligned}:H_{QQ}:&=H^{(0)}_{QQ}+H_V+H_{EX},\\ H^{(0)}_{QQ}&=H_X+H_V,\end{aligned}\right\} \tag{3·7}$$

where

$$H_X=-\frac{\chi}{4}\sum_M\sum_{abcd}Q(ab)Q(cd)A^{\dagger}_{2M}(ab)A_{2M}(cd), \tag{3·7a}$$

$$H_V=-\frac{\chi}{8}\sum_M\sum_{abcd}Q(ab)Q(cd)\{A^{\dagger}_{2M}(ab)A^{\dagger}_{2\widetilde{M}}(cd)+\text{h.c.}\}, \tag{3·7b}$$

$$H_V=-\frac{\chi}{2}\sum_M\sum_{abcd}Q(ab)R(cd)\{A^{\dagger}_{2M}(ab)B_{2M}(cd)+\text{h.c.}\}, \tag{3·7c}$$

$$\begin{aligned}H_{EX}&=-\frac{\chi}{2}\sum_M\sum_{abcd}R(ab)R(cd):B^{\dagger}_{2M}(ab)B_{2M}(cd):\\ &=\frac{5}{2}\chi\sum_{JM}\sum_{abcd}R(ab)R(cd)\begin{Bmatrix}j_a & j_b & 2\\ j_c & j_d & J\end{Bmatrix}A^{\dagger}_{JM}(ad)A_{JM}(cb)\end{aligned} \tag{3·7d}$$

with

$$Q(ab)\equiv q(ab)\xi(ab), \tag{3·8}$$

$$R(ab)\equiv q(ab)\eta(ab). \tag{3·9}$$

According to the inherent assumption underlying the P+QQ force model,[3),24)] we hereafter neglect the exchange term $H_{EX}$ in the quadrupole force $H_{QQ}$. Then each matrix element of $H_{QQ}$ is represented by one of

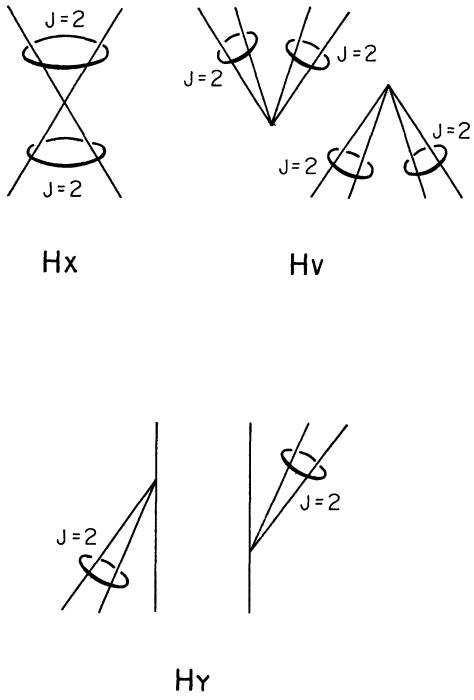



Fig. 7. Graphic representation of the matrix elements of the quadrupole force.

the diagrams in Fig. 7. The part $H_X$ represents a scattering of the pair of quasi-particles coupled to $J^\pi=2^+$. The part $H_V$ represents a pair-creation (or a pair-annihilation) of the quasi-particle pair coupled to $J^\pi=2^+$, so that it introduces the ground-state correlation. The part $H_Y$ denotes a creation (or an annihilation) of the pair of quasi-particles coupled to $J^\pi=2^+$, accompanied by the scattering of a single quasi-particle. As was discussed in Part II, the parts, $H_X$ and $H_V$, play an essential role in constructing the dressed 3QP modes as elementary excitations, while the part $H_Y$ gives rise to couplings between the different types of elementary excitation modes, for instance, a coupling between the 1QP mode and the dressed 3QP mode.

### 3–2 *Formulation of model*

Let us now consider the systems of odd-mass nuclei in the truncated shell-model space which consists of one major harmonic-oscillator shell (for both protons and neutrons) and a high-$j$ orbit with unique-parity entering into the major shell, and suppose the unique-parity orbit being filled with protons (or neutrons) in odd numbers. To explicitly specify the unique-parity orbit $j$, we use the Roman letter $p=(nlj)$*) and the corresponding Greek letters $\pi_1=(p,m_1)$, $\pi_2=(p,m_2)$, ... are used to specify the single-particle

*) We will often omit the suffix $p$ of $j_p$ hereafter.

states in the unique-parity orbit. The Roman letters $b,c,d,\ldots$ and the corresponding Greek letters $\beta=(b,m_\beta)$, $\gamma=(c,m_\gamma),\ldots$ are used for the single-particle orbits *with the exception of the unique-parity orbit* and for the corresponding single-particle states, respectively.

In this special situation of shell structure for the appearance of the AC states, the dressed 3QP mode with parity opposite to that of major shell takes on an especially simple form due to the parity-selection property of the quadrupole force: The eigenmode operators for the dressed 3QP modes defined by (2·3·1) are simply reduced to

$$\begin{aligned} C^\dagger_{nIK}=&\frac{1}{\sqrt{3!}}\sum_{m_1m_2m_3}\{\psi^p_{nIK}(\pi_1\pi_2\pi_3)\boldsymbol{P}(\pi_1\pi_2\pi_3)T_{3/2,3/2}(\pi_1\pi_2\pi_3)\\ &+\varphi^p_{nIK}(\pi_1\pi_2\pi_3)\boldsymbol{P}(\pi_1\pi_2\pi_3)T_{3/2,-1/2}(\pi_1\pi_2\pi_3)\}\\ &+\frac{1}{\sqrt{2}}\sum_{bc}{}'\sum_{mm_\beta m_\gamma}\{\psi^c_{nIK}(\beta\gamma;\pi)\boldsymbol{P}(\beta\gamma)a^\dagger_\pi a^\dagger_\beta a^\dagger_\gamma\\ &+\varphi^c_{nIK}(\beta\gamma;\pi)\boldsymbol{P}(\beta\gamma)a^\dagger_\pi a_{\tilde\beta}a_{\tilde\gamma}\}, \end{aligned}\tag{3·10}$$

where $I$ and $K$ are the angular momentum and its projection and $n$ denotes a set of additional quantum numbers to specify the eigenmode. The prime in the second term is used to emphasize that summation with respect to $b$ and $c$ should be taken by excluding the unique-parity orbit $p$. The operator $T_{3/2s_0}(\pi_1\pi_2\pi_3)$ in the first term is the quasi-spin tensor of rank $s=3/2$ and its projection $s_0$ at the unique-parity orbit $p$, the explicit form of which is given by (2·2·2); for example,

$$\begin{aligned} T_{3/2,3/2}(\pi_1\pi_2\pi_3)=&a^\dagger_{\pi_1}a^\dagger_{\pi_2}a^\dagger_{\pi_3},\\ T_{3/2,-1/2}(\pi_1\pi_2\pi_3)=&\frac{1}{\sqrt{3}}\{a^\dagger_{\pi_1}a_{\tilde\pi_2}a_{\tilde\pi_3}+a_{\tilde\pi_1}a^\dagger_{\pi_2}a_{\tilde\pi_3}+a_{\tilde\pi_1}a_{\tilde\pi_2}a^\dagger_{\pi_3}\}. \end{aligned}$$

The projection operators $\boldsymbol{P}$ in (3·10), the full definitions of which are given in Appendix 2A, guarantee the three-body-correlation amplitudes, $\psi$ and $\varphi$, to simultaneously satisfy the anti-symmetry relation and the condition requiring that the eigenmode operator must not include any $J=0$-coupled quasi-particle pair:

*anti-symmetry relations*

$$\begin{aligned} &\mathcal{P}\psi^p_{nIK}(\pi_1\pi_2\pi_3)=\delta_\mathcal{P}\psi^p_{nIK}(\pi_1\pi_2\pi_3),\quad \mathcal{P}\varphi^p_{nIK}(\pi_1\pi_2\pi_3)=\delta_\mathcal{P}\varphi^p_{nIK}(\pi_1\pi_2\pi_3),\\ &\psi^c_{nIK}(\gamma\beta;\pi)=-\psi^c_{nIK}(\beta\gamma;\pi),\quad \varphi^c_{nIK}(\gamma\beta;\pi)=-\varphi^c_{nIK}(\beta\gamma;\pi) \end{aligned}\tag{3·11}$$

with $\mathcal{P}$ being the permutation operator with respect to $(\pi_1,\pi_2,\pi_3)$ and $\delta_\mathcal{P}$ being defined by

$$\delta_\mathcal{P}=\begin{cases} 1 & \text{for even permutations,}\\ -1 & \text{for odd permutations.}\end{cases}\tag{3·12}$$

*conditions eliminating zero-coupled pairs*

$$\sum_{m_2} \psi^p_{nIK}(\pi_1\pi_2\tilde{\pi}_2)=\sum_{m_2}\varphi^p_{nIK}(\pi_1\pi_2\tilde{\pi}_2)=0,$$
$$\sum_{m_\beta} \psi^c_{nIK}(\beta\tilde{\beta};\pi)=\sum_{m_\beta}\varphi^c_{nIK}(\beta\tilde{\beta};\pi)=0. \tag{3·13}$$

The expression (3·10) for the eigenmode operators satisfying the conditions (3·11) and (3·13) implies that the dressed 3QP modes are not accompanied by the pairing "collective" components and are characterized by the amount of transferred seniority $\Delta_v=3$ to the state on which they operate.

The eigenvalue equation for the three-body-correlation amplitudes $\psi$ and $\varphi$ should be obtained so that $C^\dagger_{nIK}$ becomes an eigenmode in good approximation satisfying

$$[H_0+H^{(0)}_{QQ}, C^\dagger_{nIK}]=\omega_{nI}C^\dagger_{nIK}-Z_{nIK}, \tag{3·14}$$

where "interaction" $Z_{nIK}$ is generally composed of the normal product of quasi-spin tensors with $s=1/2$, i.e. $\Delta_v=1$, and of the higher fifth-order normal products. This is neglected in the first step which determines the dressed 3QP eigenmodes $C^\dagger_{nIK}$ (with $\Delta_v=3$).

The "physical" eigenmode operators creating the dressed 3QP states, $Y^\dagger_{nIK}$ in (2·3·7), are the ones which have large amplitudes $\psi$ and small amplitudes $\varphi$, and the other ones (with $\varphi$ larger than $\psi$), $A_{n_0IK}$ in (2·3·7), are the "special" operators which have no physical meanings. In Chap. 2, it has been shown that the correlated ground state $|\Phi_0\rangle$ satisfies the supplementary conditions under the basic approximation of the NTD method:

$$Y_{nIK}|\Phi_0\rangle=0, \quad A_{n_0IK}|\Phi_0\rangle=0. \tag{3·15}$$

It has also been shown that, within the same approximation, the dressed 3QP modes satisfy the fermion-like commutation relation,

$$\{Y_{n'I'K'}, Y^\dagger_{nIK}\}|\Phi_0\rangle=\delta_{nn'}\delta_{II'}\delta_{KK'}|\Phi_0\rangle, \tag{3·16}$$

by the use of the orthonormality relation

$$\begin{aligned}(&\boldsymbol{\Psi}_{n'I'K'}\cdot\boldsymbol{\tau}\boldsymbol{\Psi}_{nIK})\\ &\equiv\sum_{m_1m_2m_3}\{\psi^p_{n'I'K'}(\pi_1\pi_2\pi_3)\psi^p_{nIK}(\pi_1\pi_2\pi_3)-\varphi^p_{n'I'K'}(\pi_1\pi_2\pi_3)\varphi^p_{nIK}(\pi_1\pi_2\pi_3)\}\\ &\quad+\sum_{bc}{}'\sum_{mm_\beta m_\gamma}\{\psi^c_{n'I'K'}(\beta\gamma;\pi)\psi^c_{nIK}(\beta\gamma;\pi)-\varphi^c_{n'I'K'}(\beta\gamma;\pi)\varphi^c_{nIK}(\beta\gamma;\pi)\}\\ &=\epsilon_n\delta_{nn'}\delta_{II'}\delta_{KK'}.\end{aligned} \tag{3·17}$$

Let us introduce the coupled angular-momentum representation through the relations

$$\left.\begin{aligned}
&\psi^{p}_{nIK}(\pi_1\pi_2\pi_3)=\sum_{J=\text{even}}\psi_{nI}(pp(J)p)(Jj_pMm_3|IK)(j_pj_pm_1m_2|JM),\\
&\psi^{c}_{nIK}(\beta\gamma;\pi)=\sum_{J}\psi_{nI}(bc(J)p)(Jj_pMm_\pi|IK)(j_bj_cm_\beta m_\gamma|JM),\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\cdots\cdots,
\end{aligned}\right\}\tag{3·18}$$

and define the basic amplitudes (the meaning of which is precisely given in Appendix 4A) by

$$\left.\begin{aligned}
&\psi_{nI}(p^3)\equiv\sqrt{\frac{3}{2}}N_I(p^3)^{-1}\psi_{nI}(pp(2)p),\\
&\psi_{nI}(bc;p)\equiv\sqrt{2}\,N(bc)\psi_{nI}(bc(2)p),\\
&\qquad\qquad\cdots
\end{aligned}\right\}\tag{3·19}$$

with

$$N(bc)\equiv\{1+\delta_{bc}\}^{-1/2},\tag{3·20}$$

$$N_I(p^3)\equiv\{C_I/2\}^{1/2},\tag{3·21}$$

$$C_I\equiv 1+10\begin{Bmatrix} j & j & 2\\ j & I & 2\end{Bmatrix}-\delta_{Ij}\frac{20}{(2j)^2-1}.\tag{3·22}$$

Then, the eigenvalue equation for the correlation amplitudes is simply expressed in terms of only the basic amplitudes (with the intermediate angular momentum $J=2$) as follows:

$$\{2E_p-\omega'_{nI}\}\psi_{nI}(p^3)=\chi Q(pp)N_I(p^3)\{A_{nI}+B_{nI}\},\tag{3·23a}$$

$$\{2E_p+\omega'_{nI}\}\varphi_{nI}(p^3)=\frac{1}{\sqrt{3}}\chi Q(pp)N_I(p^3)\{A_{nI}+B_{nI}\},\tag{3·23b}$$

$$\{(E_b+E_c)-\omega'_{nI}\}\psi_{nI}(bc;p)=\chi Q(bc)N(bc)\{A_{nI}+B_{nI}\},\tag{3·23c}$$

$$\{(E_b+E_c)+\omega'_{nI}\}\varphi_{nI}(bc;p)=\chi Q(bc)N(bc)\{A_{nI}+B_{nI}\},\tag{3·23d}$$

where

$$\left.\begin{aligned}
&A_{nI}\equiv\sum_{(bc)}{}'\,Q(bc)N(bc)\{\psi_{nI}(bc;p)+\varphi_{nI}(bc;p)\},\\
&B_{nI}\equiv Q(pp)N_I(p^3)\left\{\psi_{nI}(p^3)+\frac{1}{\sqrt{3}}\varphi_{nI}(p^3)\right\}
\end{aligned}\right\}\tag{3·24}$$

and

$$\omega'_{nI}\equiv\omega_{nI}-E_p.\tag{3·25}$$

Here the symbol $\sum'_{(bc)}$ denotes the summation over the orbit-pair $(bc)$ excluding the unique-parity orbit $p$. In deriving Eq. (3·23), we have dropped the terms which come from the recoupling of the quadrupole force in order to keep consistency to the inherent assumption of the P+QQ force model. Formal structure of Eq. (3·23) is as simple as that of the RPA-eigenvalue equation for the phonon modes in even-even nuclei. Therefore, we can easily obtain the eigenvalues of Eq. (3·23) by using the simple dispersion equation (which is presented as Eq. (3·37)).

The normalization of the basic amplitudes (3·19) for the physical solutions becomes

$$\psi_{nI}(p^3)^2+\sum_{(bc)}{}'\psi_{nI}(bc;p)^2-\varphi_{nI}(p^3)^2-\sum_{(bc)}{}'\varphi_{nI}(bc;p)^2=1. \tag{3·26}$$

Combining Eqs. (3·23) and (3·26), we obtain the explicit expressions for the basic amplitudes:

$$\left.\begin{aligned}
&\psi_{nI}(p^3)=M_\omega Q(pp)N_I(p^3)/\{2E_p-\omega'_{nI}\},\\
&\varphi_{nI}(p^3)=\sqrt{\frac{1}{3}}M_\omega Q(pp)N_I(p^3)/\{2E_p+\omega'_{nI}\},\\
&\psi_{nI}(bc;p)=M_\omega Q(bc)N(bc)/\{(E_b+E_c)-\omega'_{nI}\},\\
&\varphi_{nI}(bc;p)=M_\omega Q(bc)N(bc)/\{(E_b+E_c)+\omega'_{nI}\}.
\end{aligned}\right\} \tag{3·27}$$

Here the normalization factor $M_\omega$ is given by

$$\begin{aligned}
M_\omega&\equiv\chi\cdot\{A_{nI}+B_{nI}\}\\
&=\left\{\frac{\partial}{\partial\omega}(S_p+S_c)\right\}^{-1/2}\\
&=\left[\frac{1}{3}Q(pp)^2C_I\frac{(2E_p)^2+8E_p\omega'_{nI}+(\omega'_{nI})^2}{\{(2E_p)^2-(\omega'_{nI})^2\}^2}\right.\\
&\qquad\left.+2\omega'_{nI}\sum_{bc}{}'\frac{Q(bc)^2(E_b+E_c)}{\{(E_b+E_c)^2-(\omega'_{nI})^2\}^2}\right]^{-1/2}
\end{aligned} \tag{3·28}$$

with

$$S_p\equiv\frac{1}{3}\frac{Q(pp)^2C_I\{4E_p+\omega'_{nI}\}}{(2E_p)^2-(\omega'_{nI})^2}, \tag{3·29}$$

$$S_c\equiv\sum_{bc}{}'\frac{Q(bc)^2\cdot(E_b+E_c)}{(E_b+E_c)^2-(\omega'_{nI})^2}. \tag{3·30}$$

### 3–3 *Mechanism of growth of 3QP correlation and relations to other approaches*

In order to see the microscopic structure of the dressed 3QP mode formulated in the preceding subsection and to discuss the relations to other approaches[6),7)] by paying attention to their underlying pictures for the AC states, let us decompose the eigenvalue equation (3·23) in the following way. Combining Eqs. (3·23a) and (3·23b), and also combining Eqs. (3·23c) and (3·23d), we obtain

$$\left.\begin{aligned}
&\{\chi_pS_p-1\}B_{nI}+\chi_{pc}S_pA_{nI}=0,\\
&\{\chi_cS_c-1\}A_{nI}+\chi_{pc}S_cB_{nI}=0,
\end{aligned}\right\} \tag{3·31}$$

with $\chi_p=\chi_c=\chi_{pc}=\chi$. Since this equation is linear and homogeneous with respect to $A_{nI}$ and $B_{nI}$, we find that the eigenvalues $\omega_{nI}$ are the solutions of

$$(\chi_pS_p-1)(\chi_cS_c-1)-\chi_{pc}^2S_pS_c=0. \tag{3·32}$$

The physical meaning of this equation is easily understood as follows.

If $\chi_{pc}$ were zero, we would have solutions when either $\chi_p S_p=1$ or $\chi_c S_c=1$. The former is merely the dispersion equation for the dressed 3QP mode in the case of restricting our shell-model space within the unique-parity orbit $p$. The eigenvalue of $\chi_p S_p=\chi S_p=1$ is

$$\omega_I=E_I^*+\sqrt{(2E_I^*)^2-\left\{\frac{1}{2\sqrt{3}}\chi_I^* Q(pp)^2\right\}^2} \tag{3·33}$$

with

$$E_I^*\equiv E_p-\frac{1}{6}\chi_I^* Q(pp)^2, \tag{3·34}$$

$$\chi_I^*\equiv\chi C_I. \tag{3·35}$$

As was pointed out by Kisslinger,[4] there exists an interesting property of $6j$-symbols, that is,

$$\left.\begin{aligned}&\begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix}>0 \quad \text{for} \quad I=j-1,\\ &\begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix}<0 \quad \text{for} \quad I\neq j-1.\end{aligned}\right\} \tag{3·36}$$

Hence, recalling the definition of $C_I$, (3·22), we can easily see that $\chi_I^*>\chi$ only when $I=j_p-1$ and $\chi_I^*<\chi$ for $I\neq j_p-1$. Thus the $(j_p-1)$ state is especially lowered in energy by the quadrupole force in contrast to the other states with $I\neq j_p-1$. It is now clear that the dressed 3QP state with spin $(j-1)$ in the unique-parity orbit $p$ is reduced to Kisslinger's 3QP "intruder" state[4] when we neglect the ground-state correlation. On the other hand, the latter equation $\chi_c S_c=1$ is exactly the same form as the well-known dispersion equation for phonon modes. Notice, however, that the "phonon" mode in this case implies the "core-excitation" which is composed of the neutron and proton quasi-particles in the truncated major shells *with the exclusion of the valence orbit p*, i.e., the unique-parity orbit. Thus we have *two* low-energy collective states (composed of the quasi-particles in the orbit $p$ and in the core, respectively), *if the "coupling" $\chi_{pc}$ is zero.*

Now let us consider the effect on these states due to the change of $\chi_{pc}$ from zero.[1] In this case, the product $(\chi_p S_p-1)\cdot(\chi_c S_c-1)$ has to be positive so that the lower level of the two $\chi_{pc}=0$ states must be lowered in order to make each factor of the product negative while the higher level is raised making each factor positive. For sufficiently large $\chi_{pc}$, as is the actual case of $\chi_{pc}=\chi_p=\chi_c=\chi$, there is essentially only one extremely lowered $\omega_{nI}$ left in the energy region satisfying

$$\omega'_{nI}\equiv(\omega_{nI}-E)<\text{the minimum value of } (E_b+E_c).$$

In this actual case of $\chi_p=\chi_c=\chi_{pc}=\chi$, Eq. (3·32) is simply reduced to

$$\chi^{-1}=S_p+S_c$$
$$=\frac{1}{3}\frac{Q(pp)^2C_I\{4E_p+\omega'_{nI}\}}{(2E_p)^2-(\omega'_{nI})^2}+\sum_{bc}{}'\frac{Q(bc)^2(E_b+E_c)}{(E_b+E_c)^2-(\omega'_{nI})^2}. \quad (3\cdot37)$$

The above consideration tells us that, in the special situation in shell structure for the appearance of the AC states, the dressed 3QP mode may be decomposed into the "valence-shell cluster"[6] and the "phonon" modes of the core. The "valence-shell cluster" now means the correlated three-quasi-particles in the unique-parity orbit $p$ and reduces to Kisslinger's (3QP) "intruder state" in the Tamm-Dancoff limit. In this respect, our underlying picture for the AC states is similar with that of the semi-microscopic model of Alaga,[6),7)] which explicitly introduces the freedom of the valence-shell cluster coupled to the quadrupole vibration of the core. According to our picture for the AC states, however, the coupling between the valence-shell cluster and the phonon of the core is so strong (because of $\chi_{pc}=\chi_p=\chi_c=\chi$) that they form a new type of collective mode, i.e., the dressed 3QP mode as a bound state.

It is now clear that the introduced model of the AC states unifies the characteristics of both the "intruder states" and the conventional QPC theory, which have been considered as distinctly different from each other in the history of investigating the AC states.

Let us look further into the lowering effect (on the excitation energies of the AC states) due to the core, by adopting the adiabatic approximation:

$$\omega'_{nI}\equiv(\omega_{nI}-E_p)\ll \text{the minimum value of } (E_b+E_c). \quad (3\cdot38)$$

Many of the AC states satisfy this condition and in this case we may write

$$\left.\begin{aligned} S_p&\equiv\mathcal{A}_p+\mathcal{B}_p(\omega_{nI}-E_p)+C_p(\omega_{nI}-E_p)^2,\\ S_c&\equiv\mathcal{A}_c+C_c(\omega_{nI}-E_p)^2,\end{aligned}\right\} \quad (3\cdot39)$$

where

$$\left.\begin{aligned} &\mathcal{A}_p\equiv\frac{2}{3}\frac{Q(pp)^2C_I}{(2E_p)}>0, \quad \mathcal{B}_p\equiv\frac{1}{3}\frac{Q(pp)^2C_I}{(2E_p)^2}>0,\\ &C_p\equiv\frac{2}{3}\frac{Q(pp)^2C_I}{(2E_p)^3}>0,\\ &\mathcal{A}_c\equiv\sum_{bc}{}'\frac{Q(bc)^2}{E_b+E_c}>0, \quad C_c\equiv\sum_{bc}{}'\frac{Q(bc)^2}{(E_b+E_c)^3}>0.\end{aligned}\right\} \quad (3\cdot40)$$

As a result we have from Eq. (3·37)

$$(\omega_{nI}-E_p)=-\frac{\mathcal{B}_p}{2(C_p+C_c)}+\sqrt{\frac{\mathcal{B}_p^2}{4(C_p+C_c)^2}+\frac{\chi^{-1}-(\mathcal{A}_p+\mathcal{A}_c)}{(C_p+C_c)}}. \quad (3\cdot41)$$

Comparing this to the adiabatic solution of $\chi S_p=1$, which is obtained by

setting $\mathcal{A}_c=C_c=0$, we can easily see the lowering effects due to the phonon excitations of the "core." Since both $\mathcal{A}_c$ and $C_c$ contain the factors $\xi(bc)\equiv(u_b v_c+v_b u_c)$ through the quantity $Q(bc)$, the larger the $\xi(bc)$ of the core, the smaller the $\omega_{nI}$ becomes. Thus the problem of whether the dressed 3QP modes appear extremely low in energy will be determined by two important factors:

i) The enhancement factor $\xi(pp)\equiv 2u_p v_p$ in the unique-parity orbit $p$,

ii) the enhancement factor $\xi(bc)$ in the core.

### 3–4 *Stability of spherical BCS vacuum against 3QP correlation*

As is well known in the case of even-even nuclei, when the enhancement factors $\xi(ab)$ become large and the excitation energy of the $2^+$ phonon (the dressed 2QP mode) becomes zero, the instability of the spherical BCS vacuum occurs toward quadrupole deformation. In an analogous way, we expect in odd-mass nuclei that, when the enhancement factors $\xi(pp)$ and $\xi(bc)$ become large and the characteristic 3QP correlation grows so that the excitation energy of the dressed 3QP mode with $I=(j-1)$ is extremely lowered, a new type of instability may occur toward deformation.

Figure 8 represents schematically the dispersion equation (3·37) from which the excitation energies $\omega_{nI}$ of the dressed 3QP modes are determined. Its gross structure resembles that of the RPA with the P+QQ force in even-even nuclei. It should be noted, however, that the characteristics of the solution of collective type in the vicinity of $\omega_{nI}\approx E_p$ differ considerably from those obtained by simply replacing $\omega_{2^+}$ in the conventional dispersion equation of the RPA (in even-even nuclei) with $\omega'_{nI}=(\omega_{nI}-E_p)$. This difference obviously comes

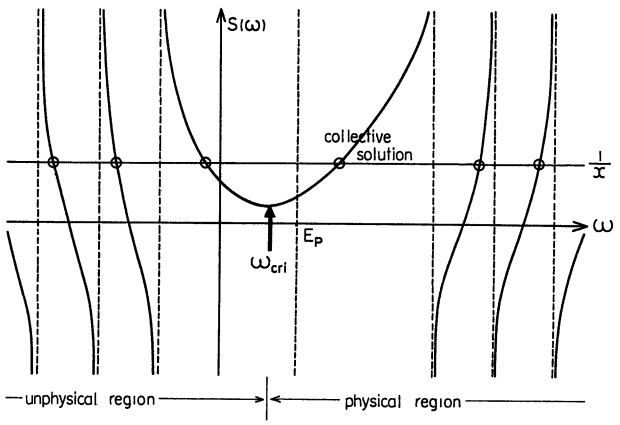



Fig. 8. Schematic representation of the dispersion equation (3·37) from which the eigenvalues of the dressed 3QP modes are determined. The arrow denotes the critical point and the right-hand side of which implies the physical region.

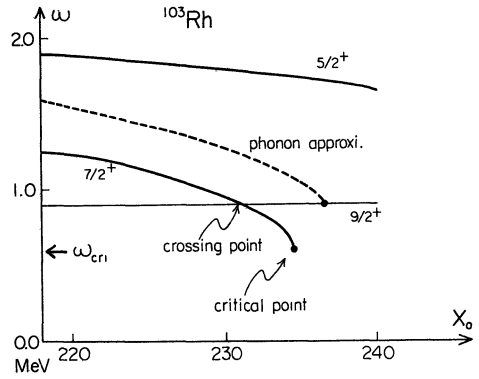



Fig. 9. Behaviour of the solutions of the dressed 3QP modes in $^{103}$Rh a functions of the quadrupole-force parameter $\chi_0 \equiv \chi b^4 A^{5/3}$. The horizontal line denotes the energy of the 1QP $9/2^+$ state. For the sake of comparison, the energies of the 1QP-plus-one-phonon states are also written by broken curves, where phonon energies are calculated by the RPA.

from the characteristic structure of the first term on the right-hand side of Eq. (3·37), $S_p$, which directly reflects the 3QP correlation at the unique-parity orbit $p$. Figure 9 shows the behaviour of the excitation energy $\omega_{nI}$ in the vicinity of $\omega_{nI} \approx E_p$ in relation to that of $(\omega_{2^+}+E_p)$ ($\omega_{2^+}$ being the energy of $2^+$-phonon given by the RPA). The critical energy $\omega'_{cri} \equiv \omega_{cri} - E_p$ may be defined at the point from which there appear a complex eigenvalue of the dressed 3QP mode with $I=(j-1)$ and is given as the solution of

$$\frac{\partial}{\partial \omega}(S_p+S_c)=0. \tag{3·42}$$

Apparently the three-body correlation amplitudes given by (3·27) diverge at the critical point.

If we neglect the part $S_p$ in Eq. (3·42) which reflects the 3QP correlation, we have $\omega^{(c)}_{cri}=E_p$. This is merely the critical energy expected from the conventional QPC theory. On the other hand, if we neglect the core contribution $S_c$, we have $\omega^{(p)}_{cri}=(2\sqrt{3}-3)E_p<E_p$. In the actual situation, in which neither $S_p$ nor $S_c$ is zero, $\omega_{cri}$ takes on the value between those of $\omega^{(c)}_{cri}$ and $\omega^{(p)}_{cri}$. The value of $\omega_{cri}$ depends on the details of shell structure and on the relative magnitude of the enhancement factors i) and ii) mentioned in the preceding subsection. It is thus interesting to see that the stability of the spherical BCS vacuum is still maintained in the region $E_p > \omega_{nI} > \omega_{cri}$, in which the crossing of the spin $(j-1)$- and the spin $j$-state has already occurred. As shown in § 5, the excitation energies of the AC states in many odd-mass nuclei fall within this region. This implies that these nuclei are lying in the "transition region," i.e., they are just before the "phase transition" from the spherical to the deformed, possessing a strong tendency toward quadrupole deformation.

Concerning the situation occuring after the "phase transition," it is interesting to recall that the aligned coupling model also gives the $(j-1)$ state as the lowest state among $j^3$ configurations in the oblately deformed potential.[5] However, the interrelation between the new type of instability and the onset of quadrupole deformation in odd-mass nuclei remains unclarified. Although there is no systematic evidence for the stable (quadrupole) deformation in the odd-mass nuclei with the spin $(j-1)$ ground state, it should be noted that the adjacent even-even nuclei exhibit the quasi-rotational spectra.[26]

### 3–5 *Mixing effects of 1QP modes on AC states*

Now let us consider the mixing effect of the 1QP mode, which lies in the next major shell, on the AC states. Following the general theory developed in Chap. 2, the effective Hamiltonian describing the system composed of the dressed 3QP modes and the 1QP modes, that is, the transcribed Hamiltonian in the quasi-particle NTD subspace

$$\{|\Phi_\alpha^{(1)}\rangle \equiv a_\alpha^\dagger|\Phi_0\rangle,\quad |\Phi_{nIK}^{(3)}\rangle \equiv Y_{nIK}^\dagger|\Phi_0\rangle\}, \tag{3·43}$$

is derived from the original P+QQ Hamiltonian as follows:

$$\left.\begin{aligned} &\boldsymbol{H}=\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})},\\ &\boldsymbol{H}^{(0)}=\sum_\alpha E_a \boldsymbol{a}_\alpha^\dagger \boldsymbol{a}_\alpha+\sum_{nIK}\omega_{nI}\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{Y}_{nIK},\\ &\boldsymbol{H}^{(\mathrm{int})}=\sum_{\substack{nIK,\alpha\\(K=m_\alpha)}}\chi_{\mathrm{int}}(a,nI)\{\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{a}_\alpha+\boldsymbol{a}_\alpha^\dagger \boldsymbol{Y}_{nIK}\}. \end{aligned}\right\} \tag{3·44}$$

Here the operators $\boldsymbol{a}_\alpha^\dagger$ and $\boldsymbol{Y}_{nIK}^\dagger$ are defined by

$$\boldsymbol{a}_\alpha^\dagger \equiv a_\alpha^\dagger|\Phi_0\rangle\langle\Phi_0|, \tag{3·45}$$

$$\boldsymbol{Y}_{nIK}^\dagger \equiv Y_{nIK}^\dagger|\Phi_0\rangle\langle\Phi_0| \tag{3·46}$$

and satisfy the condition

$$\hat{S}_-(a)\boldsymbol{a}_\alpha^\dagger=\hat{S}_-(a)\boldsymbol{Y}_{nIK}^\dagger=0 \tag{3·47}$$

for all $\hat{S}_-(a)$, where $\hat{S}_-(a)$ denotes the quasi-spin operator of orbit $a$ defined by (1·2·18). The condition (3·47) implies that $\boldsymbol{a}_\alpha^\dagger$ and $\boldsymbol{Y}_{nIK}^\dagger$ are the operators of the "intrinsic space" defined in §2-Chap. 1. The part $\boldsymbol{H}^{(\mathrm{int})}$ in (3·44) comes from the original $H_Y$-type interaction (3·7c) and represents the effective coupling Hamiltonian which is of interest. In the special situation of shell structure under consideration, the effective coupling strength $\chi_{\mathrm{int}}(p', nI)$ takes especially simple form:

$$\chi_{\text{int}}(p', nI) = -\chi\delta_{Ij_{p'}}\frac{(p\|r^2Y_2\|p')}{\sqrt{2I+1}}(u_pu_{p'}-v_pv_{p'})$$
$$\times\Big[Q(pp)N_I(p^3)\Big\{\psi_{nI}(p^3)+\frac{1}{\sqrt{3}}\varphi_{nI}(p^3)\Big\}$$
$$+\sum_{(bc)}{}'Q(bc)N(bc)\{\psi_{nI}(bc;p)+\varphi_{nI}(bc;p)\}\Big]$$
$$=-\delta_{Ij_{p'}}\sqrt{\frac{5}{2I+1}}R(pp')M_\omega, \tag{3·48}$$

where $p'$ denotes a single-particle orbit in the next major shell, and $M_\omega$ and $R(pp')$ are defined by (3·28) and (3·9), respectively. When we take account of the mixing effect of the 1QP mode on the AC states, the state vector of interest changes to*)

$$|\Phi^{(3)}_{IK}\rangle\equiv Y^\dagger_{IK}|\Phi_0\rangle \longrightarrow |\Psi^{(3)}_{IK}\rangle\equiv\sqrt{1-\zeta_I^2}\ Y^\dagger_{IK}|\Phi_0\rangle+\zeta_I a^\dagger_{\pi'}|\Phi_0\rangle, \tag{3·49}$$

where $\pi'\equiv(p', m_{\pi'})\equiv(n_{p'}l_{p'}j_{p'}, m_{\pi'})$ denotes a single-particle state with $j_{p'}=I$ and $\zeta_I$ denotes the mixing amplitude. The magnitude of the mixing amplitude $\zeta_I$ with $I=(j_p-1)$ is expected to be extremely small since the single-particle orbit $p'$ (which has the same parity with the unique-parity orbit $p$ and has $j_{p'}=I=j_p-1$) lies in the next upper major shell, and also since the effective coupling strength $\chi_{\text{int}}$ involves, in this case, the spin-flip type matrix element $(p\|r^2Y_2\|p')$ which is small when compared with the spin-non-flip ones. Thus the special physical condition of the appearance of the AC states is just the same condition as that in which the dressed 3QP modes with $I=(j_p-1)$ manifest themselves as relatively pure eigenmodes with negligible mixings of the 1QP modes. We show quantitatively in §5 that the mixing amplitude $\zeta_I$ is indeed small.

Next, let us consider the effect of the coupling Hamiltonian on the 1QP state with spin $j_p$. The state vector of the 1QP state changes, due to the mixing of the dressed 3QP mode with $I=j_p$, to

$$|\Phi^{(1)}_\pi\rangle\equiv a^\dagger_\pi|\Phi_0\rangle \longrightarrow |\Psi^{(1)}_\pi\rangle\equiv\sqrt{1-\zeta_j^2}\,a^\dagger_\pi|\Phi_0\rangle+\zeta_j Y^\dagger_{jm_\pi}|\Phi_0\rangle. \tag{3·50}$$

The mixing amplitude $\zeta_j$ is, as is shown in §5, rather small. This is partly because of the smallness of the effective coupling strength $\chi_{\text{int}}$ with $p'=p$ and $I=j_p$, and partly because of the relatively high excitation energy, $\omega_j$, of the dressed 3QP mode with $I=j_p$; both of which are due to the extreme smallness of the 3QP correlation factor $C_I$, defined by (3·22), with $I=j_p$. In this way the mixing effect on the 1QP state with spin $j_p$ is considerably reduced when

*) Hereafter we only consider the lowest energy solution of the dressed 3QP mode, which is of a collective type, and omit the suffix $n$. Needless to say, the solutions of non-collective types do not play any significant role in the following discussions.

compared to that which is given by the conventional QPC theory. The characteristic dependence of $\zeta_j$ on the $(u_p^2-v_p^2)$ factor is, however, the same as that of conventional QPC theory.

We investigate the mixing effects, both on the AC states with $I=(j-1)$ and on the 1QP state with spin $j_p$, paying special attention to the $M1$-transition property of the AC state which is very sensitive to these mixings in subsequent sections.

## §4. Electromagnetic properties of the AC states with $I=(j-1)$

In this section, we present the qualitative predictions of the introduced model for the electromagnetic properties of the AC states with $I=(j-1)$. Experimental data revealing various aspects on the structure of the AC states have now been accumulated. The main characteristics which they exhibit are as follows.

1) Strongly enhanced $E2$ transition from the $(j-1)$ state to the 1QP state with spin $j$. The magnitude of the enhancement is comparable (or somewhat larger) to that from the $2^+$ phonon states to the ground states in the adjacent even-even nuclei.

2) Moderately hindered $M1$ transition between $(j-1)$ state and $j$ state.

3) The $g$ factor of the $(j-1)$ state is approximately equal to that of the $j$ state.

Formulae on these quantities can be obtained unambiguously by using the method developed in Chap. 2. The electromagnetic multipole operators $\hat{O}^{(\pm)}_{LK}$ defined by (2·5·12) are transcribed into the quasi-particle NTD subspace under consideration as

$$\begin{aligned}\hat{O}^{(\pm)}_{LM} \longrightarrow \hat{\boldsymbol{O}}^{(\pm)}_{LM} &= \sum_{\alpha\beta}\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(1)}_\beta\rangle \boldsymbol{a}^\dagger_\alpha \boldsymbol{a}_\beta \\ &+ \sum_{IK,I'K'}\langle\Phi^{(3)}_{IK}|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{I'K'}\rangle \boldsymbol{Y}^\dagger_{IK}\boldsymbol{Y}_{I'K'} \\ &+ \sum_{\alpha,IK}\{\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{IK}\rangle \boldsymbol{a}^\dagger_\alpha \boldsymbol{Y}_{IK}+\text{h.c.}\} \quad (L\neq 0)\end{aligned} \tag{4·1}$$

with the transcription coefficients

$$\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(1)}_\beta\rangle=\langle\Phi_0|\{a_\alpha,[\hat{O}^{(\pm)}_{LM},a^\dagger_\beta]_-\}_+|\Phi_0\rangle, \tag{4·2}$$

$$\langle\Phi^{(3)}_{IK}|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{I'K'}\rangle=\langle\Phi_0|\{Y_{IK},[\hat{O}^{(\pm)}_{LM},Y^\dagger_{I'K'}]_-\}_+|\Phi_0\rangle, \tag{4·3}$$

$$\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{IK}\rangle=\langle\Phi_0|\{a_\alpha,[\hat{O}^{(\pm)}_{LM},Y^\dagger_{IK}]_-\}_+|\Phi_0\rangle. \tag{4·4}$$

Thus we obtain the reduced matrix elements of interest as follows:

$$\begin{aligned}\langle\Psi^{(1)}_p\|\hat{\boldsymbol{O}}^{(\pm)}_L\|\Psi^{(3)}_I\rangle &= \sqrt{1-\zeta_I^2}\sqrt{1-\zeta_j^2}\langle\Phi^{(1)}_p\|\hat{O}^{(\pm)}_L\|\Phi^{(3)}_I\rangle \\ &+\zeta_I\sqrt{1-\zeta_j^2}\langle\Phi^{(1)}_p\|\hat{O}^{(\pm)}_L\|\Phi^{(1)}_{p'}\rangle+\zeta_j\sqrt{1-\zeta_I^2}\langle\Phi^{(3)}_{I'=j}\|\hat{O}^{(\pm)}_L\|\Phi^{(3)}_I\rangle \\ &+\zeta_j\zeta_I\langle\Phi^{(3)}_{I'=j}\|\hat{O}^{(\pm)}_L\|\Phi^{(1)}_{p'}\rangle,\end{aligned} \tag{4·5}$$

$$\langle \Psi_I^{(3)} \| \hat{O}_L^{(\pm)} \| \Psi_I^{(3)} \rangle = (1-\zeta_I^2) \langle \Phi_I^{(3)} \| \hat{O}_L^{(\pm)} \| \Phi_I^{(3)} \rangle + 2\zeta_I \sqrt{1-\zeta_I^2} \langle \Phi_I^{(3)} \| \hat{O}_L^{(\pm)} \| \Phi_{p'}^{(1)} \rangle + \zeta_I^2 \langle \Phi_{p'}^{(1)} \| \hat{O}_L^{(\pm)} \| \Phi_{p'}^{(1)} \rangle, \cdots \quad (4\cdot6)$$

with the definitions

$$\langle \Psi_\pi^{(1)} | \hat{O}_{LM}^{(\pm)} | \Psi_{IK}^{(3)} \rangle = \frac{(ILKM|j_p m_\pi)}{\sqrt{2j_p+1}} \langle \Psi_p^{(1)} \| \hat{O}_L^{(\pm)} \| \Psi_I^{(3)} \rangle, \quad (4\cdot7)$$

$$\langle \Phi_\pi^{(1)} | \hat{O}_{LM}^{(\pm)} | \Phi_{IK}^{(3)} \rangle = \frac{(ILKM|j_p m_\pi)}{\sqrt{2j_p+1}} \langle \Phi_p^{(1)} \| \hat{O}_L^{(\pm)} \| \Phi_I^{(3)} \rangle, \text{ etc.}$$

Needless to say, since the mixing amplitudes, $\zeta_I$ and $\zeta_j$, are expected to be small, the first terms in Eqs. (4·5) and (4·6) should yield the main contribution to these reduced matrix elements unless they are strongly hindered (or forbidden). Now let us show how the experimental characteristics mentioned above can be recognized in a unified manner.

### 4–1 *E2 transitions between AC states and 1QP states*

As has been stressed, the collective nature of the AC state with $I=(j-1)$ has been recognized through the recent observation of strongly enhanced $E2$ transition from the AC state to the 1QP state with spin $j$. In the microscopic model under consideration, the reduced transition probabilities are given by

$$B(E2; I \to j_p) = \frac{1}{2I+1} |\langle \Psi_p^{(1)} \| \hat{O}_2^{(+)} \| \Psi_I^{(3)} \rangle|^2, \quad (4\cdot8)$$

where $\hat{O}_2^{(+)}$ denotes the electric quadrupole operator in the quasi-particle NTD subspace. Since the mixing effect is expected to be negligibly small, we obtain

$$\begin{aligned} B(E2; I \to j_p) &\approx \frac{1}{2I+1} |\langle \Phi_p^{(1)} \| \hat{O}_2^{(+)} \| \Phi_I^{(3)} \rangle|^2 \\ &= \left| e_{\tau(p)} Q(pp) N_I(p^3) \left\{ \psi_I(p^3) + \sqrt{\frac{1}{3}} \varphi_I(p^3) \right\} \right. \\ &\quad \left. + \sum_{(bc)}{}' e_{\tau(bc)} Q(bc) N(bc) \{ \psi_I(bc; p) + \varphi_I(bc; p) \} \right|^2. \end{aligned} \quad (4\cdot9)$$

Inserting the explicit expressions of the three-body correlation amplitudes (3·27) into (4·9), we finally obtain

$$B(E2; I \to j_p) \approx M_\omega^2 |e_\tau S_p + e_{-1/2} S_c(\text{proton}) + e_{1/2} S_c(\text{neutron})|^2, \quad (4\cdot10)$$

where $M_\omega$, $S_p$ and $S_c$ are defined by (3·28), (3·29) and (3·30), respectively. Here the contributions from proton-quasi-particles and from neutron-quasi-particles are written distinctively. The effective charges $e_\tau$ for neutrons ($\tau=1/2$) and protons ($\tau=-1/2$) are

$$e_{1/2}=\delta e \quad \text{and} \quad e_{-1/2}=e+\delta e. \tag{4·11}$$

For simplicity, we have adopted the same polarization charge $\delta e$ for protons and neutrons. It is noteworthy that Eq. (4·9) has a structure formally similar to the corresponding expression given by the RPA in even-even nuclei, in spite of the difference due to the incorporation of the 3QP correlation. For the $E2$ transition between the AC state and the 1QP state, we can, therefore, expect the well-known enhancement associated with the structure of Eq. (4·9). In particular, we have the usual relation; the lower the excitation energy of the AC state, the larger the $B(E2)$ value becomes. Such an enhancement is a direct and natural consequence of the present model in which the motions of quasi-particles at the unique-parity orbit $p$ and of quasi-particles excited from the "core" (the orbits $b$, $c$,..., etc.) are strongly coupled with each other to form a new type of collective excited state, i.e., the dressed 3QP state. It must also be emphasized that the cooperation of these two motions are strengthened further by the collective ground-state correlation which is enlarged as the excitation energy of the dressed 3QP mode, $\omega_{nI}$, is lowered. Of course, the model of the AC state as the 3QP "intruder" state (given in the TD approximation within the unique-parity orbit $p$) cannot yield such a mechanism of striking $E2$ enhancement.

### 4–2 *Magnetic dipole moments of AC states*

The magnetic dipole moment of the AC state is given by

$$\mu=\sqrt{\frac{4\pi}{3}}\langle \Psi^{(3)}_{IK}|\hat{O}^{(-)}_{10}|\Psi^{(3)}_{IK}\rangle \equiv g_I I \quad \text{with} \quad K=I, \tag{4·12}$$

where $\hat{O}^{(-)}_{10}$ denotes the magnetic dipole operator in the quasi-particle NTD subspace. The $g$ factor of the AC state, $g_I$, is expressed in our model as follows:

$$g_I=g_p^{(0)}+\frac{I(I+1)+j_p(j_p+1)-6}{2I(I+1)}g_p^{(1)}+\frac{I(I+1)+6-j_p(j_p+1)}{2I(I+1)}g_c, \tag{4·13}$$

where

$$g_p^{(0)}=g_p\{\psi_I(p^3)^2-\varphi_I(p^3)^2\}, \tag{4·14}$$

$$g_p^{(1)}=g_p\sum_{(bc)}{}'\{\psi_I(bc;p)^2-\varphi_I(bc;p)^2\} \tag{4·15}$$

and

$$g_c=\sqrt{\frac{5}{2}}\sum_{bcd}{}' M(bc)N(cd)^{-1}N(db)^{-1}\begin{Bmatrix} 2 & j_c & j_d \\ j_b & 2 & 1 \end{Bmatrix} \times[\psi_I(cd;p)\psi_I(db;p)-\varphi_I(cd;p)\varphi_I(db;p)]. \tag{4·16}$$

Here $g_p$ denotes the single-particle $g$ factor of the unique-parity orbit $p$ and

$$M(bc)\equiv\frac{1}{\sqrt{3}}(b\|\boldsymbol{\mu}\|c)\cdot(u_bu_c+v_bv_c) \tag{4·17}$$

with $\boldsymbol{\mu}=g_l\boldsymbol{l}+g_s\boldsymbol{s}$. In obtaining (4·13), we omitted the mixing effect of the 1QP mode with spin $(j-1)$, which lies in the next upper major shell, since the effect on the $g$ factor of the AC state with $I=(j-1)$ is negligibly small. The physical meaning of each term in Eq. (4·13) is clear. The first term, $g_p^{(0)}$, comes from the "cluster" of quasi-particles at the "valence shell" orbit $p$. If we restrict our shell-model space to only the unique-parity orbit $p$, which is being filled, then $g_p^{(0)}$ becomes equal to $g_p$ (because in this case $\psi_I(p^3)^2-\varphi_I(p^3)^2=1$). The second and third terms are of the same form as the Lande formula: The second term comes from the odd quasi-particle at the orbit $p$, i.e., $p$ in $\psi_I(bc;p)$ and $\varphi_I(bc;p)$, while the third term comes from the quasi-particles excited from the "core," i.e., $b$ and $c$ in $\psi_I(bc;p)$ and $\varphi_I(bc;p)$.

It is interesting to note that the geometrical factors involved in the second and third terms in (4·13) possess characteristic dependences on the value of spin $I$; in the case of $I=(j_p-1)$ with high-$j_p$,

$$\left.\begin{aligned}\frac{I(I+1)+j_p(j_p+1)-6}{2I(I+1)}&\approx 1+O\left(\frac{1}{j_p}\right),\\ \frac{I(I+1)+6-j_p(j_p+1)}{2I(I+1)}&\approx -O\left(\frac{1}{j_p}\right).\end{aligned}\right\} \tag{4·18}$$

If we neglect the quantity of order $O(1/j_p)$, and making use of the normalization condition (3·26), we have

$$g_I\approx g_p^{(0)}+g_p^{(1)}=g_p. \tag{4·19}$$

In this way, although the $g$ factor of the "phonon" (composed of the quasi-particles in the orbits $b$, $c$,...), i.e., $g_c$, may be of the order $Z/A$ (in unit of nuclear magneton $e\hbar/2Mc$), its contribution to the $g$ factor of the $(j-1)$ state is especially reduced. The experimental fact that $g_{I=j-1}\approx g_p$ has been often interpreted as an evidence of the simple $(j^n)_I$ configuration[10)~15)] for the structure of the AC states with $I=(j-1)$. However, as we have seen, the mechanism of obtaining the value of $g_I$ nearly equal to $g_p$ is distinctly different from this interpretation; in other words, in the shell model of $j^n$-configurations, we have $g_I=g_p^{(0)}=g_p$ for arbitrary values of $I$, while in the microscopic model under consideration, we have $g_I=g_p^{(0)}+g_p^{(1)}=g_p$ as a good approximation for the special case of $I=(j-1)$. It is shown in §5 that the magnitudes of $g_p^{(0)}$ and $g_p^{(1)}$ are approximately in the ratio of one to one.

### 4–3 *M1 transitions between AC states and 1QP states*

In contrast to the properties of $E2$ transition and of $M1$ moment described above, the $M1$-transition property of the AC state is very sensitive to the

mixing effect resulting from the coupling term $\boldsymbol{H}^{(\mathrm{int})}$ in the transcribed Hamiltonian (3·44), as shown below. As usual, the reduced $M1$-transition probabilities from the AC state with $I=(j-1)$ to the 1QP state with spin $j$ is given by

$$B(M1;\, I\to j_p)=\frac{1}{2I+1}|\langle\Psi_p^{(1)}\|\hat{\boldsymbol{O}}_1^{(-)}\|\Psi_I^{(3)}\rangle|^2 \tag{4·20}$$

with $I=j_p-1$. Since the creation operator of the dressed 3QP mode under consideration, (3·10), does not involve any components of quasi-particle pair with spin and parity $1^+$, we have

$$\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle=0. \tag{4·21}$$

Namely, in the first-order approximation in which the AC state with $I=(j-1)$ is regarded as a pure dressed 3QP mode, the $M1$ transition between the $(j-1)$ state and the $j$ state is forbidden. In fact the attenuation of the $M1$ transition has been observed in experiments. Then the retarded $M1$ transition must take place only through the mixing effects as follows.

$$\begin{aligned}B(M1;\, I\to j_p)=\frac{1}{2I+1}\Big|&\zeta_I\sqrt{1-\zeta_j^2}\,\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_{p'}^{(1)}\rangle\\&+\zeta_j\sqrt{1-\zeta_I^2}\,\langle\Phi_{I'}^{(3)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle\Big|^2,\end{aligned} \tag{4·22}$$

where

$$\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_{p'}^{(1)}\rangle=\sqrt{\frac{3}{4\pi}}(p\|\boldsymbol{\mu}\|p')\cdot(u_pu_{p'}+v_pv_{p'})=\frac{3}{\sqrt{4\pi}}M(pp') \tag{4·23}$$

and

$$\begin{aligned}&\langle\Phi_{I'}^{(3)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle\\&=3\sqrt{\frac{(2I+1)(2I'+1)}{4\pi}}\Big[M(pp)\begin{Bmatrix}I & j_p & 2\\ j_p & I' & 1\end{Bmatrix}\sum_{(bc)}{}'\{\psi_{I'}(bc;p)\psi_I(bc;p)\\&\quad-\varphi_{I'}(bc;p)\varphi_I(bc;p)\}+5\begin{Bmatrix}2 & I' & j_p\\ I & 2 & 1\end{Bmatrix}\sum_{bcd}{}'M(bc)N(bd)^{-1}N(cd)^{-1}\\&\quad\times\begin{Bmatrix}2 & j_b & j_d\\ j_c & 2 & 1\end{Bmatrix}\{\psi_{I'}(cd;p)\psi_I(db;p)-\varphi_{I'}(cd;p)\varphi_I(db;p)\}\Big]\end{aligned} \tag{4·24}$$

with $I=(j_p-1)=j_{p'}$ and $I'=j_p$.

The first term in Eq. (4·22) represents the contribution from the mixing of the 1QP state with spin $j_{p'}=j_p-1$, lying in the next upper major shell, in the AC state with $I=j_p-1$. The second term comes from the mixing of the dressed 3QP state (with $I'=j_p$) in the 1QP state (with spin $j_p$). Since the second term involves the $(u_p^2-v_p^2)$ factor through the mixing amplitude $\zeta_j$, the value depends sensitively on the nucleon-occupation probability of the unique-parity orbit $p$ and changes its sign on both sides of the half-shell, while

the first term in Eq. (4·22) preserves its sign through the whole range. As a consequence of the interference effect between the two, the magnitude of the $M1$ transition becomes greatly sensitive to details of these mixing effects. The magnitude itself should of course be small compared with that of a single-particle transition (because $\zeta_I$ and $\zeta_j$ are both small), unless one moves away from the particular physical situation for the appearance of the AC states.

