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Structure of Anomalous Coupling $(j-1)$ States

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In spherical odd-mass nuclei in which an opposite parity orbit of large spin j in the major shell (such as $1f_{7/2}^{-}$, $1g_{9/2}^{+}$) is presumably being filled, there is a competition between a spin j- and spin $(j-1)$-state for the ground state. The extra low-lying states with spin $(j-1)$ and with opposite parity have been called the anomalous coupling states (A.C.S.).

Low-lying excited states with spin $(j-1)$ have also been found not a few in the $1h_{11/2}^{-}$ region. Moreover, strongly enhanced $E2$ transitions between the $(j-1)$ states and the j states are observed in recent experiments.[1] The enhancements of the transitions are camparable to those of phonon transitions in neighboring even-even nuclei.

In the conventional phonon-quasi-particle coupling theory, one would expect to observe a "quintet" with $j-2\leqq I\leqq j+2$ resulting from the coupling of the odd quasi-particle (in the orbit j) to the phonon at about the energy of the 2^+-phonon state in neighboring doubly even nuclei, since there is no $j'=I\neq j$ orbit with the same parity inside the major shell.

However, experimentally one finds that $(j-1)$ states lie close in energy to the

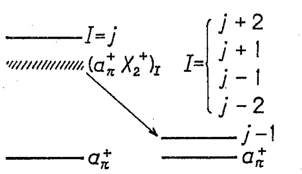

Fig. 1. Splitting of a quintet and an appearance of $(j-1)$ state close to a ground state.

ground states, which means that the concept of "quintet" is strongly violated and therefore the $(j-1)$ states should be recognized as new collective excitation modes.

In connection with the possibility of new collective modes, it should be noticed[2] that the three quasi-particle correlations illustrated in Fig. 2 have been neglected

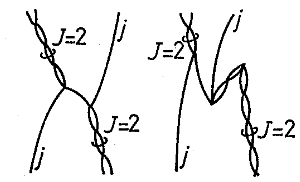

Fig. 2. Diagrams representing three quasi-particle correlations. The phonon disassociates into a pair of quasi-particle, one of which reassociates with the odd quasi-particle while the remaining quasi-particle is now the odd quasi-particle.

completely in the conventional phonon-quasi-particle coupling theory. The characteristic of this type of "phonon-quasi-particle coupling" lies in the following point. The more significant the effect becomes, the more the concept of two independent excitations (quasi-particles and phonons) becomes violated, and they are resolved to form the new collective modes.

With the idea, we have proposed the concept of dressed three quasi-particle modes as new collective modes in spherical odd-mass nuclei in a previous paper.[3] According to the theory, we construct the eigen-mode operators for A.C.S. as follows:

$$Y_{nIK}^{\dagger}=\sum_{\alpha\beta\gamma}\sum_{s_0=3/2,1/2}\psi_{nIK}^{\pi}(\alpha\beta\gamma:s_0)\,\bar{\delta}_{\alpha\pi}\bar{\delta}_{\beta\pi}\bar{\delta}_{\gamma\pi}:T_{3/2s_0}^{\pi}(\alpha\beta\gamma):+\sum_{b,c\neq\pi}\sum_{m_\pi m_\beta m_\gamma}\psi_{nIK}^{\xi}\times(\pi\beta\gamma)\,a_\pi^{\dagger}a_\beta^{\dagger}a_\gamma^{\dagger}+\sum_{b,c\neq\pi}\sum_{m_\pi m_\beta m_\gamma}\times\phi_{nIK}^{\xi}(\pi\beta\gamma)\,a_\pi^{\dagger}\tilde{a}_\beta\tilde{a}_\gamma\,,$$

where $T_{3/2s_0}^{\pi}(\alpha\beta\gamma)$ means the quasi-spin tensor of rank 3/2 composed of the quasi-particle creation and annihilation operators in a large spin opposite parity orbit π, and the summation with respect to b, c should be carried over proton and neutron orbits in a major shell except for the π orbit. It should be noticed that $Y^{\dagger}_{nIK}$ becomes very simple from parity considerations. Furthermore, because of the special situation of shell structure, this new mode appears relatively "pure" (the coupling to the single particle state is forbidden in our truncated shell model space).

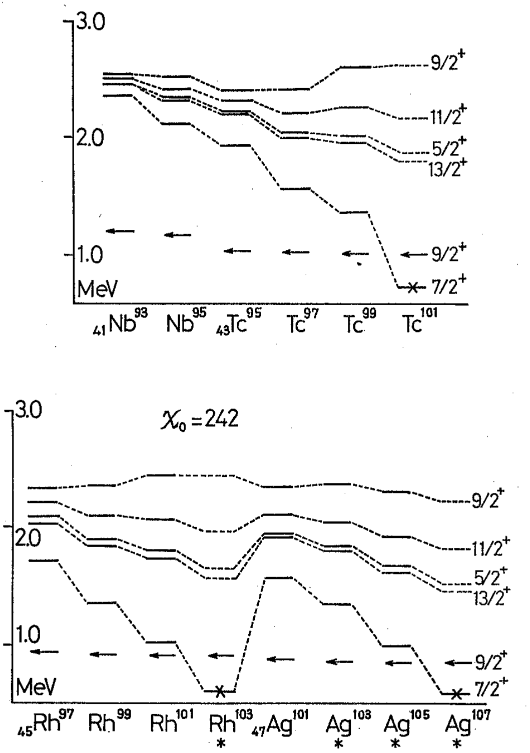

Fig. 3. Excitation energies of dressed three quasi-particle modes. Single quasi-particle energies in orbit π are written by arrows. It should be noticed that all energies are measured from the ground states of their modes. Thus the differences of these energies are those which correspond to the spectra of odd-mass nuclei. The symbol "×" means that the calculated energy of $(j-1)$ state becomes smaller than the single quasi-particle energy E_{π}. In this case other angular momentum states are written by broken lines. The nuclei whose $(j-1)$ states are found below j states are denoted by the asterisk *.

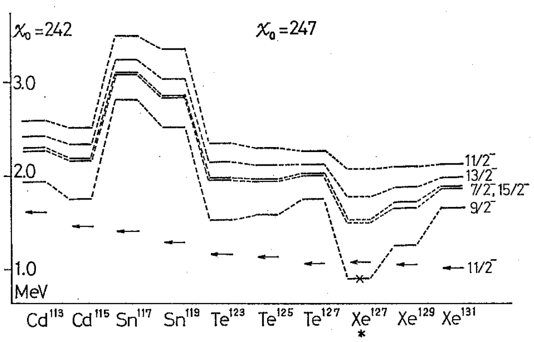

Fig. 4. Excitation energies of dressed three quasi-particle modes. Notations are the same as Fig. 3.

Adopting the pairing plus quadrupole force model, we can easily calculate the excitation energies for these modes. In Figs. 3∼4 we present some results, where pairing force strength and single particle energies are taken from the work of Kisslinger and Sorensen.[4] Quadrupole force strength χ are regarded as free parameters in each shell region except for the usual mass number dependence, i.e.,

$$\chi=\chi_0\cdot A^{-7/3}.$$

Adopted values of χ_0 are written in the Figures.

From Nb^{93} to Ag^{107}, and also in each isotope, the calculated energies of $7/2^+$ states go down as a function of mass number A, and this precisely corresponds to the experimental result. The behavior of $7/2^+$ states is something like that of RPA phonons, but the effect of quardrupole force χ is strengthened only for the $7/2^+$ states due to the three quasi-particle correlations, so the excitation energies of $7/2^+$ states go down faster than those of phonons. The growth of three quasi-particle correlations is demonstrated in these calculations to be very strong although it may be somewhat overestimated as is usual in the new-Tamm-Dancoff approximations.

The $9/2^-$ states found in Cd, Te, Xe isotopes are also well explained in these calculations with a reasonable value of χ. These states have been explained as Kisslinger's three quasi-particle states.[5]

If 9/2⁻ states are such states (extra degree of freedom out of his phonon-quasi-particle coupling theory), then we can expect 9/2⁻ states also in Sn isotopes that are single closed nuclei. But none of such a level is observed up to now.[6] The reason is well understood when we consider 9/2⁻ states as "dressed three quasi-particle states", as is shown in Fig. 4, because in such single closed nuclei collectiveness due to the other shells becomes weak and so $(j-1)$ states lie at about 1 MeV higher positions.

Interesting result is that in Te isotopes the trend of excitation energies ω_{nI} is smooth but in Xe isotopes the change of ω_{nI} is rapid and at Xe^{127}, the $9/2^-$ state cross below the $11/2^-$ (single quasi-particle) state. This indicates the instability of the spherical ground state. Experiments also show[7] that the $9/2^-$ states appear below $11/2^-$ states in neutron deficient Xe isotopes below Xe^{127} and that the spectra of adjacent even-even nuclei become more and more quasi-rotational.

Thus, there is a close connection between the growth of three quasi-particle correlation (i.e., lowering of the $(j-1)$ states) and the phase transition "spherical" to "deformed".

This work was done in collaboration with Prof. T. Marumori and Dr. A. Kuriyama. Details will appear in this journal.

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$|\Delta I|=1/2$ Rule for Nonleptonic Kaon Decays, Duality and Current Algebra

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In a previous paper,[1] we investigated weak nonleptonic kaon decays assuming duality, current algebra and absence of exotic resonances. Expressions for relevant amplitudes are obtained, which enjoy all the conditions stated above. The $|\Delta I|=1/2$ rule is then satisfied automatically, whereby the $|\Delta I|=3/2$ interaction is ruled out.[2] One may not be surprised, however, if the scheme in which only the spurion has an exotic quantum number $(I=3/2)$ and intermediate resonances have ordinary ones leads to internal contradiction.

In this letter it will be shown that $|\Delta I|=3/2$ parts are incompatible with duality and current algebra. We do not assume non-existence of exotic resonances. All the possible Regge trajectories are taken into account, that is, $I=0$, 1, 2 trajectories in $\pi\pi$ channels and $I=1/2$, 3/2 in $K\pi$ channels. The resonances with $I=2$, 3/2 have not yet been established experimentally, so

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Theory of Collective Excitations in Spherical Odd-Mass Nuclei. I

——*Basic Ideas and Concept of Dressed Three-Quasi-Particle Modes*——

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A new systematic theory of describing the collective excitations in spherical odd-mass nuclei is developed. The theory can be regarded as a direct extension of the conventional quasi-particle-new-Tamm-Dancoff method (i.e. the quasi-particle-random phase approximation) for spherical even-mass nuclei into the case of spherical odd-mass nuclei. In order to construct properly the theory within the framework of the quasi-particle-new-Tamm-Dancoff method, it is shown to be decisive to introduce a new concept which precisely specifies the "dressed" three-quasi-particle modes. The new concept is recognized in connection with the "quasi-spin space" which has been introduced through the quasi-spin formalism for the pairing correlations. It is not the purpose of this paper, part I, to go into a clear-cut formulation of the theory and into detailed quantitative calculations, but rather to put an emphasis on the explanation of basic ideas.

§ 1. Introduction

Recent accumulation of the experimental data illuminating the structure of low-lying collective excited states in spherical odd-mass nuclei has stimulated the investigation of problems on particle-vibration coupling.

An important effect of particle-vibration coupling, which has been neglected for a long time, has been emphasized by Bohr and Mottelson[1)] in the Tokyo Conference in 1967: "In the phenomenological phonon-quasi-particle coupling model, the lowest-order-perturbation effects which contribute to energies of the excited states composed of the odd quasi-particle and the one-phonon, are shown in Figs. 1A and 1B. The diagrams of type 1A are nothing but the conventional ones which have so far been treated as "phonon-quasi-particle coupling", while the diagrams of type 1B have usually been neglected so far. The physical effect underlying the diagrams 1B is that *the phonon disassociates into a pair of quasi-particle, one of which reassociates with the odd-quasi-particle while the remaining quasi-particle is now the odd-quasi-particle.* Thus this effect is essentially based on the Pauli principle between the quasi-particles composing the phonon and the extra quasi-particle outside the core. The extreme importance of the diagrams of type 1B can be recognized as follows. The diagrams of type 1A consist of the coupling with the factor $(u_1u_2 - v_1v_2)$ which can be quite small,

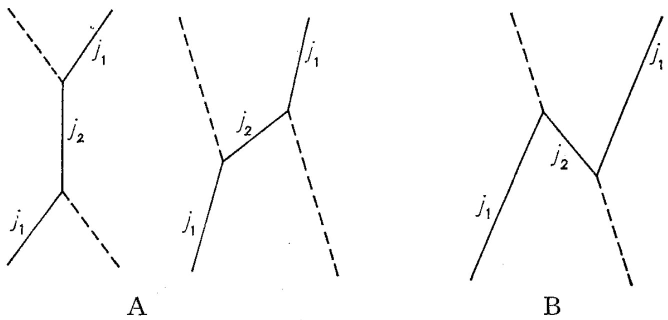

Fig. 1A. Contributions of Lowest Order. The solid and broken lines represent the quasi-particle and the phonon respectively. This type of particle-vibration coupling is accompanied by the reduction factor $(u_1u_2-v_1v_2)$.

Fig. 1B. Contributions of Lowest Order. This type of particle-vibration coupling is accompanied by the enhancement factor $(u_1v_2+v_1u_2)$. Both (A) and (B) are from reference 1).

while the diagrams of type 1B involve the coupling with the factor $(u_1v_2+v_1u_2)$ which is close to unity for low-lying states in the middle of the shell. Thus it is likely that the description of collective excited states of almost all spherical odd-mass nuclei is significantly effected by the inclusion of the effect.[1]

In the conventional phonon-quasi-particle-coupling theory,[2] the phonons are regarded as ideal bosons described by the random phase approximation (the RPA) and so are commutable with the odd-quasi-particle. Therefore, the effect which underlies the diagrams 1B and is based on the Pauli-principle between odd-quasi-particle and quasi-particles composing the phonon cannot be treated *in principle* within the framework of the theory.

Thus we are forced to construct a new theory to treat properly the essential effect responsible for the diagrams 1B. Of course, the more significant the effect becomes, the more the higher order diagrams of the type 1B must be taken into account. In constructing the theory, therefore, it may be strongly required to take the essential effect into account not by the perturbation approximation but by diagonalizing the Hamiltonian in a certain subspace, in such a way that we adopt the new Tamm-Dancoff approximation (i.e. the RPA) when constructing the phonon modes in even-mass nuclei.

The main purpose of this paper is to propose a new systematic theory which satisfies such requirements. The basic idea is as follows. Taking account of the composite nature of the phonon, let us replace the phonon line in the diagrams 1B by the conventional correlated-two-quasi-particle line shown in Fig. 2. Then the diagrams 1B can be decomposed into the corresponding microscopic diagrams in Fig. 3. The structure of the diagrams in Fig. 3 shows that they are composed of *only* two types of the quasi-particle interaction, H_X and H_V, represented in Fig. 4 in § 2, which are also well known to be responsible for the phonon modes in even-mass nuclei. The situation never changes even when we take account of any higher order diagrams of type 1B, and the excited state corresponding to any such diagram is always represented as a superposition of

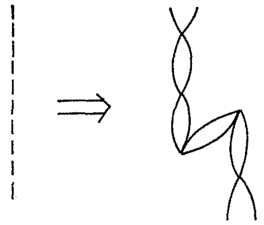

Fig. 2. Representation of the phonon as the correlated two-quasi-particles.

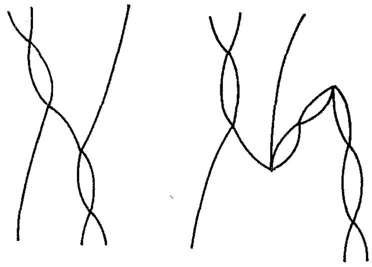

Fig. 3. Microscopic structure of diagrams 1B.

the particle states with 3, 7, 11, 15, $\cdots$ quasi-particles.*) We may thus conclude that if we succeed in constructing the correlated three-quasi-particle modes (i.e. the "dressed" three-quasi-particle modes) including the corresponding ground-state correlations in the framework of the new Tamm-Dancoff (NTD) approximation (by using the two-types of the quasi-particle interactions H_X and H_Y), the theory satisfies the above-mentioned requirements *in a very suitable way*.

A serious formal difficulty in developing the basic idea (in constructing the dressed three-quasi-particle modes in the framework of the NTD approximation) arises from the well-known spurious-state problem. Owing to the fact that any quasi-particle state $|\phi\rangle$ is not an eigenstate of nucleon-number operator $\hat{\boldsymbol{N}}$, the quasi-particle picture inevitably introduces spurious states arising from the nucleon-number fluctuation $(\hat{\boldsymbol{N}}-N_0)|\phi\rangle$, and only the states orthogonal to them correspond to those of a physical nucleus. In the case of the "dressed" two-quasi-particle modes, i.e. the phonon modes, it is a well-known and major advantage of the NTD method that both the collective excited states and the corresponding correlated ground state are orthogonal to the spurious states within the framework of the NTD approximation. In our dressed three-quasi-particle modes, however, the literal application of the NTD method never leads us to both "physical" collective excited states and the "physical" ground state orthogonal to the spurious states, because the creation operators of the dressed three-quasi-particle modes themselves generally involve some components of the number-fluctuation operator $(\hat{\boldsymbol{N}}-N_0)$.

In order to avoid this serious difficulty and to enjoy the proper advantage of the NTD method for the spurious-state problem, it is decisive to introduce a new concept to specify the dressed three-quasi-particle states as the "physical" states (orthogonal to the spurious states). In § 3, we show that recognition of the quasi-spin space, which has been introduced through the quasi-spin formalism[3] for the pairing correlations, plays an important role in introducing the new concept to specify the "physical" dressed three quasi-particle modes. Remember

*) This forms a marked contrast to the conventional "coupling diagrams" 1A, in which the quasi-particle interaction of the type H_Y represented in Fig. 4 in § 2 plays an important role as the essential coupling between odd-quasi-particle and collective modes. Thus the states corresponding to these diagrams are always represented as the superposition of particle states with $1, 3, 5, 7, \cdots$ quasi-particles.

that the quasi-spin formalism labels the states of an identical-nucleon configuration j^n by a formal angular-momentum quantum number (i.e. the quasi-spin) which is equivalent to the seniority number v, and that its generalization to the many-shell case is greatly simple.[3)]

After § 4 we show that, only when with the new concept to specify the "physical" dressed-three quasi-particle modes, we can develop the systematic theory to treat properly the essential effect responsible for the diagrams in Fig. 1B in the framework of the NTD approximation and can enjoy the proper advantage of the NTD method for the spurious-state problem.

In order to magnify the essential effect (based on the Pauli principle) responsible for the diagrams in Fig. 1B, and to illustrate the physical essence of the theory without inessential complications, throughout this paper except the final section we adopt the single j-shell model with nucleons interacting through a general effective nuclear force. In the final section (i.e. § 9) we discuss the generalization to a realistic case, and show that no conceptual difficulties are encountered in extending the essential idea developed with the single j-shell model.

§ 2. The Hamiltonian

We start with the single j-shell model*) with nucleons interacting through a general effective nuclear force. The Hamiltonian is then given by

$$H=\sum_{\alpha}(\epsilon_0-\lambda)c_\alpha{}^\dagger c_\alpha+\sum_{\alpha\beta\gamma\delta}\mathcal{V}_{\alpha\beta\gamma\delta}c_\alpha{}^\dagger c_\beta{}^\dagger c_\delta c_\gamma\,, \tag{2·1}$$

where $c_\alpha{}^\dagger$ and c_α are a creation and an annihilation operators of a nucleon in the state α and λ is a chemical potential. The matrix element of the two-body potential $\mathcal{V}_{\alpha\beta\gamma\delta}$ satisfies the relations

$$\mathcal{V}_{\alpha\beta\gamma\delta}=-\mathcal{V}_{\beta\alpha\gamma\delta}=-\mathcal{V}_{\alpha\beta\delta\gamma}=\mathcal{V}_{\gamma\delta\alpha\beta}\,. \tag{2·2}$$

After the Bogoliubov transformation

$$\left.\begin{aligned}&a_\alpha{}^\dagger=uc_\alpha{}^\dagger-s_\alpha vc_{\bar\alpha}\,,\\&a_\alpha=uc_\alpha-s_\alpha vc_{\bar\alpha}{}^\dagger\,,\\&\qquad u=\cos\theta/2\,,\quad v=\sin\theta/2\,,\quad s_\alpha\equiv(-)^{j-m_\alpha},\end{aligned}\right\} \tag{2·3}$$

our Hamiltonian may be written in terms of the quasi-particle operators, $a_\alpha{}^\dagger$ and a_α, as follows:

$$H=\sum_{\alpha}Ea_\alpha{}^\dagger a_\alpha+\sum_{\alpha\beta\gamma\delta}\mathcal{V}_{\alpha\beta\gamma\delta}:c_\alpha{}^\dagger c_\beta{}^\dagger c_\delta c_\gamma:\,, \tag{2·4}$$

*) The single particle states are then characterized by a magnetic quantum number. These single-particle states are designated by Greek subscripts. In association with the subscript $\alpha(=m_\alpha)$, we use a subscript $\bar\alpha$ which means $-m_\alpha$. For a basis of stationary states, it is possible to build the entire treatment on real quantities if the phase convention is suitably chosen. Throughout this paper, we always assume this to be the case.

where E is the quasi-particle energy, determined as usual together with the parameters u and v of the Bogoliubov transformation, and the symbol : : denotes the normal product with respect to the quasi-particles. In Eq. (2·4) we have dropped the constant term corresponding to the energy of the BCS ground state.

For the convenience of later discussion, we decompose the interaction term into the following parts:

$$H_{\rm int} \equiv \sum_{\alpha\beta\gamma\delta} \mathcal{V}_{\alpha\beta\gamma\delta} : c_\alpha{}^\dagger c_\beta{}^\dagger c_\delta c_\gamma :$$

$$= H_X + H_V + H_Y\,, \tag{2·5}$$

where

$$H_X = H_{pp} + H_{hh} + H_{ph}\,,$$

$$H_{pp} + H_{hh} = \sum_{\alpha\beta\gamma\delta} \mathcal{V}_{\alpha\beta\gamma\delta} (u^4 + v^4)\, a_\alpha{}^\dagger a_\beta{}^\dagger a_\delta a_\gamma\,,$$

$$H_{ph} = 4 \sum_{\alpha\beta\gamma\delta} \mathcal{V}_{\alpha\beta\gamma\delta} (u^2 v^2)\, s_\gamma s_\beta a_\alpha{}^\dagger a_{\bar\gamma}{}^\dagger a_{\bar\beta} a_\delta\,,$$

$$H_V = \sum_{\alpha\beta\gamma\delta} \mathcal{V}_{\alpha\beta\gamma\delta} (u^2 v^2)\, s_\gamma s_\delta \{a_\alpha{}^\dagger a_\beta{}^\dagger a_{\bar\delta}{}^\dagger a_{\bar\gamma}{}^\dagger + a_{\bar\gamma} a_{\bar\delta} a_\beta a_\alpha\}\,,$$

$$H_Y = 2 \sum_{\alpha\beta\gamma\delta} \mathcal{V}_{\alpha\beta\gamma\delta} uv (u^2 - v^2)\, s_\delta \{a_\alpha{}^\dagger a_\beta{}^\dagger a_{\bar\delta}{}^\dagger a_\gamma + a_\gamma{}^\dagger a_{\bar\delta} a_\beta a_\alpha\}\,. \tag{2·5'}$$

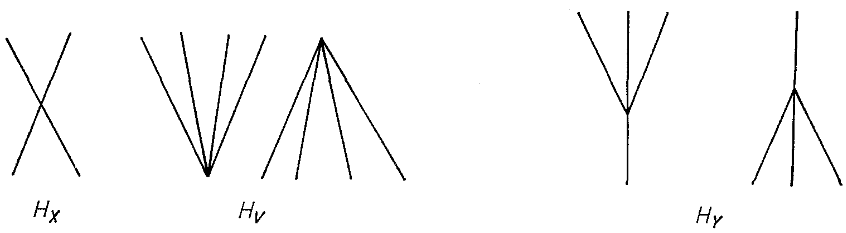

Fig. 4. Graphic representation of the interactions.

Each part is represented by one of the diagrams in Fig. 4. The interaction $H_X(\equiv H_{pp} + H_{hh} + H_{ph})$ conserves the number of quasi-particles, and is therefore the only one considered in the Tamm-Dancoff calculation for a fixed number of quasi-particles. In the conventional "pairing plus quadrupole-force model", the parts H_{pp} and H_{hh} are neglected and the part H_{ph}, which contains the factor $2uv$, plays the role as the interaction H_X. The part H_V introduces the ground-state correlations, and composes the diagrams of type 1B together with the part H_X. Contrary to the H_X and H_V, the part H_Y which involves the factor $(u^2 - v^2)$ plays an important role as an essential coupling between odd-quasi-particle and collective modes, in the conventional "coupling diagrams" of type 1A.

§ 3. Concept of "physical" dressed three-quasi-particle modes

3. 1. *Quasi-spin space*

For the convenience of a later discussion on the introduction of "physical" dressed three-quasi-particle modes, here we briefly recapitulate the quasi-spin formalism.[3)]

Let us define the nucleon-pair operators coupled to the angular momentum JM:

$$\left.\begin{aligned} \boldsymbol{A}^\dagger_{JM} &= \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \langle jjm_\alpha m_\beta | JM\rangle c_\alpha{}^\dagger c_\beta{}^\dagger, \\ \boldsymbol{B}^\dagger_{JM} &= -\sum_{\alpha\beta} \langle jjm_\alpha m_\beta | JM\rangle c_\alpha{}^\dagger \tilde{c}_\beta\,, \end{aligned}\right\} \tag{3·1}$$

where

$$\tilde{c}_\beta \equiv s_\beta c_{\bar{\beta}} = (-)^{j-m_\beta} c_{\bar{\beta}}\,. \tag{3·2}$$

Then we can easily see that the three operators

$$\begin{aligned} &\hat{\boldsymbol{S}}_+ = \Omega^{1/2} \boldsymbol{A}^\dagger_{00}\,, \qquad \hat{\boldsymbol{S}}_- = \Omega^{1/2} \boldsymbol{A}_{00}\,, \\ &\hat{\boldsymbol{S}}_0 = \left(\frac{\Omega}{2}\right)^{1/2} \left\{ \boldsymbol{B}^\dagger_{00} - \left(\frac{\Omega}{2}\right)^{1/2} \right\} = \frac{1}{2}(\hat{\boldsymbol{N}} - \Omega) \end{aligned} \tag{3·3}$$

have the commutation properties of angular momentum operators:

$$[\hat{\boldsymbol{S}}_+, \hat{\boldsymbol{S}}_-] = 2\hat{\boldsymbol{S}}_0\,, \qquad [\hat{\boldsymbol{S}}_0, \hat{\boldsymbol{S}}_\pm] = \pm \hat{\boldsymbol{S}}_\pm\,, \tag{3·4}$$

so that we call them the quasi-spin operators. In Eq. (3·3), $\hat{\boldsymbol{N}} \equiv \sum_\alpha c_\alpha{}^\dagger c_\alpha$ is the nucleon-number operator and $\Omega \equiv j + 1/2$ is the maximum allowed number of pairs.

From Eq. (3·3) the physical meaning of the quantum number S_0 is evident:

$$S_0 = \tfrac{1}{2}(N_0 - \Omega)\,, \tag{3·5}$$

where N_0 is the nucleon number. In order to understand the physical meaning of the "quasi-spin" quantum number S, let us take up the equation

$$\hat{\boldsymbol{S}}_- | S, S_0 = -S \rangle\!\rangle = 0\,, \tag{3·6}$$

which means that the state $|S, S_0 = -S\rangle\!\rangle$ includes no $J=0$ nucleon pairs. By definition, the nucleon number of this state is called the seniority v. We then have

$$\begin{aligned} \hat{\boldsymbol{S}}^2 | S, S_0 = -S\rangle\!\rangle &= S(S+1)\,|S, S_0 = -S\rangle\!\rangle \\ &= [\hat{\boldsymbol{S}}_+ \hat{\boldsymbol{S}}_- + \hat{\boldsymbol{S}}_0 (\hat{\boldsymbol{S}}_0 - 1)]\,|S, S_0 = -S\rangle\!\rangle \\ &= \hat{\boldsymbol{S}}_0 (\hat{\boldsymbol{S}}_0 - 1)\,|S, S_0 = -S\rangle\!\rangle \\ &= \tfrac{1}{2}(v - \Omega)\,\{\tfrac{1}{2}(v - \Omega) - 1\}\,|S, S_0 = -S\rangle\!\rangle. \end{aligned} \tag{3·7}$$

Thus the quantum number S is related to the seniority v through

$$S=\tfrac{1}{2}(\Omega-v). \tag{3·8}$$

With the quasi-spin operators $\hat{\boldsymbol{S}}_{\pm}$, $\hat{\boldsymbol{S}}_0$, we can define quasi-spin-tensor operators $\boldsymbol{T}_{ss_0}$ of rank s and its projection s_0 in the quasi-spin space, as usual, by the commutation relations

$$\begin{aligned}&[\hat{\boldsymbol{S}}_0,\ \boldsymbol{T}_{ss_0}]=s_0 T_{ss_0}\,,\\ &[\hat{\boldsymbol{S}}_{\pm},\ \boldsymbol{T}_{ss_0}]=\sqrt{(s\mp s_0)(s\pm s_0+1)}\boldsymbol{T}_{ss_0\pm 1}\,.\end{aligned} \tag{3·9}$$

The single nucleon operators $c_\alpha^\dagger$ and $\tilde{c}_\alpha$ are therefore regarded as spinors in the quasi-spin space:

$$\boldsymbol{T}_{1/2\ 1/2}(\alpha)=c_\alpha^\dagger\,,\qquad \boldsymbol{T}_{1/2\ -1/2}(\alpha)=\tilde{c}_\alpha\,. \tag{3·10}$$

The quasi-spin-tensor operators can be obtained from products of these elementary operators by the standard vector coupling procedures. For example, we have

$$\left.\begin{aligned}&\boldsymbol{T}_{11}(\alpha\beta)\equiv c_\alpha^\dagger c_\beta^\dagger,\\ &\boldsymbol{T}_{10}(\alpha\beta)\equiv\sqrt{\frac{1}{2}}\{c_\alpha^\dagger\tilde{c}_\beta+\tilde{c}_\alpha c_\beta^\dagger\},\\ &\boldsymbol{T}_{1-1}(\alpha\beta)\equiv\tilde{c}_\alpha\tilde{c}_\beta\,,\end{aligned}\right\} \tag{3·11}$$

and

$$\left.\begin{aligned}&\boldsymbol{T}_{3/2\ 3/2}(\alpha\beta\gamma)\equiv c_\alpha^\dagger c_\beta^\dagger c_\gamma^\dagger,\\ &\boldsymbol{T}_{3/2\ 1/2}(\alpha\beta\gamma)\equiv\sqrt{\frac{1}{3}}\{\tilde{c}_\alpha c_\beta^\dagger c_\gamma^\dagger+c_\alpha^\dagger\tilde{c}_\beta c_\gamma^\dagger+c_\alpha^\dagger c_\beta^\dagger\tilde{c}_\gamma\},\\ &\boldsymbol{T}_{3/2-1/2}(\alpha\beta\gamma)\equiv\sqrt{\frac{1}{3}}\{c_\alpha^\dagger\tilde{c}_\beta\tilde{c}_\gamma+\tilde{c}_\alpha c_\beta^\dagger\tilde{c}_\gamma+\tilde{c}_\alpha\tilde{c}_\beta c_\gamma^\dagger\},\\ &\boldsymbol{T}_{3/2-3/2}(\alpha\beta\gamma)\equiv\tilde{c}_\alpha\tilde{c}_\beta\tilde{c}_\gamma\,.\end{aligned}\right\} \tag{3·12}$$

Finally it should be noticed that there is no interference between the coupling of quasi-spins and the coupling of ordinary angular mementa, because $\hat{\boldsymbol{S}}_{\pm}$, $\hat{\boldsymbol{S}}_0$ commute with the angular momentum operators $\hat{\boldsymbol{J}}_{\pm}$, $\hat{\boldsymbol{J}}_0$.

3. 2. *Rotation in quasi-spin space*[4)]

The quasi-spin operators $\hat{\boldsymbol{S}}_{\pm}$, $\hat{\boldsymbol{S}}_0$ are associated with the transformation of states under rotations of the coordinate system in the quasi-spin space. Let us introduce a new coordinate system K' obtained from the original system K (in which the argument in § 3.1 has been done) by a rotation specified in terms of the Euler angles $\omega\equiv(\phi,\theta,\psi)$. The transformation of states is then given by

$$\begin{aligned}|SS_0\rangle&=R(\omega)\,|SS_0\rangle\!\rangle\\ &=\sum_{S_0'}|SS_0'\rangle\!\rangle\langle\!\langle SS_0'|R(\omega)\,|SS_0\rangle\!\rangle,\end{aligned} \tag{3·13}$$

where $R(\omega)$ is the unitary rotation operator in the quasi-spin space

$$\left.\begin{aligned} R(\omega) &= \exp\{-i\phi\hat{S}_z\}\exp\{-i\theta\hat{S}_y\}\exp\{-i\psi\hat{S}_z\}, \\ \hat{S}_z &\equiv \hat{S}_0, \qquad \hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-), \end{aligned}\right\} \tag{3·14}$$

and $|S, S_0\rangle\!\rangle$ is the state in the original system K while $|S, S_0\rangle$ means the state in the new coordinate system K'. Remember that the quantum numbers S and S_0 in the state $|S, S_0\rangle$ refer to the eigenvalues of $\hat{S}^2 \equiv R(\omega)\hat{\boldsymbol{S}}^2 R^{-1}(\omega)$ $(=\hat{\boldsymbol{S}}^2)$ and $\hat{S}_0 = R(\omega)\hat{\boldsymbol{S}}_0 R^{-1}(\omega)$ respectively.

The matrix elements of $R(\omega)$ define the conventional $D(\omega)$ functions*) in the quasi-spin space:

$$\begin{aligned} D^S_{S_0'S_0}(\omega) &\equiv \langle\!\langle S, S_0'|R(\omega)|S, S_0\rangle\!\rangle^* \\ &= \langle S, S_0'|R(\omega)|S, S_0\rangle^*. \end{aligned} \tag{3·15}$$

With Eq. (3·15), Eq. (3·13) becomes

$$|S, S_0\rangle = \sum_{S_0'} D^{S*}_{S_0'S_0}(\omega)\,|S, S_0'\rangle\!\rangle. \tag{3·16}$$

Since $R(\omega)$ is unitary, this can also be writen

$$|S, S_0\rangle\!\rangle = \sum_{S_0'} D^S_{S_0S_0'}(\omega)\,|S, S_0'\rangle. \tag{3·17}$$

By definition, the quasi-spin-tensor operators in the new coordinate system K', T_{ss_0}, are related to those in the original system K, $\boldsymbol{T}_{ss_0}$, through

$$\left.\begin{aligned} T_{ss_0} &= R(\omega)\boldsymbol{T}_{ss_0}R^{-1}(\omega) = \sum_{s_0'} D^{s*}_{s_0's_0}(\omega)\,\boldsymbol{T}_{ss_0'}, \\ \boldsymbol{T}_{ss_0} &= R^{-1}(\omega)T_{ss_0}R(\omega) = \sum_{s_0'} D^s_{s_0s_0'}T_{ss_0'}. \end{aligned}\right\} \tag{3·18}$$

Now let us take up a new coordinate system K'_{ω_0} specified by the Euler angles $\omega_0 \equiv (0, -\theta, 0)$. According to Eq. (3·18), we then have the elementary quasi-spin spinors $T_{1/2\,s_0}(\alpha)$ in the K'_{ω_0}-system by

$$\begin{pmatrix} T_{1/2\,1/2}(\alpha) \\ T_{1/2\,-1/2}(\alpha) \end{pmatrix} = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix} \begin{pmatrix} \boldsymbol{T}_{1/2\,1/2}(\alpha) \\ \boldsymbol{T}_{1/2\,-1/2}(\alpha) \end{pmatrix}. \tag{3·19}$$

With definition

$$T_{1/2\,1/2}(\alpha) \equiv a_\alpha^\dagger, \qquad T_{1/2\,-1/2}(\alpha) \equiv \tilde{a}_\alpha = s_\alpha a_{\bar{\alpha}}, \tag{3·20}$$

Eq. (3·19) can be written

$$\begin{aligned} a_\alpha^\dagger &= uc_\alpha^\dagger - s_\alpha v c_{\bar{\alpha}}, \\ a_\alpha &= uc_\alpha - s_\alpha v c_{\bar{\alpha}}^\dagger, \\ u &\equiv \cos\theta/2, \qquad v \equiv \sin\theta/2, \end{aligned}$$

---

*) We use a definition of the D function which is employed by A. Bohr and Mottelson.[5] (Therefore, the $D^S_{S_0S_0'}$ function here is the complex conjugate of that employed by Rose, and differs from that employed by Edomonds by the phase factor $(-)^{S_0-S_0'}$).

which is nothing but the Bogoliubov transformation (2·3). We may therefore say that the Bogoliubov transformation just corresponds to a special rotation $\omega_0 \equiv (0, -\theta, 0)$ of the coordinate system in the quasi-spin space.

In this new coordinate system K'_{ω_0}, the quasi-spin operators are given by

$$\left.\begin{aligned}
\hat{S}_+ &= R(\omega_0)\hat{\boldsymbol{S}}_+R^{-1}(\omega_0) = \Omega^{1/2}A^\dagger_{00}\,,\\
\hat{S}_- &= R(\omega_0)\hat{\boldsymbol{S}}_-R^{-1}(\omega_0) = \Omega^{1/2}A_{00}\,,\\
\hat{S}_0 &= R(\omega_0)\hat{\boldsymbol{S}}_0R^{-1}(\omega_0)\\
&= \left(\frac{\Omega}{2}\right)^{1/2}\left\{B^\dagger_{00} - \left(\frac{\Omega}{2}\right)^{1/2}\right\}\\
&= \frac{1}{2}(\hat{n}-\Omega),
\end{aligned}\right\} \tag{3·21}$$

where $A^\dagger_{JM}$ and $B^\dagger_{JM}$ are the quasi-particle-pair operators coupled to the angular momentum JM

$$\left.\begin{aligned}
A^\dagger_{JM} &= \frac{1}{\sqrt{2}}\sum_{\alpha\beta}\langle jjm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger a_\beta{}^\dagger,\\
B^\dagger_{JM} &= -\sum_{\alpha\beta}\langle jjm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger \tilde{a}_\beta\,,
\end{aligned}\right\} \tag{3·22}$$

and $\hat{n}$ means the quasi-particle-number operator

$$\hat{n} \equiv \sum_\alpha a_\alpha{}^\dagger a_\alpha\,. \tag{2·23}$$

Since $\hat{S}^2 = R(\omega_0)\hat{\boldsymbol{S}}^2R^{-1}(\omega_0) = \hat{\boldsymbol{S}}^2$, the quasi-spin quantum number S in the state $|S, S_0\rangle$ in the new system K'_{ω_0} (i.e. *in the quasi-particle representation*) *has the same physical meaning as Eq. (3·8):*

$$S = \tfrac{1}{2}(\Omega - v)\,. \tag{3·24}$$

From Eq. (3·21), however, the physical meaning of the quantum number S_0 is now

$$S_0 = \tfrac{1}{2}(n_0 - \Omega)\,, \tag{3·25}$$

where n_0 denotes the number of quasi-particles. Needless to say, the BCS ground state $|\phi_0\rangle$ is given by

$$|\phi_0\rangle \equiv |S = \tfrac{1}{2}\Omega, S_0 = -\tfrac{1}{2}\Omega\rangle. \tag{3·26}$$

Finally it should be noticed that the nucleon-number-fluctuation operator $(\hat{\boldsymbol{N}} - N_0)$, causing the troubles of spurious states, is now given by

$$\hat{\boldsymbol{N}} - N_0 = (u^2 - v^2)(2\hat{S}_0 + \Omega) + 2uv(\hat{S}_+ + \hat{S}_-)\,. \tag{3·27}$$

This implies that in the quasi-particle representation any collective vibration which involves motion of the quasi-spin operators $\hat{S}_\pm$, $\hat{S}_0$ always contains the spurious components.

