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# Symmetry-unrestricted Skyrme-HFB Calculations for Exotic Shapes in Proton-Rich $N = Z$ Nuclei in the $A = 60 - 80$ Region*

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**Abstract**

By performing a fully 3D symmetry-unrestricted Skyrme-HFB calculation, we discuss the possibility of exotic deformations violating both reflection and axial symmetries in proton-rich $N = Z$ nuclei; $^{64}$Ge, $^{68}$Se, $^{72}$Kr, $^{76}$Sr, $^{80}$Zr and $^{84}$Mo. The calculation indicates that the oblate ground state of $^{68}$Se is extremely soft against the $Y_{33}$ triangular deformation, and that the low-lying "spherical" minimum coexisting with the prolate ground state in $^{80}$Zr may be unstable against the $Y_{32}$ tetrahedral deformation.

## Introduction

In $N = Z$ nuclei with $A = 60 - 80$, proton and neutron shell effects act coherently and rich possibilities arise for coexistence/competition of different shapes. Recently, on the basis of the Skyrme Hartree-Fock(HF) plus BCS calculation with no restriction on the nuclear shape, Takami, Yabana and Matsuo suggested that the oblate ground state of $^{68}$Se is extremely soft against the $Y_{33}$ triangular deformation, and that the low-lying "spherical" minimum coexisting with the prolate ground state in $^{80}$Zr has the $Y_{32}$ tetrahedral shape [1,2]. We examine this prediction by carrying out a fully three dimensional (3D), selfconsistent Skyrme HF-Bogoliubov (HFB) calculation with the use of the density-dependent, zero-range pairing interaction.

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## A fully 3D HFB calculation

Two years ago, we had constructed a new computer code for cranked Skyrme HF calculation on a 3-dimensional Cartesian-mesh space without imposing any restrictions on the spatial symmetry. The first application of this code was made for investigation of the high-spin yrast structure of $^{32}$S [3,4]. This code has also been used (see Fig.1) for the analysis of the superdeformed (SD) band recently discovered [5] in $^{36}$Ar. We are presently investigating what will happen above the SD band termination at spin 16 [6]. This code has been extended to the HF-Bogoluibov (HFB) version including the pairing correlations by means of the algorithm called the "two basis method" [8,9]. Here, the imaginary-time evolution method is combined with a diagonalization of the HFB Hamiltonian matrix to construct the canonical basis. Single-particle wave functions and densities are represented on a 3-dimensional Cartesian mesh space in a spherical box without assuming any spatial symmetry. The radius of spherical box and mesh spacing are set to 10.0 [fm] and 1.0 [fm], respectively. Potential energy surfaces are evaluated by means of the constrained HFB procedure. As in Ref. [9], we use Skyrme III for the mean-field (particle-hole) channel and the density-dependent zero-range interaction

$$V_{pair}(\vec{r}_1, \vec{r}_2) = V_0 \left(1 - \hat{P}_\sigma\right) \left(1 - \frac{\rho\left(\vec{r}_1\right)}{\rho_c}\right) \delta\left(\vec{r}_1 - \vec{r}_2\right) \tag{1}$$

for the pairing (particle-particle) channel, where $\rho(r)$ denotes the total nuclear density and $\hat{P}_\sigma$ the spin exchange operator. The standard parameters [10], $V_0 = -1000.0$ [MeV $\cdot$ fm$^3$] and $\rho_c = 0.16$ [fm$^{-3}$], are used. We are presently examining the dependence on different versions of the Skyrme force as well as on the pairing-force strength $V_0$ [7].