### 4–4 *Additional remarks*

1) The mixing of the 1QP state from the next upper major shell in the AC state may be directly checked with the spectroscopic factor of the single-nucleon-transfer reaction. For example, in the case of $(d, p)$ reaction leading to the AC state, the direct transfer of single-neutron can only take place through the mixing effect. Therefore, the spectroscopic factor, which is given by $S_I=(\zeta_I u_{p'})^2\approx\zeta_I^2$ in the NTD approximation, must be very small as long as the AC state can be regarded as a relatively pure dressed 3QP state. In fact, the spectroscopic factor of the $(j-1)$ state has been known to be extremely small, being consistent with the theoretical prediction.

2) In a similar way as has been discussed so far, we can evaluate the other properties of the AC states on the basis of the method developed in Chap. 2. In the case of evaluating the quadrupole moment, however, we should carefully examine whether we should extend our quasi-particle NTD subspace to include the dressed 5QP modes or not, because even a small mixing of such higher collective states may yield a large effect on such a quantity.

## § 5. Comparison of calculated results with experimental data

In this section, let us make a comparison between the calculated results and available experimental data, in order to examine quantitatively the theoretical predictions stated above.

### 5–1 *Procedure of numerical calculations*

The parameters entering into the solutions of Eq. (3·23) are the quadrupole-force strength $\chi$ and the quantities related to the pairing correlations (i.e., the parameters $u_a$ and $v_a$ of the Bogoliubov transformation and the single-quasi-particle energies $E_a$), which are determined from the single-particle energies $\epsilon_a$ and the pairing-force strength $G$.

In order to see the essential effects of the 3QP correlation originated from the quadrupole force and to fix the parameters as much as possible, we use the same values for the pairing-force strength $G$ and for the single-particle energies $\epsilon_a$ as those adopted in the work of Kisslinger and Sorensen,[1] and also make the same truncation of shell-model space as they have made. On the other

hand, the quadrupole-force strength $\chi$ is regarded as a free parameter which should be determined phenomenologically except for its usual mass-number dependence;[3]

$$\chi=\chi_0 b^{-4}A^{-5/3} \quad \text{MeV}\cdot\text{fm}^{-4},$$

where $b^2$ is the harmonic-oscillator range parameter. As is usual in the P+QQ force model, the reduced matrix element of the single-particle quadrupole moment $q(ab)$ is calculated with the harmonic-oscillator-shell-model wave functions. Then, since $q(ab)$ is proportional to $b^2$, the factor $b^{-4}$ does not explicitly appear in the reduced matrix element of the quadrupole force, $1/2\cdot\chi q(ab)q(cd)$, and only $\chi_0$ is regarded as a parameter.

Numerical calculations have been performed for the three shell regions, i.e., $1h^-_{11/2}$-odd-neutron region, $1g^+_{9/2}$-odd-proton region and $1g^+_{9/2}$-odd-neutron region. To see change of the relative excitation-energies of the dressed 3QP modes, $\omega'_I\equiv\omega_I-E_p$, over a wide sequence of spherical odd-mass nuclei, we have first used a constant value of $\chi_0$ in each shell region. Secondly, in the evaluations of the mixing effects and of various electromagnetic quantities, we have chosen the value of $\chi_0$ in each nucleus, so that the calculated value of $\omega'_I$ just reproduces the experimental excitation energy of the $(j-1)$ state measured from the 1QP state with spin $j$. In this step, the mixing effects have been calculated by taking the same value of the single-particle energy of the orbit $p'$ (in the next upper major shell) as that adopted in the work of Uher and Sorensen[25] and also by putting $v_{p'}\approx 0$. The electromagnetic quantities have been calculated by using the polarization charge $\delta e=0.5\,e$ (for both proton and neutron) and the effective spin $g$ factor $g_s^{\text{eff}}=0.55\ g_s$, since we have adopted the P+QQ force model in the truncated shell-model space consisting of one major shell (for both protons and neutrons).

Thus it is evident that our choice of the parameters is merely the conventional one without any modifications.

### 5–2 *Region of $h^-_{11/2}$-odd-neutron nuclei*

This is the region in which the unique-parity orbit $h^-_{11/2}$ is being filled with neutrons. In Cd, Te and Xe isotopes, the $9/2^-$ states have been found in recent experiments at a few hundred keV in energy above the 1QP $11/2^-$ states.

In Fig. 11 are shown the calculated energy levels $\omega'_I$ for the sequences of odd-mass Cd, Sn, Te, Xe and Ba isotopes. The adopted value of $\chi_0$ is the same as has been derived by Baranger and Kumar[24] within a few percent and also as is expected from conventional arguments in the P+QQ force model.[3] It is predicted from the result of the theoretical calculations that the excitation energies, $\omega'_I=\omega_I-E_p$, of the $9/2^-$ states are decreasing as one moves from the single-closed shell Sn isotopes to the heavier Te, Xe and Ba isotopes, and in each sequence of the isotopes, they are decreasing as the

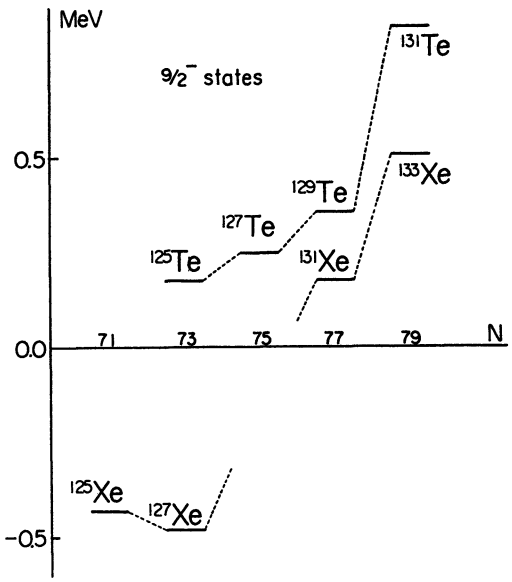



Fig. 10. Experimental trend of energy levels of the $9/2^-$ states in the $h^-_{11/2}$-odd-neutron region. The level energies are those measured from the 1QP $11/2^-$ states.
$^{125}$Te; Ref. 35), $^{127}$Te; Ref. 29), $^{129}$Te; Ref. 30), $^{131}$Te; Ref. 31), $^{125,127}$Xe; Ref. 34), $^{131}$Xe; Ref. 32), $^{133}$Xe; Ref. 33).

neutrons fill the unique-parity orbit $1h^-_{11/2}$ toward its middle. This calculated trend is naturally understood when we recall the enhancement factors of the 3QP correlation discussed at the end of §3–3: The decrease of the $9/2^-$ energy from Sn to Ba isotopes can be well understood as due to the increase of the factor $\xi(bc)$ of the core, and in each sequence of the isotopes the decrease is due to the increase of the factor $\xi(pp)$ at the unique-parity orbit $1h^-_{11/2}$.

So far, none of the low-lying $9/2^-$ states is experimentally observed in Sn isotopes.[36),78)] The reason can be explained when we consider the $9/2^-$ states as the dressed 3QP states, because in such single-closed-shell nuclei the enhancement factors $\xi(bc)$ of the core become so small that, in the theoretical calculations, the $9/2^-$ states are forced to lie at about 1 MeV above the 1QP $11/2^-$ states. In Te and Cd isotopes (in which two protons and two proton-holes are added to the proton-closed shell Sn isotopes, respectively), the $9/2^-$ states found in the experiments are well reproduced by theoretical calculations with the reasonable value of $\chi_0$. When we regard the $9/2^-$ states as the 3QP "intruder states" of Kisslinger composed of the neutrons in $(1h^-_{11/2})^n$-configuration, it is hard to understand the (above mentioned) different experimental situations between Sn isotopes and Te and Cd isotopes. Furthermore, according to the discussion made in §3–3, the fact that there are no near-lying $9/2^-$ states other than the first $9/2^-$ states under consideration also indicates

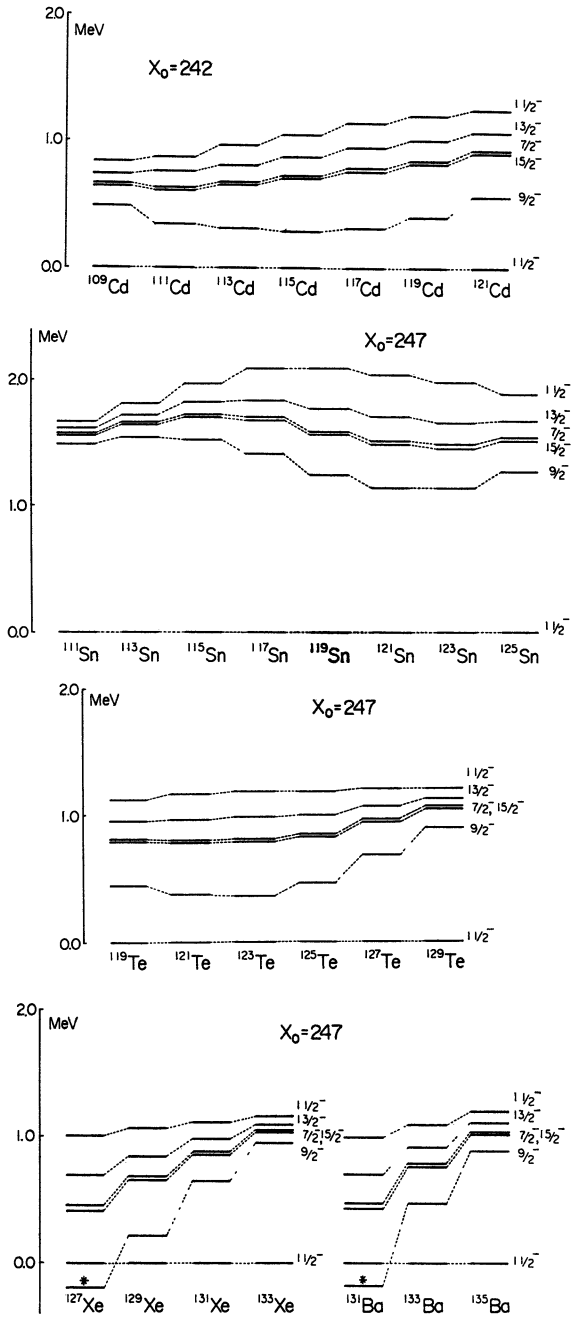



Fig. 11. Calculated excitation-energy systematics of the dressed 3QP states in the $h_{11/2}^-$-odd-neutron region. The level energies are those measured from the 1QP $11/2^-$ states. The quadrupole-force parameter $\chi_0$ is fixed in each region of isotopes. The asterisk denotes the occurence of instability for this choice of the parameter $\chi_0$.

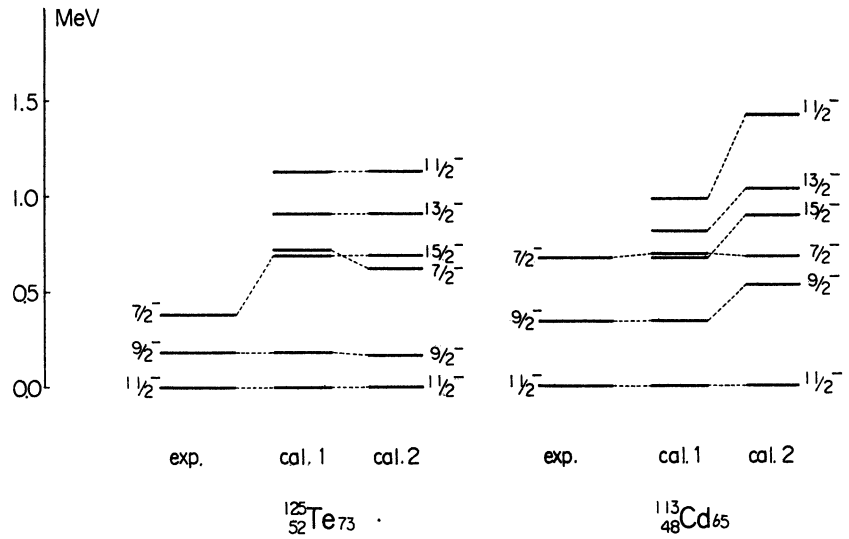



Fig. 12. Comparison between the experimental energy levels and the theoretical calculations for both cases, i.e., without the coupling effects (cal. 1) and taking account of the coupling effects (cal. 2). The energies are those measured from the 1QP $11/2^-$ states. Only the lowest-lying collective states in each spin are written in the figure. Experimental data are taken from; $^{115}$Cd, Ref. 28), $^{125}$Te, Ref. 35).

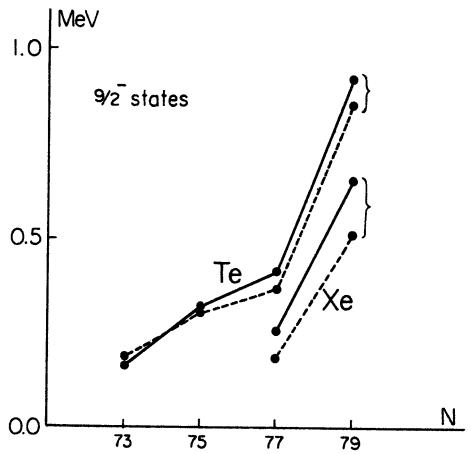



Fig. 13. Energy shifts due to the coupling effects of the dressed 3QP $9/2^-$ modes with the 1QP $h_{9/2}^-$ modes. The energies of the $9/2^-$ states in the absence of the coupling effects are connected by broken lines, while those in the presence of the coupling effects are connected by solid lines. All energies are those measured from the 1QP $11/2^-$ states.

the strong coupling between the 3QP "intruder" state and the "phonon" excited from the core.

In contrast to Te isotopes, the experimental energy change of the $9/2^-$ states in the sequence of Xe isotopes is rapid, and at the neutron deficient $^{127}$Xe the $9/2^-$ state becomes lower than the $11/2^-$ state. According to our point of view, this fact indicates the growth of instability toward quadrupole deformation as one moves toward $^{127}$Xe. These experimental facts are indeed those which are expected from the theoretical calculations, and the situation remains unchanged for a rather wide range of parameter $\chi_0$. From the theoretical calculations, similar experimental aspects can also be expected in Ba isotopes. So far there is no systematic experimental evidence that the neutron-deficient odd-mass Xe isotopes, in which the $9/2^-$ states are lower than the 1QP $11/2^-$ states, have stable deformations. It is interesting to note, however, that the adjacent even-even nuclei clearly display the quasi-rotational spectra.

The magnetic moment and electromagnetic transition rates of the $9/2^-$ state have been measured in $^{125}$Te and $^{127}$Te. The collective structure of the $9/2^-$ states exhibited in the excitation-energy systematics mentioned above is directly demonstrated in the striking enhancement of the $B(E2)$ value between the $9/2^-$ state and the $11/2^-$ state. The esssential role of the 3QP correlation (in characterizing the $9/2^-$ state) at the unique-parity orbit $1h_{11/2}^-$ is directly reflected in its $g$ factor which is approximately equal to that of the 1QP $11/2^-$ state. Here, in contrast to the case of the $B(E2)$ values, the quasi-particles

Table I. The correlation amplitudes of the dressed 3QP $9/2^-$ mode in $^{125}$Te. The adopted value of $\chi_0$ is 260 (MeV) and the calculated excitation energy, $\omega'=\omega-E_p$, is 0.08 MeV. The values of forward amplitudes $\psi(bc;p)$ are written in the second column, while the values of backward amplitudes $\varphi(bc;p)$ are written in the third column. In this state, the unique-parity orbit $p$ is specified by the set of quantum numbers (neutron; $h_{11/2}^-$), and therefore only the orbital pairs, $(bc)$'s, are written in the first column. For convenience, the notations $\psi(pp;p)$ and $\varphi(pp;p)$ are used here to denote $\psi(p^3)$ and $\varphi(p^3)$ defined by (3·19), respectively. These amplitudes are normalized to one according to Eq. (3·26) in the text.

neutron

| $bc$ | $(h_{11/2})^2$ | $(g_{7/2})^2$ | $(d_{5/2})^2$ | $(d_{3/2})^2$ | $g_{7/2}d_{5/2}$ | $g_{7/2}d_{3/2}$ | $d_{5/2}d_{3/2}$ | $d_{5/2}s_{1/2}$ | $d_{3/2}s_{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(bc;p)$ | 0.97 | 0.15 | 0.07 | 0.35 | −0.03 | 0.27 | 0.11 | 0.11 | −0.25 |
| $\varphi(bc;p)$ | 0.48 | 0.13 | 0.07 | 0.30 | −0.03 | 0.24 | 0.10 | 0.10 | −0.22 |

proton

| $bc$ | $(h_{11/2})^2$ | $(g_{7/2})^2$ | $(d_{5/2})^2$ | $(d_{3/2})^2$ | $g_{7/2}d_{5/2}$ | $g_{7/2}d_{3/2}$ | $d_{5/2}d_{3/2}$ | $d_{5/2}s_{1/2}$ | $d_{3/2}s_{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(bc;p)$ | 0.07 | 0.64 | 0.21 | 0.02 | −0.11 | 0.11 | 0.03 | 0.06 | −0.02 |
| $\varphi(bc;p)$ | 0.07 | 0.51 | 0.18 | 0.02 | −0.09 | 0.10 | 0.03 | 0.05 | −0.01 |

excited from the core give rise to only slight difference between their $g$ factors. The essential part of the values of the $B(E2)$ and $g$ factor can be determined without introducing the mixing effects. On the other hand, the mixing effects coming from the coupling between the dressed 3QP mode and the 1QP mode are sensitively reflected in the occurence of the retarded $M1$ transition from the

Table II. $B(E2; 9/2^- \to 11/2^-)$ values in the $h_{11/2}^-$-odd-neutron region. The values are written in unit of $e^2 \cdot 10^{-50}$ cm$^4$. The excitation energies of the $9/2^-$ states measured from the $11/2^-$ states are listed in the second column (in unit of MeV). The $B(E2)$ values calculated by neglecting the coupling effects are listed in the third column, while those calculated by taking account of the coupling effects in the fourth column. They are compared with experimental data $B(E2)^{\text{exp}}$ listed in the fifth column. The harmonic-oscillator-range parameter $b^2 = 1.0A^{1/3}$ and the polarization charge $\delta e = 0.5e$ are used.

| nucleus | $\omega'_{j-1}$ | $B(E2)$[1] | $B(E2)$[2] | $B(E2)^{\text{exp}}$ |
|---|---|---|---|---|
| $^{113}$Cd | 0.34 | 9.8 | 8.3 | |
| $^{115}$Cd | 0.33 | 9.4 | 8.7 | |
| $^{125}$Te | 0.18 | 10.7 | 10.6 | 11.5±0.5[a] |
| $^{127}$Te | 0.25 | 8.3 | 8.1 | 9.2±1.3[b] |
| $^{129}$Te | 0.36 | 6.2 | 5.8 | |
| $^{131}$Te | 0.85 | 2.8 | 2.6 | |
| $^{131}$Xe | 0.18 | 15.8 | 14.6 | |
| $^{133}$Xe | 0.51 | 7.5 | 6.6 | |

a) Ref. 37), b) Ref. 38).

Table III. Gyromagnetic ratio for the $9/2^-$ states in the $h_{11/2}^-$-odd-neutron region in unit of nuclear magneton ($e\hbar/2Mc$). The values calculated by neglecting the coupling effects are listed in the second column, while the values calculated by taking account of the coupling effects are listed in the third column. The experimental data for the $9/2^-$ states are listed in the fourth column, while those for the $11/2^-$ states in the fifth column. The effective spin $g$ factor $g_s^{\text{eff}} = 0.55 g_s$ is used. The $g$ factor of the 1QP $11/2^-$ state is, therefore, assumed to be $-0.19$.

| nucleus | $g_{j-1}^{1)}$ | $g_{j-1}^{2)}$ | $g_{j-1}^{\text{exp}}$ | $g_j^{\text{exp}}$ |
|---|---|---|---|---|
| $^{113}$Cd | −0.26 | −0.25 | | −0.20[c] |
| $^{115}$Cd | −0.25 | −0.25 | | |
| $^{125}$Te | −0.21 | −0.21 | −0.204±0.007[a] | −0.169±0.009[d] |
| $^{127}$Te | −0.22 | −0.22 | −0.214±0.014[b] | −0.165±0.009[e] |
| $^{129}$Te | −0.23 | −0.22 | | −0.209±0.009[e] |
| $^{131}$Te | −0.25 | −0.25 | | |
| $^{131}$Xe | −0.22 | −0.22 | | |
| $^{133}$Xe | −0.25 | −0.25 | | |

a) Ref. 39), b) Ref. 38), c) Ref. 59), d) Ref. 41), e) Ref. 42).

Table IV. $B(M1; 9/2^- \to 11/2^-)$ values in the $h_{11/2}^-$-odd-neutron region. The values are written in unit of $(e\hbar/2Mc)^2$. The mixing amplitudes $\zeta_j$ and $\zeta_{I=j-1}$, defined by (3·50) and (3·49), are listed in the second and third columns, respectively. The contributions from the first and second terms in (4·22) are shown separately in the fourth and fifth columns, respectively. The calculated values $B(M1)^{\text{cal}}$ are listed in the sixth column and are compared with the experimental data $B(M1)^{\text{exp}}$ listed in the seventh column. In this calculation, the effective spin $g$ factor $g_s^{\text{eff}}=0.55\, g_s$ is used.

| nucleus | $\zeta_j$ | $\zeta_{j-1}$ | $M_{11}$ | $M_{33}$ | $B(M1)^{\text{cal}}$ | $B(M1)^{\text{exp}}$ |
|---|---|---|---|---|---|---|
| $^{113}$Cd | −0.39 | −0.09 | −0.36 | −2.17 | $1.5\times10^{-1}$ | |
| $^{115}$Cd | −0.28 | −0.09 | −0.35 | −1.51 | $8.2\times10^{-2}$ | |
| $^{125}$Te | 0.03 | −0.08 | −0.25 | 0.11 | $4.8\times10^{-4}$ | $(6.5\pm0.3)\times10^{-3}$ a) |
| $^{127}$Te | 0.13 | −0.07 | −0.19 | 0.45 | $1.6\times10^{-3}$ | $(1.6\pm0.6)\times10^{-3}$ b) |
| $^{129}$Te | 0.24 | −0.06 | −0.13 | 0.84 | $1.2\times10^{-2}$ | |
| $^{131}$Te | 0.25 | −0.03 | −0.06 | 1.05 | $2.3\times10^{-2}$ | |
| $^{131}$Xe | 0.27 | −0.06 | −0.15 | 1.13 | $2.3\times10^{-2}$ | |
| $^{133}$Xe | 0.36 | −0.04 | −0.08 | 1.53 | $5.0\times10^{-2}$ | |

a) Ref. 37), b) Ref. 38).

Table V. Electromagnetic properties of the dressed 3QP states with negative parity in $^{125}$Te. The calculated values for two alternative approximations, i.e., without taking account of the coupling effects (cal. 1) and taking account of the coupling effects (cal. 2), are listed in the third and fourth columns, respectively. The experimental data are listed in the fifth column. The units are $e^2\cdot10^{-50}$ cm$^4$ for $B(E2)$, $e\hbar/2Mc$ for $g$ factors and $(e\hbar/2Mc)^2$ for $B(M1)$. The polarization charge $\delta e=0.5e$ and $g_s^{\text{eff}}=0.55\, g_s$ are used. The procedure of the calculation is the same as in Tables II∼IV, except that the value of $g_p$ is directly taken from the experimental value of the 1QP $11/2^-$ state ($g_p=-0.17$). The spectroscopic factors for $(d, p)$ reaction are calculated by the approximation $S_I\approx(\zeta_I)^2$.

| observable | spin | cal. 1 | cal. 2 | exp |
|---|---|---|---|---|
| $B(E2)$ | $7/2^- \to 11/2^-$ | 4.5 | 4.6 | |
| | $9/2^- \to 11/2^-$ | 10.7 | 10.6 | 11.5±0.5 a) |
| | $11/2'^- \to 11/2^-$ | 2.5 | 2.5 | |
| | $13/2^- \to 11/2^-$ | 3.5 | 3.5 | |
| | $15/2^- \to 11/2^-$ | 4.6 | 4.6 | |
| $g$ | $7/2^-$ | −0.38 | −0.38 | |
| | $9/2^-$ | −0.19 | −0.19 | −0.204±0.007 b) |
| | $11/2'^-$ | −0.11 | −0.11 | |
| | $13/2^-$ | −0.06 | −0.06 | |
| | $15/2^-$ | −0.05 | −0.05 | |
| $B(M1)$ | $9/2^- \to 11/2^-$ | 0.0 | $4.8\times10^{-4}$ | $(6.5\pm0.3)\times10^{-3}$ a) |
| $S$ | $7/2^-$ | 0.0 | 0.026 | |
| | $9/2^-$ | 0.0 | 0.006 | |

a) Ref. 37), b) Ref. 39).

$9/2^-$ state to the $11/2^-$ state. The actual value of $B(M1)$ is determined by the interference between the first and the second term in Eq. (4·23).

As shown in Table V, these characteristic properties of the $9/2^-$ states have been reproduced very well in the theoretical calculations without making any arbitrary alternation of the values of conventional parameters in the P+QQ force model. Thus the introduced model of the $9/2^-$ state as a typical manifestation of the dressed 3QP mode has been verified by the numerical results. Of course, similar characteristics are also expected in other nuclei and, therefore, systematic measurements of these electromagnetic quantities are expected to reveal further details of the structure of the $9/2^-$ states; in particular, it is interesting to focus our attention on the changes of their properties, from the nuclei with positive $\omega_I'$ toward the nuclei with negative $\omega_I'$.

### 5–3 *Region of $g_{9/2}^+$-odd-proton nuclei*

In this region, the unique-parity orbit $1g_{9/2}^+$ is being filled with protons. In the experiments, the rapid drop in energy of the $7/2^+$ state is observed as one moves from Nb to Ag. And, as is well known, the $7/2^+$ states appear below the $9/2^+$ states in Rh isotopes heavier than $^{103}$Rh and in all Ag isotopes, $^{103}$Ag$\sim$ $^{113}$Ag. In the theoretical calculations with a constant value of $\chi_0$, the energies of $7/2^+$ states, from $^{93}$Nb to $^{107}$Ag and also for each isotope, go down as functions of nucleon numbers $Z$ and $N$, and are in good agreement with the experimental trend. (See Figs. 14 and 15.) Here, the decrease of $\omega_I$ with $I=7/2$ can be understood as a result of the fact that two enhancement factors, $\xi(pp)$ and $\xi(bc)$, act coherently as one moves from Nb to the heavier odd-proton nuclei in this region.

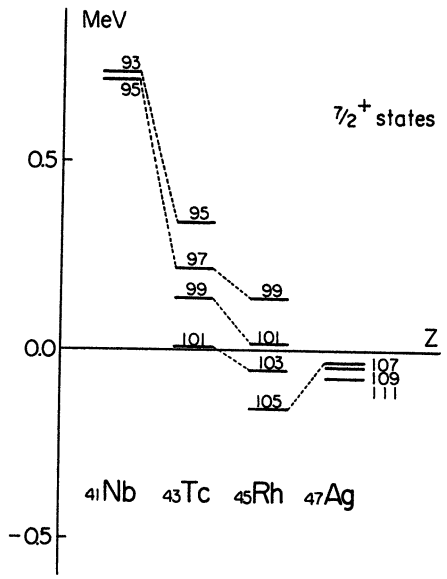



Fig. 14. Experimental trend of energy levels of the $7/2^+$ states in the $g_{9/2}^+$-odd-proton region. The level energies are those measured from the 1QP $9/2^+$ states.
$^{93}$Nb; Ref. 50), $^{95}$Tc; Ref. 52),
$^{97}$Tc; Ref. 44), $^{99}$Tc; Ref. 45),
$^{101}$Tc; Ref. 56), $^{99,101}$Rh; Ref. 46),
$^{103}$Rh; Ref. 47), $^{105}$Rh; Ref. 48),
$^{109,111}$Ag; Ref. 49).

The process of the rapid enhancement of the 3QP correlation can be explicitly seen when we compare the spectrum of $^{93}$Nb with that of $^{95}$Tc. In $^{93}$Nb we see a quintet with spins from $5/2^+$ to $13/2^+$, which is interpreted as consisting of one phonon and the odd quasi-particle. The energy splitting of the multiplet may be treated as a result of relatively small perturbations due to the 3QP correlation and the mixing effects (coming from the $H_V$-type interaction). However, in $^{95}$Tc in which only two protons are added to $^{93}$Nb, we see striking enhancement of the 3QP correlation. There, the energy splitting of the quintet amounts to the unperturbed energy of the $2^+$ phonon itself and, therefore, the splitting is beyond the limit of perturbational treatment. This enhancement is obviously caused by the increment of the factor $\xi(pp)$ in $^{95}$Tc when compared with that of $^{93}$Nb. As is shown in Fig. 16 and Table X, the theoretical calculation reproduces these changes very well, not only in the excitation energies but also in the electromagnetic properties. The appearance of different nature among the members of the quintet essentially comes from the spin dependence of the 3QP correlation, except for the $5/2^+$ state where the coupling of the 1QP mode from the next upper major shell affects its level position non-negligibly.

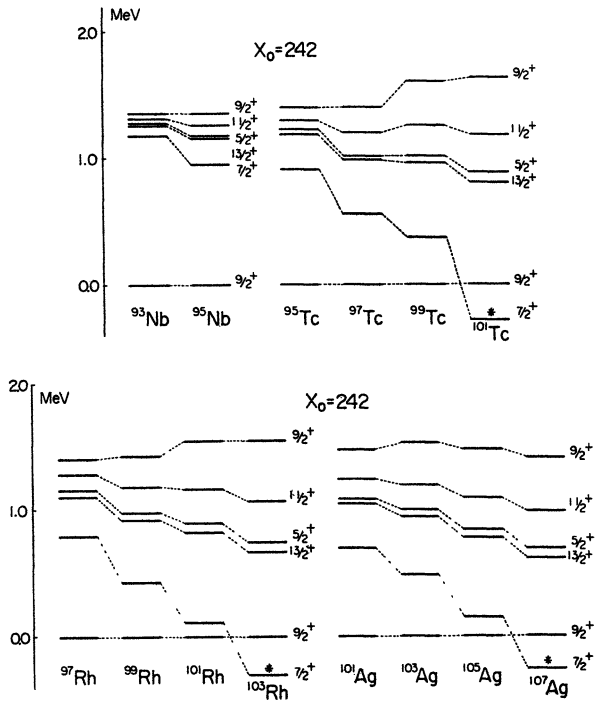



Fig. 15. Calculated excitation energy systematics of the dressed 3QP states in the $g^+_{9/2}$-odd-proton region. Notations are the same as in Fig. 11.

Let us further increase the enhancement factors, $\xi(pp)$ by adding protons and also $\xi(bc)$ of the core by adding neutrons. Then we arrive at such a quite different situation that only the $7/2^+$ states are extremely lowered in energy. For nuclei in which the anomalous coupling $7/2^+$ states appear below the 1QP $9/2^+$ states, we expect the growth of instability of the spherical BCS vacuum toward quadrupole deformation. Although, in the vicinity of the critical point $\omega_{cri}$, we cannot expect the quantitative validity of the NTD

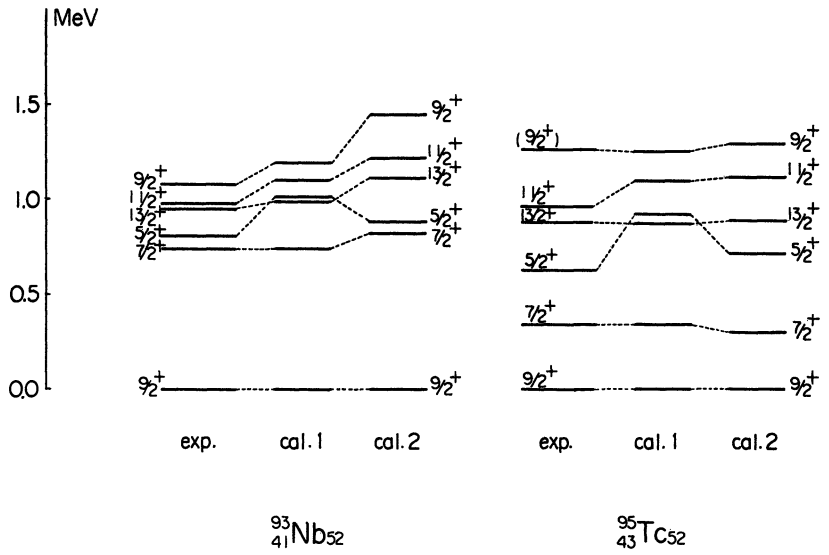



Fig. 16(a)

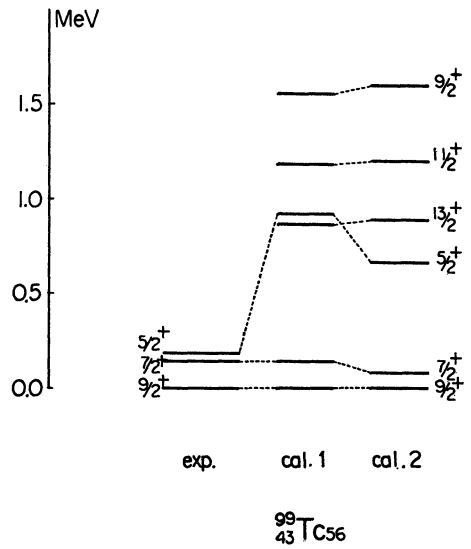



Fig. 16(b).

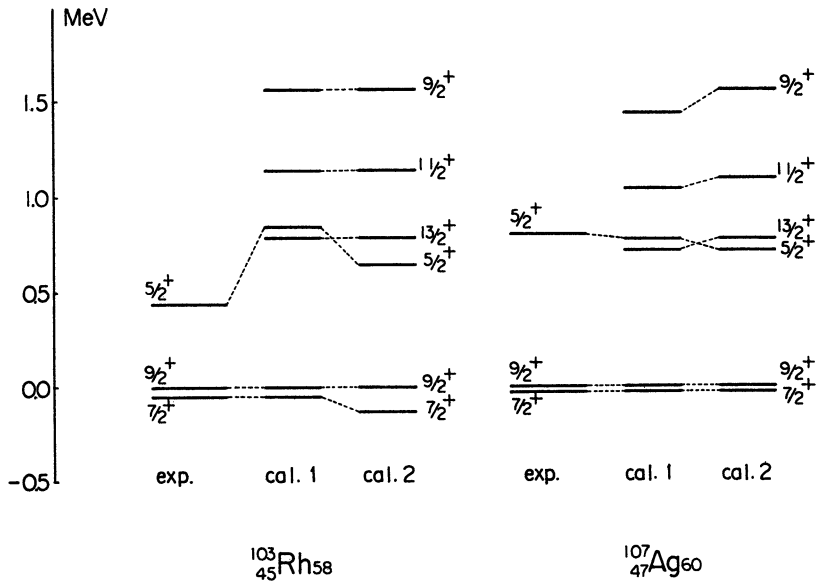



Fig. 16(c).

Fig. 16. Comparison between the experimental energy levels and the theoretical calculations in the $g^{+}_{9/2}$-odd-proton region. Notations are the same as in Fig. 12. For the experimental data, refer to the caption of Fig. 14.

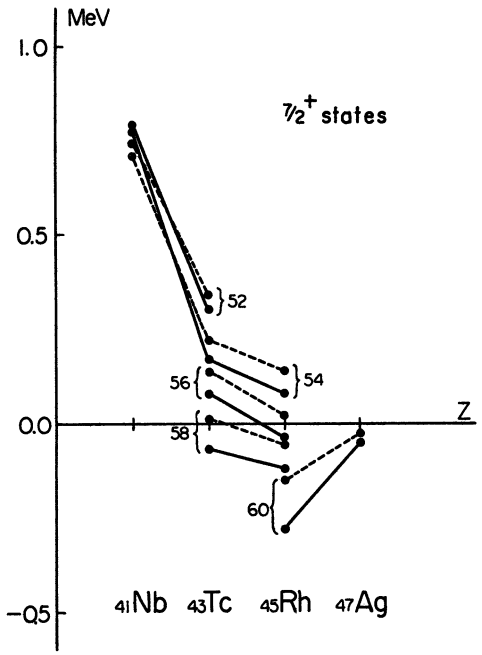



Fig. 17. Energy shifts due to the coupling effects of the dressed 3QP $7/2^{+}$ modes with the 1QP $g^{+}_{7/2}$ modes in the $g^{+}_{9/2}$-odd-proton region. Notations are the same as in Fig. 13.

approximation to be satisfactory, it is surprising that the experimental behaviours including the critical region, $E_p > \omega_I > \omega_{cri}$, have been well reproduced in the theoretical calculations, with the value of $\chi_0 = 242$ (MeV) which is just the conventional value derived from the classical method.[3] For instance, the calculated values of $B(E2; 7/2^+ \to 9/2^+)$ with polarization charge $\delta e = 0.5\, e$ are in good agreement with available experimental data even for Ag isotopes where the $7/2^+$ states appear below the $9/2^+$ states. (See Table VII.)

### 5-4 *Region of $g^+_{9/2}$-odd-neutron nuclei*

In this region the unique-parity orbit $1g^+_{9/2}$ is being filled with neutrons.

Table VI. The correlation amplitudes of the dressed 3QP $7/2^+$ mode in $^{99}$Tc. The adopted value of $\chi_0$ is 248(MeV) and the calculated excitation energy, $\omega' = \omega - E_p$, is 0.14 MeV. Notations are the same as in Table I. The unique-parity orbit $p$ denotes (proton; $1g^+_{9/2}$).

proton

| $bc$ | $(g_{9/2})^2$ | $(f_{5/2})^2$ | $(p_{3/2})^2$ | $f_{5/2}p_{3/2}$ | $f_{5/2}p_{1/2}$ | $p_{3/2}p_{1/2}$ |
|---|---|---|---|---|---|---|
| $\psi(bc; p)$ | 0.97 | 0.05 | 0.06 | −0.03 | 0.08 | 0.10 |
| $\varphi(bc; p)$ | 0.45 | 0.04 | 0.05 | −0.02 | 0.07 | 0.09 |

neutron

| $bc$ | $(h_{11/2})^2$ | $(g_{7/2})^2$ | $(d_{5/2})^2$ | $(d_{3/2})^2$ | $g_{7/2}d_{5/2}$ | $g_{7/2}d_{3/2}$ | $d_{5/2}d_{3/2}$ | $d_{5/2}s_{1/2}$ | $d_{3/2}s_{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(bc; p)$ | 0.11 | 0.13 | 0.68 | 0.04 | −0.10 | 0.07 | 0.11 | 0.36 | −0.07 |
| $\varphi(bc; p)$ | 0.11 | 0.11 | 0.53 | 0.03 | −0.09 | 0.06 | 0.10 | 0.30 | −0.06 |

Table VII. $B(E2; 7/2^+ \to 9/2^+)$ values in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table II.

| nucleus | $\omega'_{j-1}$ | $B(E2)^{1)}$ | $B(E2)^{2)}$ | $B(E2)^{exp}$ |
|---|---|---|---|---|
| $^{93}$Nb | 0.74 | 2.4 | 2.3 | 2.25±0.16 a) |
| $^{95}$Nb | 0.72 | 3.5 | 3.3 | |
| $^{95}$Tc | 0.34 | 5.2 | 5.2 | |
| $^{97}$Tc | 0.22 | 8.1 | 8.0 | |
| $^{99}$Tc | 0.14 | 11.4 | 11.2 | 13.5±1.5 b) |
| $^{101}$Tc | 0.01 | 18.7 | 18.1 | ≈30 d) |
| $^{99}$Rh | 0.14 | 9.3 | 9.2 | |
| $^{101}$Rh | 0.02 | 14.4 | 14.2 | |
| $^{103}$Rh | −0.05 | 21.0 | 20.6 | 9.5 c) |
| $^{105}$Rh | −0.15 | 39.2 | 37.7 | >31 c) |
| $^{107}$Ag | −0.03 | 20.1 | 19.1 | |
| $^{109}$Ag | −0.04 | 23.6 | 22.3 | 27.4 c) |
| $^{111}$Ag | −0.07 | 29.2 | 27.5 | 20.1 c) |

a) Ref. 51), b) Ref. 54), c) Ref. 57), d) Ref. 56).

Experimental data on the $N=47$ isotones show that the excitation energies of the $7/2^+$ states measured from those of the 1QP $9/2^+$ states become small with decreasing proton number, i.e., from $^{85}_{38}$Sr to $^{81}_{34}$Se. The same behaviour is also found in the $N=45$ isotones, i.e., from $^{83}_{38}$Sr to $^{77}_{32}$Ge. (See Fig. 18.) These behaviours are in correspondence with those of the $2^+$ phonon states in the sequences of neighbouring even-even nuclei. Then the appearance of the $7/2^+$ states below the 1QP $9/2^+$ states in many nuclei belonging to this region

Table VIII. Gyromagnetic ratio for the $7/2^+$ states in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table III.

| nucleus | $g^{1)}_{j-1}$ | $g^{2)}_{j-1}$ | $g^{\mathrm{exp}}_{j-1}$ | $g^{\mathrm{exp}}_{j}$ |
|---|---|---|---|---|
| $^{93}$Nb | 1.32 | 1.31 | | 1.37 a) |
| $^{95}$Nb | 1.31 | 1.30 | | |
| $^{95}$Tc | 1.27 | 1.26 | | |
| $^{97}$Tc | 1.27 | 1.25 | | |
| $^{99}$Tc | 1.25 | 1.23 | $0.75\pm0.26$ a) | 1.26 a) |
| $^{101}$Tc | 1.23 | 1.21 | | |
| $^{99}$Rh | 1.25 | 1.24 | | |
| $^{101}$Rh | 1.23 | 1.22 | | |
| $^{103}$Rh | 1.22 | 1.20 | | |
| $^{105}$Rh | 1.18 | 1.16 | | |
| $^{107}$Ag | 1.22 | 1.21 | | |
| $^{109}$Ag | 1.22 | 1.21 | $1.22\pm0.037$ b) | |
| $^{111}$Ag | 1.21 | 1.20 | | |

a) Ref. 59), b) Ref. 58).

Table IX. $B(M1;\ 7/2^+ \rightarrow 9/2^+)$ values in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table IV.

| nucleus | $\zeta_j$ | $\zeta_{j-1}$ | $M_{11}$ | $M_{33}$ | $B(M1)^{\mathrm{cal}}$ | $B(M1)^{\mathrm{exp}}$ |
|---|---|---|---|---|---|---|
| $^{93}$Nb | −0.29 | −0.18 | 0.68 | 1.86 | $1.9\times10^{-1}$ | $\sim1.6\times10^{-1}$ d) |
| $^{95}$Nb | −0.29 | −0.17 | 0.66 | 1.76 | $1.8\times10^{-1}$ | |
| $^{95}$Tc | −0.12 | −0.19 | 0.66 | 0.62 | $4.9\times10^{-2}$ | |
| $^{97}$Tc | −0.13 | −0.19 | 0.65 | 0.69 | $5.3\times10^{-2}$ | |
| $^{99}$Tc | −0.12 | −0.19 | 0.67 | 0.55 | $4.4\times10^{-2}$ | $(7.6\pm0.9)\times10^{-2}$ a)<br>$(9.44\pm0.1)\times10^{-2}$ b) |
| $^{101}$Tc | −0.11 | −0.21 | 0.72 | 0.43 | $3.9\times10^{-2}$ | $\sim1\times10^{-1}$ c) |
| $^{99}$Rh | 0.03 | −0.18 | 0.54 | −0.15 | $4.7\times10^{-3}$ | |
| $^{101}$Rh | 0.03 | −0.19 | 0.57 | −0.13 | $5.7\times10^{-3}$ | |
| $^{103}$Rh | 0.03 | −0.20 | 0.59 | −0.10 | $7.1\times10^{-3}$ | $(9.3\pm0.06)\times10^{-2}$ b) |
| $^{105}$Rh | 0.04 | −0.23 | 0.67 | −0.13 | $8.8\times10^{-3}$ | $<3.1\times10^{-2}$ b) |
| $^{107}$Ag | 0.19 | −0.18 | 0.40 | −0.64 | $1.8\times10^{-3}$ | $(4.19\pm0.4)\times10^{-2}$ b) |
| $^{109}$Ag | 0.20 | −0.17 | 0.38 | −0.70 | $3.0\times10^{-3}$ | $3.8\times10^{-2}$ b) |
| $^{111}$Ag | 0.20 | −0.17 | 0.38 | −0.82 | $5.6\times10^{-3}$ | $6.9\times10^{-2}$ b) |

a) Ref. 55), b) Ref. 57), c) Ref. 56), d) Ref. 50).

Table X(a). Electromagnetic properties of the "core excited" states with positive parity in $^{93}$Nb. Notations and parameters are the same as in Table V. The value of $g_p$ is taken from the experimental value of the 1QP $9/2^+$ state ($g_p=1.37$).

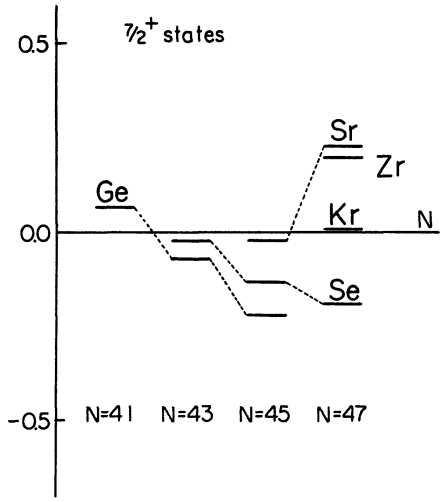
| observable | spin | cal. 1 | cal. 2 | exp |
|---|---|---|---|---|
| $B(E2)$ | $5/2^+ \to 9/2^+$ | 1.1 | 1.6 | $2.8\pm0.2$ [a] |
| | $7/2^+ \to 9/2^+$ | 2.4 | 2.3 | $2.25\pm0.16$ [a] |
| | $9/2^{+\prime} \to 9/2^+$ | 0.4 | 0.1 | $0.219\pm0.026$ [a] |
| | $11/2^+ \to 9/2^+$ | 0.7 | 0.6 | $1.06\pm0.09$ [a] |
| | $13/2^+ \to 9/2^+$ | 1.1 | 1.1 | $1.76\pm0.12$ [a] |
| $g$ | $5/2^+$ | 2.16 | 2.11 | |
| | $7/2^+$ | 1.47 | 1.45 | |
| | $9/2'^+$ | 1.17 | 1.18 | |
| | $11/2^+$ | 1.01 | 1.01 | |
| | $13/2^+$ | 0.95 | 0.95 | |
| $B(M1)$ | $7/2^+ \to 9/2^+$ | 0.0 | $1.93\times10^{-1}$ | $\sim1.6\times10^{-1}$ [b] |
| $S$ | $5/2^+$ | 0.0 | 0.10 | |
| | $7/2^+$ | 0.0 | 0.03 | |

a) Ref. 51), b) Ref. 50).

Table X(b). Electromagnetic properties of the dressed 3QP states with positive parity in $^{99}$Tc. Notations and parameters are the same as in Table V. The value of $g_p$ is taken from the experimental value of the 1QP $9/2^+$ state ($g_p=1.26$).

| observable | spin | cal. 1 | cal. 2 | exp |
|---|---|---|---|---|
| $B(E2)$ | $5/2^+ \to 9/2^+$ | 3.2 | 4.0 | $4.5\pm0.5$ [a] |
| | $7/2^+ \to 9/2^+$ | 11.4 | 11.2 | $13.5\pm1.5$ [a] |
| | $9/2'^+ \to 9/2^+$ | 1.0 | 0.9 | |
| | $11/2^+ \to 9/2^+$ | 2.1 | 2.1 | |
| | $13/2^+ \to 9/2^+$ | 3.5 | 3.5 | |
| $g$ | $5/2^+$ | 1.67 | 1.65 | $1.44\pm0.12$ [b] |
| | $7/2^+$ | 1.28 | 1.27 | $0.75\pm0.26$ [b] |
| | $9/2'^+$ | 1.10 | 1.10 | |
| | $11/2^+$ | 1.05 | 1.05 | |
| | $13/2^+$ | 1.05 | 1.05 | |
| $B(M1)$ | $7/2^+ \to 9/2^+$ | 0.0 | $4.4\times10^{-2}$ | $(7.6\pm0.9)\times10^{-2}$ [c] |
| $S$ | $5/2^+$ | 0.0 | 0.10 | |
| | $7/2^+$ | 0.0 | 0.04 | |

a) Ref. 54), b) Ref. 59), c) Ref. 55).



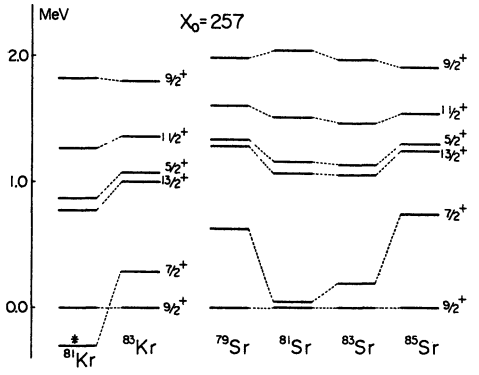




Fig. 18. Fig. 19.

Fig. 18. Experimental trend of energy levels of the $7/2^+$ states in the $g^+_{9/2}$-odd-neutron region. The level energies are those measured from the 1QP $9/2^+$ states.
$^{73}$Ge; Ref. 60), $^{75,77}$Ge; Ref. 79), $^{77,79}$Se; Ref. 61), $^{83}$Sr; Ref. 63), $^{85}$Sr; Ref. 62), $^{87}$Zr; Ref. 64).

Fig. 19. Calculated excitation-energy systematics of the dressed 3QP states in the $g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 11.

clearly has a close relationship with the fact that the neighbouring even-even nuclei possess a strong tendency of displaying the quasi-rotational spectra. In the case where the neutron number is in the vicinity of $N=40$ (the beginning of the $g^+_{9/2}$-orbit), in contrast to the cases of isotones with $N=46$ and 48, the neighbouring even-even nuclei seem to become increasingly unstable toward quadrupole deformation as the proton number increases toward $Z=38$ or 40.[69] Therefore, it is interesting to observe experimentarily, whether the correspondence between the behaviour of the excitation energies of the $7/2^+$ states and that of the $2^+$ states also holds in this case. Another marked phenomenon in this region, which is possibly in an intimate relation to the formation of the (static) quadrupole deformed field, is the appearance of the $5/2^+$ states with decreasing energy toward the nuclei with $N=41$ (which are shown in Fig. 23 and briefly discussed below).

### 5–5 *On AC states with spin $I=(j-2)$*

So far we have restricted our discussions on the AC states with spin $I=(j-1)$. Now let us briefly discuss the AC states with $I=(j-2)$.

In the experiments, special lowering of the anomalous spin $(j-2)$ states has been observed in some cases among the nuclei displaying the low-lying $(j-1)$ states. We can cite as examples the $5/2^+$ states in Tc isotopes ($Z=43$) with $N\gtrsim 56$ and the $5/2^+$ states in odd-neutron nuclei in the vicinity of $N=41$.

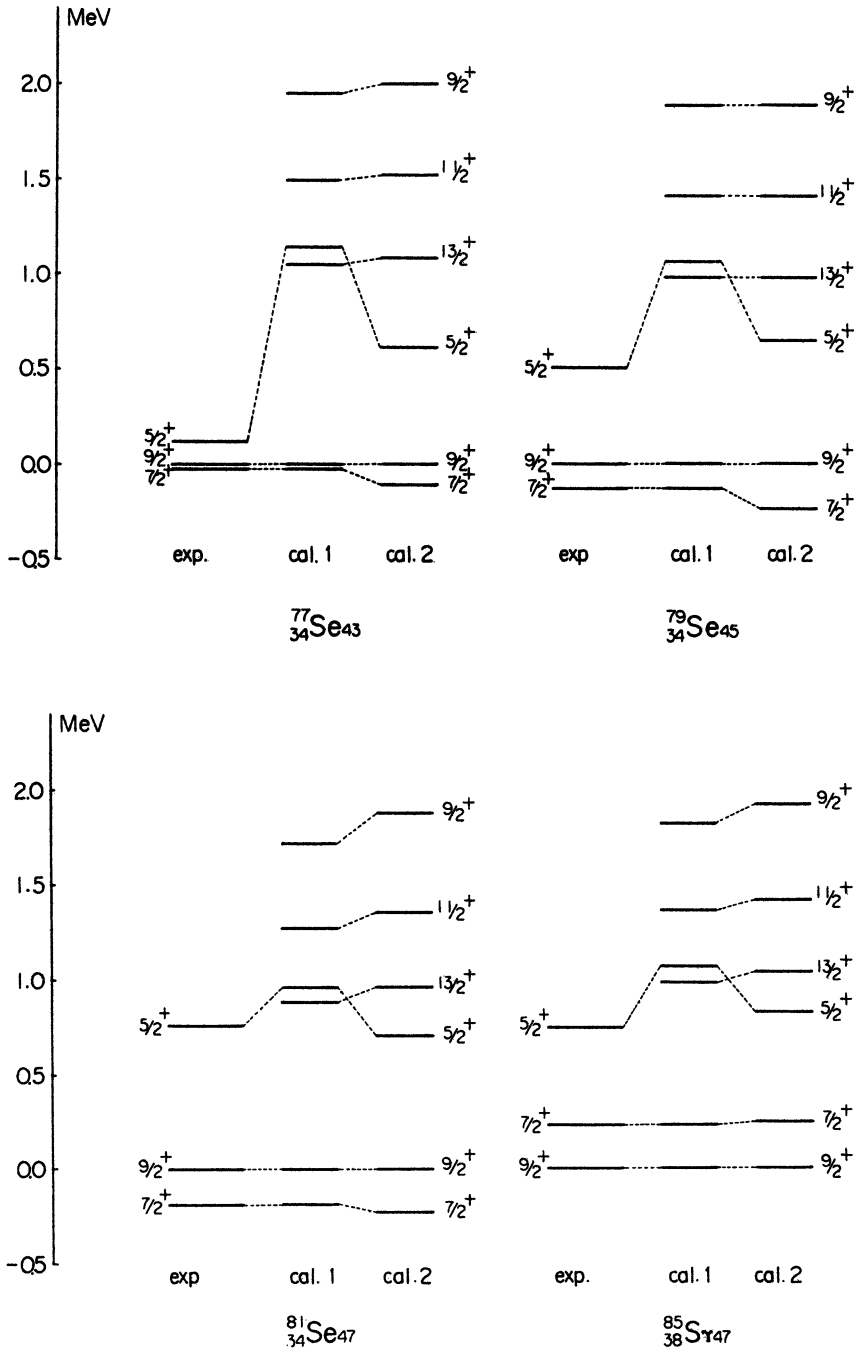



Fig. 20. Comparison between the experimental energy levels and the theoretical calculations in the $g^{+}_{9/2}$-odd-neutron region. Notations are the same as in Fig. 12. For the experimental data, refer to the caption of Fig. 18.