3.3. *Concept of "physical" dressed three-quasi-particle modes*

We are now in a position to construct the "physical" dressed three quasi-particle modes (in the NTD approximation).

It is usual to characterize the conventional spherical tensor operators by the amount of angular momentum they transfer to the state on which they act. For example, a spherical tensor of rank $\lambda, T(\lambda\mu)$, transfers an angular momentum λ to the state. (The different μ components of the tensor have to possess the same intrinsic properties.) In the completely same way, we may characterize the quasi-spin tensors T_{ss_0} by the amount of transferred quasi-spin s, i.e. by the amount of *transferred seniority* $\Delta v \equiv 2s$ to the state on which they operate. At this stage we can *precisely define* the concept of "dressed" n-quasi-particle eigenmodes which is customary used in the quasi-particle-NTD approximation: *The eigenmode operators of the dressed n-quasi-paticle modes should be expressed in terms of the quasi-spin tensor* T_{ss_0} (*composed of n quasi-particle operators*) *with the transferred seniority* $\Delta v \equiv 2s = n$. For example, the eigenmode operators of the dressed two-quasi-particle modes, i.e. phonons are known to be composed of the quasi-spin tensor $T_{s=1,s_0}(\alpha\beta)$ with transferred seniority $\Delta v \equiv 2s = 2$.

Eigenmode operators of our dressed three-quasi-particle modes (in the NTD approximation) may, therefore, be written in the following form:

$$C_{nIK}^{\dagger} = \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \sum_{s_0} \psi_{nIK}(\alpha\beta\gamma : s_0)\, T_{3/2\, s_0}(\alpha\beta\gamma), \tag{3·28}$$

where I and K are the angular momentum and its projection, and n indicates a set of additional quantum numbers to specify the modes. The expression (3·28) means that the eigenmode operators $C_{nIK}^{\dagger}$ transfer the quasi-spin $s=3/2$, i.e. $\Delta v = 3$ to the state on which they operate. With the aid of Eqs. (3·12), (3·18) and (3·20), the explicit form of the quasi-spin tensor $T_{3/2\, s_0}(\alpha\beta\gamma)$ of rank 3/2 in the quasi-particle representation is written

$$T_{3/2\, 3/2}(\alpha\beta\gamma) \equiv a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger, \tag{3·29a}$$

$$\begin{aligned} T_{3/2\, 1/2}(\alpha\beta\gamma) &\equiv \sqrt{\frac{1}{3}} \{\tilde{a}_\alpha a_\beta^\dagger a_\gamma^\dagger + a_\alpha^\dagger \tilde{a}_\beta a_\gamma^\dagger + a_\alpha^\dagger a_\beta^\dagger \tilde{a}_\gamma\} \\ &= : T_{3/2\, 1/2}(\alpha\beta\gamma) : \\ &\quad + \sqrt{\frac{1}{3}} \{\delta_{\bar{\beta}\gamma} s_\beta a_\alpha^\dagger - \delta_{\bar{\alpha}\gamma} s_\alpha a_\beta^\dagger + \delta_{\bar{\alpha}\beta} s_\alpha a_\gamma^\dagger\}, \end{aligned} \tag{3·29b}$$

$$\begin{aligned} T_{3/2\, -1/2}(\alpha\beta\gamma) &\equiv \sqrt{\frac{1}{3}} \{a_\alpha^\dagger \tilde{a}_\beta \tilde{a}_\gamma + \tilde{a}_\alpha a_\beta^\dagger \tilde{a}_\gamma + \tilde{a}_\alpha \tilde{a}_\beta a_\gamma^\dagger\} \\ &= : T_{3/2\, -1/2}(\alpha\beta\gamma) : \end{aligned}$$

$$+\sqrt{\frac{1}{3}}\{\delta_{\bar{\alpha}\beta}s_\alpha\tilde{a}_\gamma-\delta_{\bar{\alpha}\gamma}s_\alpha\tilde{a}_\beta+\delta_{\gamma\bar{\beta}}s_\beta\tilde{a}_\alpha\}, \tag{3·29c}$$

$$T_{3/2-3/2}(\alpha\beta\gamma)\equiv\tilde{a}_\alpha\tilde{a}_\beta\tilde{a}_\gamma . \tag{3·29d}$$

Since $T_{3/2\,s_0}(\alpha\beta\gamma)$ is antisymmetric with respect to (α,β,γ), the amplitudes $\psi_{nIK}(\alpha\beta\gamma;s_0)$ in Eq. (3·28) also satisfy the antisymmetry relation

$$P\psi_{nIk}(\alpha\beta\gamma:s_0)=\delta_P\psi_{nIK}(\alpha\beta\gamma:s_0), \tag{3·30}$$

where P is the permutation operator with respect to (α,β,γ) and

$$\delta_P=\begin{cases}\ \ 1 \text{ for even permutations}\\ -1 \text{ for odd permutations.}\end{cases} \tag{3·31}$$

In order that the eigenmode operators (3·28) are the "physical" ones which create the states orthogonal to the spurious states (within the framework of the NTD approximation), they are required never to contain any component of nucleon-number-fluctuation operator (3·27), i.e. never to involve the quasi-spin operators $\hat{S}_\pm$, $\hat{S}_0$. With the purpose to find a condition for this requirement, we rewrite Eq. (3·28) in the following form:

$$C^\dagger_{nIK}=\frac{1}{\sqrt{3!}}\sum_J{}'\sum_{s_0}\psi_n(j^2(J)j\}j^3I:s_0)\,T_{3/2\,s_0}(j^2(J)jIK), \tag{3·32}$$

where $\sum_J{}'$ means the summation with respect to even values of J, and

$$\psi_n(j^2(J)j\}j^3I:s_0)\equiv\sum_{\alpha\beta\gamma}\sum_M\langle JjMm_\gamma|IK\rangle\langle jjm_\alpha m_\beta|JM\rangle\psi_{nIK}(\alpha\beta\gamma:s_0), \tag{3·33}$$

$$T_{3/2\,s_0}(j^2(J)jIK)\equiv\sum_{\alpha\beta\gamma}\sum_M\langle JjMm_\gamma|IK\rangle\langle jjm_\alpha m_\beta|JM\rangle\,T_{3/2\,s_0}(\alpha\beta\gamma), \tag{3·34}$$

with the even J.

From the antisymmetry relation (3·30), the amplitudes $\psi_n(j^2(J)j\}j^3I:s_0)$ satisfy the equation

$$\psi_n(j^2(J)j\}j^3I:s_0)=-\sum_{J'}{}'\psi_n(j^2(J')j\}j^3I:s_0)\sqrt{(2J+1)(2J'+1)}\times W(jjIj:J'J)(-)^{j+I}. \tag{3·35}$$

With the definition of the quasi-spin operators (3·21), it is now clear that the condition for the eigenmode operators (3·32) to include no quasi-spin operators $\hat{S}_\pm$, $\hat{S}_0$ is

$$\psi_n(j^2(0)j\}j^3I:s_0)=0\,, \tag{3·36}$$

which is also written

$$\sum_\alpha s_\alpha\psi_{nIK}(\alpha\bar{\alpha}\gamma:s_0)=0\,. \tag{3·37}$$

Here it is of interest to know that Eq(3·35) with the condition (3·36) is, in its form, precisely the same equation as the coefficients of fractional parentage (c.f.p.) with seniority $v=3$ for j^3-configurations have to satisfy.

In the expression (3·28) of the eigenmode operators $C^\dagger_{nIK}$, the second terms in Eqs.(3·29b) and (3·29c), $\{T_{3/2\,1/2}(\alpha\beta\gamma)-:T_{3/2\,1/2}(\alpha\beta\gamma):\}$ and $\{T_{3/2-1/2}(\alpha\beta\gamma)-:T_{3/2-1/2}(\alpha\beta\gamma):\}$ (which include only one-quasi-particle operators), are automatically dropped because of the condition (3·37). Thus, our eigenmode operators of the physical dressed three-quasi-particle modes are finally defined by

$$C^\dagger_{nIK}=\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\sum_{s_0}\psi_{nIK}(\alpha\beta\gamma:s_0):T_{3/2\,s_0}(\alpha\beta\gamma): \tag{3·38}$$

with the conditions (3·30) and (3·37).

§4. Properties of eigenmode operators with $\Delta v=3$

The three-body-correlation amplitudes $\psi_{nIK}(\alpha\beta\gamma;s_0)$ in Eq.(3·38) may be determined so that $C^\dagger_{nIK}$ becomes a "good" approximate eigenmode operator satisfying

$$[H, C^\dagger_{nIK}]=\omega_{nI}C^\dagger_{nIK}-Z_{nIK}\,, \tag{4·1}$$

where "interaction" Z_{nIK} is composed of one-quasi-particle operators $a_\alpha{}^\dagger$ and a_α (i.e. $T_{1/2\,s_0}(\alpha)$ with $\Delta v=1$), third-order normal products of trilinear quasi-spin tensors $T_{1/2\,s_0}(\alpha\beta\gamma)$ with $\Delta v=1$ and fifth-order nomal products. Thus, in our NTD approximation (concerned with the physical dressed three-quasi-particle modes with $\Delta v=3$), the "interaction" Z_{nIK} is neglected in the first step. With this approximation, direct calculation of Eq.(4·1) with Eq.(3·38) leads us to the following equation which the correlation amplitudes have to satisfy:

$$\omega_{nIK}\begin{bmatrix}\psi_{nIK}(\alpha\beta\gamma:3/2)\\ \psi_{nIK}(\alpha\beta\gamma:-1/2)\\ \psi_{nIK}(\alpha\beta\gamma:1/2)\\ \psi_{nIK}(\alpha\beta\gamma:-3/2)\end{bmatrix}$$

$$=\sum_{\alpha_1\beta_1\gamma_1}\begin{bmatrix}3D_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & -A_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & 0 & 0\\ A_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & -D_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & B_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & \sqrt{3}B_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}\\ \sqrt{3}B_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & B_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & D_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & -A_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}\\ 0 & 0 & A_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}, & -3D_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}\end{bmatrix}\begin{bmatrix}\psi_{nIK}(\alpha_1\beta_1\gamma_1:3/2)\\ \psi_{nIK}(\alpha_1\beta_1\gamma_1:-1/2)\\ \psi_{nIK}(\alpha_1\beta_1\gamma_1:1/2)\\ \psi_{nIK}(\alpha_1\beta_1\gamma_1:-3/2)\end{bmatrix}, \tag{4·2}$$

where

$$A_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}=\frac{4}{\sqrt{3}}u^2v^2\{\langle\alpha\beta\gamma|V_G|\alpha_1\beta_1\gamma_1\rangle-2\langle\alpha\beta\gamma|V_F|\alpha_1\beta_1\gamma_1\rangle\},$$

$$\left.\begin{aligned}
B_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}&\equiv\frac{4}{3}uv(u^2-v^2)\{\langle\alpha\beta\gamma|V_G|\alpha_1\beta_1\gamma_1\rangle-2\langle\alpha\beta\gamma|V_F|\alpha_1\beta_1\gamma_1\rangle\},\\
D_{\alpha\beta\gamma,\alpha_1\beta_1\gamma_1}&\equiv\frac{1}{3!}\sum_{P(\alpha\beta\gamma)}\delta_P P\cdot\delta_{\alpha\alpha_1}\delta_{\beta\beta_1}\delta_{\gamma\gamma_1}\cdot E\\
&\quad+\frac{2}{3}\{(u^4+v^4)\langle\alpha\beta\gamma|V_G|\alpha_1\beta_1\gamma_1\rangle+4u^2v^2\langle\alpha\beta\gamma|V_F|\alpha_1\beta_1\gamma_1\rangle\},\\
\langle\alpha\beta\gamma|V_G|\alpha_1\beta_1\gamma_1\rangle&\equiv\frac{1}{3!}\sum_{P(\alpha\beta\gamma)}\delta_P\ P\{\mathcal{V}_{\alpha\beta\alpha_1\beta_1}\delta_{\gamma\gamma_1}+\mathcal{V}_{\alpha\beta\gamma_1\alpha_1}\delta_{\gamma\beta_1}+\mathcal{V}_{\alpha\beta\beta_1\gamma_1}\delta_{\gamma\alpha_1}\},\\
\langle\alpha\beta\gamma|V_F|\alpha_1\beta_1\gamma_1\rangle&\equiv\frac{1}{3!}\sum_{P(\alpha\beta\gamma)}\delta_P P\{\mathcal{V}_{\alpha\bar{\beta}_1\bar{\beta}\alpha_1}s_\beta s_{\beta_1}\delta_{\gamma\gamma_1}\\
&\quad+\mathcal{V}_{\alpha\bar{\alpha}_1\bar{\beta}\gamma_1}s_\beta s_{\alpha_1}\delta_{\gamma\beta_1}+\mathcal{V}_{\alpha\bar{\gamma}_1\bar{\beta}\beta_1}s_\beta s_{\gamma_1}\delta_{\gamma\alpha_1}\}.
\end{aligned}\right\}\tag{4·3}$$

In Eq. (4·3) the symbol $\sum_{P(\alpha\beta\gamma)}$ means the summation of all permutations of (α, β, γ). For simplicity, we shall often use the matrix notation with respect to (α, β, γ), with which Eq. (4·2) is written

$$\omega_{nIK}\begin{bmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\\ \psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{bmatrix}=\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A} & 0 & 0\\ \boldsymbol{A} & -\boldsymbol{D} & \boldsymbol{B} & \sqrt{3}\boldsymbol{B}\\ \sqrt{3}\boldsymbol{B} & \boldsymbol{B} & \boldsymbol{D} & -\boldsymbol{A}\\ 0 & 0 & \boldsymbol{A} & -3\boldsymbol{D}\end{bmatrix}\begin{bmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\\ \psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{bmatrix}.\tag{4·2}'$$

Of course, Eq. (4·2) is compatible with the conditions (3·30) and (3·37).

Let $C^\dagger_{n_+IK}$ with the positive eigenvalue ω_{n_+IK} (which is reduced to $3E$ in the absence of the interaction) represent the creation operator $Y^\dagger_{nIK}$ of the mode under consideration:

$$C^\dagger_{n_+IK}\equiv Y^\dagger_{nIK},\qquad \omega_{n_+IK}>0\,.\tag{4·4}$$

Then the corresponding annihilation operator Y_{nIK} also satisfies Eq. (4·1) under our approximation (to neglect the "interaction" Z) with the negative eigenvalue $\omega_{n_-IK}\equiv-\omega_{n_+IK}<0$, so that

$$C_{n_-IK}\equiv Y_{nIK}\,,\qquad \omega_{n_-IK}\equiv-\omega_{n_+IK}<0\,.\tag{4·5}$$

We thus obtain

$$C_{n_+IK}=C^\dagger_{n_-IK}\,.\tag{4·6}$$

This implies a condition for the correlation amplitudes,

$$\begin{bmatrix}\psi_{n_-IK}(\alpha\beta\gamma:3/2)\\ \psi_{n_-IK}(\alpha\beta\gamma:-1/2)\\ \psi_{n_-IK}(\alpha\beta\gamma:1/2)\\ \psi_{n_-IK}(\alpha\beta\gamma:-3/2)\end{bmatrix}=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{bmatrix}\begin{bmatrix}\psi_{n_+IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}:3/2)\\ \psi_{n_+IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}:-1/2)\\ \psi_{n_+IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}:1/2)\\ \psi_{n_+IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}:-3/2)\end{bmatrix}s_\alpha s_\beta s_\gamma\,,\tag{4·7}$$

which is consistent to Eq. (4·2).

Now it is very important to examine the compatibility of Eq. (4·2) and another decisive condition for the correlation amplitudes which is essential to the NTD approximation. This condition is implicitly related to the orthonormality problem of the various states $Y^{\dagger}_{nIK}|\Phi_0\rangle$ (within the framework of the NTD approximation):

$$\langle\Phi_0|\, Y_{mI'K'} Y^{\dagger}_{nIK}|\Phi_0\rangle = \delta_{nm}\delta_{II'}\delta_{KK'}, \tag{4·8}$$

where $|\Phi_0\rangle$ is the correlated ground state (in the NTD approximation). With the aid of Eqs. (4·4) and (4·5), Eq. (4·8) may be written

$$\left.\begin{array}{l}\langle\Phi_0|\,\{C_{m_+I'K'}, C^{\dagger}_{n_+IK}\}_+|\Phi_0\rangle = \delta_{nm}\delta_{II'}\delta_{KK'},\\ \langle\Phi_0|\,\{C_{m_-I'K'}, C^{\dagger}_{n_-IK}\}_+|\Phi_0\rangle = \delta_{nm}\delta_{II'}\delta_{KK'},\end{array}\right\} \tag{4·9}$$

where we have used the fact $Y_{nIK}|\Phi_0\rangle=0$, i.e. $C_{n_+IK}|\Phi_0\rangle = C^{\dagger}_{n_-IK}|\Phi_0\rangle=0$, and the symbol $\{A, B\}_+$ means the conventional anticommutation relation

$$\{A, B\}_+ = AB+BA\,. \tag{4·10}$$

(Here it should be noticed that, contrary to the case of phonon modes, in Eq. (4·9) we have used the Fermion-type anticommutation relations for our eigenmode operators of the physical dressed three-quasi-particle modes.) As is recognized from the formal structure of the NTD theory for conventional phonon modes, the fundamental essence of the NTD approximation is to define the inner product and its orthogonality of correlation amplitudes in such a way that they become equivalent to the ground-state-expectation values of commutation relations of the eigenmode operators under consideration. We may, therefore, set up the form of the inner product and its orthogonality of our correlation amplitudes in a consistent way to Eq. (4·9):*)

$$(\boldsymbol{\Psi}_{m_\pm I'K'}\cdot\boldsymbol{\Psi}_{n_\pm IK}) \equiv \sum_{\alpha\beta\gamma}(\psi_{m_\pm I'K'}(\alpha\beta\gamma:3/2),\, \psi_{m_\pm I'K'}(\alpha\beta\gamma:-1/2),\, \psi_{m_\pm I'K'}(\alpha\beta\gamma:1/2),\, \psi_{m_\pm I'K'}(\alpha\beta\gamma:-3/2)) \times \boldsymbol{\tau}\begin{bmatrix}\psi_{n_\pm IK}(\alpha\beta\gamma:3/2)\\ \psi_{n_\pm IK}(\alpha\beta\gamma:-1/2)\\ \psi_{n_\pm IK}(\alpha\beta\gamma:1/2)\\ \psi_{n_\pm IK}(\alpha\beta\gamma:-3/2)\end{bmatrix} = \delta_{nm}\delta_{II'}\delta_{KK'}\,, \tag{4·11}$$

where $\boldsymbol{\tau}$ is the metric matrix, the 4×4 matrix elements of which are certain *real numerical constants.* Since $(\boldsymbol{\Psi}_{n_\pm IK}\cdot\boldsymbol{\Psi}_{n_\pm IK})$ is real by definition, $\boldsymbol{\tau}$ must be Hermitian

$$\boldsymbol{\tau} = \boldsymbol{\tau}^{\dagger}. \tag{4·12}$$

*) An explicit justification of the equivalence of Eqs. (4·9) and (4·11) will be given in § 7.

From Eq. (4·7) and Eq. (4·11) with $n=m$, $I=I'$, $K=K'$, we also have

$$\boldsymbol{\tau}=\begin{bmatrix}0 & 0 & 0 & -1\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}\boldsymbol{\tau}\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0\end{bmatrix}. \qquad (4\cdot13)$$

Equation (4·11) with the metric $\boldsymbol{\tau}$ satisfying Eqs. (4·12) and (4·13) is the decisive condition which our correlation amplitudes (in the framework of the NTD approximation) have to satisfy. Thus the compatibility of Eqs. (4·2) and (4·11) is essential in constructing our NTD theory.

The compatibility may be satisfied if we could have

$$\boldsymbol{\tau M}=\boldsymbol{M}^{+}\boldsymbol{\tau}^{+}=\boldsymbol{M}^{+}\boldsymbol{\tau}\,, \qquad (4\cdot14)$$

where $\boldsymbol{M}$ is the matrix on the right-hand side of Eq. (4·2)′:

$$\boldsymbol{M}=\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A} & 0 & 0\\ \boldsymbol{A} & -\boldsymbol{D} & \boldsymbol{B} & \sqrt{3}\boldsymbol{B}\\ \sqrt{3}\boldsymbol{B} & \boldsymbol{B} & \boldsymbol{D} & -\boldsymbol{A}\\ 0 & 0 & \boldsymbol{A} & -3\boldsymbol{D}\end{bmatrix}. \qquad (4\cdot15)$$

Equation (4·14) means that, under the definition of inner product (4·11), the matrix $\boldsymbol{M}$ beomes a self-adjoint operator, so that the eigenvalues ω_{nIK} are generally real and the correlation amplitudes $\psi_{nIK}(\alpha\beta\gamma; s_0)$ satisfy the orthogonality relation $(\boldsymbol{\Psi}_{mI'K'}\cdot\boldsymbol{\Psi}_{nIK})=0$ if $n\neq m$.

To examine Eq. (4·14), let us write the matix $\boldsymbol{\tau}$ as

$$\boldsymbol{\tau}=\begin{bmatrix}a & b & c & 0\\ b & d & 0 & -c\\ c & 0 & d & b\\ 0 & -c & b & a\end{bmatrix}, \qquad (4\cdot16)$$

(a, b, c, d: real numerical constant)

which satisfies the conditions (4·12) and (4·13). Then Eq. (4·14) leads us to

$$\left.\begin{aligned}&a=-d\,,\qquad b=0\,,\qquad c=0\,,\\ &\sqrt{3}\boldsymbol{B}d=0\,.\end{aligned}\right\} \qquad (4\cdot17)$$

This implies that it is necessary to reject $\sqrt{3}\boldsymbol{B}$ from $\boldsymbol{M}$ in order that the matrix $\boldsymbol{M}$ satisfies Eq. (4·14) together with the metric $\boldsymbol{\tau}$ having a simple form

$$\boldsymbol{\tau}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}. \qquad (4\cdot18)$$

Here it should be emphasized that, by definition in Eq. (4·3), $\boldsymbol{B}$ comes from *ony* the interaction part H_Y (in Eq. (2·5)) which has no *responsibility* to the type of diagrams in Fig. 1B. Therefore, it is consistent to the purpose of this paper discussed in § 1 to reject $\boldsymbol{B}$ from the matrix $\boldsymbol{M}$. The rejection of $\boldsymbol{B}$ from $\boldsymbol{M}$ means that in Eq. (4·1) all terms coming from $[H_Y, C^\dagger_{nIK}]$ should be newly included into the "interaction" Z_{nIK}.

Thus we finally arrive at the eigenvalue equation (to determine the correlation amplitudes) which is *very suitable* for our purpose discussed in § 1 and is compatible with the definition of the inner product (4·11) with the simple metric (4·18):

$$\omega_{nIK}\begin{pmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\\ \psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{pmatrix}=\begin{pmatrix}3\boldsymbol{D} & -\boldsymbol{A} & 0 & 0\\ \boldsymbol{A} & -\boldsymbol{D} & 0 & 0\\ 0 & 0 & \boldsymbol{D} & -\boldsymbol{A}\\ 0 & 0 & \boldsymbol{A} & -3\boldsymbol{D}\end{pmatrix}\begin{pmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\\ \psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{pmatrix}$$

$$=\begin{pmatrix}3\boldsymbol{D} & \boldsymbol{A} & 0 & 0\\ \boldsymbol{A} & \boldsymbol{D} & 0 & 0\\ 0 & 0 & -\boldsymbol{D} & -\boldsymbol{A}\\ 0 & 0 & -\boldsymbol{A} & -3\boldsymbol{D}\end{pmatrix}\boldsymbol{\tau}\begin{pmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\\ \psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{pmatrix}. \qquad (4\cdot19)$$

§ 5. Physical meaning of eigenmodes with $\Delta v=3$

Except for the case $\omega_{nIK}=0$, the eigenvalue equation (4·19) is simply reduced to

$$\omega^{(1)}_{nIK}\begin{pmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\end{pmatrix}=\begin{pmatrix}3\boldsymbol{D} & \boldsymbol{A}\\ \boldsymbol{A} & \boldsymbol{D}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\psi_{nIK}(3/2)\\ \psi_{nIK}(-1/2)\end{pmatrix}, \qquad (5\cdot1a)$$

$$\omega^{(2)}_{nIK}\begin{pmatrix}\psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{pmatrix}=\begin{pmatrix}-\boldsymbol{D} & -\boldsymbol{A}\\ -\boldsymbol{A} & -3\boldsymbol{D}\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\psi_{nIK}(1/2)\\ \psi_{nIK}(-3/2)\end{pmatrix}, \qquad (5\cdot1b)$$

where $\omega^{(1)}_{nIK}=-\omega^{(2)}_{nIK}$. Thus the eigenmode operators with the positive energy solutions $\omega^{(1)}_{n_+IK}>0$ (which are reduced to $3E$ in the absence of the interaction) in Eq. (5·1a) are now written

$$C^\dagger_{n_+IK}\equiv Y^\dagger_{n_+IK}$$
$$=\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\{\psi_{n_+IK}(\alpha\beta\gamma:3/2):T_{3/2\ 3/2}(\alpha\beta\gamma):$$
$$+\psi_{n_+IK}(\alpha\beta\gamma:-1/2):T_{3/2-1/2}(\alpha\beta\gamma):\}. \qquad (5\cdot2)$$

With the aid of Eq. (4·7), the corresponding annihilation operators Y_{nIK} are expressed as

$$Y_{nIK} \equiv C^{\dagger}_{n\_IK}$$
$$= \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \{\psi_{n\_IK}(\alpha\beta\gamma: -3/2): T_{3/2-3/2}(\alpha\beta\gamma):$$
$$+ \psi_{n\_IK}(\alpha\beta\gamma: 1/2): T_{3/2\ 1/2}(\alpha\beta\gamma):\}, \qquad (5\cdot3)$$

which correspond to the eigenmode operators with the negative energy solutions $\omega^{(2)}_{n\_IK} = -\omega^{(1)}_{n\_IK} < 0$ in Eq. (5·1b). The solutions of Eq. (5·1b), therefore, leads us to the Hermitian conjugate operators to the eigenmode operators obtained with the solutions of Eq. (5·1a).

For simplicity, hereafter we use the following notations:

$$\left.\begin{aligned} \psi_{nIK}(\alpha\beta\gamma) &\equiv \psi_{n_+IK}(\alpha\beta\gamma: 3/2), \\ \varphi_{nIK}(\alpha\beta\gamma) &\equiv \psi_{n_+IK}(\alpha\beta\gamma: -1/2). \end{aligned}\right\} \qquad (5\cdot4)$$

Equation (4·7) is then written

$$\left.\begin{aligned} \psi_{nIK}(\alpha\beta\gamma) &= \psi_{n\_IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}: -3/2)\, s_\alpha s_\beta s_\gamma, \\ \varphi_{nIK}(\alpha\beta\gamma) &= \psi_{n\_IK}(\bar{\alpha}\bar{\beta}\bar{\gamma}: 1/2)\, s_\alpha s_\beta s_\gamma, \end{aligned}\right\} \qquad (5\cdot5)$$

and the orthonormality relations (4·11) are simply expressed as

$$\left.\begin{aligned} (\Psi_{m_+I'K'}\cdot\Psi_{n_+IK}) &\equiv \sum_{\alpha\beta\gamma} \{\psi_{m_+I'K'}(\alpha\beta\gamma: 3/2)\,\psi_{n_+IK}(\alpha\beta\gamma: 3/2) \\ &\qquad - \psi_{m_+I'K'}(\alpha\beta\gamma: -1/2)\,\psi_{n_+IK}(\alpha\beta\gamma: -1/2)\} \\ &= \sum_{\alpha\beta\gamma} \{\psi_{mI'K'}(\alpha\beta\gamma)\,\psi_{nIK}(\alpha\beta\gamma) - \varphi_{mI'K'}(\alpha\beta\gamma)\,\varphi_{nIK}(\alpha\beta\gamma)\} \\ &= \delta_{nm}\delta_{II'}\delta_{KK'}, \\ (\Psi_{m\_I'K'}\cdot\Psi_{n\_IK}) &\equiv \sum_{\alpha\beta\gamma} \{-\psi_{m\_I'K'}(\alpha\beta\gamma: 1/2)\,\psi_{n\_IK}(\alpha\beta\gamma: 1/2) \\ &\qquad + \psi_{m\_I'K'}(\alpha\beta\gamma: -3/2)\,\psi_{n\_IK}(\alpha\beta\gamma: -3/2)\} \\ &= \sum_{\alpha\beta\gamma} \{\psi_{mI'K'}(\alpha\beta\gamma)\,\psi_{nIK}(\alpha\beta\gamma) - \varphi_{mI'K'}(\alpha\beta\gamma)\,\varphi_{nIK}(\alpha\beta\gamma)\} \\ &= \delta_{nm}\delta_{II'}\delta_{KK'}. \end{aligned}\right\} \qquad (5\cdot6)$$

The physical interpretation of the operator

$$Y^{\dagger}_{nIK} = \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \{\psi_{nIK}(\alpha\beta\gamma): T_{3/2\ 3/2}(\alpha\beta\gamma): + \varphi_{nIK}(\alpha\beta\gamma): T_{3/2-1/2}(\alpha\beta\gamma):\} \qquad (5\cdot2)'$$

is clear: $Y^{\dagger}_{nIK}$ creates three-quasi-particles with the *large amplitudes* $\psi_{nIK}(\alpha\beta\gamma)$, accompanying the *small* (one-quasi-particle-creation and two-quasi-particle-annhilation) *amplitudes* $\varphi_{nIK}(\alpha\beta\gamma)$. In the absence of ground-state correlations, $Y^{\dagger}_{nIK}$ becomes the operator which creates an exact three-quasi-particle eigenstate with $v=3$ in the sense of the Tamm-Dancoff method.

So far we have discussed only the eigenmode operators $Y^{\dagger}_{nIK}$ which have physical meaning. At this stage, it is important to emphasize that the eigenvalue equation (5·1a) has solutions which inevitably lead us to "special" eigenmode operators:

$$C^\dagger_{n_0IK}=\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\{\psi_{n_0IK}(\alpha\beta\gamma):T_{3/2\,3/2}(\alpha\beta\gamma):+\varphi_{n_0IK}(\alpha\beta\gamma):T_{3/2-1/2}(\alpha\beta\gamma):\}$$
$$=A_{n_0IK}, \tag{5·7}$$

which have the *large amplitudes* $\varphi_{n_0IK}(\alpha\beta\gamma)$ and the *small amplitudes* $\psi_{n_0IK}(\alpha\beta\gamma)$, and have *no physical meaning.* Needless to say, the conjugate creation operators $A^\dagger_{n_0IK}$ arise from Eq. (5·1b). The appearence of the "special" eigenmode operators $A^\dagger_{n_0IK}$ (having no physical meaning) is essentially based on a special situation of the ground-state correlations due to our physical dressed three-quasi-particle modes. The original interaction responsible for the dressed three-quasi-particle modes (and so responsible for the ground-state correlations) is not a three-body interaction but the two-body interaction. Therefore, the "bare" three-quasi-particles can be "dressed" (by the ground-state correlations) only through the amplitudes $\varphi_{nIK}(\alpha\beta\gamma)$. The existence of the amplitudes $\varphi_{nIK}(\alpha\beta\gamma)$ inevitably leads to the appearence of the "special" eigenmode operator $A^\dagger_{n_0IK}$ having the *large amplitudes* $\varphi_{n_0IK}(\alpha\beta\gamma)$. In the absence of the ground-state correlations, which means that $\varphi_{nIK}(\alpha\beta\gamma)$ vanish, the "special" eigenmodes do not appear.

As one of the inherent properties of Eq. (5·1a), we have the orthogonality relations concerning the "special" solutions

$$(\boldsymbol{\Psi}_{n_0'I'K'}\cdot\boldsymbol{\Psi}_{n_0IK})\equiv\sum_{\alpha\beta\gamma}\{\psi_{n_0'I'K'}(\alpha\beta\gamma)\psi_{n_0IK}(\alpha\beta\gamma)-\varphi_{n_0'I'K'}(\alpha\beta\gamma)\varphi_{n_0IK}(\alpha\beta\gamma)\}$$
$$=0 \quad \text{if} \quad (n_0'I'K')\neq(n_0IK), \tag{5·8a}$$
$$(\boldsymbol{\Psi}_{n,I'K'}\cdot\boldsymbol{\Psi}_{n_0IK})\equiv\sum_{\alpha\beta\gamma}\{\psi_{nI'K'}(\alpha\beta\gamma)\psi_{n_0IK}(\alpha\beta\gamma)-\varphi_{nI'K'}(\alpha\beta\gamma)\varphi_{n_0IK}(\alpha\beta\gamma)\}$$
$$=0\,. \tag{5·8b}$$

The existence of "special" eigenmode operators $A^\dagger_{n_0IK}$, which have no physical meaning, imposes an important condition upon the state vectors $|\Phi\rangle$ in the framework of the NTD approximation: Any state vector $|\Phi\rangle$ *which actually has physical meaning* must satisfy the supplementary condition

$$A_{n_0IK}|\Phi\rangle=0\,. \tag{5·9}$$

§ 6. Structure of the ground-state correlations

Characteristics of structure of the ground-state correlations due to the physical dressed three-quasi-particle modes with $\Delta v=3$ should be determined *in principle* through properties of the fundamental equation (4·19) (with the condition (3·37)) which defines the three-quasi-particle modes. As is seen from Eq. (4·3), the fundamental equation (4·19) does not contain any matrix element of the part H_Y defined in Eq. (2·5). The diagrams considered in the correlated ground state $|\Phi_0\rangle$ are therefore closed diagrams which are composed by combining only the matrix elements of H_X and H_V, so that $|\Phi_0\rangle$ may be generally written as a superposition of 0, 4, 8, 12, $\cdots$ quasi-particle states:

$$|\Phi_0\rangle = C_0|\phi_0\rangle + \sum_{\alpha\beta\gamma\delta} C_1(\alpha\beta\gamma\delta)\, a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger a_\delta{}^\dagger|\phi_0\rangle$$
$$+ \sum_{\substack{\alpha\beta\gamma\delta \\ \varepsilon\varphi\mu\lambda}} C_2(\alpha\beta\gamma\delta\epsilon\varphi\mu\nu)\, a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger a_\delta{}^\dagger a_\varepsilon{}^\dagger a_\varphi{}^\dagger a_\mu{}^\dagger a_\nu{}^\dagger|\phi_0\rangle$$
$$+\cdots\cdots, \tag{6·1}$$

where $|\phi_0\rangle$ is the BCS ground state and C_0 is the constant related to the normalization of $|\phi_0\rangle$.

The coefficients C in Eq. (6·1) should be determined in a consistent way to the framework of the approximation which we have used in obtaining the fundamental equation (4·19), by the conditions $Y_{nIK}\ |\Phi_0\rangle=0$ and $A_{n_0IK}|\Phi_0\rangle=0$. From the supplementary condition (5·9) for the ground state $|\Phi_0\rangle$, we have a set of recurrence relations connecting C_n to C_{n-1}, the first of which is

$$C_0\psi_{n_0IK}(\alpha\beta\gamma) - 12\sqrt{3}\left(\frac{1}{3!}\right)\sum_{P(\alpha\beta\gamma)}\delta_P P\{\sum_{\alpha_1\beta_1\gamma_1}\varphi_{n_0IK}(\alpha_1\beta_1\gamma_1)\,\delta_{\gamma\gamma_1}s_{\alpha_1}s_{\beta_1}C_1(\bar{\alpha}_1\bar{\beta}_1\alpha\beta)\} = 0\,. \tag{6·2}$$

Combining Eq. (6·2) and Eq. (5·8b) and using the antisymmetry property of $C_1(\alpha\beta\gamma\delta)$ with respect to the permutation of $(\alpha, \beta, \gamma, \delta)$, we obtain a relation to determine $C_1(\alpha\beta\gamma\delta)$ in terms of the physical amplitudes:

$$\varphi_{nIK}(\alpha\beta\gamma) - 12\sqrt{3}\left(\frac{1}{3!}\right)\sum_{P(\alpha\beta\gamma)}\delta_P P\{\sum_{\alpha_1\beta_1\gamma_1}\psi_{nIK}(\alpha_1\beta_1\gamma_1)\,\delta_{\gamma\gamma_1}s_\alpha s_\beta C_1(\bar{\alpha}\bar{\beta}\alpha_1\beta_1)/C_0\} = 0\,. \tag{6·3}$$

After determining C_1, we may proceed to the next relation connecting C_2 to C_1 in order to determine C_2 and so on.*)

This procedure within the framework of our approximation suggests that the correlated ground state $|\Phi_0\rangle$ in question is of the form:

$$|\Phi_0\rangle = C_0 \exp\{kK_{J=0}^\dagger\}|\phi_0\rangle \equiv C_0 \exp\{W^\dagger\}|\phi_0\rangle, \tag{6·4}$$

where

$$\left.\begin{aligned} &K_{J=0}^\dagger \equiv \frac{1}{\sqrt{4!}}\sum_{\alpha\beta\gamma\delta}\chi_{J=0}(\alpha\beta\gamma\delta)\, a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger a_\delta{}^\dagger, \\ &\sum_{\alpha\beta\gamma\delta}\chi_{J=0}^2(\alpha\beta\gamma\delta) = 1\,, \end{aligned}\right\} \tag{6·5}$$

*) We show in §7 that the ground state $|\Phi_0\rangle$ obtained from the relation (6·3) satisfies also the relation $Y_{nIK}|\Phi_0\rangle=0$ under the basic approximation of RPA.

and the constant k and $\chi_{J=0}(\alpha\beta\gamma\delta)$ are defined through the relation

$$\frac{1}{\sqrt{4!}}k\cdot\chi_{J=0}(\alpha\beta\gamma\delta)\equiv\frac{C_1(\alpha\beta\gamma\delta)}{C_0}. \tag{6·6}$$

Needless to say, $\chi_{J=0}(\alpha\beta\gamma\delta)$ is antisymmetric with respect to the permutation of $(\alpha, \beta, \gamma, \delta)$.