## Shapes coexisting near the ground states

Results of calculation are shown in Figs. 2-4 and Table 1. As is expected, the calculated ground-state shape changes from triaxial ($^{64}$Ge), oblate ($^{68}$Se, $^{72}$Kr) to large prolate shape ($^{76}$Sr, $^{80}$Zr) with increasing $Z(= N)$. As seen in Fig.2, we obtain two or three local minima close in energy, indicating shape coexistence, in $^{68}$Se, $^{72}$Kr, $^{76}$Sr and $^{80}$Zr. Quite recently, a prolate excited band ($\beta_2 = 0.27$) was found near the oblate ground band ($\beta_2 = -0.27$) in $^{68}$Se [11]. Our result of calculation is consistent with these data. The calculated barrier height (about 300 keV) between the oblate and prolate minima, shown in Fig.3, might seem too low to sustain the shape coexistence. We would like to stress, however, that careful consideration of dynamics is required to evaluate the mixing between the two minima.. This is a quite challenging open subject. As seen in Fig.4, our calculation indicates that the oblate solution of $^{68}$Se is unstable (extremely soft) with respect to the triangular $Y_{33}$ deformation, and that the spherical solution of $^{80}$Zr is unstable (extremely soft) against the tetrahedral $Y_{32}$ deformation. As mentioned in Ref. [1], the instability toward the triangular deformation in the oblate regime in $^{68}$Se is caused by the strong $Y_{33}$ coupling between the high $\Omega$ [404]9/2 and [413]7/2 levels (stemming from $g_{9/2}$ ) and the [301]3/2 and [310]1/2 levels (associated with $p_{3/2}$). On the other hand, the tendency toward the tetrahedral $Y_{32}$ shape in $^{80}$Zr is associated with the fact that $N = Z = 40$ is a magic number for this shape [12].

### Concluding remarks

For $N = Z$ nuclei in the $^{68}$Se - $^{80}$Zr region, we have presented some examples of the HFB solution that indicate instabilities toward non-axial octupole deformations. The result is consistent with that of Refs. [1,2]. Investigation of excitation spectra associated with the new type of symmetry breaking in the selfconsistent mean fields remains as a challenging subject of coming years for both theorists and experimentalists.

## References


[1] S. Takami, K. Yabana and M. Matsuo, Phys. Lett. B431 (1998) 242.

[2] M. Matsuo, S. Takami and K. Yabana, *Nuclear Structure '98*, AIP Conference Proceedings 481, ed. C. Baktash (1999), p.345.

[3] M. Yamagami and K. Matsuyanagi, *Nuclear Structure '98*, AIP Conference Proceedings 481, ed. C. Baktash (1999), p.327.

[4] M. Yamagami and K. Matsuyanagi, Nucl. Phys. A672 (2000) 123.

[5] C. E. Svensson et al., these proceedings.

[6] S. Mizutori, T. Inakura, M. Yamagami and K. Matsuyanagi, in preparation.

[7] M. Yamagami, K. Matsuyanagi and M. Matsuo, in preparation.

[8] B. Gall, P. Bonche, J. Dobaczewski, H. Flocard, P.-H. Heenen, Z. Phys. A348 (1994) 183.

[9] J. Terasaki, P.-H. Heenen, H. Flocard and P. Bonche, Nucl. Phys. A600 (1996) 371.

[10] J. Terasaki, H. Flocard, P.-H. Heenen, P. Bonche, Nucl. Phys. A621 (1997) 706.

[11] S.M. Fischer et al., Phys. Rev. Lett. 84 (2000) 4064.

[12] I. Hamamoto, B. Mottelson, H. Xie and X.Z. Zhang, Z. Phys. D21 (1991) 163.

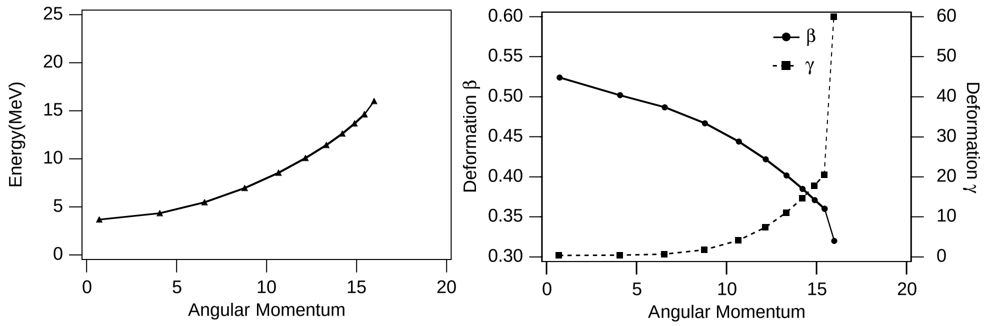


Figure 1: Excitation energies and deformation parameters of the superdeformed solution in $^{36}$Ar, obtained in the cranked Skyrme HF calculation [6].