From the theoretical point of view based on the P+QQ force model, it is clear from the characteristic of the 3QP-correlation factor $C_I$ defined by (3·22) that, in contrast to the case of $I=(j-1)$, the excitation energy of the dressed 3QP mode having $I=(j-2)$ cannot be lowered by the action of the 3QP correlation. Therefore, as long as we stand on the P+QQ force model, the observed $(j-2)$ states cannot be regarded as the appearances of the dressed 3QP modes with $I=(j-2)$ in their pure forms. Hence, from our point of view, the energy-lowering of the $(j-2)$ states should be attributed to the effects of couplings among the modes with different transferred seniority quantum numbers.

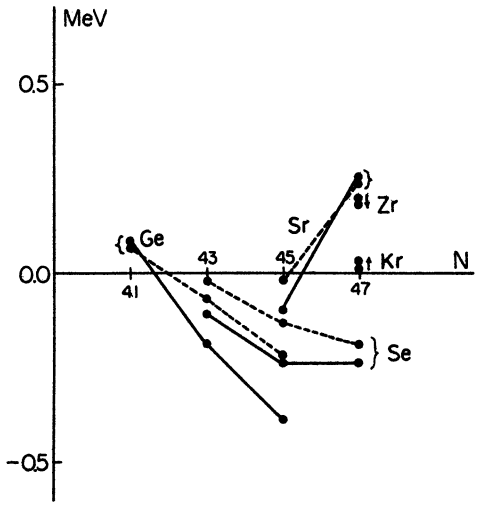



Fig. 21. Energy shifts due to the coupling effects of the dressed 3QP 7/2+ modes with the 1QP $g^{+}_{7/2}$ modes in the $g^{+}_{9/2}$-odd-neutron region. Notations are the same as in Fig. 13.

Table XI. The correlation amplitudes of the dressed 3QP 7/2+ mode in $^{85}$Sr. The adopted value of $\chi_0$ is 295 (MeV) and the calculated excitation energy, $\omega'=\omega-E_p$, is 0.23 MeV. Notations are the same as in Table I. The unique-parity orbit $p$ denotes (neutron; $1g^{+}_{9/2}$).

neutron

| $bc$ | $(g_{9/2})^2$ | $(f_{7/2})^2$ | $(f_{5/2})^2$ | $(p_{3/2})^2$ | $f_{7/2}f_{5/2}$ | $f_{7/2}p_{3/2}$ | $f_{5/2}p_{3/2}$ | $f_{5/2}p_{1/2}$ | $p_{3/2}p_{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(bc;p)$ | 1.05 | 0.01 | 0.02 | 0.02 | 0.01 | 0.02 | −0.01 | 0.04 | 0.05 |
| $\varphi(bc;p)$ | 0.48 | 0.01 | 0.02 | 0 02 | 0.01 | 0.01 | −0.01 | 0.04 | 0.04 |

proton

| $bc$ | $(g_{9/2})^2$ | $(f_{7/2})^2$ | $(f_{5/2})^2$ | $(p_{3/2})^2$ | $f_{7/2}f_{5/2}$ | $f_{7/2}p_{3/2}$ | $f_{5/2}p_{3/2}$ | $f_{5/2}p_{1/2}$ | $p_{3/2}p_{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\psi(bc;p)$ | 0.24 | 0.02 | 0.20 | 0.18 | 0.02 | 0.07 | −0.09 | 0.29 | 0.31 |
| $\varphi(bc;p)$ | 0.21 | 0.02 | 0.17 | 0.16 | 0.02 | 0.06 | −0.08 | 0.25 | 0.26 |

Table XII. $B(E2; 7/2^+ \to 9/2^+)$ values in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table II.

| nucleus | $\omega'_{j-1}$ | $B(E2)^{1)}$ | $B(E2)^{2)}$ | $B(E2)^{\mathrm{exp}}$ |
|---|---|---|---|---|
| $^{73}$Ge | 0.07 | 19.8 | 17.4 | $9.1\pm0.9$ [a] |
| $^{75}$Ge | −0.07 | 19.0 | 18.3 | |
| $^{77}$Ge | −0.22 | 31.6 | 30.3 | |
| $^{77}$Se | −0.02 | 18.6 | 18.0 | |
| $^{79}$Se | −0.13 | 23.1 | 22.5 | |
| $^{81}$Se | −0.19 | 37.1 | 34.3 | |
| $^{83}$Kr | 0.01 | 13.5 | 12.8 | $\begin{cases}5.8\pm1.3\ [b)]\\2.6\pm1.5\ [c)]\end{cases}$ |
| $^{83}$Sr | −0.02 | 11.1 | 10.9 | |
| $^{85}$Sr | 0.23 | 6.0 | 5.9 | |
| $^{87}$Zr | 0.20 | 5.2 | 5.1 | |

a) Ref. 65), b) Ref. 66), c) Ref. 68).

Table XIII. Gyromagnetic ratio for the $7/2^+$ states in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table III.

| nucleus | $g^{1)}_{j-1}$ | $g^{2)}_{j-1}$ | $g^{\mathrm{exp}}_{j-1}$ | $g^{\mathrm{exp}}_{j}$ |
|---|---|---|---|---|
| $^{73}$Ge | −0.25 | −0.22 | | −0.20 [c] |
| $^{75}$Ge | −0.22 | −0.20 | | |
| $^{77}$Ge | −0.17 | −0.15 | | |
| $^{77}$Se | −0.23 | −0.21 | | |
| $^{79}$Se | −0.21 | −0.19 | | |
| $^{81}$Se | −0.16 | −0.14 | | |
| $^{83}$Kr | −0.24 | −0.23 | $\begin{cases}-0.268\pm0.001\ [a)]\\-0.271\pm0.016\ [b)]\end{cases}$ | −0.215 [a] |
| $^{83}$Sr | −0.23 | −0.22 | | |
| $^{85}$Sr | −0.25 | −0.24 | | |
| $^{87}$Zr | −0.24 | −0.24 | | |

a) Ref. 67), b) Ref. 68), c) Ref. 59).

Table XIV. $B(M1; 7/2^+ \to 9/2^+)$ values in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table IV.

| nucleus | $\zeta_j$ | $\zeta_{j-1}$ | $M_{11}$ | $M_3$ | $B(M1)^{\mathrm{cal}}$ | $B(M1)^{\mathrm{exp}}$ |
|---|---|---|---|---|---|---|
| $^{73}$Ge | −0.30 | −0.22 | −0.84 | −1.64 | $1.8\times10^{-1}$ | |
| $^{75}$Ge | −0.11 | −0.20 | −0.70 | −0.51 | $4.3\times10^{-2}$ | |
| $^{77}$Ge | 0.04 | −0.22 | −0.66 | 0.22 | $5.8\times10^{-3}$ | |
| $^{77}$Se | −0.11 | −0.19 | −0.65 | −0.51 | $4.0\times10^{-2}$ | |
| $^{79}$Se | 0.04 | −0.19 | −0.55 | 0.19 | $4.0\times10^{-3}$ | |
| $^{81}$Se | 0.21 | −0.20 | −0.46 | 1.20 | $1.6\times10^{-2}$ | |
| $^{83}$Kr | 0.19 | −0.14 | −0.32 | 0.71 | $4.5\times10^{-3}$ | $(2.04\pm0.5)\times10^{-2}$ [a] |
| $^{83}$Sr | 0.03 | −0.15 | −0.46 | 0.08 | $4.2\times10^{-3}$ | |
| $^{85}$Sr | 0.15 | −0.12 | −0.27 | 0.37 | $2.8\times10^{-4}$ | |
| $^{87}$Zr | 0.10 | −0.12 | −0.27 | 0.36 | $2.2\times10^{-4}$ | |

a) Ref. 68).

Now it is noteworthy that the excitation-energy systematics of the $(j-2)$ states and of the $(j-1)$ states are distinctly different from each other. In the examples of $1g^{+}_{9/2}$-odd-neutron nuclei, the energy of the $5/2^+$ state decreases as the neutron number approaches to $N=41$ (the beginning of the $g^{+}_{9/2}$-orbit), while the energy of the $7/2^+$ state decreases as the neutrons fill the $g^{+}_{9/2}$-orbit

Table XV. Electromagnetic properties of the dressed 3QP states with positive parity in $^{83}$Kr. Notations and parameters are the same as in Table V. The value of $g_p$ is taken from the experimental value of the 1QP $9/2^+$ state ($g_p=-0.22$).

| observable | spin | cal. 1 | cal. 2 | exp |
|---|---|---|---|---|
| $B(E2)$ | $5/2^+ \to 9/2^+$ | 3.7 | 3.2 | $\begin{cases}5.8\pm1.3\ ^{a)}\\2.6\pm1.5\ ^{b)}\end{cases}$ |
| | $7/2^+ \to 9/2^+$ | 13.5 | 12.8 | |
| | $9/2'^+ \to 9/2^+$ | 2.6 | 2.1 | |
| | $11/2^+ \to 9/2^+$ | 3.0 | 2.9 | |
| | $13/2^+ \to 9/2^+$ | 3.9 | 3.7 | |
| $g$ | $5/2^+$ | $-0.46$ | $-0.44$ | $-0.268\pm0.001$ [c] |
| | $7/2^+$ | $-0.22$ | $-0.22$ | |
| | $9/2^+$ | $-0.14$ | $-0.14$ | |
| | $11/2^+$ | $-0.10$ | $-0.10$ | |
| | $13/2^+$ | $-0.09$ | $-0.09$ | |
| $B(M1)$ | $7/2^+ \to 9/2^+$ | 0.0 | $4.5\times10^{-3}$ | $(2.04\pm0.5)\times10^{-2}$ b) |
| $S$ | $5/2^+$ | 0.0 | 0.20 | |
| | $7/2^+$ | 0.0 | 0.02 | |

a) Ref. 66), b) Ref. 68), c) Ref. 67).

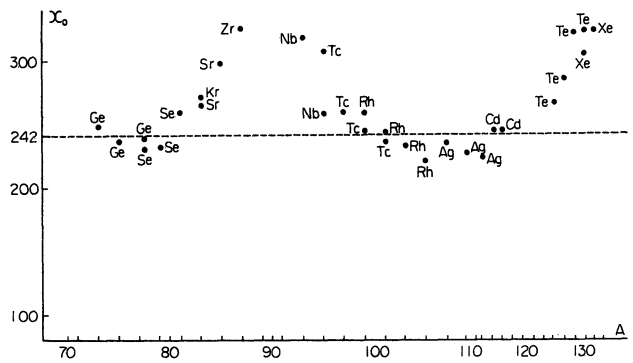



Fig. 22. Values of parameter $\chi_0$ chosen to bring the energies of the AC state with spin $(j-1)$ into agreement with the experimental data. The parameter $\chi_0$ is related to the quadrupole-force strength $\chi$ through $\chi=\chi_0 b^{-4}A^{-5/3}$, where $b^2$ is the harmonic-oscillator-range parameter and is taken to be $1.0A^{1/3}$. The broken line shows the value expected by the classical arguments.

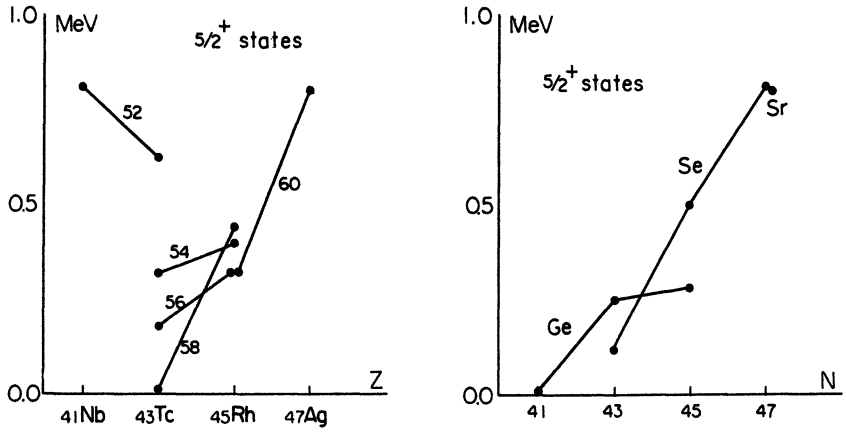



Fig. 23(a). Experimental trend of energy levels of the $5/2^+$ states in the region of $g^+_{9/2}$-odd-proton nuclei. The level energies are those measured from the 1QP $9/2^+$ states. For the experimental data, refer to the caption of Fig. 14.

Fig. 23(b). Experimental trend of energy levels of the $5/2^+$ states in the region of $g^+_{9/2}$-odd-neutron nuclei. The level energies are those measured from the 1QP $9/2^+$ states. For the experimental data, refer to the caption of Fig. 18.

toward its middle. (Compare Fig. 18 with Fig. 23.) The nuclei having extremely low-lying $(j-2)$ states seem to possess such a common feature that their Fermi surfaces (the chemical potentials) lie below the unique-parity level $p$ and, at the same time, the nuclei have relatively large enhancement factors of the "core," $\xi(bc)$. The former situation is just the one in which the effect of the $H_Y$-interaction becomes strong, because of its well-known dependence on the $\eta(ab)$ factor. (See Eqs. (3·7c) and (3·9).) The latter corresponds to the situation which is responsible for the lowering of the $2^+$ phonon states in the adjacent even-even nuclei. They are merely the conditions in which the effect of the coupling Hamiltonian $\boldsymbol{H}^{(\mathrm{int})}$ can become strong. Thus the special situation for the appearance of the extremely low-lying $(j-2)$ states seems to just correspond to the situations in which we can expect relatively strong couplings among the modes having different transferred seniority quantum numbers.

In Fig. 24 is shown the effect of coupling of the 1QP $2d^+_{5/2}$ mode (which lies in the next upper major shell) on the dressed 3QP mode with $5/2^+$. The calculated energy shift due to this type of coupling is rather large; in particular, remarkable energy-lowering of the $(j-2)$ states has been obtained in the numerical calculations for the nuclei in the vicinity of $N=41$. The magnitude of the energy shift is still insufficient to yield full explanation of the extreme lowering of $(j-2)$ states as observed in the experiments. This is a matter of course since we have neglected the other types of coupling, for example, that of

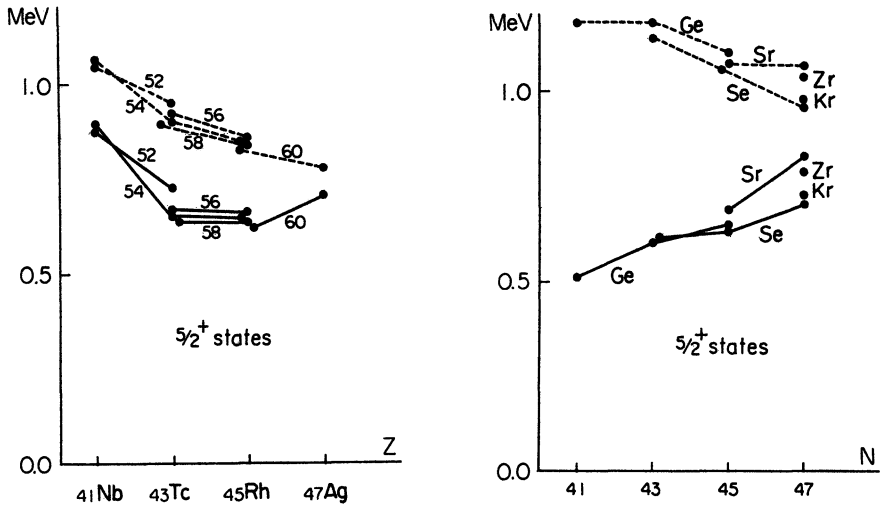



Fig. 24(a). Energy shifts due to the coupling effects of the dressed 3QP $5/2^+$ modes with the 1QP $d^+_{5/2}$ modes in the $g^+_{9/2}$-odd-proton region. Notations are the same as in Fig. 13.

Fig. 24(b). Energy shifts due to the coupling effects of the dressed 3QP $5/2^+$ modes with the 1QP $d^+_{5/2}$ modes in the $g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 13.

the dressed 5QP mode. Nevertheless, it is interesting to note that the energy shifts considered here bring the theoretical trend in the excitation-energy systematics of the $5/2^+$ states toward agreement with the experimental one (in which the energy of the $5/2^+$ state decreases toward the nuclei with $N=41$). Furthermore, the following data seem to be consistent with the theoretical prediction on the $(j-2)$ states mentioned above: In low-energy excitations through the $(d, p)$ reactions on nuclei in Ge-Se region, several states with anomalous spin $5/2^+$ have been observed in each nucleus, with spectroscopic factors being fragmented over these states.[70),71)]

We can easily find one of the reasons for the difference between the coupling effects for the $(j-1)$ states and for the $(j-2)$ states as follows: Let us note the matrix element $(p'\|r^2Y_2\|p)$ comprized in the effective coupling strength $\chi_{\rm int}(p', I)$. In the case of $I=(j_p-1)$, i.e., $j_{p'}=j_p-1$, the matrix element should be of spin-flip type, while in the case of $I=(j_p-2)$, i.e., $j_{p'}=j_p-2$, it is of spin-non-flip type. Since the latter is considerably larger than the former, the effective coupling strength $\chi_{\rm int}(p', I)$ for the mode with $I=(j-2)$ is larger than that for the mode with $I=(j-1)$.

In the semi-phenomenological models in which the stable quadrupole deformation is assumed, it has been known that the $(j-2)$ states as well as the $(j-1)$ states can be lowered in energy if the Coriolis interaction is suitably taken into account.[72)~74)] However, it seems difficult, to reproduce correctly the

(above mentioned) different trends in the systematics between the $5/2^+$ and $7/2^+$ states (as a function of $N$) without any ambiguity in fixing the sign of deformation parameter $\beta_0$.[72] Nevertheless, since almost all nuclei under consideration are regarded as lying just before the critical point of phase transition (from spherical to deformed), it may be very interesting to investigate a possible "unknown effect" which presumably corresponds to the Coriolis force and persists through the transition region. Thus the characteristic difference between the nature of the $(j-1)$ and $(j-2)$ states must be of great significance in further clarifying the mechanism of growth of the quadrupole instability. In this connection, the following experimental fact may be noteworthy: The extremely low-lying $(j-2)$ states appear in odd-mass nuclei, with $N$ or $Z$ being about 40 or 42, whose even-even neighbours exhibit the striking dip in energy of the first excited $0^+$ states.[75)~77)]

## §6. Concluding remarks

Starting from the new type of quasi-particle-phonon coupling (producing the 3QP correlation), we have investigated the mechanism of forming a new type of collective excitation mode (the dressed 3QP mode) in the special condition of shell structure for the appearance of the AC states with spin $I=(j-1)$. As was emphasized by Bohr and Mottelson, the new type of quasi-particle-phonon coupling originates from the composite nature of the phonon mode and plays an increasingly important role as the phonon energy decreases. Therefore, in the situation of odd-mass nuclei in which the new type of coupling is highly developed, the phonon mode can no longer be regarded as an elementary mode. Namely, the phonon mode is strongly coupled with the 3QP "intruder" state to form the dressed 3QP mode as a new kind of elementary excitation mode in spherical odd-mass nuclei. We can then point out the existence of an intimate relationship between the process of extreme energy-lowering of the dressed 3QP mode with spin $(j-1)$ and that of developing the quasi-rotational spectra in the neighbouring even-even nuclei, in connection with the growth of instability of the spherical BCS vacuum toward quadrupole deformation.

The microscopic model of the AC states with spin $(j-1)$ as the dressed 3QP modes has been shown to yield a consistent explanation of their various characteristics; i.e., the excitation-energy systematics, the $E2$- and $M1$-transition properties, the $g$ factor, the $S$ factor in $(d, p)$ reaction, etc. On the basis of the P+QQ force model, the predictions of the theory have also been examined with the numerical calculations. Thus we have arrived at the conclusion that the AC states with spin $(j-1)$ are nothing but the typical phenomena in which the dressed 3QP modes manifest themselves in their relatively pure and simple forms. In the succeeding chapters, the implication of this conclusion

is investigated under general situations of shell structure, by putting special attention to the physical condition of shell structure for the striking enhancement of the 3QP correlation.

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# Chapter 4. Persistency of AC State-Like Structure in Collective Excitations

*—Odd-Mass Mo, Ru, I, Cs and La Isotopes—*

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## § 1. Introduction

From among the complicated spectra of the low-lying excitations in odd-mass nuclei with mass numbers around $A \approx 100$, recent experiments reveal noticeable collective behaviour of the first $3/2^+$ states in odd-neutron nuclei and that of the second $5/2^+$ states in odd-proton nuclei, which are difficult to understand within the framework of the conventional quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen:[1] In odd-neutron Mo and Ru isotopes with $N=53$, 55 and 57, there systematically appear collective $3/2^+$ states with the enhanced $E2$- and hindered $M1$-transitions to the single-quasi-particle (1QP) $5/2^+$ states. (See Fig. 1.) In odd-proton I, Cs and La isotopes, the second $5/2^+$ states display the enhanced $E2$- and retarded $M1$-transitions to the 1QP $7/2^+$ states, characteristically indicating their strong collective nature. The excitation energies of the second $5/2^+$ states measured from the 1QP $7/2^+$ states decrease as the neutron number goes from the magic number $N=82$ to $N=72$. (See Fig. 2.) Furthermore, the first and the second $5/2^+$ states lie close to each other, proposing the interesting problem of clarifying the difference of their microscopic structures.

The main purpose of this chapter*) is to propose an interpretation which identifies the first $3/2^+$ states (in odd-neutron Mo and Ru isotopes) and the second $5/2^+$ states (in odd-proton I, Cs and La isotopes) as evidences for the fact that the appearance of the dressed 3QP mode is not specific to the anomalous coupling (AC) states but more general in odd-mass nuclei. The first motive for this identification is directly obtained when we notice a similarity between the above-mentioned electromagnetic properties of the $3/2^+$

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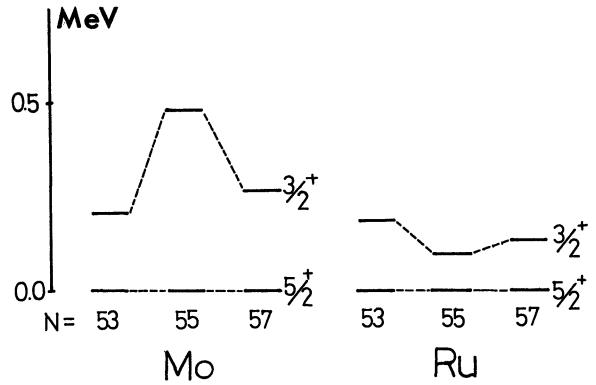


Fig. 1. Experimental trend of the excitation energies of the $3/2_1^+$ states in odd-mass Mo and Ru isotopes.
$^{95}$Mo; Ref. 2), $^{97}$Mo; Ref. 2), $^{99}$Mo; Ref. 5), $^{97}$Ru; Ref. 6), $^{99}$Ru; Ref. 12), $^{101}$Ru; Ref. 12).

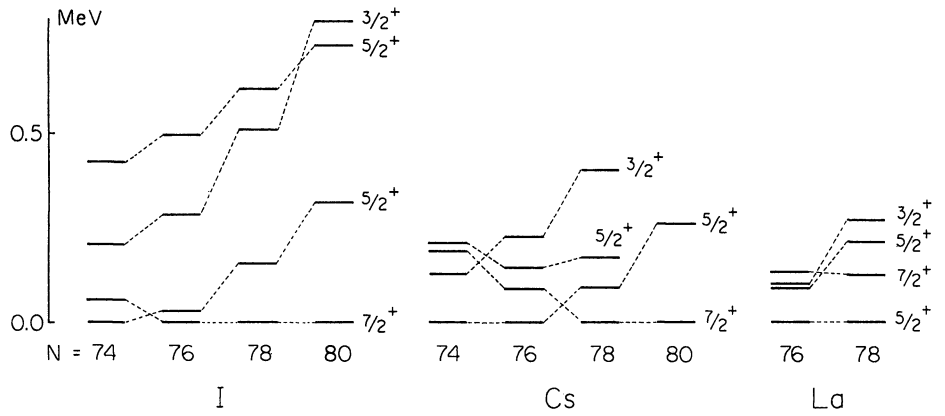


Fig. 2. Experimental trends of the excitation energies of the $5/2_2^+$ states and of the $3/2_1^+$ states in odd-mass I, Cs and La isotopes.
$^{127}$I; Ref. 13), $^{129}$I; Ref. 19), $^{131}$I; Ref. 14), $^{133}$I; Ref. 15), $^{129}$Cs; Ref. 22), $^{131}$Cs; Ref. 23), $^{133}$Cs; Ref. 27), $^{133}$ La; Ref. 24), $^{135}$La; Ref. 28).

and $5/2^+$ states and those of the AC states with spin $I=(j-1)$. As was mentioned in the preceding chapter, the main characteristics of the electromagnetic properties of the AC states are 1) strikingly enhanced $E2$ transitions to the 1QP states with spin $j$, which are comparable in magnitude with those of $2^+$-phonon transitions in neighboring even-even nuclei and 2) hindered corresponding $M1$ transitions. Of course, there is an important difference in shell structure between the collective $3/2^+$ and $5/2^+$ states under consideration and the AC states: In the case of the AC state the special situation of shell structure is the existence of a high-spin, unique-parity orbit which is being filled with several nucleons, while in the case of the collective $3/2^+$ and $5/2^+$ states many shell orbits with the same even parity are lying close to one another.

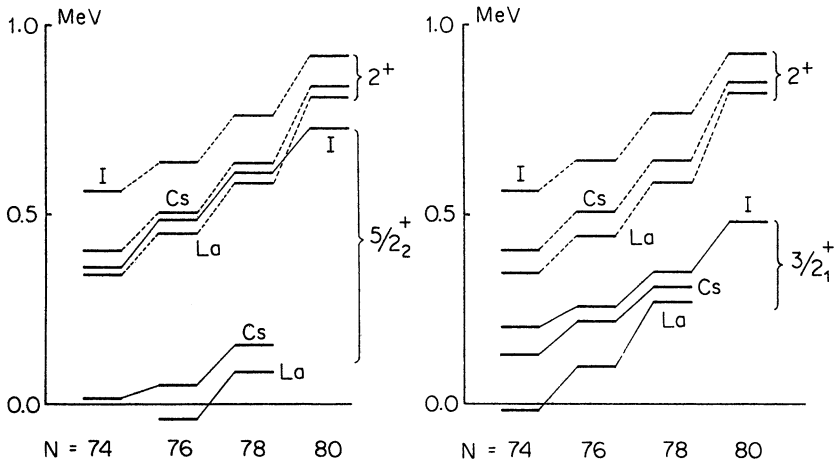


Fig. 3. Comparison between the experimental excitation-energy systematics of the $2^+$ phonon states in even-even nuclei and of the $5/2^+_2$ and $3/2^+_1$ states measured from the 1QP $7/2^+_1$ and $5/2^+_1$ states, respectively. The excitation energies of the $2^+$ phonons are those averaged over the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(Z, N) = 1/2\{\omega_{2^+}(Z-1, N)+\omega_{2^+}(Z+1, N)\}$.

Since these orbits with the same even parity are expected to actively participate in the 3QP correlation, it is quite interesting to investigate to what extent the similarity between the AC states and the collective $3/2^+$ and $5/2^+$ states can persist. In this chapter, therefore, the mechanism of the formation of AC state-like structure in the collective excitations of Mo, Ru, I, Cs and La isotopes will be clarified, by paying special attention to its relation with the conditions of shell structure.

In § 2, the formulation of the dressed 3QP mode as the new type of collective mode in these nuclei is presented by the use of the conventional pairing-plus-quadrupole (P+QQ) force model. In § 3 a criterion to investigate the similarity and difference between the 3QP correlations characterizing these states and those characterizing the AC states is given. With the aid of this criterion, the microscopic structure of the collective $3/2^+$ and $5/2^+$ states is discussed on the basis of calculated results. Here, the change of microscopic structure of these states depending on the mass number is also investigated in relation to the shell structure. In § 4, the coupling effects between the 1QP modes and the dressed 3QP modes are examined. In contrast to the case of the AC states, the dressed 3QP mode investigated in this chapter lies close, in energy, to the 1QP mode with the same spin and parity. For instance, the first $5/2^+$ states and the collective second $5/2^+$ states in I, Cs and La isotopes lie especially close to each other. At first sight, therefore, these two $5/2^+$ states seem to couple strongly with each other. However, it will be clarified that there exists an interesting mechanism to make the coupling effects weak. Taking into account the coupling effects, an analysis of the various properties

is made for the second $5/2^+$ states of I, Cs and La isotopes in § 5-a) and for the first $3/2^+$ states of Mo and Ru isotopes in § 5-b). The concluding remark is given in § 6.

## § 2. Preliminaries

In order to explicitly specify the freedom of protons and neutrons, we use Greek letters ($\alpha$, $\beta$, $\gamma$) and ($\rho$, $\sigma$): For the odd-proton (neutron) nuclei, $\alpha$, $\beta$ and $\gamma$ are respectively used to denote a set of quantum numbers of the single-particle states for protons (neutrons) and $\rho$ and $\sigma$ are respectively used for neutrons (protons). Then, according to the general theory developed in Chap. 2, the creation operator of the dressed 3QP mode in the P+QQ force model is given in terms of the quasi-particle operators as follows:

$$\begin{aligned} Y^{\dagger}_{nIK} = & \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \psi_{nI}(\alpha\beta\gamma) \boldsymbol{P}(\alpha\beta\gamma) a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a^{\dagger}_{\gamma} \\ & + \sum_{(\rho\sigma)\gamma} \{1+\delta_{rs}\}^{-1/2} \psi_{nI}(\rho\sigma;\gamma) \boldsymbol{P}(\rho\sigma) a^{\dagger}_{\rho} a^{\dagger}_{\sigma} a^{\dagger}_{\gamma} \\ & + \frac{1}{\sqrt{3!}} \sum_{a_1 a_2 a_3} \varphi^{(1)}_{nI}(a_1 a_2 a_3) \boldsymbol{P}(a_1 a_2 a_3) T_{3/2,-1/2}(a_1 a_2 a_3) \\ & + \frac{1}{\sqrt{2}} \sum_{\substack{a_1 a_2 \gamma \\ (a \neq c)}} \varphi^{(2)}_{nI}(a_1 a_2;\gamma) \boldsymbol{P}(a_1 a_2) T_{10}(a_1 a_2) a_{\tilde{\gamma}} \\ & + \sum_{\substack{(\alpha\beta)\gamma \\ (a \neq c, b \neq c)}} \{1+\delta_{ab}\}^{-1/2} \varphi^{(3)}_{nI}(\alpha\beta;\gamma) \boldsymbol{P}(\alpha\beta) a^{\dagger}_{\gamma} a_{\tilde{a}} a_{\tilde{\beta}} \\ & + \sum_{(\rho\sigma)\gamma} \{1+\delta_{rs}\}^{-1/2} \varphi^{(3)}_{nI}(\rho\sigma;\gamma) \boldsymbol{P}(\rho\sigma) a^{\dagger}_{\gamma} a_{\tilde{\rho}} a_{\tilde{\sigma}}. \end{aligned} \tag{2·1}$$

Here, the subscript $i(=1,2,3)$ of $a$ is used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit $a$ is necessary. The symbol $\sum_{(\alpha\beta)\gamma}$ represents the summation with respect to the orbital pair $(ab)$, $m_a$, $m_\beta$ and $\gamma$, and the operators $\boldsymbol{P}$'s denote the projection operators by which any angular-momentum-zero-coupled-pair component is removed from $(a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a^{\dagger}_{\gamma})$ and $(a_{\tilde{\alpha}} a_{\tilde{\beta}})$. The projection operators $\boldsymbol{P}$, the explicit form of which are given in Appendix 2A, guarantee the dressed 3QP modes to be orthogonal to both the spurious states (due to the nucleon-number non-conservation) and the pairing vibrational modes.

The collective mode given by Eq. (2·1) is characterized by the amount of seniority $\Delta v=3$ which it transfers to the correlated ground state. The first two terms on the right-hand side of Eq. (2·1) represent the forward-going components and the others are the backward-going components which originate from the ground-state correlation. It is evident that the ground-state correlation is essential to bring about the collectivity of excitation modes in the doubly-open-proton-neutron system such as the nuclei under consideration. As was shown in § 3 of Chap. 2, within the framework of the new-Tamm-Dancoff

approximation we obtain the eigenvalue equation which the correlation amplitudes should satisfy:

$$\begin{bmatrix} 3\boldsymbol{D} & -\boldsymbol{A} \\ \boldsymbol{A}^T & -\boldsymbol{d} \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}_{nI} \\ \boldsymbol{\varphi}_{nI} \end{bmatrix} = \omega_{nI} \begin{bmatrix} \boldsymbol{\psi}_{nI} \\ \boldsymbol{\varphi}_{nI} \end{bmatrix}, \tag{2·2}$$

where $\boldsymbol{\psi}_{nI}$ and $\boldsymbol{\varphi}_{nI}$ denote the matrix notations symbolizing the sets of the forward amplitudes $\{\psi_{nI}(\alpha\beta\gamma)$ and $\psi_{nI}(\rho\sigma;\gamma)\}$ and the sets of the backward amplitudes $\{\varphi^{(1)}_{nI}(\alpha_1\alpha_2\alpha_3)$, $\varphi^{(2)}_{nI}(\alpha_1\alpha_2;\gamma)$, $\varphi^{(3)}_{nI}(\alpha\beta;\gamma)$ and $\varphi^{(3)}_{nI}(\rho\sigma;\gamma)\}$, respectively. The explicit forms of matrices $3\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ (and its transpose $\boldsymbol{A}^T$) are given in Appendices 2B and 6A. When compared with the eigenvalue equation for the AC state given by (3·23) of the preceding chapter, the main complexity of Eq. (2·2) comes from the amplitudes of the type $\psi_{nI}(\alpha\beta\gamma)$: In the case of the AC states, because of the parity selection property of the quadrupole force the amplitudes $\psi_{nI}(\alpha\beta\gamma)$ are reduced to particularly simple forms, while in the case of the dressed 3QP mode under consideration (which has the same parity as that of the major shell) such a parity selection rule does not work for the main amplitudes. Of course the corresponding backward amplitudes also become relatively complicated. Therefore it is evident that we cannot expect, from the outset, the formation of such simple structure as the AC states, in the dressed 3QP mode given by (2·1).

As usual, the reduced $E2$-transition probability from the dressed 3QP state $|\Phi^{(3)}_{nIK}\rangle = Y^{\dagger}_{nIK}|\Phi_0\rangle$ with spin $I$ to the 1QP state $|\Phi^{(1)}_{\delta}\rangle = a^{\dagger}_{\delta}|\Phi_0\rangle$ with spin $j_d$ is defined by

$$B(E2; I \to j_d) = \frac{1}{2I+1} |\langle \Phi^{(1)}_{d} \| \hat{\boldsymbol{O}}^{(+)}_2 \| \Phi^{(3)}_{nI} \rangle|^2. \tag{2·3}$$

Following the general method developed in § 5 of Chap. 2, the reduced matrix element of the electric quadrupole operator $\hat{\boldsymbol{O}}^{(+)}_2$ in Eq. (2·3) is given as follows:

$$\begin{aligned}
&\langle \Phi^{(1)}_{d} \| \hat{\boldsymbol{O}}^{(+)}_2 \| \Phi^{(3)}_{nI} \rangle \\
&= \sqrt{\frac{3}{2}} \sum_{\substack{abc \\ a'b'c'J'}} e_\tau Q(ab)\delta_{c\bar{d}} P_I(ab(2)c|a'b'(J')c')\ \psi_{nI}[a'b'(J')c'] \\
&\quad + \sum_{(rs)c} e_\tau Q(rs)\delta_{c\bar{d}} \{1+\delta_{rs}\}^{-1/2} \psi_{nI}[rs(2)c] \\
&\quad + \frac{1}{\sqrt{2}} \sum_{aJ} e_\tau Q(aa)\delta_{a\bar{d}} P_I(aa(2)a|aa(J)a) \varphi^{(1)}_{nI}[aa(J)a] \\
&\quad + \sum_{\substack{ac,\ J=\text{even} \\ (a\neq c,\ J\neq 0)}} e_\tau Q(ca)\delta_{a\bar{d}} \sqrt{5(2J+1)} \begin{Bmatrix} j_a & j_a & J \\ j_c & I & 2 \end{Bmatrix} \varphi^{(2)}_{nI}[aa(J)c] \\
&\quad + \sum_{\substack{(ab)c \\ (a\neq c, b\neq c)}} e_\tau Q(ab)\delta_{c\bar{d}} \{1+\delta_{ab}\}^{-1/2} \varphi^{(3)}_{nI}[ab(2)c] \\
&\quad + \sum_{(rs)c} e_\tau Q(rs)\delta_{c\bar{d}} \{1+\delta_{rs}\}^{-1/2} \varphi^{(3)}_{nI}[rs(2)c],
\end{aligned} \tag{2·4}$$

where

$$Q(ab)=\frac{1}{\sqrt{5}}(a\|r^2 Y_2\|b)\cdot(u_a v_b+v_a u_b) \tag{2·5}$$

and $P$'s are the projection operators defined in Appendix 2A. In Eq. (2·4), $\psi$ and $\varphi$ are the coupled-angular-momentum representation of the forward amplitudes and backward amplitudes, respectively, which are straightforwardly obtained through the conventional procedure of angular-momentum coupling and defined in Appendix 6A; for instance,

$$\psi_{nI}(\alpha\beta\gamma)=\sum_{JI}(j_a j_b m_\alpha m_\beta|JM)(Jj_c M m_\gamma|IK)\psi_{nI}[ab(J)c]. \tag{2·6}$$

Needless to say, the parts in (2·4) involving the backward amplitudes $\varphi$ represent the effect of collective enhancement originating from the ground-state correlation. The characteristic property of the $E2$-transition matrix element given by (2·4) will be discussed in § 3 as a measure of clarifying the microscopic structure of the collective $5/2_2^+$ and $3/2_1^+$ states of interest.

A computer program named BARYON-1 was constructed to solve Eq. (2·2) and to calculate various electromagnetic properties of the dressed 3QP modes. Since our aim is not to obtain a detailed quantitative fitting with experimental data but to get an essential understanding of structures of collective $3/2_1^+$ and $5/2_2^+$ states which are difficult to understand within the framework of the conventional QPC theory, we have used exactly the same values for the single-particle energies and for the pairing-force strength $G$ in the numerical calculations as those adopted in the calculations of Kisslinger and Sorensen.[1] We have also made the same truncation of shell-model space as Kisslinger and Sorensen have made: The shell-model subspace for I, Cs and La isotopes consists of the orbits

$$\{\pi;\ 1g^+_{7/2},\ 2d^+_{5/2},\ 1h^-_{11/2},\ 2d^+_{3/2},\ 3s^+_{1/2}\},$$
$$\{\nu;\ 2d^+_{5/2},\ 1g^+_{7/2},\ 3s^+_{1/2},\ 1h^-_{11/2},\ 2d^+_{3/2}\},$$

and the subspace for Mo and Ru isotopes is composed of the orbits

$$\{\pi;\ 1f^-_{5/2},\ 2p^-_{3/2},\ 2p^-_{1/2},\ 1g^+_{9/2}\},$$
$$\{\nu;\ 2d^+_{5/2},\ 1g^+_{7/2},\ 3s^+_{1/2},\ 1h^-_{11/2},\ 2d^+_{3/2}\}.$$

The quadrupole-force strengths $\chi$ have been fixed at the values which reproduce the average energies of the $2^+$ phonon states in the adjacent even-even nuclei, e.g., $\bar{\omega}_{2^+}(Z,N)\equiv 1/2\{\omega_{2^+}(Z-1,N)+\omega_{2^+}(Z+1,N)\}$ for odd-$Z$ nuclei. (For the sake of comparison, the excitation spectra calculated with a constant value of $\chi_0\equiv\chi b^4 A^{5/3}$ for each isotopes are presented in Figs. 4(a) and 6(a).) Thus no adjustment of parameters has been attempted in the course of the calculations.

As is described in Appendix 4A, the method of solving Eq. (2·2) consists

of the following steps. First, the components of the 3QP correlation amplitudes $\psi$ and $\varphi$ (in the coupled-angular-momentum representation) are orthonormalized by diagonalizing the projection operators $\boldsymbol{P}$'s entering into the eigenvalue equation (2·2). By this diagonalization, the submatrices $3\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ in Eq. (2·2) are transformed into new submatrices $3\bar{\boldsymbol{D}}$, $\bar{\boldsymbol{d}}$ and $\bar{\boldsymbol{A}}$ (the elements of which are defined between the orthonormalized components). The calculation of thus obtained eigenvalue equation (4A·5) may be further performed in two steps: In the first step, the submatrices $3\bar{\boldsymbol{D}}$ and $\bar{\boldsymbol{d}}$ are diagonalized respectively. Needless to say, the eigenvalues of $3\bar{\boldsymbol{D}}$ represent the excitation energies of "bare" 3QP states in the quasi-particle Tamm-Dancoff approximation. Then, in the second step, the resulting total matrix given by (4A·8) is diagonalized to obtain the excitation energies and correlation amplitudes of the dressed 3QP modes. In the calculation of this chapter, we have adopted the following approximation: In the second step, the diagonal matrices, $\boldsymbol{\omega}^f$ and $\boldsymbol{\omega}^b$ (which are obtained from $3\bar{\boldsymbol{D}}$ and $-\bar{\boldsymbol{d}}$ respectively in the first step) have been truncated within 10 and 40 dimensions, respectively, with the former being of increasing order and the latter being of decreasing order in energy. Accuracy of this approximation has been checked by comparing some results with the corresponding results of full calculations, and has been satisfactory except for the special case where the excitation energy of the dressed 3QP mode lies extremely close to the critical point for the instability of the spherical BCS vacuum.

## § 3. Microscopic structure of collective excitations in odd-mass Mo, Ru, I, Cs and La isotopes

### 3–1 *Collective* $5/2^+$ *states in I, Cs and La*

In Figs. 4(a) and 4(b) are shown the calculated results for the excitation energies of the collective $5/2_2^+$ states as the dressed 3QP modes. In these figures, the first $7/2^+$ and $5/2^+$ states correspond to the 1QP states related to the orbits $1g_{7/2}^+$ and $2d_{5/2}^+$, respectively. All the energies are measured from those of the 1QP $7/2^+$ states. The systematic appearance of the low-lying collective $5/2_2^+$ states in I, Cs and La isotopes is reproduced very well. The experimental trend that the excitation energies of the collective $5/2_2^+$ states (measured from those of the 1QP $7/2^+$ states) decrease as the neutron number goes away from the magic number is also well reproduced. This trend is seen in a rather magnified way in Fig. 4(a) in which a constant value of the quadrupole-force parameter $\chi_0$ is used for each isotopes. Such magnification is the same as that well known in the conventional RPA with the P+QQ force in even-even nuclei, i.e., the $2^+$ phonon energies calculated with the constant value of $\chi_0$ decrease more rapidly than the experimental trend as the nucleon numbers go away from the magic number.[1)]

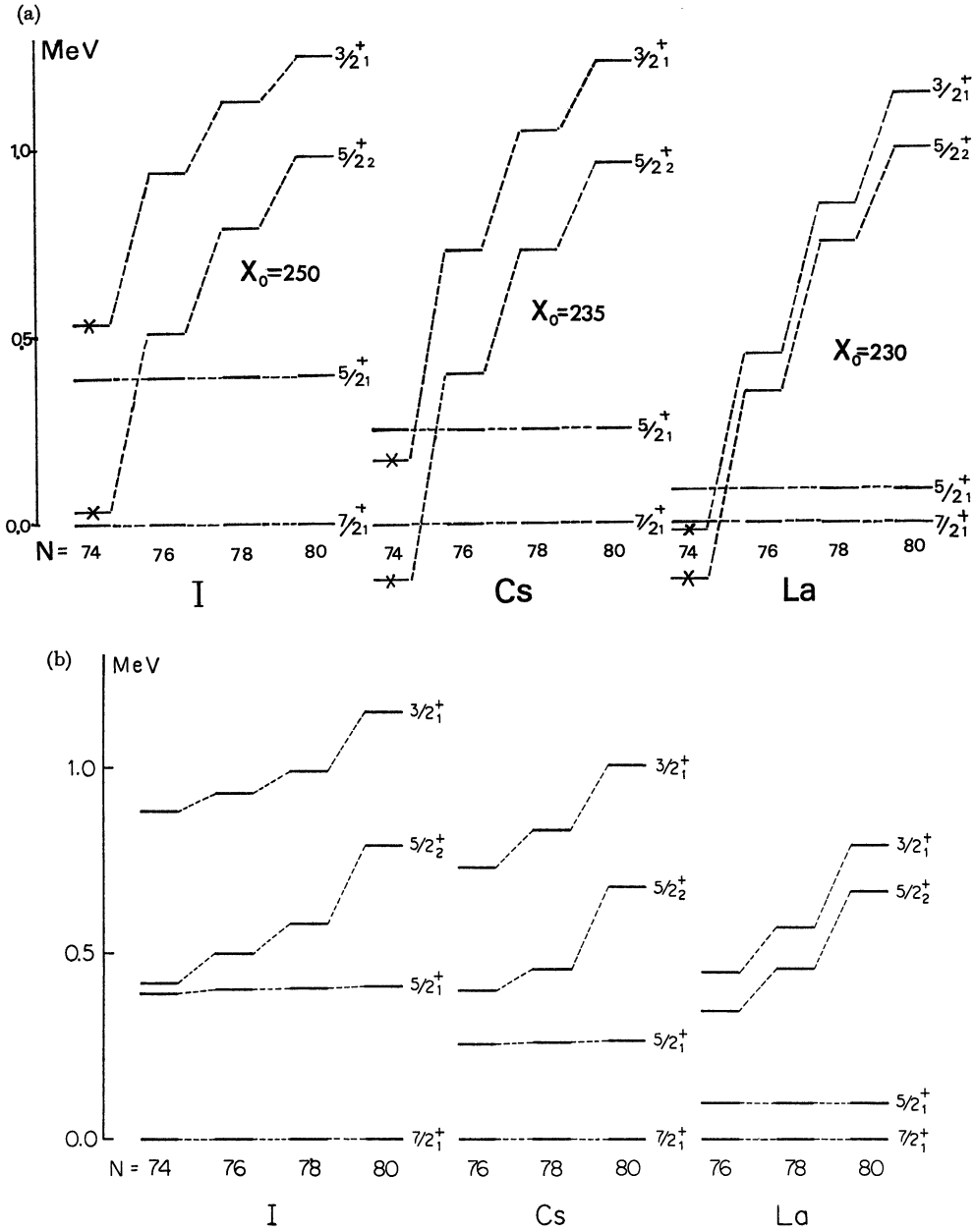


Fig. 4. Calculated excitation energies of the dressed 3QP states with $I^\pi=5/2^+$ and with $I^\pi=3/2^+$ in odd-mass I, Cs and La isotopes. They are measured from those of the lowest 1QP states.
(a) The constant values of the quadrupole-force-strength parameter $\chi_0$(defined by $\chi_0=\chi b^4 A^{5/3}$, $b^2$ being the harmonic-oscillator-range parameter) are used for each isotopes.
(b) The values of $\chi_0$ are chosen to reproduce the average energies of the $2^+$ phonons in the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(Z, N)=1/2\{\omega_{2^+}(Z-1, N)+\omega_{2^+}(Z+1, N)\}$.

Before discussing the calculated $B(E2)$ values in Table I, it may be important to set up a criterion for investigating the similarity and difference between the 3QP correlations characterizing the new dressed 3QP modes and those characterizing the AC states. As was shown in the preceding chapter, in the case of the AC states, the triggering effect of the 3QP correlations which strongly violates the concept of phonon in odd-mass nuclei is restricted among quasi-particles in a specific unique-parity orbit, because of the parity-selection property of the quadrupole force. On the contrary, in the case of the dressed 3QP mode under consideration (which has the same parity as that of the major shell), many shell orbits having the same even parity (such as $g^+_{7/2}$, $d^+_{5/2}$, $d^+_{3/2}$, and $s^+_{1/2}$) lie close and are expected to be equally active for the 3QP correlations. A criterion useful for discussing the new 3QP correlations occuring under such a condition of shell structure is given as follows: If the 3QP correlations mainly come from a specific orbit, for instance, the $1g_{7/2}$ orbit when we consider the collective $5/2^+_2$ states, we may say that the structure of the $5/2^+_2$ states is similar to that of the AC states. As was shown in the preceding chapter, an important characteristic in this case is that the value of $B(E2;\ 5/2^+_2 \to 7/2^+_1)$ belonging to the class of $B(E2;\ I=j-1\to j)$ is greatly enhanced compared to the ones of $E2$ transitions to the other 1QP states, for instance, $B(E2;\ 5/2^+_2 \to 5/2^+_1)$ belonging to the class of $B(E2;\ I=j-1\to j'\neq j)$. In this case, a picture of the low-energy excitation structure may be given as illustrated in Fig. 5. On the other hand, if the 3QP correlations among quasi-particles in different orbits are of importance, the $E2$ transitions to the other 1QP states, for instance, the $B(E2;\ 5/2^+_2 \to 5/2^+_1)$ must also be strongly enhanced. Therefore, in this case, we may say that the structure of $5/2^+_2$ states differs considerably from that of the AC states. Needless to say, the form of the eigenmode operator given by (2·1) allows us to treat these two cases on an equal footing, so that with the aid of this criterion we can achieve an essential understanding of the microscopic structure of the new collective excited states.

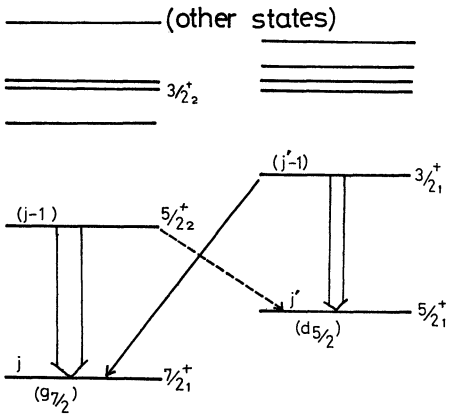


Fig. 5. Graphic explanation of the $E2$-transition properties.

Table I. Calculated $B(E2)$ values from the dressed 3QP $5/2_2^+$ states in odd-mass I, Cs and La isotopes (in unit of $e^2 \cdot 10^{-50}\,\mathrm{cm}^4$). The results of exact calculation are given in the second and fourth columns denoted "exact", while those calculated by adopting the ACS approximation are listed in the third and fifth columns denoted "ACS". The harmonic-oscillator-range parameter $b^2 = 1.0 A^{1/3}\,\mathrm{fm}^2$ and the effective charges, $e_p^{\mathrm{eff}} = 1.5e$ and $e_n^{\mathrm{eff}} = 0.5e$, are used. The adopted values of $\chi_0$ are the same as in Fig. 4(b) and are listed in the fifth column.

| | $5/2_2^+ \longrightarrow 7/2_1^+$ | | $5/2_2^+ \longrightarrow 5/2_1^+$ | $\chi_0$ |
|---|---|---|---|---|
| | exact | ACS | exact | |
| $^{127}$I | 14.3 | 13.5 | 0.3 | 234.2 |
| $^{129}$I | 6.9 | 9.6 | 0.1 | 251.5 |
| $^{131}$I | 5.2 | 6.8 | 0.1 | 280.6 |
| $^{133}$I | 3.8 | 4.7 | 0.0 | 315.0 |
| $^{131}$Cs | 10.7 | 20.0 | 0.4 | 235.0 |
| $^{133}$Cs | 8.8 | 12.9 | 0.3 | 264.7 |
| $^{135}$Cs | 6.9 | 8.4 | 0.1 | 307.0 |
| $^{133}$La | 17.0 | 48.1 | 0.7 | 230.2 |
| $^{135}$La | 9.7 | 19.7 | 0.4 | 259.0 |
| $^{137}$La | 7.0 | 11.3 | 0.2 | 300.0 |

Now, the calculated $B(E2)$ values in Table I demonstrate that the $B(E2;\ 5/2_2^+ \to 7/2_1^+)$'s are stronger by about one order in magnitude than the other $B(E2;\ 5/2_2^+ \to 5/2_1^+)$'s. Thus we can conclude that the structure of the $5/2_2^+$ states is similar to that of the AC states with spin $I = j - 1$. (In the present case, $j$ corresponds to the orbit $1g_{7/2}$.) In fact, microscopic structure of the calculated amplitudes of the $5/2_2^+$ states (as the dressed 3QP modes) is very similar to that of the AC states which was investigated in Chap. 3: The forward-going amplitudes of $(\pi\pi\pi)$-type with the largest $\{(\pi g_{7/2})^3\}$ component and of $(\nu\nu\pi)$-type are strongly coupled with each other. The backward-going amplitudes of $(\nu\nu\pi)$-type become larger as the neutron number decreases. Some examples of the main amplitudes in I, Cs and La are shown in Table II. The amplitudes in this Table are those which are orthonormalized by diagonalizing the projection operators, $\boldsymbol{P}$'s, entering into the eigenvalue equation (2·2) in the coupled-angular-momentum representation. (See Appendices 4A and 6A.)

It is rather a wonder that the overall similarity between the $5/2_2^+$ states and the AC states persists in spite of their different situation in shell structures. To investigate the reason for this, let us look into the characteristics of the calculated amplitudes in more detail. In I isotopes, the chemical potential for protons lies close to the $1g_{7/2}$ orbit and the energy difference between the 1QP $1g_{7/2}$ and the 1QP $2d_{5/2}$ states is relatively large (i.e. $\Delta E \approx 400$ keV), so that the component $\{(\pi g_{7/2})^3\}$ in the forward amplitudes $\psi_{n I=5/2}$ reaches the maximum. As the proton number increases, the chemical potential shifts

Table II. Main correlation amplitudes of the dressed 3QP $5/2_2^+$ modes in $^{127}$I(a), $^{133}$Cs (b) and $^{135}$La(c). They are those orthonormalized according to the procedure described in Appendix 4A. The first and second rows specify the types and the components in the coupled-angular-momentum representation, respectively. Abbreviations such as

$\{(d_{5/2})^2 g_{7/2}\} = \{d_{5/2} d_{5/2}\,(J=2) g_{7/2}\}$,

$\{(d_{5/2})^2_4 g_{7/2}\} = \{d_{5/2} d_{5/2}\,(J=4) g_{7/2}\}$,

etc., are used. The values of the forward-going amplitudes are listed in the third row, while those of the backward-going amplitudes are listed in the fourth row. To distinguish the different kinds of backward-going amplitudes, we use the superscripts, (2) and (3), that specify the kinds of amplitudes in Eq. (2·1) when necessary.