In order to see the physical essence of the approximation used in obtaining the expression (6·4) for the ground state $|\Phi_0\rangle$, it is convenient to take up

$$\begin{aligned}
&\langle\Phi_0|[K_{J=0}, K^\dagger_{J=0}]|\Phi_0\rangle\\
&\equiv 1-4\sum_{\alpha\beta}\sum_{\alpha_1\beta_1\gamma_1}\chi_{J=0}(\alpha_1\beta_1\gamma_1\alpha)\chi_{J=0}(\alpha_1\beta_1\gamma_1\beta)\langle\Phi_0|a_\alpha{}^\dagger a_\beta|\Phi_0\rangle\\
&\quad -3\sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1\beta_1}\chi_{J=0}(\alpha_1\beta_1\alpha\beta)\chi_{J=0}(\alpha_1\beta_1\gamma\delta)\langle\Phi_0|a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma a_\delta|\Phi_0\rangle\\
&\quad +\tfrac{2}{3}\sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1\beta_1\gamma_1}\chi_{J=0}(\alpha\beta\gamma\delta)\chi_{J=0}(\alpha_1\beta_1\gamma_1\delta)\langle\Phi_0|a^\dagger_{\alpha_1}a^\dagger_{\beta_1}a^\dagger_{\gamma_1}a_\alpha a_\beta a_\gamma|\Phi_0\rangle.
\end{aligned} \tag{6·7}$$

Using the expression (6·4), we then can calculate the expectation values on the right-hand side of Eq. (6·7), the largest terms of which are of the order of k^2, i.e.

$$\begin{aligned}
\langle\Phi_0|a_\alpha{}^\dagger a_\beta|\Phi_0\rangle&\equiv\frac{n}{2\Omega}\delta_{\alpha\beta}\\
&\cong 4k^2\sum_{\alpha_1\beta_1\gamma_1}\chi_{J=0}(\alpha_1\beta_1\gamma_1\alpha)\chi_{J=0}(\alpha_1\beta_1\gamma_1\beta)\\
&\approx O(k^2/2\Omega)\cdot\delta_{\alpha\beta}\,,
\end{aligned} \tag{6·8a}$$

$$\begin{aligned}
&\langle\Phi_0|a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma a_\delta|\Phi_0\rangle\\
&\qquad\cong -12k^2\sum_{\alpha_1\beta_1}\chi_{J=0}(\alpha_1\beta_1\alpha\beta)\chi_{J=0}(\alpha_1\beta_1\gamma\delta)\\
&\qquad\approx O\left(\frac{n}{(2\Omega)^2}\right)\cdot\delta_{\alpha+\beta,\ \gamma+\delta}\,,
\end{aligned} \tag{6·8b}$$

$$\begin{aligned}
&\langle\Phi_0|a^\dagger_{\alpha_1}a^\dagger_{\beta_1}a^\dagger_{\gamma_1}a_\alpha a_\beta a_\gamma|\Phi_0\rangle\\
&\qquad\cong -24k^2\sum_{\delta_1}\chi_{J=0}(\alpha\beta\gamma\delta_1)\chi_{J=0}(\alpha_1\beta_1\gamma_1\delta_1)\\
&\qquad\approx O\left(\frac{n}{(2\Omega)^3}\right)\cdot\delta_{\alpha_1+\beta_1+\gamma_1,\alpha+\beta+\gamma}\,,
\end{aligned} \tag{6·8c}$$

where n is the average number of quasi-particles in the ground state and $2\Omega \equiv 2j+1$ is the total number of particle states under consideration. With the use of Eqs. (6·8), Eq. (6·7) is reduced to

$$\langle\Phi_0|[K_{J=0}, K^\dagger_{J=0}]|\Phi_0\rangle\approx 1+O\left(\frac{n}{2\Omega}\right). \tag{6·9}$$

It is well known that *basic approximation* in the NTD method is $n \ll 2\Omega$, i.e. $O(n/2\Omega) \approx 0$. We may also expect this property to be held not only for the ground state$|\Phi_0\rangle$ but for all low-lying excited states under consideration, so that we have

$$[K_{J=0}, K^\dagger_{J=0}] \cong 1\,. \tag{6·10}$$

With the basic approximation which underlies Eq. (6·10), we can easily see that the recurrence relations connecting C_n to C_{n-1} obtained by the condition $A_{n_0IK}|\Phi_0\rangle = 0$ are simply reduced to one equation

$$\begin{aligned} A_{n_0IK}|\Phi_0\rangle &\equiv C_0 A_{n_0IK} e^{W^\dagger}|\phi_0\rangle \\ &\cong \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \Big\{\psi_{n_0IK}(\alpha\beta\gamma) - 12\sqrt{3}\Big(\frac{1}{\sqrt{4!}}\Big)k \\ &\qquad \times \sum_{\alpha_1\beta_1\gamma_1} \varphi_{n_0IK}(\alpha_1\beta_1\gamma_1)\,\delta_{\gamma\gamma_1} s_{\alpha_1} s_{\beta_1} \chi_{J=0}(\bar{\alpha}_1\bar{\beta}_1\alpha\beta)\Big\} \\ &\qquad \times : T_{3/2\ 3/2}(\alpha\beta\gamma) : |\Phi_0\rangle = 0\,, \end{aligned} \tag{6·11}$$

from which we obtain Eq.(6·2).

§7. Quasi-fermion approximation

We are now in a position to discuss the equivalence of Eq. (4·9) and Eq. (5·6) which has so far been assumed as the fundamental essence of the formal structure of NTD theory. With the aid of Eq. (5·2)′, Eq. (4·9) can be written

$$\begin{aligned} \langle\Phi_0|\{Y_{mI'K'}, Y^\dagger_{nIK}\}_+|\Phi_0\rangle &\equiv \sum_{\alpha\beta\gamma} \psi_{mI'K'}(\alpha\beta\gamma)\psi_{nIK}(\alpha\beta\gamma) \\ &- \sum_{\alpha\beta\gamma} (3\psi_{mI'K'}(\alpha\beta\gamma)\psi_{nIK}(\alpha\beta\gamma) - \varphi_{mI'K'}(\alpha\beta\gamma)\varphi_{nIK}(\alpha\beta\gamma)\}\langle\Phi_0|a_\alpha{}^\dagger a_\alpha|\Phi_0\rangle \\ &- \sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1} \{\tfrac{3}{2}\psi_{mI'K'}(\alpha_1\gamma\delta)\psi_{nIK}(\alpha_1\alpha\beta) + 2\varphi_{mI'K'}(\alpha_1\delta\bar{\beta})\varphi_{nIK}(\alpha_1\alpha\bar{\gamma})s_\beta s_\gamma \\ &\qquad + \tfrac{1}{2}\varphi_{mI'K'}(\alpha_1\bar{\alpha}\bar{\beta})\varphi_{nIK}(\alpha_1\bar{\gamma}\bar{\delta})s_\alpha s_\beta s_\gamma s_\delta\}\langle\Phi_0|a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma a_\delta|\Phi_0\rangle \\ &- \tfrac{1}{2}\sqrt{3}\sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1}\psi_{mI'K'}(\alpha_1\alpha\beta)\varphi_{nIK}(\alpha_1\gamma\delta)\langle\Phi_0|a_\alpha a_\beta \tilde{a}_\gamma \tilde{a}_\delta|\Phi_0\rangle \\ &- \tfrac{1}{2}\sqrt{3}\sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1}\varphi_{mI'K'}(\alpha_1\alpha\beta)\psi_{nIK}(\alpha_1\gamma\delta)\langle\Phi_0|\tilde{a}_\alpha{}^\dagger \tilde{a}_\beta{}^\dagger a_\gamma{}^\dagger a_\delta{}^\dagger|\Phi_0\rangle. \end{aligned} \tag{7·1}$$

In evaluating the right-hand side of Eq. (7·1) within the framework of the NTD approximation, we adopt the following procedures: (i) We first calculate the expectation values by using the expression (6·4), and take up the *largest terms* which are of the order of k^2 or more, i.e. Eqs. (6·8a), (6·8b) and

$$\langle\Phi_0|a_\alpha a_\beta a_\gamma a_\delta|\Phi_0\rangle \cong 24\frac{1}{\sqrt{4!}}k\cdot\chi_{J=0}(\alpha\beta\gamma\delta)\,. \tag{7·2}$$

(ii) We then use the relation (6·3). (iii) Afterwords we drop all terms with

$O(n/2\Omega)$ according to the basic approximation in the NTD method. Procedures (i), (ii) and (iii) lead to

$$\begin{aligned}\langle\Phi_0|\{Y_{mI'K'}Y^\dagger_{nIK}\}_+|\Phi_0\rangle \\ &\cong \sum_{\alpha\beta\gamma\delta}\{\psi_{mI'K'}(\alpha\beta\gamma)\psi_{nIK}(\alpha\beta\gamma)-\varphi_{mI'K'}(\alpha\beta\gamma)\varphi_{nIK}(\alpha\beta\gamma)\} \\ &=\delta_{nm}\delta_{II'}\delta_{KK'}\,. \end{aligned} \tag{7·3}$$

We can therefore see that, with the basic approximation in the NTD method, the equivalence of Eq. (4·9) and Eq. (5·6) is justified.

Now it may be natural to expect that the relation (7·3) would also approximately hold for all low-lying excited states under consideration. If this is allowable approximation, we have

$$\{Y_{mI'K'},\ Y^\dagger_{nIK}\}_+=\delta_{nm}\delta_{II'}\delta_{KK'}\,, \tag{7·4}$$

which means that the eigenmode operators of the dressed physical three-quasi-particles are regarded as Fermion operators. We may thus call it the "quasi-Fermion approximation".

Adopting the same procedures as used in obtaining Eq. (7·3), we can also see that the various eigenstates $|\Phi_{nIK}\rangle\equiv Y^\dagger_{nIK}|\Phi_0\rangle$ are orthogonal and normalized

$$\langle\Phi_{mI'K'}|\Phi_{nIK}\rangle\equiv\langle\Phi_0|Y_{mI'K'}Y^\dagger_{nIK}|\Phi_0\rangle=\delta_{nm}\delta_{II'}\delta_{KK'}\,. \tag{7·5}$$

From Eqs. (7·3) and (7·5), we obtain

$$\langle\Phi_0|Y^\dagger_{nIK}Y_{mI'K'}|\Phi_0\rangle=0 \tag{7·6}$$

within the basic approximation $O(n/2\Omega)\approx0$. In the quasi-Fermion approximation, we therefore have

$$Y_{nIK}|\Phi_0\rangle=0\,, \tag{7·7}$$

since the inner product of the state vector $Y_{nIK}|\Phi_0\rangle$ is of the order of $O(n/2\Omega)$.*)

Here it is quite interesting to note that with the same procedure as used in obtaining Eq. (6·11), the excited states $|\Phi_{nIK}\rangle\equiv Y^\dagger_{nIK}|\Phi_0\rangle$ is expressed by

$$\begin{aligned}Y^\dagger_{nIK}|\Phi_0\rangle=\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\Big\{\psi_{nIK}(\alpha\beta\gamma)-12\sqrt{3}\Big(\frac{1}{\sqrt{4!}}\Big)k \\ \times\sum_{\alpha_1\beta_1\gamma_1}\varphi_{nIK}(\alpha_1\beta_1\gamma_1)\delta_{\gamma\gamma_1}s_{\alpha_1}s_{\beta_1}\chi_{J=0}(\bar{\alpha}_1\bar{\beta}_1\alpha\beta)\Big\} \\ \times:T_{3/2\ 3/2}(\alpha\beta\gamma):|\Phi_0\rangle.\end{aligned} \tag{7·8}$$

Equation (7·8) clearly shows the following important fact: *The collective excited states are always described by three-quasi-particle creations with $\Delta v=3$ from the correlated ground state $|\Phi_0\rangle$*, although they are always represented as

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*) Needless to say, we have $\langle\Phi_{mI'K'}|Y_{nIK}|\Phi_0\rangle=0$, and $\langle\Phi_0|Y_{nIK}|\Phi_0\rangle=0$, so that any state under the NTD approximation is orthogonal to $Y_{nIK}|\Phi_0\rangle$.

superpositions of 3, 7, 11, 15, $\cdots$ quasi-particle states.

§ 8. The spurious states

The function $\chi_{J=0}(\alpha\beta\gamma\delta)$ defined by Eq. (6·5) is antisymmetric with respect to $(\alpha, \beta, \gamma, \delta)$, so that we have

$$\chi(j^2(J_1)j^2(J_1)\}j^4J=0)$$
$$=\sum_{J_2}{}'\chi(j^2(J_2)j^2(J_2)\}j^4J=0)\sqrt{(2J_1+1)(2J_2+1)}W(jjjj:J_1J_2)$$
$$\text{for even } J_1, \tag{8·1}$$

where $(j^2(J_1)j^2(J_2)\}j^4J=0)$ is defined through

$$\chi_{J=0}(\alpha\beta\gamma\delta)=\sum_{J_1M_1}{}'\sum_{J_2M_2}{}'\chi(j^2(J_1)j^2(J_2)\}j^4J=0)$$
$$\times\langle jjm_\alpha m_\beta|J_1M_1\rangle\langle jjm_\gamma m_\delta|J_2M_2\rangle(-)^{J_1-M_1}\delta_{J_1J_2}\delta_{M_1\bar{M}_2} \tag{8·2}$$

and $\sum_J'$ means the summation with respect to even values of J. Now let us impose a condition on $\chi_{J=0}(\alpha\beta\gamma\delta)$;

$$\chi(j^2(J_1=0)j^2(J_2=0)\}j^4J=0)=0\,, \tag{8·3}$$

with which Eq. (8·1) becomes of the same form as the equation which the coefficient of fractional parentage (c.f.p) with seniority $v=4$ and $J=0$ for j^4-configulations have to satisfy. We then can examine that, within the basic approximation $O(n/2\Omega)\approx 0$, the condition (8·3) is compatible with Eqs. (6·2) and (6·3) which determine the function $\chi_{J=0}(\alpha\beta\gamma\delta)$.

With the condition (8·3), we have under the basic approximation

$$\hat{S}_-|\Phi_0\rangle=-\frac{1}{\sqrt{4!}}12k\frac{1}{\sqrt{2\Omega}}\sum_{\alpha\beta}\langle jjm_\alpha m_\beta|00\rangle\sum_{\alpha_1\beta_1}\chi_{J=0}(\alpha_1\beta_1\alpha\beta)\,a_{\alpha_1}^\dagger a_{\beta_1}^\dagger|\Phi_0\rangle$$
$$=0\,, \tag{8·4}$$

where $\hat{S}_-$ is defined in Eq. (3·21). This means an important fact that the *correlated ground state* $|\Phi_0\rangle$ *has no zero-coupled quasi-particle pairs**). With the aid of Eq. (8·4), we obtain

$$\hat{S}_-|\Phi_{nIK}\rangle=[\hat{S}_-, Y_{nIK}^\dagger]|\Phi_0\rangle$$
$$=\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\{\psi_{nIK}(\alpha\beta\gamma)[\hat{S}_-, T_{3/2\,3/2}(\alpha\beta\gamma)]$$
$$+\varphi_{nIK}(\alpha\beta\gamma)[\hat{S}_-, T_{3/2-1/2}(\alpha\beta\gamma)]\}|\Phi_0\rangle$$

*) If Eq. (8·4) were exactly valid, $|\Phi_0\rangle$ would be represented as $|\Phi_0\rangle=\sum_S C(S)|S, S_0=-S\rangle$ so that the seniority number v in each component has the very same value as the number of quasi-particles n_0. (See Eqs. (3·24) and (3·25).)

$$=\frac{1}{\sqrt{3!}}\sqrt{3}\sum_{\alpha\beta\gamma}\{\varphi_{nIK}(\alpha\beta\gamma):T_{3/2-3/2}(\alpha\beta\gamma):$$
$$+\psi_{nIK}(\alpha\beta\gamma):T_{3/2\ 1/2}(\alpha\beta\gamma):\}|\Phi_0\rangle. \qquad (8\cdot5)$$

Since the inner product of the state vector on the right-hand side of Eq. (8·5) is of the order of $O(n/2\Omega)$, we can further see that *the excited states* $|\Phi_{nIK}\rangle$ *also have no zero-coupled pairs* under the basic approximation $O(n/2\Omega)\approx0$:

$$\hat{S}_-|\Phi_{nIK}\rangle=0\,. \qquad (8\cdot6)$$

It is now quite clear that, within the framework of the NTD approximation, both the ground state $|\Phi_0\rangle$ and the collective excited states are orthogonal to the spurious states,

$$\left.\begin{aligned}|\Phi_{sp}(0)\rangle&\equiv\text{const}(\hat{N}-N_0)|\Phi_0\rangle\\&=\text{const}\{(u^2-v^2)\hat{n}+2uv(\hat{S}_++\hat{S}_-)\}|\Phi_0\rangle,\\|\Phi_{sp}(nIK)\rangle&\equiv\text{const}(\hat{N}-N_0)|\Phi_{nIK}\rangle\\&=\text{const}\{(u^2-v^2)\hat{n}+2uv(\hat{S}_++\hat{S}_-)\}|\Phi_{nIK}\rangle,\end{aligned}\right\} \qquad (8\cdot7)$$

which arise from the nucleon-number fluctuation introduced inevitably by the use of the quasi-particle picture: With the use of Eqs. (8·4) and (8·6) and of the fact $(u^2-v^2)\langle\phi_i|\hat{n}|\phi_j\rangle\approx O(n/2\Omega)\approx0$ with $|\Phi_j\rangle=|\Phi_0\rangle$ or $|\Phi_{nIK}\rangle$, we obtain

$$\langle\Phi_{sp}(i)|\Phi_j\rangle=0\,, \qquad (8\cdot8)$$

where $|\Phi_{sp}(i)\rangle=|\Phi_{sp}(0)\rangle$ or $|\Phi_{sp}(nIK)\rangle$. We therefore may conclude that our theory can leads us to both the "physical" collective excited state and the "physical" ground state, and with this theory we can enjoy the proper advantage of the NTD method to remove the troubles of spurious states.

§ 9. Generalization to the realistic case

With the purpose of illustrating the physical essence of our method, we have so far used the single j-shell model. However, the extension of the essential idea to the realistic case possesses no difficulties.

It is well known that the use of the quasi-particle-Tamm-Dancoff approximation on the basis of the BCS theory can be regarded as an attempt to describe both the ground state and the low-lying excited states in terms of the seniority eigenstates: The BCS ground state and the Tamm-Dancoff excited states in terms of quasi-particles are given by

$$\left.\begin{aligned}|\phi_0\rangle&=|S(a)=\tfrac{1}{2}\Omega_a,\,S_0(a)=-\tfrac{1}{2}\Omega_a;\,S(b)=\tfrac{1}{2}\Omega_b,\,S_0(b)=-\tfrac{1}{2}\Omega_b;\cdots\rangle,\\|\phi_{\text{excit}}\rangle&=|S(a),\,S_0(a)=-S(a);\,S(b),\,S_0(b)=-S(b);\cdots:\Gamma JM\rangle.\end{aligned}\right\} \qquad (9\cdot1)$$

In Eq. (9·1), Γ means a set of additional quantum numbers to specify the Tamm-

Dancoff excited states, and $S(a)$ and $S_0(a)$ are the "quasi-spin" quantum number and its projection belonging to the single-particle level a*)

$$S(a)=\tfrac{1}{2}(\Omega_a-v_a), \qquad S_0(a)=\tfrac{1}{2}(n_a-\Omega_a), \tag{9·2}$$

where $2\Omega_a\equiv 2j_a+1$ and n_a means the number of quasi-particles in the level a. Thus the total seniority v of the state in question is well defined as

$$v=\sum_a v_a, \tag{9·3}$$

the value of which is the same as the number of quasi-paricles $n_0=\sum_a n_a$, except for a special class of excited states associated withe pairing excitations with $J=0$.**)

Corresponding to such a quasi-particle-Tamm-Dancoff approximation with $v=n_0=3$ and extending the essential idea so far developed into the realistic case we now can define the eigenmode operators of "physical" dressed three-quasi-particles, which should be used in the quasi-particle-NTD approximation:

$$\begin{aligned} C^\dagger_{nIK}=&\sum_{\alpha\beta\gamma}\psi^{(1)}_{nIK}(\alpha\beta\gamma)a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger+\sum_{\alpha\beta\gamma}\varphi^{(1)}_{nIK}(\alpha\beta\gamma)a_\alpha{}^\dagger\tilde{a}_\beta\tilde{a}_\gamma \\ &+\sum_{\alpha\beta\gamma}\varphi^{(2)}_{nIK}(\alpha\beta\gamma)a_\gamma{}^\dagger a_\beta{}^\dagger\tilde{a}_\alpha+\sum_{\alpha\beta\gamma}\psi^{(2)}_{nIK}(\alpha\beta\gamma)\tilde{a}_\alpha\tilde{a}_\beta\tilde{a}_\gamma, \end{aligned} \tag{9·4}$$

where $\varphi^{(i)}_{nIK}(\alpha\beta\gamma)$ $(i=1$ and $2)$ satisfy the two conditions

$$\left.\begin{aligned} &P\psi^{(i)}_{nIK}(\alpha\beta\gamma)=\delta_P\psi^{(i)}_{nIK}(\alpha\beta\gamma), \\ &\sum_{m_\alpha m_\beta}\sum_{m_\gamma M}\langle Jj_cMm_\gamma|IK\rangle\langle j_aj_bm_\alpha m_\beta|00\rangle\psi^{(i)}_{nIK}(\alpha\beta\gamma)=0, \end{aligned}\right\} \tag{9·5}$$

while $\varphi^{(k)}_{nIK}(\alpha\beta\gamma)$ $(k=1$ and $2)$, which satisfy

$$\left.\begin{aligned} &\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=-\varphi^{(k)}_{nIK}(\alpha\gamma\beta), \\ &\sum_{m_\beta m_\gamma}\sum_{m_\alpha M}\langle Jj_aMm_\alpha|IK\rangle\langle j_bj_cm_\beta m_\gamma|00\rangle\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=0, \end{aligned}\right\} \tag{9·6a}$$

have to obey the following conditions:

$$\left.\begin{aligned} &\sum_{m_\alpha m_\beta}\sum_{m_\gamma M}\langle Jj_cMm_\gamma|IK\rangle\langle jjm_\alpha m_\beta|JM\rangle\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=0 \\ &\qquad\qquad \text{for } J=0 \text{ and odd-}J,\ \text{if } a=b\neq c\,(j_a=j_b\equiv j), \\ &\sum_{m_\alpha m_\gamma}\sum_{m_\beta M}\langle Jj_bMm_\beta|IK\rangle\langle j'j'm_\alpha m_\gamma|J'M'\rangle\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=0 \\ &\qquad\qquad \text{for } J'=0 \text{ and odd-}J',\ \text{if } a=c\neq b\,(j_a=j_c\equiv j'), \end{aligned}\right\} \tag{9·6b}$$

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*) In the realistic case, the single-particle states are characterized by the quantum numbers: the charge q, n, l, j, m. The single-particle state with a set of these quantum numbers is then designated by a Greek subscript α. We further use a Latin letter a to mean all the quantum numbers in α except the magnetic quantum number m.

**) For the quasi-particle-Tamm-Dancoff states defined by Eq. (9·1), we have $\hat{S}_-(a)|\phi_{\rm excit}\rangle=0$. For the class of excited states $|\phi_{\rm pair}\rangle$ associated with the pairing excitations, however, we have $\hat{S}_-(b)|\phi_{\rm pair}\rangle\neq 0$ for some levels b, which means $v\neq n_0$ so that the quasi-particle picture loses its original merit. Needless to say, such a class of excitations arising from the motion of $\hat{S}_\pm(a)$ are closely related to the spurious states, since the nucleon number fluctuation is given by $\hat{N}-N_0=\sum_a(u_a{}^2-v_a{}^2)\hat{n}_a+2\sum_a u_av_a(\hat{S}_+(a)+\hat{S}_-(a))$.

and

$$\left.\begin{aligned} &P\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=\delta_P\varphi^{(k)}_{nIK}(\alpha\beta\gamma),\\ &\sum_{m_\alpha m_\beta}\sum_{m_\gamma M}\langle Jj_cMm_\gamma|IK\rangle\langle j_aj_bm_\alpha m_\beta|00\rangle\varphi^{(k)}_{nIK}(\alpha\beta\gamma)=0 \end{aligned}\right\} \quad (9\cdot6c)$$

$$\text{if } a=b=c.$$

In Eq. (9·4), the terms on the right-hand side *with* $a=b=c$ are composed of $T_{s(a)=3/2,s_0(a)}(\alpha\beta\gamma;a=b=c)$ with the transferred seniority $\Delta v_a=2s(a)=3$, and for example the terms with $a=b\neq c$ consist of products of quasi-spin tensors $T_{s(a)=1,s_0(a)}(\alpha\beta;a=b)$ with $\Delta v_a=2$ (in the level a) and $T_{s(c)=1/2,s_0(c)}(\gamma)$ with $\Delta v_c=1$ (in the level c), and so on. Therefore it is quite evident that the amount of seniority which the eigenmode operators $C^\dagger_{nIK}$ transfers to the state on which they operate is now $\Delta v\equiv\sum_a\Delta v_a=3$. It is also clear from the conditions (9·5) and (9·6) that the eigenmode operators never contains the "quasi-spin" operators $\hat{S}_\pm(a)$, $\hat{S}_0(a)$ defined at each level a. Within the framework of the NTD approximation, both our collective excited states $|\Phi_{nIK}\rangle$ and the corresponding correlated ground state $|\Phi_0\rangle$ are thus orthogonal to the spurious states as well as to the special class of collective excited states associated with the pairing vibrations (in superconducting nuclei) which arise mainly from the motion of $\hat{S}_\pm(a)$.

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Theory of Collective Excitations in Spherical Odd-Mass Nuclei. II

——*Structure of the Anomalous Coupling States with Spin* $I=(j-1)$ *)——

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A new point of view on the structure of the anomalous coupling states is proposed. In this point of view, the main component of the anomalous coupling states with spin $I=(j-1)$ is considered to be the dressed three-quasi-particle mode, which has been developed in a previous paper, part I, as a new collective mode in spherical odd-mass nuclei. The excitation-energy systematics of the anomalous coupling states is theoretically made with the use of the pairing-plus-quadrupole force. From the numerical results, it is concluded that the excited anomalous coupling states with spin $I=(j-1)$ can be recognized very well as the dressed three-quasi-particle states. The role of the three-quasi-particle correlation in characterizing the anomalous coupling states is investigated, and the importance of the three-quasi-particle correlation in the collective excitations in odd-mass nuclei is demonstrated.

§ 1. Introduction

For a long time it has been difficult to understand the following experimental fact. In spherical odd-mass nuclei in which an *opposite-parity* level of large spin j in the major shell (such as $1f_{7/2}^-$ and $1g_{9/2}^+$) is presumably being filled, there occurs a competition between a spin j- and a spin $(j-1)$-state for the ground state. Such extra low-lying states with spin $I=(j-1)$ and with opposite parity have been called the anomalous coupling states. Recently the low-lying oppsite-parity states with spin $I=(j-1)$ have also been discovered, not a few, in the $1h_{11/2}^-$ region. One of the most characteristic features of the anomalous coupling states is that $E2$ transitions from the anomalous coupling states to the one-quasi-particle states with spin j are strongly enhanced while $M1$ transitions are moderately hindered.[1] The amount of enhancement of the $E2$ transitions is comparable to that from the phonon states to the ground states in neighboring even-even nuclei. Thus, it has been recognized that the anomalous coupling states have a strong collective nature.

The systematic studies of the low-lying states of spherical odd-mass nuclei have been made with the use of the phonon-quasi-particle-coupling theory.[2]

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*) A preliminary report of this work has been published in this journal, **46** (1971), 996.

Although the theory has succeeded in systematic explanation of the energy levels, transition rates and other properties of many low-lying levels, the anomalous coupling states have remained still unsolved.

The main purpose of this paper is to propose a new point of view in understanding the structure of anomalous coupling states with the use of the pairing-plus-quadrupole-force ($P+QQ$) model. The basic idea underlying the new point of view is as follows: In the conventional phonon-quasi-particle-coupling theory[2] for spherical odd-mass nuclei, the elementary excitation modes are assumed to be one-quasi-particle modes, one-phonon mode, two-phonon modes, etc., and the low-lying states are described as superposed coupling states of these elementary excitation modes. Contrary to this assumption, we suppose that the elementary excitation modes characterizing the low-lying states are one-quasi-particle modes, "dressed" three-quasi-particle modes, "dressed" five-quasi-particle modes, etc. The concept of the "dressed" n-quasi-particle modes has been proposed in the previous paper,[3] part I, in connection with the quasi-spin tensors of each orbit. Now let us consider the systems of odd-mass nuclei in the truncated shell-model space consisting of one major harmonic-oscillator shell (for both the protons and the neutrons) and of a large spin, *opposite-parity* level j which enters into the major shell, and suppose the oppsite-parity level j being is filled. (See Fig. 1.) Then, the dressed three-quasi-particle modes, proposed in part I, with the opposite parity (to that of the major shell) and with spin $I\neq j$ (for instance $I=j-1$) do not couple to any one-quasi-particle modes, because there is no single-particle level with opposite parity and with spin $j'=I\neq j$ in the truncated space. (Such a single-particle level is generally lying in the next upper major shell. See Fig. 1.) Thus, the special physical condition of the appearance of anomalous coupling states becomes just the same condition that the dressed three-quasi-particle modes manifest themselves as relatively pure eigenmodes without coupling to the one-quasi-particle modes.

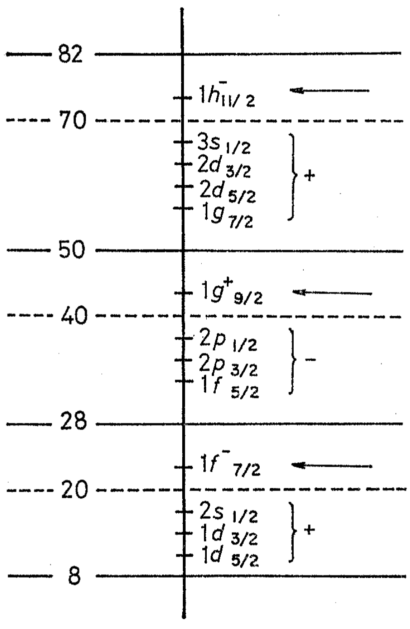

Fig. 1. Schematic representation of shell structure. The symbol "←" denotes the high spin orbit with opposite-parity in each major shell.

It is now clear that the proposed new point of view is to consider the main component of the anomalous coupling states with spin $I=(j-1)$ to be just the dressed three-quasi-particle modes, which have been proposed in part I as new collective modes in spherical odd-mass nuclei. If we neglect the ground-state correlations and restrict ourselves only within the opposite parity level j which is being filled, the dressed three-quasi-particle state with spin $I=j-1$ in the

$P+QQ$ model is reduced to Kisslinger's (Tamm-Dancoff-) three-quasi-particle "intruder" state,[4] which is very much lowered in energy by the quadrupole force and has been considered by him to be the state due to an extra degree of freedom in the phonon-quasi-particle-coupling states. On the other hand, if we neglect the characteristic three-quasi-particle correlation[3] in the dressed three-quasi-particle mode, the mode is decomposed into the odd quasi-particle and the phonon. We may therefore expect that the dressed three-quasi-particle mode with spin $I=(j-1)$ can involve two essential characteristic aspects of the anomalous coupling states in a unified manner: One characteristic aspect which is represented by Kisslinger's "intruder" states[4] and another characteristic aspect of strong collectiveness (which underlies the phonon-quasi-particle-coupling states).

In the case of even-even nuclei, as is well known, when the strength of the quadrupole force becomes large and reaches a critical value which brings the excitation energy of the phonon (i.e., the dressed two-quasi-particle mode) to be zero, there occurs the instability of the (spherical) BCS ground state toward deformation. In the same way, when the strength of the quadrupole force becomes large, the characteristic three-quasi-particle correlation grows up so that the excitation energy of the dressed three-quasi-particle mode with spin $I=(j-1)$ is very much lowered. Also, when the excitation energy (of the mode with spin $(j-1)$) becomes equal to the energy of the one-quasi-particle (ground) state with spin j, there may occur an instability of spherical odd-mass nuclei toward deformation. Therefore, our new point of view of the anomalous coupling states as the dessed three-quasi-particle modes seems to us to be strongly supported by Bohr-Mottelson's old suggestion[5] concerning the possible connection between the appearance of the spin $(j-1)$ state as ground state and the onset of deformation. Although there is no systematic evidence for such stable deformation in these nuclei (with the spin $(j-1)$ ground states), it should be noticed that the adjacent even-even nuclei exhibit the quasi-rotational spectra.[6]

In § 3, the "physical" dressed three-quasi-particle modes proposed in part I are recapitulated in the single j-shell model with the $P+QQ$ force, and the three-quasi-particle correlation characterizing the modes is explicitly shown. In § 4, the dressed three-quasi-particle modes in the $P+QQ$ model are investigated under the special physical condition of shell structure for the appearance of anomalous coupling states. Enhancement factors of the characteristic three-quasi-particle correlation (involved in these modes) are also discussed. Solving the eigenvalue equation of the modes, we make excitation-energy systematics in § 5 in order to check the proposed new point of view of the anomalous coupling states. The result of numerical calculations indicates that, in the first order approximation, the anomalous coupling excited states with spin $I=(j-1)$ can be recognized very well as the dressed three-quasi-particle states.

§ 2. The Hamiltonian

Let us start with the spherically symmetric *j-j* coupling-shell-model Hamiltonian*) with the $P+QQ$ force. Then, after the Bogolyubov transformation, our Hamiltonian may be written in terms of the quasi-particle operators, $a_\alpha{}^\dagger$ and a_α, as follows:

$$H=H_0+H_{QQ}$$
$$=\sum_\alpha E_a a_\alpha{}^\dagger a_\alpha - \tfrac{1}{2}\chi \sum_M :\hat{Q}^\dagger_{2M}\hat{Q}_{2M}:, \qquad (2\cdot1)$$

where χ is the strength of the quadrupole force, and E_a is the quasi-particle energy, determined as usual together with the parameters v_a and u_a of the Bogolyubov transformation. The symbol : : denotes the normal product with respect to the quasi-particles, and the quantity $\hat{Q}_{2M}$ is the mass-quadrupole-moment operator in terms of quasi-particles,

$$\hat{Q}_{2M}=\sum_{ab} q(ab)\,[\xi(ab)\,\{A^\dagger_{2M}(ab)+(-)^{2-M}A_{2-M}(ab)\}$$
$$+\eta(ab)\,\{B^\dagger_{2M}(ab)+(-)^{2-M}B_{2-M}(ab)\}], \qquad (2\cdot2)$$

where $q(ab)$ is the reduced matrix element of the single-particle quadrupole moment, defined through

$$\left.\begin{aligned}\langle\alpha|r^2Y_{2M}(\theta,\varphi)|\beta\rangle &= q(ab)\,(-)^{j_b-m_\beta}\langle j_aj_bm_\alpha-m_\beta|2M\rangle\\ q(ab) &= -(-)^{j_a+j_b+2}q(ba),\end{aligned}\right\} \qquad (2\cdot3)$$

and

$$\left.\begin{aligned}\xi(ab) &\equiv \frac{1}{\sqrt{2}}(u_av_b+v_au_b),\\ \eta(ab) &\equiv \frac{1}{2}(u_au_b-v_av_b).\end{aligned}\right\} \qquad (2\cdot4)$$

The operators $A^\dagger_{JM}(ab)$ and $B^\dagger_{JM}(ab)$ in Eq. (2·2) are the conventional pair operators defined by

$$\left.\begin{aligned}A^\dagger_{JM}(ab) &= \frac{1}{\sqrt{2}}\sum_{m_\alpha m_\beta}\langle j_aj_bm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger a_\beta{}^\dagger,\\ B^\dagger_{JM}(ab) &= -\sum_{m_\alpha m_\beta}\langle j_aj_bm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger \tilde{a}_\beta,\end{aligned}\right\} \qquad (2\cdot5)$$

*) The single-particle states are then characterized by a set of quantum numbers: the charge *q, n, l, j, m*. Throughout this paper, these states are designated by Greek letters. In association with the letter α, we use a Roman letter *a* to denote the same set except the magnetic quantum number *m*. We further use a subscript $-\alpha$, which is obtained from α by changing the sign of the magnetic quantum number. For a basis of stationary states, it is possible to build the entire treatment on real quantities if the phase convention is suitably chosen. In this paper we always assume this to be the case.

where

$$\tilde{a}_\beta \equiv (-)^{j_b - m_\beta} a_{-\beta}\,. \tag{2·6}$$

The quadrupole force H_{QQ} in Eq. (2·1) represents the interaction causing the breakup of the Cooper pair. We divide it in the following way:

$$H_{QQ} = H_X + H_V + H_Y + H_4\,,$$

where

$$H_X = -\chi \sum_M \sum_{a_1 b_1 a_2 b_2} q(a_1 b_1) q(a_2 b_2) \xi(a_1 b_1) \xi(a_2 b_2) A^\dagger_{2M}(a_1 b_1) A_{2M}(a_2 b_2)\,, \tag{2·7a}$$

$$H_V = -\tfrac{1}{2}\chi \sum_M \sum_{a_1 b_1 a_2 b_2} q(a_1 b_1) q(a_2 b_2) \xi(a_1 b_1) \xi(a_2 b_2) \times \{A^\dagger_{2M}(a_1 b_1)(-)^{2-M} A^\dagger_{2-M}(a_2 b_2) + \text{h.c.}\}\,, \tag{2·7b}$$

$$H_Y = -2\chi \sum_M \sum_{a_1 b_1 a_2 b_2} q(a_1 b_1) q(a_2 b_2) \xi(a_1 b_1) \eta(a_2 b_2) \times \{A^\dagger_{2M}(a_1 b_1) B_{2M}(a_2 b_2) + \text{h.c.}\}\,, \tag{2·7c}$$

$$\begin{aligned} H_4 &= -2\chi \sum_M \sum_{a_1 b_1 a_2 b_2} q(a_1 b_1) q(a_2 b_2) \eta(a_1 b_1) \eta(a_2 b_2) : B^\dagger_{2M}(a_1 b_1) B_{2M}(a_2 b_2) : \\ &= +4\chi \sum_{J_1 M_1} \sum_{a_1 b_1 a_2 b_2} q(a_1 b_1) q(a_2 b_2) \eta(a_1 b_1) \eta(a_2 b_2) \\ &\quad \times 5 \cdot \begin{Bmatrix} j_{a_1} & j_{b_1} & 2 \\ j_{a_2} & j_{b_2} & J_1 \end{Bmatrix} A^\dagger_{J_1 M_1}(a_1 b_2) A_{J_1 M_1}(a_2 b_1)\,. \end{aligned} \tag{2·7d}$$

The essential advantage of using the $P+QQ$ model is its great simplicity in the treatment. The *inherent assumption* underlying the model is the following:[7)]

i) The contribution of the pairing force to the Hartree-Fock field is neglected.

ii) The contribution of the quadrupole force to the pairing potential, which comes from the recoupling of a pair of single-particle states in the force itself, is neglected.

iii) The exchange term arising from the quadrupole force is also neglected for the same reason that it involves the recoupling.