| | Oblate | Spherical | Prolate |
|---|---|---|---|
| $^{64}$Ge | | | g.s.<br>$\beta, \gamma = 0.27, 25°$ (triaxial)<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.25, \Delta_n = 1.12$ |
| $^{68}$Se | g.s.<br>$\beta, \gamma = 0.28, 60°$<br>$\beta_3 = \beta_{33} \approx 0.08$<br>$\Delta_p = 1.28, \Delta_n = 1.13$ | | 0.52<br>$\beta, \gamma = 0.26, 0°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.29, \Delta_n = 1.15$ |
| $^{72}$Kr | g.s.<br>$\beta, \gamma = 0.32, 60°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.03, \Delta_n = 1.23$ | | 0.92<br>$\beta, \gamma = 0.40, 0°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.25, \Delta_n = 0.92$ |
| $^{76}$Sr | 1.79<br>$\beta, \gamma = 0.30, 60°$<br>$\beta_3 = \beta_{33} \approx 0.0$<br>$\Delta_p = 1.47, \Delta_n = 1.43$ | | g.s.<br>$\beta, \gamma = 0.51, 0°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 0.67, \Delta_n = 0.50$ |
| $^{80}$Zr | 0.86<br>$\beta, \gamma = 0.20, 60°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.02, \Delta_n = 0.82$ | 1.01<br>$\beta, \gamma = 0.0, 0°$<br>$\beta_3 = \beta_{32} \approx 0.15$<br>$\Delta_p = 0.68, \Delta_n = 0.39$ | g.s.<br>$\beta, \gamma = 0.51, 0°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 0.79, \Delta_n = 0.78$ |
| $^{84}$Mo | 0.20<br>$\beta, \gamma = 0.16, 60°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 1.46, \Delta_n = 1.42$ | g.s.<br>$\beta, \gamma = 0.0, 0°$<br>$\beta_3 = \beta_{30} \approx 0.0$<br>$\Delta_p = 0.74, \Delta_n = 0.72$ | 1.52<br>$\beta, \gamma = 0.66, 0°$<br>$\beta_3 = 0.0$<br>$\Delta_p = 0.0, \Delta_n = 0.0$ |

Table 1: HFB solutions for proton-rich $N = Z$ nuclei in the $A = 60-80$ region. The numbers in the first lines indicate excitation energies measured from the ground state solutions. The symbol $\approx$ indicates that the potential energy surface is extremely shallow about the equilibrium value. Pairing gaps $\Delta_p$ and $\Delta_n$ are here defined as averages of diagonal elements $\Delta_{i\bar{i}}$ over 5 MeV interval in the vicinity of the Fermi surface, and their values at the equilibrium deformations are listed.