(a) $5/2_2^+$ state in $^{127}$I

| $(\pi\pi\pi)$ type | | | | | $(\nu\nu\pi)$ type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^3$ | $(d_{5/2})^2 g_{7/2}$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{7/2})^2_4 d_{5/2}$ | $(g_{7/2})^2 d_{3/2}$ | $(h_{11/2})^2 g_{7/2}$ | $(d_{3/2})^2 g_{7/2}$ | $(s_{1/2} d_{3/2}) g_{7/2}$ | $(d_{3/2} g_{7/2}) g_{7/2}$ | $(h_{11/2})^2 d_{5/2}$ |
| 0.89 | 0.23 | −0.14 | 0.09 | −0.08 | 0.67 | 0.30 | −0.09 | 0.19 | −0.08 |
| 0.31 | $\begin{cases} 0.01^{(2)} \\ 0.17^{(3)} \end{cases}$ | $\begin{cases} -0.03^{(2)} \\ -0.07^{(3)} \end{cases}$ | 0.04 | $\begin{cases} -0.03^{(2)} \\ -0.01^{(3)} \end{cases}$ | 0.48 | 0.21 | −0.14 | 0.10 | 0.08 |

(b) $5/2_2^+$ state in $^{133}$Cs

| $(\pi\pi\pi)$ type | | | | | $(\nu\nu\pi)$ type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^3$ | $(d_{5/2})^2 g_{7/2}$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{7/2})^2_4 d_{5/2}$ | $(g_{7/2})^2 d_{3/2}$ | $(h_{11/2})^2 g_{7/2}$ | $(d_{3/2})^2 g_{7/2}$ | $(s_{1/2} d_{3/2}) g_{7/2}$ | $(d_{3/2} g_{7/2}) g_{7/2}$ | $(h_{11/2})^2 d_{5/2}$ |
| 0.82 | 0.28 | −0.17 | 0.11 | −0.10 | 0.57 | 0.32 | −0.06 | 0.11 | −0.09 |
| 0.27 | $\begin{cases} 0.01^{(2)} \\ 0.18^{(3)} \end{cases}$ | $\begin{cases} -0.03^{(2)} \\ -0.07^{(3)} \end{cases}$ | 0.05 | $\begin{cases} -0.03^{(2)} \\ 0.01^{(3)} \end{cases}$ | 0.36 | 0.19 | −0.14 | 0.05 | −0.07 |

(c) $5/2_2^+$ state in $^{135}$La

| $(\pi\pi\pi)$ type | | | | | $(\nu\nu\pi)$ type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^3$ | $(d_{5/2})^2 g_{7/2}$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{7/2})^2 d_{3/2}$ | $(g_{7/2})^2_4 d_{3/2}$ | $(h_{11/2})^2 g_{7/2}$ | $(d_{3/2})^2 g_{7/2}$ | $(s_{1/2} d_{3/2}) g_{7/2}$ | $(d_{3/2} g_{7/2}) g_{7/2}$ | $(h_{11/2})^2 d_{5/2}$ |
| 0.76 | 0.40 | −0.20 | −0.13 | −0.13 | 0.59 | 0.33 | −0.20 | 0.15 | −0.12 |
| 0.26 | $\begin{cases} 0.01^{(2)} \\ 0.24^{(3)} \end{cases}$ | $\begin{cases} -0.04^{(2)} \\ -0.08^{(3)} \end{cases}$ | $\begin{cases} -0.04^{(2)} \\ 0.11^{(3)} \end{cases}$ | −0.04 | 0.38 | 0.20 | −0.16 | 0.08 | −0.09 |

up and the energy difference between the 1QP $1g_{7/2}$- and 1QP $2d_{5/2}$-states decreases till about $\Delta E \approx 100$keV in La isotopes. In La isotopes, therefore, we may expect that the components $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ and $\{(\pi d_{5/2})^2 \pi g_{7/2}\}$ grow up appreciably to break the AC state-like structure of the dressed 3QP mode. However, this trend is actually not as appreciable as has been expected. As is seen from Table II-(c), in particular, the component $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ which is directly connected with the largest component $\{(\pi g_{7/2})^3\}$ through the 3QP correlation among different orbits, $1g_{7/2}$ and $2d_{5/2}$, does not become extremely large. On the other hand, although the component $\{(\pi d_{5/2})^2 \pi g_{7/2})\}$ increases non-negligibly, it does not bring about the breaking of the similarity with the AC states. Here it should be pointed out that, while the component $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ contributes directly to the $B(E2;\ 5/2_2^+ \to 5/2_1^+)$, the component $\{(\pi d_{5/2})^2 \pi g_{7/2}\}$ can contribute only through angular-momentum recoupling. (Such a classification of the amplitudes is obtained by interpreting them in connection with the concept of phonon-band, as will be described in the succeeding chapter.) Associated with these characteristics in the $(\pi\pi\pi)$-type, it is also seen that the components containing the $g_{7/2}$-proton-quasi-particle play a dominant role among those of $(\nu\nu\pi)$-type. In this way, contrary to the $B(E2;\ 5/2_2^+ \to 7/2_1^+)$, the $B(E2;\ 5/2_2^+ \to 5/2_1^+)$ cannot be enhanced. Now, the origins of these characteristics in the microscopic structure of the collective $5/2^+$ state (as the dressed 3QP mode) may be found in the following facts:

1) The 3QP correlation among quasi-particles in different orbits, $1g_{7/2}$ and $2d_{5/2}$, involves the spin-flip matrix element $(2d_{5/2} \| r^2 Y_2 \| 1g_{7/2})$ which is considerably small, compared to the spin-non-flip one $(1g_{7/2} \| r^2 Y_2 \| 1g_{7/2})$ contributing the 3QP correlation at the specific orbit $1g_{7/2}$.

2) The component $\{(\pi g_{7/2})^3\}$ is increasing due to the special favouring of the $I=(j-1)$-coupling, while the corresponding component $\{(\pi d_{5/2})^3\}$ is forbidden for spin $I=5/2$.

It is interesting to note that the above-mentioned condition of shell structure (for the realization of AC state-like structure) is common to almost all major shells. Thus we can expect the picture (for low-lying excited states) illustrated in Fig. 5 to hold over several regions of spherical odd-mass nuclei. To examine the theoretical prediction, the experimental data on the electromagnetic transition rates, especially the ones between excited states, are highly desired. For the nuclei under consideration, more systematic measurement on the values of $B(E2;\ 5/2_2^+ \to 5/2_1^+)$ should confirm more definitely the conclusion given here.

### 3-2 *Collective* $3/2^+$ *states in I, Cs and La*

Here it is interesting to note that, in addition to the collective $5/2_2^+$ states discussed above, experimental data reveal the systematic presence of the $3/2_1^+$ states in I, Cs and La isotopes. With the same trend as in the case of collective

Table III. Calculated $B(E2)$ values from the dressed 3QP $3/2_1^+$ states in odd-mass I, Cs and La isotopes (in unit of $e^2 \cdot 10^{-50}$ cm$^4$). The notations and parameters adopted are the same as in Table I.

| | $3/2_1^+ \longrightarrow 5/2_1^+$ | | $3/2_1^+ \longrightarrow 7/2_1^+$ |
|---|---|---|---|
| | exact | ACS | exact |
| $^{127}$I | 3.1 | 10.4 | 2.6 |
| $^{129}$I | 2.0 | 7.8 | 2.4 |
| $^{131}$I | 0.1 | 5.6 | 1.0 |
| $^{133}$I | 0.2 | 3.6 | 1.4 |
| $^{131}$Cs | 9.8 | 16.0 | 1.7 |
| $^{133}$Cs | 10.4 | 11.1 | 2.4 |
| $^{135}$Cs | 4.2 | 7.2 | 1.7 |
| $^{133}$La | 13.2 | 67.7 | 0.9 |
| $^{135}$La | 9.3 | 20.5 | 0.7 |
| $^{137}$La | 6.5 | 11.4 | 0.6 |

$5/2_2^+$ states, the excitation energies of the $3/2_1^+$ states measured from the 1QP $5/2_1^+$ states decrease as the neutron number changes from $N=82$ to $N=74$. (See Figs. 2 and 3.)

Regarding the $3/2_1^+$ states to be the dressed 3QP states, we have also calculated their excitation energies and $B(E2)$ values. The results are shown in Fig. 4 and Table III. In Fig. 4, we see that the above-mentioned experimental characteristics of the $3/2_1^+$ states are well reproduced in the calculation. The main components of the $3/2_1^+$ states are therefore identified as the dressed 3QP states. It is noted, however, that the relative level positions between the $3/2_1^+$ and $5/2_2^+$ states are not well reproduced in the calculation, especially for I isotopes and lighter Cs and La isotopes. This may indicate the necessity of modifying the adopted single-particle energies (of Kisslinger and Sorensen) and of taking the coupling effect of the 1QP $2d_{3/2}$ mode into account. However, since this disagreement does not essentially affect the discussion below, we do not attempt such improvements of the calculation here.

According to the criterion given in the preceding subsection (on the structure of the dressed 3QP modes), the calculated $B(E2)$ values in Table III suggest that the 3QP correlations among quasi-particles in different orbits are rather strong in the $3/2_1^+$ states compared to the collective $5/2_2^+$ states: The values of $B(E2;\ 3/2_1^+ \rightarrow 5/2_1^+)$ and of $B(E2;\ 3/2_1^+ \rightarrow 7/2_1^+)$ are both enhanced, with ratios changing from I isotopes to La isotopes. Main amplitudes of the $3/2_1^+$ states in I isotopes as the dressed 3QP modes are shown in Table IV-(a), from which we can easily see why the competition between the two $E2$ transitions (to the 1QP $5/2^+$ and 1QP $7/2^+$ states) is remarkable in I isotopes. Namely, among the components of both $(\pi\pi\pi)$- and $(\nu\nu\pi)$-types, the two sets of components containing the $g_{7/2}$- and $d_{5/2}$-proton-quasi-particle, respectively,

Table IV. Main correlation amplitudes of the dressed 3QP $3/2_1^+$ modes in $^{131}$I(a), $^{133}$Cs(b) and $^{135}$La(c). Notations are the same as in Table II.

(a) $3/2_1^+$ state in $^{131}$I

| ($\pi\pi\pi$) type | | | | | ($\nu\nu\pi$) type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^3$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{7/2})^2_4 d_{5/2}$ | $(d_{5/2})^2 g_{7/2}$ | $(d_{5/2})^3$ | $(h_{11/2})^2 g_{7/2}$ | $(h_{11/2})^2 d_{5/2}$ | $(d_{3/2})^2 g_{7/2}$ | $(d_{3/2})^2 d_{5/2}$ | $(s_{1/2} d_{3/2}) g_{7/2}$ |
| −0.65 | 0.35 | 0.10 | 0.19 | 0.11 | 0.02 | 0.22 | 0.34 | 0.13 | −0.13 |
| −0.07 | $\begin{cases} 0.02^{(2)} \\ 0.11^{(3)} \end{cases}$ | 0.02 | $\begin{cases} 0.01^{(2)} \\ 0.06^{(3)} \end{cases}$ | 0.04 | 0.18 | 0.12 | 0.10 | 0.06 | −0.08 |

(b) $3/2_1^+$ state in $^{133}$Cs

| ($\pi\pi\pi$) type | | | | | ($\nu\nu\pi$) type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^3$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{7/2})^2_4 d_{5/2}$ | $(d_{5/2})^2 g_{7/2}$ | $(d_{5/2})^3$ | $(h_{11/2})^2 g_{7/2}$ | $(d_{3/2})^2 g_{7/2}$ | $(h_{11/2})^2 d_{5/2}$ | $(d_{3/2} g_{7/2}) d_{5/2}$ | $(d_{3/2})^2 d_{5/2}$ |
| −0.40 | 0.84 | 0.10 | 0.28 | 0.46 | 0.39 | 0.23 | 0.65 | 0.16 | 0.37 |
| −0.08 | $\begin{cases} 0.03^{(2)} \\ 0.07^{(3)} \end{cases}$ | 0.03 | $\begin{cases} 0.04^{(2)} \\ 0.09^{(3)} \end{cases}$ | 0.16 | 0.17 | 0.09 | 0.37 | 0.09 | 0.19 |

(c) $3/2_1^+$ state in $^{135}$La

| ($\pi\pi\pi$) type | | | | | ($\nu\nu\pi$) type | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(g_{7/2})^2 d_{5/2}$ | $(d_{5/2})^3$ | $(d_{5/2})^2 g_{7/2}$ | $(d_{5/2})^2_4 g_{7/2}$ | $(g_{7/2})^3$ | $(h_{11/2})^2 d_{5/2}$ | $(d_{3/2})^2 d_{5/2}$ | $(s_{1/2} d_{3/2}) d_{5/2}$ | $(h_{11/2})^2 g_{7/2}$ | $(d_{3/2} g_{7/2}) d_{5/2}$ |
| 0.62 | 0.56 | 0.21 | 0.15 | −0.15 | 0.60 | 0.33 | −0.15 | 0.19 | 0.15 |
| $\begin{cases} 0.02^{(2)} \\ 0.34^{(3)} \end{cases}$ | 0.20 | $\begin{cases} 0.04^{(2)} \\ 0.02^{(3)} \end{cases}$ | 0.06 | −0.04 | 0.37 | 0.19 | −0.15 | 0.11 | 0.09 |

play equally important roles in I isotopes. As the proton number increases, the chemical potential for protons shifts up toward the $2d_{5/2}$ orbit (from the $1g_{7/2}$ orbit), so that in La isotopes the component $\{(\pi g_{7/2})^3\}$ in the forward amplitudes $\psi_{nI=3/2}$ is diminished and the component $\{(\pi d_{5/2})^3\}$ is enlarged. (See Table IV-(c).) The increase in $B(E2; 3/2_1^+ \rightarrow 5/2_1^+)$ in La isotopes is clearly due to the growth of the 3QP correlation in the $2d_{5/2}$ orbit. Although the component $\{(\pi d_{5/2})^3\}$ is still not the largest one, the components containing the single $d_{5/2}$-proton-quasi-particle become the dominant ones among the other components of both $(\pi\pi\pi)$- and $(\nu\nu\pi)$-types. Associated with this, the value of $B(E2; 3/2_1^+ \rightarrow 7/2_1^+)$ becomes smaller in clear contrast to the increasing $B(E2; 3/2_1^+ \rightarrow 5/2_1^+)$. In this sense, we may say that the $3/2_1^+$ states in La

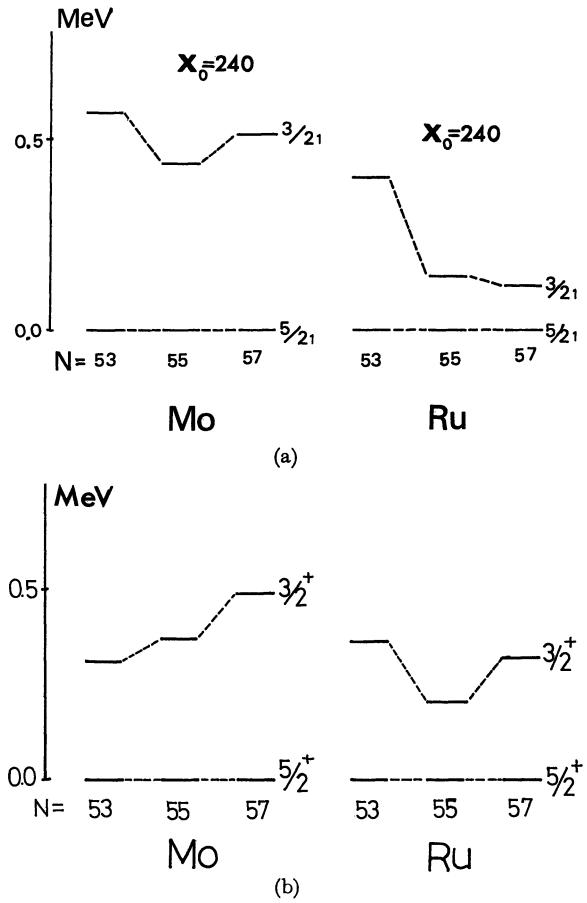


Fig. 6. Calculated excitation energies of the dressed 3QP $3/2_1^+$ states in odd-mass Mo and Ru isotopes. They are measured from those of the lowest 1QP states.
(a) The constant values of $\chi_0$ are used for each isotopes.
(b) The values of $\chi_0$ are chosen to reproduce the average energies of the $2^+$ phonons in the adjacent even-even nuclei.

isotopes have a structure similar to that of the AC states with $I=j-1$ (where $j$ corresponds to the 1QP $5/2^+$ state). Since quantitative prediction depends rather sensitively on the single-particle energies adopted, the numerical values presented in Table III should not be taken too strictly. Nevertheless, we can expect such a structure change of the $3/2_1^+$ states (from I isotopes to La isotopes) in a qualitative sense.

### 3–3 *Collective $3/2^+$ states in Mo and Ru*

The calculated results of the collective $3/2_1^+$ states (in odd-neutron Mo and Ru isotopes) as the dressed 3QP modes are shown in Fig. 6 and Table V. The systematic appearance of the low-lying collective $3/2_1^+$ states in the isotopes with $N=53$, 55 and 57 are reproduced very well, together with the values of $B(E2;\ 3/2_1^+\to 5/2_1^+)$. Thus, the $5/2_1^+$ and $3/2_1^+$ states are identified as the 1QP and dressed 3QP states, respectively. Furthermore, the special enhancement of $B(E2;\ 3/2_1^+\to 5/2_1^+)$ shown in Table V (when compared to the other $B(E2)$'s) suggests that the structure of the $3/2_1^+$ states is similar to that of the AC states with $I=j-1$. (In this case, $j$ corresponds to the 1QP $5/2^+$ states.) From Table VI, we can easily see the similarity of the $3/2_1^+$ states to the AC states, although the fine structure is appreciably different as a result of the 3QP correlations among quasi-particles in different orbits with the same parity ($d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$). The gradual change of the microscopic structure of the $3/2_1^+$ states with increasing neutron number will be further discussed in § 5.

## § 4. New reduction effect of couplings between dressed 3QP and 1QP modes

In contrast to the case of the AC states, the dressed 3QP states, especially the collective $5/2^+$ states in I, Cs and La isotopes, lie close, in energy, to the 1QP states with the same spin and parity. It is, therefore, necessary to examine

Table V. Calculated $B(E2)$ values from the dressed 3QP $3/2_1^+$ states in odd-mass Mo and Ru isotopes (in unit of $e^2\cdot 10^{-50}$ cm$^4$). The adopted values of $\chi_0$ are the same as in Fig. 6(b) and are listed in the sixth column. The notations and the other parameters are the same as in Table I.

| | $3/2_1^+\longrightarrow 5/2_1^+$ | | $3/2_1^+\longrightarrow 7/2_1^+$ | $3/2_1^+\longrightarrow 1/2_1^+$ | $\chi_0$ |
|---|---|---|---|---|---|
| | exact | ACS | exact | exact | |
| $^{95}$Mo | 3.1 | 5.2 | 0.3 | 0.3 | 263.0 |
| $^{97}$Mo | 10.1 | 8.0 | 0.6 | 0.8 | 244.3 |
| $^{99}$Mo | 10.3 | 8.6 | 0.1 | 0.5 | 237.2 |
| $^{97}$Ru | 5.6 | 5.7 | 0.2 | 0.8 | 245.0 |
| $^{99}$Ru | 8.6 | 15.4 | 1.9 | 2.9 | 236.0 |
| $^{101}$Ru | 12.8 | 20.3 | 0.7 | 0.8 | 231.0 |

Table VI. Main correlation amplitudes of the dressed 3QP $3/2_1^+$ modes in $^{97}$Ru(a), $^{99}$Ru(b) and $^{101}$Ru(c). Notations are the same as in Table II.

(a) $3/2_1^+$ state in $^{97}$Ru

| $(\nu\nu\nu)$ type | | $(\pi\pi\nu)$ type | |
|---|---|---|---|
| $(d_{5/2})^3$ | $(d_{5/2})^2 s_{1/2}$ | $(g_{9/2})^2 d_{5/2}$ | $(g_{9/2})^2 s_{1/2}$ |
| 0.94 | −0.60 | 0.75 | −0.20 |
| 0.38 | $\begin{cases} -0.25^{(2)} \\ -0.38^{(3)} \end{cases}$ | 0.55 | −0.42 |

(b) $3/2_1^+$ state in $^{99}$Ru

| $(\nu\nu\nu)$ type | | | | | $(\pi\pi\nu)$ type | | | |
|---|---|---|---|---|---|---|---|---|
| $(d_{5/2})^3$ | $(d_{5/2})^2 g_{7/2}$ | $(d_{5/2})_4^2 g_{7/2}$ | $(d_{5/2})^2 s_{1/2}$ | $(g_{7/2})^2 d_{5/2}$ | $(g_{9/2})^2 d_{5/2}$ | $(p_{3/2} p_{1/2}) d_{5/2}$ | $(g_{9/2})^2 s_{1/2}$ | $(g_{9/2})^2 g_{7/2}$ |
| 0.86 | 0.22 | 0.12 | −0.77 | 0.17 | 0.85 | −0.10 | −0.18 | 0.21 |
| 0.41 | $\begin{cases} 0.05^{(2)} \\ 0.28^{(3)} \end{cases}$ | 0.08 | $\begin{cases} -0.39^{(2)} \\ -0.19^{(3)} \end{cases}$ | $\begin{cases} 0.04^{(2)} \\ 0.17^{(3)} \end{cases}$ | 0.69 | −0.12 | −0.27 | 0.42 |

(c) $3/2_1^+$ state in $^{101}$Ru

| $(\nu\nu\nu)$ type | | | | | $(\pi\pi\nu)$ type | | | |
|---|---|---|---|---|---|---|---|---|
| $(d_{5/2})^2 s_{1/2}$ | $(d_{5/2})^3$ | $(g_{7/2})^2 d_{5/2}$ | $(h_{11/2})^2 d_{5/2}$ | $(d_{5/2})^2 g_{7/2}$ | $(g_{9/2})^2 d_{5/2}$ | $(g_{9/2})^2 s_{1/2}$ | $(g_{9/2})^2 g_{7/2}$ | $(p_{1/2} p_{3/2}) d_{5/2}$ |
| −0.98 | 0.74 | 0.33 | 0.21 | 0.19 | 1.13 | −0.34 | 0.18 | −0.13 |
| $\begin{cases} -0.43^{(2)} \\ -0.12^{(3)} \end{cases}$ | 0.31 | $\begin{cases} 0.02^{(2)} \\ 0.28^{(3)} \end{cases}$ | 0.05 | $\begin{cases} 0.05^{(2)} \\ 0.07^{(3)} \end{cases}$ | 0.73 | −0.24 | 0.16 | −0.15 |

their coupling effects. According to the general formulation given in § 5-Chap. 2, the original Hamiltonian is transcribed unambiguously into the quasi-particle new-Tamm-Dancoff subspace, the basis vectors of which are $\{|\Phi_\delta^{(1)}\rangle \equiv a_\delta^\dagger|\Phi_0\rangle,\ |\Phi_{nIK}^{(3)}\rangle \equiv Y_{nIK}^\dagger|\Phi_0\rangle\}$. The transcribed Hamiltonian is of the form

$$\begin{aligned} \boldsymbol{H} = & \sum_\delta E_d \boldsymbol{a}_\delta^\dagger \boldsymbol{a}_\delta + \sum_{nIK} \omega_{nI} \boldsymbol{Y}_{nIK}^\dagger \boldsymbol{Y}_{nIK} \\ & + \sum_{\substack{\delta, nIK \\ (K=m_\delta)}} V_{\text{int}}(d, nI)\{\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{a}_\delta + \boldsymbol{a}_\delta^\dagger \boldsymbol{Y}_{nIK}\}, \end{aligned} \tag{4·1}$$

where

$$\boldsymbol{a}_\delta^\dagger \equiv a_\delta^\dagger |\Phi_0\rangle\langle\Phi_0|, \quad \boldsymbol{Y}_{nIK}^\dagger \equiv Y_{nIK}^\dagger |\Phi_0\rangle\langle\Phi_0|. \tag{4·2}$$

The third term of the transcribed Hamiltonian (4·1) represents the interaction between the dressed 3QP and 1QP modes, and comes from the $H_Y$-type original interaction (shown in Fig. 7 of Chap. 3) which has not played any role in constructing the dressed 3QP modes. The effective coupling strength $V_{\text{int}}(d, nI)$ is thus composed of the matrix elements of $H_Y$ accompanied by the amplitudes of the dressed 3QP mode $Y_{nIK}^\dagger$. In the P+QQ force model, the explicit form is given as follows:

$$\begin{aligned} V_{\text{int}}(d, nI) = & -\chi \delta_{Ij_d} \sqrt{\frac{10}{2I+1}} \\ & \times \Bigg[ \sqrt{\frac{3}{2}} \sum_{\substack{abc \\ a'b'c'J'}} Q(ab) R(cd) P_I(ab(2)c|a'b'(J')c')\ \psi_{nI}[a'b'(J')c'] \\ & + \sum_{(rs)c} Q(rs) R(cd) \{1+\delta_{rs}\}^{-1/2}\ \psi_{nI}[rs(2)c] \\ & + \frac{1}{\sqrt{2}} \sum_{aJ} Q(aa) R(ad) P_I(aa(2)a|aa(J)a)\ \varphi_{nI}^{(1)}[aa(J)a] \\ & + \sum_{\substack{ac, J=\text{even} \\ (a \neq c, J \neq 0)}} Q(ca) R(ad) \sqrt{5(2J+1)} \begin{Bmatrix} j_a & j_a & J \\ j_c & I & 2 \end{Bmatrix}\ \varphi_{nI}^{(2)}[aa(J)c] \\ & + \sum_{\substack{(ab)c \\ (a \neq c, b \neq c)}} Q(ab) R(cd) \{1+\delta_{ab}\}^{-1/2}\ \varphi_{nI}^{(3)}[ab(2)c] \\ & + \sum_{(rs)c} Q(rs) R(cd) \{1+\delta_{rs}\}^{-1/2}\ \varphi_{nI}^{(3)}[rs(2)c] \Bigg], \end{aligned} \tag{4·3}$$

where

$$R(cd) = \frac{1}{\sqrt{5}} (c \| r^2 Y_2 \| d) \cdot (u_c u_d - v_c v_d) \tag{4·4}$$

and $Q(ab)$ is defined by (2·5). The formal structure of (4·3) is very similar

to that of the reduced matrix element for $E2$ transition given by (2·4). In fact, Eq. (4·3) can be obtained from (2·4) by the following replacement:

$$e_\tau Q(ab)\delta_{cd} \Longrightarrow -\chi\delta_{Ijd}\sqrt{\frac{10}{2I+1}}\ Q(ab)R(cd).$$

Such a formal similarity is a characteristic of the P+QQ force model adopted.

The result of calculations of the coupling effects due to the interaction

Table VII. Calculated results for the coupling effects. The mixing amplitude of the 1QP modes are given in the third column, while those of the lowest dressed 3QP modes and of the next higher ones are given in the fourth and fifth columns, respectively. In the sixth column are listed the values of the energy shifts due to the coupling effects in unit of MeV.
(a) The $5/2_1^+$ and $5/2_2^+$ states in odd-mass I, Cs and La isotopes.
(b) The $3/2_1^+$ states in odd-mass Mo and Ru isotopes.

(a)

| nucleus | state | $\zeta^{(1)}(d)$ | $\zeta^{(3)}(n=1, I)$ | $\zeta^{(3)}(n=2, I)$ | $\Delta\omega$ |
|---|---|---|---|---|---|
| $^{127}$I | $5/2_1^+$ | 0.87 | −0.28 | 0.39 | −0.35 |
| | $5/2_2^+$ | 0.24 | 0.96 | 0.15 | 0.02 |
| $^{129}$I | $5/2_1^+$ | 0.90 | −0.18 | 0.39 | −0.23 |
| | $5/2_2^+$ | 0.15 | 0.98 | 0.10 | 0.01 |
| $^{131}$I | $5/2_1^+$ | 0.92 | −0.20 | −0.34 | −0.16 |
| | $5/2_2^+$ | 0.18 | 0.98 | −0.10 | 0.01 |
| $^{133}$I | $5/2_1^+$ | 0.96 | −0.15 | 0.24 | −0.08 |
| | $5/2_2^+$ | 0.14 | 0.99 | 0.06 | 0.01 |
| $^{131}$Cs | $5/2_1^+$ | 0.93 | −0.01 | −0.35 | −0.15 |
| | $5/2_2^+$ | 0.01 | 1.00 | −0.01 | 0.00 |
| $^{133}$Cs | $5/2_1^+$ | 0.95 | −0.03 | 0.33 | −0.13 |
| | $5/2_2^+$ | 0.02 | 1.00 | 0.01 | 0.00 |
| $^{135}$Cs | $5/2_1^+$ | 0.97 | 0.00 | −0.23 | −0.06 |
| | $5/2_2^+$ | 0.00 | 1.00 | 0.00 | 0.00 |
| $^{133}$La | $5/2_1^+$ | 0.95 | 0.16 | −0.26 | −0.15 |
| | $5/2_2^+$ | −0.15 | 0.99 | 0.06 | 0.01 |
| $^{135}$La | $5/2_1^+$ | 0.96 | 0.07 | −0.27 | −0.08 |
| | $5/2_2^+$ | −0.07 | 1.00 | 0.03 | 0.00 |
| $^{137}$La | $5/2_1^+$ | 1.00 | 0.02 | −0.10 | −0.01 |
| | $5/2_2^+$ | −0.02 | 1.00 | 0.00 | 0.00 |

(b)

| nucleus | state | $\zeta^{(1)}(d)$ | $\zeta^{(3)}(n=1, I)$ | $\zeta^{(3)}(n=2, I)$ | $\Delta\omega$ |
|---|---|---|---|---|---|
| $^{95}$Mo | $3/2_1^+$ | 0.05 | 1.00 | 0.00 | −0.01 |
| $^{97}$Mo | $3/2_1^+$ | 0.16 | 0.98 | 0.03 | −0.01 |
| $^{99}$Mo | $3/2_1^+$ | −0.51 | 0.77 | −0.30 | −0.17 |
| $^{97}$Ru | $3/2_1^+$ | −0.09 | 1.00 | −0.00 | −0.02 |
| $^{99}$Ru | $3/2_1^+$ | −0.25 | 0.96 | −0.10 | −0.10 |
| $^{101}$Ru | $3/2_1^+$ | 0.44 | 0.83 | 0.30 | −0.19 |

term in (4·1) are shown in Table VII. It is noticeable that the coupling effects are very small even in situation where the two $5/2^+$ states are close to each other in energy (i.e., $\Delta\omega \approx 0.01$ MeV). The mechanism to make the coupling effects (between the dressed 3QP and 1QP modes) so small must be found in the microscopic structure of the effective coupling strength $V_{\text{int}}(d, nI)$. From the microscopic structure of the effective coupling strength between the dressed 3QP $5/2_2^+$ and 1QP $5/2_1^+$ modes in the case of the P+QQ force model, we can find the following origins to weaken the coupling:

1) The important matrix elements of $H_Y$ (of the quadrupole force) in the effective coupling strength, which are accompanied by large components of the amplitudes of the dressed 3QP $5/2_2^+$ mode, always contain the spin-flip matrix element $(2d_{5/2}\|r^2Y_2\|1g_{7/2})$, which is considerably smaller compared to the diagonal matrix element $(1g_{7/2}\|r^2Y_2\|1g_{7/2})$. In this connection, it is interesting to recall that the considerable smallness of the ratio of $(2d_{5/2}\|r^2Y_2\|1g_{7/2})$ to $(1g_{7/2}\|r^2Y_2\|1g_{7/2})$ is also one of the important origins to bring about the AC state-like structure for the collective $5/2_2^+$ state.

2) The pairs of matrix elements of $H_Y$ (such as Figs. 7(a) and 7(b)), which are in the relation of exchange diagrams with each other, must always be involved in the effective coupling strength, because the antisymmetrization among the three quasi-particles composing the dressed 3QP $5/2^+$ mode is properly taken into account. (See the projection operator $P_I$ entering into $V_{\text{int}}(d, nI)$.) Actual calculations tell us that the effects of such exchange parts on the effective coupling strength between the $5/2_1^+$ and $5/2_2^+$ states are not constructive but rather destructive to each other.

3) The effective coupling strength is determined by the contributions of many components of the amplitudes. For example, the components $\{(\pi d_{5/2})^2\pi g_{7/2}\}$ and $\{(\pi g_{7/2})^2\pi d_{5/2}\}$ represented by Figs. 8(a) and 8(b) both contribute to $V_{\text{int}}(d, nI)$. (Notice that the summation in the first term of Eq. (4·3) should be taken with respect to the orbital triad $(a, b, c)$.) The calculated result shows that such different components of the amplitudes generally contribute in random phase, namely, they cancel one another. For the collective $5/2_2^+$ states under consideration, this effect is extremely strong since the contributions from Figs. 8(a) and 8(b) are of approximately equal magnitudes with different signs.

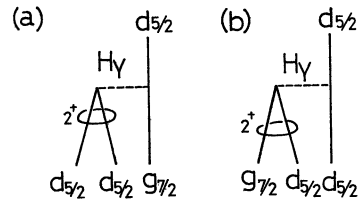


Fig. 7. Example of exchange diagrams both of which contribute to the effective coupling strength with $I^\pi = 5/2^+$.

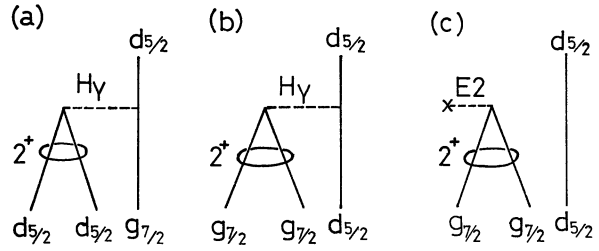


Fig. 8. Graphic representation of two kinds of diagrams both of which contribute to the effective coupling strength with $I^\pi=5/2^+$. The two kinds of diagrams, (a) and (b), contribute destructively to the effective coupling strengths of the lowest dressed 3QP $5/2_2^+$ states in I, Cs and La isotopes. It should be noted that, in the case of calculating the $E2$ transition (c), such a destructive effect never appear because one of them (a) is forbidden.

It should be pointed out, here, that Fig. 8(a) involves the small matrix element $R(g_{7/2}\ d_{5/2})$ accompanied with the large component $\{(\pi d_{5/2})^2\ \pi g_{7/2}\}$, while Fig. 8(b) involves the large matrix element $R(d_{5/2}\ d_{5/2})$ accompanied with the small component $\{(\pi g_{7/2})^2\ \pi d_{5/2}\}$. We should also note that, in the case of calculating the $E2$ transitions, such a destructive effect never appear because the matrix element corresponding to Fig. 8(a) is forbidden. (See the $\delta_{cd}$-factor in Eq. (2·4).)

4) In addition to these effects, it should also be pointed out that the effective coupling strength depends characteristically on the reduction factors $(u_c u_d - v_c v_d)$ entering into the $R(cd)$ in $V_{\text{int}}(d, nI)$. The reduction factor of the orbital pair, $g_{7/2}$ and $d_{5/2}$, becomes particularly small in La isotopes.

All of these effects cooperate in weakening the effective coupling strength between the 1QP $5/2_1^+$ state and the *lowest* dressed 3QP $5/2_2^+$ state. As a result, the 1QP $5/2_1^+$ mode couples rather with the *next higher* dressed 3QP $5/2_3^+$ mode, as shown in Table VII.

It is worthy to emphasize that the reduction effects 2) and 3) of the effective coupling strength never appear in the conventional QPC theory, because the antisymmetrization between the odd quasi-particle and the quasi-particle-pair composing the phonon is not at all taken into account in the QPC theory. The physical significance of these new reduction effects will be further discussed in Chap. 5 (where the necessity of alteration of the conventional picture on the low-energy excitations in odd-mass nuclei will be discussed *en bloc*).

## § 5. Analysis of electromagnetic properties of AC state-like collective excited states

In this section, we investigate the electromagnetic properties of the AC state-like collective excited states (the collective $5/2_2^+$ states in I, Cs and La isotopes and the collective $3/2_1^+$ states in Mo and Ru isotopes) taking the

coupling effect between the dressed 3QP and 1QP modes into account. We first evaluate the extent to which the $B(E2)$ values presented in § 3 are affected by the coupling effect. Since the couplings between the dressed 3QP $5/2_2^+$ states and the 1QP $5/2_1^+$ states are very weak, it is expected that the coupling effect does not cause a large deviation from the $E2$-transition property of the $5/2_2^+$ states in the absence of the coupling effect. Secondly, we examine whether the similarity between the collective $5/2_2^+$ and $3/2_1^+$ states and the AC states holds also in their magnetic properties. If the structures of the collective $5/2_2^+$ and $3/2_1^+$ states are similar to the AC states, we expect, according to the discussion in § 4 of Chap. 3, that 1) the values of $B(M1; 5/2_2^+ \to 7/2_1^+)$ and $B(M1; 3/2_1^+ \to 5/2_1^+)$ are nearly zero and 2) $g(5/2_2^+) \approx g(7/2_1^+)$ and $g(3/2_1^+) \approx g(5/2_1^+)$. In the case of applying these criteria for the magnetic properties to the collective states under consideration, we should be careful in that the properties of $M1$-transitions and -moments are quite sensitive to the coupling effects. On the other hand, the magnitude of the coupling effect may be examined more carefully by comparing the calculated $B(M1)$ and $g$ factors with the experimental data.

When the transcribed Hamiltonian (4·1) containing the coupling term $\boldsymbol{H}^{(\mathrm{int})}$ is diagonalized, any eigenstate in the quasi-particle NTD subspace under consideration takes the following form as a superposition of the dressed 3QP and 1QP states:

$$|IK;\nu\rangle = \zeta_\nu^{(1)}(d)\delta_{Ij_d}\delta_{Km_\delta}a_\delta^\dagger|\Phi_0\rangle + \sum_n \zeta_\nu^{(3)}(nI)\,Y_{nIK}^\dagger|\Phi_0\rangle \tag{5·1}$$

with the normalization

$$\{\zeta_\nu^{(1)}(d)\}^2 + \sum_n \{\zeta_\nu^{(3)}(nI)\}^2 = 1. \tag{5·2}$$

Here the index $\nu$ is used to label the eigenstates having the same values of angular momentum $I$ and its projection $K$. The summation with respect to $n$, in Eqs. (5·1) and (5·2), should be taken, in principle, over all (physical) eigensolutions of the dressed 3QP modes with definite $I$ and $K$ ( , several of which have collective nature while the others are of non-collective nature). In practice, of course, the mixing amplitudes $\zeta_\nu^{(3)}(nI)$ with $n>1$ are negligibly small in the low-lying collective $3/2_1^+$ and $5/2_2^+$ states under consideration.

The matrix elements of any electromagnetic multipole operators between the eigenstates (5·1) are given as follows:

$$\begin{aligned}
&\langle IK;\nu|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|I'K';\nu'\rangle \\
&\quad = \zeta_\nu^{(1)}(d)\zeta_{\nu'}^{(1)}(d')\delta_{Ij_d}\delta_{I'j_{d'}}\langle\Phi_\delta^{(1)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{\delta'}^{(1)}\rangle \\
&\qquad + \zeta_\nu^{(1)}(d)\delta_{Ij_d}\sum_{n'}\zeta_{\nu'}^{(3)}(n'I')\,\langle\Phi_\delta^{(1)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{n'I'K'}^{(3)}\rangle \\
&\qquad + \zeta_{\nu'}^{(1)}(d')\delta_{I'j_{d'}}\sum_n\zeta_\nu^{(3)}(nI)\,\langle\Phi_{nIK}^{(3)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{\delta'}^{(1)}\rangle
\end{aligned} \tag{5·3}$$

$$+\sum_{nn'}\zeta_{\nu}^{(3)}(nI)\zeta_{\nu'}^{(3)}(n'I')\ \langle\Phi_{nIK}^{(3)}|\hat{O}_{LM}^{(\pm)}|\Phi_{n'I'K'}^{(3)}\rangle,$$

where $\hat{O}_{LM}^{(\pm)}$ denotes the transcribed electromagnetic operator in the quasi-particle NTD subspace defined by Eq. (5·14) of Chap. 2, and its matrix elements appearing in (5·3) are given in Appendix 2D. We use the symbolical notations, such as $E_{11}$, $E_{13}$, $E_{31}$ and $E_{33}$, for the first, second, third, and fourth terms on the right-hand side of (5·3), respectively. Using Eq. (5·3), we obtain for example,

$$\begin{aligned} B(E2;\, I_\nu \to I'_{\nu'}) &= \frac{1}{2I+1}|\langle I;\nu\|\hat{O}_2^{(+)}\|I';\nu'\rangle|^2 \\ &\equiv \frac{1}{2I+1}|E_{11}+E_{13}+E_{31}+E_{33}|^2, \end{aligned} \tag{5·4}$$

$$\begin{aligned} B(M1;\, I_\nu \to I'_{\nu'}) &= \frac{1}{2I+1}|\langle I;\nu\|\hat{O}_1^{(-)}\|I';\nu'\rangle|^2 \\ &\equiv \frac{1}{2I+1}|M_{11}+M_{13}+M_{31}+M_{33}|^2, \end{aligned} \tag{5·5}$$

$$\begin{aligned} \mu(I_\nu) = g(I_\nu)I &= \sqrt{\frac{4\pi}{3}}\langle IK;\nu|\hat{O}_{10}^{(-)}|IK;\nu\rangle \quad (\text{with } K=I) \\ &\equiv (g_{11}+2g_{13}+g_{33})\cdot I, \end{aligned} \tag{5·6}$$

where $\hat{O}_{2M}^{(+)}$ and $\hat{O}_{1M}^{(-)}$ are the electric quadrupole and magnetic dipole operators, respectively, and their reduced matrix elements are defined by

$$\langle IK;\nu|\hat{O}_{LM}^{(\pm)}|I'K';\nu'\rangle = \frac{(I'LK'M|IK)}{\sqrt{2I+1}}\langle I;\nu\|\hat{O}_L^{(\pm)}\|I';\nu'\rangle. \tag{5·7}$$

Following these general formulae, the calculations of the $B(E2)$, $B(M1)$ and magnetic $g$ factors of the collective $5/2_2^+$ and $3/2_1^+$ states have been performed. In this numerical work, any modification of the Kisslinger and Sorensen's parameters which were adopted in §3 has not been attempted. We

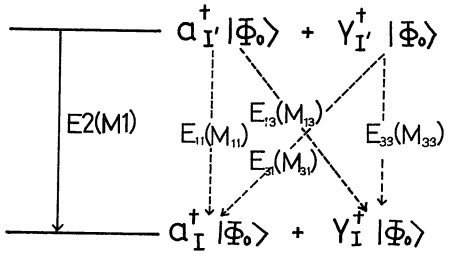


Fig. 9. Graphic explanation of the contributions from each term in Eqs. (5·4) and (5·5) to the reduced matrix element of electromagnetic operator.

have also adopted a simple choice of the effective charges and the effective spin $g$ factors:

$$e_p=e+\delta e, \quad e_n=\delta e=0.5e,$$

$$g_s^{\mathrm{eff}}=\begin{cases}0.55\, g_s, \text{ (for I, Cs, La)}\\ 0.50\, g_s. \text{ (for Mo, Ru)}\end{cases}$$

5-*a*) *The region of I, Cs and La isotopes*

The results of calculations in which the coupling effects are taken into account are tabulated from Table VIII to XI. Table VIII shows that the $E2$-transition properties discussed in §3 (of the collective $5/2_2^+$ states) are well conserved even when the coupling effects are taken into account. The reason is understood as follows: Since the mixing amplitude, $\zeta^{(1)}_{\nu=2}(5/2^+)$ defined by (5·1), in the collective $5/2_2^+$ state is small, the main contributions to the $B(E2;5/2_2^+\to 7/2_1^+)$ are expected to come from the third and the fourth terms on the right-hand side of Eq. (5·3). Furthermore, since the value of $\langle\Phi^{(3)}_{nIK}|\hat{O}^{(+)}_{2M}|\Phi^{(3)}_{n'I'K'}\rangle$ is not large, the contribution of the $E_{33}$ part is also small. Then the coupling effect is expected to enter mainly through the factor $\zeta^{(1)}_{\nu=1}(7/2^+)$ in the $E_{31}$ part. (Here, it should be noted that the $\zeta^{(3)}_{\nu=2}(n\ 5/2^+)$ factors approximately take the values $\zeta^{(3)}_{\nu=2}(n\ 5/2^+)\approx\delta_{n1}$.) The change in the value of $\zeta^{(1)}_{\nu=1}(7/2^+)$ from unity, in turn, comes from the admixtures of the dressed 3QP modes with $I=7/2^+$ in the 1QP $7/2_1^+$ state. Since such coupling effects on the 1QP $7/2_1^+$ state are also reduced when compared to the estimation of the QPC theory, no essential change in the enhancement property of the $B(E2;5/2_2^+\to 7/2_1^+)$ occurs. Following similar arguments, we can see that the hindrance property of the

Table VIII. $B(E2)$ values of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effects. The unit is $e^2\,10^{-50}$ cm$^4$. The adopted parameters are the same as those in Table I.

| | $5/2_2^+\longrightarrow 7/2_1^+$ | | $5/2_2^+\longrightarrow 5/2_1^+$ | |
|---|---|---|---|---|
| | $B(E2)_{\mathrm{cal}}$ | $B(E2)_{\mathrm{exp}}$ | $B(E2)_{\mathrm{cal}}$ | $B(E2)_{\mathrm{exp}}$ |
| $^{127}$I | 7.6 | | 0.6 | 0.7±0.1 [a] |
| $^{129}$I | 6.6 | 2.1±0.4 [b] | 0.1 | |
| $^{131}$I | 5.1 | | 0.2 | |
| $^{133}$I | 3.8 | | 0.1 | |
| $^{131}$Cs | 10.2 | 23.6±2.5 [c] | 0.4 | 0.95 [c] |
| $^{133}$Cs | 8.0 | 10.4±1.2 [b] | 0.2 | 3.5 [d] |
| $^{135}$Cs | 6.7 | | 0.1 | |
| $^{133}$La | 13.4 | | 0.2 | |
| $^{135}$La | 9.6 | 22 [e] | 0.2 | 1.7 [e] |
| $^{137}$La | 7.0 | | 0.2 | |

[a] Ref. 18), [b] Ref. 19), [c] Ref. 25), [d] Ref. 27), [e] Ref. 28).

Table IX. Calculated $g$ factors of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes. The calculated values of $g(5/2_2^+)$ listed in the second column are compared to the experimental values given in the third column. In the fourth column are listed the values calculated by adopting the ACS approximation. The calculated and experimental $g$ factors of the 1QP $7/2_1^+$ states are given in the fifth and sixth columns, respectively, for the sake of comparison. In these calculations, effective spin $g$ factor $g_s^{\text{eff}}=0.55\,g_s$ is used. The unit is nuclear magneton $e\hbar/2Mc$.

| | $g(5/2_2^+)_{\text{cal}}$ | $g(5/2_2^+)_{\text{exp}}$ | $g(5/2_2^+)_{\text{ACS}}$ | $g(7/2_1^+)_{\text{cal}}$ | $g(7/2_1^+)_{\text{exp}}$ |
|---|---|---|---|---|---|
| $^{127}$I | 0.80 | | 0.78 | 0.77 | 0.73[a] |
| $^{129}$I | 0.80 | | 0.78 | 0.77 | 0.747[b] |
| $^{131}$I | 0.81 | | 0.78 | 0.77 | 0.751[c] |
| $^{133}$I | 0.80 | | 0.78 | 0.77 | |
| $^{131}$Cs | 0.85 | $\begin{cases}0.79\pm0.05^{d)}\\0.74\pm0.03^{e)}\end{cases}$ | 0.78 | 0.77 | |
| $^{133}$Cs | 0.74 | | 0.78 | 0.77 | |
| $^{135}$Cs | 0.86 | | 0.78 | 0.73 | |
| $^{133}$La | 0.83 | | 0.77 | 0.74 | |
| $^{135}$La | 0.78 | | 0.77 | 0.77 | |
| $^{137}$La | 0.78 | | 0.78 | 0.77 | |

a) Ref. 20), b) Ref. 29), c) Ref. 30), d) Ref. 25), e) Ref. 26).

$B(E2;5/2_2^+\to5/2_1^+)$ is also conserved. Of course, their magnitudes change appreciably in a quantitative sense, since the $E_{31}$ part itself is a small quantity in this transition.

The AC state-like structure of the collective $5/2_2^+$ states is clearly exhibited in the calculated $g$ factors in Table IX which shows the property $g(5/2_2^+)\approx g(7/2_1^+)$. In these calculated values for $g(5/2_2^+)$, the coupling effect is negligibly small and they are determined essentially from the fourth term in Eq. (5·3). Namely, the $g$ factors in Table IX represent those of the $5/2_2^+$ states as the pure dressed 3QP states. Now, the experimental value of the $g(5/2_2^+)$ in $^{131}$Cs is, in fact, nearly equal to those of the $g(7/2_1^+)$ available in neighbouring nuclei. This fact is in excellent agreement with the theoretical prediction and, therefore, is regarded as a further evidence for the AC state-like structure of the collective $5/2_2^+$ states.

In Table IX, for the sake of comparison, are also presented the $g$ factors calculated by completely neglecting the 3QP correlations among different orbits. In this approximation (called the ACS approximation hereafter) in which the 3QP correlation is taken into account only within the specific orbit $p$, the formula for the $g$ factor is simply reduced to Eq. (4·13) of Chap. 3. Of course, for the collective $5/2_2^+$ states, the specific orbit $p$ denotes the $1g_{7/2}$ orbit. By comparing the results of the ACS approximation to those of the exact calculation, we see that the former give qualitatively the same characteristics for the values of $g(5/2_2^+)$ as the latter. This fact implies that the 3QP correlation in the specific orbit $1g_{7/2}$ plays a decisive role in the collective $5/2_2^+$ states, compared to the 3QP correlations among different orbits.

Table X. $B(M1)$ values of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effect. The unit is $(e\hbar/2Mc)^2$. The calculated results for transitions to the 1QP $7/2_1^+$ and $5/2_1^+$ states are listed in the second and sixth columns, respectively. The corresponding experimental values are given in the third and seventh columns. The contributions from the $M_{11}$ and $M_{33}$ parts in Eq. (5·5) are explicitly shown in the fourth and fifth columns, respectively, since both parts frequently become the same order of magnitudes. In these calculations, $g_s^{\mathrm{eff}}=0.55\,g_s$ is used.

| | $5/2_2^+\longrightarrow 7/2_1^+$ | | $5/2_2^+\longrightarrow 5/2_1^+$ | | | |
|---|---|---|---|---|---|---|
| | $B(M1)_{\mathrm{cal}}$ | $B(M1)_{\mathrm{exp}}$ | $M_{11}$ | $M_{33}$ | $B(M1)_{\mathrm{cal}}$ | $B(M1)_{\mathrm{exp}}$ |
| $^{127}$I | $1.4\times10^{-2}$ | $3.0\times10^{-2}$ a) | −0.28 | 0.04 | $9.5\times10^{-3}$ | $1.4\times10^{-1}$ a) |
| $^{129}$I | $1.0\times10^{-2}$ | | −0.23 | 0.003 | $8.6\times10^{-3}$ | |
| $^{131}$I | $4.9\times10^{-3}$ | | −0.83 | 0.41 | $2.9\times10^{-2}$ | |
| $^{133}$I | $9.9\times10^{-4}$ | | −0.68 | 0.35 | $1.9\times10^{-2}$ | |
| $^{131}$Cs | $1.7\times10^{-3}$ | $4.28\times10^{-3}$ b) | −0.17 | 0.07 | $1.7\times10^{-3}$ | $6.08\times10^{-4}$ b) |
| $^{133}$Cs | $1.0\times10^{-3}$ | $5.28\times10^{-3}$ c) | −0.12 | 0.04 | $1.1\times10^{-3}$ | $3.52\times10^{-1}$ c) |
| $^{135}$Cs | $9.5\times10^{-4}$ | | 0.01 | 0.02 | $1.3\times10^{-4}$ | |
| $^{133}$La | $5.3\times10^{-4}$ | | −0.63 | 0.39 | $1.0\times10^{-2}$ | |
| $^{135}$La | $2.1\times10^{-3}$ | $1.4\times10^{-3}$ d) | −0.31 | 0.19 | $2.6\times10^{-3}$ | $1.77\times10^{-3}$ d) |
| $^{137}$La | $8.1\times10^{-5}$ | | −0.11 | 0.07 | $2.4\times10^{-3}$ | |

a) Ref. 17), b) Ref. 25), c) Ref. 27), d) Ref. 28).

Table X shows the calculated values of $B(M1)$. As for the $M1$ transitions between the $5/2_2^+$ and 1QP $7/2_1^+$ states, the first term in (5·3) vanishes since it is the $l$-forbidden quasi-particle-transition matrix element. Furthermore, the matrix element $\langle\Phi^{(3)}_{nIK}|\hat{O}^{(-)}_{1M}|\Phi^{(1)}_{\delta'}\rangle$ is very small since the dressed 3QP mode in the P+QQ force model does not contain the "$J^\pi=1^+$" quasi-particle pairs except for their appearance through angular-momentum recoupling (due to the Pauli principle among constituting three quasi-paritcles). Therefore, the values of $B(M1;\ 5/2_2^+\to 7/2_1^+)$ are mainly determined by the fourth term in (5·3), i.e., the $M_{33}$ part. By comparing these to the available experimental data for $^{131,133}$Cs and $^{135}$La, we see that the calculated results reproduce the retarded $M1$ transition very well. On the other hand, the values of $B(M1;\ 5/2_2^+\to 5/2_1^+)$ are determined by the competition between the $M_{11}$ and $M_{33}$ parts, both of which are small quantities. The hindrance of this transition observed in some experiments may also be considered as an additional evidence for the AC state-like structure of the $5/2_2^+$ states. Although complete agreement between theoretical and experimental values should not be expected for such small quantities, we may thus assert that the $M1$-transition probabilities are of significance in examining the magnitude of the mixing amplitudes under investigation.

As we have seen, almost all characteristics of the collective $5/2_2^+$ states can be reproduced by the calculations based on the proposed theory. However,

Table XI. $B(E2)$ values of the $3/2_1^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effects. The unit is $e^2\,10^{-50}\,\mathrm{cm}^4$. The adopted parameters are the same as in Table III.

| | $3/2_1^+ \longrightarrow 5/2_1^+$ | | $3/2_1^+ \longrightarrow 7/2_1^+$ | |
|---|---|---|---|---|
| | $B(E2)_{\mathrm{cal}}$ | $B(E2)_{\mathrm{exp}}$ | $B(E2)_{\mathrm{cal}}$ | $B(E2)_{\mathrm{exp}}$ |
| $^{127}$I | 0.04 | 6.5±0.8 [a] | 1.12 | 11.2±0.2 [b] |
| $^{129}$I | 0.05 | | 0.65 | 7.0±0.8 [a] |
| $^{131}$I | 0.60 | | 0.25 | |
| $^{133}$I | 0.10 | | 1.91 | |
| $^{131}$Cs | 2.06 | | 0.04 | |
| $^{133}$Cs | 0.90 | | 1.05 | 7.2±0.8 [a] |
| $^{135}$Cs | 3.44 | | 1.29 | |
| $^{133}$La | 11.27 | | 0.17 | |
| $^{135}$La | 8.40 | ≥4.8 [c] | 0.32 | ≥0.1 [c] |
| $^{137}$La | 6.46 | | 0.43 | |

[a] Ref. 19), [b] Ref. 16), [c] Ref. 28).

it should also be pointed out that the following experimental facts still need investigation: 1) In the ($^3$He, $d$) reactions performed by Auble et al.,[21] the spectroscopic factors of the collective $5/2_2^+$ states in I isotopes were found to vary from 0.12 in $^{127}$I to 0.47 in $^{131}$I, which are both significantly larger than the calculated values. 2) The observed values of $B(E2; 5/2_1^+ \rightarrow 7/2_1^+)$ are very large in $^{127}$I and $^{129}$I.[16),19)] In our calculation, since this $E2$ transition is largely of 1QP transition, such a strong enhancement has not been obtained.