According to the *inherent assumption* of the model, we hereafter neglect the exchange term H_4, (2·7d), in the quadrupole force H_{QQ}. Thus we may write the interaction as

$$H'_{QQ} = H_X + H_V + H_Y\,,$$

each matrix element of which is represented by one of the diagrams in Fig. 2. The part H_X represents a scattering of the pair of quasi-particles coupled to $J^\pi = 2^+$. The part H_V represents a pair-creation and a pair-annihilation of the pair of quasi-particles coupled to $J^\pi = 2^+$, so that it introduces the ground-state correla-

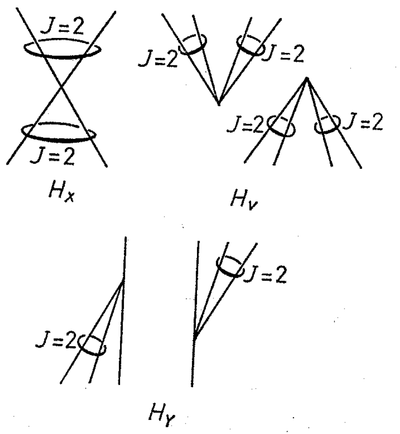

Fig. 2. Graphic representation of the matrix elements of the interaction.

tions. The part H_Y denotes the coupling between a quasi-particle and the pair of quasi-particles coupled to $J^\pi = 2^+$. As has been discussed in part I, the part H_Y does not play any important roles in constructing the dresssed three-quasi-particle modes as elementary excitation modes. The H_Y changes the number of quasi-particles, so that it may play an essential role in constructing interactions between the different types of elementary excitation modes, for instance, interactions between the one-quasi-particle modes and the dressed three-quasi-particle modes, etc. Throughout this paper, in which the anomalous coupling states are regarded as relatively pure dressed three-quasi-particle modes (in the first order approximation), we do not touch the interaction between the one-quasi-particle modes and the dressed three-quasi-particle modes, so that we drop the part H_Y. Thus, our Hamiltonian of the $P+QQ$ model for the dressed three-quasi-particle modes is finally given by

$$H^{(0)} = H_0 + H^{(0)}_{QQ}\,,\qquad H^{(0)}_{QQ} \equiv H_X + H_V\,, \tag{2·8}$$

which is the same as one used in constructing the phonon modes (i.e., the dressed two-quasi-particle modes) in spherical even-even nuclei.

§ 3. "Physical" dressed three-quasi-particle modes in the single j-shell model

For the convenience of later discussion, let us start with the single j-shell model.*) The "physical" dressed three-quasi-particle modes (with excitation energy $\omega_{nI} > 0$), which have been introduced in part I, are then

$$\begin{aligned} Y^\dagger_{nIK} &= \frac{1}{\sqrt{3!}} \sum_{m_\pi m_\rho m_\sigma} \{\psi_{nIK}(\pi\rho\sigma): T_{3/2\,3/2}(\pi\rho\sigma): \\ &\qquad + \varphi_{nIK}(\pi\rho\sigma): T_{3/2-1/2}(\pi\rho\sigma):\} \\ &= \frac{1}{\sqrt{3!}} \sum_J{}' [\psi_n(j_p^2(J)j_p\} j_p^3 I): T_{3/2\,3/2}(j_p^2(J)j_p IK): \\ &\qquad + \varphi_n(j_p^2(J)j_p\} j_p^3 I): T_{3/2-1/2}(j_p^2(J)j_p IK):]\,, \end{aligned} \tag{3·1}$$

*) The single-particle states in this single j-shell model are denoted by Greek letters, π, ρ, σ, etc., and a set of the quantum numbers specifying the single j-level is characterized by a Roman letter $p(=r=s)$.

with

$$T_{3/2\,s_0}(j_p^2(J)j_pIK) \equiv \sum_{m_\pi m_\rho m_\sigma} \langle Jj_pMm_\sigma|IK\rangle\langle j_pj_pm_\pi m_\rho|JM\rangle T_{3/2\,s_0}(\pi\rho\sigma), \tag{3·2}$$

and

$$\left.\begin{aligned} \psi_{nIK}(\pi\rho\sigma) &\equiv \sum_J{}' \psi_n(j_p^2(J)j_p\}j_p^3I)\langle Jj_pMm_\sigma|IK\rangle\langle j_pj_pm_\pi m_\rho|JM\rangle, \\ \varphi_{nIK}(\pi\rho\sigma) &\equiv \sum_J{}' \varphi_n(j_p^2(J)j_p\}j_p^3I)\langle Jj_pMm_\sigma|IK\rangle\langle j_pj_pm_\pi m_\rho|JM\rangle. \end{aligned}\right\} \tag{3·3}$$

Here I and K are the angular momentum and its projection and n denotes a set of additional quantum numbers to specify the mode, and the symbols : : and $\sum_J{}'$ denotes the normal product with respect to the quasi-particles and the summation with respect to even values of J, respectively. The operator $T_{3/2,s_0}(\pi\rho\sigma)$ (with its components $s_0=3/2, 1/2, -1/2, -3/2$) is the quasi-spin tensor of rank $s=3/2$ in the level p, the explicit form of which is given in Eq. (I. 3·29) of part I, for instance,

$$T_{3/2\;3/2}(\pi\rho\sigma) \equiv a_\pi{}^\dagger a_\rho{}^\dagger a_\sigma{}^\dagger,$$

$$T_{3/2\,-1/2}(\pi\rho\sigma) \equiv \sqrt{\tfrac{1}{3}}\{a_\pi{}^\dagger\tilde{a}_\rho\tilde{a}_\sigma + \tilde{a}_\pi a_\rho{}^\dagger\tilde{a}_\sigma + \tilde{a}_\pi\tilde{a}_\rho a_\sigma{}^\dagger\},$$

etc.

Since the quasi-spin tensor $T_{3/2s_0}(\pi\rho\sigma)$ is antisymmetric with respect to the permutation of (π, ρ, σ), the three-body-correlation amplitudes $\psi_{nIK}(\pi\rho\sigma)$ and $\varphi_{nIK}(\pi\rho\sigma)$ also have the same property. As a result, the amplitudes $\psi_n(j_p^2(J)j_p\}j_p^3I)$ and $\varphi_n(j_p^2(J)j_p\}j_p^3I)$ (with the even value of J) satisfy the equations

$$\left.\begin{aligned} \psi_n(j_p^2(J)j_p\}j_p^3I) &= \sum_{J'}{}' \sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} j_p & j_p & J \\ j_p & I & J' \end{Bmatrix}\psi_n(j_p^2(J')j_p\}j_p^3I), \\ \varphi_n(j_p^2(J)j_p\}j_p^3I) &= \sum_{J'}{}' \sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} j_p & j_p & J \\ j_p & I & J' \end{Bmatrix}\varphi_n(j_p^2(J')j_p\}j_p^3I). \end{aligned}\right\} \tag{3·4}$$

As has been emphasized in part I, the eigenmode operator (3·1) must include *no zero-coupled quasi-particle pairs*, i.e., *no quasi-spin generators* ($\hat{S}_\pm$, $\hat{S}_0$ defined in Eq. (I. 3·21) in part I), so that the amplitudes have to satisfy the condition

$$\left.\begin{aligned} \psi_n(j_p^2(0)j_p\}j_p^3I) &= 0\,, \\ \varphi_n(j_p^2(0)j_p\}j_p^3I) &= 0\,. \end{aligned}\right\} \tag{3·5}$$

The expression (3·1) with the explicit use of the quasi-spin tensor of rank $s=3/2$ means that *the dressed three-quasi-particle modes are characterized by the amount of transfered quasi-spin $s=3/2$, i.e., by the amount of transfered seniority $\Delta v \equiv 2s=3$ to the state on which they operate.* Thus, the eigenvalue equation for the three-body-correlation amplitudes should be obtained so that $Y^{\dagger}_{nIK}$ becomes a "good" approximate eigenmode operator satisfying

$$\left.\begin{array}{c}[H^{(0)},\ Y^{\dagger}_{nIK}]=\omega_{nI}Y^{\dagger}_{nIK}-Z_{nIK},\\ (\text{with } \omega_{nI}>0)\end{array}\right\} \tag{3·6}$$

where "interaction" Z_{nIK} is generally composed of the normal product of quasi-spin tensors with $s=1/2$, i.e., $\Delta v=1$ and of the higher fifth-order normal products, and is neglected in the first step (which determines the dressed three-quasi-particle eigenmodes $Y^{\dagger}_{nIK}$.) In the $P+QQ$ model under consideration, the interaction Z_{nIK} which is neglected should also include the other third-order normal products which come from the recoupling of the quadrupole force, in order to keep consistency to the *inherent assumption* of the $P+QQ$ model mentioned in § 2.

In part I, it has been shown that, under the basic approximation $n\ll 2\Omega$, i.e., $O(n/2\Omega)\approx 0$ (where n is the average number of quasi-particles in the ground state and $2\Omega\equiv 2j+1$), the correlated ground state $|\Phi_0\rangle$ (satisfying $Y_{nIK}|\Phi_0\rangle=0$) and the dressed three-quasi-particle states $Y^{\dagger}_{nIK}|\Phi_0\rangle$ have no zero-coupled quasi-particle pairs. It has also been shown that, under the basic approximation $O(n/2\Omega)\approx 0$, the dressed three-quasi-particle modes satisfy the *quasi-Fermion approximation*, i.e.,

$$\langle\Phi_0|\{Y_{mI'K'},\ Y^{\dagger}_{nIK}\}_+|\Phi_0\rangle=\delta_{nm}\delta_{II'}\delta_{KK'}, \tag{3·7}$$

with the use of the orthogonality relation of the correlation amplitudes

$$\sum_{m_\pi m_\rho m_\sigma}\{\psi_{mI'K'}(\pi\rho\sigma)\psi_{nIK}(\pi\rho\sigma)-\varphi_{mI'K'}(\pi\rho\sigma)\varphi_{nIK}(\pi\rho\sigma)\}=\delta_{mn}\delta_{II'}\delta_{KK'}, \tag{3·8}$$

which is obtained from properties of the eigenvalue equation ((I. 5·1) in part I) for the correlation amplitudes.

In the $P+QQ$ model, the eigenvalue equation for the correlation amplitudes becomes especially simple: It is written as

$$\omega_{nI}\begin{pmatrix}\psi_n(j_p^2(J_1)j_p\}j_p^3I)\\ \varphi_n(j_p^2(J_1)j_p\}j_p^3I)\end{pmatrix}=\sum_{J_1'}{}'\begin{pmatrix}3D(J_1J_1'), & -A(J_1J_1')\\ A(J_1J_1'), & -D(J_1J_1')\end{pmatrix}\begin{pmatrix}\psi_n(j_p^2(J_1')j_p\}j_p^3I)\\ \varphi_n(j_p^2(J_1')j_p\}j_p^3I)\end{pmatrix}, \tag{3·9}$$

where

$$\left.\begin{array}{l}D(J_1J_1')=E_pP_I(J_1:J_1')-2\chi q^2(pp)u_p^2v_p^2P_I(J_1:2)P_I(J_1':2),\\ A(J_1J_1')=\dfrac{6}{\sqrt{3}}\chi q^2(pp)u_p^2v_p^2P_I(J_1:2)P_I(J_1':2),\end{array}\right\} \tag{3·10}$$

and

$$P_I(J_1:J_1')\equiv\frac{1}{3}(\delta_{J_1J_1'}+K_{J_1J_1'}-\delta_{j_pI}L_{J_1J_1'}),$$

$$\left.\begin{aligned} K_{J_1J_1'} &\equiv 2\sqrt{(2J_1+1)(2J_1'+1)}\begin{Bmatrix} j_p & j_p & J_1 \\ j_p & I & J_1' \end{Bmatrix} = K_{J_1'J_1}, \\ L_{J_1J_1'} &\equiv \frac{1}{1+K_{00}}(\delta_{J_10}+K_{J_10})(\delta_{J_1'0}+K_{J_1'0}) = L_{J_1'J_1}. \end{aligned}\right\} \tag{3·11}$$

It is easily seen that $P_I(J_1\colon J_1')$ has the following properties:

$$\left.\begin{aligned} &\text{(i)} \quad \psi_n(j_p^2(J_1)j_p\}j_p^3I) = \sum_{J_1'}{}' P_I(J_1\colon J_1')\psi_n(j_p^2(J_1')j_p\}j_p^3I), \\ &\qquad \varphi_n(j_p^2(J_1)j_p\}j_p^3I) = \sum_{J_1'} P_I(J_1\colon J_1')\varphi_n(j_p^2(J_1')j_p\}j_p^3I), \\ &\text{(ii)} \quad P_I(J_1\colon J_1') = P_I(J_1'\colon J_1), \\ &\text{(iii)} \quad P_I(J_1\colon 0) = P_I(0\colon J_1') = 0, \\ &\text{(iv)} \quad \sum_J{}' P_I(J_1\colon J)P_I(J, J_1') = P_I(J_1\colon J_1'), \end{aligned}\right\} \tag{3·12}$$

so that the solutions of Eq. (3·9) automatically satisfy the conditions (3·4) and (3·5).

With the aid of Eq. (3·12), we have the following eigen-value equation from Eq. (3·9):

$$\omega_{nI}\begin{pmatrix} \psi_n(j_p^2(2)j_p\}j_p^3I) \\ \varphi_n(j_p^2(2)j_p\}j_p^3I) \end{pmatrix} = \begin{pmatrix} 3E_p - 2\chi_I'q^2(pp)u_p^2v_p^2, & \frac{2}{\sqrt{3}}\chi_I'q^2(pp)u_p^2v_p^2 \\ \frac{2}{\sqrt{3}}\chi_I'q^2(pp)u_p^2v_p^2, & E_p - \frac{2}{3}\chi_I'q^2(pp)u_p^2v_p^2 \end{pmatrix}$$
$$\times\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \psi_n(j_p^2(2)j_p\}j_p^3I) \\ \varphi_n(j_p^2(2)j_p\}j_p^3I) \end{pmatrix}, \tag{3·13}$$

where

$$\chi_I' \equiv \chi(1+K_{22}-\delta_{j_pI}L_{22}) = 3\chi P_I(2:2). \tag{3·14}$$

After solving Eq. (3·13), we obtain the amplitudes $\psi_n(j_p^2(J_1)j_p\}j_p^3I)$ and $\varphi_n(j_p^2(J_1)j_p\}j_p^3I)$ with $J_1 \neq 2$ through the equation

$$\begin{pmatrix} \omega_{nI}-3E_p, & 0 \\ 0, & \omega_{nI}+E_p \end{pmatrix}\begin{pmatrix} \psi_n(j_p^2(J_1)j_p\}j_p^3I) \\ \varphi_n(j_p^2(J_1)j_p\}j_p^3I) \end{pmatrix}$$
$$= \begin{pmatrix} -6\chi q^2(pp)u_p^2v_p^2P_I(J_1\colon 2), & \frac{6}{\sqrt{3}}\chi q^2(pp)u_p^2v_p^2P_I(J_1\colon 2) \\ \frac{6}{\sqrt{3}}\chi q^2(pp)u_p^2v_p^2P_I(J_1\colon 2), & -2\chi q^2(pp)u_p^2v_p^2P_I(J_1\colon 2) \end{pmatrix}$$
$$\times\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \psi_n(j_p^2(2)j_p\}j_p^3I) \\ \varphi_n(j_p^2(2)j_p\}j_p^3I) \end{pmatrix}, \tag{3·15}$$

which also comes from Eq. (3·9), and indicates that $\psi_n(j_p^2(J_1)j_p\}j_p^3I)$ and $\varphi_n(j_p^2(J_1)j_p\}j_p^3I)$ with $J_1\neq 2$ can be expressed by the special amplitudes $\psi_n(j_p^2(2)j_p\}j_p^3I)$ and $\varphi_n(j_p^2(2)j_p\}j_p^3I)$. In the single j-shell model with the $P+QQ$ force, therefore, the dressed three-quasi-particle modes $Y_{nIK}^\dagger$ in Eq. (3·1) can be simply expressed in terms of only the special amplitudes $\psi_n(j_p^2(2)j_p\}j_p^3I)$ and $\varphi_n(j_p^2(2)j_p\}j_p^3I)$:

$$
\begin{aligned}
Y_{nIK}^\dagger &= \frac{1}{\sqrt{3!}}\cdot\frac{1}{P_I(2:2)}\cdot\{\psi_n(j_p^2(2)j_p\}j_p^3I):T_{3/2\,3/2}(j_p^2(2)j_pIK): \\
&\qquad\qquad + \varphi_n(j_p^2(2)j_p\}j_p^3I):\ T_{3/2-1/2}(j_p^2(2)j_pIK):\} \\
&= N_I\{\psi_n'(j_p^2(2)j_p\}j_p^3I):T_{3/2\,3/2}j_p^2(2)j_pIK): \\
&\qquad\qquad + \varphi_n'(j_p^2(2)j_p\}j_p^3I):T_{3/2-1/2}(j_p^2(2)j_pIK):\},
\end{aligned}
\tag{3·16}
$$

where N_I is the normalization constant given by

$$N_I=\{6P_I(2:2)\}^{-1/2} \tag{3·17}$$

and

$$
\left.
\begin{aligned}
\psi_n'(j_p^2(2)j_p\}j_p^3I) &= \sqrt{3!}\cdot N_I\psi_n(j_p^2(2)j_p\}j_p^3I), \\
\varphi_n'(j_p^2(2)j_p\}j_p^3I) &= \sqrt{3!}\cdot N_I\varphi_n(j_p^2(2)j_p\}j_p^3I),
\end{aligned}
\right\}
\tag{3·18}
$$

with which Eq. (3·8) is reduced to

$$
\begin{aligned}
&\psi_n'^2(j_p^2(2)j_p\}j_p^3I)-\varphi_n'^2(j_p^2(2)j_p\}j_p^3I) \\
&\quad = \frac{1}{P_I(2:2)}\cdot\{\psi_n^2(j_p^2(2)j_p\}j_p^3I)-\varphi_n^2(j_p^2(2)j_p\}j_p^3I)\} \\
&\quad = 1\,.
\end{aligned}
\tag{3·19}
$$

From Eq. (3·13) the eigenvalue ω_{nI} is easily obtained:

$$
\left.
\begin{aligned}
&\omega_I=E_I'+\sqrt{4E_I'^2-\left\{\frac{2}{\sqrt{3}}\chi_I'q^2(pp)u_p^2v_p^2\right\}^2}, \\
&\text{with} \\
&E_I'\equiv E_p-\frac{2}{3}\chi_I'q^2(pp)u_p^2v_p^2.
\end{aligned}
\right\}
\tag{3·20}
$$

Notice that I-dependence of the solution is completely expressed through χ_I' defined in Eq. (3·14). In this sense, we can regard χ_I' as the quantity which represents an effective change of the quadrupole-force strength due to the characteristic three-quasi-particle correlation. As was pointed out by Kisslinger[4] as the essence characterizing his "intruder" states (which correspond to the Tamm-Dancoff counterpart of the $I=j_p-1$ modes under consideration), there exists the simple property of $6j$-symbols, i.e.,

$$\left.\begin{array}{l}\begin{Bmatrix} j_p\, j_p\, 2 \\ j_p\; I\; 2 \end{Bmatrix} > 0 \quad \text{for } I = j_p - 1\,, \\ \begin{Bmatrix} j_p\, j_p\, 2 \\ j_p\; I\; 2 \end{Bmatrix} < 0 \quad \text{for } I \neq j_p - 1\,. \end{array}\right\} \tag{3·21}$$

So, we can easily see that $\chi_I' > \chi$ only when $I=j_p-1$ and $\chi_I' < \chi$ for $I \neq j_p-1$. Thus, as is shown in Fig. 3, the $I=j_p-1$ state is especially lowered in energy in contrast to the other states with $I \neq j_p-1$.

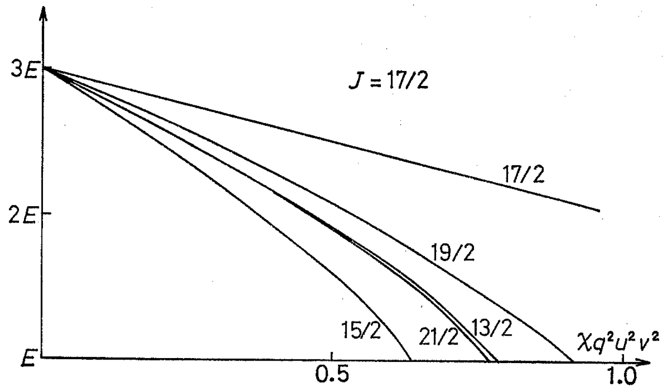

Fig. 3. Single j-shell-model solutions of dressed three-quasi-particle modes in the case of $j=17/2$. Excitation energies are written in unit of the quasi-particle energy E as functions of $\chi q^2 u^2 v^2$.

§ 4. Anomalous coupling states as dressed three-quasi-particle modes

We are now in a position to discuss the anomalous coupling states in odd-mass nuclei. As discussed in § 1, we consider the odd-mass system in the truncated shell-model space consisting of one major harmonic-oscillator shell (for both the protons and the neutrons) and of a large spin, *opposite-parity* (proton or neutron) level which enters into the (proton or neutron) major shell, and suppose that the opposite-parity level is being filled (by the protons or the neutrons). When we especially need to specify the opposite parity level and the single-particle-shell-model state in the level, we use Roman letter p and its associated Greek letter $\pi \equiv (p, m_\pi)$, respectively.

In this special situation in shell structure (for the appearance of anomalous coupling states), the dressed three-quasi-particle modes become especially simple from the parity consideration: In the $P+QQ$ model, the "physical" eigenmode operators $Y_{nIK}^\dagger$ with $\omega_{nI}>0$ (in Eq. (I. 9·4) in part I), which have the *opposite parity* to that of the major shell, are simply reduced to

$$\begin{aligned} Y_{nIK}^\dagger = & N_I \psi_{nI}(p^3) : T_{3/2\,3/2}(j_p{}^2(2) j_p IK) : \\ & + N_I \varphi_{nI}(p^3) : T_{3/2\,-1/2}(j_p{}^2(2) j_p IK) : \\ & + \sum_{(ab) \neq p} N(ab)\, \psi_{nI}(p\,; ab)\, [a_p{}^\dagger A_2{}^\dagger(ab)]_{IK} \\ & - \sum_{(ab) \neq p} N(ab)\, \varphi_{nI}(p\,; ab)\, [a_p{}^\dagger \tilde{A}_2(ab)]_{IK}\,, \end{aligned} \tag{4·1}$$

where $\sum_{(ab)\neq p}$ indicates the summation with respect to the set of levels a and b except the opposite-parity level p. In Eq. (4·1), N_I is given by (3·17), and the correlation amplitudes $\psi_{nI}(p_3)$ and $\varphi_{nI}(p^3)$ are the same as those used in Eq. (3·16), i.e.,

$$\left.\begin{aligned}\psi_{nI}(p^3)&\equiv\psi_n'(j_p^2(J=2)j_p\}j_p^3I),\\ \varphi_{nI}(p^3)&\equiv\varphi_n'(j_p^2(J=2)j_p\}j_p^3I),\end{aligned}\right\}\tag{4·2}$$

and

$$N(ab)\equiv\{2/(1+\delta_{ab})\}^{1/2},\tag{4·3}$$

$$\left.\begin{aligned}[a_p^\dagger A_2^\dagger(ab)]_{IK}&\equiv\sum_{m_\pi M}\langle 2j_pMm_\pi|IK\rangle a_\pi^\dagger A_{2M}^\dagger(ab),\\ [a_p^\dagger\tilde{A}^\dagger_2(ab)]_{IK}&\equiv\sum_{m_\pi M}\langle 2j_pMm_\pi|IK\rangle a_\pi^\dagger A_{2-M}(ab)\cdot(-)^{2-M}.\end{aligned}\right\}\tag{4·4}$$

With definition (4·1), the normalization of correlation amplitudes becomes

$$\psi_{nI}^2(p^3)+\sum_{(ab)\neq p}\psi_{nI}^2(p;ab)-\varphi_{nI}^2(p^3)-\sum_{(ab)\neq p}\varphi_{nI}^2(p;ab)=1\,.\tag{4·5}$$

It should be noticed that the eigenmode operators $Y_{nIK}^\dagger$ obviously transfer the *total seniority* $\Delta v\equiv\sum_a\Delta v_a=3$ to the state on which they operate. (See § 9 in part I.) The eigenvalue equation for the correlation amplitudes is now written as follows:

$$\begin{aligned}\omega_{nI}\psi_{nI}(p^3)=&3E_p\psi_{nI}(p^3)-\chi\{Q(pp)\sqrt{C_I}\}^2\left\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\}\\ &-\chi Q(pp)\sqrt{C_I}\sum_{(a_1b_1)\neq p}Q(a_1b_1)N(a_1b_1)\{\psi_{nI}(p;a_1b_1)+\varphi_{nI}(p;a_1b_1)\},\end{aligned}\tag{4·6a}$$

$$\begin{aligned}\omega_{nI}\varphi_{nI}(p^3)=&-E_p\varphi_{nI}(p^3)+\sqrt{\frac{1}{3}}\chi\{Q(pp)\sqrt{C_I}\}^2\left\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\}\\ &+\sqrt{\frac{1}{3}}\chi Q(pp)\sqrt{C_I}\sum_{(a_1b_1)\neq p}Q(a_1b_1)N(a_1b_1)\\ &\times\{\psi_{nI}(p;a_1b_1)+\varphi_{nI}(p;a_1b_1)\},\end{aligned}\tag{4·6b}$$

$$\begin{aligned}\omega_{nI}\psi_{nI}(p;ab)=&(E_p+E_a+E_b)\psi_{nI}(p;ab)\\ &-\chi Q(ab)N(ab)\sum_{(a_1b_1)\neq p}Q(a_1b_1)N(a_1b_1)\\ &\times\{\psi_{nI}(p;a_1b_1)+\varphi_{nI}(p;a_1b_1)\}\\ &-\chi Q(ab)N(ab)Q(pp)\sqrt{C_I}\left\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\},\end{aligned}\tag{4·6c}$$

$$\begin{aligned}\omega_{nI}\varphi_{nI}(p;ab)=&(E_p-E_a-E_b)\varphi_{nI}(p;ab)\\ &+\chi Q(ab)N(ab)\sum_{(a_1b_1)\neq p}Q(a_1b_1)N(a_1b_1)\\ &\times\{\psi_{nI}(p;a_1b_1)+\varphi_{nI}(p;a_1b_1)\}\\ &+\chi Q(ab)N(ab)Q(pp)\sqrt{C_I}\left\{\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\},\end{aligned}$$

$$(\text{with } a, b \neq p) \tag{4·6d}$$

where

$$Q(ab) \equiv q(ab)\xi(ab), \tag{4·7}$$

$$C_I \equiv 3P_I(2:2) = (1+K_{22} - \delta_{j_pI}L_{22})$$
$$= 1 + 10\begin{Bmatrix} j_p j_p 2 \\ j_p\ I\ 2 \end{Bmatrix} - \delta_{j_pI}\cdot 20 \begin{Bmatrix} j_p j_p 0 \\ j_p j_p 2 \end{Bmatrix}^2 \left[1 + 2\begin{Bmatrix} j_p j_p 0 \\ j_p j_p 0 \end{Bmatrix}\right]^{-1}. \tag{4·8}$$

In deriving Eq. (4·6), we have dropped the terms which come from the recoupling of the quadrupole force, as usual, in accordance with the *inherent assumption* on the $P+QQ$ model mentioned in § 2. Formal structure of Eq. (4·6) is as simple as that for the phonon modes in even-even nuclei, except for the fact that only matrix elements concerned with the oppsite-parity level p are changed due to the three-quasi-particle correlation.

Combining Eqs. (4·6a) and (4·6b) and also combining Eqs. (4·6c) and (4·6d), we obtain

$$\left.\begin{aligned} \{\chi_p S_p - 1\} B_I + \chi_{pc} S_p A_I = 0 \,, \\ \{\chi_c S_c - 1\} A_I + \chi_{pc} S_c B_I = 0 \,, \end{aligned}\right\} \tag{4·9}$$

where

$$\chi_p = \chi_c = \chi_{pc} = \chi \,, \tag{4·10}$$

$$\left.\begin{aligned} A_I &\equiv \sum_{(ab)\neq p} Q(ab) N(ab) \{\psi_{nI}(p; ab) + \varphi_{nI}(p; ab)\}, \\ B_I &\equiv Q(pp)\sqrt{C_I}\left\{\psi_{nI}(p^3) + \sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\}, \end{aligned}\right\} \tag{4·11}$$

and

$$S_p \equiv 2\cdot\frac{Q^2(pp)\cdot C_I\cdot\{E_p + (1/3)\omega_{nI}\}}{(2E_p)^2 - (\omega_{nI} - E_p)^2}, \tag{4·12a}$$

$$S_c \equiv 2\cdot\sum_{a,b\neq p}\frac{Q^2(ab)\cdot(E_a+E_b)}{(E_a+E_b)^2 - (\omega_{nI} - E_p)^2}. \tag{4·12b}$$

Since Eq. (4·9) is linear and homogenious with respect to A_I and B_I, we find that the eigenvalue ω_{nI} are the solutions of

$$(\chi_p S_p - 1)(\chi_c S_c - 1) - \chi_{pc}^2 S_p S_c = 0 \,. \tag{4·13}$$

The physical meaning of Eq. (4·13) is easily understood in the following way: If χ_{pc} were zero, we would have solutions when either $\chi_p S_p = 1$ or $\chi_c S_c = 1$. The former leads us to the solutions of Eq. (3·20) in the single opposite-parity level p. The latter is, in its form, the very same as the well-known dispersion equation for phonon modes of the "core", which is composed of the neutrons and protons in the truncated major shells with the exception of the opposite-parity level p. Thus we would have *two* low-energy collective states (due to the quasi-particles in the level p and due to the core, respectively), if the "coupling" χ_{pc} were zero. Now let us consider the effect on these states due to the change of χ_{pc} from zero.[2] In this case, the product $(\chi_p S_p - 1)(\chi_c S_c - 1)$ has to be positive so that the lower level of the two $\chi_{pc}=0$ states must be lowered in order to make each factor of the product negative, while the higher level is raised making each factor positive. For sufficiently large χ_{pc}, such as the actual case of Eq. (4·10), there will be essentially only one extremely lowered ω_{nI} left in the energy region satisfying $(\omega_{nI}-E_p) <$ the minimum value of (E_a+E_b).

In the actual case in which $\chi_p=\chi_c=\chi_{pc}=\chi$, Eq. (4·13) is simply reduced to

$$\chi S_p + \chi S_c = 1 . \qquad (4\cdot 14)$$

In order to compare solutions of Eq. (4·14) with those of the equation $\chi S_p = 1$ for the opposite parity level p, and to see the lowering effect on the level position due to the core, let us adopt the adiabatic approximation:

$$(\omega_{nI}-E_p) \ll \text{ the minimum value of } (E_a+E_b). \qquad (4\cdot 15)$$

In this case we may write

$$\left.\begin{aligned} S_p &= \mathcal{A}_p + \mathcal{B}_p(\omega_{nI}-E_p) + \mathcal{C}_p(\omega_{nI}-E_p)^2, \\ S_c &= \mathcal{A}_c + \mathcal{C}_c(\omega_{nI}-E_p)^2, \end{aligned}\right\} \qquad (4\cdot 16)$$

where

$$\left.\begin{aligned} &\mathcal{A}_p \equiv \frac{2}{3}\cdot\frac{Q^2(pp)C_I}{E_p} > 0, \qquad \mathcal{B}_p \equiv \frac{2}{3}\cdot\frac{Q^2(pp)C_I}{4E_p^2} > 0, \\ &\mathcal{C}_p \equiv \frac{2}{3}\cdot\frac{Q^2(pp)C_I}{4E_p^3} > 0, \\ &\mathcal{A}_c \equiv 2\sum_{a,b\neq p}\frac{Q^2(ab)}{E_a+E_b} > 0, \\ &\mathcal{C}_c \equiv 2\sum_{a,b\neq p}\frac{Q^2(ab)}{(E_a+E_b)^3} > 0. \end{aligned}\right\} \qquad (4\cdot 17)$$

As a result we have from Eq. (4·14)

$$(\omega_{nI}-E_p) = \frac{-\mathcal{B}_p}{2(\mathcal{C}_p+\mathcal{C}_c)} + \sqrt{\frac{\mathcal{B}_p^2}{4(\mathcal{C}_p+\mathcal{C}_c)^2} + \frac{(\chi^{-1}-\mathcal{A}_p-\mathcal{A}_c)}{(\mathcal{C}_p+\mathcal{C}_c)}} . \qquad (4\cdot 18)$$

Comparing Eq. (4·18) with the adiabatic solutions of $\chi S_p = 1$, which correspond to Eq. (4·18) with $\mathcal{A}_c = \mathcal{C}_c = 0$, we can easily see lowering effects due to the core. Since both $\mathcal{A}_c$ and $\mathcal{C}_c$ contain the factor $\xi(ab) \equiv 1/\sqrt{2}(u_a v_b + v_a u_b)$ through the quantity $Q(ab)$, the larger the $\xi(ab)$ of the core, the lower the ω_{nI} becomes. Thus in real nuclei under consideration, the problem of whether the dressed three-quasi-particle modes appear extremely low in energy will be determined by two important factors: i) the enhancement factor $\xi(pp)=\sqrt{2}u_pv_p$ in the opposite parity level p which is being filled and ii) the enhancement factor $\xi(ab)$ in the core.

§ 5. Excitation-energy systematics

We are now in a position to solve the eigenvalue equation (4·6) for the dressed three-quasi-particle modes and discuss to what extent the proposed viewpoint of the anomalous coupling states is supported by the results of numerical calculations.

The parameters which enter into the determination of the solutions of Eq. (4·6) are the quadrupole-force strength χ and the quantities related to the pairing correlations (i.e., the parameters u_a and v_a of the Bogolyubov transformation and the single-quasi-particle energies E_a), which are determined from the single-particle energies ϵ_a and the pairing-force strength G.

In order to see the essential effects of the three-quasi-particle correlation originated from the quadrupole force and to fix the parameters as far as possible, we use the same values of the pairing-force strength G and of the single-particle energies ϵ_a as those adopted in the work of Kisslinger and Sorensen,[2] and also make the same truncation of shell-model space as they have made. On the other hand, the quardrupole-force strength χ is regarded as a free parameter (in each shell region) which should be determined phenomenologically except for its usual mass-number depencence,[8]

$$\chi=\chi_0\cdot b^{-4}\cdot A^{-5/3}\ \mathrm{MeV},$$

where b is the harmonic-oscillator range parameter.*)

By the use of the FACOM 230–60 computer of the Kyushu-University Computer Center, numerical calculations have been performed for the three shell regions, i.e., $1h_{11/2}^{-}$-odd-neutron region, $1g_{9/2}^{+}$-odd-proton region and $1g_{9/2}^{+}$-odd-neutron region.

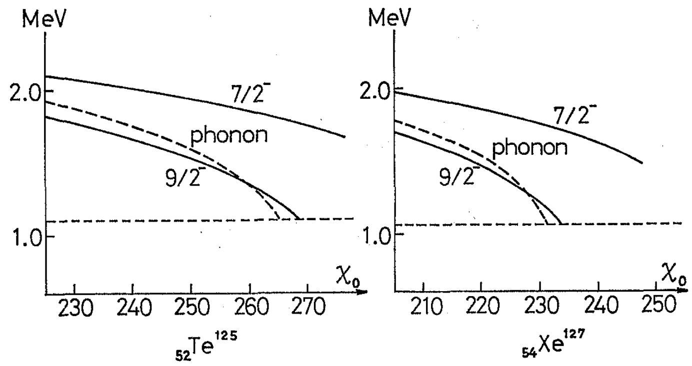

Fig. 4. (a)

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*) Usually the reduced matrix element of the single-particle quadrupole moment, $q(ab)$ in Eq. (2·3), is calculated with the harmonic-oscillator-shell-model wave functions. Since $q(ab)$ is proportional to b^2, the factor b^{-4} does not explicitly appear in the reduced matrix element of the quadrupole force, $1/2\cdot\chi q(ab)q(cd)$, and so only χ_0 is regarded as a parameter.

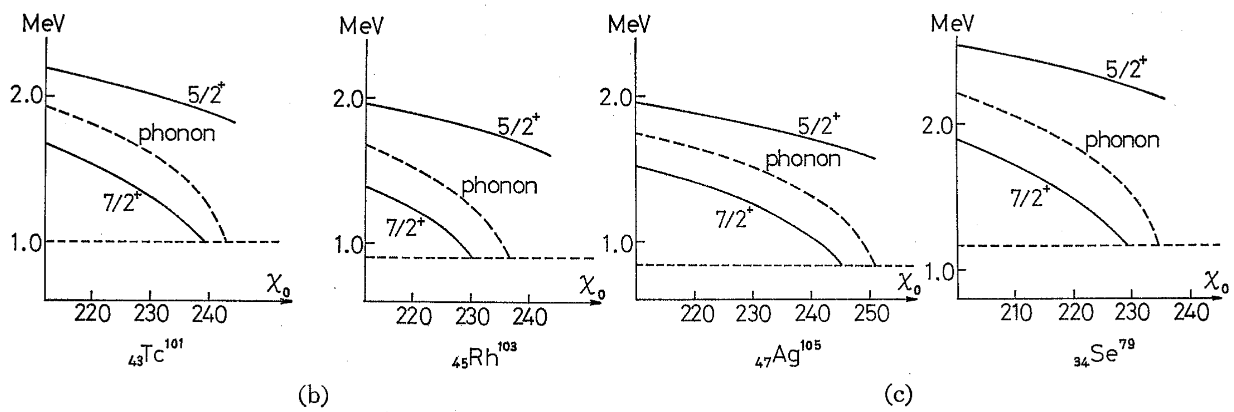


Fig. 4. Calculated energies of dressed three-quasi-particle modes as functions of χ_0. Only the $(j-1)$ and $(j-2)$ states are presented for simplicity. Single quasi-particle energies E_p are denoted by broken lines. Energies of one-quasi-particle-plus-one-phonon states are also written by broken curves, where phonon energies are calculated by the RPA.

In Fig. 4 the calculated excitations energies ω_{nI} as functions of the quadrupole-fore strength χ (i.e., χ_0) are shown in some examples, together with the one-quasi-particle-plus-one-phonon energies, $\omega_{\rm phon}+E_p$, (in which $\omega_{\rm phon}$ are calculated by the conventional RPA for the neighbouring even-even nuclei) for reference. It is seen that, due to the characteristic three-quasi-particle correlation, the $I=(j_p-1)$ states can be lowered naturally with reasonable values of χ, and energy splittings between the $I=(j_p-1)$ states and the $I\neq(j_p-1)$ states become large according to the increase of χ. The amount of the splitting also depends on the magnitude of j_p as expected from the discussions in § 3.