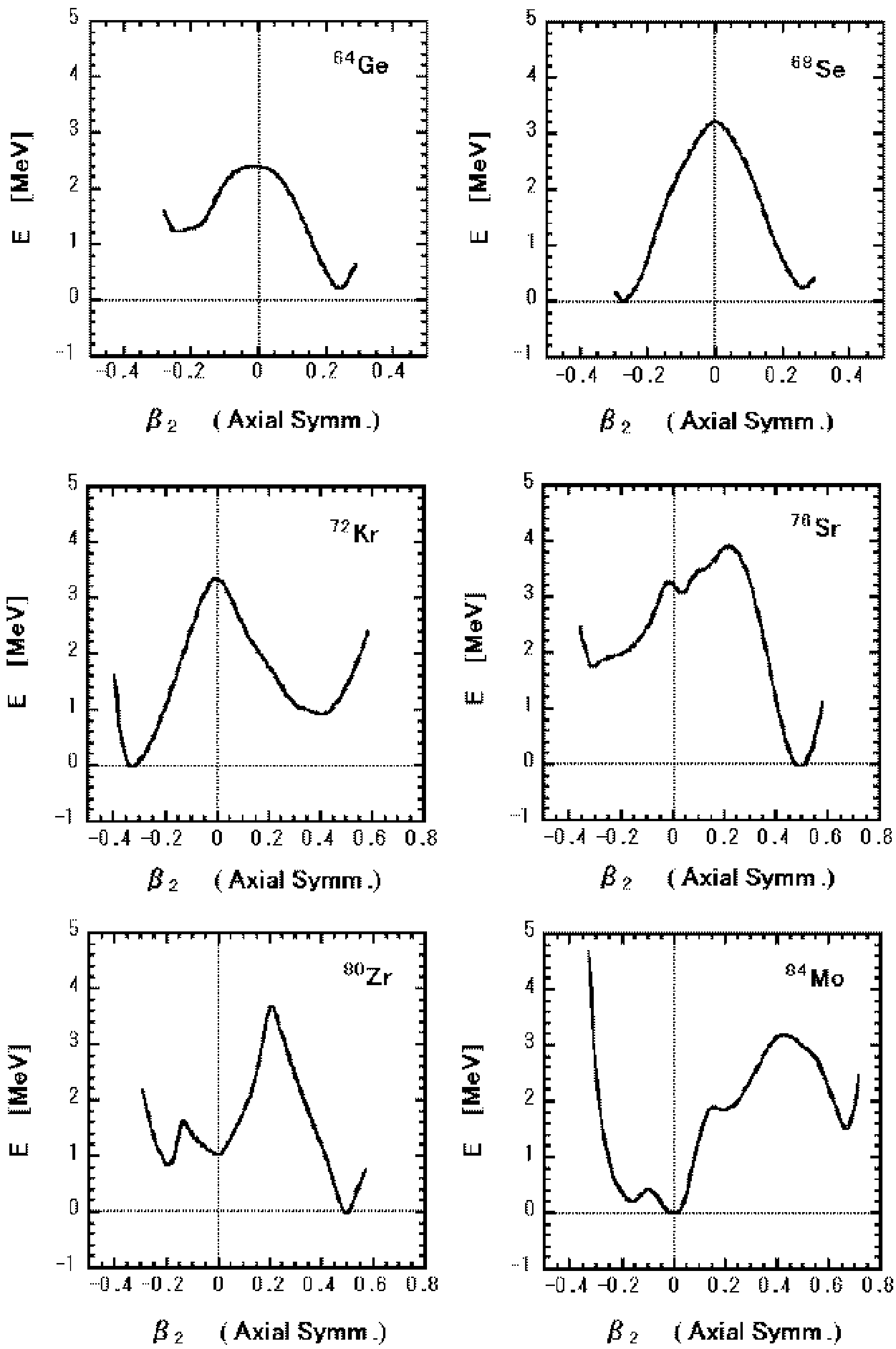


Figure 2: Potential energies for $^{64}$Ge, $^{68}$Se, $^{72}$Kr, $^{76}$Sr, $^{80}$Zr and $^{84}$Mo drawn as functions of the quadrupole deformation $\beta_2$.