### 5-*b*) *The region of Mo and Ru isotopes*

In contrast to the case of collective $5/2_2^+$ states discussed above, the coupling effect of the 1QP mode is not negligible for the collective $3/2_1^+$ states in Mo and Ru isotopes. (See Table VII.) One of the reasons is that the new reduction effect coming from the exchange effects originated from the Pauli principle (discussed in §4) is not so drastic in the case of collective $3/2_1^+$ states. Another reason is that, since the 1QP $d_{3/2}^+$ mode which couples to the low-lying dressed 3QP $3/2_1^+$ mode lies *higher* in energy, the reduction $(u_c u_{\tilde{d}} - v_c v_{\tilde{d}})$ factors appearing in the main matrix elements in $V_{\mathrm{int}}(d, nI)$ are not as small as in the case of collective $5/2_2^+$ states in I, Cs and La isotopes (in which the 1QP $d_{5/2}^+$ mode lies *lower* in energy). The calculated trend that the mixing amplitude of the 1QP $d_{3/2}^+$ mode in the collective $3/2_1^+$ state becomes larger as $N$ increases seems to be in good agreement with the experimental trend that the spectroscopic factor of the ($d$, $p$) reaction leading to the $3/2_1^+$ state changes from 0.019 in $^{95}$Mo ($N$=53) to 0.11 in $^{99}$Mo ($N$=57).[5]

The result of calculations for the collective $3/2_1^+$ states are tabulated in Tables XII, XIII and XIV. By comparing the calculated $B(E2; 3/2_1^+ \rightarrow$

Table XII. $B(E2; 3/2_1^+ \to 5/2_1^+)$ values in odd-mass Mo and Ru isotopes, calculated by taking account of the coupling effects. The unit is $e^2\,10^{-50}\,\mathrm{cm}^4$. The adopted parameters are the same as in Table V.

$3/2_1^+ \longrightarrow 5/2_1^+$

| | $B(E2)_{\mathrm{cal}}$ | $B(E2)_{\mathrm{exp}}$ |
|---|---|---|
| $^{95}$Mo | 2.9 | $5.70 \pm 0.36$ [a] |
| $^{97}$Mo | 4.5 | $3.07 \pm 0.17$ [a] |
| $^{99}$Mo | 2.5 | |
| $^{97}$Ru | 5.5 | $7^{+12}_{-7}$ [b] |
| $^{99}$Ru | 7.5 | 13.05 [c] |
| $^{101}$Ru | 8.8 | 5.7 [c] |

[a] Ref. 2), [b] Ref. 6), [c] Ref. 7).

$5/2_1^+$) values (in Table XII) to those in Table V, we see the extent to which the coupling effects (between the dressed 3QP and 1QP modes) reduce their enhancements. Corresponding to the increasing coupling effect with $N$, the reduction of the $B(E2)$ values from those in the absence of the coupling effect also becomes appreciable in $^{99}$Mo and $^{101}$Ru with $N=57$. Here, of course, the coupling effect is not so strong as to break the zeroth-order picture of the collective $3/2_1^+$ states as the dressed 3QP states.

The magnetic dipole moments of the $3/2_1^+$ states have been known in $^{95}$Mo, $^{99}$Ru and $^{101}$Ru. The observed values of $g(3/2_1^+)$ show a small deviation from the property $g(3/2_1^+)=g(5/2_1^+)$. Concerning the origins of this deviation, we can point out the following: 1) The deviation of about 10% (from the property $g_{j-1}=g_j$) should be expected even if the collective $3/2_1^+$ states under consideration have exactly the same structure as the AC states. (See the geometrical factors in Eq. (4·13) of Chap. 3.) 2) The mixing of the 1QP $d_{3/2}$ state in the collective $3/2_1^+$ state brings about a destructive effect on the $g$ factor, since the values of $g(d_{3/2})$ and $g(d_{5/2})$ are of opposite signs. 3) The growth of the 3QP correlations among different orbits bring about the deviation from the property $g_{j-1}=g_j$. In the calculated $g$ factors shown in Table XIII, the $3s_{1/2}$ quasiparticle participating in the dressed 3QP $3/2_1^+$ mode plays an important role in bringing about the deviation.

The magnitude of the difference between the values of $g(3/2_1^+)$ and $g(5/2_1^+)$ observed in experiments seems to be consistently accounted for by the effects 1) and 2). However, as shown in Table XIII, the calculated results show that the values of $g(3/2_1^+)$ are considerably affected by the effect 3). Due to the fact that the effect 3) depends quite sensitively on the single-particle energies adopted in the calculation, the calculated results are not in good agreement with the experimental data. A more accurate evaluation on the contribution of the

Table XIII. Calculated $g$ factors of the $3/2_1^+$ states in odd-mass Mo and Ru isotopes. The calculated values of $g(3/2_1^+)$ listed in the fourth column are compared to the experimental values given in the fifth column. In the sixth column are listed the values calculated by adopting the ACS approximation. The calculated and experimental $g$ factors of the 1QP $5/2_1^+$ states are given in the seventh and eighth columns, respectively, for the sake of comparison. The contributions from the $g_{11}$ and $g_{33}$ parts in Eq. (5·6) are explicity given in the second and third columns, respectively, with the aim of showing the coupling effects on the calculated $g(3/2_1^+)$ values listed in the fourth column. In these calculations, $g_s^{\mathrm{eff}}=0.50\,g_s$ is adopted. The unit is $e\hbar/2Mc$.

| | $g_{11}$ | $g_{33}$ | $g(3/2_1^+)_{\mathrm{cal}}$ | $g(3/2_1^+)_{\mathrm{exp}}$ | $g(3/2_1^+)_{\mathrm{ACS}}$ | $g(5/2_1^+)_{\mathrm{cal}}$ | $g(5/2_1^+)_{\mathrm{exp}}$ |
|---|---|---|---|---|---|---|---|
| $^{95}$Mo | 0.001 | −0.62 | −0.62 | −0.26±0.02 [a]<br>−0.24±0.03 [b] | −0.35 | −0.37 | −0.365 [c] |
| $^{97}$Mo | 0.01 | −0.26 | −0.25 | | −0.33 | −0.37 | −0.373 [a] |
| $^{99}$Mo | 0.10 | −0.10 | −0.01 | | −0.28 | −0.36 | |
| $^{97}$Ru | 0.003 | −0.60 | −0.60 | | −0.32 | −0.37 | |
| $^{99}$Ru | 0.02 | −0.31 | −0.29 | −0.189±0.004 [d]<br>−0.20 ±0.02 [e] | −0.34 | −0.36 | −0.25 [f] |
| $^{101}$Ru | 0.08 | −0.59 | −0.48 | −0.207±0.017 [g] | −0.29 | −0.35 | −0.28 [c] |

[a] Ref. 3), [b] Ref. 4), [c] Ref. 31), [d] Ref. 9), [e] Ref. 8), [f] Ref. 11), [g] Ref. 10).

$3s_{1/2}$ quasi-particle seems necessary.

As for the systematics of $B(M1;\ 3/2_1^+\to 5/2_1^+)$, a curious trend in the sequence of odd-mass Ru isotopes has been observed: The value decreases from $^{97}$Ru ($N=53$) to $^{99}$Ru ($N=55$) and then increases from $^{99}$Ru to $^{101}$Ru ($N=57$). (See Table XIV.) The origin of such a curious trend may be understood as follows: Since the transition matrix element of the type $\langle \Phi_\delta^{(1)} | \hat{O}_{1M}^{(-)} | \Phi_{n'I'K'}^{(3)} \rangle$ is extremely small, the $M_{31}$ and $M_{13}$ parts in (5·3) are negligibly small. Then, the $M1$ transition of interest can take place mainly through the $M_{11}$

Table XIV. $B(M1;\ 3/2_1^+\to 5/2_1^+)$ values in odd-mass Mo and Ru isotopes, calculated by taking account of the coupling effect. The unit is $(e\hbar/2Mc)^2$. The contributions from the $M_{11}$ and $M_{33}$ parts in Eq. (5·5) are explicitly given in the second and third columns, with the aim of showing that the $B(M1)$ values depend sensitively on the relative signs of them. In these calculations, $g_s^{\mathrm{eff}}=0.50\,g_s$ is used.

$3/2_1^+ \longrightarrow 5/2_1^+$

| | $M_{11}$ | $M_{33}$ | $B(M1)_{\mathrm{cal}}$ | $B(M1)_{\mathrm{exp}}$ |
|---|---|---|---|---|
| $^{95}$Mo | −0.06 | −0.05 | $3.0\times10^{-3}$ | $(4.21\pm0.02)\times10^{-3}$ [a] |
| $^{97}$Mo | −0.17 | 0.23 | $7.3\times10^{-4}$ | $(2.2\pm0.4)\times10^{-2}$ [a] |
| $^{99}$Mo | −0.47 | 0.32 | $4.2\times10^{-3}$ | |
| $^{97}$Ru | −0.11 | 0.01 | $2.4\times10^{-3}$ | $2.20\times10^{-2}$ [b] |
| $^{99}$Ru | −0.26 | 0.43 | $6.9\times10^{-3}$ | $3.33\times10^{-4}$ [c] |
| $^{101}$Ru | 0.42 | 0.70 | $2.8\times10^{-1}$ | $2.78\times10^{-1}$ [c] |

[a] Ref. 2), [b] Ref. 6), [c] Ref. 7).

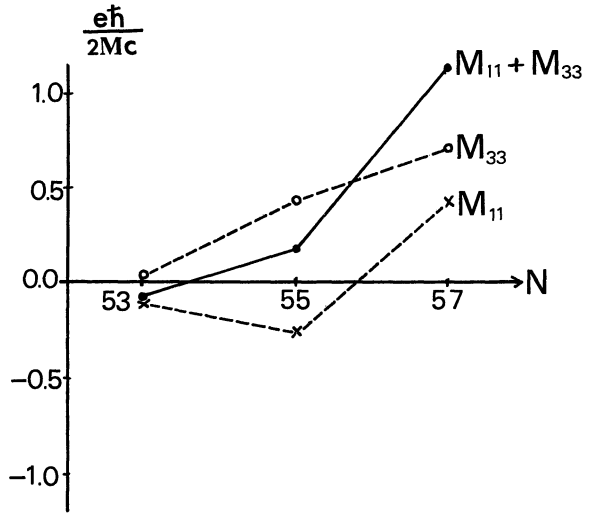


Fig. 10. The contributions from the $M_{11}$ and $M_{33}$ parts to the $B(M1; 3/2_1^+ \to 5/2_1^+)$ in Ru isotopes. (See also Table XIV.)

and $M_{33}$ parts. The $M_{11}$ part, which comes from the mixing effect of the 1QP $3/2^+$ state in the dressed 3QP $3/2_1^+$ state, represents the 1QP transition between the spin-flip single-particle orbits, $2d_{3/2}$ and $2d_{5/2}$. The $M_{33}$ part, which comes from the mixing effect of the dressed 3QP $5/2^+$ states in the 1QP $5/2_1^+$ state, represents the transition between the dressed 3QP $3/2^+$ and $5/2^+$ states. The sign of the mixing amplitude $\zeta^{(1)}_{\nu=1}(3/2^+)$ involved in the $M_{11}$ part depends essentially on the sign of the effective coupling strength $V_{\rm int}(a, nI)$ with $a=(2d_{3/2})$, $n=1$ and $I=3/2$, while the sign of the mixing amplitude $\zeta^{(3)}_{\nu=1}(5/2^+)$ involved in the $M_{33}$ part depends on that of $V_{\rm int}(a, nI)$ with $a=(2d_{5/2})$, $n=1$ and $I=5/2$. Now, from the microscopic structure of the $V_{\rm int}(a, nI)$ given by Eq. (4·3) we can observe the following: The value of $V_{\rm int}(d_{3/2}, 3/2^+)$ changes sign as one moves from $^{97}$Ru to $^{101}$Ru, while the value of $V_{\rm int}(d_{5/2}, 5/2^+)$ conserves sign. The sign change in the former comes from the increase of the component $\{(\nu d_{5/2})^2 \nu s_{1/2}\}$ in the dressed 3QP $3/2_1^+$ mode, which has a sign opposite to the sign of the main component $\{(\nu d_{5/2})^3\}$. Thus, as is seen from Table XIV and Fig. 10, the phase relation between the $M_{11}$ and $M_{33}$ parts changes from destructive to constructive as one moves from $^{97}$Ru to $^{101}$Ru. Here the increase of the absolute magnitudes of the $M_{11}$ and $M_{33}$ parts represents the increasingly important role of the coupling effects as the neutron number goes from $N=53$ to $N=57$. In conclusion, we can say that the curious trend in $B(M1; 3/2_1^+ \to 5/2_1^+)$ comes from the change in the coherent property between the $M_{11}$ and $M_{33}$ parts, which is essentially determined by the structure change of the dressed 3QP $3/2_1^+$ mode. Since the values of the $M_{11}$ and $M_{33}$ parts depend rather sensitively on the adopted single-

particle energies, a better agreement between theoretical and experimental $B(M1)$ values is obtainable within the framework of the introduced model if the adopted parameters are changed slightly.

For $^{95}$Mo with $N=53$, it has been known that the shell-model calculation with the subspace consisting of the orbit $1g_{9/2}$ for protons and the orbit $2d_{5/2}$ for neutrons yields the low-lying $3/2_1^+$ state.[32),33)] Of course, the collective nature of the $3/2_1^+$ state cannot be fully accounted for in the Tamm-Dancoff approximation with restricted shell-model subspace. Recently, the collective structure of the $3/2_1^+$ state in $^{95}$Mo has been investigated in terms of the semi-microscopic model in which the three-neutron valence-shell cluster is interacting with the quadrupole vibration of the "core".[38),39)] Similar investigations have also been done for odd-proton I isotopes with $Z=53$.[34)~37)] The results of these investigations indicate the remarkable improvements over the conventional particle-vibration-coupling model; namely, the appearance of the low-lying $3/2_1^+$ state (in $^{95}$Mo) and $5/2_2^+$ states (in I isotopes) is well reproduced in these calculations, together with their enhanced $E2$-transition properties. This fact implies the importance of explicitly taking into account the three-particle correlations in the valence-shell orbits. As was discussed in § 3–3 of Chap. 3, the dressed 3QP modes under consideration are capable of decomposing into the form in which the direct relation with this semi-microscopic model is visualized. However, it should be emphasized that the essential role of the 3QP correlations (characterizing the collective excitations in spherical odd-mass nuclei) is not by any means specific to the single-closed-shell plus three-nucleon system such as $^{95}$Mo and I isotopes. In fact, as we have seen, the collective $3/2_1^+$ and $5/2_2^+$ states appear quite regularly in nuclei with $N$ or $Z$ being 53, 55 and 57. The following fact should also be noted: In our model, for example, the collective $E2$ enhancements of the $3/2_1^+$ states (in odd-neutron Mo and Ru isotopes) are caused not only by the forward-going amplitudes of $(\pi\pi\nu)$-type but also by the backward-going amplitudes of $(\pi\pi\nu)$-type which represent the ground-state correlation. (See Table VI.) In particular, the ground-state correlation originating from the quadrupole force acting between proton- and neutron-quasi-particle-pairs plays an important role in enlarging the backward-going amplitudes of $(\pi\pi\nu)$-type. This implies that the internal structure of the quadrupole vibration (phonon) of the core is considerably affected by the interaction between the quasi-particles in the valence-shell orbits and the quasi-particles excited from the core. It seems difficult to take such an effect into account within the semi-microscopic model mentioned above.

## § 6. Concluding remarks

We have shown that the collective $5/2_2^+$ states in odd-proton I, Cs and La isotopes and the collective $3/2_1^+$ states in odd-neutron Mo and Ru isotopes are identified as the new elementary mode of collective excitation, i.e., the dressed

3QP mode. We have also shown that the physical condition for the appearance of the dressed 3QP modes is not specific to the AC states but quite general in spherical odd-mass nuclei. The presence of a high-spin orbit having parity opposite to that of the major shell, such as in the case of the AC states, is not a necessary condition for the realization of the dressed 3QP modes. Rather, the important condition is found in the shell structure of the orbits lying in the neighbourhood of the chemical potential. Even if many orbits having the same parity lie close to one another and the energy spacings between the orbit of interest and the others (with the same parity) are not so large as in the case of the AC states, one cannot expect a less dominant role of the 3QP correlation at the specific orbit lying in the vicinity of the chemical potential. Furthermore, the physical condition (in shell structure) weakening the effective coupling strength between the 1QP mode and the collective dressed 3QP mode (with the same spin and parity) is common to the condition for the realization of the AC state-like dressed 3QP mode. Thus, the dressed 3QP modes similar to the AC states can exist as relatively pure elementary excitation modes over a wide region of spherical odd-mass nuclei.

The essential roles of the 3QP correlation will not be restricted to characterize the AC state-like collective excitation modes having spin $(j-1)$ which have been investigated thus far. Rather, we should expect various roles of the 3QP correlation of which we know little at present. For example, the role of the 3QP correlation among quasi-particles in different orbits should be investigated further. In this chapter, the importance of such effects has only been briefly mentioned for the case of the $3/2_1^+$ states. In the succeeding chapter, standing on the new point of view acquired here, we will investigate microscopic structure of breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture." Our discussion will be extended to all low-lying collective excited states, including those having spins other than $(j-1)$. The present status of our picture of the low-energy excitations in spherical odd-mass nuclei will then be summarized.

## Appendix 4A. Procedure of numerical calculation

Here, we describe a calculational method in solving the eigenvalue equation of the dressed 3QP mode, (3·3) of Chap. 2, full expression of which is given in Appendix 6A.

### 4A–1 *Orthonormal basis vectors*

In solving Eq. (3·3) of Chap. 2, we should first prepare the orthonormal basis vectors in the coupled-angular-momentum representation defined in Appendix 6A. Such a requirement is easily achieved by diagonalizing the projection operator $\boldsymbol{P}_I$, the matrix elements of which are $P_I(ab(J)c|a'b'(J')c')$ explicitly defined by (2A·6). From the property of the projection operator,

$\boldsymbol{P}_I^2=\boldsymbol{P}_I$, it is clear that the eigenvalues take only the value $+1$ or $0$:

$$\boldsymbol{U}_I\boldsymbol{P}_I\boldsymbol{U}_I^{-1}=\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \tag{4A·1}$$

where $\boldsymbol{U}_I$ denotes the unitary transformation which diagonalize $\boldsymbol{P}_I$. The eigenvectors belonging to the eigenvalue 1 just coincide with the coefficients of fractional parentage (cfp) for $(j_aj_bj_c)$-configurations with seniority $v=3$ and total angular momentum $I$. The orthonormalized basis vectors are then obtained as

$$\bar{\psi}_{iI}[abc]=\boldsymbol{U}_I(i)\boldsymbol{\psi}_I[abc], \tag{4A·2}$$

where the vector $\boldsymbol{\psi}_I[abc]$ is constructed from the elements

$$\{\psi_I[ab(J)c];\, J,\, \mathcal{P}(abc)\}, \tag{4A·3}$$

with $\mathcal{P}(abc)$ denoting all the permutation with respect to $(abc)$. In (4A·2), $\boldsymbol{U}_I(i)$ denotes a row vector of the matrix $\boldsymbol{U}_I$ and the letter $i$ labels the independent basis vectors. Needless to say, the projection operator $\boldsymbol{P}_I$ and the matrix $\boldsymbol{U}_I$ are both diagonal with respect to different sets of the orbital triad $(abc)$.

4A–2 *Eigenvalue equation in terms of orthonormal vectors*

When the projection operator $\boldsymbol{P}_I$ is diagonalized, the matrix elements of the eigenvalue equation are reduced to the following form:

$$\begin{aligned}&\bar{\boldsymbol{D}}_I[abc|a'b'c']\\ &\quad\equiv\boldsymbol{U}_I[abc]\cdot\boldsymbol{D}_I[abc|a'b'c']\cdot\boldsymbol{U}_I[a'b'c']^{-1}\\ &\quad=\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\boldsymbol{U}_I[abc]\cdot\boldsymbol{M}_I[abc|a'b'c']\cdot\boldsymbol{U}_I[a'b'c']^{-1}\begin{bmatrix}\mathbf{1}' & \mathbf{0}'\\ \mathbf{0}' & \mathbf{0}'\end{bmatrix},\end{aligned} \tag{4A·4}$$

where $\boldsymbol{D}_I[abc|a'b'c']$ denotes the matrix composed of the elements $D_I[ab(J)c|a'b'(J')c']$ and $\boldsymbol{M}_I[abc|a'b'c']$ the corresponding matrix excluding the projection operator $\boldsymbol{P}_I$. In (4A·4), the matrix $\boldsymbol{U}_I$ which corresponds to a particular set of orbital triad $(abc)$ is explicitly denoted as $\boldsymbol{U}_I[abc]$. Thus, the vectors belonging to the zero-eigenvalue of the projection operator do not couple to the physical vectors having eigenvalue 1. We can also obtain the matrices $\bar{\boldsymbol{A}}_I$ and $\bar{\boldsymbol{d}}_I$ from $\boldsymbol{A}_I$ and $\boldsymbol{d}_I$ by the same procedure as above. In this way we obtain the eigenvalue equation written in terms of the orthonormal basis vectors as follows:

$$\omega_{nI}\begin{bmatrix}\bar{\boldsymbol{\varphi}}_{nI}\\ \bar{\boldsymbol{\varphi}}_{nI}\end{bmatrix}=\begin{bmatrix}3\bar{\boldsymbol{D}}_I & -\bar{\boldsymbol{A}}_I\\ \bar{\boldsymbol{A}}_I^T & -\bar{\boldsymbol{d}}_I\end{bmatrix}\begin{bmatrix}\bar{\boldsymbol{\varphi}}_{nI}\\ \bar{\boldsymbol{\varphi}}_{nI}\end{bmatrix}. \tag{4A·5}$$

4A–3 *Two-step diagonalization of secular matrix*

We now diagonalize the secular matrix (4A·5) by the following two-step procedure. First we independently diagonalize the forward and backward matrices, $\bar{\boldsymbol{D}}_I$ and $\bar{\boldsymbol{d}}_I$:

$$\boldsymbol{V}_I^{(f)\cdot}(3\bar{\boldsymbol{D}}_I)\cdot\boldsymbol{V}_I^{(f)-1}=\begin{pmatrix}\omega_{1I}^f & & 0\\ & \omega_{2I}^f & \\ 0 & & \ddots\end{pmatrix}\equiv\boldsymbol{\omega}_I^f, \tag{4A·6}$$

$$\boldsymbol{V}_I^{(b)\cdot}(-\bar{\boldsymbol{d}}_I)\cdot\boldsymbol{V}_I^{(b)-1}=\begin{pmatrix}\omega_{1I}^b & & \\ & \omega_{2I}^b & \\ 0 & & \ddots\end{pmatrix}\equiv\boldsymbol{\omega}_I^b. \tag{4A·7}$$

The new secular matrix thus obtained, i.e.,

$$\begin{bmatrix}\boldsymbol{\omega}_I^f & -\boldsymbol{V}_I^{(f)}\bar{\boldsymbol{A}}_I\boldsymbol{V}_I^{(b)-1}\\ \boldsymbol{V}_I^{(b)}\bar{\boldsymbol{A}}_I^T\boldsymbol{V}_I^{(f)-1} & \boldsymbol{\omega}_I^b\end{bmatrix}, \tag{4A·8}$$

is then diagonalized as the second step. The two-step-diagonalization procedure is, of course, equivalent to the direct diagonalization. This method, however, possesses the following merits:

(1) The diagonal matrix $\boldsymbol{\omega}_I^f$ obtained in the first step gives us the solutions of the corresponding Tamm-Dancoff approximation, i.e., the solutions of the "bare" 3-quasi-particle states.

(2) In the second step, the secular matrix (4A·8) can be truncated in such a way that some restricted eigenvectors of $\bar{\boldsymbol{D}}_I$ and $\bar{\boldsymbol{d}}_I$ are sufficient to yield a good approximation for the full calculation. It should be noted here that the normalization of the correlation amplitudes given by (3·6) of Chap. 2 does not change at all by this truncation.

4A–4 *Another method for providing orthonormal basis vectors*

We can adopt an alternative method for providing the orthonormal basis vectors by rewriting the forward-going components of the eigenmode operator (3·1) of Chap. 2 as follows:

$$\begin{aligned}C_{nI}^\dagger=&\frac{1}{\sqrt{3!}}\sum_{aJ}\psi_{nI}[aa(J)a]\sum_{m_{a_1}m_{a_2}m_{a_3}M}(j_aj_am_{a_1}m_{a_2}|JM)(Jj_aMm_{a_3}|IK)a_{a_1}^\dagger a_{a_2}^\dagger a_{a_3}^\dagger\\ &+\frac{1}{\sqrt{2!}}\sum_{\substack{acJ\\(a\neq c)}}\psi_{nI}[aa(J)c]\sum_{m_{a_1}m_{a_2}m_{\gamma}M}(j_aj_am_{a_1}m_{a_2}|JM)(Jj_cMm_\gamma|IK)a_{a_1}^\dagger a_{a_2}^\dagger a_\gamma^\dagger\\ &+\sum_{\substack{abcJ\\(a<b<c)}}\psi_{nI}[ab(J)c]\sum_{m_am_\beta m_\gamma M}(j_aj_bm_am_\beta|JM)(Jj_cMm_\gamma|IK)a_a^\dagger a_\beta^\dagger a_\gamma^\dagger\\ &+\sum_{(rs)cJ}\frac{\psi_{nI}[rs(J)c]}{\sqrt{1+\delta_{rs}}}\sum_{m_\rho m_\sigma m_\gamma M}(j_rj_sm_\rho m_\sigma|JM)(Jj_cMm_\gamma|IK)a_\rho^\dagger a_\sigma^\dagger a_\gamma^\dagger\\ &+\{\text{backward components being similar to the above definitions}\}.\end{aligned} \tag{4A·9}$$

In this method, the matrix of the secular equation becomes more complicated than that in the preceding subsections, hence we do not give its explict form. In the calculations of Chaps. 4, 5 and 6, we have independently adopted both methods for providing the basis vectors. They have been used to check the numerical calculations done there.

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# Chapter 5. Microscopic Structure of Breaking and Persistency of "Phonon-plus-Odd-Quasi-Particle Picture"

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## §1. Introduction

In the preceding chapter, we have obtained a conclusion that, in almost all spherical odd-mass nuclei, the dressed three-quasi-particle (3QP) states with spin $I=(j-1)$ are expected to appear in the neighbourhood of the 1QP states with spin $j\geqq 5/2$. Furthermore, we have emphasized that the roles of the 3QP correlations should be regarded not only as to bring about the general presence of the $(j-1)$ states but also as to play an essential role in characterizing the low-energy excitation structure in almost all spherical odd-mass nuclei.

This conclusion leads us inevitably to change the customarily used "phonon-plus-odd-quasi-particle picture" in which elementary modes of excitation (characterizing low-lying states in spherical odd-mass nuclei) are assumed to be odd-quasi-particle modes and phonon modes.[1),2)] In the conventional quasi-particle-phonon-coupling (QPC) theory,[1),2)] as is well known, the phonons are described by random-phase approximation (RPA) assuming them to be ideal bosons (and hence are commutable with the odd quasi-particles). Boson expansion methods for odd-mass nuclei[3)~5)] can also be regarded as perturbational approaches to describe the system starting from these (independent) elementary excitation modes.

In contrast to these approaches, the theory developed in Chap. 2 (which may be called the "method of new-Tamm-Dancoff (NTD) space") is free from introducing the concept of phonon to odd-mass nuclei and, furthermore, includes the QPC theory as a specially approximated version (in which the kinematical effects due to the Pauli principle among quasi-particles more than two are all neglected). The proposed theory enables us to classify both the complicated "anharmonicity effects" and the roles of residual interactions in a systematic way. Furthermore, by using the theory, we are able to investigate the mutual relationships between various aspects of "anharmonicity effects." Thus it now becomes possible to investigate the microscopic structure of break-

ing and persistency of the conventional "phonon-plus-odd-quasi-particle picture," from the new point of view obtained in the previous chapters. In this chapter, special emphasis will be put on extracting (from the complicated "anharmonicity effects") the essential correlations which necessarily lead us to adopt a new picture for the low-lying collective excited states in spherical odd-mass nuclei.

In §2, starting from the basic picture of the QPC theory, some criteria for investigating the breaking and persistency of the conventional phonon picture in spherical odd-mass nuclei are set up. In §3, characteristics of the collective 3QP correlations in many $j$-shell model are discussed exemplifying the results calculated for odd-proton $^{133}$Cs and $^{135}$La nuclei. Here, with the aid of the criteria set up in §2, various aspects of the 3QP correlations are investigated by paying attention to their relation with shell structure. It will be shown that, although simple phonon picture is drastically changed due to the action of collective 3QP correlations, one element of the phonon picture which is characterized by the concept of "phonon-band" can persist under a certain condition of shell structure. In §4, we briefly discuss the roles of correlations between proton- and neutron-quasi-particles (in characterizing the dressed 3QP modes) by showing the results calculated for $^{97}$Mo and $^{105}$Pd nuclei. The results calculated for $^{117}$Sn and $^{115}$Cd nuclei are also presented in §5 in order to supplement the statement given in §4 and to show the possibility of complete breakdown of the phonon-band character under another situation in shell structure.

In §6, after the investigations on the microscopic structure of the eigenmodes themselves, we turn to estimate the effect of the *interactive force* $H_Y$. In the conventional QPC theory, as is well known, the coupling between the odd quasi-particle and the phonon comes entirely from the interactive force $H_Y$ and plays a role changing the number of phonons by one. However, in this section, an important difference between the evaluation of the $H_Y$ effect in the "ideal-boson-fermion space" (implicitly assumed in the QPC theory) and that in the "quasi-particle NTD space" (characterizing the proposed theory) will be shown.

In order to keep a close contact with the conventional QPC theory, all discussions in this chapter will be made by adopting the pairing-plus-quadrupole ($P+QQ$) force model.[6]

## §2. Criteria for breaking and persistency of phonon-bands

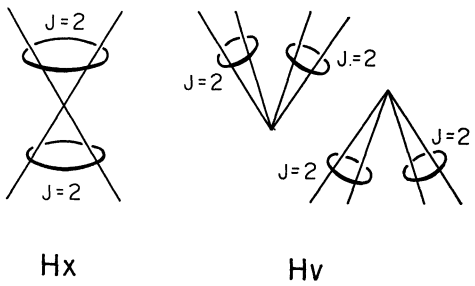
In this section, in order to investigate the microscopic structure of breaking and persistency of "phonon-plus-odd-quasi-particle picture," we first recapitulate the characteristics of the excitation spectrum and of $E2$-transition properties given by the unperturbed Hamiltonian $\mathcal{H}^{(0)}$ of the QPC theory.[1),2)]

The $P+QQ$ Hamiltonian can be divided into the following parts in the quasi-particle representation:

$$\left.\begin{aligned} &H=H_0+:H_{QQ}:, \\ &:H_{QQ}:=H_X+H_V+H_Y, \end{aligned}\right\} \tag{2·1}$$

where $H_0$ denotes the free quasi-particle Hamiltonian and each part of $:H_{QQ}:$ is schematically represented in Fig. 1. In the conventional QPC theory, two-quasi-particle (2QP) correlation diagrams originated from the $H_X$- and $H_V$-type interactions are summed up to all orders in the sense of NTD approximation. Then the part, $H_0+H_X+H_V$, is transformed into the free Hamiltonian $\mathcal{H}^{(0)}$ which describes a system composed of (non-interacting) odd quasi-particle plus phonons. On the other hand, the part $H_Y$ is considered to give rise to the coupling between the odd quasi-particle and phonon in the "ideal-boson-fermion space." Thus the model Hamiltonian of the conventional QPC theory takes the following form:

$$\left.\begin{aligned} &\mathcal{H}=\mathcal{H}^{(0)}+\mathcal{H}^{(\mathrm{int})}, \\ &\mathcal{H}^{(0)}=\sum_{a} E_a \dot{a}_a^{\dagger}\dot{a}_a+\sum_{JM(J\neq 2)}(E_a+E_b)\dot{A}_{JM}^{\dagger}(ab)\dot{A}_{JM}(ab) \\ &\qquad\qquad +\sum_{\nu}\sum_{M}\omega_\nu \Gamma_{2M}^{\dagger}(\nu)\Gamma_{2M}(\nu), \\ &\mathcal{H}^{(\mathrm{int})}=\sum_{\nu}\sum_{M a\beta}\bar{\chi}_{a\beta}(\nu)\{\Gamma_{2\widetilde{M}}^{\dagger}(\nu)+\Gamma_{2M}(\nu)\}\dot{a}_a^{\dagger}\dot{a}_{\tilde{\beta}}, \end{aligned}\right\} \tag{2·2}$$



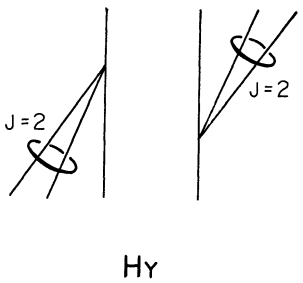




Fig. 1. Graphic representation of the matrix elements of the quadrupole force. Part $H_X$ represents a scattering of the pair of quasi-particles coupled to $J^{\pi}=2^{+}$, while part $H_Y$ represents a pair-creation and a pair-annihilation of quasi-particles coupled to $J^{\pi}=2^{+}$. Parts $H_X$ and $H_V$ are called the *constructive force*. Part $H_Y$ represents a creation and an annihilation of the quasi-particles coupled to $J^{\pi}=2^{+}$, accompanying a scattering of a quasi-particle. Part $H_Y$ is called the *interactive force*.

where $\dot{a}^{\dagger}_{\alpha}$, $\dot{A}^{\dagger}_{JM}(ab)$ and $\Gamma^{\dagger}_{2M}(\nu)=\sum_{ab}\{\psi_{\nu}(ab)\dot{A}^{\dagger}_{2M}(ab)-\varphi_{\nu}(ab)\dot{A}\widetilde{}_{2M}(ab)\}$ represent the creation operators of ideal-odd-quasi-particle, ideal boson corresponding to quasi-particle pair and ideal-phonon, respectively. In Eq. (2·2), the Greek letter $\nu$ distinguishes various eigensolutions of phonon modes and, as an additional approximation,[1),2)] non-collective phonons have often been neglected together with the second term of $\mathcal{H}^{(0)}$. In the same way the mass-quadrupole operator is expressed as

$$\hat{Q}_{2M}=\sum_{\nu}Q_{M}(\nu)\{\Gamma^{\dagger}_{2M}(\nu)+\Gamma\widetilde{}_{2M}(\nu)\}+\sum_{\alpha\beta}q_{M}(\alpha\beta)\dot{a}^{\dagger}_{\alpha}\dot{a}_{\tilde{\beta}}, \tag{2·3}$$

where $Q_{M}(\nu)$ and $q_{M}(\alpha\beta)$ represent the collective and single-quasi-particle matrix elements, respectively. A theoretical foundation for deriving the model operators in the QPC theory has been known as boson expansion methods in odd-mass nuclei,[3)~5)] in which the unperturbed Hamiltonian $\mathcal{H}^{(0)}$ and the expression (2·3) are considered as a zeroth-order approximation for boson expansion.

Now, one of the characteristics of the basis states given by $\mathcal{H}^{(0)}$ is that they are classified into definite sets of states each of which can be called *a phonon-band*. The phonon-bands are distinguished with one another by the quantum numbers of the single-particle orbit $a=(nlj)$, to which the odd quasi-particle belongs. As illustrated in Fig. 2, each phonon-band consists of a series of degenerate multiplets in which the odd quasi-particle is coupled with some number of phonons. The excitation spectrum and the $E2$-transition properties obey the well known pattern of the harmonic oscillators. It should be emphasized here that the $E2$ transitions between different phonon-bands (inter-band transitions) are forbidden if we neglect the second term in (2·3).

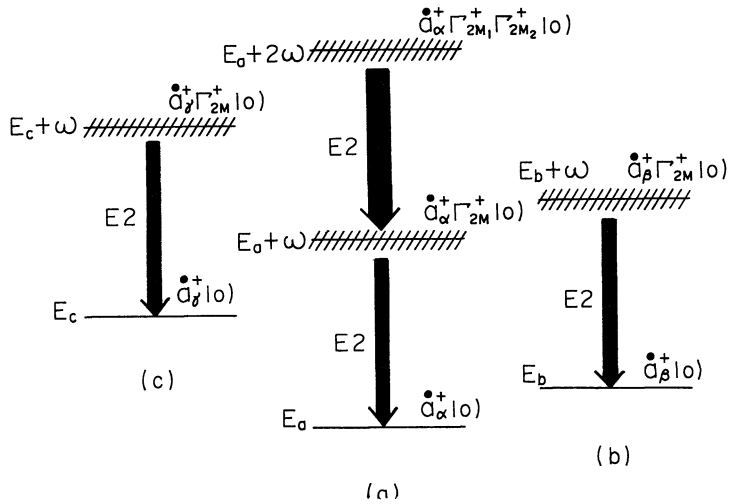



Fig. 2. Schematic representation of the concept of phonon-bands.

In particular, the inter $E2$ transitions from the multiplet (composed of the odd quasi-particle *in the orbit a* coupled with one phonon) to the band-head (the odd quasi-particle state in *the orbit b*) which belongs to the other phonon-bands are strictly forbidden.

Starting from this zeroth-order picture of the QPC theory, let us switch on the 3QP correlations and follow up the process of breaking of the picture. Then we can consider two cases for the way of the breaking;

A) the case where the 3QP correlation among quasi-particles in the same single-particle orbit plays a predominant role (Fig. 3-A),

B) the case where the 3QP correlation among quasi-particles in different single-particle orbits plays a predominant role (Fig. 3-B).

A typical example of case A is the AC states. As was shown in Chap. 3, in the case of the AC states the triggering effect of the 3QP correlations (which strongly violate the concept of phonon in odd-mass nuclei) is restricted among quasi-particles in a specific high-spin and unique-parity orbit, because of the parity-selection property of the quadrupole force. Hence, in this case, we can look into the breaking of the *simple phonon picture* within a "(isolated) single phonon-band." (See Fig. 16(a) in Chap. 3, which shows the splitting of the "quintet" composed of the $g_{9/2}$ odd quasi-particle coupled with the $2^+$ phonon.)

As was discussed in Chap. 4, we can also expect the other situation which belongs to case A, that is, in spite of the fact that many orbits with the same parity lie close and equally active for the 3QP correlations, effect B is highly reduced compared to effect A. An important characteristic in this case is that, although the energy splittings of the multiplets are very large (due to effect A), the $E2$ transitions between different phonon-bands (the inter-band transitions) are hindered compared to the intra-band transitions. (See Fig. 4.) In this sense, we can say that the concept of phonon-band is preserved in case A.

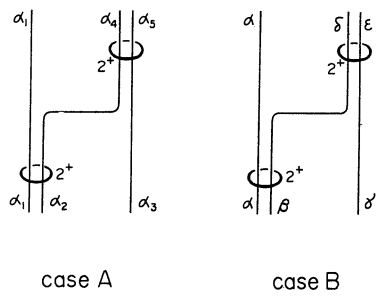



Fig. 3. Illustrations for two types of the 3QP correlations.
case A: 3QP correlation among quasi-particles in the same orbit.
case B: 3QP correlation among quasi-particles in different orbits ($a \neq b \neq c$, $a \neq b = c$, $a = b \neq c$ *or* $b \neq c = a$).
The subscript $i$ ($=1, 2, \ldots, 5$) of $\alpha$ is used to specify the single-particle states with different magnetic quantum numbers in the same orbit $a$.

The situation belonging to case A seems to resemble to that given by the phenomenological core-excitation model,[7] in the point that the inter-band transitions are forbidden approximately. However, there exist important differences, which are; 1) the center of gravity theorem is violated, 2) the $B(E2)$ values for the transitions from the multiplet to their band-head (one-quasi-particle) state are different from one another and 3) they are also not equal to the phonon transition $B(E2; 2^+ \to 0^+)$ in the even-even core. This is because of the fact that, from our viewpoint, the microscopic structure of the core excitation (phonon) itself is changed in a way which depends on the spin $I$ of the multiplet, due to the 3QP correlation at a specific orbit.

On the other hand, in case B, the members with the same spin which belong to different phonon-bands couple with one another strongly (due to the 3QP correlation among different orbits), as is evident from Fig. 3-B. Then the (approximate) selection rule for the $E2$ transitions mentioned above is violated and, therefore, in this case we cannot identify the phonon-bands. It should be emphasized that such an effect of "band-mixing" never occur in the conventional QPC theory. Namely the "band-mixing" due to the 3QP correlation is a "direct mixing" originated from the $H_X$ and $H_V$ interactions, whereas in the QPC theory the "band-mixing" can occur only through "indirect mixing" mediated by the coupling to the 1QP states (originated from the $H_V$ interaction).

The way and the extent of the breaking of the "phonon-plus-odd-quasi-particle picture" will depend on various conditions. For instance, depending on shell structure and on the spins of collective states, there may exist a phonon-band which couples easily (or hardly) to the other bands. Therefore, we will investigate, in the following sections, the microscopic structure of breaking and persistency of the "phonon-plus-odd-quasi-particle picture" with the aid of the criteria given here.

## §3. Persistency of phonon-band character and breaking of simple phonon picture

In the theory of the intrinsic excitations in spherical odd-mass nuclei, which is formulated in Chap. 2, the original $P+QQ$ Hamiltonian is transcribed into the "quasi-particle NTD space" as follows:

$$\left.\begin{aligned} &\boldsymbol{H}=\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}, \\ &\boldsymbol{H}^{(0)}=\mathbf{1}\cdot(H_0+H_X+H_V)\cdot\mathbf{1}=\sum_{\delta} E_\delta \boldsymbol{a}_\delta^\dagger \boldsymbol{a}_\delta+\sum_{nIK}\omega_{nI}\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{Y}_{nIK}, \\ &\boldsymbol{H}^{(\mathrm{int})}=\mathbf{1}\cdot H_V\cdot\mathbf{1}=\sum_{\delta,nIK} V_{\mathrm{int}}(\delta, nI)\{\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{a}_\delta+\boldsymbol{a}_\delta^\dagger \boldsymbol{Y}_{nIK}\}, \end{aligned}\right\} \quad (3\cdot1)$$

where $\boldsymbol{a}_\delta^\dagger$ and $\boldsymbol{Y}_{nIK}^\dagger$ denote the creation operators of the 1QP and dressed 3QP modes in the space, respectively. Here we have adopted the projection operator onto the "quasi-particle NTD subspace," $\mathbf{1}$, by which the modes with trans-

ferred seniority higher than three are neglected.

In the same way as in the conventional QPC theory, we have a free Hamiltonian $\boldsymbol{H}^{(0)}$ for the new type of elementary excitation modes if the (original) interactive force $H_Y$ is neglected. However, since the collective 3QP correlations have already been taken into account in constructing the dressed 3QP modes, the spectra given by $\boldsymbol{H}^{(0)}$ now acquire abundant structures. The dressed 3QP mode can of course be decomposed into a phonon coupled with an odd quasi-particle in the limit where various 3QP correlation diagrams are all neglected. Hence, by comparing the characteristics of the low-energy excitation structures given by $\boldsymbol{H}^{(0)}$ with those of $\mathcal{H}^{(0)}$, we can see the breaking and persistency of the phonon-band in the QPC picture due to the collective 3QP correlations.

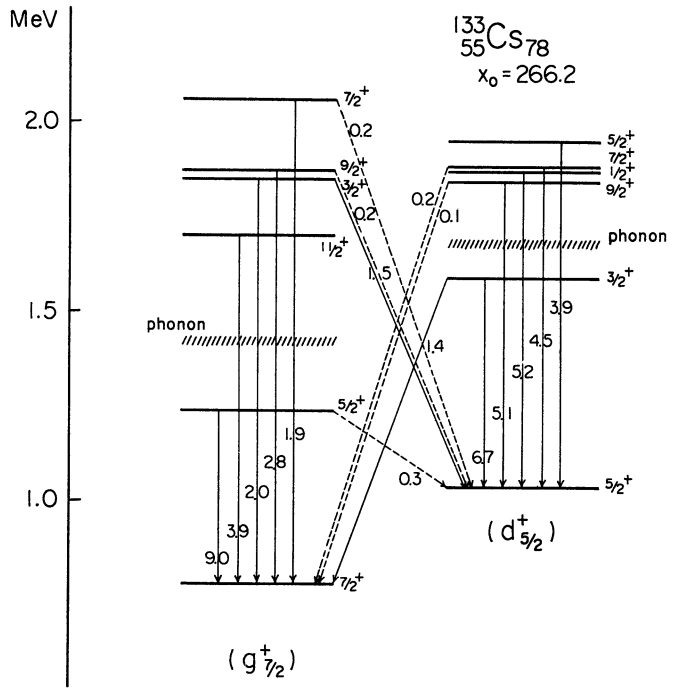



Fig. 4(a). Result of the calculations for the dressed 3QP states in $^{133}$Cs. They are presented to show the breaking and persistency of the quintet structures based on the 1QP states with orbits $1g_{7/2}$ and $2d_{5/2}$. The presented level energies are those measured from the correlated ground state. The numbers appearing on the transition arrows give the $B(E2)$ values in unit of $e^2 10^{-50}$ cm$^4$, which are calculated with polarization charge $\delta e = 0.5e$ and with harmonic-oscillator-range parameter $b^2 = 1.0A^{1/3}$. The adopted value of $\chi_0$ is related with the quadrupole-force strength $\chi$ through $\chi_0 = \chi b^4 A^{5/3}$ (MeV). For simplicity, the $E2$ transitions smaller than 0.1 and the other higher-lying states are both omitted from the figure. The parameters of the shell-model space used in this calculation are the same as those adopted by Kisslinger and Sorensen.[1]

With this aim, the calculated excitation spectra for odd-proton $^{133}$Cs and $^{135}$La are presented in Fig. 4. In this figure, the calculated $B(E2)$ values are also written on the transition arrows. Method of calculations and the adopted parameters are the same as are described in §2-Chap. 4. The quadrupole-force strengths are fixed so as to reproduce, by means of the RPA, the average energies of the $2^+$ phonon states in the adjacent even-even nuclei. It should be noted here that the numerical examples are presented for $^{133}$Cs and $^{135}$La whose even-even neighbours are considered as exhibiting vibrational spectra.

The format of Fig. 4 is made so that the relationship to the spectrum characterized by the concept of phonon-band is visible. In odd-proton $^{133}$Cs and $^{135}$La, the $1g_{7/2}$ and $2d_{5/2}$ orbits lie near the chemical potential of protons. In this figure we are able to identify two families of states which belong to the phonon-band based on the $1g_{7/2}$- and the $2d_{5/2}$-1QP states, respectively. Needless to say, the two "quintets" (composed of the $2^+$ phonon coupled with the $g_{7/2}$- and $d_{5/2}$-odd-quasi-particle, respectively) should be degenerated in energy in the hatched regions, *if we neglect the collective* $3QP$ *correlations completely*.

From Fig. 4, we can see the following characteristics:

(1) The energy-splittings of the quintets are very large, i.e., the level structure

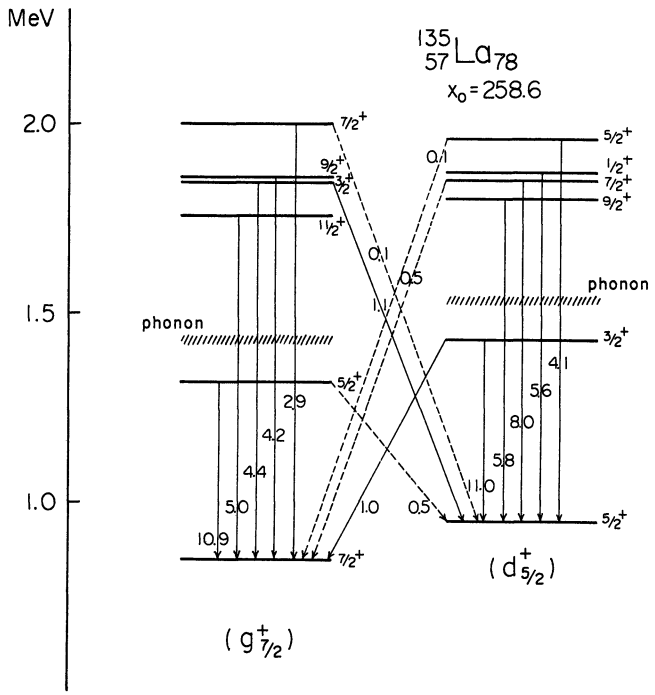



Fig. 4(b). Result of calculation for the dressed 3QP states in $^{135}$La. Notations and parameters used are the same as in Fig. 4(a).

shows a drastic change from that of the simple phonon picture. The magnitudes of the splittings are comparable to (or even larger than) the excitation energies of phonons themselves. Clearly we are in a situation far from the zeroth-order picture of the QPC theory. Of course, from the viewpoint of the boson expansion methods, this fact implies that the couplings between the ideal-odd-quasi-particles and the ideal-phonons are too strong to be treated within a perturbational method.

(2) The splitting of the quintet which belongs to the ground band is larger than that of the excited band. The magnitude of the splitting decreases as the excitation energy of the band-head 1QP state becomes higher.

(3) Corresponding to the changes in level structure, the $E2$-transition probabilities (from the quintet to their band-head) also become different among the members of the quintet. As a gross property, the lower the excitation energy of the level, the larger the $B(E2)$ value.

(4) In each quintet with band-head spin $j$, the sum of $B(E2)$, $\sum_I B(E2; I\to j)$, becomes smaller than $5\times B(E2; 2^+\to 0^+)$ of the phonon transition calculated by means of the RPA. (In the phenomenological core-excitation model, we have $\sum_I B(E2; I\to j)=5\times B(E2; 2^+\to 0^+)$.)

In spite of these drastic changes of the excitation structure which evidently show the breaking of the simple phonon picture, we can still find the following characteristic:

(5) The property (characterizing the concept of phonon-band) that the inter $E2$ transitions are hindered compared to the intra $E2$ transitions is seen to persist rather well (aside from the $3/2^+$ states of $^{133}$Cs in which the inter-transitions compete with the intra-transitions).

In the region of Cs and La isotopes, the low-energy-excitation structure is determined mainly by the competitions among the three effects; effect A (shown in Fig. 3-A) in the orbit $g_{7/2}$, effect A in the orbit $d_{5/2}$ and effect B (shown in Fig. 3-B) involving the orbits $g_{7/2}$ and $d_{5/2}$. Therefore, characteristic (5) suggests that, according to the criterion given in §2, effects A are dominant to effect B.

We can find the origin of this trend as follows: In the 3QP correlations among quasi-particles in different orbits (the effects B), the one which couples the $g_{7/2}$-band to the $d_{5/2}$-band contains, as a major part, the spin-flip matrix element $(d_{5/2}\|r^2Y_2\|g_{7/2})$ which is considerably smaller than the diagonal matrix elements, $(g_{7/2}\|r^2Y_2\|g_{7/2})$ and $(d_{5/2}\|r^2Y_2\|d_{5/2})$, contributing to effects A. Therefore, in spite of the drastic breaking of the simple phonon picture mentioned as characteristics (1)~(4), the concept of phonon-band is expected to persist in such a situation for shell structure. It is also interesting to recall that this condition of shell structure is common to that for the appearance of the dressed 3QP modes having ACS-like structure, e.g., the $5/2_2^+$ states in Cs and La isotopes.

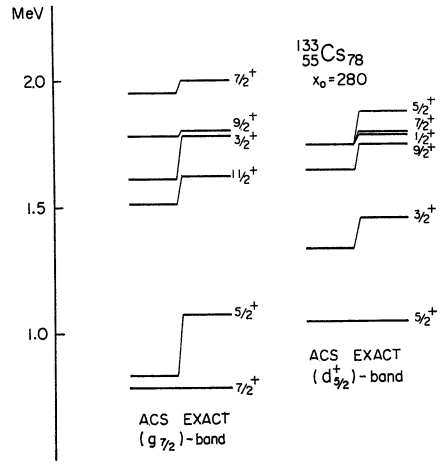



Fig. 5. Comparison between the result of exact calculation ("EXACT") and that of approximated one ("ACS") for the dressed 3QP states in $^{133}$Cs. See the text for interpretation.

Figure 5 has been made to show the dominant role of effect A. Here the result of the exact calculation for the dressed 3QP modes is compared with that of the approximated one in which effects B are completely neglected. When effects B are neglected, the eigenvalue equation for the dressed 3QP modes is reduced to Eq. (3·23) of Chap. 3 with the orbit $p$ now denoting the orbit of each band-head 1QP mode. We call such an approximation as "single-band approximation" or "ACS approximation." From Fig. 5, we can see that the excitation structure is determined in a major way by effects A, that is, the characteristics coming from effects A persist clearly even when effects B are included.

Then, as shown in §3-Chap. 3, the splitting of the multiplet depends on three factors; (i) the enhancement factor $u_jv_j$ in the orbit $j$ of the band-head 1QP mode, (ii) the enhancement factors $(u_bv_c+v_bu_c)$ in the core and (iii) the value of spin $j$ of the band-head 1QP mode which is involved in the 3QP-correlation factor defined by Eq. (3·22) of Chap. 3.

Among these, an important consequence of the effect (i) is seen as the characteristic property (2) mentioned above. Remembering the fact that the more the orbit $j$ of the odd quasi-particle becomes close to the chemical potential $\lambda$ the larger the $u_jv_j$ factor becomes, we can easily understand the origin of this property. Thus the role of the 3QP correlations, especially the one among quasi-particles in the same orbit lying near the chemical potential, is essential to determine the low-energy-excitation structure and becomes less important for high-energy excitations.

In concluding this section, however, it should also be stressed that the competition between effects A and B depends rather sensitively on the spin of the dressed 3QP mode*) and the quasi-particle-energy difference between

*) For instance, notice that effect A in the orbit $d_{5/2}$ is forbidden for the modes with spins 1/2, 5/2 and 7/2.

the shell-model orbits of interest. For instance, the breaking of the band structure for the specific states with spin $3/2^+$ in Cs isotopes can be understood to be a result of the balance of these effects. In fact, by comparing Fig. 4(a) with Fig. 4(b), we can see that the band character for the $3/2^+$ states is enhanced in La isotopes where effect A in the orbit $d_{5/2}$ becomes dominant for the first $3/2^+$ state (i.e., the ACS-like structure of the $3/2^+_1$ state is realized).