It is now interesting to see experimental trend of energy levels of the anomalous coupling $I=(j_p-1)$ states in sequences of the odd-mass isotopes since the anomalous coupling states are found extensively in excited states according to the recent progress of experiment. (See Fig. 5.) The experimental trend is somewhat similar to that of the 2^+ phonon states in the sequences of even-mass isotopes in the sense that the excitation energies are rapidly lowered when one moves away from the closed shells. It should also be emphasized that, at certain nucleon numbers, there often occurs the crossing of energy levels between the anomalous coupling state and the single-quasi-particle $I=j_p$ state. (See Fig. 5.) As has been mentioned in § 1, this may be expected to be the appearance of an instability of the spherical-odd mass nuclei and to be the onset of deformation.

General trend of the calculated ω_{nI} (for the anomolous coupling (j_p-1) states in the sequences of the odd-mass isotopes) with reasonable fixed values of χ_0 is in good agreement with the above-mentioned experimental trend, if not in fine detail. (See Figs. 6, 7 and 9.) we now proceed to discuss the results of the theoretical calculations in each shell region in some detail.

i) *The region of $h_{11/2}^-$-odd-neutron nuclei*

This is the region in which the opposite parity level $1h_{11/2}^-$ is being filled by neu-

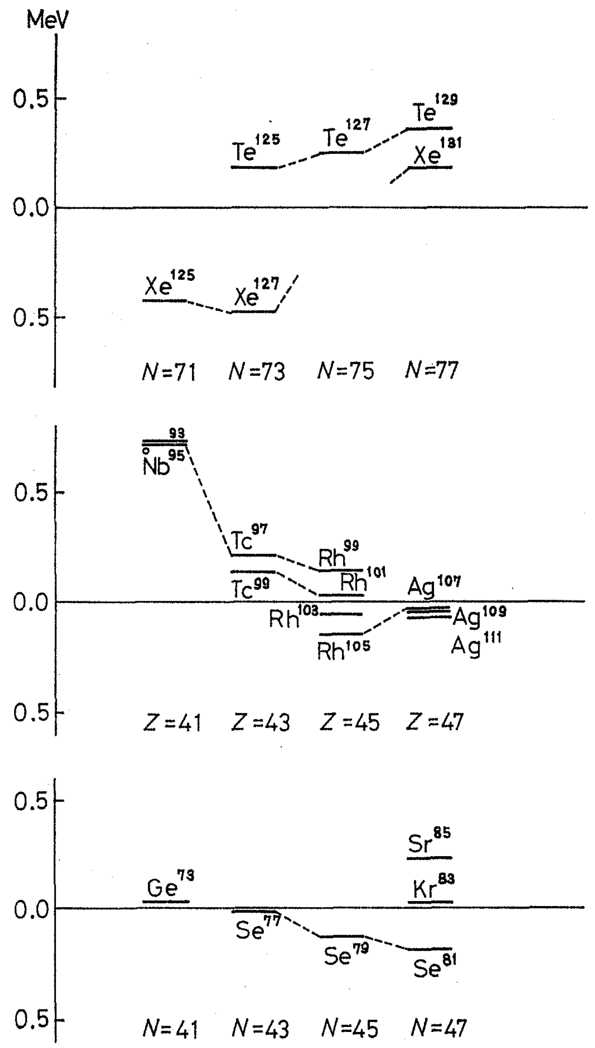


Fig. 5. Experimental trend of energy levels of the anomalous coupling states with spin $I=(j_p-1)$. The level energies are presented relative to those of single quasi-particle j_p states. The states that have not finally been assigned are denoted by the symbol ∘.
Te^{125}, Ref. 1).
Te^{127}, Ref. 10).
Te^{129}, Ref. 11).
$Xe^{125,127}$, J. Rezanka et al., Nucl. Phys. **A141** (1970), 130.
Xe^{131}, Ref. 12).
Nb^{93}, H. C. Sharma and N. Nath, Nucl. Phys. **A142** (1970), 291.
Tc^{97}, G. Graeffe, Nucl. Phys. **A127** (1969), 65.
Tc^{99}, W. B. Cook et al., Nucl. Phys. **A139** (1969), 277.
$Rh^{99,101}$, M. E. Phelps et al., Nucl. Phys. **A135** (1969), 116; **A159** (1970), 113.
Rh^{103}, W. H. Zoller et al., Nucl. Phys. **A130** (1969), 293.
Rh^{105}, S. O. Schriber and M. W. Johns, Nucl. Phys. **A96** (1967), 337.
$Ag^{109,111}$, W. C. Schick and W. L. Talbert, Nucl. Phys. **A128** (1969), 353.
Ge^{73}, J. Kyles et al., Nucl. Phys. **A150** (1970), 143.
$Se^{77,79}$, D. G. Sarantites and B. R. Erdal, Phys. Rev. **177** .(1969), 1631.
Sr^{85}, D. J. Horen and W. H. Kelly, Phys. Rev. **145** (1966), 988.

trons. In the Cd, Te and Xe isotopes, $9/2^-$ states are found in experiments[9)~12)] at a few hundred keV in energy above the $11/2^-$ single-neutron-quasi-particle states.

In Fig. 6 are shown the calculated energy levels ω_{nI} for the sequences of odd-mass Cd, Sn, Te, Xe and Ba isotopes, respectively. The adopted values of χ_0 in this region are the same as derived by Baranger and Kumar[13)] within a few percent. It is predicted by the results of the theoretical calculations that the excitation energies $(\omega_{nI}-E_p)$ of the $9/2^-$ states are on the decrease as one moves from the single-closed shell Sn isotopes to the heavier Te, Xe and Ba isotopes, and in each sequence of the isotopes they are on the decrease as the neutrons fill the opposite-parity $1h_{11/2}$ shell toward its middle. This calculated trend is naturally understood when we remember the enhancement factors of the three-quasi-particle correlation discussed at the end of § 4: The decrease of the $9/2^-$ energy from Sn to Ba isotopes can be well understood as due to the increase of the factor $\xi(ab)$ of the core, and in each sequence of the isotopes the decrease is due to the increase of the factor $\xi(pp)$ in the opposite-parity level $1h_{11/2}^{-}$.

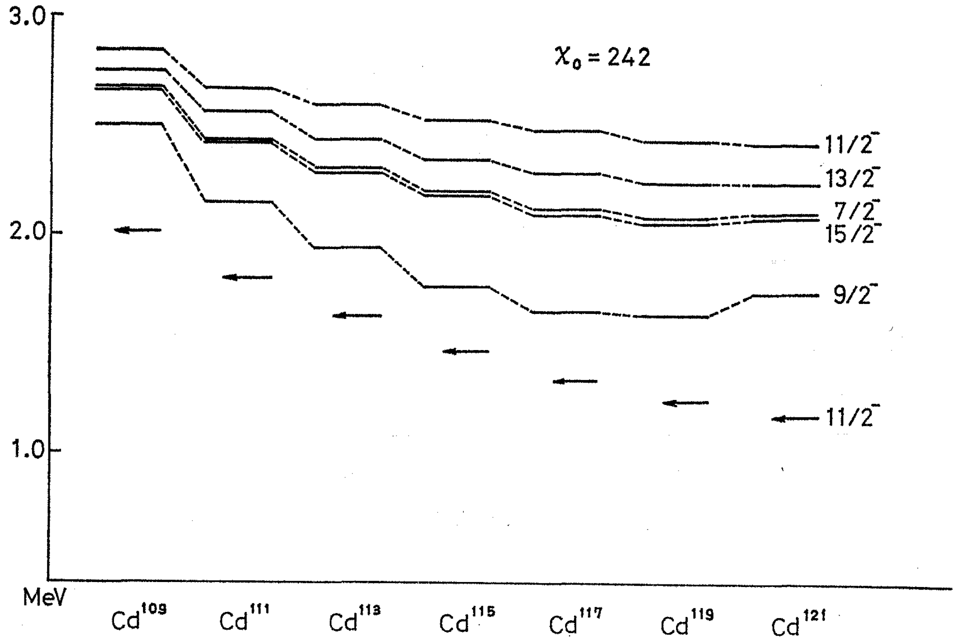


Fig. 6. (a)

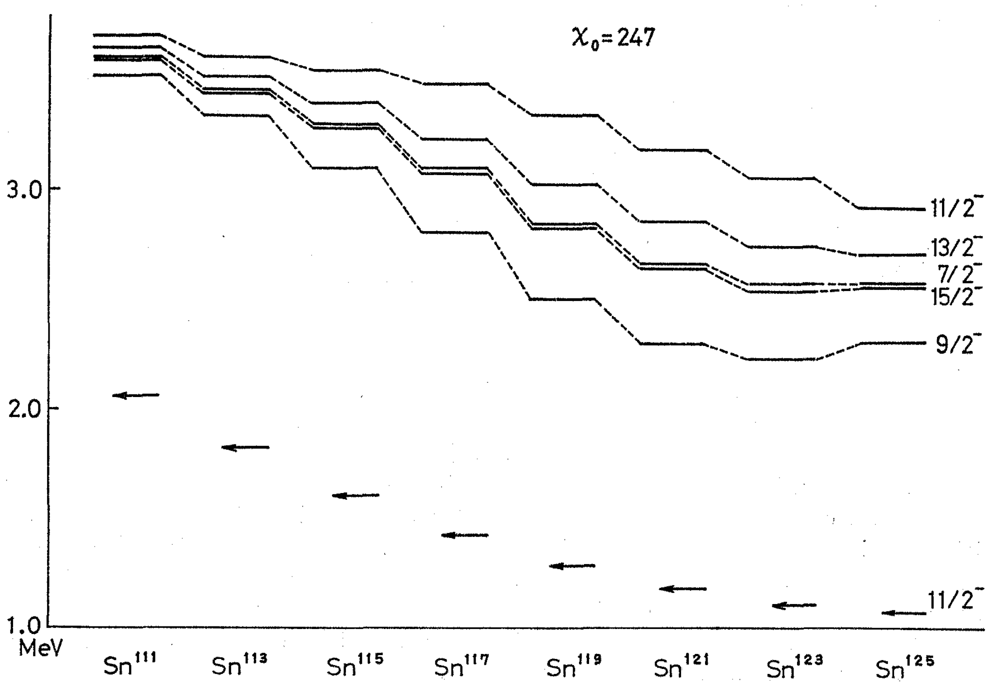


Fig. 6. (b)

Figure captions are printed on the next page below.

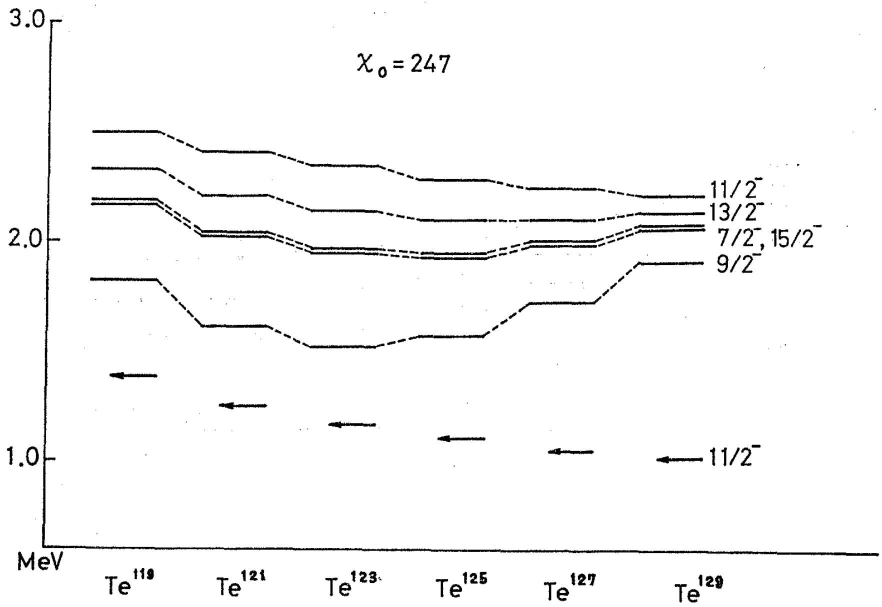


Fig. 6. (c)

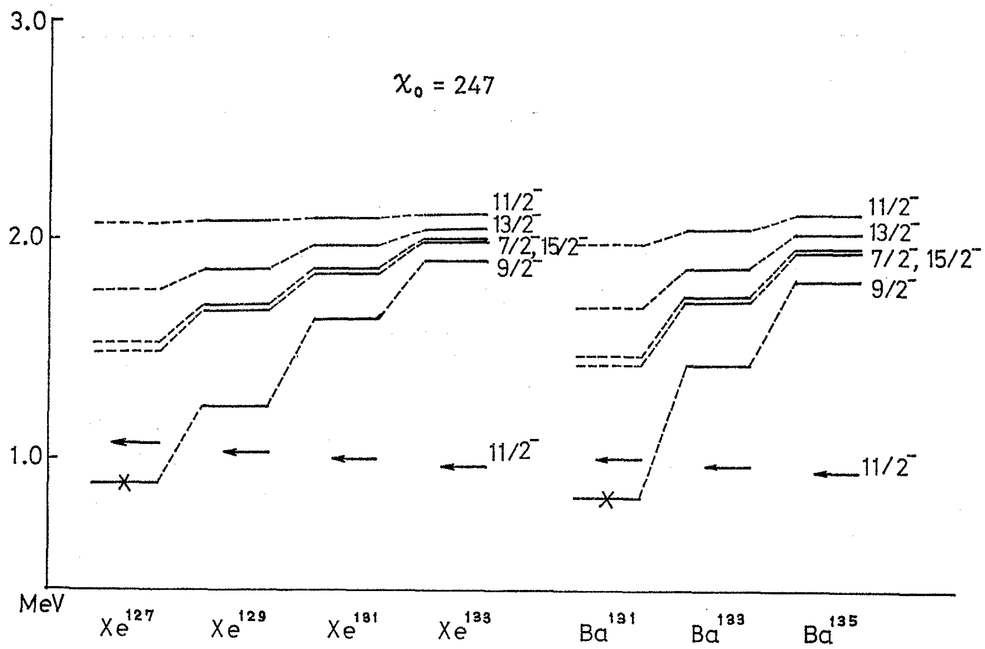


Fig. 6. (d)

Fig. 6. Calculated energies of the dressed three-quasi-particle states in the $1h_{\overline{11}/2}$-odd-neutron region. Adopted values of the quadrupole-force parameter χ_0 are written in the figures. Single-quasi-particle energies of the level j_p are written by arrows. It shold be noticed that all energies are measured from the vacuum of their modes. Thus, the differences of these energies correspond to the spectra of odd-mass nuclei. The symbol " × " means that the calculated energy of the (j_p-1) state becomes smaller than the single-quasi-particle energy E_p, and in this case the other angular momentum states are written by broken lines.

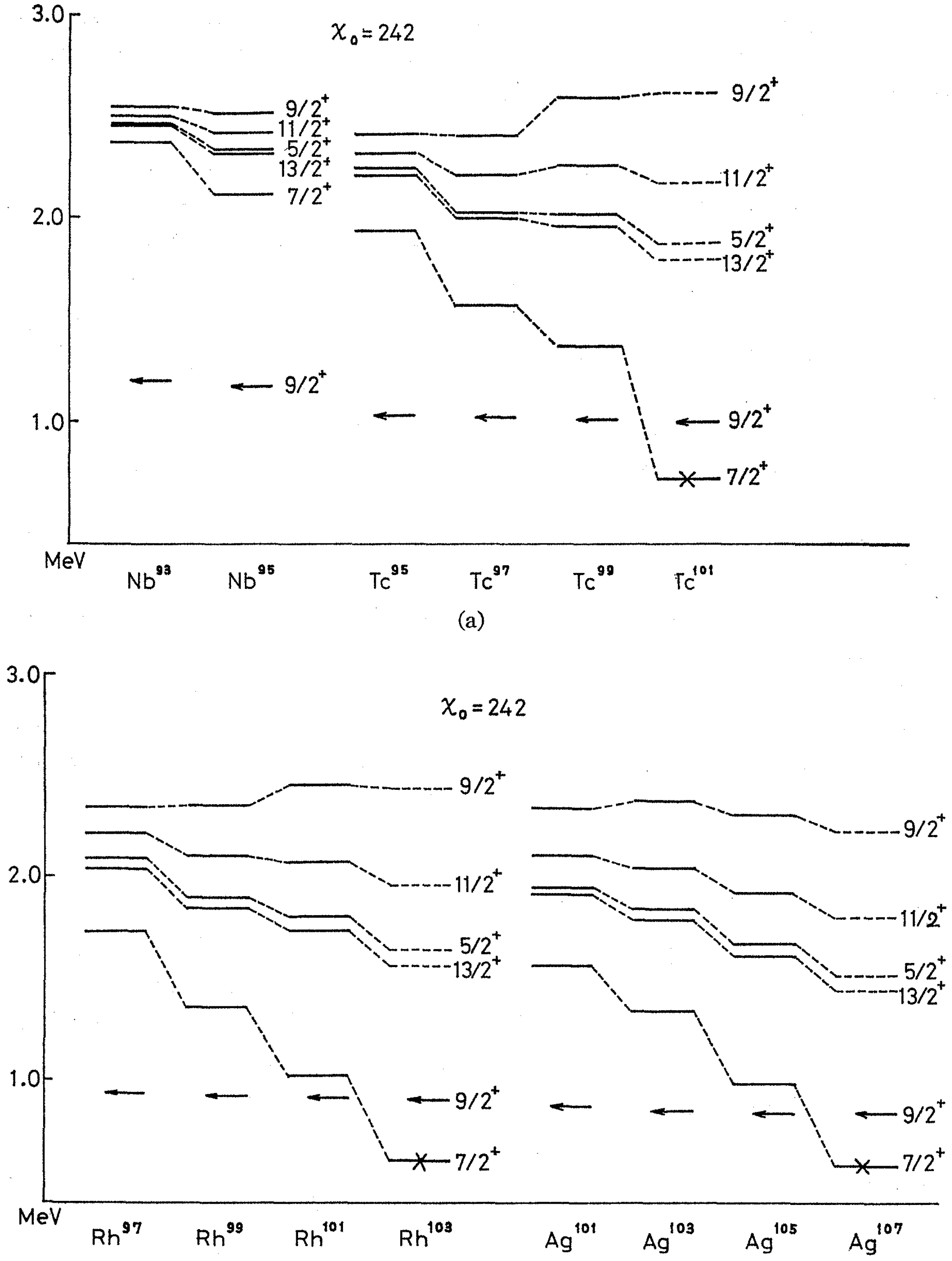
In the Sn isotopes, none of the low-lying $9/2^-$ states is experimentally observed up to now.[14] The reason is well explained when we consider the $9/2^-$

states as the dressed three-quasi-particle states, because in such single-closed-shell nuclei the enhancement factor $\xi(ab)$ of the core (on the three-quasi-particle correlation) becomes so small that, in the theoretical calculations, the $9/2^-$ states are forced to lie at high energy, about 1MeV, above the $11/2^-$ single-quasi-particle states. In the Te and Cd isotopes (in which the two protons and the two proton-holes are added, respectively, to the proton-closed shell in the Sn isotopes) the low lying $9/2^-$ states found in experiment are well explained by theoretical calculations with a reasonable value of χ. When we regard the $9/2^-$ states as Kisslinger's "intruder states" composed of the neutrons in $(1h_{11/2}^-)^3$-configuration, it is hard to understand the (above mentioned) different experimental situations between the Sn isotopes and the Te and Cd isotopes.

Contrary to the Te isotopes, the experimental energy change of the $9/2^-$ states in the Xe isotopes is rapid, and at the neutron deficient Xe^{127} the $9/2^-$ state becomes lower than the $11/2^-$ single-quasi-particle states. According to our point of view, this may indicate the onset of deformation. These experimental facts are just ones expected from the theoretical calculations, and the situation can be seen to remain unchanged for a wide range of the parameter χ_0. From the theoretical calculations, similar experimental aspects may be also expected in the Ba isotopes. So far there is no systematic experimental evidence that the neutron-deficient odd-mass Xe isotopes, in which the $9/2^-$ states are lower than the $11/2^-$ single-quasi-particle states, have stable deformations. However, it is interesting to notice that the adjacent even-even nuclei manifest quasi-rotational spectra clearly.

ii) *The region of $g_{9/2}^+$-odd-proton nuclei*

In this region the opposite-parity $1g_{9/2}^+$ level is being filled by the protons. In experiments, the rapid drop of the $7/2^+$ state in energy is observed as one moves from Nb to Ag. And, as is well known, the $7/2^+$ states appear below the $9/2^+$ states in the heavier Rh isotopes than Rh^{103} and in all the Ag isotopes, $Ag^{103}\sim Ag^{113}$. In the theoretical calculations, the energies of $7/2^+$ states, from Nb^{93} to Ag^{107} and also for each isotope, go down as functions of the nucleon number with fixed value of χ_0 (Fig. 7), and so are in good agreement with the experimental trend. The growth of three-quasi-particle correlation (i.e., the decrease of ω_{nI} with $I=7/2$) can be understood as due to the fact that two enhancement factors ($\xi(pp$ and $\xi(ab)$) act coherently as one moves from Nb to the heavier odd-proton nuclei in this region. For nuclei in which the anomalous coupling $7/2^+$ state, appears as the ground state, we may expect instability of the spherical shape. And, for such nuclei, we should be careful to take into account the limit of applicability of the theory based on the BCS approximation with the spherical base. Although the quantitative comparison between the theory and the experiment near the critical point, $\omega_{nI}=E_p$ with $I=7/2$, is not so meaningful, it is somewhat surprising that, with the value of $\chi_0=242$ MeV



(b)

Fig. 7. Calculated energies of the dressed three-quasi-particle states in the $1g_{9/2}^{+}$-odd-proton region. Notations are the same as in Fig. 6.

which is just the value derived by the classical method[8] with the nuclear radius parameter $r_0=1.2$ fm, the experimental behavior near the critical point can be reproduced rather well by the theoretical calculations, as is seen from Fig. 7.

For some nuclei in which both $7/2^+$ states and $5/2^+$ states are observed in

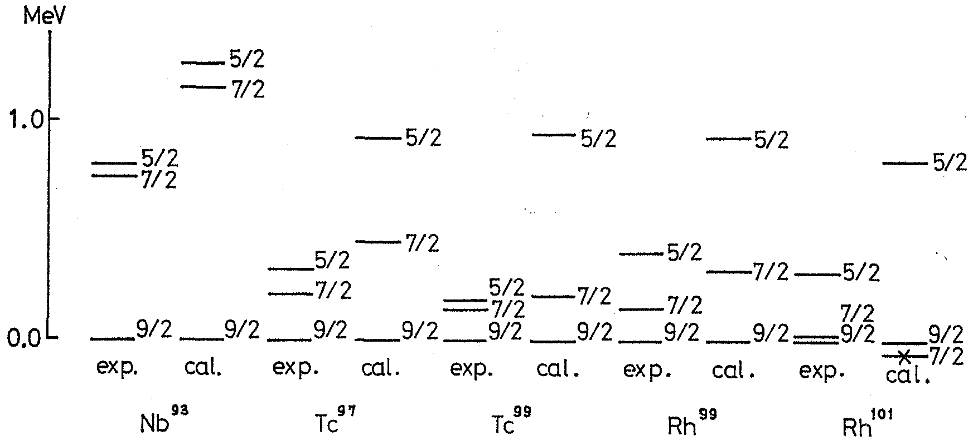


Fig. 8. Comparison of the experimental energy levels with the theoretical calculations in the case where both the (j_p-1) and the (j_p-2) states are found above the j_p states. For the quadrupole-force parameter χ_0, the same values as Baranger and Kumar's are adopted in the calculations of this figure. The energies are presented relative to the $9/2^+$ states and only the lowest (j_p-1) and (j_p-2) states with positive parity are written. Here, the energies of $9/2^+$ states are approximated to be those of "pure" single quasi-particle states.

experiments, a comparison between the theoretical and the experimental values is presented in Fig. 8, where the values of χ_0 are the same as those of Baranger and Kumar.[13] Although it is always possible to fit the calculated energies of the $7/2^+$ states to the experimental values by a suitable choice of χ_0, the $5/2^+$ states cannot be lowered to the values required by experiments with the same value of χ_0. For the $5/2^+$ states, therefore, we may expect a possibility of considerable admixture of the "dressed five-quasi-particle states", as is suggested from the recent experiment[15] in Tc^{99} where considerable $E2$ enhancement from the $5/2^+$ state to the $7/2^+$ state is observed. In this connection, it is worthy of notice that, in the conventional phonon-quasi-particle-coupling theory, effects of the so-called two-phonon states affect most strongly the $I=(j-2)$ states. Thus, when one tries to accout in detail for the whole spectrum of both the $I=(j_p-1)$ state and the $I\neq(j_p-1)$ states, it may be necessary to take into account such mixing effects of the elementary excitation modes (such as the one-quasi-particle modes, the dressed three-quasi-particle modes and the dressed five-quasi-particle modes) which are essentially based on the H_Y type interaction and have so far been neglected in our present first step approximation. In this case, it might be necessary to extend the truncated shell-model space to include the next major shell, as was asserted by Ikegami and Sano.[16]

iii) *The region of $g^+_{9/2}$-odd-neutron nuclei*

In this region, the anomalous coupling $7/2^+$ states are observed below the $9/2^+$ states in almost all nuclei, and therefore the instability of the spherical shape may be expected. In some nuclei near $Z=40$, the excited $7/2^+$ states above the $9/2^+$ states are experimentally found. This means that in such nuclei the

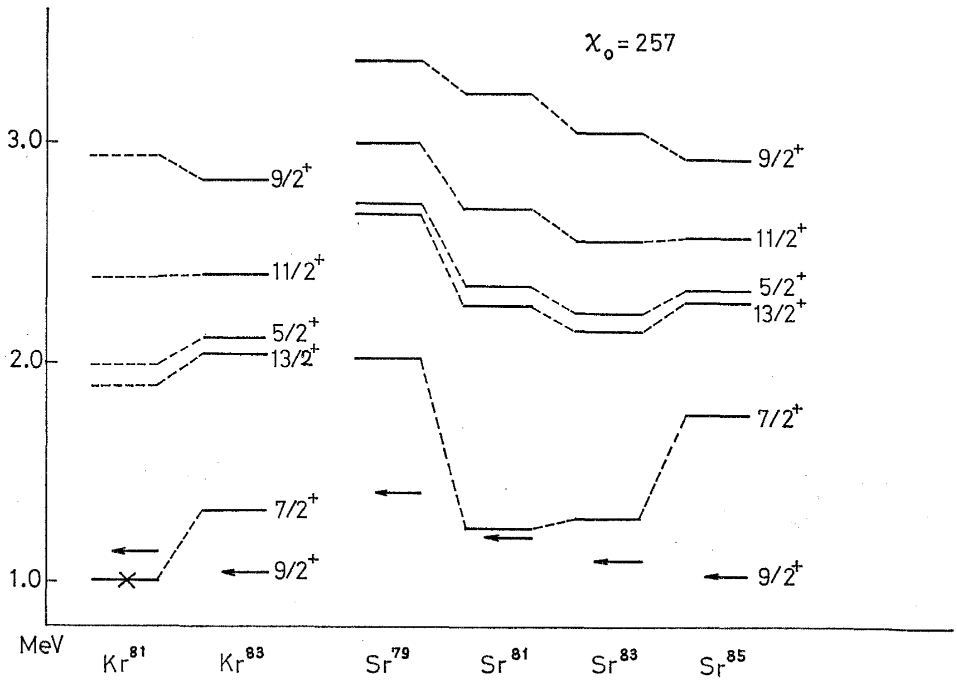


Fig. 9. Calculated energies of the dressed three-quasi-particle states in the $1g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 6.

spherical shape remains still stable. The above mentioned experimental trends are also well reproduced by the theoretical calculations with the reasonable value of χ_0. (See Fig. 9.)

Concerning the $5/2^+$ states, experiments show that these states also appear as the low-lying states, especially in the nuclei around $N=41$. Within our first step approximation (in which the mixing effects of the elementary excitation modes are neglected) the $5/2^+$ states cannot go down to such low-lying experimental positions in energy. The situation is the same as the case of the $g^+_{9/2}$-odd-proton nuclei discussed in ii).

§ 6. Concluding remarks

Based on the characteristic experimetal fact that $E2$ transitions from the anomalous coupling states to the one-quasi-particle states with spin j_p are strongly enhanced while $M1$ transitions are moderately hindered, we have proposed a new point of view on the structure of the anomalous coupling states. In order to check on the proposed new point of view, we have made excitation-energy systematics by using the theory of collective excitations in spherical odd-mass nuclei proposed in part I. From the numerical results, it is concluded that the excited anomalous coupling $I=(j_p-1)$ states can be recognized very well as the dressed three-quasi-particle states and so the proposed new point of view is strongly supported by the experiments. Thus, the importance of the three-quasi-particle correlation in the collective modes in odd-mass nuclei has been demonstrated.

The effects of the three-quasi-particle correlation (based on the Pauli-principle between the odd quasi-particle and the quasi-particles composing the phonon) have so far been neglected by the argument that a phonon contains only a small amplitude for the presence of any particular quasi-particle.[2] However, this argument is not correct. The addition of the odd quasi-particle induces the three-quasi-particle correlation, which strongly violates the concept of "phonon" in odd-mass nuclei. Furthermore, the more the collectiveness of the "phonon" increases, the more the new correlation grows up.

It has also been shown that Kisslinger's "intruder" states[4] composed of $(j_p)^3$-configuration can never exist purely, because of their strong interaction with

Table I(a). Amplitudes of the dressed three-quasi-particle mode with $I=7/2^-$ in Te^{125}. Adopted value of χ_0 is 247 (MeV) and the calculated excitation energy is 1.95 MeV. The values of forward amplitudes $\psi_{nI}(p;ab)$ are written in the second column, while the values of backward amplitudes $\varphi_{nI}(p;ab)$ are written in the third column, where $\psi_{nI}(p;pp)\equiv\psi_{nI}(p^3)$ and $\varphi_{nI}(p;pp)\equiv\varphi_{nI}(p^3)$ in the text. In this state, the opposite-parity level p is specified by the set of quantum numbers (neutron: $1h_{11/2}$), and only the quantum numbers a, b are written in the first column. These amplitudes are normalized to one according to Eq. (4·5) in the text.

neutron

ab	$h_{11/2}^2$	$g_{7/2}^2$	$d_{5/2}^2$	$d_{3/2}^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi_{nI}(p;ab)$	0.607	0.091	0.041	0.274	0.025	0.255	−0.101	0.092	0.249
$\varphi_{nI}(p;ab)$	0.153	0.060	0.030	0.121	0.017	0.147	−0.064	0.062	0.131

proton

ab	$h_{11/2}^2$	$g_{7/2}^2$	$d_{5/2}^2$	$d_{3/2}^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi_{nI}(p;ab)$	0.042	0.701	0.143	0.009	0.122	0.092	−0.027	0.047	0.012
$\varphi_{nI}(p;ab)$	0.031	0.190	0.069	0.007	0.049	0.062	−0.019	0.033	0.009

Table I(b). Amplitudes of the dressed three-quasi-particle mode with $I=9/2^-$ in Te^{125}. Adopted value of χ_0 is 247 (MeV) and the calculated excitation energy is 1.57 MeV. Notations are the same as Table I(a).

neutron

ab	$h_{11/2}^2$	$g_{7/2}^2$	$d_{5/2}^2$	$d_{3/2}^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi_{nI}(p;ab)$	0.806	0.111	0.052	0.293	0.031	0.300	−0.123	0.114	0.285
$\varphi_{nI}(p;ab)$	0.300	0.089	0.044	0.189	0.026	0.223	−0.096	0.092	0.202

proton

ab	$h_{11/2}^2$	$g_{7/2}^2$	$d_{5/2}^2$	$d_{3/2}^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi_{nI}(p;ab)$	0.053	0.601	0.158	0.012	0.125	0.114	−0.034	0.059	0.015
$\varphi_{nI}(p;ab)$	0.045	0.312	0.107	0.010	0.077	0.092	−0.028	0.049	0.014

the "phonon" to form the new collective states, i.e., the dressed three-quasi-particle states. The situation is illustrated in Table I, where one can see that the amounts of the $(j_p)^3$-component and of the other components in the correlation amplitudes are in the ratio of about one to one.

Throughout this paper, we do not touch the mixing effects of the dressed three-quasi-particle modes with the other elementary excitation modes (such as the one-quasi-particle modes and the dressed five-quasi-particle modes.) The mixing effects, which essentially come from the H_Y type interaction in Fig. 2, may be expected to become important for the $I \neq (j_p - 1)$ collective states, especially for the $I = (j_p - 2)$ states. A more general calculation including the mixing effects will be reported in another paper, together with various electromagnetic properties of the anomalous coupling states.

Finally the following should be emphasized: When the anomalous coupling $I = (j_p - 1)$ state becomes the ground state, an instability toward deformation is expected, and so these nuclei are outside of the applicability of the present theory with spherical bases. Starting from this point of view, it would be desirable to investigate theoretically the stability of the spherical BCS ground state against the characteristic three-quasi-particle correlation.

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Theory of Collective Excitations in Spherical Odd-Mass Nuclei. III

——*Electromagnetic Properties of the Anomalous Coupling States**)——

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Various electromagnetic properties of the anomalous coupling states with spin $(j-1)$ are shown to be well explained by the new viewpoint of the dressed n-quasi-particle modes proposed in a previous paper. In this point of view, the anomalous coupling collective states with spin $(j-1)$ are considered as the dressed three-quasi-particle modes, which are regarded as a kind of elementary excitation modes in odd-mass nuclei. The effects of couplings between dressed three-quasi-particle modes and one-quasi-particle modes are also discussed in this connection, including those with spin j, $(j-1)$ and $(j-2)$.

§ 1. Introduction

In a previous paper, part II,[1),**)] we have introduced a new point of view on the structure of anomalous coupling states (ACS) with spin $I=(j-1)$. In this point of view, the ACS are considered as typical manifestation of the "*dressed*" *three-quasi-particle modes* which can be regarded as a kind of elementary modes of excitations in odd-mass nuclei. The concept of the dressed n-quasi-particle modes has been proposed in part I[2),***)] and related papers.[3)] With the use of the conventional pairing-plus-quadrupole-force $(P+QQ)$ model, it has been shown in II that the excitation-energy systematics of the ACS with spin $j-1$ (the $7/2^+$ states in the Kr-Sr region, the $7/2^+$ states in the Tc-Rh-Ag region and the $9/2^-$ states in the Te-Xe region) can be well explained by the proposed new point of view.

Extensive data on the electromagnetic properties of the ACS are now being accumulated and are providing us with important information which reveals various aspects on the structure of the ACS.[12)~26)] The characteristics of the electromagnetic properties of the ACS with spin $j-1$ may be summarized as follows:
1) Strongly enhanced $E2$-transitions between the $(j-1)$ states and the one-quasi-particle (1QP) states with spin j. The enhancements of the $E2$ transitions are

*) A preliminary version of this work was reported at the Int. Conf. on Nuclear Moments and Nuclear Structure; J. Phys. Soc. Japan **34** Suppl. (1973), 407.
**) This work is referred to as II.
***) This work is referred to as I.

comparable (or somewhat larger) to those of the phonon transitions in neighbouring even-even nuclei.

2) Moderately hindered $M1$-transitions between $(j-1)$ states and j states. In some experiments, however, they are only weakly retarded.

3) The g factor of the $(j-1)$ states are nearly equal to (or slightly deviates from) those of the 1QP states with spin j.

The main purpose of the present paper is to show how various electromagnetic properties of the ACS mentioned above can be explained in a unified manner. In order to evaluate any physical quantities unambiguously within the framework of new-Tamm-Dancoff (NTD) approximation, we at first define the concept of *quasi-particle NTD space* spanned by the introduced elementary excitation modes.[2),3)] The quasi-particle NTD space is constructed in a complete one-to-one correspondence to the quasi-particle-Tamm-Dancoff (TD) space (characterizing the conventional quasi-particle representation). Therefore the dressed n-quasi-particle modes are reduced to the Tamm-Dancoff n-quasi-particle states if the ground state correlations are neglected. Then, any physical operators are transcribed into the quasi-particle NTD space, so that any ambiguity does not appear in evaluating the electromagnetic quantities of interest. In this way, the coupling Hamiltonian between different types of excitation modes is also derived unambiguously.

Within the framework of the $P+QQ$ force model, the theory is formulated in an explicit form in §§ 2 and 3. In § 4, the mechanism of appearance of the collective 3QP correlations, which is responsible for the special lowering of the $(j-1)$ states, and the process of their growing up are clarified. Furthermore, the excitation-energy systematics of the ACS with spin $(j-2)$ is explained by taking into account the effects of the interplay between dressed 3QP modes and 1QP modes. In §§ 5 and 6, we present numerical results on the electromagnetic properties of the ACS for cases without and with the coupling effects. The results will clearly show how various electromagnetic properties of the ACS mentioned above can be recognized in a unified way within the framework of the proposed theory.

§ 2. Preliminaries

In this section we briefly recapitulate the theory and model on the ACS introduced in I and II, as a necessary step toward discussion in the following section.

2-1 *The Hamiltonian*

We start with the spherically symmetric j-j coupling shell model*) with the

*) The single-particle states are then characterized by a set of quantum numbers: the charge q, n, l, j, m. Throughout this paper, these states are designated by Greek letters. In association with the letter α, we use a Roman letter a to denote the same set except for the magnetic quantum number m. We further use a subscript $\bar{\alpha}$, which is obtained from α by changing the sign of the magnetic quantum number.