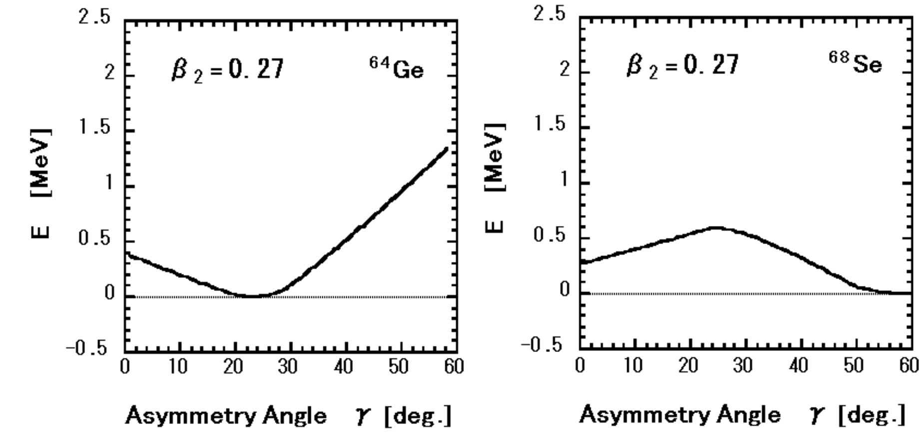


Figure 3: Potential energies for $^{64}$Ge and $^{68}$Se drawn at fixed $\beta_2$ as functions of the triaxial deformation parameter $\gamma$.

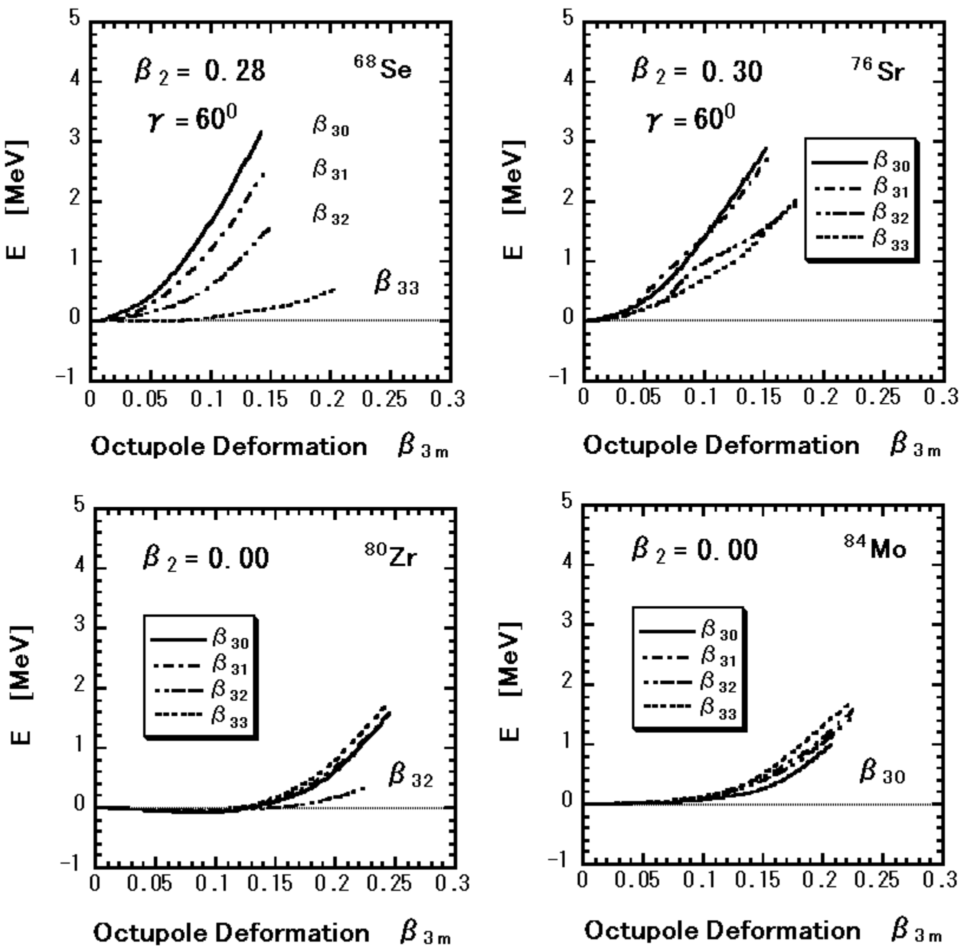


Figure 4: Potential energies for the oblate solutions of $^{68}$Se, $^{76}$Sr, and for the spherical solutions of $^{80}$Zr, $^{84}$Mo, drawn as functions of the octupole deformation parameters $\beta_{3m}(m = 0, 1, 2, 3;$ see Ref. [4] for their definitions).