## §4. Roles of correlation between proton- and neutron-quasi-particles

In this section, we investigate the microscopic structure of the dressed 3QP modes from the viewpoint of correlation between proton- and neutron-quasi-particles. At first, it should be mentioned that a strong correlation between proton- and neutron-quasi-particles is implicitly assumed in setting up the criterion given in §2. When we consider the difference in the effective charges of protons and neutrons, it is easy to understand that, without this strong correlation, the criterion given in conjunction with $E2$-transition property cannot be applied irrespective of odd-proton or odd-neutron nuclei. The reason is based on the fact that the motions of proton-quasi-particle-pairs and of neutron-quasi-particle-pairs are coupled strongly with each other due

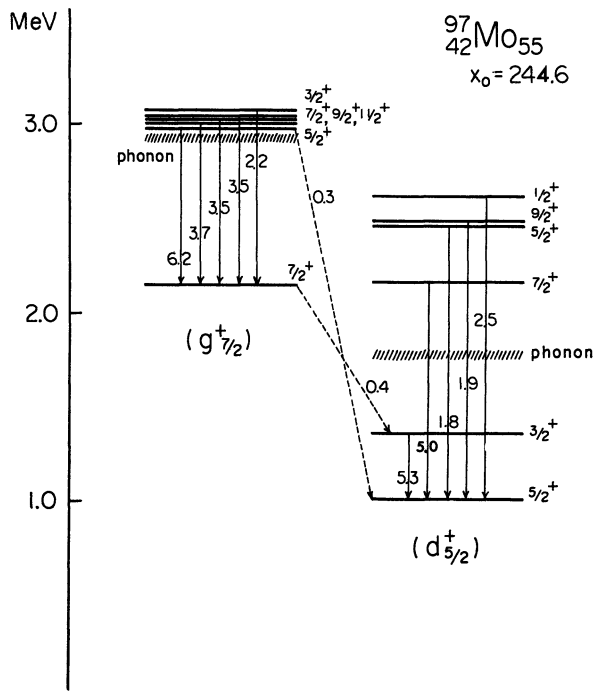



Fig. 6(a). Result of calculation for the dressed 3QP states in $^{97}$Mo. Notations and parameters used are the same as in Fig. 4(a).

to the $QQ$ force and the ground-state correlations play a role to enhance the cooperative effect between the motions of proton- and neutron-quasi-particle-pairs. This situation is just the same as in the case of the $2^+$ phonon modes (in the doubly open even-even nuclei) described by means of the RPA with the $P+QQ$ force. Hence, as long as the shell structures in the vicinity of the chemical potential (for odd-number nucleons) are the same, we expect similar $E2$-transition property irrespective of odd-proton or odd-neutron nuclei.

Figure 6 shows the calculated results for odd-neutron $^{97}$Mo and $^{105}$Pd nuclei. The orbits participating most actively in the 3QP correlations in $^{97}$Mo and $^{105}$Pd are the $2d_{5/2}$ and $1g_{7/2}$ orbits which are the same as in $^{133}$Cs and $^{135}$La nuclei discussed in §3. By comparing the excitation structure shown in Fig. 6 with that of the odd-proton $^{133}$Cs and $^{135}$La nuclei shown in Fig. 4, we can see that the characteristics similar to those described as (1)~(5) in §3 hold also in the case of odd-neutron $^{97}$Mo and $^{105}$Pd nuclei. For $^{105}$Pd nucleus, the picture of core-excitation model[7] has sometimes been used in interpreting

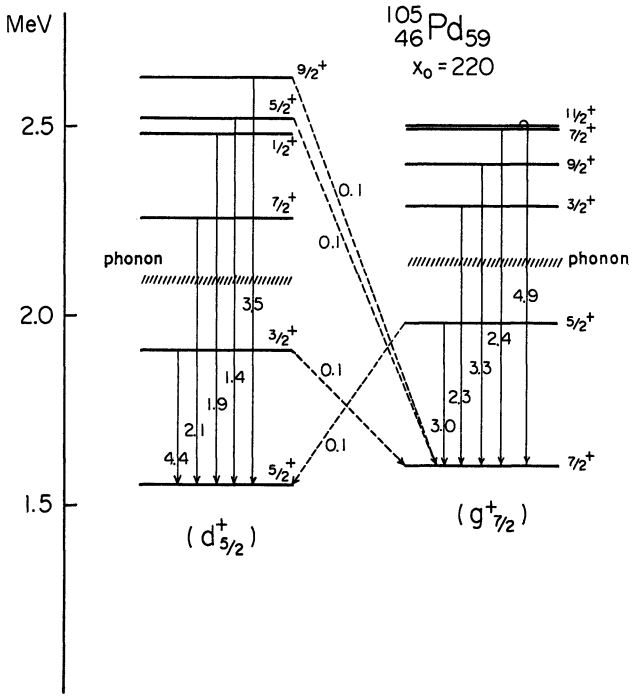



Fig. 6(b). Result of calculation for the dressed 3QP states in $^{105}$Pd. Notations and parameters used are the same as in Fig. 4(a) except the following:
neutron single-particle energies;
$\epsilon(d_{5/2})=0.0$, $\epsilon(g_{7/2})=1.6$, $\epsilon(s_{1/2})=1.9$,
$\epsilon(h_{11/2})=2.0$ and $\epsilon(d_{3/2})=2.5$.
pairing-force strengths;
$G_p=29/A$ and $G_n=21/A$. (all in MeV)

the experimental data.[8),9)] From the present point of view, however, it is clear that such experimental facts can be considered to reflect the persistency of phonon-band character in $^{105}$Pd and never to imply a realization of the weak coupling picture underlying the core-excitation model.

Now let us consider in more detail the role of proton-quasi-particles in the dressed 3QP modes in odd-neutron nuclei. (The same consideration can also be made for the role of neutron-quasi-particles in odd-proton nuclei.) As was already emphasized, the 3QP correlations can be regarded, in the language of quasi-particle-phonon-coupling picture, as arising from Pauli principle between the neutron-odd-quasi-particle and the quasi-particles constructing the phonon. Consequently, because of the absence of Pauli principle between proton- and neutron-quasi-particles, the triggering effects of the 3QP correlations (which violate the concept of phonon in odd-mass nuclei) are restricted within the neutron-quasi-particles. The following fact should be noted however. In the dressed 3QP mode, the component $\psi_{nI}(\rho\sigma;\alpha)$ and $\varphi_{nI}(\rho\sigma;\alpha)$,*) composed of proton-quasi-particle-pair $(\rho\sigma)$ plus a neutron-quasi-particle $\alpha$ *in the orbit a, can couple* with the component of the same type, $\psi_{nI}(\rho\sigma;\beta)$ and $\varphi_{nI}(\rho\sigma;\beta)$, having the neutron-quasi-particle $\beta$ *in the orbit b* different from the orbit $a(b\neq a)$.**) This kind of "coupling" is mediated by the 3QP correlations among neutron-quasi-particles. (See Fig. 7.) Since the motion of proton-quasi-particles couples strongly with that of neutron-quasi-particles (through the $QQ$ force), the magnitude of the mixing among the components, $\psi_{nI}(\rho\sigma;\alpha)$ and $\varphi_{nI}(\rho\sigma;\alpha)$ with various $a$, depends quite obediently on the magnitude of effects B in the 3QP correlations among neutrons. In this way, the fact that there exists a strong correlation between proton- and neutron-quasi-particles makes it possible to say as follows: "If effects B in the 3QP correlations are sufficiently strong among neutron-quasi-particles, then we can observe the breaking of the phonon-band structure in terms of the $E2$-transition properties." We can confirm this point from the comparison between Fig. 6(b) and Fig. 8(b). As will be discussed in §5, effect B in odd-neutron $^{115}$Cd nucleus is considerably stronger than that in odd-neutron $^{105}$Pd nucleus. As a con-

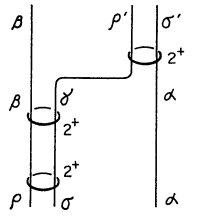



Fig. 7. Illustration of the 3QP correlation discussed in the text. Here, $\alpha$, $\beta$, $\gamma$, .... denote the quantum numbers of the single-particle states for neutrons and $\rho$, $\sigma$, .... those for protons.

*) For definition of the amplitudes of the dressed 3QP mode, see Eq. (2.1) in Chap. 4.

**) Within the limit of neglecting the 3QP correlations, these two sets of components belong to different multiplets in the QPC theory, $(\dot{a}^{\dagger}_{a}\Gamma^{\dagger}_{2M})_{IK}|0)$ and $(\dot{a}^{\dagger}_{\beta}\Gamma^{\dagger}_{2M})_{IK}|0)$, characterized by the quantum numbers $a$ and $b$, respectively.

sequence, we can see that the inter-band transitions in $^{115}$Cd are larger than those in $^{105}$Pd.

Now, recalling the enhancement factor (ii) mentioned in §3 (which is applicable for both cases A and B, of the 3QP correlations), we can say as follows: The major role of the proton-quasi-particles in the odd-neutron dressed 3QP mode is to enhance the collectivity of the mode, accompanying a rapid growth of the ground-state correlation. The growth of the collectivity due to such an effect is clearly seen by comparing the spectrum of single-closed-shell nucleus $^{117}$Sn (Fig. 8(a)) with that of $^{115}$Cd (Fig. 8(b)) in which two proton-holes are added to $^{117}$Sn. Thus, although the proton-quasi-particles do not produce any 3QP correlations by themselves, they play an indispensable role to form the concept of dressed 3QP mode (in odd-neutron nuclei) as a collective mode of excitation.

## §5. Case of low-spin orbits

Let us consider the calculated result for $^{115}$Cd shown in Fig. 8(b). In this nucleus, the chemical potential for neutrons lies in the vicinity of the orbits $3s_{1/2}$ and $2d_{3/2}$. Since effects A are strictly forbidden in the orbit with spin

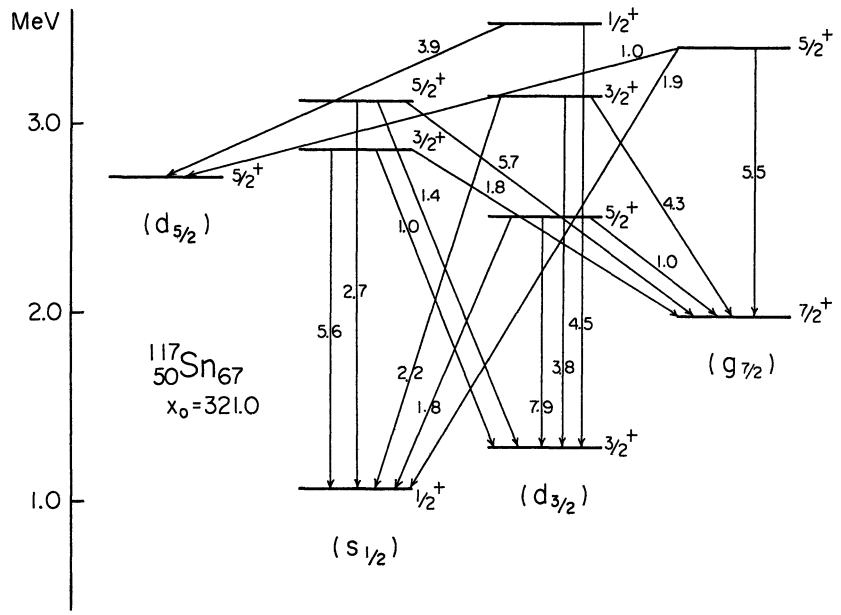



Fig. 8(a). Result of calculation for the dressed 3QP states in $^{117}$Sn. Notations and parameters used are the same as in Fig. 4(a) except the following:
neutron single-particle energies;
$\epsilon(d_{5/2})=0.0$, $\epsilon(g_{7/2})=1.27$, $\epsilon(s_{1/2})=2.55$,
$\epsilon(h_{11/2})=3.25$, $\epsilon(d_{3/2})=3.24$. (all in MeV)
These values are taken from Ref. 10). The unit of $B(E2)$ values is $e^2\cdot10^{-51}$ cm$^4$.

$j<5/2$, effects B are expected to manifest themselves, in a relatively pure form, in the dressed 3QP modes which largely involve the low-spin orbits such as $3s_{1/2}$ and $2d_{3/2}$. In fact, Fig. 8(b) shows that we cannot definitely classify these dressed 3QP modes in terms of the criterion given in §2, since the many inter-band transitions are of the same order in magnitude with the intra-band transitions.*) This implies that, for the collective excitations standing on the 1QP states with low-spin ($j<5/2$) and with normal parity, the concept of phonon-band is broken down completely due to effect B.

It is noticeable in Fig. 8(b) that the "doublet" with spins $3/2^+$ and $5/2^+$, belonging to the $s_{1/2}$ phonon-band, is considerably shifted up in energy. The reason is understood as follows: In nuclei in which the $s_{1/2}$ orbit lies close to the chemical potential, the $2^+$ phonon is largely composed of the quasi-particle-pair involving the $s_{1/2}$ quasi-particle. When the odd quasi-particle is lying just at the $s_{1/2}$ orbit, however, the excitations of such quasi-particle-pairs are strictly forbidden. This is easily understood when we recall the fact that the 3QP configurations with seniority $v=3$ are forbidden due to the Pauli principle if there exist *two* quasi-particles at the $s_{1/2}$ orbit. Thus, the excitation of the

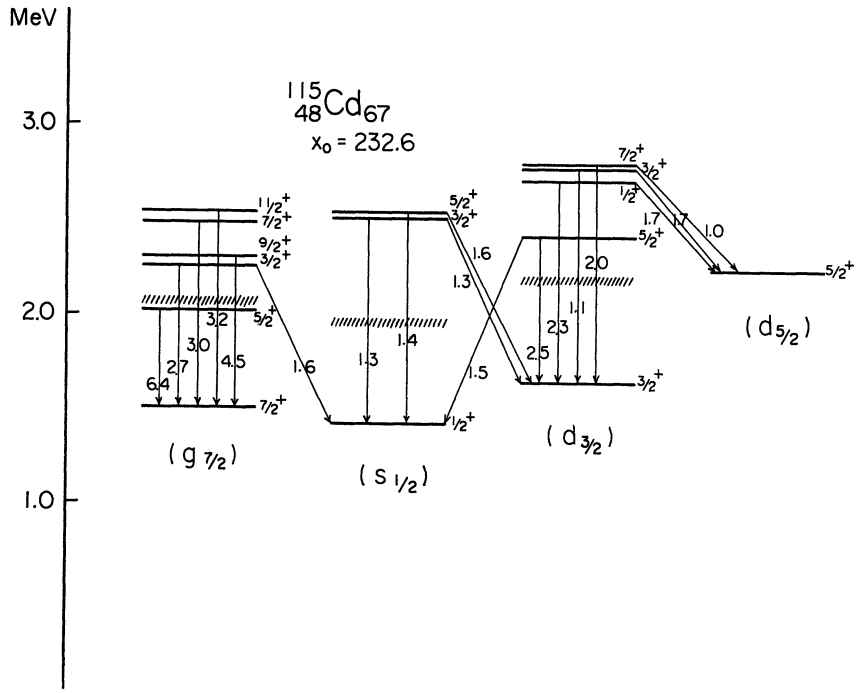



Fig. 8(b). Result of calculation for the dressed 3QP states in $^{115}$Cd. Notations and parameters are the same as in Fig. 4(a).

*) Since the phonon-bands are difficult to identify in this case, the dressed 3QP states are classified in a rather arbitrary way in Fig. 8.

$2^+$ phonon of the core is highly hindered when there already exists the $s_{1/2}$ odd quasi-particle. Furthermore, in this case, we can expect the following trend: The microscopic structure of the $2^+$ phonon itself should be changed drastically, since the main components of the $2^+$ phonon are forbidden. As a consequence, the 3QP correlation among quasi-particles in different orbits tends to play an increasingly important role.

We have seen two typical examples in which either effect A or effect B is playing an essential role to govern the low-energy-excitation spectrum: In nuclei in which the orbits with spin $j \geqq 5/2$, such as $d_{5/2}$ and $g_{7/2}$, lie in the vicinity of the chemical potential, the 3QP correlation in the same orbit plays a dominant role (case A). On the other hand, in nuclei in which the orbits with spin $j<5/2$, such as $s_{1/2}$ and $d_{3/2}$, lie in the vicinity of the chemical potential, the 3QP correlation among different orbits plays a dominant role (case B). The degree of the breaking of the phonon-band character is determined by the competition between effects A and B. Although present accumulation of the experimental

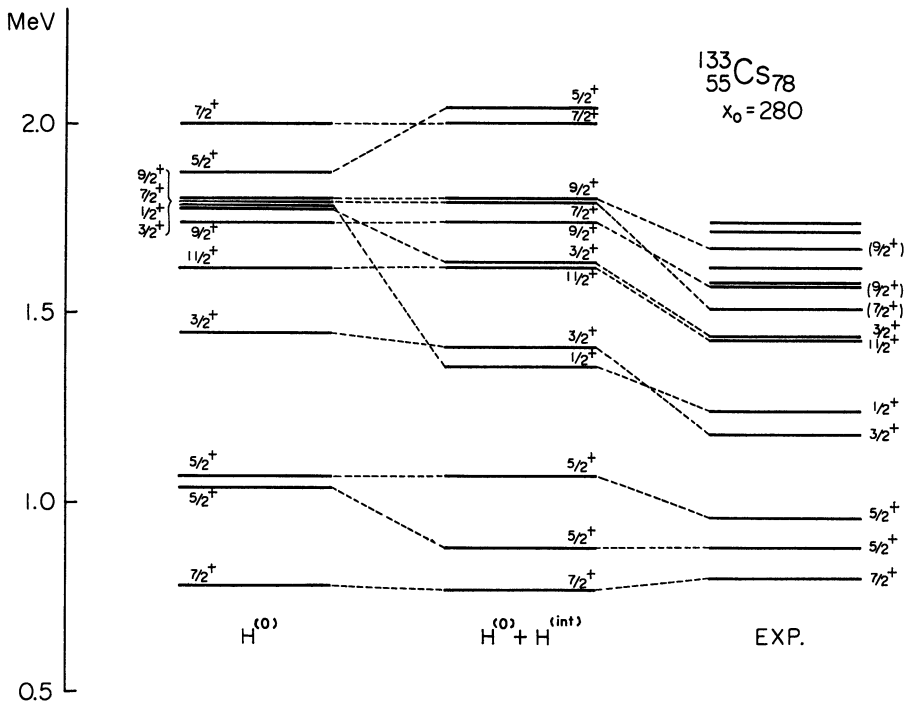



Fig. 9(a). Energy shifts due to the coupling effects between the dressed 3QP- and 1QP-modes in $^{133}$Cs. The energy levels denoted by $\boldsymbol{H}^{(0)}$ show the result calculated by neglecting the coupling effects, while those denoted by $\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}$ show the result calculated by taking the coupling effects into account. The experimental energy levels denoted by EXP are taken from Ref. 11). The parameters of the calculations are the same as in Fig. 4(a) except that the adopted quadrupole-force parameter $\chi_0$ is a little stronger.

data on the $E2$ transitions between excited states is not sufficient to allow us a systematic comparison with the theoretical calculations, the current rapid progress in the measurement of these transitions is expected to elucidate further many interesting structures of the 3QP correlations.

## §6. Couplings between dressed 3QP- and 1QP-modes

So far, we have neglected the effects originating from the interactive force $H_Y$. The essential role of this type of interactions is to produce couplings between different kinds of elementary excitation modes. In the QPC theory, the effects are represented by $\mathcal{H}^{(\mathrm{int})}$ in Eq. (2·2), which change the number of phonons by one accompanying a scattering of odd quasi-particle. On the other hand, in the theory developed in Chap. 2, the effects manifest themselves as couplings between the dressed 3QP modes and the 1QP modes, $\boldsymbol{H}^{(\mathrm{int})}$ in Eq. (3·1), in the quasi-particle NTD subspace. Since in the QPC theory, the change of excitation spectrum from that given by $\mathcal{H}^{(0)}$ is attributed entirely to this special type of couplings $\mathcal{H}^{(\mathrm{int})}$, and also since the low-lying spectrum given

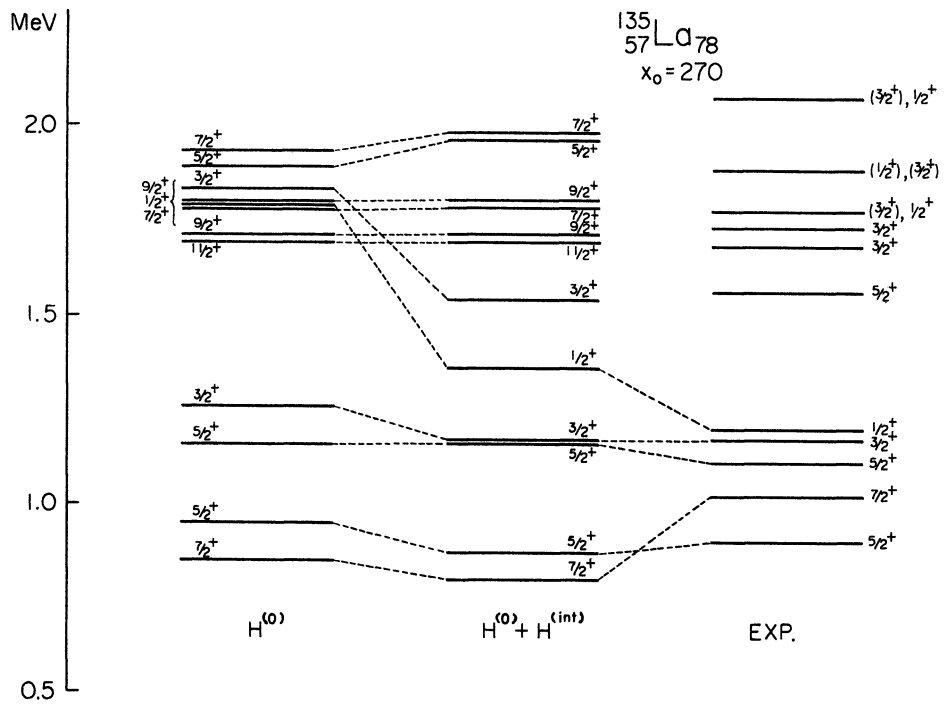



Fig. 9(b). Energy shifts due to the coupling effects between the dressed 3QP- and 1QP-modes in $^{135}$La. Notations are the same as in Fig. 9(a). The experimental energy levels are taken from Ref. 12). The parameters of the calculation are the same as in Fig. 4(b) except that the adopted quadrupole-force parameter $\chi_0$ is a little stronger.

by $\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}$ corresponds to that given by $\mathcal{H}^{(0)}+\mathcal{H}^{(\mathrm{int})}$ *if we neglect the 3QP correlations completely,* the problem of how the effects of $H_Y$ are changed (from those evaluated in the QPC theory) by the inclusion of the 3QP correlations will be of great significance.

Figure 9 and Tables I and II show the calculated results for $^{133}$Cs and $^{135}$La. From the comparison between the spectrum of $\boldsymbol{H}^{(0)}$ and that of $\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}$, we can see that $\boldsymbol{H}^{(\mathrm{int})}$ does not change the low-energy-excitation spectrum given by $\boldsymbol{H}^{(0)}$ so drastically, except for some states with low-spins $1/2^+$ and $3/2^+$. This is because of the newly arised reduction effect which is absent in the QPC theory and makes the effects of $H_Y$ interaction to be less important for low-lying states.

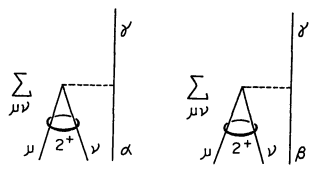
The mechanism of this reduction effect can be understood as follows: Let us consider two sets of diagrams (with $a \neq b$) shown in Fig. 10. In the QPC theory, each sum of the diagrams, Figs. 10(a) and 10(b), contributes *separately* to the effective coupling strength, $\bar{\chi}_{\delta a}(\nu)$ or $\bar{\chi}_{\delta\beta}(\nu)$, in $\mathcal{H}^{(\mathrm{int})}$. However, when we take the 3QP correlations into account, these two sets of diagrams *both* contribute to the *single* effective coupling strength $V_{\mathrm{int}}(d; nI)$ in $\boldsymbol{H}^{(\mathrm{int})}$. As is seen from the expression of $V_{\mathrm{int}}(d; nI)$ given by (4·3) in

Table I. Calculated $B(E2)$ values for $^{133}$Cs. The states in the first column are labeled according to the level ordering given in Fig. 9(a). The parameters used are the same as in Fig. 9(a). The $B(E2)$ values calculated by neglecting the coupling effects are listed in the second column, while those calculated by taking account of the coupling effects are listed in the third column. They are compared to the experimental values listed in the fourth column. The unit is $e^2\cdot 10^{-50}\,\mathrm{cm}^4$. The polarization charge $\delta e=0.5e$ and the harmonic-oscillater-range parameter $b^2=1.0A^{1/3}$ are used in the calculation. Experimental data are taken from Ref. 11).

| transitions | $B(E2)^{1)}$ | $B(E2)^{2)}$ | $B(E2)^{\mathrm{exp}}$ |
|---|---|---|---|
| $5/2_2^+ \to 7/2_1^+$ | 11.69 | 11.62 | 10.4±1.2 |
| $11/2_1^+ \to 7/2_1^+$ | 4.25 | 4.21 | 10.0±1.1 |
| $3/2_2^+ \to 7/2_1^+$ | 2.41 | 3.83 | 1.4±0.2 |
| $9/2_2^+ \to 7/2_1^+$ | 3.09 | 3.06 | 7.4±0.8 |
| $7/2_3^+ \to 7/2_1^+$ | 1.94 | 1.90 | 1.42±0.17 |
| $3/2_1^+ \to 5/2_1^+$ | 9.48 | 9.52 | |
| $9/2_1^+ \to 5/2_1^+$ | 5.94 | 5.10 | |
| $1/2_1^+ \to 5/2_1^+$ | 5.89 | 7.27 | |
| $7/2_2^+ \to 5/2_1^+$ | 5.24 | 4.46 | |
| $5/2_3^+ \to 5/2_1^+$ | 4.28 | 1.42 | |
| $3/2_2^+ \to 5/2_1^+$ | 1.27 | 0.11 | |
| $3/2_1^+ \to 7/2_1^+$ | 1.33 | 0.31 | 7.2±0.8 |
| $5/2_2^+ \to 5/2_1^+$ | 0.46 | 0.35 | |
| $5/2_1^+ \to 7/2_1^+$ | 0.02 | 0.27 | |

Table II. Calculated $B(E2)$ values for $^{135}$La. Notations and the parameters used are the same as in Table I and Fig. 9(b), respectively. Experimental data are taken from Ref. 12).

| transitions | $B(E2)^{1)}$ | $B(E2)^{2)}$ | $B(E2)^{exp}$ |
|---|---|---|---|
| $5/2_2^+ \to 7/2_1^+$ | 18.86 | 18.13 | 22.6 |
| $11/2_1^+ \to 7/2_1^+$ | 5.45 | 5.21 | |
| $3/2_2^+ \to 7/2_1^+$ | 5.47 | 7.47 | |
| $9/2_2^+ \to 7/2_1^+$ | 4.55 | 4.35 | |
| $7/2_3^+ \to 7/2_1^+$ | 4.16 | 1.37 | |
| $3/2_1^+ \to 5/2_1^+$ | 15.41 | 14.09 | $\geqq$4.9 |
| $9/2_1^+ \to 5/2_1^+$ | 6.46 | 5.79 | |
| $7/2_2^+ \to 5/2_1^+$ | 5.78 | 5.18 | |
| $1/2_1^+ \to 5/2_1^+$ | 6.07 | 7.68 | $\geqq$20.2 |
| $5/2_3^+ \to 5/2_1^+$ | 4.71 | 2.85 | |
| $3/2_2^+ \to 5/2_1^+$ | 0.62 | 0.32 | |
| $3/2_1^+ \to 7/2_1^+$ | 1.15 | 0.002 | $\geqq$0.1 |
| $5/2_2^+ \to 5/2_1^+$ | 0.92 | 0.25 | 1.7 |
| $5/2_1^+ \to 7/2_1^+$ | 0.003 | 0.31 | 1.8 |




(a) (b)

Fig. 10. Two sets of the matrix elements of interactive force $H_Y$. The sets (a) and (b) are distinguished with each other by the difference in one of the shell-model orbits, i.e., $a \neq b$. The quantum numbers written in this figure, $\alpha \equiv (a, m_a) \equiv (n_a l_a j_a, m_a)$, $\beta, \gamma, \ldots$ can be interpreted as; for instance, $\alpha$, $\beta$ and $\gamma$ denote the quantum numbers of the single-particle states for neutrons and $\mu$ and $\nu$ those for protons. The diagrams in the sets (a) and (b) contribute separately to the effective coupling strength $\bar{\chi}_{\gamma\alpha}$ and $\bar{\chi}_{\gamma\beta}$, respectively, in the QPC theory. On the other hand, they both contribute simultaneously to the effective coupling strength $V_{\text{int}}(c; nI)$ in the proposed theory.

Chap. 4, the phase relations between the matrix elements belonging to different sets of diagrams (each of which is represented in Fig. 10(a) or Fig. 10(b)) are governed by the relative phases of the amplitudes, e.g., $\psi_{nI}[rs(2)a]$ and $\psi_{nI}[rs(2)b]$, and also by the quadrupole matrix elements, $R(ad) \equiv 5^{-1/2} (a \| r^2 Y_2 \| d) \cdot (u_a u_d - v_a v_d)$ and $R(bd)$. Consequently, even when all the diagrams belonging to a single set (with definite $a$) contribute in phase, we have no guarantee of the coherent property among different sets of diagrams (with various $a$). In fact, the calculated results show that they contribute generally

*in random phase,* namely, they cancell one another. This is the case especially for the lowest-lying mode with a given spin $I>1/2$.

Additional reasons to weaken the effective coupling strength $V_{\text{int}}(d; nI)$ were already stated in §4-Chap. 4.

Now let us recall the characteristic dependence of the $H_Y$ interaction on the reduction factor, $(u_a u_{\bar{a}}-v_a v_{\bar{a}})$. This factor is *small* for the quasi-particles lying near the chemical potential. In the special cases where a pair of orbits $a$ and $d$ are occupied just so as to make the factor nearly zero, e.g., each just half full, the effect of $H_Y$ vanishes. Within the framework of the QPC theory, a consequence of such a property has indeed been confirmed experimentarily in the $E2$-transition properties between the low-lying states which are both mainly composed of the 1QP states.[6] Since this property of $H_Y$ is endowed to the effective coupling strengths, both in the QPC theory and in the present theory, it is evident that the latter conserves the major success of the former. Furthermore, in the proposed theory, we have the (afore-mentioned) new reduction effect originated from the 3QP correlations which depend on the enhancement factor $(u_a v_{\bar{a}}+v_a u_{\bar{a}})$ becoming large in the neighbourhood of the chemical potential. Obviously, this fact magnifies the above-mentioned property of reducing the $H_Y$ effect.

Thus we can say that the couplings of the dressed 3QP modes to the 1QP modes are significantly hindered *if they are both in low-lying states near the ground state.* This fact is in accord with the general principle: "If eigenmodes were properly chosen, couplings to different eigenmodes should be weak."

On the other hand, for the higher excitations, the (above-mentioned) mechanism becomes less effective. Thereby, if the 1QP modes lie in higher excited states, their couplings to the dressed 3QP modes (lying below them) become relatively significant. This is the case for the $1/2^+$ and $3/2^+$ states in $^{133}$Cs and $^{135}$La nuclei shown in Fig. 9.

A tentative comparison between the calculated results and the experimental data (Fig. 9) shows that the proposed theory can reproduce qualitative characteristics of the low-energy excitations quite well. Here it should be stressed that we have adopted no systematical fitting-procedures by adjusting parameters. Therefore, in view of the rapid accumulation of experimental data, more detailed analysis based on the proposed theory should be very promising.

We conclude this section by observing the following point which is exceptional to this promising results of calculations: Although level structures are nicely reproduced, the calculated excitation energies of the dressed 3QP states are, in average, higher than the corresponding experimental ones, if we choose the strength of the $QQ$ force so as to fit the average energy of $2^+$ phonons in the adjacent even-even nuclei, $\bar{\omega}_{2^+}(N,Z)=1/2\{\omega_{2^+}(N,Z-1)+\omega_{2^+}(N,Z+1)\}$.

This rather general tendency may be due to the present limited truncation of the quasi-particle NTD space, that is, the neglect of modes with transferred seniority higher than three. In addition to this, the following fact should also be emphasized: What we have discussed up to now is the intrinsic excitation modes in the quasi-spin space, and the couplings between intrinsic- and (pairing-) collective-modes have been completely neglected.

## §7. Concluding remarks

Microscopic structure of breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture" has been investigated by putting special emphasis on the deviations of $E2$-transition properties from those expected by the concept of phonon-band. It has been demonstrated that the simple phonon picture for spherical odd-mass nuclei is seriously broken due to the collective 3QP correlations among quasi-particles lying near the chemical potential. In particular, the breaking and persistency of the phonon-band character have been shown to be essentially dependent on the characteristics of the 3QP correlation. The effect of $H_V$ interaction has also been shown to be significantly affected by the inclusion of this correlation. The microscopic structure of the 3QP correlation depends, in turn, on details of the shell structure in the vicinity of the chemical potential. Accordingly, results of the calculation have been exemplified for two classes of nuclei in which either high-spin or low-spin orbits lie near the chemical potential. From these investigations, it is now clear that the 3QP correlation should be regarded as an elementary correlation in low-energy excitations. In fact, a large body of experimental data illuminating rich aspects of the many-quasi-particle correlations is now accumulating. (See, for instance, the progress report by Meyer.[13])

The effects of the 3QP correlation (based on the Pauli principle between the odd quasi-particle and the quasi-particles composing the phonon) have so far been neglected by the argument that a phonon contains only a small amplitude for the presence of any particular quasi-particle.[6] However, this argument is not correct. The 3QP correlation is essentially different from the "static" effects such as the blocking effect. Rather, it is a "dynamical" correlation induced by the presence of the odd quasi-particle: In such low-energy-excitation mode as the $2^+$ phonon, as is well known, the quasi-particles lying near the chemical potential play an essential role in constructing the mode. When the "odd quasi-particle" is also added near the chemical potential, the quadrupole force ($H_X$ and $H_V$) acts upon both the quasi-particles constructing the $2^+$ phonon and the "odd quasi-particle" without discrimination. Therefore, the collective 3QP correlation also tends to grow significantly, as the collective 2QP correlation becomes stronger (,i.e., as the excitation energy of the $2^+$ phonon becomes smaller). What we have investigated from Chap. 3

to this chapter can be regarded as clarifying the actual physical situation for this process. The conclusions obtained here are, therefore, closely connected with the dynamics of the $P+QQ$ force model.[6] Accordingly, the problem whether they are specific to the $P+QQ$ force model or more general will be examined in the succeeding chapter.

Of course, these conclusions do not exclude a possibility of a decomposition among many-quasi-particles if, e.g., some of them lie far from the chemical potential: In some cases of physical situations in shell structure, there may be frequent occurrence of a possibility that the dressed $n$-quasi-particle mode with $n>3$ can be approximately decomposed into the correlated cluster in the valence shell and the phonons of the "core." Recall here that such a possibility was already pointed out in Chaps. 3 and 4 in relating the picture of the dressed 3QP mode to that of the Alaga model.[14),15)]

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# Chapter 6. Comparison between Results with the $P+QQ$ Force and with More Complex Residual Force

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## § 1. Introduction

From Chap. 3 to Chap. 5, microscopic structure of collective excitations in spherical odd-mass nuclei has been discussed with the use of the pairing-plus-quadrupole ($P+QQ$) force. Since we have widely employed characteristic properties of the quadrupole force, it is indispensable to examine whether or not the conclusions obtained from Chap. 3 to Chap. 5 are specific to the $P+QQ$ force. The aim of this chapter is to examine the effects of the other components of residual interaction which are neglected in the $P+QQ$ force model. With this aim, we make a comparison between the results calculated by using the quadrupole force and those calculated by using the central force with Gaussian radial dependence. In §§ 2 and 3, comparisons between the results of the $P+QQ$ force and those of the Gaussian force are made for the case of collective excited states with positive parity in Se isotopes. These states provide a good example in which we can see the effects of the other components of residual interaction in a relatively simple way. In § 4, as an alternative example, we present the results for single closed shell Sn isotopes in which quadrupole collectivity of the excited states is not so strong as in the case of Se isotopes. Hence, we can learn from these examples the relative importance of the neglected components of residual interaction in relation to the quadrupole collectivity of the states of interest. Needless to say, the theory developed in Chap. 2 is applicable for any residual interaction. However, we do not extend our present purpose to looking into the details of residual interactions themselves. In § 5, we add a few remarks concerning further refinements of the analysis.

## § 2. Dressed three-quasi-particle $7/2^+$ states in Se isotopes

We solve the eigenvalue equation for the dressed three-quasi-particle (3QP) mode given by Eq. (3·3) of Chap. 2 with the use of conventional Gaussian

force. The full expression of the matrix elements entering into the eigenvalue equation is given in Appendix 6A. The Gaussian force adopted here is the one without any exchange mixture, i.e.,

$$V(r) = -V_0 \exp(-r^2/r_0^2) \tag{2·1}$$

with $r=|\boldsymbol{r}_1-\boldsymbol{r}_2|$. As usual, the matrix elements of the Gaussian force are calculated by using harmonic oscillator wave functions with $\hbar\omega=41A^{-1/3}$ MeV. Then the matrix elements depend only on the ratio of the force-range $r_0$ to the range-parameter of the harmonic oscillator potential $b=(\hbar/M\omega)^{1/2}$. We adopt the method of Horie and Sasaki[1] in calculating these matrix elements.

Figure 1 shows the result of calculations for the "anomalous coupling" $7/2^+$ states in Se isotopes as the dressed 3QP states. The shell-model space adopted here is composed of the orbits $\{1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2}\}$ for both protons and neutrons. The single-particle energies used are the same as those of Kisslinger and Sorensen.[2] In the conventional treatment, where we use the Gaussian force as an effective interaction, the BCS equation determining the quasi-particle energies $E_a$ and the coefficients of the Bogoliubov transformation $(u_a, v_a)$ is also solved by using the Gaussian force. However, in this calculation, we have used the constant pairing force in the BCS equation, since our aim is to compare the results of the Gaussian-force case with those of the $P+QQ$ force case. This implies that the Gaussian force is regarded

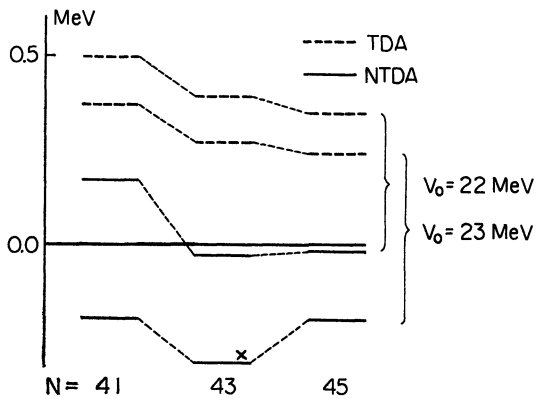



Fig. 1. Excitation energies of the dressed 3QP $7/2^+$ states in Se isotopes calculated with the use of the Gaussian force. The energies are measured from the 1QP $9/2^+$ states. The range parameter of the Gaussian force defined by (2·1) is fixed at 2.0 fm, while the calculated results for two choices of the force-strength $V_0$, i.e., $V_0$=22MeV and 23MeV, are shown. The solid lines represent the results in the new-Tamm-Dancoff approximation, while the broken lines in the Tamm-Dancoff approximation. The symbol × denotes the occurrence of complex eigenenergy.

here as a residual interaction among quasi-particles. This is in accord with our present aim of looking into the difference between the Gaussian and quadrupole forces in characterizing the microscopic structure of the dressed 3QP mode.

From Fig. 1, we can see that the $7/2^+$ states become the lowest-lying dressed 3QP states for a reasonable choice of the force-range parameter $r_0$. The reason for this particular favouring of the $7/2^+$ state is very similar to that in the $P+QQ$ force model (discussed in detail in Chap. 3): Since the square cfp of the type $[j^2(2), j;\ I|\}j^3;\ I\ v=3]^2$ takes the maximum value when $I=j-1$, the $7/2^+$ state involves the component $\{g^2_{9/2}(J=2)g_{9/2}\}_{I=7/2}$ as the maximum one among the components of the type $\{g^2_{9/2}(J)g_{9/2}\}_I$.*) Accordingly, the $7/2^+$ state has a large energy gain due to the relatively large matrix elements of the force, $G((\nu g_{9/2})^2\ (\nu g_{9/2})^2;\ 2)$ and $F(rs(\nu g_{9/2})^2;\ 2)$.**) The matrix element $G((\nu g_{9/2})^2\ (\nu g_{9/2})^2;\ 2)$ mainly contributes to increase the diagonal matrix element in the eigenvalue equation (3·3) of Chap. 2, while the matrix elements $F(rs(\nu g_{9/2})^2;\ 2)$ mainly contributes to increase the components of the type $\{rs(2^+)\nu g_{9/2}\}$.

One of the important characteristics of the quadrupole force is the special parity-selection property, i.e.,

$$(a\|r^2 Y_2\|b)=0 \qquad \text{when } (-)^{l_a+l_b}=-1. \tag{2·2}$$

This property of the quadrupole force plays an efficient role in the discussion of the $7/2^+$ state in terms of the $P+QQ$ force, owing to the special situation of shell structure in which the high-spin, unique-parity orbit is being filled with odd-number nucleons. In fact, the parity-selection property greatly simplified the discussions in Chap. 3. In the case of the Gaussian force, we have no such property. Accordingly, the dressed 3QP $7/2^+$ mode under consideration contains various kinds of components, for example, the components corresponding to the $p_{1/2}$ quasi-particle coupled with the $3^-$ phonon. In spite of the inclusion of such kinds of components into the eigenvalue equation, we can see that the predominant role of the quadrupole correlation in characterizing the microscopic structure of the $7/2^+$ state does not change in any significant way. Namely, these components neglected in the $P+QQ$ force model contribute only as a small perturbation to the low-lying $7/2^+$ state.

Now, in order to see the effect of the backward-going diagrams (originated from the ground-state correlation) on the excitation energy of the $7/2^+$ state, let us consider the quantity

*) The components of the amplitudes of the dressed 3QP modes are defined in Appendix 6A; see also Appendix 4A for the method of providing the orthonormal basis vectors in the coupled-angular-momentum representation.

**) The matrix elements of the force, $G$ and $F$, are defined in Appendix 1A.

$$\delta B=\frac{\omega_{TD}-\omega_{NTD}}{3E-\omega_{TD}}, \tag{2·3}$$

where $E$ denotes the energy of the $g_{9/2}$ neutron quasi-particle and $\omega_{TD}(\omega_{NTD})$ the energy of the (dressed) 3QP mode in the (new) Tamm-Dancoff approximation. For the parameters $V_0=23$ MeV and $r_0=2$ fm, the ratio of $\delta B$ (Gaussian) to $\delta B$ (quadrupole) takes the following value:

$$\frac{\delta B(\text{Gaussian})}{\delta B(\text{quadrupole})}=\frac{0.21}{0.94}=0.22 \tag{2·4}$$

for the $7/2^+$ state in $^{79}$Se. Thus, in the case of the Gaussian force, the effect of the backward-going diagrams becomes smaller compared to the case of the quadrupole force. However, it should be emphasized that they sensitively affect both the excitation energy and the amplitudes (of the dressed 3QP mode), since the $7/2^+$ states lie very near to the critical point for the instability of the spherical BCS vacuum.

Figure 2 shows the main amplitudes of the dressed 3QP $7/2^+$ mode under consideration. In this figure, the corresponding amplitudes calculated by using the quadrupole force are also written for the sake of comparison. We can see that the correspondence between the amplitudes given by the Gaussian force and those given by the quadrupole force is remarkable in both their relative phases and magnitudes.*) For instance, the main components are, for both cases, of the types $\{(g_{9/2})^3\}$, $\{ab(2^+)g_{9/2}\}$ and $\{rs(2^+)g_{9/2}\}$. Furthermore, their relative magnitudes show the similar trend for both cases.

As for the difference between the two cases, we can see in Fig. 2 that, in the $^{79}$Se nucleus, the backward-going amplitudes in the case of the quadrupole force are larger than those in the case of the Gaussian force. Correspondingly, the forward-going amplitudes in this case are amplified as a whole. In Fig. 2, we can also see that the backward-going amplitude of the type $\{(g_{9/2})^3\}$ becomes notably smaller in the case of the Gaussian force. The reason for this may be found in the fact that, in the Gaussian-force case, the $G$-type and $F$-type matrix elements cancel each other[3] in the submatrix $\boldsymbol{A}$ in the eigenvalue equation (3·3) of Chap. 2. Therefore, the phase relation among different components connecting neutron-quasi-particle pairs in the submatrix $\boldsymbol{A}$ becomes quite random in comparison with that among corresponding components in the submatrix $\boldsymbol{D}$ for the forward-going amplitudes. This trend is similar to that in the RPA with the use of the Gaussian force (for the single-closed shell nuclei). However, we should be careful in the fact that the relative magnitude of the $G$-type and $F$-type matrix elements depends sensitively on the range parameter $r_0$. On the other hand, such a destructive

*) In the comparison, we should be careful in treating the phase convention of the single-particle wave-functions.

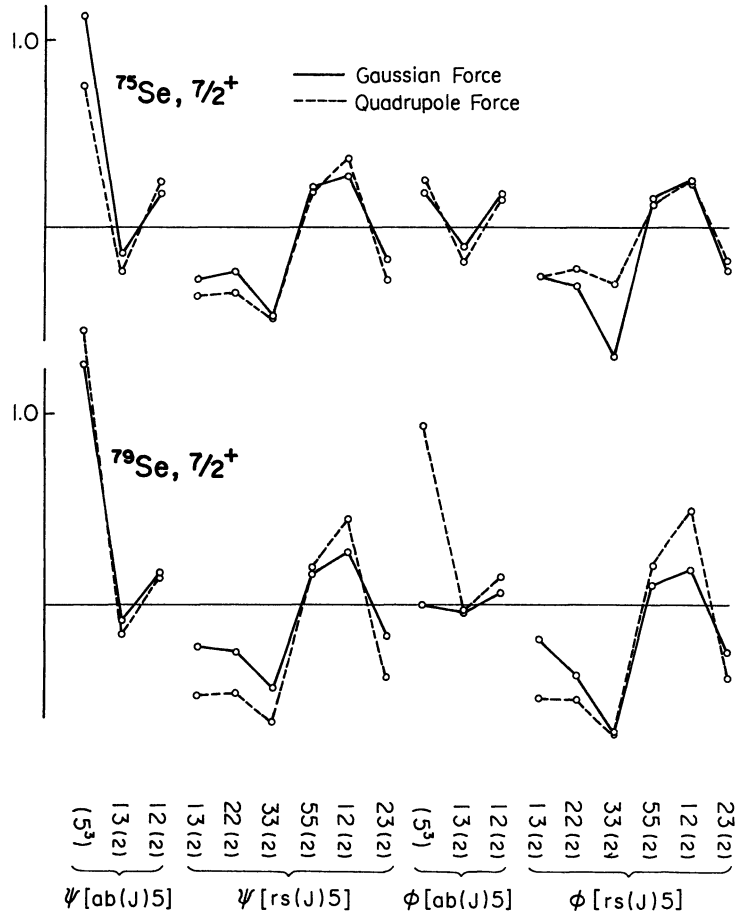



Fig. 2. Main amplitudes of the dressed 3QP $7/2^+$ modes in Se isotopes. The results calculated with the Gaussian force are connected by solid lines, while those with the quadrupole force are connected by broken lines. The range parameter and force strength of the Gaussian force are fixed at $r_0=2.0$ fm and $V_0=23$MeV, respectively. (These values are always adopted in the discussion in §§2 and 3.) The $P+QQ$ force-parameters are; $G=24/A$ MeV for both protons and neutrons, $\chi_0=230$ MeV. The definition of the amplitudes is given in Appendices 6A and 4A. In this figure, the following abbreviation is used to specify the shell-model orbit:

$1\equiv(2p_{1/2})$, $2\equiv(2p_{3/2})$, $3\equiv(1f_{5/2})$, $5\equiv(1g_{9/2})$.

effect between the $G$-type and $F$-type matrix elements is absent in the matrix elements connecting proton and neutron quasi-particle pairs.[3] Consequently, the backward-going amplitudes of the type $\{rs(2^+)g_{9/2}\}$ become significantly large in the Gaussian-force case as well as in the quadrupole-force case.

Although we can find such a few differences in fine structure which are dependent on details of the force, the correspondence of the main amplitudes between the two cases is remarkable. Thus, we can conclude that the micro-

scopic structure of the collective $7/2^+$ state in the Gaussian-force case is very similar to that in the quadrupole-force case.

## § 3. Other collective states with positive parity in Se isotopes

Next, let us consider other collective states with positive parity in Se isotopes. These states are the ones corresponding to the quintet composed of one-quasi-particle and one-phonon in the quasi-particle-phonon-coupling theory. Considering these states also as the dressed 3QP states, we have carried out the calculation with the use of the Gaussian force. The parameters used are the same as in the preceding section.

Figure 3 shows the result of the calculation. In this figure, the result obtained by using the quadrupole force is also shown for the sake of comparison. We can clearly see that the calculated level sequence is the same as that in the case of the quadrupole force. This fact indicates that the microscopic structure of these states obtained by using the Gaussian force is very similar to that obtained by using the quadrupole force. The main amplitudes of the dressed 3QP modes are shown in Figs. 4 and 5. In Fig. 4 are shown the main amplitudes of the $5/2^+$ mode. From the comparison between the Gaussian-force case (solid line) and the quadrupole-force case (broken line), we can say that the correspondence between the two cases holds fairly well not

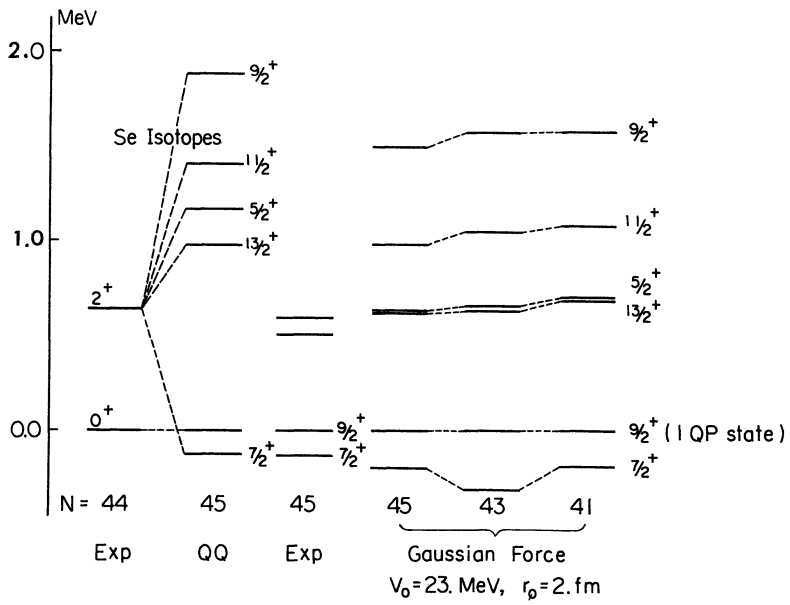



Fig. 3. Excitation energies of the dressed 3QP states with positive parity in Se isotopes. Results calculated by using the Gaussian force are compared with those by the quadrupole force. Parameters used are the same as in Fig. 2.

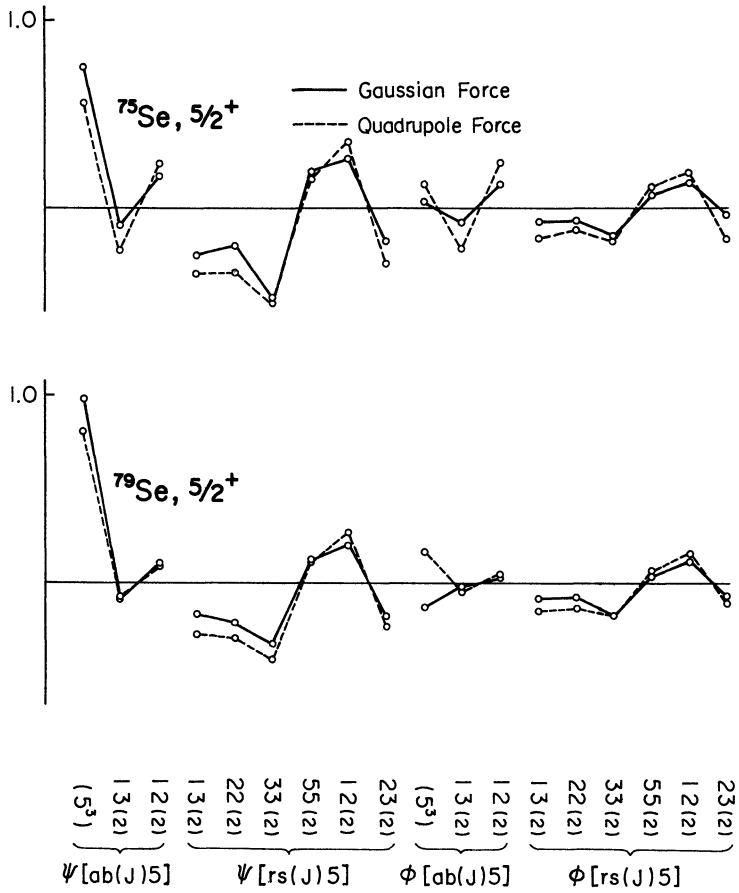



Fig. 4. Main amplitudes of the dressed 3QP 5/2$^+$ modes in Se isotopes. Notations and parameters used are the same as in Fig. 2.

only for the forward-going amplitudes but also for the backward-going amplitudes. The main amplitudes of the modes with spins from 5/2$^+$ to 13/2$^+$ are compared in Figs. 5 (a) and 5 (b). Figure 5 (a) shows the result calculated by using the quadrupole force, while Fig. 5 (b) shows the result calculated by using the Gaussian force. In these figures, the magnitudes of the amplitudes of the modes with spins from 5/2$^+$ to 13/2$^+$ are collectively shown on each position representing a specific component of the amplitudes. From these figures, we can see that, in each component, the relative magnitudes among the amplitudes of the modes with spins from 5/2$^+$ to 13/2$^+$ are similar between the Gaussian-force and quadrupole-force cases. This fact shows the reason why we have obtained the same level sequence irrespective of the forces adopted.