$P+QQ$ force. Taking into account the special importance of pairing correlations, first we perform the Bogoliubov transformation. Our Hamiltonian may be written in terms of the quasi-particle operators $a_\alpha^\dagger$ and a_α as follows:

$$H=H_0+H_{QQ}$$
$$=\sum_\alpha E_a a_\alpha{}^\dagger a_\alpha-\tfrac{1}{2}\chi\sum_M :\widehat{Q}_{2M}^\dagger\widehat{Q}_{2M}:, \tag{2·1}$$

where χ is the strength of the quadrupole force, and E_a is the quasi-particle energy, determined as usual together with the parameters v_a and u_a of the Bogoliubov transformation. The symbol :: denotes the normal product with respect to the quasi-particles, and the quantity $\widehat{Q}_{2M}$ is the mass-quadrupole-moment operator in terms of quasi-particles,

$$\widehat{Q}_{2M}=\sum_{ab} q(ab)\,[\xi(ab)\,\{A_{2M}^\dagger(ab)+\widetilde{A}_{2M}(ab)\}$$
$$+\eta(ab)\,\{B_{2M}^\dagger(ab)+\widetilde{B}_{2M}(ab)\}]\,, \tag{2·2}$$

where

$$q(ab)\equiv\frac{1}{\sqrt{5}}\langle a\|r^2Y_2\|b\rangle \tag{2·3}$$

and

$$\left.\begin{aligned}\xi(ab)&\equiv\frac{1}{\sqrt{2}}(u_av_b+v_au_b)\,,\\ \eta(ab)&\equiv\frac{1}{2}(u_au_b-v_av_b)\,.\end{aligned}\right\} \tag{2·4}$$

The operators $A_{JM}^\dagger(ab), \widetilde{A}_{JM}(ab), B_{JM}^\dagger(ab)$ and $\widetilde{B}_{JM}(ab)$ are the conventional pair operators defined by

$$\left.\begin{aligned}A_{JM}^\dagger(ab)&\equiv\frac{1}{\sqrt{2}}\sum_{m_\alpha m_\beta}\langle j_aj_bm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger a_\beta{}^\dagger\,,\\ B_{JM}^\dagger(ab)&\equiv-\sum_{m_\alpha m_\beta}\langle j_aj_bm_\alpha m_\beta|JM\rangle a_\alpha{}^\dagger\tilde{a}_\beta\,,\\ \widetilde{A}_{JM}(ab)&\equiv(-)^{J-M}A_{J\bar{M}}(ab),\\ \widetilde{B}_{JM}(ab)&\equiv(-)^{J-M}B_{J\bar{M}}(ab)\,,\end{aligned}\right\} \tag{2·5}$$

where

$$\tilde{a}_\beta\equiv s_\beta a_{\bar{\beta}}\equiv(-)^{j_b-m_\beta}a_{\bar{\beta}}\,. \tag{2·6}$$

The quadrupole force H_{QQ} in Eq. (2·1), which represents the interaction causing the breakup of the Cooper pair, can be divided into three parts depending on roles they play in constructing the elementary excitation modes:

$$\left.\begin{aligned}H_{QQ}&=H_{QQ}^{(0)}+H_Y\,,\\ H_{QQ}^{(0)}&\equiv H_X+H_V\,,\end{aligned}\right\} \tag{2·7}$$

where*)

$$H_X = -\chi \sum_M \sum_{\substack{a_1b_1 \\ a_2b_2}} Q(a_1b_1) Q(a_2b_2) A^\dagger_{2M}(a_1b_1) A_{2M}(a_2b_2) \ , \tag{2·7a}$$

$$H_V = -\frac{\chi}{2} \sum_M \sum_{\substack{a_1b_1 \\ a_2b_2}} Q(a_1b_1) Q(a_2b_2) \{A^\dagger_{2M}(a_1b_1) \tilde{A}^\dagger_{2M}(a_2b_2) + \text{h.c.}\}, \tag{2·7b}$$

$$H_Y = -2\chi \sum_M \sum_{\substack{a_1b_1 \\ a_2b_2}} Q(a_1b_1) q(a_2b_2) \eta(a_2b_2) \{A^\dagger_{2M}(a_1b_1) B_{2M}(a_2b_2) + \text{h.c.}\} \tag{2·7c}$$

with

$$Q(ab) \equiv q(ab) \xi(ab) \ . \tag{2·8}$$

The first part $H^{(0)}_{QQ}$ plays an essential role in constructing the dressed three-quasi-particle modes as elementary excitations, while the latter part H_Y plays an essential role as coupling between the different types of elementary excitation modes, for instance, interactions between the 1QP modes and the dressed 3QP modes, etc. For detailed explanation of the classification of various roles of the interaction, see Ref. 3).

2-2 *Model of the ACS as the dressed 3QP modes*

Let us consider the odd-mass system in the truncated shell model space consisting of one major harmonic-oscillator shell (for both the protons and the neutrons) and of a unique-parity level which enters into the (proton or neutron) major shell, and suppose that the unique-parity level is being filled with several protons or neutrons. When we especially need to specify the unique-parity level and the single-particle states at the level, we use the Roman letter p and the Greek letters π, ρ, σ, etc., respectively. The Roman letters $a, b, c, \cdots$ and the associated Greek letters $\alpha, \beta, \gamma, \cdots$ are used for the levels except the unique-parity level and for the states at the levels, respectively. In this special situation in shell structure (for the appearance of the ACS), the dressed 3QP modes with parity opposite to that of the major shell become especially simple from the parity consideration: In the $P+QQ$ model, the eigenmode operators for the dressed 3QP modes (defined in I-§ 9), which have parity opposite to that of the major shell, are simply reduced to

$$C^\dagger_{nIK} = \frac{1}{\sqrt{3!}} \sum_{\pi\rho\sigma} \{\psi^p_{nIK}(\pi\rho\sigma) : T'_{3/2\,3/2}(\pi\rho\sigma) : + \varphi^p_{nIK}(\pi\rho\sigma) : T_{3/2\,\overline{1/2}}(\pi\rho\sigma) :\}$$
$$+ \frac{1}{\sqrt{2!}} \sum_{\pi\beta\gamma} \{\psi^c_{nIK}(\pi\beta\gamma) a_\pi{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger + \varphi^c_{nIK}(\pi\beta\gamma) a_\pi{}^\dagger \tilde{a}_\beta \tilde{a}_\gamma\} \ , \tag{2·9}$$

where the operator $T_{3/2 s_0}(\pi\rho\sigma)$ is the quasi-spin tensor of rank $s=3/2$ and its

*) In the same way as in II, we have neglected the exchange term of the interaction H_{QQ} in Eq. (2·7), according to the inherent assumption of the $P+QQ$ model.

projection s_0 the explicit form of which is given in I-§ 3, for instance,

$$T_{3/2\,3/2}(\pi\rho\sigma)=a_\pi^\dagger a_\rho^\dagger a_\sigma^\dagger\,,$$

$$T_{3/2\,\overline{1/2}}(\pi\rho\sigma)=\sqrt{\frac{1}{3}}\{a_\pi^\dagger\tilde{a}_\rho\tilde{a}_\sigma+\tilde{a}_\pi a_\rho^\dagger\tilde{a}_\sigma+\tilde{a}_\pi\tilde{a}_\rho a_\sigma^\dagger\}\ .$$

The three-body-correlation amplitudes satisfy the relations

$$P\psi^p_{nIK}(\pi\rho\sigma)=\delta_P\psi^p_{nIK}(\pi\rho\sigma)\ ,\ P\varphi^p_{nIK}(\pi\rho\sigma)=\delta_P\varphi^p_{nIK}(\pi\rho\sigma)\ ,$$
$$\psi^c_{nIK}(\pi\gamma\beta)=-\psi^c_{nIK}(\pi\beta\gamma)\ ,\quad \varphi^c_{nIK}(\pi\gamma\beta)=-\varphi^c_{nIK}(\pi\beta\gamma)\ ,\qquad (2\cdot 10)$$

where P is the permutation operator with respect to (π,ρ,σ) and δ_P is defined by

$$\delta_P=\begin{cases}1 & \text{for even permutations,}\\ -1 & \text{for odd permutations.}\end{cases}\qquad (2\cdot 11)$$

As was emphasized in I, the eigenmode operator (2·9) *must include no zero-coupled quasi-particle pairs*, so that the correlation amplitudes have to satisfy the conditions

$$\sum_{m_\pi}\psi^p_{nIK}(\pi\bar{\pi}\sigma)s_\pi=\sum_{m_\pi}\varphi^p_{nIK}(\pi\bar{\pi}\sigma)s_\pi=0\ ,$$
$$\sum_{m_\beta}\psi^c_{nIK}(\pi\beta\bar{\beta})s_\beta=\sum_{m_\beta}\varphi^c_{nIK}(\pi\beta\bar{\beta})s_\beta=0\ .\qquad (2\cdot 12)$$

The expression (2·9) for the eigenmode operators with the conditions (2·10) and (2·12) clearly means that the dressed 3QP modes are characterized by the amount of transferred seniority $\Delta v=3$ to the state on which they operate.

The eigenvalue equations for the three-body-correlation amplitude should be obtained so that $C^\dagger_{nIK}$ becomes a "good" approximate eigenmode satisfying

$$[H_0+H^{(0)}_{QQ},C^\dagger_{nIK}]=\omega_{nI}C^\dagger_{nIK}-Z_{nIK}\ .\qquad (2\cdot 13)$$

Thus, in our NTD approximation, the "interaction" Z_{nIK} whose composition has been explained in II is neglected in the first step in which the dressed 3QP eigenmode $C^\dagger_{nIK}$ is determined.

It is convenient to distinguish the eigenmode operators according to the corresponding energy eigenvalues ω_{n_+} and ω_{n_0}:

$$C^\dagger_{nIK}=\begin{cases}Y^\dagger_{nIK} & \text{for}\quad \omega_n=\omega_{n_+};\ \lim_{\chi\to 0}\omega_{n_+}=E_p+E_a+E_b\,,\\ A_{n_0IK} & \text{for}\quad \omega_n=\omega_{n_0};\ \lim_{\chi\to 0}\omega_{n_0}=E_p-E_a-E_b\,.\end{cases}\qquad (2\cdot 14)$$

As has been discussed in I, the operators $Y^\dagger_{nIK}$ are "physical" operators to create the dressed 3QP states. The existence of "special" operators A_{n_0IK} means that any state vector $|\Phi\rangle$ which actually has physical meaning must satisfy the supplementary condition

$$A_{n_0IK}|\Phi\rangle=0\,. \tag{2·15}$$

Thus, the correlated ground state $|\Phi_0\rangle$ should be determined by the equations

$$Y_{nIK}|\Phi_0\rangle=0\,,\quad A_{n_0IK}|\Phi_0\rangle=0\,. \tag{2·16}$$

It has also been shown in I, within the framework of the NTD approximation, that the dressed 3QP modes satisfy the quasi-Fermion approximation, i.e.,

$$\langle\Phi_0|\{Y_{n'I'K'},\,Y^\dagger_{nIK}\}|\Phi_0\rangle=\delta_{nn'}\delta_{II'}\delta_{KK'}\,, \tag{2·17}$$

by the use of the orthogonality relation

$$\begin{aligned}(\Psi_{n'I'K'}\cdot\Psi_{nIK}) &= \sum_{\pi\rho\sigma}\{\psi^p_{n'I'K'}(\pi\rho\sigma)\psi^p_{nIK}(\pi\rho\sigma)-\varphi^p_{n'I'K'}(\pi\rho\sigma)\varphi^p_{nIK}(\pi\rho\sigma)\}\\ &\quad+\sum_{\pi\beta\gamma}\{\psi^c_{n'I'K'}(\pi\beta\gamma)\psi^c_{nIK}(\pi\beta\gamma)-\varphi^c_{n'I'K'}(\pi\beta\gamma)\varphi^c_{nIK}(\pi\beta\gamma)\}\\ &=\epsilon_n\delta_{nn'}\delta_{II'}\delta_{KK'}\end{aligned} \tag{2·18}$$

with

$$\epsilon_n=\begin{cases}1 & \text{for}\quad \omega_n=\omega_{n_+}\,,\\ -1 & \text{for}\quad \omega_n=\omega_{n_0}\,,\end{cases} \tag{2·19}$$

which is obtained from the properties of the eigenmode equation for the correlation amplitudes.

The basic operators of the eigenmode, $:T_{3/2\,3/2}(\pi\rho\sigma):$, $:T_{3/2\,\overline{1/2}}(\pi\rho\sigma):$, $a_\pi{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger$ and $a_\pi{}^\dagger\tilde{a}_\beta\tilde{a}_\gamma$, are expanded in terms of the eigenmode operators $C^\dagger_{nIK}$ uniquely:

$$\left.\begin{aligned}&:T_{3/2\,3/2}(\pi\rho\sigma):=\sqrt{3!}\sum_{nI}\psi^p_{nIK}(\pi\rho\sigma)\,(Y^\dagger_{nIK}-A_{nIK})\,,\\ &:T_{3/2\,\overline{1/2}}(\pi\rho\sigma):=-\sqrt{3!}\sum_{nI}\varphi^p_{nIK}(\pi\rho\sigma)\,(Y^\dagger_{nIK}-A_{nIK})\,,\\ &a_\pi{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger=\sqrt{2}\sum_{nI}\psi^c_{nIK}(\pi\beta\gamma)\,(Y^\dagger_{nIK}-A_{nIK})\,,\\ &a_\pi{}^\dagger\tilde{a}_\beta\tilde{a}_\gamma=-\sqrt{2}\sum_{nI}\varphi^c_{nIK}(\pi\beta\gamma)\,(Y^\dagger_{nIK}-A_{nIK})\,.\end{aligned}\right\} \tag{2·20}$$

The part A_{nIK} in (2·20) does not play any role in our physical space because of the subsidiary conditions (2·16).

In actual calculations, it is convenient to use the coupled angular-momentum representation. Then, the physical eigenmode operators may be written as

$$\begin{aligned}Y^\dagger_{nIK}=&N_I\psi_{nI}(p^3)\;:T_{3/2\,3/2}(j_p{}^2(2)j_pIK):\\ &+N_I\varphi_{nI}(p^3)\;:T_{3/2\,\overline{1/2}}(j_p{}^2(2)j_pIK):\\ &+\sum_{(ab)}N(ab)\psi_{nI}(p;ab)\,[a_p{}^\dagger A_2{}^\dagger(ab)]_{IK}\\ &-\sum_{(ab)}N(ab)\varphi_{nI}(p;ab)\,[a_p{}^\dagger\tilde{A}_2(ab)]_{IK}\,,\end{aligned} \tag{2·21}$$

where $\sum_{(ab)}$ means the summation with respect to the set of levels (ab) and the following notations are used:

$$N(ab)\equiv\sqrt{2/(1+\delta_{ab})}\ , \tag{2·22}$$

$$N_I\equiv 1/\sqrt{2C_I}\ , \tag{2·23}$$

$$C_I\equiv 1+10\begin{Bmatrix} j_p & j_p & 2 \\ j_p & I & 2 \end{Bmatrix}-\delta_{j_pI}\cdot\frac{20}{4j_p^2-1} \tag{2·24}$$

and

$$\left.\begin{aligned} &:T_{3/2\,3/2}(j_p^2(2)j_pIK):=\sqrt{2}[A_2^\dagger(pp)a_p^\dagger]_{IK} \\ &:T_{3/2\,\overline{1/2}}(j_p^2(2)j_pIK):=-\sqrt{\frac{2}{3}}[a_p^\dagger\tilde{A}_2(pp)]_{IK}-\sqrt{\frac{4}{3}}[B_2^\dagger(pp)\tilde{a}_p]_{IK}\ . \end{aligned}\right\} \tag{2·25}$$

The correlation amplitudes ψ_{nIK} in the coupled angular-momentum representation are related with those defined in Eq. (2·9) by

$$\left.\begin{aligned} \psi_{nIK}(\pi\rho\sigma)&\equiv\sum_{J=\text{even}}\psi_{nI}(j_p^2(J)j_p\}j_p^3I)\langle Jj_pMm_\sigma|IK\rangle\langle j_pj_pm_\pi m_\rho|JM\rangle\ , \\ \psi_{nIK}(\pi\beta\gamma)&\equiv\psi'_{nI}(p;bc)\langle 2j_pMm_\pi|IK\rangle\langle j_bj_cm_\beta m_\gamma|2M\rangle\ , \end{aligned}\right\} \tag{2·26}$$

$$\left.\begin{aligned} \psi_{nI}(p^3)&\equiv\sqrt{3!}N_I\psi_{nI}(j_p^2(2)j_p\}j_p^3I)\ , \\ \psi_{nI}(p;bc)&\equiv N(bc)\psi'_{nI}(p;bc)\ , \end{aligned}\right\} \tag{2·27}$$

and the same relations are obtained for the backward amplitudes φ_{nI}. It should be noticed that the dressed 3QP modes $Y^\dagger_{nIK}$ can be simply expressed in terms of only the special amplitudes with the intermediate angular momentum $J=2$, as was shown in II. The eigenvalue equation for the correlation amplitudes is now written as follows:

$$\left.\begin{aligned} &\{2E_p-\omega'_{nI}\}\psi_{nI}(p^3)=\chi Q(pp)\sqrt{C_I}(A_I+B_I)\ , \\ &\{2E_p+\omega'_{nI}\}\varphi_{nI}(p^3)=\sqrt{\frac{1}{3}}\chi Q(pp)\sqrt{C_I}(A_I+B_I)\ , \\ &\{(E_a+E_b)-\omega'_{nI}\}\psi_{nI}(p;ab)=\chi Q(ab)N(ab)(A_I+B_I)\ , \\ &\{(E_a+E_b)+\omega'_{nI}\}\varphi_{nI}(p;ab)=\chi Q(ab)N(ab)(A_I+B_I)\ , \end{aligned}\right\} \tag{2·28}$$

where

$$\left.\begin{aligned} A_I&\equiv\sum_{(ab)}Q(ab)N(ab)\{\psi_{nI}(p;ab)+\varphi_{nI}(p;ab)\}\ , \\ B_I&\equiv Q(pp)\sqrt{C_I}\left\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\} \end{aligned}\right\} \tag{2·29}$$

and

$$\omega'_{nI}\equiv\omega_{nI}-E_p\ . \tag{2·30}$$

The eigenvalue of Eq. (2·28) can be easily obtained by writing it in the following form:

$$\chi^{-1}=S_p+S_c$$
$$=\frac{2}{3}\frac{Q^2(pp)C_I\{4E_p+\omega'_{nI}\}}{(2E_p)^2-(\omega'_{nI})^2}+2\sum_{ab}\frac{Q^2(ab)\cdot(E_a+E_b)}{(E_a+E_b)^2-(\omega'_{nI})^2}. \qquad (2\cdot31)$$

In the coupled angular-momentum representation, the normalization of correlation amplitudes (2·18) for the physical solutions becomes

$$\psi^2_{nI}(p^3)+\sum_{(ab)}\psi^2_{nI}(p;ab)-\varphi^2_{nI}(p^3)-\sum_{(ab)}\varphi^2_{nI}(p;ab)=1. \qquad (2\cdot32)$$

Combining Eq. (2·28) and Eq. (2·32), we obtain explicit expressions for the correlation amplitudes:

$$\left.\begin{aligned}\psi_{nI}(p^3)&=M_{I\omega}Q(pp)\sqrt{C_I}/\{2E_p-\omega'_{nI}\},\\ \varphi_{nI}(p^3)&=\sqrt{\frac{1}{3}}M_{I\omega}Q(pp)\sqrt{C_I}/\{2E_p+\omega'_{nI}\},\\ \psi_{nI}(p;ab)&=M_{I\omega}Q(ab)N(ab)/\{(E_a+E_b)-\omega'_{nI}\},\\ \varphi_{nI}(p;ab)&=M_{I\omega}Q(ab)N(ab)/\{(E_a+E_b)+\omega'_{nI}\},\end{aligned}\right\} \qquad (2\cdot33)$$

where the normalization factor $M_{I\omega}$ is given by

$$M_{I\omega}\equiv\chi\cdot(A_I+B_I)$$
$$=\left(\frac{d}{d\omega}(S_p+S_c)\right)^{-1/2}$$
$$=\left(\frac{2}{3}Q^2(pp)C_I\frac{(2E_p)^2+8E_p\omega'_{nI}+(\omega'_{nI})^2}{\{(2E_p)^2-(\omega'_{nI})^2\}^2}\right.$$
$$\left.+4\omega'_{nI}\sum_{ab}\frac{Q^2(ab)(E_a+E_b)}{\{(E_a+E_b)^2-(\omega'_{nI})^2\}^2}\right)^{-1/2}. \qquad (2\cdot34)$$

As will be discussed in the next section, our eigenvalue equation (2·28) possesses real solutions in so far as the condition

$$\frac{d}{d\omega}(S_p+S_c)>0 \qquad (2\cdot35)$$

is satisfied.

§ 3. Coupling between single- and dressed-three-quasi-particle modes and the transcription of physical operators into "quasi-particle NTD space"

So far we have treated only the parts H_X and H_V, i.e., the *constructive force* (of the elementary modes of excitations), in the interaction H_{QQ}. They have determined the dressed 3QP modes, which inherently introduce a model subspace called the quasi-particle NTD subspace. The next problem is to treat the part H_Y, i.e., the *interactive force* (between the different types of elementary excitation modes), by finding a method of transcription of any physical operators into the quasi-particle NTD subspace, within the same NTD approximation employed to construct the modes.

The "quasi-particle NTD subspace" under consideration should be composed of orthonormalized basis vectors,

$$\{|\Phi_\alpha^{(1)}\rangle \equiv a_\alpha^\dagger|\Phi_0\rangle,\ |\Phi_{nIK}^{(3)}\rangle \equiv Y_{nIK}^\dagger|\Phi_0\rangle\}\ . \tag{3·1}$$

Within the basic approximation of the NTD method

$$O(n/\Omega) = \langle\Phi_0|a_\alpha^\dagger a_\alpha|\Phi_0\rangle \approx 0\ , \tag{3·2}$$

the orthonormality of the basis vectors,

$$\langle\Phi_{\alpha'}^{(1)}|\Phi_\alpha^{(1)}\rangle = \delta_{\alpha\alpha'}\ ,\quad \langle\Phi_{n'I'K'}^{(3)}|\Phi_{nIK}^{(3)}\rangle = \delta_{nn'}\delta_{II'}\delta_{KK'}\ ,$$
$$\langle\Phi_\alpha^{(1)}|\Phi_{nIK}^{(3)}\rangle = 0\ ,\quad \text{etc.}, \tag{3·3}$$

is proved to be satisfied. The unit operator of the extended subspace, that is, the projection operator onto the subspace is defined by

$$\mathbf{1} = \sum_\alpha |\Phi_\alpha^{(1)}\rangle\langle\Phi_\alpha^{(1)}| + \sum_{nIK} |\Phi_{nIK}^{(3)}\rangle\langle\Phi_{nIK}^{(3)}|\ . \tag{3·4}$$

It must be emphasized that the single-quasi-particle (1QP) states $a_\alpha^\dagger|\Phi_0\rangle$ in the *quasi-particle NTD subspace* should possess the eigenvalues E_a, in spite of the presence of the interactions H_X and H_Y,*) and are orthogonal to the spurious states (due to the nucleon number fluctuation in the quasi-particle representation). This may be recognized by noticing that

$$\mathbf{1}\cdot[H_0 + H_{QQ}^{(0)}, a_\alpha^\dagger]|\Phi_0\rangle = \mathbf{1}\cdot E_a a_\alpha^\dagger|\Phi_0\rangle \tag{3·5}$$

and

$$\mathbf{1}\cdot\hat{S}_- a_\alpha^\dagger|\Phi_0\rangle = \mathbf{1}\cdot[\hat{S}_-, a_\alpha^\dagger]|\Phi_0\rangle = \mathbf{1}\cdot\tilde{a}_\alpha|\Phi_0\rangle = 0\ , \tag{3·6}$$

where $\hat{S}_-$ denotes the generators of the quasi-spin space defined by I-(3·21). Equations (3·5) and (3·6) hold because the correlated ground state $|\Phi_0\rangle$ satisfies the conditions (2·16) and $\hat{S}_-|\Phi_0\rangle = 0$, under the basic approximation (3·2).

Let $\hat{O}_{\lambda\mu}$ be any single-particle operator of rank $\lambda(\neq 0)$. This operator can always be written in terms of quasi-particle operators,

$$\hat{O}_{\lambda\mu} = \hat{O}_{\lambda\mu}^{(0)} + \hat{O}_{\lambda\mu}^{(1)}$$
$$= \sum_{ab} \frac{\langle a\|\hat{O}_\lambda\|b\rangle}{\sqrt{2\lambda+1}} \cdot \frac{(u_a v_b + (-)^T v_a u_b)}{\sqrt{2}} (A_{\lambda\mu}^\dagger(ab) + (-)^T \tilde{A}_{\lambda\mu}(ab))$$

*) The details on this point will be discussed in a forthcoming paper.[4]

$$+\sum_{ab}\frac{\langle a\|\widehat{O}_\lambda\|b\rangle}{\sqrt{2\lambda+1}}(u_a u_b-(-)^T v_a v_b)B^\dagger_{\lambda\mu}(ab)\ , \tag{3·7}$$

where $(-)^T$ denotes the phase factor of operator $\widehat{O}_{\lambda\mu}$ under time reversal defined by

$$T\widehat{O}_{\lambda\mu}T^\dagger=(-)^T\widehat{O}_{\lambda-\mu}(-)^\mu\ . \tag{3·8}$$

The transcription of operator $\widehat{O}_{\lambda\mu}$ into the subspace can be done unambiguously:

$$\begin{aligned}\widehat{O}_{\lambda\mu}\rightarrow\widehat{\boldsymbol{O}}_{\lambda\mu}&\equiv\mathbf{1}\widehat{O}_{\lambda\mu}\mathbf{1}\\&=\sum_{\alpha,\alpha'}\langle\Phi^{(1)}_\alpha|\widehat{O}_{\lambda\mu}|\Phi^{(1)}_{\alpha'}\rangle\boldsymbol{a}_\alpha{}^\dagger\boldsymbol{a}_{\alpha'}+\sum_{\substack{nIK\\n'I'K'}}\langle\Phi^{(3)}_{nIK}|\widehat{O}_{\lambda\mu}|\Phi^{(3)}_{n'I'K'}\rangle\boldsymbol{Y}^\dagger_{nIK}\boldsymbol{Y}_{n'I'K'}\\&\quad+\sum_{\alpha,nIK}\{\langle\Phi^{(1)}_\alpha|\widehat{O}_{\lambda\mu}|\Phi^{(3)}_{nIK}\rangle\boldsymbol{a}_\alpha{}^\dagger\boldsymbol{Y}_{nIK}+\text{h.c.}\}\ ,\end{aligned} \tag{3·9}$$

where we have used the fact that the eigenmode operators $a_\alpha{}^\dagger$ and $Y^\dagger_{nIK}$ can be expressed in the "quasi-particle NTD subspace" as

$$\begin{aligned}\boldsymbol{a}_\alpha{}^\dagger&\equiv|\Phi^{(1)}_\alpha\rangle\langle\Phi_0|=a_\alpha{}^\dagger|\Phi_0\rangle\langle\Phi_0|\ ,\\\boldsymbol{Y}^\dagger_{nIK}&\equiv|\Phi^{(3)}_{nIK}\rangle\langle\Phi_0|=Y^\dagger_{nIK}|\Phi_0\rangle\langle\Phi_0|\ .\end{aligned} \tag{3·10}$$

The transcription coefficients in Eq. (3·9) are evaluated by using the quasi-Fermion approximation (2·17):

$$\begin{aligned}\langle\Phi^{(1)}_\alpha|\widehat{O}_{\lambda\mu}|\Phi^{(1)}_{\alpha'}\rangle&=\langle\Phi_0|\{a_\alpha,[\widehat{O}_{\lambda\mu},a^\dagger_{\alpha'}]_-\}_+|\Phi_0\rangle\ ,\\\langle\Phi^{(1)}_\alpha|\widehat{O}_{\lambda\mu}|\Phi^{(3)}_{nIK}\rangle&=\langle\Phi_0|\{a_\alpha,[\widehat{O}_{\lambda\mu},Y^\dagger_{nIK}]_-\}_+|\Phi_0\rangle\ ,\\\langle\Phi^{(3)}_{nIK}|\widehat{O}_{\lambda\mu}|\Phi^{(3)}_{n'I'K'}\rangle&=\langle\Phi_0|\{Y_{nIK},[\widehat{O}_{\lambda\mu},Y^\dagger_{n'I'K'}]_-\}_+|\Phi_0\rangle\ .\end{aligned} \tag{3·11}$$

For details of the theory, see Ref. 3) and also a forthcoming paper.[4)]

In completely the same way, our original Hamiltonian H is expressed, after transcription, as

$$\begin{aligned}H\rightarrow\boldsymbol{H}&\equiv\mathbf{1}\cdot H\cdot\mathbf{1}=\mathbf{1}(H_0+H^{(0)}_{QQ}+H_Y)\mathbf{1}\\&=\sum_\alpha E_\alpha\boldsymbol{a}_\alpha{}^\dagger\boldsymbol{a}_\alpha+\sum_{nIK}\omega_{nI}\boldsymbol{Y}^\dagger_{nIK}\boldsymbol{Y}_{nIK}\\&\quad+\sum_{nIK,\alpha}\bar{\chi}_{nI}(\boldsymbol{Y}^\dagger_{nIK}\boldsymbol{a}_\alpha+\boldsymbol{a}_\alpha{}^\dagger\boldsymbol{Y}_{nIK})\ ,\end{aligned} \tag{3·12}$$

where we have dropped the constant term related to the energy of the correlated ground state. The third term of the transcribed Hamiltonian represents the interaction between the 1QP modes and the dressed 3QP modes, and comes from the H_Y-type (original) interaction which has not played any role in constructing the elementary excitation modes. This shows that the H_Y-type interaction manifests itself, after the transcription, as a coupling between the different types of excitation modes. It should be noticed that we have obtained exactly hermitian property of the coupling term. Thus, the theory can clearly overcome the non-hermiticity difficulties of the eigenvalue equation of the conventional higher

RPA. The reason is that we have properly taken into account the relations between the eigenmodes and the structure of the correlated ground state (prescribed by the eigenmode), in the evaluation of the coupling Hamiltonian.

In the $P+QQ$ model, the coupling strength $\bar{\chi}_{nI}$ is given by

$$\begin{aligned}\bar{\chi}_{nI}=&-\chi\cdot\delta_{j_{p'}I}\cdot\frac{\langle p\|r^2Y_2\|p'\rangle}{\sqrt{2I+1}}\cdot(u_{p'}u_p-v_{p'}v_p)\\&\times\Big[Q(pp)\sqrt{C_I}\Big\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\Big\}\\&+\sum_{bc}Q(bc)\{\psi'_{nI}(p;bc)+\varphi'_{nI}(p;bc)\}\Big]\\&=-\delta_{j_{p'}I}\cdot\frac{\langle p\|r^2Y_2\|p'\rangle}{\sqrt{2I+1}}\cdot(u_{p'}u_p-v_{p'}v_p)M_{I\omega}\,,\end{aligned}\tag{3·13}$$

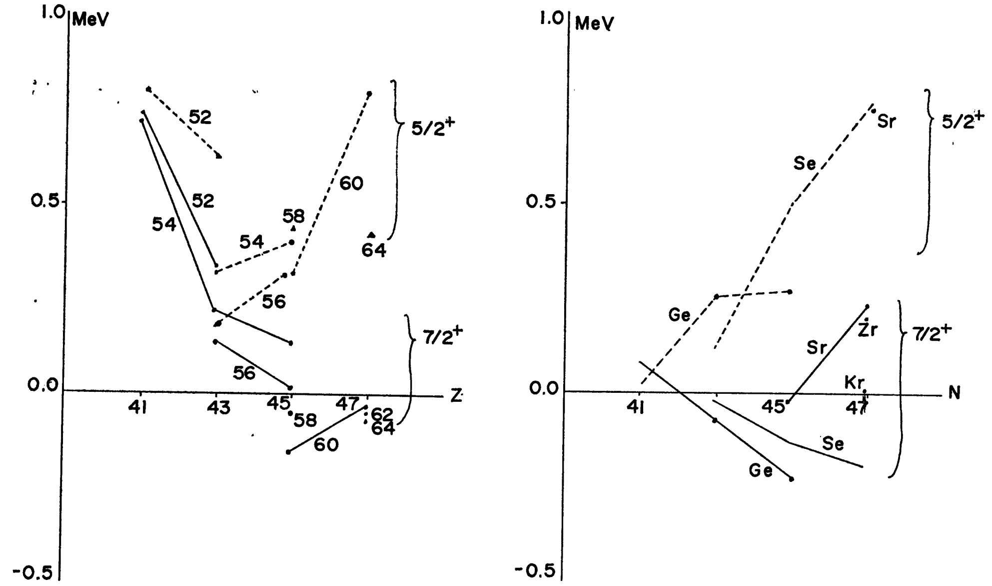
where $M_{I\omega}$ is defined by (2·34). We will consider the effects of this coupling term in § 4, where an important difference with the conventional phonon-quasiparticle coupling theory[5] will be shown.

§ 4. Characteristics of the excitation-energies

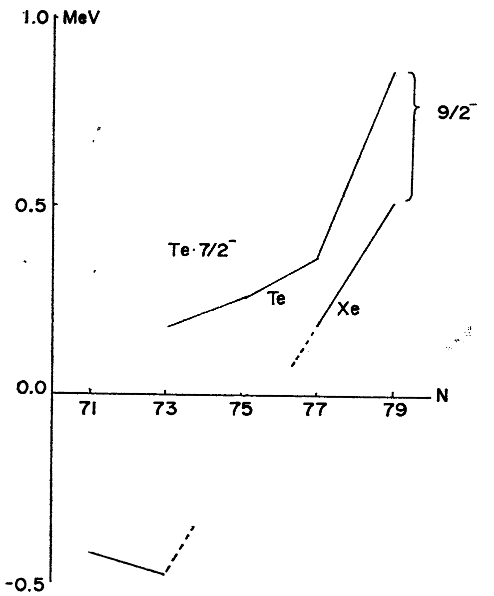
As was already shown in II, the excitation-energy systematics of the ACS with spin $(j-1)$ can be well reproduced in theoretical calculations. We present, therefore, only some numerical examples here. Figures 1 and 6 show the growth of 3QP correlations in the $g_{9/2}$-odd-proton region. The addition of only two protons to Nb^{93} is sufficient to bring on the strong growth of 3QP-correlations in Tc^{95} and, finally, we can clearly see completely different situations in Ag-isotopes, where only the $7/2^+$ states are extremely low in energy. The collective character of the $7/2^+$ states may also be recognized when we compare the excitation-energy systematics of the $7/2^+$ states with those of the 2^+ phonon states in even-even nuclei. Figure 2 clearly shows the similarity (as functions of neutron number N) between them. These two characteristics are, as was discussed in II, due to the two enhancement factors, $\xi(pp)$ at the unique parity level p and $\xi(ab)$ in the core, respectively. Depending on these enhancement factors in the eigenvalue equation (2·28), the excitation-energies ω_{nI} of the dressed 3QP modes become small and, finally, they arrive at a critical point from which there appear complex eigenvalues. The critical energy $\omega'_{\rm cri}\equiv\omega_{\rm cri}-E_p$ is determined from the characteristic properies of our eigenvalue equation (2·28) and is given as the solution of

$$\frac{d}{d\omega}(S_p+S_c)=0\,.\tag{4·1}$$

The lowering of the excitation energy (of the mode with spin $(j-1)$) clearly shows the growth of collective correlations in the state considered, and the appearance of the complex eigenvalue indicates that there may occur a new type of instability of spherical BCS vacuum due to the characteristic 3QP correlations. Figure 3 shows the behaviour of the dispersion equation (2·31) schematically.



(a) (b)



(c)

Fig. 1. Experimental trend of energy levels of the ACS with spin $(j-1)$ and with spin $(j-2)$. The level energies are presented in relation to those of 1QP states with spin j. For the nuclei where several $(j-2)$ states are known, only the lowest $(j-2)$ states ever known are presented; therefore the broken lines which connect the isotopes (isotones) are somewhat arbitrary.

Nb^{93}, Ref. 11).
Tc^{95}, Ref. 27).
$Ge^{75,77}$, Ref. 28).
Sr^{83}, R. W. Bercaw and R. E. Warner, Phys. Rev. **C2** (1970), 297.
Zr^{87}, R. Turcotte et al., Nucl. Phys. **A198** (1972), 67.
Cd^{115}, D. J. Hnatowich and C. D. Coryel, Nucl. Phys. **A143** (1970), 289.
Te^{127}, R. C. Ragaini, G. E. Gordon and W. B. Walters, Nucl. Phys. **A99** (1967), 547.
Te^{131}, J. Blachot et al., Phys. Rev. **C4** (1971), 214.
Xe^{133}, R. N. Saxena and H. D. Sharma, Nucl. Phys. **A171** (1971), 593.
The other references are listed in Ref. 1).

From the figure we can see that stable solutions of Eq. (2·28) extist as long as the condition $\omega'_{\rm cri} < \omega'_{nI}$ is satisfied, that is, all eigensolutions of Eq. (2·28) are real even if the energies of ACS with spin $(j-1)$ are lower than those of 1QP states with spin j.*) In the following, therefore, we shall present numerical results not only for the ex-

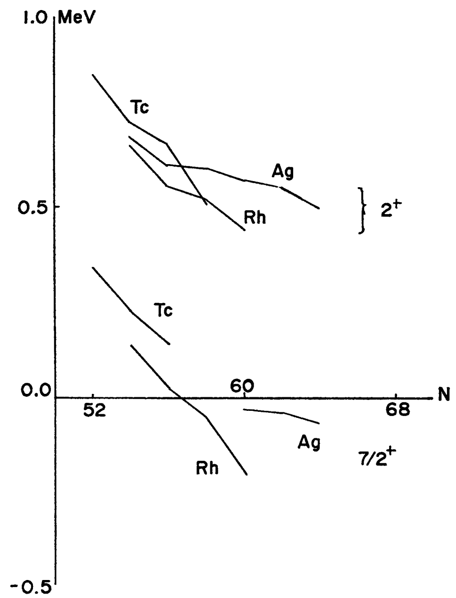


Fig. 2. Comparison between the excitation-energy systematics of the ACS with spin $(j-1)$ and those of 2^+-phonon states. The presented phonon-energies are the averaged values between the neighbouring even-even nuclei, i.e., $\omega_{2^+}(N,Z) \equiv \frac{1}{2}\{\omega_{2^+}(N,Z-1)+\omega_{2^+}(N,Z+1)\}$. The energies of the ACS with spin $7/2^+$ are presented in relation to those of 1QP states with spin $9/2^+$.

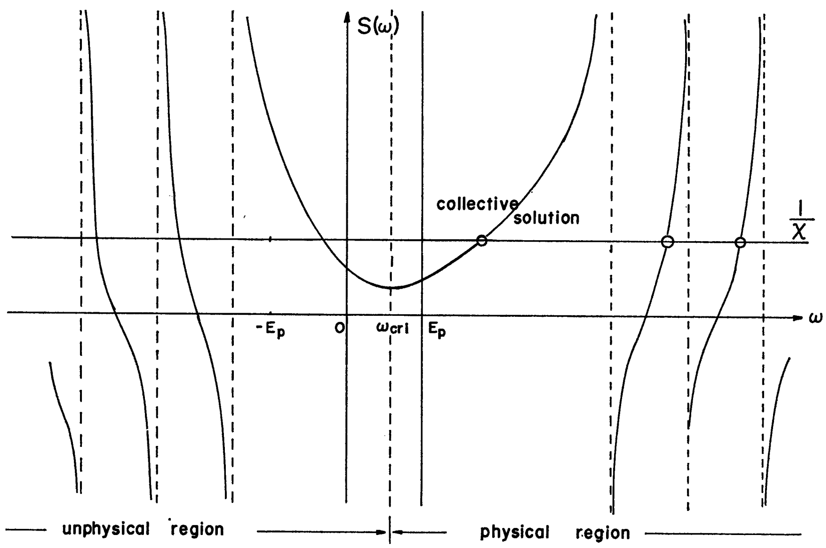


Fig. 3. Schematic representation of the dispersion equation (2·31) from which the eigenvalues of the dressed 3QP modes are determined.

*) Of course, in such a situation, the quantitative validity of the NTD approximation is not expected to be satisfactory, and rather strong "non-linear effects" (the effects of the "interaction" term Z_{nIK} in Eq. (2·13)) may become important. Nevertheless we can expect some persistency of the qualitative characteristics (the stability of spherical BCS vacuum holds barely in the very limited region $\omega'_{\rm cri} < \omega'_{nI} < 0$).

cited ACS but also for the ACS that appear below the 1QP states with spin j.

Now let us turn to consider the mixing effects due to the coupling term $\sum_{nIK,\alpha}\bar{\chi}_{nI}(\boldsymbol{Y}^{\dagger}_{nIK}\boldsymbol{a}_{\alpha}+\boldsymbol{a}_{\alpha}{}^{\dagger}\boldsymbol{Y}_{nIK})$ in the transcribed Hamiltonian (3·12), with the effective coupling constant $\bar{\chi}_{nI}$ given by Eq. (3·13). The characteristic of the coupling term is its inclusion of the $\delta_{j_{p'}I}\cdot(u_{p'}u_p-v_{p'}v_p)$ factor, which comes from the (original) H_Y-type interaction. For the modes with $I=j_p$, we have

$$u_p{}^2-v_p{}^2\approx 0$$

in the special physical situation in which high-spin, unique-parity level p is half-filled. Remember that the lowering of $(j-1)$ states occurs in odd-mass nuclei in which a level of high-spin with opposite parity in the major shell is being filled. For the modes with $I\neq j_p$, the mixing effects are expected to be rather small, since a single-particle level p' (which has the same parity with the level p) with spin $j_{p'}=I\neq j_p$ does not exist in the same major shell but lies in the next upper major shell.

Including the mixing effects, the states of interest are changed as

$$\left.\begin{aligned} Y^{\dagger}_{IK}|\Phi_0\rangle &\to \sqrt{1-\alpha_I{}^2}\,Y^{\dagger}_{IK}|\Phi_0\rangle \\ &\quad +\alpha_I a^{\dagger}_{\pi'}|\Phi_0\rangle\,, \\ a_{\pi}{}^{\dagger}|\Phi_0\rangle &\to \sqrt{1-\alpha_j{}^2}\,a_{\pi}{}^{\dagger}|\Phi_0\rangle \\ &\quad +\alpha_j Y^{\dagger}_{jm_{\pi}}|\Phi_0\rangle \end{aligned}\right\} \tag{4·2}$$

with $I=j_{p'}=(j-1)$ or $(j-2)$, where we have considered only

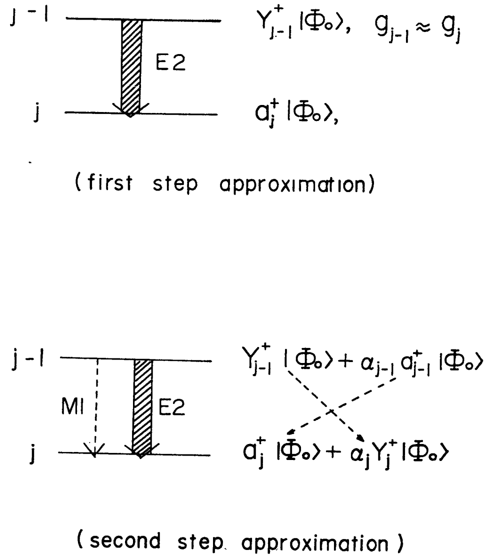
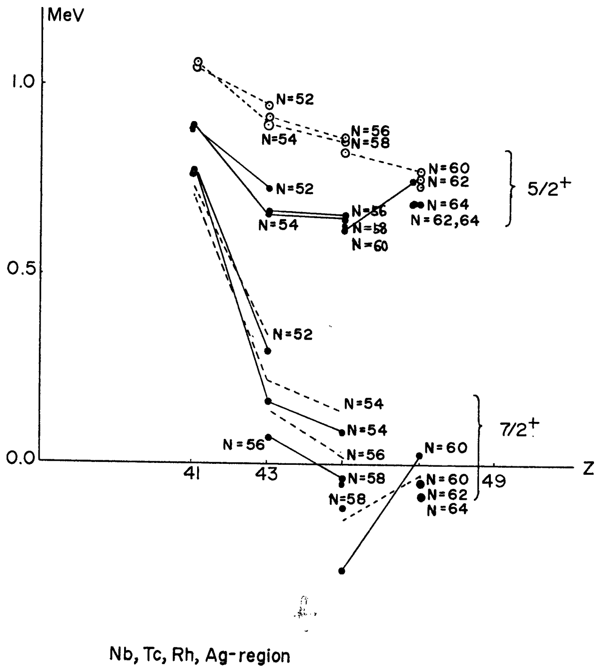


Fig. 4. Schematic representation of the basis states and of the coupling effects, which we consider in this step of approximations.

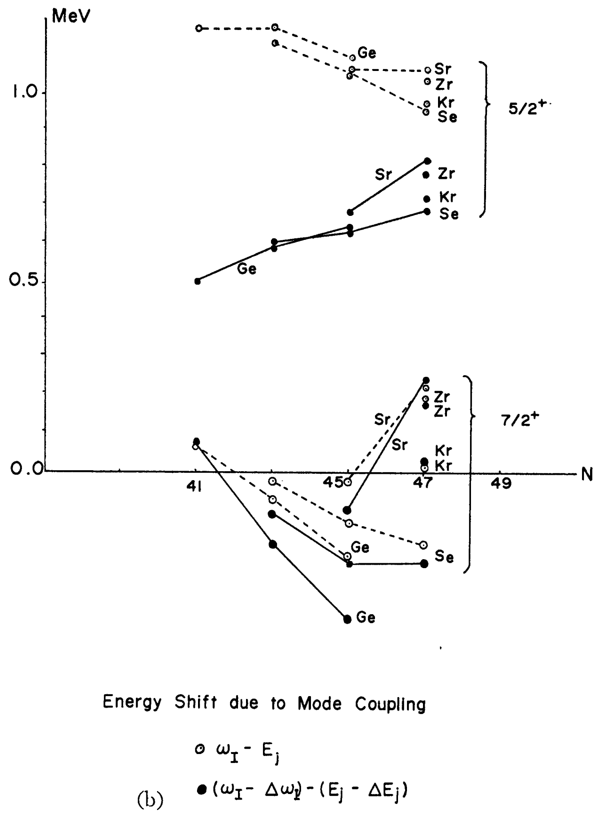


(a)

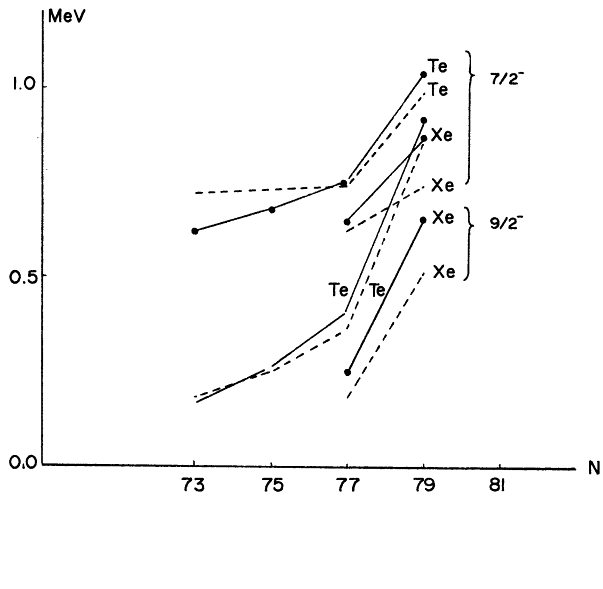
Fig. 5.

(Figure caption is printed below on the next page.)

the lowest (collective) solution for the dressed 3QP modes and omitted the suffix n. The energy shifts in this approximation due to the coupling effects are shown in Fig. 5 and the mixing amplitudes α are presented in Tables III and V. There are three points to be noticed: 1) The mixing amplitudes α_j of dressed 3QP modes for the 1QP states are, generally, about half as large as those given by phonon-quasi-particle-coupling calculations of Kisslinger and Sorensen.[5)] Correspondingly, we have obtained much reduced energy shifts of the 1QP states. This is because the effective coupling constant $\bar{\chi}_I$ with $I=j$ should become about half as large as that of the conventional phonon-quasi-particle coupling theory, if we take into account 3QP correlations accurately and remove the spurious states due to the nucleon number nonconservation. The characteristic de-



(b)



Te-Xe region

(c)

Fig. 5. Energy shifts due to the coupling effects of the dressed 3QP modes with the 1QP modes. The energies of ACS in the absence of the coupling effects are given by white circles and are connected by broken lines, while the energies of the ACS in the presence of the coupling effects are given by black circles and are connected by solid lines. All energies are presented in relation to those of 1QP states with spin j, in each approximation.

pendence of α_j on the $(u_p^2-v_p^2)$ factor is, however, the same as that of Kisslinger and Sorensen.[5] 2) The mixing amplitudes α_{j-1} are generally the small as expected and the assumption of the purity of the ACS with spin $(j-1)$ as the dressed 3QP modes is not violated by the coupling effects. 3) The mixing amplitudes α_{j-2} are rather large and, correspondingly, notable energy shifts due to the coupling effects are obtained. This is because of the large matrix-elements $\langle p'\|r^2Y_2\|p\rangle$ with

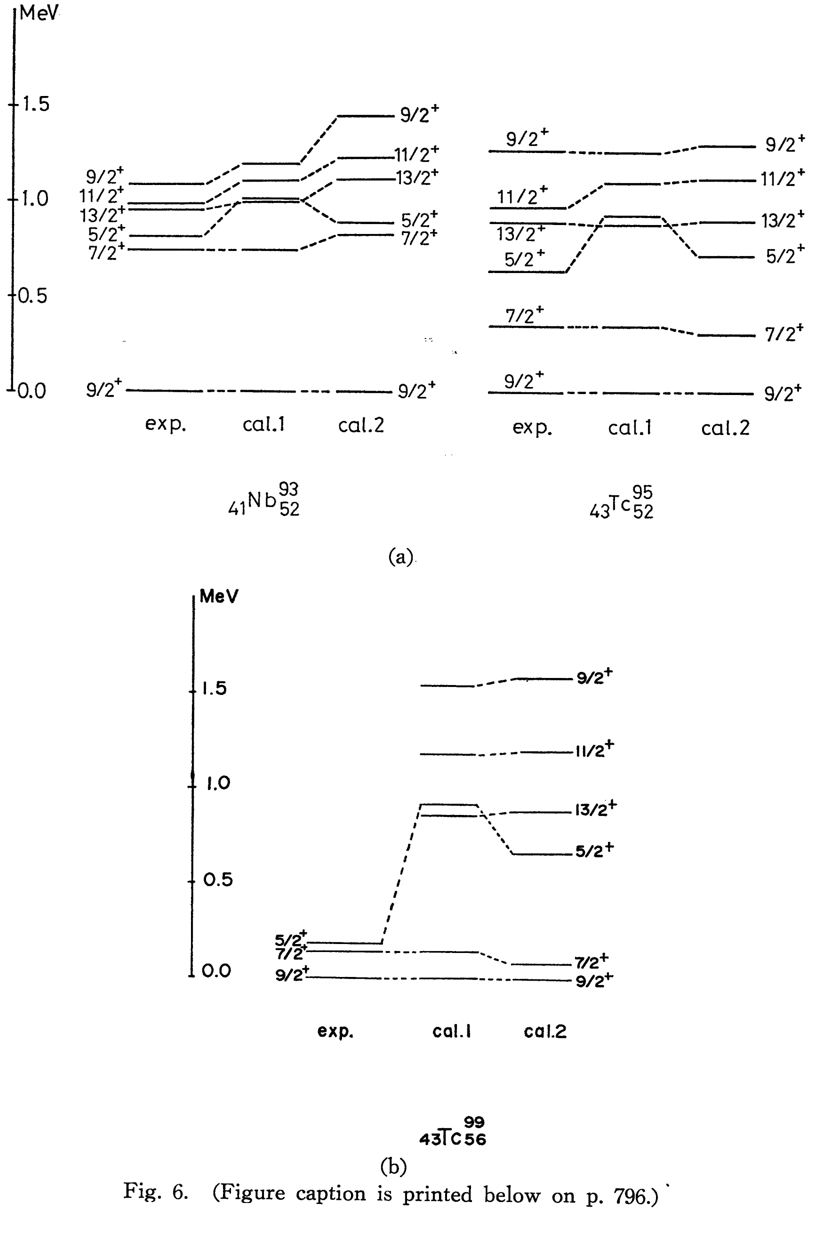
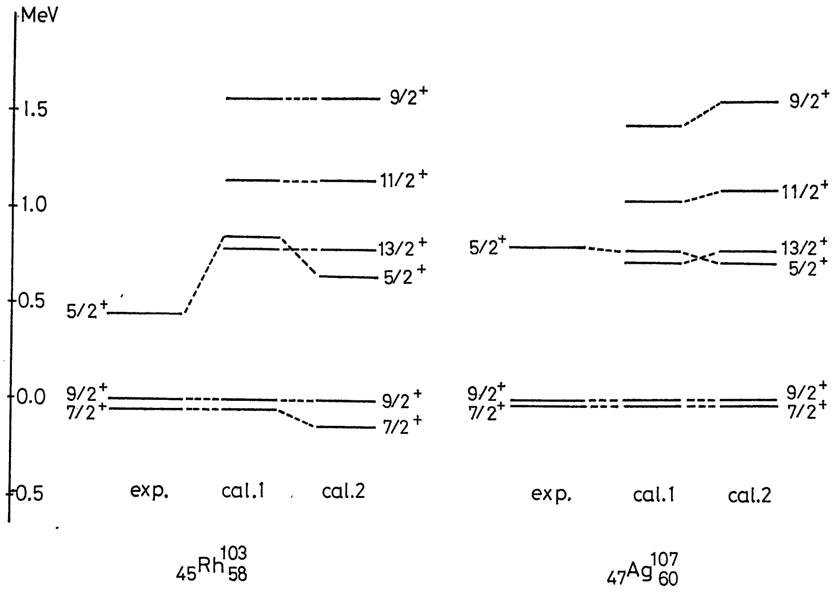
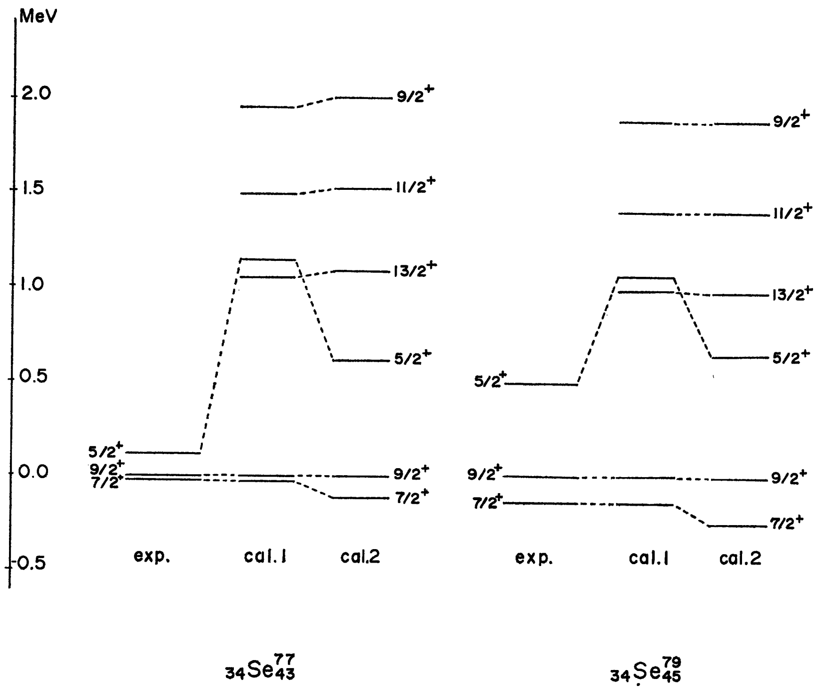


Fig. 6. (Figure caption is printed below on p. 796.)

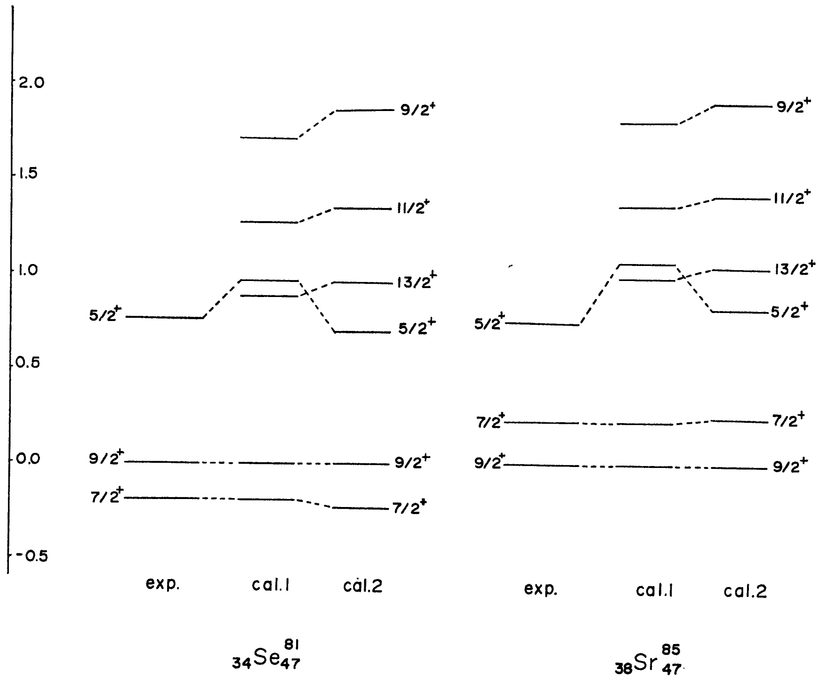


(c)

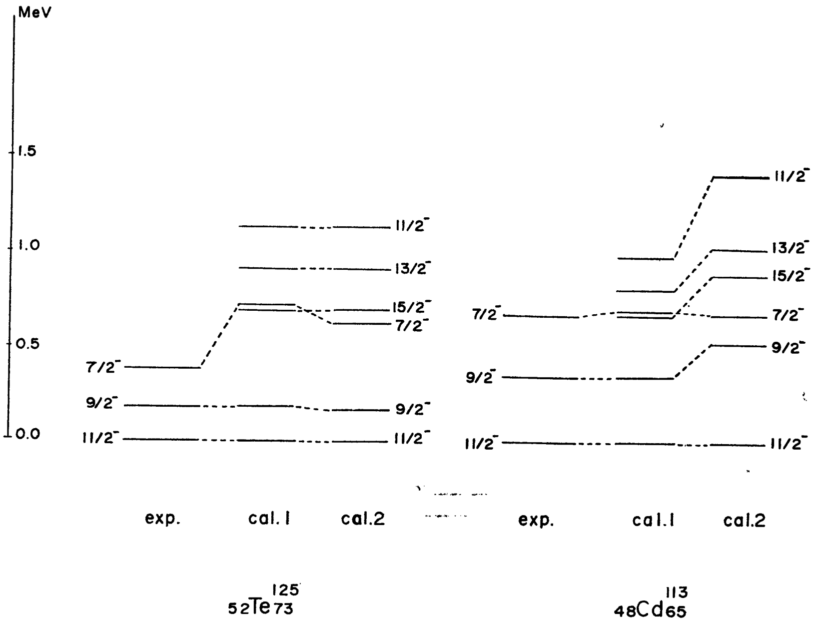


(d)

Fig. 6. (Figure caption is printed below on the next page.)



(e)



(f)

Fig. 6. Comparison between the experimental energy levels and the theoretical calculations for both cases, i.e., without the coupling effects (cal. 1) and with including the coupling effects (cal. 2). The energies are presented in relation to the 1QP states with spin j. Only the calculated results on the lowest-lying collective states in each spin are written in the figure. For reference to the experimental data, see the caption of Fig. 1.

$|j_{p'}-j_p|=2$ (when compared with the case $|j_{p'}-j_p|=1$) in the effective coupling constant $\bar{\chi}_I$. In particular, remarkable energy-lowering of the $(j-2)$ states has been obtained in numerical calculations for the nuclei around neutron number $N=41$ (the beginning of the $g^+_{9/2}$ shell). This trend comes from the increase of the $(u_pu_{p'}-v_pv_{p'})$ factor toward the beginning of the shell, just corresponding to the experimental trend. At the beginning of the unique parity shell, however, the degree of energy-lowerings is not so large as observed in experiments, presumably because of the present limited truncation of the quasi-particle NTD subspace. Thus, in contrast to the ACS with spin $(j-1)$, rather strong mixing effects operating between different excitation modes may become important for the ACS with spin $(j-2)$, as was asserted by Ikegami and Sano.[6]

In Fig. 6, we present some calculated results on energy-spectra and compare them with experimental data. In all calculations, including those of electromagnetic properties, we have adopted the following procedure: We use the same values of pairing-force strength and of single-particle energies as those adopted in the work of Kisslinger and Sorensen,[5] and also make the same truncation of shell-model space as they have made. With these parameters, firstly we fix the strength of the quadrupole force χ so as to reproduce the experimental value of ω_I' with $I=(j-1)$ and, secondly, calculate the energies ω_I' with $I\neq(j-1)$ by the value of χ thus obtained in each nucleus. Consequently we obtain calculated energy spectra denoted by cal. 1) in which the energies of the states with $I=(j-1)$ are fitted to the experimental values and those with $I\neq(j-1)$ are not fitted. In the next step, we include the coupling effects with thus determined

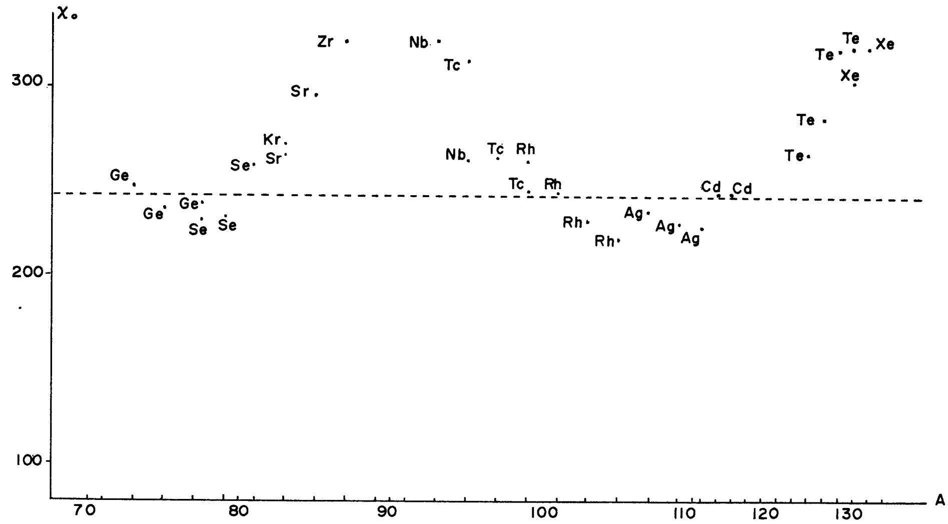


Fig. 7. Values of parameter χ_0 chosen so as to bring the energies of the ACS with spin $(j-1)$ into agreement with the experimental data. The parameter χ_0 is related to the quadrupole force strength χ through $\chi=\chi_0 b^{-4}A^{-5/3}$, where b^2 is the harmonic-oscillator range parameter and is taken to be $1.2A^{1/3}$. The broken line shows the value expected by the classical arguments.

parameters and then obtain energy spectra denoted by cal. 2). In this step, the mixing amplitudes α are estimated by taking the single-particle energies of level p' from the work of Uher and Sorensen[7] and also by approximating the occupation probability of level p' to be zero, i.e., $v_{p'}\approx 0$. In Fig. 7 are shown the values χ which reproduce the experimental values of ω_I' with $I=(j-1)$. It is seen that the values $\chi_0(\equiv\chi\cdot b^4A^{5/3})$ approach the value $\chi_0=242\,(\mathrm{MeV})$ expected by the conventional arguments in the regions far from closed shells.[8] On the other hand, they become large in the neighbourhood of closed shells. This may be a natural consequence of the one-major-shell truncation in constructing the dressed 3QP modes. It should be noted that the values of χ_0 thus determined are the same as used in fitting the 2^+ phonon states in even nuclei within a few percent. This means that any unreasonable parameters have not been used in the course of the calculations.

With the energy-eigenvalues mentioned above, the values of $B(E2)$ and also of the other electromagnetic quantities will be evaluated in the followings.

§ 5. Electromagnetic properties ——first order approximation——

In this section, we consider the electromagnetic properties of the ACS without including coupling effects, that is, by regarding the ACS as the pure dressed 3QP modes defined in § 2.

5-1 *E2-transitions between the ACS and the 1QP states*

As was already stressed in the introduction, the collective structure of the ACS with spin $I=(j-1)$ has been recognized through the recent observations of the strongly enhanced $E2$-transitions between the ACS and the 1QP states with spin j. As usual, electric quadrupole operator is given by

$$\begin{aligned}\hat{Q}_\mu &= \hat{Q}_\mu^{(0)} + \hat{Q}_\mu^{(1)}\\ &= \sum_{ab} e_\tau Q(ab)\{A_{2\mu}^\dagger(ab) + \tilde{A}_{2\mu}(ab)\}\\ &\quad + 2\sum_{ab} e_\tau q(ab)\eta(ab)B_{2\mu}^\dagger(ab)\ ,\end{aligned} \tag{5·1}$$

where e_τ $(\tau=1/2, -1/2)$ are effective charges for neutrons and protons respectively;

$$\begin{aligned} e_{1/2} &\equiv e_n = e\cdot\alpha\ ,\\ e_{\overline{1/2}} &\equiv e_p = e(1+\alpha)\end{aligned} \tag{5·2}$$

with the polarization charge $e\cdot\alpha$. Following the procedure given in § 3, we get the reduced $E2$-transition probabilities under consideration:

$$B(E2;I\to j_p) = \frac{1}{2I+1}|\langle\Phi_p^{(1)}\|\hat{Q}\|\Phi_{nI}^{(3)}\rangle|^2$$

$$=|e_{\tau(p)}Q(pp)\sqrt{C_I}\left\{\psi_{nI}(p^3)+\sqrt{\frac{1}{3}}\varphi_{nI}(p^3)\right\}$$
$$+\sum_{(ab)}e_{\tau(ab)}Q(ab)N(ab)\{\psi_{nI}(p;ab)+\varphi_{nI}(p;ab)\}|^2 . \tag{5·3}$$

Inserting the solutions for the amplitudes (2·33) into (5·3), we finally obtain

$$B(E2;I\to j_p)=M_{I\omega}^2|e_{\tau(p)}S_p+e_pS_c(\text{proton})+e_nS_c(\text{neutron})|^2 , \tag{5·4}$$

where $M_{I\omega}$ and $S_{p(c)}$ are given by (2·34) and (2·31), respectively. It is interesting to notice that formally Eq. (5·3) has a structure similar to the corresponding expression obtained by the conventional RPA in even-even nuclei, in spite of the essential difference due to the incorporation of the 3QP correlations. For the $E2$-transitions between the ACS and 1QP states, we may, therefore, expect the well-known enhancement associated with the structure of Eq. (5·3). In particular, we may have the usual relation; the lower the excitation energy of the ACS, the larger the $B(E2)$ values. Such an enhancement, caused by the collective ground-state correlations due to the QQ-force is a direct and natural consequence of the present theory, contrary to Kisslinger's (Tamm-Dancoff) 3QP "intruder" states.[9] As are shown in Table I, the calculated values of $B(E2;I\to j_p)$ are of the same order of magnitude as those of the phonon transitions in the neighbouring even-even nuclei. As a consequence of the 3QP correlations, the $B(E2;I\to j_p)$ with $I=(j_p-1)$ are stronger than the other $I\neq(j_p-1)$ transitions (see Table V). Although the present accumulation of experimental data on these transitions is not sufficient to allow us systematic comparison, we can see that the calculated results are in good agreement with the experimental values known at the present time, with the value $\alpha\approx0.5$. The current rapid growth in the measurement of these transitions may be expected to elucidate further the many interesting systematic trends.

5-2 *M1-transitions between the ACS and the 1QP states*

The magnetic dipole operator in the quasi-particle representation is given by

$$\hat{\mu}_M=\hat{\mu}_M^{(0)}+\hat{\mu}_M^{(1)}$$
$$=\frac{1}{\sqrt{2}}\sum_{ab}m(ab)(u_av_b-v_au_b)\{A_{1M}^\dagger(ab)-\tilde{A}_{1M}(ab)\}$$
$$+\sum_{ab}M(ab)B_{1M}^\dagger(ab) , \tag{5·5}$$

where

$$M(ab)\equiv m(ab)(u_au_b+v_av_b) ,$$
$$m(ab)\equiv\frac{1}{\sqrt{3}}\langle a\|\boldsymbol{\mu}\|b\rangle \tag{5·6}$$

Table I. $B(E2)$ values for transitions from the ACS with spin $(j-1)$ to the 1QP states with spin j. The second column labeled ω'_{j-1} lists the excitation energies of the $(j-1)$ states in unit of MeV. The third column labeled $B(E2)^{1)}$ lists the calculated values of $B(E2; j-1\to j)$ in the absence of the coupling effects and the fourth column labeled $B(E2)^{2)}$ lists the calculated values of $B(E2; j-1\to j)$ in the presence of the coupling effects, both in unit of $e^2\times 10^{-50}$ cm^4 for polarization charge $\alpha=0.5$ and are compared with experimental data $B(E2)^{\exp}$.

Isotope	ω'_{j-1}	$B(E2)^{1)}$	$B(E2)^{2)}$	$B(E2)^{\exp}$
$_{41}Nb^{93}$	0.74	2.4	2.3	$2.25\pm0.16^{a)}$
Nb^{95}	0.72	3.5	3.3	
$_{43}Tc^{95}$	0.34	5.2	5.2	
Tc^{97}	0.22	8.1	8.0	
Tc^{99}	0.14	11.4	11.2	$13.5\pm1.5^{b)}$
$_{45}Rh^{99}$	0.14	9.3	9.2	
Rh^{101}	0.02	14.4	14.2	
Rh^{103}	−0.05	21.0	20.6	$9.5^{c)}$
Rh^{105}	−0.15	39.2	37.7	$>31^{c)}$
$_{47}Ag^{107}$	−0.03	20.1	19.1	
Ag^{109}	−0.04	23.6	22.3	$27.4^{c)}$
Ag^{111}	−0.07	29.2	27.5	$20.1^{c)}$
$_{32}Ge^{73}$	0.07	19.8	17.4	$9.1\pm0.9^{d)}$
Ge^{75}	−0.07	19.0	18.3	
Ge^{77}	−0.22	31.6	30.3	
$_{34}Se^{77}$	−0.02	18.6	18.0	
Se^{79}	−0.13	23.1	22.5	
Se^{81}	−0.19	37.1	34.3	$5.8\pm1.3^{e)}$
$_{36}Kr^{83}$	0.01	13.5	12.9	$2.6\pm1.5^{f)}$
$_{38}Sr^{83}$	−0.02	11.1	10.9	
Sr^{85}	0.23	6.0	5.9	
$_{40}Zr^{87}$	0.20	5.2	5.1	
$_{48}Cd^{113}$	0.34	9.8	8.3	
Cd^{115}	0.33	9.4	8.7	
$_{52}Te^{125}$	0.18	10.7	10.6	$11.5\pm0.5^{g)}$
Te^{127}	0.25	8.3	8.1	
Te^{129}	0.36	6.2	5.8	
Te^{131}	0.85	2.8	2.6	
$_{54}Xe^{131}$	0.18	15.8	14.6	
Xe^{133}	0.51	7.5	6.6	

a) Ref. 10) b) Ref. 13) c) Ref. 15) d) Ref. 17) e) Ref. 18) f) Ref. 20) g) Ref. 25)

and $\boldsymbol{\mu}=g_l\boldsymbol{l}+g_s\boldsymbol{s}$, in unit of nuclear magneton $e\hbar/2Mc$. The reduced $M1$-transition probabilities from the ACS to 1QP states are expressed as

$$B(M1; I\to j_p)=\frac{3}{4\pi}\cdot\frac{1}{2I+1}|\langle\Phi_p^{(1)}\|\hat{\mu}\|\Phi_{nI}^{(3)}\rangle|^2. \quad (5\cdot7)$$

Since the dressed 3QP modes $Y^{\dagger}_{nIK}$ in the $P+QQ$ force model contain no components of the type of $\hat{\mu}^{(0)}_{M}\cdot a^{\dagger}_{\pi}$, it is obvious that the matrix elements in (5·7) should vanish. We thus obtain an important property of the ACS:

$$B(M1;I\to j_p)=0\,, \tag{5·8}$$

that is, in the first-order approximation in which the ACS with $I=(j-1)$ are regarded as pure dressed 3QP modes, the $M1$-transition between the ACS and 1QP states with spin j is forbidden. The attenuation of the $M1$-transitions is indeed observed in experiments and is a sensitive criterion for the purity of the ACS as the dressed 3QP states. In some nuclei, however, it is only weakly retarded. In order to explain the $M1$-transitions, therefore, we must consider the coupling effects of the dressed 3QP modes with 1QP modes. The effects will be considered in the next section.

5-3 *Magnetic dipole moments of the ACS*

The magnetic dipole moments of the ACS are measured by

$$\mu=\langle\Phi^{(3)}_{nIK}|\hat{\mu}_0|\Phi^{(3)}_{nIK}\rangle \qquad \text{with} \quad K=I\,. \tag{5·9}$$

The procedure given in § 3 leads to the following expression for the magnetic moments under consideration:

$$\mu=g_I\cdot I\,, \tag{5·10}$$

with

$$g_I=g^{(0)}_p+\frac{I(I+1)+j_p(j_p+1)-6}{2I(I+1)}g^{(1)}_p+\frac{I(I+1)+6-j_p(j_p+1)}{2I(I+1)}g_c\,. \tag{5·11}$$

The partial g-factors in this equation are given by

$$g^{(0)}_p=g_p\{\psi^2_{nI}(p^3)-\varphi^2_{nI}(p^3)\}\,, \tag{5·12}$$

$$g^{(1)}_p=g_p\sum_{bc}\{\psi'_{nI}(p;bc)^2-\varphi'_{nI}(p;bc)^2\} \tag{5·13}$$

and

$$g_c=\sqrt{10}\sum_{abc}M(bc)\begin{Bmatrix}2 & j_c & j_a\\ j_b & 2 & 1\end{Bmatrix}$$
$$\times[\psi'_{nI}(p;ca)\psi'_{nI}(p;ab)-\varphi'_{nI}(p;ca)\varphi'_{nI}(p;ab)]\,, \tag{5·14}$$

respectively. Here g_p indicates the g-factor of a single-particle at the high-spin, unique parity level p. The meaning of each term in Eq. (5·11) is clear. The first term, $g^{(0)}_p$, comes from the quasi-particles at the unique parity level p. If we restrict our shell-model space only within the unique parity level p, which is being filled, $g^{(0)}_p$ becomes equal to g_p (because in this case $\psi^2_{nI}(p^3)-\varphi^2_{nI}(p^3)=1$). The second and third terms are of the same form as in the Lande formula; the second term comes from the odd-particle at level p and the third term comes from the quasi-particles in the core, respectively. It is important to notice that the contributions from the quasi-particles in the core (the amplitude of which is

Table II. Gyromagnetic ratio g_{j-1} for the ACS with spin $(j-1)$. The values are written in unit of nuclear magneton $e\hbar/2Mc$. The calculations were done using the effective spin g factor $g_s^{eff}=0.55g_s$, therefore, the g factors of the 1QP states with spin j were assumed as

$1g_{9/2}$ proton state: $g_j=1.23$, $1g_{9/2}$ neutron state: $g_j=-0.23$,
$1h_{11/2}$ neutron state: $g_j=-0.19$.

The column labeled $g_{j-1}^{1)}$ lists the calculated values in the absence of the coupling effects, while the column labeled g_{j-1}^{c1} lists the calculated values by replacing the values g_c defined in (5·14) with $g_c=Z/A$. The column labeled $g_{j-1}^{2)}$ lists the calculated values in the presence of the coupling effects and are compared with the experimental data g_{j-1}^{exp}.

Isotope	g_{j-1}^{c1}	$g_{j-1}^{1)}$	$g_{j-1}^{2)}$	g_{j-1}^{exp}	g_j^{exp}
$_{41}Nb^{93}$	1.27	1.32	1.31		1.37[a]
Nb^{95}	1.26	1.31	1.30		
$_{43}Tc^{95}$	1.22	1.27	1.26		
Tc^{97}	1.22	1.27	1.25		
Tc^{99}	1.21	1.25	1.23	0.75±0.26[a]	1.26[a]
$_{45}Rh^{99}$	1.21	1.25	1.24		
Rh^{101}	1.19	1.23	1.22		
Rh^{103}	1.18	1.22	1.20		
Rh^{105}	1.14	1.18	1.16		
$_{47}Ag^{107}$	1.18	1.22	1.21		
Ag^{109}	1.18	1.22	1.21	1.22±0.037[b]	
Ag^{111}	1.17	1.21	1.20		
$_{32}Ge^{73}$	−0.28	−0.25	−0.22		−0.20[a]
Ge^{75}	−0.27	−0.22	−0.20		
Ge^{77}	−0.26	−0.17	−0.15		
$_{34}Se^{77}$	−0.28	−0.23	−0.21		
Se^{79}	−0.27	−0.21	−0.19		
Se^{81}	−0.26	−0.16	−0.14		
$_{36}Kr^{83}$	−0.28	−0.24	−0.23	{−0.268±0.001[c] {−0.271±0.016[g]	−0.215[c]
$_{38}Sr^{83}$	−0.28	−0.23	−0.22		
Sr^{85}	−0.28	−0.25	−0.24		
$_{40}Zr^{87}$	−0.28	−0.24	−0.24		
$_{48}Cd^{113}$	−0.25	−0.26	−0.25		−0.20[a]
Cd^{115}	−0.25	−0.25	−0.25		
$_{52}Te^{125}$	−0.24	−0.21	−0.21	−0.204±0.007[e]	−0.17±0.03[d]
Te^{127}	−0.24	−0.22	−0.22		−0.17±0.01[f]
Te^{129}	−0.24	−0.23	−0.22		−0.21±0.01[f]
Te^{131}	−0.24	−0.25	−0.25		
$_{54}Xe^{131}$	−0.24	−0.22	−0.22		
Xe^{133}	−0.24	−0.25	−0.25		

a) Ref. 12) b) Ref. 16) c) Ref. 19) d) Ref. 21) e) Ref. 22) f) Ref. 26) g) Ref. 20) represented by $\psi'_{nI}(p;bc)$ and $\varphi'_{nI}(p;bc)$) are accompanied with the kinematical factor, which becomes small especially for $I=(j-1)$. This means that, in some situation, the quasi-particles in the core give rise to only small effects to the total-g-factor and the magnetic moments of the ACS are determined mainly from the quasi-particles at the unique parity level p (i.e., by the first and the second term). Therefore, the observed value of g_I nearly equal to g_p does not necessarily means the simple $(j^n)_I$ configuration. Even if the wave functions under consideration be far from those of the simple $(j^n)_I$ configurations, the calculated magnetic moments give nearly the same values in experiments. In Table II are shown the calculated g-factors of the ACS. In this calculation, we have used the effective spin g-factor $g_s^{\text{eff}}=0.55\ g_s$. Of course, better values should be obtained by taking the empirical values for g_p directly. However, our present aim is to show qualitative equality between g_{j-1} and g_p, Table II should read to show to what extent the values of g_{j-1} deviate from the values of g_p. Also the values of g_{j-1} calculated using the classical approximation on g_c, $g_c=Z/A$, are presented for the same purpose.