It has been expected that the neutron-proton short-range interaction, which has been neglected in the $P+QQ$ force model (except for its field

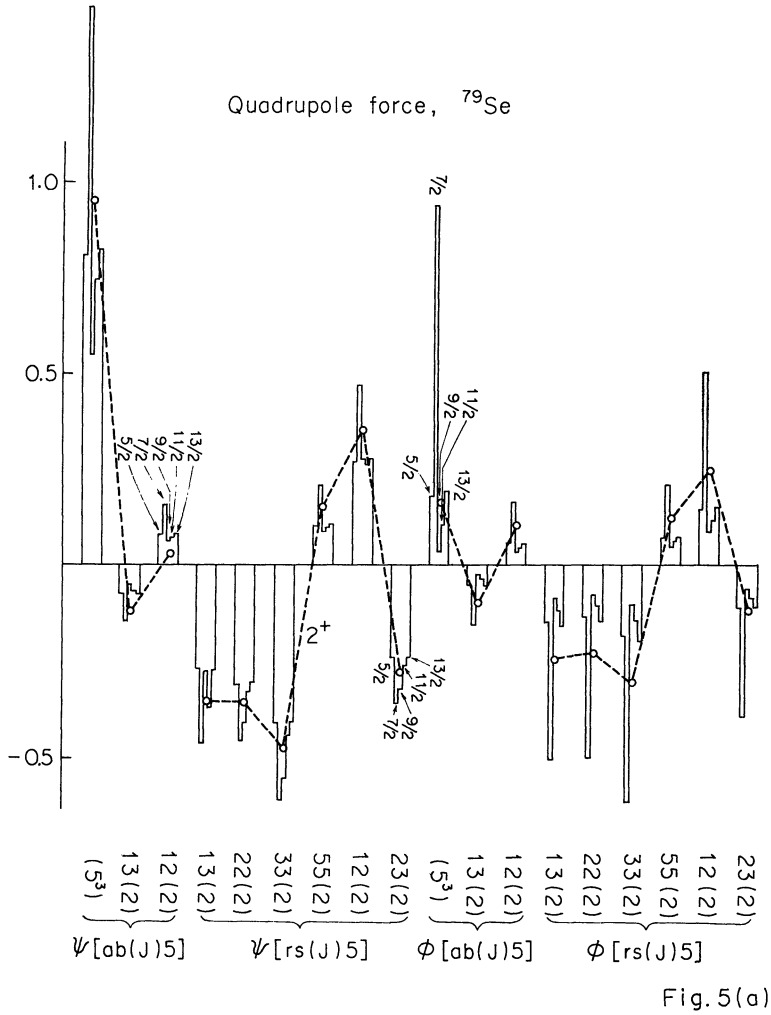
producing parts), becomes important in the region $28 \leq Z \leq 50$ and $28 \leq N \leq 50$.[2] The effect of this kind is automatically taken into account in this calculation with the Gaussian force. The result of this calculation shows, however, that the effect does not bring about appreciable differences from the result of the $P+QQ$ force case, at least for low-lying collective states with positive parity in Se isotopes.

As for the difference between the two cases, we can point out that the energy-splitting of the "quintet" in the Gaussian-force case is somewhat smaller than that in the quadrupole-force case. The main reason for this is that the effect of the backward-going diagrams is especially strong for the $7/2^+$ state in the quadrupole-force case. (See the values in (2·4) which show the difference between the two cases in lowering the energy of the $7/2^+$ state.) Another different point is that the mass-number dependence of the excitation energies becomes smooth in the Gaussian-force case, compared to that in the quadrupole-force case. This is the trend similar to that well known in the RPA for even-even nuclei. Therefore, its origin may be attributed to the fact that, in the Gaussian-force case, not only the force-element of the type $F(abcd;\ 2^+)$ but also of the type $G(abcd;J)$ are effective. (The $F$ and $G$ type force-elements are defined in Appendix 1A.) The more detailed differences between the two cases can be seen when we look into the fine structure of the amplitudes. For example, in the Gaussian-force case, the backward-going component $\phi[(g_{9/2})^3]$ in the $5/2^+$ mode becomes very small (in $^{75}$Se) and has even the phase opposite to that in the quadrupole-force case (in $^{79}$Se). (See Fig. 4.) Besides this, in the $9/2^+_2$ state in $^{79}$Se which lies in the relatively higher energy region, the forward-going component $\{(\pi p_{3/2}\pi f_{5/2})_4{}^+\nu g_{9/2}\}$ in the Gaussian-force case becomes non-negligible, i.e., $\psi[23(4)5]=0.109$.

Although we can find some differences between the two cases as mentioned above, it should be emphasized that the correspondence of the main amplitudes between the two cases does hold fairly well. Thus we can say that the nature of the dressed 3QP modes discussed in terms of the $P+QQ$ force model is also maintained in the Gaussian-force model, as long as they are low-lying in energy.

## § 4. Collective excited states in Sn isotopes

In the preceding sections, we have seen that the characteristics of the dressed 3QP mode derived from theoretical calculations are essentially the same between the quadrupole-force case and the Gaussian-force case. This conclusion implies that, for such collective states as the low-lying excited states in Se isotopes, the quadrupole collectivity is so dominant that the other correlations do not play significant roles. Accordingly, the other components of residual interaction which are neglected in the $P+QQ$ force model are expected to play appreciable roles when we consider the nuclei in which the




(a) The quadrupole-force case.

Fig. 5. Main amplitudes of the dressed 3QP modes with positive parity in $^{79}$Se. The magnitudes of the amplitudes of the modes with spins from $5/2^+$ to $13/2^+$ are collectively shown on each position representing a specific component of the amplitudes. The set of states with spins from $5/2^+$ to $13/2^+$ corresponds to the "quintet" composed of the 1QP $9/2^+$ mode coupled with the $2^+$ phonon in the quasi-particle-phonon-coupling theory. For the sake of comparison, the corresponding amplitudes of the $2^+$ phonon calculated by the RPA in $^{78}$Se are

quadrupole collectivity is not strong. In this section, in order to show this possibility, we present the results for single-closed shell $^{117}$Sn and $^{119}$Sn calculated by using the Gaussian force. As was pointed out in § 5-Chap. 5, the Sn isotopes belong to the situation where the chemical potential lies near the low-spin orbits so that the phonon-band character is expected to be broken

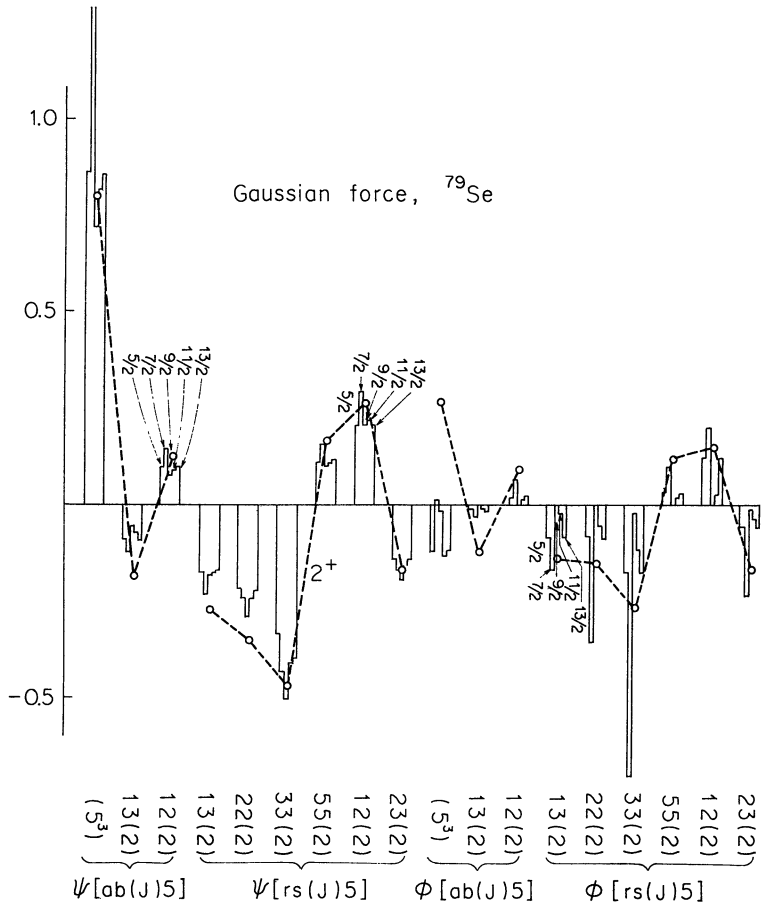



Fig. 5(b)

(b) The Gaussian-force case.

shown by the symbol ∘ connected with broken lines. The strengths of the quadrupole force and of the Gaussian force are chosen to approximately reproduce the experimental excitation energies of the $7/2^+$ and $2^+$ states, i.e., the energy difference between the collective $7/2^+$ and 1QP $9/2^+$ states in $^{79}$Se and the excitation energy of the $2^+$ state in $^{78}$Se. They are; $\chi_0=230$ (MeV) for $^{79}$Se, $\chi_0=220$ (MeV) for $^{78}$Se, $V_0=23$ MeV and $r_0=2.0$ fm for $^{79}$Se, $V_0=21$ MeV and $r_0=2.0$ fm for $^{78}$Se.

remarkably. If this is the case, then we can also expect that the relative magnitude between the inter and intra phonon-band $E2$ transitions is sensitively dependent on details of many conditions, for example, such as relative occupation probabilities among shell-model orbits. Because of this situation and also because of the relative weakness of the quadrupole collectivity, we can expect that the properties of the collective excited states in Sn isotopes

are very sensitive to details of the wave functions.

Figures 6 and 7 show the calculated results for $^{117}$Sn and $^{119}$Sn. These figures are made in a form in which the breaking and persistency of the phonon-band character is easy to see. By comparing the results of the Gaussian-force case to those of the quadrupole-force case, we see that the level sequence within a "phonon-band" is almost the same between the two cases. Although the gross structure of the excitation spectra displays some similarity, the energy shift due to the ground-state correlation differs in magnitude between the two cases. For example, the energy shift for the $3/2_2^+$ state in $^{119}$Sn is about 300 keV in the quadrupole-force case, while the corresponding energy shift is reduced to about 40 keV in the Gaussian-force case. This reduction of energy shift in the Gaussian-force case is rather special for the single-closed shell nuclei such as Sn isotopes under consideration, since, as we have seen in § 3, the ground-state correlation in the Gaussian-force case is mainly caused by the $F$-type matrix elements between proton and neutron quasi-particle pairs.

A more interesting difference between the two cases is found when we

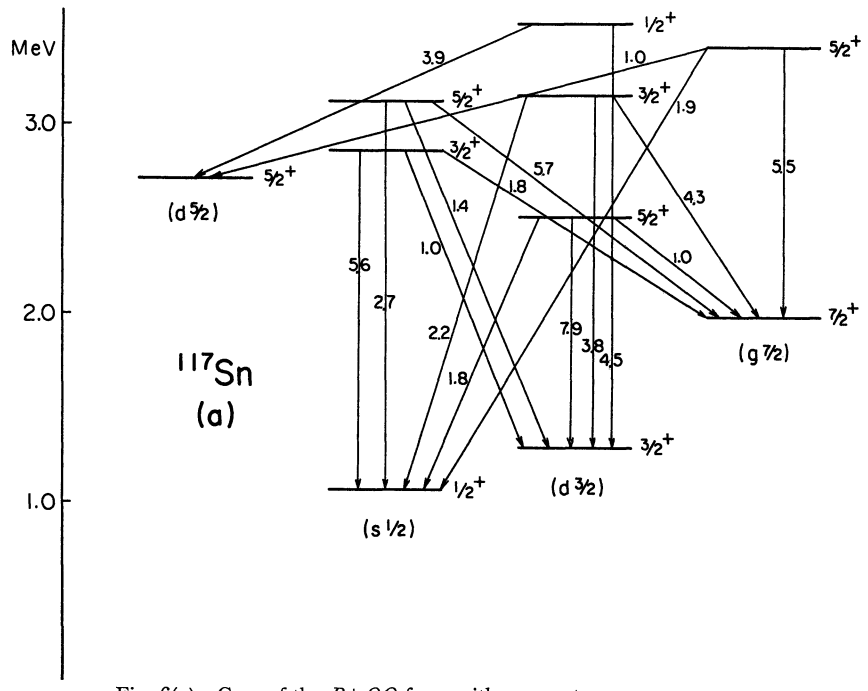



Fig. 6(a) Case of the $P+QQ$ force with parameters;
$G=0.205$, $\chi_0=\chi b^4 A^{5/3}=321.0$ (MeV)

Fig. 6. Results of calculation for the dressed 3QP states with spin $I \leqq 5/2^+$ in $^{117}$Sn. They are presented to show the breaking and persistency of the multiplet structures standing over the 1QP states with orbits $3s_{1/2}$, $2d_{3/2}$ and $1g_{7/2}$. The presented level energies are those measured from the correlated ground state. The numbers appearing on the transition arrows give the $B(E2)$ values in unit of $e^2 10^{-51}$ cm$^4$, which are calculated with polarization charge

compare the magnitudes of the inter-band transitions. These figures show that many inter-band transitions compete with intra-band transitions. This trend is clearly seen for both $^{117}$Sn and $^{119}$Sn in the quadrupole-force case (Figs. 6(a) and 7(a)). Similar trend is seen for $^{119}$Sn also in the Gaussian-force case (Fig. 7(b)). On the other hand, the inter-band transitions are relatively smaller for $^{117}$Sn in this case (Fig. 6(b)). In this way, the nucleon-number dependence of the inter-band transitions seems to be very sensitive to the residual interaction adopted. (Similar property is seen in the relative magnitudes among some inter-band transitions.) Such a situation is exactly the expected one: Since the magnitude of the inter-band transition depends sensitively on the relative magnitudes among many components of the amplitudes of the dressed 3QP mode, the magnitude of the inter-band transition tends to change significantly from one isotope to another isotope. The fine structure of the relative magnitude among many components, in turn, depends on the details of the residual interaction. (Note that the $(u, v)$ dependence is more complex in the Gaussian-force case than in the quadrupole-force case, since, in the former case, $G$-type and $F$-type matrix elements enter into the

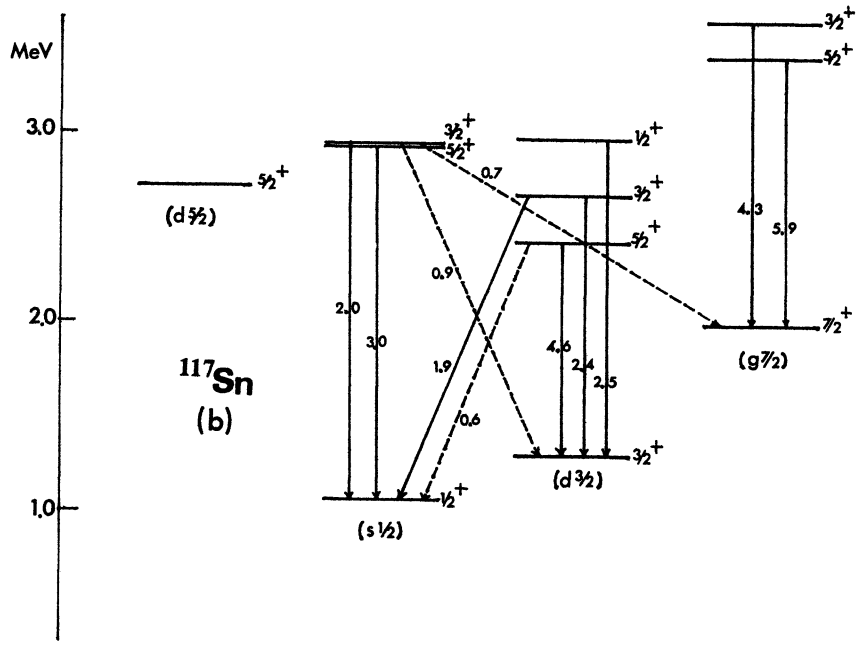



Fig. 6(b) Case of the Gaussian force with parameters;
$V_0$=35 MeV, $r_0$=1.720 fm.

$\delta e$=1.0 and harmonic-oscillator-range parameter $b^2$=1.0$A^{1/3}$. For simplicity, the $E2$ transitions smaller than 1.0 (in case (a)) or 0.5 (in case (b)) and other higher-lying states are omitted from the figure. The single-particle energies are taken from Ref. 5):
$\varepsilon(d_{5/2})$=0.0, $\varepsilon(g_{7/2})$=0.83, $\varepsilon(s_{1/2})$=2.29, $\varepsilon(h_{11/2})$=3.53, $\varepsilon(d_{3/2})$=3.26. (all in MeV)

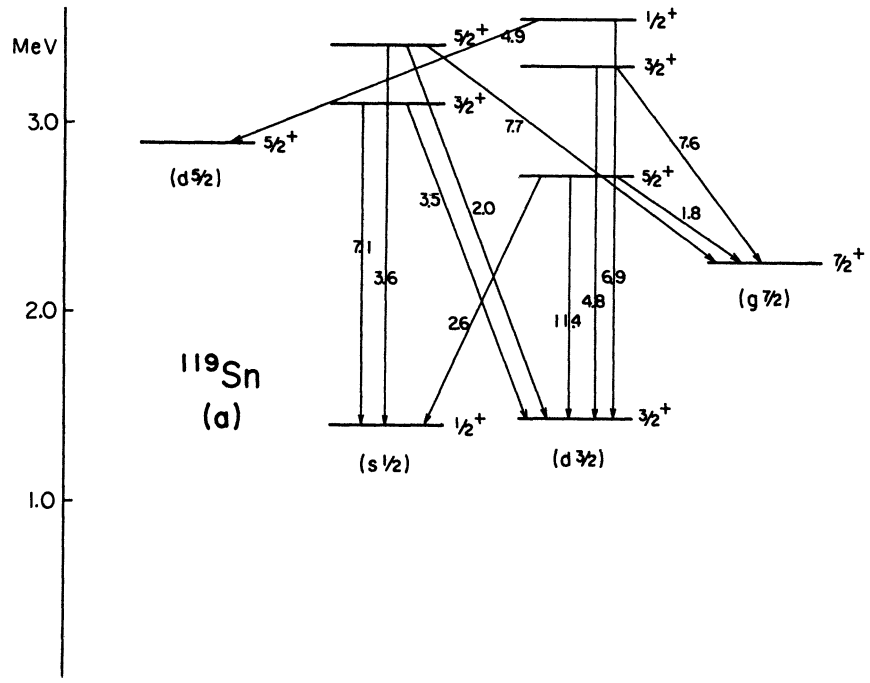



Fig. 7(a) Case of the $P+QQ$ force with parameters;
$G=0.227$, $\chi_0=322.3$. (MeV)

Fig. 7. Result of calculation for the dressed 3QP states with spin $I \leqq 5/2^+$ in $^{119}$Sn. Notations are the same as in Fig. 6. The single-particle energies are taken from Ref. 5):
$\varepsilon(d_{5/2})=0.0$, $\varepsilon(g_{7/2})=0.75$, $\varepsilon(s_{1/2})=2.39$, $\varepsilon(h_{11/2})=3.15$, $\varepsilon(d_{3/2})=2.87$.
(all in MeV)

eigenvalue equation with different $(u, v)$ dependence.)

Some inter $E2$ transitions between the $s_{1/2}$ and $d_{3/2}$ phonon-bands have been measured by Stelson et al.[4] For the inter $E2$ transition between the $5/2^+$ state (belonging approximately to the $d_{3/2}$ "phonon-band") and the 1QP $1/2^+$ state, the data indicate that its magnitude drastically changes from $^{117}$Sn to $^{119}$Sn; i.e., the ratio $B(E2; 5/2^+_{(d_{3/2})} \to 1/2^+_1)/B(E2; 5/2^+_{(d_{3/2})} \to 3/2^+_1)$ is smaller than 0.1 in $^{117}$Sn, whereas it is about 0.9 in $^{119}$Sn. Concerning this specific transition, the result calculated by the Gaussian force agrees with the data better than the result calculated by the quadrupole force. For the inter $E2$ transitions from the $3/2^+$ and $5/2^+$ states (belonging approximately to the $s_{1/2}$ "phonon-band") to the 1QP $3/2^+$ states, the calculated $B(E2)$ values, especially in the quadrupole-force case, seem to be larger than the corresponding experimental data.[4] We furthermore see that the splitting of the "doublet" ($3/2^+$ and $5/2^+$) is very small in the Gaussian-force case and seems to agree better with the data. However, it does not necessarily imply that the "weak-coupling character" holds, since the inter-band transitions

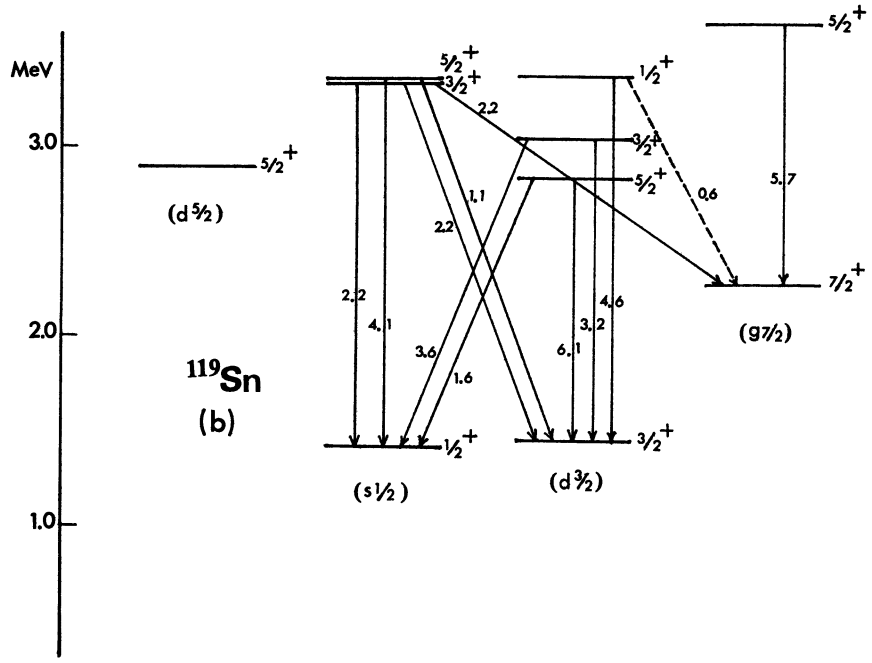



Fig. 7(b) Case of the Gaussian force with parameters;
$V_0$=35 MeV, $r_0$=1.725 fm.

to the 1QP $7/2^+$ and $3/2^+$ states can become large even in this case. (See Fig. 7(b).) Even when the inter-band transitions become relatively smaller as in the case of Fig. 6(b), the structure of the excited states under consideration differs from that of the odd-quasi-particle plus $2^+$ phonon; this property is merely a consequence of the dominance of the component $\{\nu s_{1/2}(\nu h_{11/2})^2\}$ in this case. Thus, the definite conclusion for the properties of a particular state should be made only after we carefully examine the parameters to be adopted in the calculation. Furthermore, the coupling effect of the pairing-collective excitations and also the coupling effect between the dressed 3QP- and 1QP-modes should be taken into account, since the effects of such kinds are expected to be appreciable for the states under consideration.[6]

## § 5. Further refinements

So far, we have seen that the effects of the other components which are neglected in the $P+QQ$ force model are not appreciable, if the states of interest are of sufficiently (quadrupole-) collective character. However, when we consider a physical quantity which sensitively reflects such small components, we must be careful in treating the fine details of such components and their relations to the dominant components (i.e., here the quadrupole components).

In this section, we briefly discuss such a situation by exemplifying the cases of treating allowed Gamow-Teller (GT) beta decay between odd-mass nuclei and of calculating the $M1$ moments.

The $P+QQ$ force model is by itself not capable of accounting for the retardation of GT transition rates which are regularly observed in medium-mass nuclei. It has been shown that the proton-neutron residual interaction of charge-exchange and spin-flip type, i.e., $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})$ type, is responsible for the hindrance of non-$l$-forbidden GT transition rates between one-quasi-particle states.[7] The $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})$ type residual interaction (which is called the GT force hereafter) brings about the coupling of the "$J^\pi=1^+$" proton-neutron quasi-particle pairs to the one-quasi-particle state. Although the mixing of this kind of quasi-particle pairs in the one-quasi-particle state is small, they contribute so coherently that their effects on the beta-decay rate become significant.[7] Thus, we are forced to simultaneously take account of both the GT type correlation and the quadrupole correlation, since we are interested in the GT transition between (non-deformed) odd-mass nuclei exhibiting the quadrupole collective character.

A simple way of simultaneously treating these two kinds of correlations may be to introduce the GT force in addition to the $P+QQ$ force. In the method of quasi-particle-phonon-coupling theory, the "$J^\pi=1^+$" phonon composed of proton-neutron quasi-particle pairs is introduced by treating the GT force with the unlike-particle RPA (describing the odd-odd nuclei).[8] With this method, the effect of the GT force is treated independently to that of the quadrupole force responsible for the low-lying $2^+$ phonon. Consequently, a difficulty of this method arises from the non-commutability between the (higher-lying) $1^+$ phonon and the (low-lying) $2^+$ phonon. It should be noted here that the excitation of the "$J^\pi=1^+$" quasi-particle pairs takes place mostly in the same shell-model space as that for the "$J^\pi=2^+$" quasi-particle pairs. (When the shell-model space is enlarged so as to include the spin-orbit partners of all single-particle orbits in the filling major shell, the shell-model space becomes exactly the same for these excitations.) The difficulty in evaluating the GT matrix element (, for example, between the one-proton-quasi-particle state and the one-neutron-quasi-particle-plus-one-$2^+$-phonon state,) is closely connected with the basic approximation of treating the two correlations independently.[8] This is seen when we attempt to unambiguously expand the GT transition operator in terms of the quasi-particle, $2^+$ phonon and $1^+$ phonon operators which are assumed to be mutually commutable.

The difficulty of this kind can be overcome when we adopt the method developed in Chap. 2. With this method, by introducing the GT force in addition to the $P+QQ$ force, we can simply achieve the aim of treating the GT and quadrupole correlations on an equal footing.[10] When calculation of this kind is performed by this method, the $J^\pi=1^+$ proton-neutron quasi-

particle pairs participate in the dressed 3QP mode as one of the constituents. Then we have a new dressed 3QP state predominantly exhibiting the GT correlated character at a higher energy region. On the other hand, the low-lying dressed 3QP states which predominantly are of quadrupole character remains essentially the same as in the $P+QQ$ force model except for a small perturbative effect due to the GT force. Of course, the one-quasi-particle state couples with both kinds of the dressed 3QP states. After diagonalizing the effective Hamiltonian defined by (5·11) of Chap. 2 in the quasi-particle new-Tamm-Dancoff space, we can evaluate any GT transition matrix element between the obtained eigenstates by making use of the transcription rule developed in § 5-Chap. 2. This procedure is exactly the same as in the case of evaluating the electromagnetic transition matrix elements. As for the difference of nucleon numbers between the initial and final states, we can take it into account by using the method described in Appendix 6B. For details of this kind of application, see Ref. 10).

An analogous situation occurs in the case of evaluating the $M1$ moments. In the calculation of $M1$ moments, we used the effective spin $g$ factor, $g_s^{\text{eff}}$, in Chaps. 3 and 4. The use of $g_s^{\text{eff}}$ is regarded as representing the effect of coupling of the $J^\pi=1^+$ quasi-particle pairs to the one-quasi-particle mode (i.e., the effect of $M1$ core polarization). As is well known, one of the important assumptions in using such an "effective quantity" is that the excitation of the $J^\pi=1^+$ quasi-particle pairs is approximately independent of the other kind of excitations. On the other hand, we can explicitly take into account such a kind of excitations by adopting, for example, a delta-function force with suitable spin-dependence in the calculation.[11] In this case, the ($M1$-type) $J^\pi=1^+$ quasi-particle pairs participate in the dressed 3QP mode as one of the constituents. Consequently, we will have a new dressed 3QP state predominantly exhibiting the $M1$-type correlated character at a higher energy region, and the 1QP state couples with this state. (In this calculation, of course, the shell-model space should be chosen so as to include the spin-orbit partners of all single-particle orbits in the filling major shell.) Then there may be no need to use the "effective quantity" such as $g_s^{\text{eff}}$. Hence, the investigation in which this $M1$-type correlation is explicitly taken into account together with the quadrupole correlation will enable us to examine the validity of the use of the effective spin $g$ factor $g_s^{\text{eff}}$. Of course, the introduction of such small components (representing the $J^\pi=1^+$ quasi-particle pairs) will bring about no essential change in the nature of the low-lying dressed 3QP states which have been described in terms of the $P+QQ$ force model. We can expect this trend from the results in §§ 2 and 3, since the calculation with the use of the Gaussian force have already included such small components. The detailed investigation into the direction remarked here remains to be done.

## § 6. Concluding remarks

With the use of a central force with Gaussian radial dependence, we have investigated whether or not the nature of the dressed 3QP modes in the $P+QQ$ force model is essentially dependent on the special properties of the quadrupole force. It has been shown that the microscopic structure of the dressed 3QP modes obtained by the Gaussian force is very similar to that obtained by the $P+QQ$ force, at least for the low-excited states with positive parity in Se isotopes. Thus we expect that the conclusions obtained in Chaps. 3, 4 and 5 do not always rely on the special properties of the quadrupole force but possess more general significance. On the other hand, we also expect that other types of correlation which cannot emerge from the $P+QQ$ force model itself becomes appreciable when we consider, for example, the excited state in which the dominant role of the quadrupole correlation is relatively relaxed.

It should also be noted that the situations, in which the quadrupole collectivity is dominant but the simple phonon-band character tends to be broken, remain to be investigated in more details for both the $P+QQ$ force case and the Gaussian-force case. Needless to say, the framework of the proposed theory is general enough to be used with any residual interaction. Thus, it is very interesting to investigate the relation between the effective interaction and the microscopic structure of the dressed 3QP mode by adopting more complex effective interactions than the $P+QQ$ force or the Gaussian force without any exchange mixture.

## Appendix 6A. Matrix elements of the secular equation for the dressed 3QP modes in the coupled-angular-momentum representation

Here, we give the explicit form of the matrix elements of the secular equation for the dressed 3QP modes, Eq. (3·3) in Chap. 2, in the coupled-angular-momentum representation. We can straightforwardly obtain the explicit form by transforming the matrix elements (in the $m$-scheme) given in Appendix 2B into the coupled-angular-momentum representation.

### 6A-1 *The case of general interaction*

In this representation, the creation operator of the dressed 3QP mode given by Eq. (3·1) in Chap. 2 is represented as

$$C^{\dagger}_{nIK}=\frac{1}{\sqrt{3!}}\sum_{abcJ}\psi_{nI}[ab(J)c]\sum_{m_\alpha m_\beta m_\gamma M}(j_a j_b m_\alpha m_\beta|JM)$$
$$\times(Jj_c M m_\gamma|IK)a^{\dagger}_\alpha a^{\dagger}_\beta a^{\dagger}_\gamma$$

$$
\begin{aligned}
&+\sum_{(rs)cJ}\frac{\psi_{nI}[rs(J)c]}{\sqrt{1+\delta_{rs}}}\sum_{m_\rho m_\sigma m_\gamma M}(j_r j_s m_\rho m_\sigma|JM)(Jj_c M m_\gamma|IK)a_\rho^\dagger a_\sigma^\dagger a_\gamma^\dagger \\
&+\frac{1}{\sqrt{3!}}\sum_{aJ}\phi_{nI}^{(1)}[aa(J)a]\sum_{m_{a_1}m_{a_2}m_{a_3}M}(j_a j_a m_{a_1}m_{a_2}|JM) \\
&\qquad\times(Jj_a M m_{a_3}|IK)\mathring{T}_{3/2,-1/2}(\alpha_1\alpha_2\alpha_3) \\
&+\frac{1}{\sqrt{2!}}\sum_{\substack{acJ\\(a\neq c)}}\phi_{nI}^{(2)}[aa(J);c]\sum_{m_{a_1}m_{a_2}m_\gamma M}(j_a j_a m_{a_1}m_{a_2}|JM) \\
&\qquad\times(Jj_c M m_\gamma|IK)\mathring{T}_{10}(\alpha_1\alpha_2)a_{\tilde{\gamma}} \\
&+\sum_{\substack{(ab)cJ\\(a,b\neq c)}}\frac{\phi_{nI}^{(3)}[ab(J);c]}{\sqrt{1+\delta_{ab}}}\sum_{m_\alpha m_\beta m_\gamma M}(j_a j_b m_\alpha m_\beta|JM)\,(Jj_c M m_\gamma|IK)a_\gamma^\dagger a_{\tilde{\alpha}}a_{\tilde{\beta}} \\
&+\sum_{(rs)cJ}\frac{\phi_{nI}^{(3)}[rs(J);c]}{\sqrt{1+\delta_{rs}}}\sum_{m_\rho m_\sigma m_\gamma M}(j_r j_s m_\rho m_\sigma|JM)\,(Jj_c M m_\gamma|IK)a_\gamma^\dagger a_{\tilde{\rho}}a_{\tilde{\sigma}} \\
&+\frac{1}{\sqrt{2}}\sum_{rcJ}\phi_{nI}^{(2)}[rr(J);c]\sum_{m_{\rho_1}m_{\rho_2}m_\gamma M}(j_r j_r m_{\rho_1}m_{\rho_2}|JM)\,(Jj_c M m_\gamma|IK) \\
&\qquad\times\mathring{T}_{10}(\rho_1\rho_2)a_{\tilde{\gamma}} \\
&+\sum_{\substack{rscJ\\(r\neq s)}}\phi_{nI}^{(3)}[rc(J);s]\sum_{m_\rho m_\sigma m_\gamma M}(j_r j_c m_\rho m_\gamma|JM)\,(Jj_s M m_\sigma|IK)a_\sigma^\dagger a_{\tilde{\rho}}a_{\tilde{\gamma}},
\end{aligned}
\tag{6A·1}
$$

where the Greek letters $\alpha=(a,\ m_a)$, $\beta$, $\gamma,\cdots$ are used to designate the single-particle states for neutrons (protons), and $\rho=(r,\ m_\rho)$, $\sigma=(s,\ m_\sigma),\cdots$ for protons (neutrons). In the text, we have called, for example, the amplitudes $\psi_{nI}[rs(J)c]$ and $\phi_{nI}^{(3)}[rs(J)c]$ the components of the type $\{rs(2)\nu g_{9/2}\}$ when $c$ denotes the $1g_{9/2}$ orbit for neutrons and $J=2$. (In this case, $r$ and $s$ denote the orbits for protons.) The antisymmetric properties of the three-body correlation amplitudes are then expressed as follows:

$$
\begin{aligned}
&\psi_{nI}[ab(J)c]=\sum_{a'b'c'J'}P_I(ab(J)c|a'b'(J')c')\psi_{nI}[a'b'(J')c'], \\
&\phi_{nI}^{(1)}[aa(J)a]=\sum_{J'}P_I(aa(J)a|aa(J')a)\phi_{nI}^{(1)}[aa(J')a], \\
&\phi_{nI}^{(2)}[aa(J);c]=\frac{(1-\delta_{J0})\,(1+(-)^J)}{2}\phi_{nI}^{(2)}[aa(J);c], \\
&\phi_{nI}^{(3)}[ab(J);c]=-(-)^{j_a+j_b-J}(1-\delta_{J0})\phi_{nI}^{(3)}[ba(J);c], \\
&\phi_{nI}^{(3)}[rs(J);c]=-(-)^{j_r+j_s-J}(1-\delta_{J0})\phi_{nI}^{(3)}[sr(J);c],
\end{aligned}
\tag{6A·2}
$$

where the projection operators $P_I(ab(J)c|a'b'(J')c')$ defined in Appendix 2A are used.

The matrix elements in the coupled-angular-momentum representation are explicitly given as follows:

$$3D_I[ab(J)c|a'b'(J')c']=(E_a+E_b+E_c)P_I(ab(J)c|a'b'(J')c')$$
$$+3\sum_{a_1b_1c_1J_1}\sum_{a_1'b_1'c_1'J_1'}P_I(ab(J)c|a_1b_1(J_1)c_1)V^{(f)}(a_1b_1a_1'b_1';J_1)\delta_{J_1J_1'}\delta_{c_1c_1'}$$
$$\times P_I(a_1'b_1'(J_1')c_1'|a'b'(J')c'),$$

$$3D_I[ab(J)c|r's'(J')c']=\frac{\sqrt{6}}{\sqrt{1+\delta_{r's'}}}\sum_{a_1b_1c_1}P_I(ab(J)c|a_1b_1(J')c_1)$$
$$\times V^{(f)}(a_1b_1r's';J')\delta_{c_1c'},$$

$$3D_I[rs(J)c|r's'(J')c']=(E_r+E_s+E_c)\delta_{rr'}\delta_{ss'}\delta_{cc'}\delta_{JJ'}$$
$$+\frac{2}{\sqrt{(1+\delta_{rs})(1+\delta_{r's'})}}\Big[V^{(f)}(rsr's';J)\delta_{JJ'}\delta_{cc'}$$
$$+(-)^{j_s+j_c+j_{s'}+j_{c'}}\hat{J}\hat{J}'\sum_{J''}(2J''+1)\begin{Bmatrix}j_r & j_s & J\\ j_c & I & J''\end{Bmatrix}\begin{Bmatrix}j_{r'} & j_{s'} & J'\\ j_{c'} & I & J''\end{Bmatrix}V^{(f)}(scs'c';J'')\delta_{rr'}$$
$$+(-)^{j_s+j_c+j_{s'}+j_{c'}+J'}\hat{J}\hat{J}'\sum_{J''}(2J''+1)\begin{Bmatrix}j_r & j_s & J\\ j_c & I & J''\end{Bmatrix}\begin{Bmatrix}j_{s'} & j_{r'} & J'\\ j_{c'} & I & J''\end{Bmatrix}V^{(f)}(scr'c';J'')\delta_{rs'}$$
$$+(-)^{j_s+j_c+j_{s'}+j_{c'}+J}\hat{J}\hat{J}'\sum_{J''}(2J''+1)\begin{Bmatrix}j_s & j_r & J\\ j_c & I & J''\end{Bmatrix}\begin{Bmatrix}j_{r'} & j_{s'} & J'\\ j_{c'} & I & J''\end{Bmatrix}V^{(f)}(rcs'c';J'')\delta_{sr'}$$
$$+(-)^{j_s+j_c+j_{s'}+j_{c'}+J+J'}\hat{J}\hat{J}'\sum_{J''}(2J''+1)\begin{Bmatrix}j_s & j_r & J\\ j_c & I & J''\end{Bmatrix}\begin{Bmatrix}j_{s'} & j_{r'} & J'\\ j_{c'} & I & J''\end{Bmatrix}V^{(f)}(rcr'c';J'')\delta_{ss'}\Big],$$
(6A·3a)

$$d_I[aa(J)a|a'a'(J')a']=E_aP_I(aa(J)a|a'a'(J')a')$$
$$+\sum_{a_1J_1}\sum_{a_1'J_1'}P_I(aa(J)a|a_1a_1(J_1)a_1)V^{(f)}(a_1'a_1'a_1a_1;J_1)\delta_{J_1J_1'}\delta_{a_1a_1'}$$
$$\times P_I(a_1'a_1'(J_1')a_1'|a'a'(J')a'),$$

$$d_I[aa(J)a|a'a'(J');c']=-\sqrt{2}\sum_{J_1}P_I(aa(J)a|aa(J_1)a)$$
$$\times(-)^{j_{a'}+j_{c'}}\hat{J}_1\hat{J}'\begin{Bmatrix}j_{a'} & j_{a'} & J'\\ j_{c'} & I & J_1\end{Bmatrix}V^{(f)}(a'c'aa;J_1)\delta_{aa'},$$

$$d_I[aa(J)a|a'b'(J');c']=\frac{\sqrt{2}}{\sqrt{1+\delta_{a'b'}}}P_I(aa(J)a|aa(J')a)V^{(f)}(a'b'aa;J')\delta_{ac'},$$

$$d_I[aa(J)a|rs(J');c']=\frac{\sqrt{2}}{\sqrt{1+\delta_{rs}}}P_I(aa(J)a|aa(J')a)V^{(f)}(rsaa;J')\delta_{ac'},$$

$$d_I[aa(J);c|a'a'(J');c']=E_c\delta_{aa'}\delta_{cc'}\delta_{JJ'}+2(-)^{j_a+j_c+j_{a'}+j_{c'}}\hat{J}\hat{J}'\sum_{J''}(2J''+1)$$
$$\times\begin{Bmatrix}j_a & j_a & J\\ j_c & I & J''\end{Bmatrix}\begin{Bmatrix}j_{a'} & j_{a'} & J'\\ j_{c'} & I & J''\end{Bmatrix}V^{(f)}(a'c'ac;J'')\delta_{aa'},$$

$$d_I[aa(J);c|a'b'(J');c']=-\frac{2}{\sqrt{1+\delta_{a'b'}}}(-)^{j_a+j_c+J'}\hat{J}\hat{J}'\begin{Bmatrix}j_a & j_a & J\\ j_c & I & J'\end{Bmatrix}V^{(f)}(a'b'ac;J')\delta_{ac'},$$

$$d_I[aa(J);\,c|rs(J');\,c']=-\frac{2}{\sqrt{1+\delta_{rs}}}(-)^{j_a+j_c+J'}\hat{J}\hat{J}'\begin{Bmatrix} j_a & j_a & J \\ j_c & I & J' \end{Bmatrix}V^{(f)}(rsac;J')\delta_{ac'},$$

$$d_I[ab(J);\,c|a'b'(J');\,c']=(E_a+E_b-E_c)\delta_{aa'}\delta_{bb'}\delta_{cc'}\delta_{JJ'}$$
$$+\frac{2}{\sqrt{(1+\delta_{ab})(1+\delta_{a'b'})}}V^{(f)}(a'b'ab;J)\delta_{JJ'}\delta_{cc'},$$

$$d_I[ab(J);\,c|rs(J');\,c']=\frac{2}{\sqrt{(1+\delta_{ab})\,(1+\delta_{rs})}}V^{(f)}(rsab;J)\delta_{JJ'}\delta_{cc'},$$

$$d_I[rs(J);\,c|r's'(J');\,c']=(E_r+E_s-E_c)\delta_{rr'}\delta_{ss'}\delta'_{cc'}\delta_{JJ'}$$
$$+\frac{2}{\sqrt{(1+\delta_{rs})(1+\delta_{r's'})}}V^{(f)}(r's'rs;J)\delta_{JJ'}\delta_{cc'},$$

$$d_I[rr(J);c|r'r'(J');\,c']=E_c\delta_{rr'}\delta_{cc'}\delta_{JJ'}+2(-)^{j_r+j_c+j_{r'}+j_{c'}}\hat{J}\hat{J}'\sum_{J''}(2J''+1)$$
$$\times\begin{Bmatrix} j_r & j_r & J \\ j_c & I & J'' \end{Bmatrix}\begin{Bmatrix} j_{r'} & j_{r'} & J' \\ j_{c'} & I & J'' \end{Bmatrix}V^{(f)}(r'c'rc;J'')\delta_{rr'},$$

$$d_I[rr(J);\,c|r'c'(J');\,s']=-2(-)^{j_r+j_c+J'}\hat{J}\hat{J}'\begin{Bmatrix} j_r & j_r & J \\ j_c & I & J' \end{Bmatrix}V^{(f)}(r'c'rc;J')\delta_{rs'},$$

$$d_I[rc(J);\,s|r'c'(J');\,s']=(E_r+E_c-E_s)\delta_{rr'}\delta_{cc'}\delta_{ss'}\delta_{JJ'}+2V^{(f)}(r'c'rc;J)\delta_{JJ'}\delta_{ss'},$$

(6A·3b)

$$A_I[ab(J)c|a'a'(J')a']=\sqrt{3}\sum_{a_1b_1c_1J''}P_I(ab(J)c|a_1b_1(J'')c_1)V^{(b)}(a_1b_1a'a';J'')\delta_{c_1a'}$$
$$\times P_I(a'a'(J'')a'|a'a'(J')a'),$$

$$A_I[ab(J)c|a'a'(J');\,c']=-\sqrt{6}\sum_{a_1b_1c_1J''}P_I(ab(J)c|a_1b_1(J'')c_1)(-)^{j_{a'}+j_{c'}+J''}\hat{J}'\hat{J}''$$
$$\times\begin{Bmatrix} j_{a'} & j_{a'} & J' \\ j_{c'} & I & J'' \end{Bmatrix}V^{(b)}(a_1b_1a'c';J'')\delta_{c_1a'},$$

$$A_I[ab(J)c|a'b'(J');\,c']=\frac{\sqrt{6}}{\sqrt{1+\delta_{a'b'}}}\sum_{a_1b_1c_1}P_I(ab(J)c|a_1b_1(J')c_1)V^{(b)}(a_1b_1a'b';J')\delta_{c_1c'},$$

$$A_I[ab(J)c|rs(J');\,c']=\frac{\sqrt{6}}{\sqrt{1+\delta_{rs}}}\sum_{a_1b_1c_1}P_I(ab(J)c|a_1b_1(J')c_1)V^{(b)}(a_1b_1rs;J')\delta_{c_1c'},$$

$$A_I[rs(J)c|a'a'(J')a']=\frac{\sqrt{2}}{\sqrt{1+\delta_{rs}}}V^{(b)}(rsa'a';J)\delta_{ca'}P_I(a'a'(J)a'|a'a'(J')a'),$$

$$A_I[rs(J)c|a'a'(J');\,c']=-\frac{2}{\sqrt{1+\delta_{rs}}}(-)^{j_{a'}+j_{c'}+J}\hat{J}\hat{J}'\begin{Bmatrix} j_{a'} & j_{a'} & J' \\ j_{c'} & I & J \end{Bmatrix}V^{(b)}(rsa'c';J)\delta_{ca'},$$

$$A_I[rs(J)c|a'b'(J');\,c']=\frac{2}{\sqrt{(1+\delta_{rs})\,(1+\delta_{a'b'})}}V^{(b)}(rsa'b';J)\delta_{JJ'}\delta_{cc'},$$

$$A_I[rs(J)c|r's'(J');\,c']=\frac{2}{\sqrt{(1+\delta_{rs})\,(1+\delta_{r's'})}}V^{(b)}(rsr's';J)\delta_{JJ'}\delta_{cc'},$$

$$A_I[rs(J)c|r'r'(J');\,c']=\frac{2}{\sqrt{1+\delta_{rs}}}(-)^{j_s+j_c+j_{r'}+j_{c'}}\hat{J}\hat{J}'$$
$$\times\Sigma_{J''}(2J''+1)\left[\begin{Bmatrix} j_r & j_s & J \\ I & j_c & J'' \end{Bmatrix}\begin{Bmatrix} j_{r'} & j_{r'} & J' \\ I & j_{c'} & J'' \end{Bmatrix}V^{(b)}(scr'c';J'')\delta_{rr'}\right.$$
$$\left.+(-)^J\begin{Bmatrix} j_r & j_s & J \\ j_c & I & J'' \end{Bmatrix}\begin{Bmatrix} j_{r'} & j_{r'} & J' \\ j_{c'} & I & J'' \end{Bmatrix}V^{(b)}(rcr'c';J'')\delta_{sr'}\right],$$
$$A_I[rs(J)c|r'c'(J');\,s']=-\frac{2}{\sqrt{1+\delta_{rs}}}(-)^{j_s+j_c+J'}\hat{J}'\hat{J}\left[\begin{Bmatrix} j_r & j_s & J \\ j_c & I & J' \end{Bmatrix}V^{(b)}(scr'c';J')\delta_{rs'}\right.$$
$$\left.+(-)^J\begin{Bmatrix} j_r & j_s & J \\ I & j_c & J' \end{Bmatrix}V^{(b)}(rcr'c';J')\delta_{ss'}\right], \tag{6A·3c}$$

where $\hat{J}=\sqrt{2J+1}$. We have used the following notations for the matrix elements of the interaction:

$$\begin{aligned} V^{(f)}(abcd;J)&=-(u_au_bu_cu_d+v_av_bv_cv_d)G(abcd;J)\\ &\quad-(u_au_bu_cv_d+v_au_bv_cu_d)F(abcd;J)\\ &\quad+(-)^{j_a+j_b+J}(v_au_bu_cv_d+u_av_bv_cu_d)F(bacd;J),\\ V^{(b)}(abcd;J)&=-(u_au_bv_cv_d+v_av_bu_cu_d)G(abcd;J)\\ &\quad+(u_av_bv_cu_d+v_au_bu_cv_d)F(abcd;J)\\ &\quad-(-)^{j_a+j_b+J}(v_au_bv_cu_d+u_av_bu_cv_d)F(bacd;J), \end{aligned} \tag{6A·4}$$

where the $G$ and $F$ type matrix elements are defined in Appendix 1A. In the text, we have expressed, for example, $F(rsaa;J)$ as $F(rs(\nu g_{9/2})^2;2)$ when $a$ denotes the $1g^+_{9/2}$ orbit for neutrons and $J=2$.

### 6A-2 *The case of pairing-plus-quadrupole force*

When we adopt the pairing-plus-quadrupole force, the matrix elements are simply obtained from Eq. (6A·3) by the following replacement:

$$\begin{aligned} &V^{(f)}(abcd;J)=-V^{(b)}(abcd;J)\\ &\quad\Longrightarrow-\frac{1}{2}\chi Q(ab)Q(cd)\delta_{J2}, \end{aligned} \tag{6A·5}$$

where

$$Q(ab)=\frac{1}{\sqrt{5}}(a\|r^2Y_2\|b)\cdot(u_av_b+v_au_b).$$

## Appendix 6B. Method of calculating transition matrix elements between nuclei with different nucleon numbers[9)]

In the case of evaluating beta-decay matrix elements or nucleon-transfer matrix elements, we must calculate the matrix elements between nuclei with different nucleon numbers. Since the quasi-particle representation is introduced by solving the BCS equations, i.e., (3·5) in Chap. 1, with given nucleon numbers (proton and neutron numbers) corresponding to a specific nucleus of interest, we must know the method of calculating transition matrix element between the states expressed by different quasi-particle representations. Here, within the intrinsic subspace in the quasi-spin space, we give a method suitable for this purpose.

Since Bogoliubov transformation is nothing but the rotation of the coordinate system around the $y$-axis in the quasi-spin space (composed of the direct product of quasi-spin subspace defined in each single-particle orbit), any quasi-particle representation is characterized by the set of rotation angles $\boldsymbol{\theta}=(\theta_a, \theta_b, \cdots)$, $\theta_a$ being the rotation angle for the subspace in the orbit $a$. (See Chap. 1.) The initial and final state vectors are therefore represented by $|\Gamma^{(i)}, \boldsymbol{\theta}^{(i)}\rangle$ and $|\Gamma^{(f)}, \boldsymbol{\theta}^{(f)}\rangle$, respectively. Here, $\Gamma^{(i)}$ and $\Gamma^{(f)}$ denote the sets of quantum numbers characterizing the initial and final states, respectively, and $\boldsymbol{\theta}^{(i)}$ and $\boldsymbol{\theta}^{(f)}$ the corresponding sets of rotation angles. In the intrinsic subspace (in the quasi-spin space), these states satisfy the condition

$$\hat{S}_{-}^{(i)}(a)|\Gamma^{(i)}, \boldsymbol{\theta}^{(i)}\rangle=\hat{S}_{-}^{(f)}(a)|\Gamma^{(f)}, \boldsymbol{\theta}^{(f)}\rangle=0, \tag{6B·1}$$

where superscript $(i)$ or $(f)$ of the quasi-spin operator $\hat{S}_{-}(a)$ denotes that it is represented in terms of the quasi-particle representation corresponding to the initial or final states, respectively. (See Chap. 1.)

Let us first notice that the initial state vector can also be expressed in the quasi-particle representation corresponding to the final state as

$$|\Gamma^{(i)}, \boldsymbol{\theta}^{(f)}\rangle=R(\boldsymbol{\theta}^{(f)}-\boldsymbol{\theta}^{(i)})|\Gamma^{(i)}, \boldsymbol{\theta}^{(i)}\rangle, \tag{6B·2}$$

where $R(\boldsymbol{\theta})=\exp\{i\sum_a \theta_a \hat{S}_y(a)\}$. (Since $\hat{S}_y(a)$ is invariant under this transformation, the superscript $(i)$ or $(f)$ is unnecessary for $\hat{S}_y(a)$.) The condition (6B·1) for the initial state is then re-expressed as

$$\hat{S}_{-}^{(f)}(a)|\Gamma^{(i)}, \boldsymbol{\theta}^{(f)}\rangle=0 \tag{6B·3}$$

for all $\hat{S}_{-}^{(f)}(a)$. Let $\hat{O}$ be the transition operator which is generally written as

$$\begin{aligned}\hat{O}&=\hat{O}([c_\alpha^\dagger], [c_\beta^\dagger], \cdots, [c_\gamma], [c_\delta], \cdots)\\ &=\sum\langle\alpha\beta\cdots|O|\gamma\delta\cdots\rangle\, c_\alpha^\dagger c_\beta^\dagger\cdots c_\gamma c_\delta\cdots.\end{aligned}$$

When $\hat{O}$ is expressed in terms of the quasi-particle operators in the representation corresponding to the final state, the transition matrix element under consideration takes the following form:

$$\begin{aligned}&\langle \Gamma^{(f)}, \boldsymbol{\theta}^{(f)}|\hat{O}|\Gamma^{(i)}, \boldsymbol{\theta}^{(i)}\rangle \\ &=\langle \Gamma^{(f)}, \boldsymbol{\theta}^{(f)}|\hat{O}([u_a^{(f)}a_a^{(f)\dagger}+v_a^{(f)}a_{\bar{a}}^{(f)}], \cdots \\ &\qquad [u_c^{(f)}a_\gamma^{(f)}+v_c^{(f)}a_{\bar{\gamma}}^{(f)\dagger}], \cdots)R^{-1}(\boldsymbol{\theta}^{(f)}-\boldsymbol{\theta}^{(i)})|\Gamma^{(i)}, \boldsymbol{\theta}^{(f)}\rangle,\end{aligned} \tag{6B·4}$$

where we have used the inverse transformation of (6B·2). In the above expression, all quasi-particle operators refer to the representation corresponding to the final state. By making full use of the conditions (6B·1) and (6B·3) with the aid of the identity

$$\begin{aligned}&\exp[\Delta\theta_a\{\hat{S}_+(a)-\hat{S}_-(a)\}/2] \\ &\quad=\exp[\hat{S}_+(a)\tan(\Delta\theta_a/2)]\cdot\exp[-2\hat{S}_0(a)\log\cos(\Delta\theta_a/2)] \\ &\qquad\times\exp[-\hat{S}_-(a)\tan(\Delta\theta_a/2)],\end{aligned} \tag{6B·5}$$

we finally obtain the expression suitable for the present purpose:

$$\langle \Gamma^{(f)}, \boldsymbol{\theta}^{(f)}|\hat{O}|\Gamma^{(i)}, \boldsymbol{\theta}^{(i)}\rangle=\langle \Gamma^{(f)}, \boldsymbol{\theta}|\hat{\mathcal{O}}|\Gamma^{(i)}, \boldsymbol{\theta}\rangle, \tag{6B·6}$$

$$\begin{aligned}\hat{\mathcal{O}}&=\hat{O}([u_a^{(i)}D(a)^{-1}a_a^\dagger+v_a^{(f)}a_{\bar{a}}], \cdots, \\ &\qquad [u_c^{(f)}a_\gamma+v_c^{(i)}D(c)^{-1}a_{\bar{\gamma}}^\dagger], \cdots)\cdot\hat{G} \\ &=\hat{G}\cdot\hat{O}([u_a^{(i)}a_a^\dagger+v_a^{(f)}D(a)^{-1}a_{\bar{a}}], \cdots, \\ &\qquad [u_c^{(f)}D(c)^{-1}a_\gamma+v_c^{(i)}a_{\bar{\gamma}}^\dagger], \cdots)\end{aligned} \tag{6B·7}$$

with

$$\hat{G}=\exp[-2\textstyle\sum_a\hat{S}_0(a)\log D(a)],$$

where $D(a)=\cos(\Delta\theta_a/2)=u_a^{(i)}u_a^{(f)}+v_a^{(i)}v_a^{(f)}$, $\Delta\theta_a$ being defined by $\Delta\theta_a=\theta_a^{(i)}-\theta_a^{(f)}$.

In the right-hand side of Eq. (6B·6), we have omitted the superscript $(f)$ for the set of rotation angles $\boldsymbol{\theta}$ and the quasi-particle operators; since, as is easily proved by the same procedure as above, the set of rotation angles to which we refer in evaluating the transition matrix element can be chosen arbitrarily. Thus, by replacing $\hat{O}$ with the "effective transition operator" $\hat{\mathcal{O}}$, we can exactly take account of the difference of nucleon numbers between the initial and final states. In the actual calculations, it may be sufficient to consider only the first order terms in $\Delta\theta_a$. Then, the effective operator $\hat{\mathcal{O}}$ takes much simplified form:

$$\hat{O} \simeq \hat{O}([u_a^{(i)} a_a^\dagger + v_a^{(f)} a_{\bar{a}}], \cdots, [u_c^{(f)} a_\gamma + v_c^{(i)} a_{\bar{\gamma}}^\dagger], \cdots). \tag{6B·8}$$

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# Part IV. A Next Subject

## Chapter 7. Coupling between Collective and Intrinsic Modes of Excitation

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### § 1. Introduction

In the preceding chapters we have shown that the low-lying collective excited states in spherical odd-mass nuclei can be successfully described in terms of the dressed $n$QP modes prescribed by the concept of transferred seniority. Since the dressed $n$QP modes are defined in the "intrinsic space," which does not involve any $J=0$-coupled quasi-particle pair, they are independent of the "collective" modes of pairing correlation within the NTD approximation. In this chapter we investigate the coupling between such independent modes of excitation.

Now, according to the canonical transformation method developed in Chap. 1, we can regard the space of states in terms of quasi-particles as a product space consisting of the "intrinsic" space and the "collective" (boson) space. In this representation, the original quasi-particle interaction is classified into three types: The first represents an interaction causing the mixing among the "intrinsic" states, the second among the "collective" states and the last between "collective" and "intrinsic" states. As has been shown in § 2 of Chap. 2, the first-type interaction in the intrinsic space can furthermore be divided into two parts, i.e., the constructive force which is responsible for constructing the dressed $n$QP modes, and the interactive force which manifests itself as the coupling among the different $n$QP modes. What we have investigated in Part III as the coupling effect is nothing but the effect originating from this interactive force.

The other new type of coupling effect may arise from the third-type interaction which causes the mixing between the collective and intrinsic states. Since the collective space involves all of the quantum fluctuations of the

pairing field, i.e., the excitation modes of $J=0$-coupled quasi-particle pairs, the third-type interaction manifests itself as a coupling between the pairing vibrational modes and the dressed $n$QP modes. In treating the mutual interweaving of such composite modes of excitation, there are well-known difficulties such as the overcompleteness in the degrees of freedom and the violation of Pauli principle. However, the independency of the "collective" pairing modes and the dressed $n$QP modes enables us to overcome these difficulties.

The main purpose of this chapter is to investigate the formal structure and physical implication of this coupling, leaving the detailed analysis of its effect in comparison with experiment as a next subject.

## § 2. The pairing Hamiltonian in collective representation

The original quasi-particle interaction $H_{\text{int}}$ given by Eq. (1·3·4)*) may be divided into two parts:

$$H_{\text{int}}=H_{\text{int}}^{(1)}+H_{\text{int}}^{(2)}, \tag{2·1}$$

where the first-kind interaction $H_{\text{int}}^{(1)}$ satisfies

$$[\hat{\boldsymbol{S}}(a)^2, H_{\text{int}}^{(1)}]=0 \text{ for each orbit } a, \tag{2·2}$$

and the second-kind interaction is defined by

$$[\hat{\boldsymbol{S}}(a)^2, H_{\text{int}}^{(2)}]\neq 0. \tag{2·3}$$

Here $\hat{\boldsymbol{S}}(\text{a})^2$ is the quasi-spin operator of orbit $a$,

$$\hat{\boldsymbol{S}}(a)^2=\hat{S}_+(a)\hat{S}_-(a)+\hat{S}_0(a)^2-\hat{S}_0(a),$$

where $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ are defined in Eq. (1·2·18). Since the quasi-spin quantum number $S(a)$ is known to be related to the seniority number $v_a$ of orbit $a$ through

$$S(a)=\frac{1}{2}(\Omega_a-v_a), \quad \left(\Omega_a\equiv j_a+\frac{1}{2}\right),$$

Eqs. (2·2) and (2·3) imply that the first-kind interaction $H_{\text{int}}^{(1)}$ does not violate the seniority number $v_a$ of each orbit, while the second-kind interaction $H_{\text{int}}^{(2)}$ changes the seniority number $v_a$.

In this section, we investigate the coupling between the "collective" and "intrinsic" degrees of freedom which originates from the first-kind interaction $H_{\text{int}}^{(1)}$. A typical example of the first-kind interaction is known to be the pairing interaction. Therefore, we adopt the pairing Hamiltonian given

*) We cite the equations in different chapters by adding the chapter number to the first place of the equation number.

by Eq. (1B·2) in Appendix 1B as an illustrative example. As a matter of convenience, we here leave the parameters $(u_a, v_a)$ of Bogoliubov transformation undetermined, although it is custom to determine the parameters $(u_a, v_a)$ so as to eliminate the "dangerous term" $H_1^{(p)}$ in Eq. (1B·2).

Applying the canonical transformation (1·5·8), we obtain the pairing Hamiltonian in the collective representation:

$$
\begin{aligned}
&\boldsymbol{H}^{(p)} = U^{(p)} + \boldsymbol{H}_0^{(p)} + \boldsymbol{H}_1^{(p)} + \boldsymbol{H}_{\mathrm{int}}^{(p)}, \qquad (2\cdot4)\\
&U^{(p)} = \sum_a 2\Omega_a \left\{ \left( \eta_a + \frac{1}{2} G v_a^2 \right) v_a^2 - \frac{1}{2} u_a v_a \Delta \right\},\\
&\boldsymbol{H}_0^{(p)} = \sum_a \{ \eta_a (u_a^2 - v_a^2) + 2 u_a v_a \Delta \} \{ \hat{n}_a + 2\hat{\boldsymbol{N}}(a) \},\\
&\boldsymbol{H}_1^{(p)} = \sum_a \{ 2\eta_a u_a v_a - (u_a^2 - v_a^2)\Delta \} \{ \boldsymbol{b}_a^\dagger \sqrt{\Omega_a - \hat{n}_a - \hat{\boldsymbol{N}}(a)} + \sqrt{\Omega_a - \hat{n}_a - \hat{\boldsymbol{N}}(a)}\, \boldsymbol{b}_a \},\\
&\boldsymbol{H}_{\mathrm{int}}^{(p)} = \boldsymbol{H}_X^{(p)} + \boldsymbol{H}_{V'}^{(p)} + \boldsymbol{H}_{V}^{(p)} + \boldsymbol{H}_{\mathrm{exch}}^{(p)},
\end{aligned}
$$

where

$$
\begin{aligned}
&\eta_a = \epsilon_a - \lambda - G v_a^2, \qquad \Omega_a = j_a + \frac{1}{2}, \qquad \Delta = G \sum_a \Omega_a u_a v_a,\\
&\hat{n}_a = \sum_{m_a} a_a^\dagger a_a, \qquad \hat{\boldsymbol{N}}(a) = \boldsymbol{b}_a^\dagger \boldsymbol{b}_a,
\end{aligned}
\qquad (2\cdot5)
$$

and

$$
\begin{aligned}
&\boldsymbol{H}_X^{(p)} = -G \sum_{ac} (u_a^2 u_c^2 + v_a^2 v_c^2) \boldsymbol{b}_a^\dagger \sqrt{\Omega_a - \hat{n}_a - \hat{\boldsymbol{N}}(a)} \cdot \sqrt{\Omega_c - \hat{n}_c - \hat{\boldsymbol{N}}(c)}\, \boldsymbol{b}_c \},\\
&\boldsymbol{H}_{V'}^{(p)} = \frac{1}{2} G \sum_{ac} (u_a^2 v_c^2 + v_a^2 u_c^2) \{ \boldsymbol{b}_a^\dagger \sqrt{\Omega_a - \hat{n}_a - \hat{\boldsymbol{N}}(a)} \cdot \boldsymbol{b}_c^\dagger \sqrt{\Omega_c - \hat{n}_c - \hat{\boldsymbol{N}}(c)} + \mathrm{h.c.} \},\\
&\boldsymbol{H}_{V}^{(p)} = G \sum_{ac} (u_a^2 - v_a^2) u_c v_c [ \boldsymbol{b}_a^\dagger \sqrt{\Omega_a - \hat{n}_a - \hat{\boldsymbol{N}}(a)} \cdot \{ \hat{n}_c + 2\hat{\boldsymbol{N}}(c) \} + \mathrm{h.c.} ],\\
&\boldsymbol{H}_{\mathrm{exch}}^{(p)} = -G \sum_{ac} u_a v_a u_c v_c [ \{ \hat{n}_a + 2\hat{\boldsymbol{N}}(a) \} \{ \hat{n}_c + 2\hat{\boldsymbol{N}}(c) \} - \{ \hat{n}_a + 2\hat{\boldsymbol{N}}(a) \} \delta_{ac} ].
\end{aligned}
\qquad (2\cdot6)
$$

In this collective representation, the boson operators $(\boldsymbol{b}_a^\dagger, \boldsymbol{b}_a)$ and the quasi-particle operators $(a_a^\dagger, a_a)$ describe the collective and intrinsic degrees of freedom respectively, and therefore their mutual interweaving is clearly visualized. Needless to say, the quasi-particle number $n_a$ of each orbit $a$ in this representation must be the same as the seniority number $v_a$, because of the supplementary condition (1·6·10) for the "intrinsic" state. In the case of the Hamiltonian with the first-kind interaction satisfying (2·2), therefore, we always have

$$
[\hat{n}_a, \boldsymbol{H}^{(p)}] = 0, \qquad (2\cdot7)
$$

which shows that the "intrinsic" state $|\phi_{\mathrm{intr}}\rangle$ must be an eigenstate of the quasi-particle number $\hat{n}_a$ of each orbit, i.e.,

$$
|\phi_{\mathrm{intr}}\rangle = |S(a), S_0(a) = -S(a); S(b), S_0(b) = -S(b); \cdots; \Gamma\rangle \qquad (2\cdot8)
$$

where $\Gamma$ denotes a set of additional quantum numbers to specify the intrinsic

eigenstate. This implies that the Tamm-Dancoff basis vectors in the "intrinsic" space discussed in §2-1 of Chap. 2 themselves become the intrinsic eigenstates of the Hamiltonian with the first-kind interaction, and they are never mixed with each other through their interweaving with collective pairing modes.

Now, according to the method developed in § 6 of Chap. 1, let us expand the pairing Hamiltonian (2·4) in terms of the creation and annihilation operators of the pairing vibrations

$$\begin{aligned} \boldsymbol{X}_{\mu}^{\dagger} &= \sum_{a}\{\psi_{\mu}(a)\boldsymbol{b}_{a}^{\dagger}+\phi_{\mu}(a)\boldsymbol{b}_{a}\}, \\ \boldsymbol{X}_{\mu} &= \sum_{a}\{\psi_{\mu}(a)\boldsymbol{b}_{a}+\phi_{\mu}(a)\boldsymbol{b}_{a}^{\dagger}\}, \end{aligned} \tag{2·9}$$

the details of which are given in Appendix 1B. Then, the expanded Hamiltonian takes the form given by Eq. (1·6·7):

$$\begin{aligned} \boldsymbol{H}^{(p)} &= H_{\mathrm{intr}}^{(p)}+\boldsymbol{H}_{\mathrm{I}}^{(p)}+\boldsymbol{H}_{\mathrm{II}}^{(p)}, \\ H_{\mathrm{intr}}^{(p)} &= \hat{h}_{00}^{(p)}, \\ \boldsymbol{H}_{\mathrm{I}}^{(p)} &= \sum_{\mu}\{\boldsymbol{X}_{\mu}^{\dagger}\hat{h}_{10}^{(p)}(\mu)+\boldsymbol{X}_{\mu}\hat{h}_{01}^{(p)}(\mu)\}, \\ \boldsymbol{H}_{\mathrm{II}}^{(p)} &= \frac{1}{2}\sum_{\mu\nu}\{\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}^{\dagger}\hat{h}_{20}^{(p)}(\mu\nu)+\boldsymbol{X}_{\nu}\boldsymbol{X}_{\mu}\hat{h}_{02}^{(p)}(\mu\nu)+2\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}\hat{h}_{11}^{(p)}(\mu\nu)\}, \end{aligned} \tag{2·10}$$

where we have consistently neglected all terms which involve commutators of $\mathring{H}^{(p)}$ with $X_{\mu}^{\dagger}$ (or $X_{\mu}$) higher than double. The operators $\hat{h}_{ij}^{(p)}$ only involve the intrinsic degrees of freedom represented in terms of the quasi-particles and are defined by Eq. (1·6·8). According to the same procedure as used in deriving Eq. (1·6·14b), the pairing Hamiltonian (2·10) can be effectively reduced to the form

$$\begin{aligned} \boldsymbol{H}^{(p)} &= H_{\mathrm{intr}}^{(p)}+\boldsymbol{H}_{\mathrm{I}}^{(p)}+\boldsymbol{H}_{\mathrm{II}}^{(p)}, \\ H_{\mathrm{intr}}^{(p)} &= \mathring{H}^{(p)}-\frac{1}{2}\sum_{\mu\nu a}\phi_{\mu}(a)\phi_{\nu}(a)\{[X_{\mu},[\mathring{H}^{(p)},X_{\nu}^{\dagger}]]+\mathrm{h.c.}\} \\ &\quad -\frac{1}{2}\sum_{\mu\nu a}\phi_{\mu}(a)\psi_{\nu}(a)\{[X_{\mu},[X_{\nu},\mathring{H}^{(p)}]]+[[\mathring{H}^{(p)},X_{\nu}^{\dagger}],X_{\mu}^{\dagger}]\}, \\ \boldsymbol{H}_{\mathrm{I}}^{(p)} &= \sum_{\mu}\{\boldsymbol{X}_{\mu}^{\dagger}[X_{\mu},\mathring{H}^{(p)}]+\boldsymbol{X}_{\mu}[\mathring{H}^{(p)},X_{\mu}^{\dagger}]\}, \\ \boldsymbol{H}_{\mathrm{II}}^{(p)} &= \frac{1}{2}\sum_{\mu\nu}\{\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}^{\dagger}[X_{\mu},[X_{\nu},\mathring{H}^{(p)}]]+\boldsymbol{X}_{\nu}\boldsymbol{X}_{\mu}[[\mathring{H}^{(p)},X_{\nu}^{\dagger}],X_{\mu}^{\dagger}] \\ &\qquad +2\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}[X_{\mu},[\mathring{H}^{(p)},X_{\nu}^{\dagger}]\}. \end{aligned} \tag{2·11}$$

In the case of the pairing Hamiltonian, the commutators of $\mathring{H}^{(p)}$ with $X_{\mu}^{\dagger}$ (or $X_{\mu}$) which appear in Eq. (2·11) are easily calculated: Provided that the supplementary condition (1·6·10) is always kept to be satisfied properly, we obtain

$$[X_\mu, \mathring{H}^{(p)}]=[\mathring{H}^{(p)}, X_\mu^\dagger]$$
$$=\sum_a\{\psi_\mu(a)-\phi_\mu(a)\}\{2\eta_a u_a v_a-(u_a^2-v_a^2)\hat{\Delta}'\}\sqrt{\Omega_a-\hat{n}_a}\,, \quad (2\cdot12a)$$

$$[X_\mu, [\mathring{H}^{(p)}, X_\nu^\dagger]]=(\boldsymbol{\phi}_\mu^T, \boldsymbol{\psi}_\mu^T)\begin{bmatrix}\hat{\boldsymbol{d}}' & \hat{\boldsymbol{a}}'\\ \hat{\boldsymbol{a}}' & \hat{\boldsymbol{d}}'\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_\nu\\ \boldsymbol{\phi}_\nu\end{bmatrix}, \quad (2\cdot12b)$$

$$[X_\mu, [X_\nu, \mathring{H}^{(p)}]]=[[\mathring{H}^{(p)}, X_\nu^\dagger], X_\mu^\dagger]=-(\boldsymbol{\phi}_\mu^T, \boldsymbol{\psi}_\mu^T)\begin{bmatrix}\hat{\boldsymbol{d}}' & \hat{\boldsymbol{a}}'\\ \hat{\boldsymbol{a}}' & \hat{\boldsymbol{d}}'\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_\nu\\ \boldsymbol{\phi}_\nu\end{bmatrix}, \quad (2\cdot12c)$$

where the matrices $\hat{\boldsymbol{a}}'$ and $\hat{\boldsymbol{d}}'$ are defined by

$$\begin{aligned}\hat{d}'_{ab}&=2(\hat{E}'_a-Gu_a^2v_a^2)\delta_{ab}-G(u_a^2u_b^2+v_a^2v_b^2)\sqrt{(\Omega_a-\hat{n}_a)(\Omega_b-\hat{n}_b)}\,,\\ \hat{a}'_{ab}&=2Gu_a^2v_a^2\delta_{ab}-G(u_a^2v_b^2+v_a^2u_b^2)\sqrt{(\Omega_a-\hat{n}_a)(\Omega_b-\hat{n}_b)}\end{aligned} \quad (2\cdot13)$$

with

$$\begin{aligned}\hat{\Delta}'&\equiv G\sum_a u_a v_a(\Omega_a-\hat{n}_a),\\ \hat{E}'_a&\equiv\eta_a(u_a^2-v_a^2)+2u_a v_a\hat{\Delta}'.\end{aligned} \quad (2\cdot14)$$

At this step, let us self-consistently determine the parameters $(u_a, v_a)$ of Bogoliubov transformation and the amplitudes $(\psi_\mu(a), \phi_\mu(a))$ of the pairing vibrational modes in Eq. (2·7). The parameters $(u_a, v_a)$ are determined, with the aid of the intrinsic eigenstate (2·8), by the condition

$$\langle\phi_{\text{intr}}|\boldsymbol{H}_{\text{I}}^{(p)}|\phi_{\text{intr}}\rangle=0, \quad (2\cdot15)$$

$$2\eta_a u_a v_a-G(u_a^2-v_a^2)\sum_b(\Omega_b-n_b)u_b v_b=0, \quad (2\cdot16)$$

which is just the gap equation with the blocking effects. The amplitudes $(\psi_\mu(a), \phi_\mu(a))$ are then determined in order to diagonalize the matrix $\langle\phi_{\text{intr}}|\boldsymbol{H}_{\text{II}}^{(p)}|\phi_{\text{intr}}\rangle$, i.e.,

$$\begin{bmatrix}\boldsymbol{d}' & \boldsymbol{a}'\\ -\boldsymbol{a}' & -\boldsymbol{d}'\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_\mu\\ \boldsymbol{\phi}_\mu\end{bmatrix}=\omega_\mu\begin{bmatrix}\boldsymbol{\psi}_\mu\\ \boldsymbol{\phi}_\mu\end{bmatrix}, \quad \omega_\mu\geq 0, \quad (2\cdot17)$$

where the $c$-number matrices $\boldsymbol{a}'$ and $\boldsymbol{d}'$ are given by replacing the quasi-particle number operator $\hat{n}_a$ in the matrices $\hat{\boldsymbol{a}}'$ and $\hat{\boldsymbol{d}}'$ with its eigenvalue $n_a$ in the intrinsic eigenstate (2·8). Equation (2·17) is just the eigenvalue equation of the pairing vibrational modes with the blocking effects. Here it is worthy of note that, in spite of the inclusion of the blocking effects, Eq. (2·17) certainly has the zero-energy solution just as the usual eigenvalue equation (without the blocking effects) does.

It is now clear that the main effects of the coupling between the "collective" and "intrinsic" degrees of freedom, which originates from the first-kind

interaction $H_{\mathrm{int}}^{(1)}$, can be renormalized into both the quasi-particle field and the pairing vibrational modes.

## §3. Expressions of the Hamiltonian and electromagnetic multipole operators in terms of pairing vibrational modes and dressed $n$QP modes

In this section we develop a theory in which the mutual interweaving of the "collective" and "intrinsic" degrees of freedom can be treated in terms of the "collective" pairing vibrational modes and the "intrinsic" dressed $n$QP modes. The independency of the pairing vibrational modes and the dressed $n$QP modes (within the NTD approximation) enables us to overcome the well-known difficulties in determining the coupling between composite modes, such as the overcompleteness in the degrees of freedom and the violation of Pauli principle.

### 3-1 *Coupling Hamiltonian*

Contrary to the first-kind interaction $H_{\mathrm{int}}^{(1)}$, the second-kind interaction $H_{\mathrm{int}}^{(2)}$ changes the seniority number of each orbit. Hence the second-kind interaction remarkably affects the structure of the "intrinsic" space, the basis vectors of which are characterized by the seniority number $v_a$ of each orbit. In fact, as shown in Part II, this kind of interaction constitutes the main part of the quasi-particle interaction in the intrinsic Hamiltonian $H_{\mathrm{intr}}$ given by Eq. (1·6·14). In Part III we have investigated the various effects on the structure of the "intrinsic" states, that are caused mainly by the second-kind interaction, and shown that many properties of the spherical odd-mass nuclei are characterized by these effects. Also in the coupling between the "collective" and "intrinsic" degrees of freedom, we expect that the second-kind interaction causes more complex effects than those originating from the first-kind interaction.

When the second-kind interaction becomes effective, the eigenvector of the quasi-spin operator (2·8) is no longer the eigenvector of the intrinsic space. In this case, the coupling between the collective and intrinsic degrees of freedom, which originates from the first-kind interaction, becomes difficult to be simply renormalized into the quasi-particle field or the pairing vibrational modes. Therefore, in this section we treat the original quasi-particle interaction as a whole, without insisting on such a separation of the interaction into the first- and second-kind interactions.

We start our discussion with the collective representation of the original Hamiltonian $H$ given by Eq. (1·6·14):

$$H \longrightarrow \text{const}+\boldsymbol{H}_{\mathrm{col}}+H_{\mathrm{intr}}+H_{\mathrm{coupl}},$$
$$\boldsymbol{H}_{\mathrm{col}}=\sum_{\mu}\omega_{\mu}\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\mu},$$

$$H_{\text{intr}}=H-\frac{1}{2}\sum_{\mu\nu a}\phi_{\mu}(a)\phi_{\nu}(a)\{[X_{\mu}, Z_{\nu}]+\text{h.c.}\}$$
$$-\frac{1}{2}\sum_{\mu\nu a}\phi_{\mu}(a)\psi_{\nu}(a)\{[X_{\mu}, Z_{\nu}^{\dagger}]+[Z_{\mu}, X_{\nu}^{\dagger}]\},$$

$$H_{\text{coupl}}=\sum_{\mu}(\boldsymbol{X}_{\mu}^{\dagger}Z_{\mu}^{\dagger}+\boldsymbol{X}_{\mu}Z_{\mu})+\frac{1}{2}\sum_{\mu\nu}\{2\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}[X_{\mu}, Z_{\nu}]$$
$$+\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\nu}^{\dagger}[X_{\mu}, Z_{\nu}^{\dagger}]+\boldsymbol{X}_{\nu}\boldsymbol{X}_{\mu}[Z_{\nu}, X_{\mu}^{\dagger}]\},$$

where $Z$ denotes the interaction which is neglected in constructing the pairing vibrational modes within the RPA, and where all terms which involve the commutators of $Z_{\nu}$ (or $Z_{\nu}^{\dagger}$) with $X_{\mu}^{\dagger}$ (or $X_{\mu}$) higher than single are neglected. In § 5-2 of Chap. 2, we have given the transcription rule, by which any physical operator depending only on the intrinsic degrees of freedom can be unambiguously transcribed into the quasi-particle NTD space. With the aid of the transcription rule, the intrinsic Hamiltonian $H_{\text{intr}}$ has already been expressed in terms of the dressed $n$QP modes (see § 5-3 of Chap. 2). Now we express the coupling Hamiltonian $H_{\text{coupl}}$ in terms of the pairing vibrational modes and the dressed $n$QP modes. The parts written by thin letters in the coupling Hamiltonian $H_{\text{coupl}}$ only involve the intrinsic degrees of freedom represented in terms of the quasi-particle operators. Thus, with the aid of the transcription rule, these parts can also be transcribed into the quasi-particle NTD space as the intrinsic space: With the creation and annihilation operators of the dressed $n$QP modes ($Y_{s\lambda}^{\dagger}$, $Y_{s\lambda}$), we have

$$H_{\text{coupl}}\longrightarrow\sum_{s\lambda s'\lambda'\mu}\langle\Phi_0|Y_{s\lambda}Z_{\mu}Y_{s'\lambda'}^{\dagger}|\Phi_0\rangle(\boldsymbol{Y}_{s\lambda}^{\dagger}\boldsymbol{Y}_{s'\lambda'}\boldsymbol{X}_{\mu}+\text{h.c.})$$
$$+\sum_{s\lambda s'\lambda'\mu\mu'}\langle\Phi_0|Y_{s\lambda}[X_{\mu}, Z_{\mu'}]Y_{s'\lambda'}^{\dagger}|\Phi_0\rangle\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{Y}_{s\lambda}^{\dagger}\boldsymbol{Y}_{s'\lambda'}\boldsymbol{X}_{\mu'}$$
$$+\frac{1}{2}\sum_{s\lambda s'\lambda'\mu\mu'}\langle\Phi_0|Y_{s\lambda}[Z_{\mu'}, X_{\mu}^{\dagger}]Y_{s'\lambda'}^{\dagger}|\Phi_0\rangle(\boldsymbol{Y}_{s\lambda}^{\dagger}\boldsymbol{Y}_{s'\lambda'}\boldsymbol{X}_{\mu'}\boldsymbol{X}_{\mu}+\text{h.c.}). \quad (3\cdot1)$$

The matrix elements in the above expression are then easily evaluated using the transcription rule (2·5·8).

Thus, the original Hamiltonian is expressed in terms of the pairing vibrational modes and the dressed $n$QP modes as follows:

$$H\longrightarrow\text{const}+\boldsymbol{H}_{\text{col}}+\boldsymbol{H}_{\text{intr}}+\boldsymbol{H}_{\text{coupl}}, \quad (3\cdot2a)$$
$$\boldsymbol{H}_{\text{col}}=\sum_{\mu}\omega_{\mu}\boldsymbol{X}_{\mu}^{\dagger}\boldsymbol{X}_{\mu},$$
$$\boldsymbol{H}_{\text{intr}}=\sum_{\alpha}E_{\alpha}\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\alpha}+\sum_{\lambda}\omega_{\lambda}\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda}+\sum_{\alpha\lambda}V_{\text{int}}(\alpha, \lambda)(\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{a}_{\alpha}+\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{Y}_{\lambda}), \quad (3\cdot2b)$$
$$\boldsymbol{H}_{\text{coupl}}=\sum_{\alpha\mu}E_{\alpha}(\mu)(\boldsymbol{X}_{\mu}^{\dagger}+\boldsymbol{X}_{\mu})\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\alpha}+\sum_{\lambda\lambda'\mu}V_{\text{int}}(\mu; \lambda, \lambda')(\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda'}\boldsymbol{X}_{\mu}+\text{h.c.})$$
$$+\sum_{\alpha\lambda\mu}V_{\text{int}}(\mu; \lambda, \alpha)(\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{a}_{\alpha}\boldsymbol{X}_{\mu}+\text{h.c.})+\sum_{\alpha\lambda\mu}V_{\text{int}}(\mu; \alpha, \lambda)(\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{Y}_{\lambda}\boldsymbol{X}_{\mu}+\text{h.c.}).$$

In the above expression we have adopted the same quasi-particle NTD subspace as employed in § 5 of Chap. 2, which consists of the 1QP and dressed

3QP modes, and furthermore we have given explicitly the effects of lowest order of the coupling between the collective and intrinsic degrees of freedom. The explicit forms of $E_a(\mu)$ and $V_{\rm int}(\mu;\cdots)$ are given in Appendix 7A.

### 3-2 *Effective electromagnetic multipole operators in collective-intrinsic-coupled system*

For the investigation of the system in which the collective and intrinsic modes are coupled to each other, it is necessary to express physical operators such as electromagnetic multipole operators in terms of the collective and intrinsic modes of excitation. This is easily performed in the same way as was done for the Hamiltonian. Any physical operator $\hat{F}$ is first transformed into the collective representation in terms of the pairing-vibration modes $(\boldsymbol{X}^\dagger_\mu,\ \boldsymbol{X}_\mu)$:

$$\begin{aligned}\hat{\boldsymbol{F}} &\equiv U_{\rm col}\cdot\mathring{F}\cdot U^{-1}_{\rm col}\\ &=\mathring{F}+\sum_\mu\{\boldsymbol{X}^\dagger_\mu[X_\mu,\mathring{F}]+\boldsymbol{X}_\mu[\mathring{F},X^\dagger_\mu]\}+\cdots,\end{aligned}\tag{3·3}$$

where $U_{\rm col}$ is the canonical transformation defined by Eq. (1·5·8) and $\mathring{F}$ denotes the operator extended into the extended quasi-spin space discussed in § 4 of Chap. 1. The first term depends only on the intrinsic degrees of freedom represented in terms of the quasi-particle operators. Therefore, in the same way as was done in § 5-3 of Chap. 2, the first term can be expressed in terms of the dressed $n$QP modes by the use of the transcription rule. The parts written by thin letters in the second term are composed of the quasi-particle operators representing the intrinsic degrees of freedom. Consequently, these parts can also be easily expressed in terms of the dressed $n$QP modes, with the aid of the transcription rule (2·5·8). Here, we give the expression for the case where the operator $\hat{F}$ represent the electromagnetic multipole operator $\hat{O}^{(\pm)}_{LM}$: In this case, the term corresponding to the first term of Eq. (3·3) has already been expressed by Eq. (2·5·14). For the corresponding second term, we obtain

$$\begin{aligned}&\sum_\mu\{\boldsymbol{X}^\dagger_\mu[X_\mu,\mathring{O}^{(\pm)}_{LM}]+\boldsymbol{X}_\mu[\mathring{O}^{(\pm)}_{LM},X^\dagger_\mu]\}\\ &\quad\longrightarrow\sum_{\alpha\beta\mu}\{\langle\Phi_0|a_\alpha[X_\mu,\mathring{O}^{(\pm)}_{LM}]a^\dagger_\beta|\Phi_0\rangle\boldsymbol{X}^\dagger_\mu\boldsymbol{a}^\dagger_\alpha\boldsymbol{a}_\beta\\ &\qquad+\langle\Phi_0|a_\alpha[\mathring{O}^{(\pm)}_{LM},X^\dagger_\mu]a^\dagger_\beta|\Phi_0\rangle\boldsymbol{a}^\dagger_\alpha\boldsymbol{a}_\beta\boldsymbol{X}_\mu\}\\ &\qquad+\sum_{\lambda\lambda'\mu}\{\langle\Phi_0|Y_\lambda[X_\mu,\mathring{O}^{(\pm)}_{LM}]Y^\dagger_{\lambda'}|\Phi_0\rangle\boldsymbol{X}^\dagger_\mu\boldsymbol{Y}^\dagger_\lambda\boldsymbol{Y}_{\lambda'}\\ &\qquad+\langle\Phi_0|Y_\lambda[\mathring{O}^{(\pm)}_{LM},X^\dagger_\mu]Y^\dagger_{\lambda'}|\Phi_0\rangle\boldsymbol{Y}^\dagger_\lambda\boldsymbol{Y}_{\lambda'}\boldsymbol{X}_\mu\},\end{aligned}\tag{3·4a}$$

$$\begin{aligned}&\sum_\mu\{\boldsymbol{X}^\dagger_\mu[X_\mu,\bar{O}^{(\pm)}_{LM}]+\boldsymbol{X}_\mu[\bar{O}^{(\pm)}_{LM},X^\dagger_\mu]\}\\ &\quad\longrightarrow\sum_{\alpha\lambda\mu}\{\langle\Phi_0|Y_\lambda[X_\mu,\bar{O}^{(\pm)}_{LM}]a^\dagger_\alpha|\Phi_0\rangle\boldsymbol{X}^\dagger_\mu\boldsymbol{Y}^\dagger_\lambda\boldsymbol{a}_\alpha\\ &\qquad+\langle\Phi_0|a_\alpha[X_\mu,\bar{O}^{(\pm)}_{LM}]Y^\dagger_\lambda|\Phi_0\rangle\boldsymbol{X}^\dagger_\mu\boldsymbol{a}^\dagger_\alpha\boldsymbol{Y}_\lambda\}\\ &\qquad+\sum_{\alpha\lambda\mu}\{\langle\Phi_0|Y_\lambda[\bar{O}^{(\pm)}_{LM},X^\dagger_\mu]a^\dagger_\alpha|\Phi_0\rangle\boldsymbol{Y}^\dagger_\lambda\boldsymbol{a}_\alpha\boldsymbol{X}_\mu\\ &\qquad+\langle\Phi_0|a_\alpha[\bar{O}^{(\pm)}_{LM},X^\dagger_\mu]Y^\dagger_\lambda|\Phi_0\rangle\boldsymbol{a}^\dagger_\alpha\boldsymbol{Y}_\lambda\boldsymbol{X}_\mu\}.\end{aligned}\tag{3·4b}$$

The explicit forms of the matrix elements in the above expressions are given in Appendix 7B.

Thus we have derived all necessary expressions for the Hamiltonian and the electromagnetic multipole operators, in terms of the "collective" pairing vibrational modes and the "intrinsic" dressed $n$QP modes. In this way, we have obtained a theory, by which we can systematically study the structure of the coupling between the pairing vibrational modes and the dressed $n$QP modes.

## §4. Concluding remarks

We have studied some physical implications of the coupling between the "collective" and "intrinsic" degrees of freedom, according to the method developed in § 6 of Chap. 1. In this method, all physical operators such as the Hamiltonian and the electromagnetic multipole operators are expressed in a form of expansion in terms of the creation and annihilation operators of the pairing vibrational modes. When the interaction of the original Hamiltonian does not violate the seniority number $v_a$ of each orbit, the coupling between the collective and intrinsic degrees of freedom becomes very simple. We have shown by adopting the pairing Hamiltonian that the coupling can be renormalized into both the quasi-particle field and the pairing vibrational modes as the blocking effects. On the other hand, as shown in Part III, the interaction which does change the seniority number $v_a$ of each orbit causes various significant effects on the structure of the intrinsic states. Such a kind of interaction causes also the coupling between the collective and intrinsic degrees of freedom bringing about abundant effects on the structure of the spherical odd-mass nuclei.

Since the dressed $n$QP modes are defined in the intrinsic space, which does not involve any $J=0$-coupled quasi-particle pair, they are independent of the "collective" modes of pairing correlation within the NTD approximation. This independency of the dressed $n$QP modes and the pairing vibrational modes enables us to overcome the well-known difficulties in treating the mutual interweaving of the composite modes, such as the overcompleteness in the degrees of freedom and the violation of Pauli principle. Thus, using this independency, we have developed a theory, by which the coupling between the collective and the intrinsic degrees of freedom can be systematically studied in terms of the interplay between the pairing vibrational modes and the dressed $n$QP modes.

Recent accumulation of various kinds of experimental data is illuminating the structure of the couplings among composite modes of excitation. In the light of experimental development, the method of mode-mode coupling becomes one of hopeful approaches to understand the mechanism of the change in the structure of nuclei. The "collective" pairing vibrational modes represent

just the fluctuation of the "spherical" quasi-particle field. Hence, the coupling between the "collective" pairing vibrational modes and the "intrinsic" dressed $n$QP modes is expected to provide a wealth of information on the mechanism of the change of the "spherical" quasi-particle field into, for example, a "deformed" field. Thus, it is one of the subjects of growing interest to systematically study the coupling between the pairing vibrational modes and the dressed $n$QP modes in comparison with experimental data.

## Appendix 7A. Coupling between pairing vibrational modes and dressed $n$QP modes

We give the explicit forms of the matrix elements appearing in the coupling Hamiltonian, $\boldsymbol{H}_{\text{coupl}}$, defined by Eq. (3·2b).

### 7A-1 *Coupling originating from part $H_Y$ of original interaction*

The coupling originating from the part $H_Y$ of the original interaction $H_{\text{int}}$ given by (1·3·4) is obtained by using the commutator of $H_Y$ with the pairing vibrational mode $X_\mu^\dagger$ (or $X_\mu$):

$$[H_Y, X_\mu^\dagger]=\Sigma_\alpha E_\alpha(\mu)\hat{n}_\alpha+H_X(\mu)+H_V(\mu), \tag{7A·1}$$

$$H_X(\mu)\equiv\Sigma_{\alpha\beta\gamma\delta}V_X(\mu;\,\alpha\beta\gamma\delta)a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma,$$

$$H_V(\mu)\equiv\Sigma_{\alpha\beta\gamma\delta}\{V_{V1}(\mu;\,\alpha\beta\tilde{\gamma}\tilde{\delta})a_\alpha^\dagger a_\beta^\dagger a_\delta^\dagger a_\gamma^\dagger+V_{V2}(\mu;\,\alpha\beta\tilde{\gamma}\tilde{\delta})a_\gamma a_\delta a_\beta a_\alpha\},$$

where the following notations are used;

$$E_\alpha(\mu)\equiv-\sqrt{2}\,\Sigma_{\varepsilon_1\varepsilon_2}[\{V_Y(\varepsilon_1\varepsilon_2\alpha\tilde{\alpha})+2V_Y(\alpha\varepsilon_1\alpha\tilde{\varepsilon}_2)\}\{\psi_\mu(\varepsilon_1\varepsilon_2)-\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2)\}], \tag{7A·2a}$$

$$V_X(\mu;\,\alpha\beta\gamma\delta)\equiv-\frac{1}{\sqrt{2}}\Sigma_{\varepsilon_1\varepsilon_2}[\{V_Y'(\gamma\delta\alpha\tilde{\varepsilon}_1)\delta_{\beta\varepsilon_2}-V_Y'(\gamma\delta\beta\tilde{\varepsilon}_1)\delta_{\alpha\varepsilon_2}\}\psi_\mu(\varepsilon_1\varepsilon_2)$$
$$-\{V_Y'(\alpha\beta\gamma\tilde{\varepsilon}_1)\delta_{\delta\varepsilon_2}-V_Y'(\alpha\beta\delta\tilde{\varepsilon}_1)\delta_{\gamma\varepsilon_2}\}\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2)], \tag{7A·2b}$$

$$V_Y'(\alpha\beta\gamma\tilde{\delta})\equiv V_Y(\alpha\beta\gamma\tilde{\delta})+V_Y(\delta\alpha\gamma\tilde{\beta})-V_Y(\delta\beta\gamma\tilde{\alpha}),$$

$$V_{V1}(\mu;\,\alpha\beta\tilde{\gamma}\tilde{\delta})\equiv-\frac{1}{\sqrt{2}}\Sigma_{\varepsilon_1\varepsilon_2}\{V_Y(\alpha\beta\varepsilon_1\tilde{\delta})\delta_{\gamma\varepsilon_2}-V_Y(\alpha\beta\varepsilon_1\tilde{\gamma})\delta_{\delta\varepsilon_2}\}\psi_\mu(\varepsilon_1\varepsilon_2), \tag{7A·2c}$$

$$V_{V2}(\mu;\,\alpha\beta\tilde{\gamma}\tilde{\delta})\equiv-\frac{1}{\sqrt{2}}\Sigma_{\varepsilon_1\varepsilon_2}\{V_Y(\alpha\beta\varepsilon_1\tilde{\gamma})\delta_{\delta\varepsilon_2}-V_Y(\alpha\beta\varepsilon_1\tilde{\delta})\delta_{\gamma\varepsilon_2}\}\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2). \tag{7A·2d}$$

Here the amplitudes of the pairing vibrational modes, $\psi_\mu(\varepsilon_1\varepsilon_2)$ and $\phi_\mu(\varepsilon_1\varepsilon_2)$, are related to those defined by Eq. (1B·9) in Appendix 1B through

$$\psi_\mu(\varepsilon_1\varepsilon_2)=(j_e j_e m_{\varepsilon_1} m_{\varepsilon_2}|00)\psi_\mu(e),$$

$$\phi_\mu(\varepsilon_1\varepsilon_2)=(j_e j_e m_{\varepsilon_1} m_{\varepsilon_1}|00)\phi_\mu(e),$$

and the matrix element of the original $H_Y$-type interaction, $V_Y(\alpha\beta\gamma\delta)$, are

given after Eq. (1·3·4). In Eq. (7A·1), we have adopted the notations similar to those for the original Hamiltonian, such as $E_a$, $H_X$ and $H_V$, paying attention to their formal similarity. However, it should be noted that, contrary to the original Hamiltonian, the $H_X(\mu)$ and $H_V(\mu)$ are not hermitian and hence the order of the indices of the matrix element $V_X(\mu;\ \alpha\beta\gamma\delta)$ and the indices $i$ of $V_{Vi}(\mu;\ \alpha\beta\gamma\delta)$ have important meanings.

In the same way as in the case of the intrinsic Hamiltonian, the transcription of the operator (7A·1) into the quasi-particle NTD space can be easily performed by the use of the transcription rule (2·5·8). Using the matrices $\boldsymbol{D}^i$, $\boldsymbol{d}^i$ and $\boldsymbol{A}^i$ defined in Appendix 2B, the matrix element $V_{\rm int}(\mu;\ \lambda,\ \lambda')$ in Eq. (3·2b) is given by

$$\begin{aligned} V_{\rm int}(\mu;\ \lambda,\lambda') &= \langle \Phi_0 |\, Y_\lambda [H_V,\ X_\mu^\dagger]\, Y_{\lambda'}^\dagger |\, \Phi_0 \rangle \\ &= (\boldsymbol{\phi}_\lambda^T,\ \boldsymbol{\phi}_\lambda^T) \begin{bmatrix} 3\boldsymbol{D}^i & -\boldsymbol{A}^1 \\ (-\boldsymbol{A}^2)^T & \boldsymbol{d}^i \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}_{\lambda'} \\ \boldsymbol{\phi}_{\lambda'} \end{bmatrix}, \end{aligned} \tag{7A·3}$$

with the following replacements in the matrix elements of $\boldsymbol{D}^i$, $\boldsymbol{d}^i$ and $\boldsymbol{A}^i$ $(i=1, 2)$:

$$\begin{aligned} &E_a^i \Rightarrow E_a(\mu), \quad V_{\alpha\beta\gamma\delta}^{(fi)} \Rightarrow 2V_X(\mu;\ \alpha\beta\gamma\delta), \quad V_{V1}(\alpha\beta\gamma\delta) \Rightarrow V_{V1}(\mu;\ \alpha\beta\gamma\delta), \\ &V_{V2}(\alpha\beta\gamma\delta) \Rightarrow V_{V2}(\mu;\ \alpha\beta\gamma\delta). \end{aligned} \tag{7A·4}$$

7A-2 *Coupling originating from parts $H_X$ and $H_V$ of original interaction*

The coupling resulting from the parts $H_X$ and $H_V$ of the original interaction $H_{\rm int}$ is derived from the commutator of $H_X+H_V$ with $X_\mu^\dagger$ (or $X_\mu$):

$$\begin{aligned} [H_X+H_V,\ X_\mu^\dagger] &\Rightarrow \sum_{\alpha\beta\gamma\delta} \{ V_{V1}(\mu;\ \alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_{\tilde\delta}^\dagger a_\gamma + V_{V2}(\mu;\ \alpha\beta\gamma\delta) a_\gamma^\dagger a_{\tilde\delta} a_\beta a_\alpha \}, \\ &\equiv H_V(\mu), \end{aligned} \tag{7A·5}$$

where

$$\begin{aligned} V_{V1}(\mu;\ \alpha\beta\gamma\tilde\delta) \equiv & -2\sqrt{2} \sum_{\varepsilon_1\varepsilon_2} [ V_X(\alpha\beta\gamma\varepsilon_1)\delta_{\delta\varepsilon_2}\psi_\mu(\varepsilon_1\varepsilon_2) \\ & - \{ V_V(\tilde\alpha\tilde\beta\varepsilon_1\delta) + V_V(\varepsilon_1\delta\tilde\alpha\tilde\beta) \} \delta_{\gamma\varepsilon_2}\phi_\mu(\tilde\varepsilon_1\tilde\varepsilon_2) ], \end{aligned} \tag{7A·6a}$$

$$\begin{aligned} V_{V2}(\mu;\ \alpha\beta\gamma\tilde\delta) \equiv & 2\sqrt{2} \sum_{\varepsilon_1\varepsilon_2} [ V_X(\alpha\beta\gamma\varepsilon_1)\delta_{\delta\varepsilon_2}\phi_\mu(\tilde\varepsilon_1\tilde\varepsilon_2) \\ & - \{ V_V(\tilde\alpha\tilde\beta\varepsilon_1\delta) + V_V(\varepsilon_1\delta\tilde\alpha\tilde\beta) \} \delta_{\gamma\varepsilon_2}\psi_\mu(\varepsilon_1\varepsilon_2) ], \end{aligned} \tag{7A·6b}$$

and the matrix elements of the original $H_X$- and $H_V$-type interactions, $V_X(\alpha\beta\gamma\delta)$ and $V_V(\alpha\beta\gamma\delta)$, are given after Eq. (1·3·4). Needless to say, the operator $H_V(\mu)$ is not hermitian.

Paying attention to the formal similarity of $H_V(\mu)$ to $H_V$, we can express the operator (7A·5) in terms of the 1QP and dressed 3QP modes, with the aid of the transcription rule (2·5·8). Using the vector $\boldsymbol{B}(\alpha)$ defined in Appendix 2C, the matrix elements $V_{\rm int}(\mu;\ \lambda,\ \alpha)$ and $V_{\rm int}(\mu;\ \alpha,\ \lambda)$ in Eq. (3·2b) are given by

$$V_{\rm int}(\mu;\,\lambda,\,\alpha)=\langle\Phi_0|\,Y_\lambda[H_X+H_V,\,X_\mu^\dagger]a_\alpha^\dagger|\Phi_0\rangle$$
$$=(\boldsymbol{\psi}_\lambda^T,\,\boldsymbol{\phi}_\lambda^T)\cdot\boldsymbol{B}(\alpha),\qquad(7\text{A}\cdot7\text{a})$$

$$V_{\rm int}(\mu;\,\alpha,\,\lambda)=\langle\Phi_0|a_\alpha[H_X+H_V,\,X_\mu^\dagger]\,Y_\lambda^\dagger|\Phi_0\rangle$$
$$=\boldsymbol{B}^T(\alpha)\cdot\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\phi}_\lambda\end{bmatrix}\qquad(7\text{A}\cdot7\text{b})$$

with the following replacements in the elements of $\boldsymbol{B}(\alpha)$:

$$V_{Y1}(\alpha\beta\gamma\delta)\Rightarrow V_{Y1}(\mu;\,\alpha\beta\gamma\delta),\quad V_{Y2}(\alpha\beta\gamma\delta)\Rightarrow V_{Y2}(\mu;\,\alpha\beta\gamma\delta),\qquad(7\text{A}\cdot8)$$

for the former relation (7A·7a), and

$$V_{Y1}(\alpha\beta\gamma\delta)\Rightarrow V_{Y2}(\mu;\,\alpha\beta\gamma\delta),\quad V_{Y2}(\alpha\beta\gamma\delta)\Rightarrow V_{Y1}(\mu;\,\alpha\beta\gamma\delta),\qquad(7\text{A}\cdot9)$$

for the latter relation (7A·7b).

## Appendix 7B. Matrix elements of electromagnetic multipole operators in collective-intrinsic-coupled system

For the study of the system in which the collective and intrinsic modes of excitation are coupled to each other, it is necessary to express the electromagnetic multipole operators in terms of the collective and intrinsic modes of excitation. Here, we give the explicit forms of the matrix elements involved in the expressions (3·4a) and (3·4b).

We first take the commutator of the electromagnetic operator $\hat{O}_{LM}^{(\pm)}$ with the pairing vibrational mode $X_\mu^\dagger$ (or $X_\mu$):

$$[\mathring{O}_{LM}^{(\pm)},\,X_\mu^\dagger]=C_{LM}^{(\pm)}(\mu)+\sum_{\alpha\beta}\bar{O}_{LM}^{(\pm)}(\mu;\,\alpha\beta)a_\alpha^\dagger a_\beta,\qquad(7\text{B}\cdot1\text{a})$$

$$[\bar{O}_{LM}^{(\pm)},\,X_\mu^\dagger]=\sum_{\alpha\beta}\{\mathring{O}_{1LM}^{(\pm)}(\mu;\,\alpha\beta)a_\alpha^\dagger a_{\tilde{\beta}}^\dagger\pm\mathring{O}_{2LM}^{(\pm)}(\mu;\,\alpha\beta)a_{\tilde{\beta}}a_{\tilde{\alpha}}\},\qquad(7\text{B}\cdot1\text{b})$$

where $\mathring{O}_{LM}^{(\pm)}$ and $\bar{O}_{LM}^{(\pm)}$ denote the first and second terms of $\hat{O}_{LM}^{(\pm)}$ defined by Eq. (1·5·12), respectively, and the following notations are used;

$$C_{LM}^{(\pm)}(\mu)\equiv\sqrt{8}\sum_{\varepsilon_1\varepsilon_2}\{\mathring{O}_{LM}^{(\pm)}(\varepsilon_1\varepsilon_2)\{\psi_\mu(\varepsilon_1\varepsilon_2)\mp\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2)\}\frac{1\pm1}{2},\qquad(7\text{B}\cdot2\text{a})$$

$$\bar{O}_{LM}^{(\pm)}(\mu;\,\alpha\beta)\equiv\mp2\sqrt{2}\sum_{\varepsilon_1\varepsilon_2}\{\mathring{O}_{LM}^{(\pm)}(\tilde{\varepsilon}_1\tilde{\beta})\delta_{\alpha\varepsilon_2}\psi_\mu(\varepsilon_1\varepsilon_2)\pm\mathring{O}_{LM}^{(\pm)}(\tilde{\varepsilon}_1\alpha)\delta_{\tilde{\beta}\varepsilon_2}\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2)\},\qquad(7\text{B}\cdot2\text{b})$$

$$\mathring{O}_{1LM}^{(\pm)}(\mu;\,\alpha\beta)\equiv\frac{1}{\sqrt{2}}\sum_{\varepsilon_1\varepsilon_2}\{\bar{O}_{LM}^{(\pm)}(\alpha\varepsilon_1)\delta_{\beta\varepsilon_2}-\bar{O}_{LM}^{(\pm)}(\beta\varepsilon_1)\delta_{\alpha\varepsilon_2}\}\psi_\mu(\varepsilon_1\varepsilon_2),\qquad(7\text{B}\cdot2\text{c})$$

$$\mathring{O}_{2LM}^{(\pm)}(\mu;\,\alpha\beta)\equiv-\frac{1}{\sqrt{2}}\sum_{\varepsilon_1\varepsilon_2}\{\bar{O}_{LM}^{(\pm)}(\alpha\varepsilon_1)\delta_{\beta\varepsilon_2}-\bar{O}_{LM}^{(\pm)}(\beta\varepsilon_1)\delta_{\alpha\varepsilon_2}\}\phi_\mu(\tilde{\varepsilon}_1\tilde{\varepsilon}_2).\qquad(7\text{B}\cdot2\text{d})$$

Here we have used notations similar to the original electromagnetic multipole matrix elements $\mathring{O}^{(\pm)}_{LM}(\alpha\beta)$ and $\bar{O}^{(\pm)}_{LM}(\alpha\beta)$ defined by Eq. (1·5·13). It should be noted, however, that the time-reversal property of the quantities defined by Eq. (7B·2) is different from that of the original matrix elements $\mathring{O}^{(\pm)}_{LM}(\alpha\beta)$ and $\bar{O}^{(\pm)}_{LM}(\alpha\beta)$.

In the same way as was done in § 5-3 of Chap. 2, the operators (7B·1a) and (7B·1b) can be easily expressed in terms of the 1QP and dressed 3QP modes, because they involve only the intrinsic degrees of freedom represented in terms of the quasi-particle operators. Thus, using the matrix elements explicitly defined by Eq. (2D·3) in Appendix 2D, the matrix elements in the expression (3·4) are given by

$$\langle\Phi_0|\, Y_\lambda[\bar{O}^{(\pm)}_{LM}, X^\dagger_\mu]a^\dagger_\alpha|\Phi_0\rangle=\langle\Phi_0|\, Y_\lambda\mathring{F}^{(\pm)}_{LM}a^\dagger_\alpha|\Phi_0\rangle, \tag{7B·3a}$$

$$\langle\Phi_0|a_\alpha[\bar{O}^{(\pm)}_{LM}, X^\dagger_\mu]\, Y^\dagger_\lambda|\Phi_0\rangle=\langle\Phi_0|a_\alpha\mathring{F}^{(\pm)}_{LM}\, Y^\dagger_\lambda|\Phi_0\rangle, \tag{7B·3b}$$

$$\begin{aligned}\langle\Phi_0|a_\alpha[\mathring{O}^{(\pm)}_{LM}, X^\dagger_\mu]a^\dagger_\beta|\Phi_0\rangle&=C^{(\pm)}_{LM}(\mu)\delta_{\alpha\beta}+\langle\Phi_0|a_\alpha\bar{F}^{(\pm)}_{LM}a^\dagger_\beta|\Phi_0\rangle\\&=C^{(\pm)}_{LM}(\mu)\delta_{\alpha\beta}+\bar{O}^{(\pm)}_{LM}(\mu;\alpha\beta),\end{aligned} \tag{7B·3c}$$

$$\langle\Phi_0|\, Y_\lambda[\mathring{O}^{(\pm)}_{LM}, X^\dagger_\mu]\, Y^\dagger_{\lambda'}|\Phi_0\rangle=C^{(\pm)}_{LM}(\mu)\delta_{\lambda\lambda'}+\langle\Phi_0|\, Y_\lambda\bar{F}^{(\pm)}_{LM}\, Y^\dagger_{\lambda'}|\Phi_0\rangle \tag{7B·3d}$$

with the following replacements in the matrix elements of $\mathring{F}^{(\pm)}_{LM}$ and $\bar{F}^{(\pm)}_{LM}$:

$$\mathring{F}^{(\pm)}_{\{\}LM}(\alpha\beta)\Rightarrow\mathring{O}^{(\pm)}_{\{\}LM}(\mu;\alpha\beta),\qquad\bar{F}^{(\pm)}_{LM}(\alpha\beta)\Rightarrow\bar{O}^{(\pm)}_{LM}(\mu;\alpha\beta). \tag{7B·4}$$

The other matrix elements in the expressions (3·4a) and (3·4b) are given in a similar form.