§ 6. Electromagnetic properties ——including coupling effects——

In this section, we explicitly take into account the interplay between dressed 3QP modes and 1QP modes and consider various corrections to the electromagnetic properties of the ACS evaluated in the preceding section.

6-1 *M1-transitions between the ACS and the 1QP states*

The interplay of dressed 3QP modes with 1QP modes originates from the third term of the effective Hamiltonian (3·12) in the quasi-particle NTD subspace under consideration. As a result of the coupling effects, the wave functions should be changed into the form (4·2) and, therefore, we can expect that the $M1$-transition may take place through the part $\mu^{(1)}$ in the $M1$-operator, as shown in Fig. 4. Including the coupling effects, the reduced $M1$-transition probabilities are now given as

$$B(M1;j-1\to j)=\frac{3}{4\pi}\cdot\frac{1}{2I+1}|\alpha_{j-1}\sqrt{1-\alpha_j^2}\langle\Phi_p^{(1)}\|\hat{\mu}\|\Phi_{p'}^{(1)}\rangle + \alpha_j\sqrt{1-\alpha_{j-1}^2}\langle\Phi_{I'}^{(3)}\|\hat{\mu}\|\Phi_I^{(3)}\rangle|^2, \tag{6·1}$$

where

$$\langle\Phi_p^{(1)}\|\hat{\mu}\|\Phi_{p'}^{(1)}\rangle=\sqrt{3}M(pp') \tag{6·2}$$

and

$$\langle\Phi_{I'}^{(3)}\|\hat{\mu}\|\Phi_I^{(3)}\rangle=\sqrt{3(2I+1)(2I'+1)}$$
$$\times\left[M(pp)\begin{Bmatrix} I & j_p & 2 \\ j_p & I' & 1 \end{Bmatrix}\cdot\sum_{bc}\{\psi'_{I'}(p;bc)\psi'_I(p;bc)\right.$$

$$-\varphi'_{I'}(p;bc)\varphi'_I(p;bc)\} + 10\begin{Bmatrix}2 & I' & j_p\\ I & 2 & 1\end{Bmatrix}\cdot\sum_{abc} M(bc)\begin{Bmatrix}2 & j_b & j_a\\ j_c & 2 & 1\end{Bmatrix}$$

$$\times\left[\psi'_{I'}(p;ca)\psi'_I(p;ab) - \varphi'_{I'}(p;ca)\varphi'_I(p;ab)\right]\Big], \qquad (6\cdot3)$$

Table III. $B(M1)$ values for the transitions from the ACS with spin $(j-1)$ to the 1QP states with spin j. The values are written in unit of $(e\hbar/2Mc)^2$. In this calculations, the single-particle reduced matrix elements were calculated using $g_s^{\mathrm{eff}}=0.55g_s$. Columns 2 and 3 list the mixing amplitudes α_j and α_{j-1} defined by (4·2), respectively. Columns 4 and 5 list the values of the first and second terms in (6·1), respectively. The column labeled by $B(M1)^{\mathrm{cal}}$ lists the calculated values of $B(M1;\ j-1\to j)$ and are compared with the experimental data $B(M1)^{\mathrm{exp}}$.

Isotope	α_j	α_{j-1}	M_{11}	M_{33}	$B(M1)^{\mathrm{cal}}$	$B(M1)^{\mathrm{exp}}$
$_{41}$Nb^{93}	−0.29	−0.18	0.68	1.86	0.193	
Nb^{95}	−0.29	−0.17	0.66	1.76	0.175	
$_{43}$Tc^{95}	−0.12	−0.19	0.66	0.62	0.049	
Tc^{97}	−0.13	−0.19	0.65	0.69	0.053	
Tc^{99}	−0.12	−0.19	0.67	0.55	0.044	{0.076±0.009 [a] {0.0944±0.001 [b]
$_{45}$Rh^{99}	0.03	−0.18	0.54	−0.15	0.005	
Rh^{101}	0.03	−0.19	0.57	−0.13	0.006	
Rh^{103}	0.03	−0.20	0.59	−0.10	0.007	0.093±0.0006 [b]
Rh^{105}	0.04	−0.23	0.67	−0.13	0.009	$<$0.031 [b]
$_{47}$Ag^{107}	0.19	−0.18	0.40	−0.64	0.002	0.0419±0.004 [b]
Ag^{109}	0.20	−0.17	0.38	−0.70	0.003	0.038 [b]
Ag^{111}	0.20	−0.17	0.38	−0.82	0.006	0.069 [b]
$_{32}$Ge^{73}	−0.30	−0.22	−0.84	−1.64	0.182	
Ge^{75}	−0.11	−0.20	−0.70	−0.51	0.043	
Ge^{77}	0.04	−0.22	−0.66	0.22	0.006	
$_{34}$Se^{77}	−0.11	−0.19	−0.65	−0.51	0.040	
Se^{79}	0.04	−0.19	−0.55	0.19	0.004	
Se^{81}	0.21	−0.20	−0.46	1.20	0.016	
$_{36}$Kr^{83}	0.19	−0.14	−0.32	0.71	0.005	0.0204±0.005 [c]
$_{38}$Sr^{83}	0.03	−0.15	−0.46	0.08	0.004	
Sr^{85}	0.15	−0.12	−0.27	0.37	0.0003	
$_{40}$Zr^{87}	0.10	−0.12	−0.27	0.36	0.0002	
$_{48}$Cd^{113}	−0.39	−0.09	−0.36	−2.17	0.153	
Cd^{115}	−0.28	−0.09	−0.35	−1.51	0.082	
$_{52}$Te^{125}	0.03	−0.08	−0.25	0.11	0.0005	0.0065±0.0003 [d]
Te^{127}	0.13	−0.07	−0.19	0.45	0.002	
Te^{129}	0.24	−0.06	−0.13	0.84	0.012	
Te^{131}	0.25	−0.03	−0.06	1.05	0.023	
$_{54}$Xe^{131}	0.27	−0.06	−0.15	1.13	0.023	
Xe^{133}	0.36	−0.04	−0.08	1.53	0.050	

a) Ref. 14) b) Ref. 15) c) Ref. 20) d) Ref. 25) with $I=j_{p'}=(j-1)$ and $I'=j$. The first term in Eq. (6·1) represents the contribution due to the admixture of the 1QP modes with spin $j_{p'}=j_p-1$ (from the next upper major shell) to the ACS with spin $I=j_p-1$. The second term comes from the admixture of the dressed 3QP mode with spin $I'=j_p$ to the 1QP state with spin j_p. Because the second term contains the $(u_p^2-v_p^2)$ factor through the mixing amplitude α_j, the value depends quite sensitively on the nucleon-occupation probability of the unique-parity level p. The mixing amplitude α_j in the second term in Eq. (6·1) becomes large as one moves away from the special physical situation (for the appearance of the ACS) mentioned before and, furthermore, changes its sign on both sides of the half-shell, while the first term, in (6·1) preserves its sign through the whole range. As a result, we can expect relatively large $M1$-transition probabilities at the beginning of the shell, as is seen from Table III. In this calculation, we have not directly considered the spin-polarization effects and, instead, have used the effective spin g-factor $g_s^{\text{eff}}=0.55g_s$ for the single-particle matrix elements, because we are interested in the qualitative trends of the $M1$-transition systematics, rather than numerical agreement with the experimental value in each nucleus.

6-2 *Electric quadrupole moments of the 1QP states*

As a result of the coupling effects, the admixtures of dressed 3QP modes to 1QP states with spin j give rise to important contributions to the electric quadrupole moments of 1QP states. The corresponding effects have been extensively studied in the framework of the quasi-particle-phonon-coupling theory.[5] As was pointed out in § 4, however, the mixing amplitudes α_j of the dressed 3QP modes should become about half as large as those of the conventional theory. The quadrupole moments are given by

$$Q_j=\sqrt{\frac{16\pi}{5}\cdot\frac{j(2j-1)}{(j+1)(2j+1)(2j+3)}}\,[(1-\alpha_j^2)\langle\Phi_p^{(1)}\|\hat{Q}\|\Phi_p^{(1)}\rangle$$
$$+2\alpha_j\sqrt{1-\alpha_j^2}\langle\Phi_j^{(3)}\|\hat{Q}\|\Phi_p^{(1)}\rangle+\alpha_j^2\langle\Phi_j^{(3)}\|\hat{Q}\|\Phi_j^{(3)}\rangle]\,. \qquad (6\cdot4)$$

The second term in Eq. (6·4), which is related to the $E2$-transition probability given in § 5-1 as

$$\langle\Phi_j^{(3)}\|\hat{Q}\|\Phi_p^{(1)}\rangle=\sqrt{(2j+1)\cdot B(E2;j\to j)}\,, \qquad (6\cdot5)$$

gives the main contributions to the modified quadrupole moments of the 1QP states and the effects of the third term are negligibly small. Therefore, only the values of the second term are presented in Table IV. The effects are, in general, about half of those of Kisslinger and Sorensen.[5]

6-3 *Some comments*

1) In the same way, corrections to the $B(E2)$ and the g factors (evaluated in § 5) due to the coupling effects can be calculated and are presented in Tables

Table IV. Quadrupole moments of the 1QP states. The values are written in unit of $e\times10^{-24}$ cm^2. Columns 2 and 3 lists the values of quasi-particle moments and the contributions from the admixed dressed 3QP states, respectively. The fourth column labeled Q_{tot} list the calculated moments for polarization charge $\alpha=0.5$ in unit of $e\times10^{-24}\,cm^2$ and are compared with the experimental data Q_{exp}.

Isotope	$Q_{s.p}$	Q_{coll}	Q_{tot}	Q_{exp}
$_{41}Nb^{93}$	−0.16	−0.08	−0.24	−0.2[a]
Nb^{95}	−0.16	−0.11	−0.27	
$_{43}Tc^{95}$	−0.08	−0.03	−0.11	
Tc^{97}	−0.08	−0.05	−0.13	
Tc^{99}	−0.08	−0.06	−0.13	+0.34[a]
$_{45}Rh^{99}$	0.02	0.01	0.03	
Rh^{101}	0.02	0.01	0.04	
Rh^{103}	0.03	0.02	0.04	
Rh^{105}	0.03	0.02	0.05	
$_{47}Ag^{107}$	0.13	0.10	0.22	
Ag^{109}	0.13	0.11	0.23	
Ag^{111}	0.13	0.12	0.25	
$_{32}Ge^{73}$	−0.04	−0.26	−0.31	−0.2[a]
Ge^{75}	−0.02	−0.08	−0.10	
Ge^{77}	0.01	0.03	0.04	
$_{34}Se^{77}$	−0.02	−0.10	−0.12	
Se^{79}	0.01	0.03	0.04	
Se^{81}	0.04	0.16	0.20	
$_{36}Kr^{83}$	0.04	0.14	0.18	+0.27[a]
$_{38}Sr^{83}$	0.01	0.02	0.03	
Sr^{85}	0.04	0.08	0.12	
$_{40}Zr^{87}$	0.04	0.05	0.09	
$_{48}Cd^{113}$	−0.06	−0.33	−0.39	−0.71[b]
Cd^{115}	−0.05	−0.23	−0.28	−0.55[b]
$_{52}Te^{125}$	0.01	0.03	0.03	
Te^{127}	0.03	0.10	0.13	
Te^{129}	0.06	0.17	0.23	
Te^{131}	0.08	0.16	0.24	
$_{54}Xe^{131}$	0.06	0.26	0.33	
Xe^{133}	0.08	0.33	0.41	

a) Ref. 12) b) Ref. 29)

I and II, respectively. In general, these corrections are not important as to change any qualitative conclusions given in § 5.

2) The admixture of the 1QP states from the next upper major shell in the ACS may be directly checked with the spectroscopic factors of one-nucleon-transfer reaction. For the (d,p) reaction leading to the ACS, it is easy to show that the spectroscopic factors are given by $(\alpha_I u_{p'})^2 \approx \alpha_I^2$ within the approximation

of the NTD method. Concerning the ACS with $I=(j-1)$, it is well known in experiments that the spectroscopic factors are very small and, therefore, are consistent with the theoretical predictions (see Table III). On the other hand, several states with anomalous spin $(j-2)$ have been observed in low-energy excitations by the (d, p) reaction (especially in Ge-Se region), with fragmented spectroscopic factors in each nucleus.[28] This experimental fact is consistent with the result of § 5, in which a rather strong coupling effect operating between the different types of excitation modes has been expected for the states with $I=(j-2)$.

3) In order to estimate the quandrupole moment of the ACS, we should extend our quasi-particle NTD subspace to include the dressed 5QP modes, because even a small mixing of such higher collective states is sufficient to produce a large quadrupole moment in a way similar to the case of quadrupole moment of the 1QP states, discussed in § 6-2.

4) In order to show various electromagnetic properties of the ACS with $I=(j-1)$ in comparison with the other $I\neq(j-1)$ states, we have summarized numerical results on some typical nuclei in Table VI. In our theory, no artificial division of the collective degree-of-freedom and the particle degree-of-feedom are done *a priori*, and the theory includes both shell-model like state and fully collective states in a unified way. Furthermore, since no optimum choice of the $P+QQ$ force parameters has been tried in this calculations, the Tables should read to show the qualitative predictions of the theory.

Table V. Energy shifts of the ACS with spin $(j-2)$ due to the coupling effects of the dressed 3QP modes with the 1QP modes in the next upper major shells. The second column lists the mixing amplitudes α_{j-2} and the third column lists the calculated values of the energy shifts $\Delta\omega_{j-2}$. The values of Uher and Sorensen[7] were used for the single-particle energies in the calculations.

	α_{j-2}	$\Delta\omega_{j-2}$
$_{41}Nb^{93}$	−0.31	0.25
Nb^{95}	−0.32	0.27
$_{43}Tc^{95}$	−0.31	0.23
Tc^{97}	−0.31	0.25
Tc^{99}	−0.32	0.28
$_{45}Rh^{99}$	−0.30	0.19
Rh^{101}	−0.30	0.20
Rh^{103}	−0.29	0.19
Rh^{105}	−0.28	0.19
$_{47}Ag^{107}$	−0.25	0.14
Ag^{109}	−0.24	0.13
Ag^{111}	−0.23	0.12
$_{32}Ge^{73}$	−0.50	0.88
Ge^{75}	−0.48	0.60
Ge^{77}	−0.47	0.45
$_{34}Se^{77}$	−0.47	0.56
Se^{79}	−0.46	0.42
Se^{81}	−0.46	0.34
$_{36}Kr^{83}$	−0.45	0.32
$_{38}Sr^{83}$	−0.44	0.38
Sr^{85}	−0.44	0.29
$_{40}Zr^{87}$	−0.43	0.27
$_{48}Cd^{113}$	−0.22	0.23
Cd^{115}	−0.20	0.19
$_{52}Te^{125}$	−0.16	0.10
Te^{127}	−0.14	0.07
Te^{129}	−0.12	0.05
Te^{131}	−0.08	0.02
$_{54}Xe^{131}$	−0.13	0.06
Xe^{133}	−0.10	0.03

§ 7. Concluding remarks

The proposed new point of view on structure of the ACS has been checked through the analysis of their electromagnetic properties by the use of the $P+QQ$ force. The effects of the couplings of the dressed 3QP modes with spin I and the 1QP modes with spin $j_{p'}=I$ in the next upper major shells were also discussed in order to investigate the 1QP components in the ACS. From the numerical results, it was shown that the various electromagnetic properties of the ACS with spin $(j-1)$, especially $B(E2)$, g factor and $B(M1)$,

Table VI. Some numerical examples of the electromagnetic properties of the ACS. The first three columns list the isotope, the observables and the spin of the state, respectively. Column 4 and 5 list the calculated values for two alternative approximations:

cal. 1) The electromagnetic properties in the absence of the coupling effects.

cal. 2) The electromagnetic properties in the presence of the coupling effects.

The units are $e^2\times10^{-50}\,\mathrm{cm}^4$ for $B(E2)$, $e\hbar/2MC$ for g factors and $(e\hbar/2Mc)^2$ for $B(M1)$. The polarization charge $\alpha=0.5$ and the effective spin g factor $g_s^{\mathrm{eff}}=0.55g_s$ were used. The procedure of the calculations are the same as in Table I∼V, except that the values of g_p are taken directly from the experiments on the 1QP states with spin j_p for the calculation of the g factors of the ACS; 1.37 (Nb^{93}), 1.26 (Tc^{99}), −0.22 (Kr^{83}) and −0.17 (Te^{125}). The spectroscopic factors for (d, p) reactions are calculated by the approximation $S_{j-1}\approx(\alpha_{j-1})^2$.

Isotope	Observable	Spin	cal. 1	cal. 2	exp.
$_{41}Nb^{93}$	$B(E2)$	$5/2^+\to9/2^+$	1.1	1.6	2.8±0.2[a]
		$7/2^+\to9/2^+$	2.4	2.3	2.25±0.16[a]
		$9/2'^+\to9/2^+$	0.4	0.1	0.219±0.026[a]
		$11/2^+\to9/2^+$	0.7	0.6	1.06±0.09[a]
		$13/2^+\to9/2^+$	1.1	1.1	1.76±0.12[a]
	g	$5/2^+$	2.16	2.11	
		$7/2^+$	1.47	1.45	
		$9/2'^+$	1.17	1.18	
		$11/2^+$	1.01	1.01	
		$13/2^+$	0.95	0.95	
	$B(M1)$	$7/2^+\to9/2^+$	0.0	0.193	
	S	$7/2^+$	0.0	0.03	
$_{43}Tc^{99}$	$B(E2)$	$5/2^+\to9/2^+$	3.2	4.0	4.5±0.5[b]
		$7/2^+\to9/2^+$	11.4	11.2	13.5±1.5[b]
		$9/2'^+\to9/2^+$	1.0	0.9	
		$11/2^+\to9/2^+$	2.1	2.1	
		$13/2^+\to9/2^+$	3.5	3.5	
	g	$5/2^+$	1.67	1.65	1.44±0.12[c]
		$7/2^+$	1.28	1.27	0.75±0.26[c]
		$9/2'^+$	1.10	1.10	
		$11/2^+$	1.05	1.05	
		$13/2^+$	1.05	1.05	
	$B(M1)$	$7/2^+\to9/2^+$	0.0	0.044	0.076±0.009[d]
	S	$7/2^+$	0.0	0.04	

a) Ref. 10) b) Ref. 13) c) Ref. 12) d) Ref. 14)

Table VI. (continued)

Isotope	Observable	Spin	cal. 1	cal. 2	exp.
$_{36}Kr^{83}$	$B(E2)$	$5/2^+ \to 9/2^+$	3.7	3.2	$\{5.8\pm1.3^{a)}$, $2.6\pm1.5^{b)}$
		$7/2^+ \to 9/2^+$	13.5	12.8	
		$9/2'^+ \to 9/2^+$	2.6	2.1	
		$11/2^+ \to 9/2^+$	3.0	2.9	
		$13/2^+ \to 9/2^+$	3.9	3.7	
	g	$5/2^+$	−0.46	−0.44	
		$7/2^+$	−0.22	−0.22	$-0.268\pm0.001^{c)}$
		$9/2'^+$	−0.14	−0.14	
		$11/2^+$	−0.10	−0.10	
		$13/2^+$	−0.09	−0.09	
	$B(M1)$	$7/2^+ \to 9/2^+$	0.0	0.005	$0.0204\pm0.005^{b)}$
	S	$7/2^+$	0.0	0.02	
$_{52}Te^{125}$	$B(E2)$	$7/2^- \to 11/2^-$	4.5	4.6	
		$9/2^- \to 11/2^-$	10.7	10.6	$11.5\pm0.5^{d)}$
		$11/2'^- \to 11/2^-$	2.5	2.5	
		$13/2^- \to 11/2^-$	3.5	3.5	
		$15/2^- \to 11/2^-$	4.6	4.6	
	g	$7/2^-$	−0.38	−0.38	
		$9/2^-$	−0.19	−0.19	$-0.204\pm0.007^{e)}$
		$11/2'^-$	−0.11	−0.11	
		$13/2^-$	−0.06	−0.06	
		$15/2^-$	−0.05	−0.05	
	$B(M1)$	$9/2^- \to 11/2^-$	0.0	0.0005	$0.0065\pm0.0003^{d)}$
	S	$9/2^-$	0.0	0.006	

a) Ref. 18) b) Ref. 20) c) Ref. 19) d) Ref. 25) e) Ref. 22)

can be explained in a unified manner within the framework of the proposed microscopic model.

We have been considering the ACS with spin $(j-1)$ as typical phenomena in which the introduced new elementary excitation modes (the dressed 3QP modes) manifest themselves as relatively pure eigenmodes, owing to the special situation in shell structure. The physical condition for the enhancement of the 3QP correlations, however, is not specific to the ACS. Rather, the existence of the odd quasi-particle near the Fermi surface (the chemical potential in the sense of BCS theory) is responsible for the enhancement of the 3QP correlations. Thus it is to be expected that the 3QP correlations may play significant roles in low-energy collective excitations of almost all spherical odd-mass nuclei. Together with the excitation-energy systematics made in the previous paper II, therefore, the present results on the analysis of the electromagnetic properties of the ACS strongly supports our essential assumption, (that is, the elementary excitation modes characterizing the low-lying states in spherical odd-mass nuclei are 1QP modes, dressed 3QP modes, dressed 5QP modes, etc.). A general formulation of the theory and its application will be made in a forthcoming paper.[4]

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Theory of Collective Excitations in Spherical Odd-Mass Nuclei. IV

——*Formulation in the General Many-j-Shell Model**)——

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The aim of this paper is to give a systematic formulation of microscopic description of the collective excitations in spherical odd-mass nuclei. The theory can be regarded as a natural extension of the conventional quasi-particle-random-phase approximation for spherical even-mass nuclei into the case of spherical odd-mass nuclei. In the same manner as the conventional random-phase approximation for even-mass nuclei leads us to the concept of "phonon", the theory necessarily leads us to the concept of a new kind of fermion-type collective excitation mode. Recent rapid accumulation of experimental data seems to be revealing the systematic presence of this kind of collective mode in many odd-mass nuclei.

§ 1. Introduction

In previous papers[2)~4)] we have concluded that the appearance of the low-lying anomalous coupling states in odd-mass nuclei have to be regarded as the typical phenomena in which a new kind of fermion-type collective excitation mode (i.e., the "dressed" three-quasi-particle mode) manifests itself as a relatively pure eigenmode. It has also been emphasized that the three-quasi-particle correlation characterizing this new collective mode is not specific for the anomalous coupling states but more general in odd-mass nuclei. Thus, we have suggested that the new collective mode may also be expected to exist in almost all spherical odd-mass nuclei and to play an important role in their low-lying collective excitations. In fact, recent rapid accumulation of experimental data seems to be revealing the systematic presence of such a kind of collective excited state in many odd-mass nuclei.[5)]

The aim of this paper is to give a theoretical foundation to such a new concept of elementary excitation mode as the general one in spherical odd-mass nuclei, by developing a systematic formulation of the theory of collective excitations (the essential idea of which was proposed in Part I[2)] with the single-j-

*) A part of this paper was reported[1)] in the "Symposium on In-Beam Spectroscopy" held at the Institute of Physical and Chemical Research, August 31 and September 1, 1972.

shell model). The theory can be regarded as a natural extension of the conventional quasi-particle-random-phase approximation (RPA) for spherical even-even nuclei into the case of spherical odd-mass nuclei. In the same manner as the conventional RPA for even-even nuclei leads us to the concept of "phonon" as a boson, the theory necessarily leads us to the concept of a new kind of fermion-type collective excitation mode.

The formulation of the theory is developed in a general form as far as possible, starting with the *j-j* coupling-shell-model Hamiltonian*) in the quasi-particle representation:

$$H=H_0+H_{\rm int}\,,\quad H_{\rm int}=H_X+H_V+H_Y\,,$$
$$H_0=\sum_\alpha E_a a_\alpha^\dagger a_\alpha\,,$$
$$H_X=\sum_{\alpha\beta\gamma\delta} V_X(\alpha\beta,\gamma\delta)\,a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma\,,$$
$$H_V=\sum_{\alpha\beta\gamma\delta} V_V(\alpha\beta,\gamma\delta)\,\{a_\alpha^\dagger a_\beta^\dagger a_{\tilde\delta}^\dagger a_{\tilde\gamma}^\dagger+{\rm h.c.}\}\,,$$
$$H_Y=\sum_{\alpha\beta\gamma\delta} V_Y(\alpha\beta,\gamma\delta)\,\{a_\alpha^\dagger a_\beta^\dagger a_{\tilde\delta}^\dagger a_\gamma+{\rm h.c.}\}\,, \tag{1·1}$$

where

$$V_X(\alpha\beta,\gamma\delta)=V_X^{(1)}(\alpha\beta,\gamma\delta)+V_X^{(2)}(\alpha\beta,\gamma\delta)$$
$$\equiv\mathcal{V}_{\alpha\beta\gamma\delta}\cdot(u_au_bu_cu_d+v_av_bv_cv_d)$$
$$+2\mathcal{V}_{\alpha\tilde\delta\tilde\beta\gamma}\cdot(u_av_bu_cv_d+v_au_bv_cu_d)\,,$$
$$V_V(\alpha\beta,\gamma\delta)\equiv\mathcal{V}_{\alpha\beta\gamma\delta}\cdot(u_au_bv_cv_d)\,,$$
$$V_Y(\alpha\beta,\gamma\delta)\equiv2\mathcal{V}_{\alpha\beta\gamma\delta}\cdot(u_au_bu_cv_d-v_av_bv_cu_d)\,. \tag{1·2}$$

Here E_a is the quasi-particle energy, determined as usual together with the parameters v_a and u_a of the Bogoliubov transformation, and $a_{\tilde\alpha}^\dagger\equiv(-)^{j_a-m_\alpha}a_{-\alpha}^\dagger$. The matrix element of a general effective nuclear potential $\mathcal{V}_{\alpha\beta\gamma\delta}$ satisfies the antisymmetry relations

$$\mathcal{V}_{\alpha\beta\gamma\delta}=-\mathcal{V}_{\beta\alpha\gamma\delta}=-\mathcal{V}_{\alpha\beta\delta\gamma}=\mathcal{V}_{\beta\alpha\delta\gamma}\,.$$

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*) The single-particle states are then characterized by a set of quantum numbers; the charge q, n, l, j, m. Throughout this paper, these states are designated by Greek letters. In association with a letter α, we use a Roman letter a to denote the same set except for the magnetic quantum number m. We also use a subscript $-\alpha$, which is obtained from α by changing the sign of the magnetic quantum number. We further use the notation $f(\tilde\alpha)\equiv(-)^{j_a-m_\alpha}f(-\alpha)$, where $f(\alpha)$ is an arbitrary function of α. It is possible to treat all matrix elements of the Hamiltonian as real quantities if the phase convention is suitably chosen. In this paper, we always assume this to be the case.

§ 2. Quasi-particle-new-Tamm-Dancoff space

2·1 *The quasi-particle TD space*

An essence of our theory is to make the explicit use of a concept of *quasi-particle-new-Tamm-Dancoff (NTD) space*. To obtain a first understanding of the concepts of quasi-particle NTD space and of physical operators defined in it, let us start with the *quasi-particle-Tamm-Dancoff (TD) space* characterizing the conventional quasi-particle representation.

It is well-known that the use of the quasi-particle representation (based on the BCS theory) can be regarded as an attempt to characterize both the ground state and the excited states in terms of the seniority number $v=\sum_a v_a$, the value of which corresponds to the number of quasi-particles. Thus, the energy spectrum of H_0 in odd-mass nuclei is quite characteristic as shown in Fig. 1 and the corresponding states with a fixed *odd* number of quasi-particles, $n(=\sum_a n_a)=v$, span the n-quasi-particle TD subspace. The quasi-particle TD space for odd-mass nuclei may therefore be characterized by the orthonormal state vectors with odd numbers of quasi-particles:

$$\left.\begin{aligned}
&|v=1;\alpha\rangle\equiv a_\alpha{}^\dagger|0\rangle\,,\\
&|v=3;\alpha\beta\gamma\rangle\equiv\frac{1}{\sqrt{3!}}a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger|0\rangle\,,\\
&|v=5;\alpha\beta\gamma\delta\varepsilon\rangle\equiv\frac{1}{\sqrt{5!}}a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger a_\delta{}^\dagger a_\varepsilon{}^\dagger|0\rangle\,,\\
&\qquad\vdots
\end{aligned}\right\}\tag{2·1}$$

where $|0\rangle$ is the BCS ground state.

In order to require explicitly that any state in the quasi-particle TD space must be orthogonal to any spurious state arising from the nucleon-number non-conservation (in the quasi-particle representation), it is convenient to define the quasi-particle TD space precisely by adopting the concept of the quasi-spin tensors. The concept has been introduced through the quasi-spin formalism[2),6)] of the seniority coupling scheme. Let us define the quasi-spin operators of the orbit a in the quasi-particle representation:

$$\begin{aligned}
&\hat{S}_+(a)=2^{-1/2}\cdot\Omega_a{}^{1/2}\sum_{m_{\alpha_1}m_{\alpha_2}}\langle j_aj_am_{\alpha_1}m_{\alpha_2}|J=0,M=0\rangle a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}\,,\\
&\hat{S}_-(a)=2^{-1/2}\cdot\Omega_a{}^{1/2}\sum_{m_{\alpha_1}m_{\alpha_2}}\langle j_aj_am_{\alpha_1}m_{\alpha_2}|J=0,M=0\rangle a_{\alpha_2}a_{\alpha_1}\,,\\
&\hat{S}_0(a)=2^{-1}\cdot\{\sum_{m_\alpha}a_\alpha{}^\dagger a_\alpha-\Omega_a\}\,,\quad \Omega_a\equiv j_a+2^{-1}\,,
\end{aligned}\tag{2·2}$$

which satisfy the commutation properties of angular momentum operators

$$\begin{aligned}
&[\hat{S}_+(a),\hat{S}_-(b)]=2\delta_{ab}\hat{S}_0(a)\,,\\
&[\hat{S}_0(a),\hat{S}_\pm(b)]=\pm\delta_{ab}\hat{S}_\pm(a)\,.
\end{aligned}\tag{2·3}$$

The eigenvalues $S(a)\{S(a)+1\}$ and $S_0(a)$ of the operators $\hat{S}^2(a)=[\hat{S}_+(a)\hat{S}_-(a)+\hat{S}_0(a)\{\hat{S}_0(a)-1\}]$ and $\hat{S}_0(a)$ are known to be related to the seniority number v_a and the quasi-particle number n_a of the orbit a, respectively, through

$$S(a)=\tfrac{1}{2}(\Omega_a-v_a)\,,\quad S_0(a)=\tfrac{1}{2}(n_a-\Omega_a)\,. \tag{2·4}$$

The quasi-spin operators thus defined characterize the transformation properties of physical operators under rotations*) in the quasi-spin space belonging to the orbit a: We can define the quasi-spin-tensor operators (in the quasi-particle representation), T_{ss_0} of rank s (with its component s_0) in the quasi-spin space of the orbit a, as usual, by the commutation relations

$$[\hat{S}_0(a),T_{ss_0}]=s_0T_{ss_0}\,,$$
$$[\hat{S}_\pm(a),T_{ss_0}]=\sqrt{(s\mp s_0)(s\pm s_0+1)}\cdot T_{ss_0\pm1}\,. \tag{2·5}$$

The single quasi-particle operators $a_\alpha^\dagger$ and $a_{\tilde{\alpha}}(\equiv(-)^{j_a-m_a}a_{-\alpha})$ are therefore regarded as spinors in the quasi-spin space of the orbit a:

$$T_{1/2,1/2}(\alpha)\equiv a_\alpha^\dagger\,,\quad T_{1/2,-1/2}(\alpha)\equiv a_{\tilde{\alpha}}\,. \tag{2·6}$$

With the quasi-spin spinors, we can construct a quasi-spin tensor of rank s in the quasi-spin space of the orbit a, $T_{ss_0}(\alpha_1\alpha_2\cdots\alpha_{2s})$,**) which is composed of products of $n=2s$ quasi-particle operators, by the standard vector-coupling procedures; for example,

$$T_{3/2,3/2}(\alpha_1\alpha_2\alpha_3)=a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}a^\dagger_{\alpha_3}\,,$$
$$T_{3/2,1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}a_{\tilde{\alpha}_3}+a^\dagger_{\alpha_1}a_{\tilde{\alpha}_2}a^\dagger_{\alpha_3}+a_{\tilde{\alpha}_1}a^\dagger_{\alpha_2}a^\dagger_{\alpha_3})\,,$$
$$T_{3/2,-1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{\alpha_1}a_{\tilde{\alpha}_2}a_{\tilde{\alpha}_3}+a_{\tilde{\alpha}_1}a^\dagger_{\alpha_2}a_{\tilde{\alpha}_3}+a_{\tilde{\alpha}_1}a_{\tilde{\alpha}_2}a^\dagger_{\alpha_3})\,,$$
$$T_{3/2,-3/2}(\alpha_1\alpha_2\alpha_3)=a_{\tilde{\alpha}_1}a_{\tilde{\alpha}_2}a_{\tilde{\alpha}_3}\,.$$

Now, the quasi-particle TD space for odd-mass nuclei characterized by (2·1) is precisely defined in terms of a set of state vectors

$$|v=2s;\alpha_1\alpha_2\cdots\alpha_{2s_a},\beta_1\beta_2\cdots\beta_{2s_b},\cdots\rangle$$
$$=O_s^\dagger[\alpha_1\alpha_2\cdots\alpha_{2s_a},s_0(a)=s_a;\beta_1\beta_2\cdots\beta_{2s_b},s_0(b)=s_b;\cdots]|0\rangle\,, \tag{2·7}$$

*) It is well-known that the Bogoliubov transformation

$$c_\alpha^\dagger=u_a a_\alpha^\dagger+v_a a_{\tilde{\alpha}}\equiv Ua_\alpha^\dagger U^{-1}$$

simply corresponds to a special one of rotations in the quasi-spin space of the orbit a through an angle $\theta_a(u_a\equiv\cos\theta_a/2,\ v_a\equiv\sin\theta_a/2)$:

$$U=\prod_a\exp\{-i\theta_a\hat{S}_y(a)\}=\exp\{-i\sum_a\theta_a\hat{S}_y(a)\}\,,$$

where

$$\hat{S}_y(a)\equiv(1/2i)\{\hat{S}_+(a)-\hat{S}_-(a)\}\,.$$

**) The subscript $i=1,2,3,\cdots$ of α are used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit a is necessary.

where

$$O_s^\dagger[\alpha_1\alpha_2\cdots\alpha_{2s_a}, s_0(a); \beta_1\beta_2\cdots\beta_{2s_b}, s_0(b); \cdots; \delta_1\delta_2\cdots\delta_{2s_d}, s_0(d)]$$
$$\equiv[(2s_a)!(2s_b)!\cdots(2s_d)!]^{-1/2}\mathring{T}_{s_a,s_0(a)}(\alpha_1\alpha_2\cdots\alpha_{2s_a})$$
$$\times\mathring{T}_{s_b,s_0(b)}(\beta_1\beta_2\cdots\beta_{2s_b})\cdots\cdot\mathring{T}_{s_d,s_0(d)}(\delta_1\delta_2\cdots\delta_{2s_d}) \quad (2\cdot8)$$

with $2s=2(s_a+s_b+\cdots+s_d)\equiv v$ in odd numbers. In Eq. (2·8), we have used a definition

$$\mathring{T}_{ss_0}(\alpha_1\alpha_2\cdots\alpha_{2s})\equiv\sum_{\alpha_1'\alpha_2'\cdots\alpha_{2s}'}P(\alpha_1\alpha_2\cdots\alpha_{2s}|\alpha_1'\alpha_2'\cdots\alpha_{2s}')T_{ss_0}(\alpha_1'\alpha_2'\cdots\alpha_{2s}'), \quad (2\cdot9)$$

where the operator $\boldsymbol{P}$ (the matrix elements of which are $P(\alpha_1\alpha_2\cdots\alpha_{2s}|\alpha_1'\alpha_2'\cdots\alpha_{2s}')$) is a projection operator by which the quasi-spin operators $\hat{S}_\pm(a)$, $\hat{S}_0(a)$ are removed out of the quasi-spin tensor $T_{ss_0}(\alpha_1\alpha_2\cdots\alpha_{2s})$ completely. Therefore $P(\alpha_1\alpha_2\cdots\alpha_{2s}|\alpha_1'\alpha_2'\cdots\alpha_{2s}')$ is closely related to the coefficient of fractional parentage (c.f.p.) with seniority $v_a=2s$ for $(j_a)^{2s}$-configuration, and its explicit form for $s=3/2$ is given in Appendix I. By definition, the operators $O_s^\dagger$ in (2·8) never contain any component of the nucleon-number-fluctuation operator*)

$$\hat{N}-N=\sum_a(u_a^2-v_a^2)\{2\hat{S}_0(a)+\Omega_a\}+2\sum_a u_a v_a\{\hat{S}_+(a)+\hat{S}_-(a)\},$$

and we obtain

$$\hat{S}_-(a)|v=2s;\alpha_1\alpha_2\cdots\alpha_{2s_a},\beta_1\beta_2\cdots\beta_{2s_b},\cdots\rangle=0. \quad (2\cdot10)$$

This means that any state in the quasi-particle TD space never contains '$J^\pi=0^+$' quasi-particle pairs. In this sense we may call the quasi-particle TD space an "intrinsic" space. Here it should be noticed that, for such a class of excited states $|\phi_{\rm pair}\rangle$ as the pairing excitations arising from the motion of $\hat{S}_\pm(a)$, we have $\hat{S}_-(a)|\phi_{\rm pair}\rangle\neq0$. This means that the class of states $|\phi_{\rm pair}\rangle$ are always orthogonal to any state in the quasi-particle TD space. We therefore may call the class of states of the pairing excitations *which do not carry any seniority number* as "collective" states. Needless to say, a special one of "collective" vibrations with zero energy is known as due to the nucleon-number non-conservation. What we are considering in this paper are

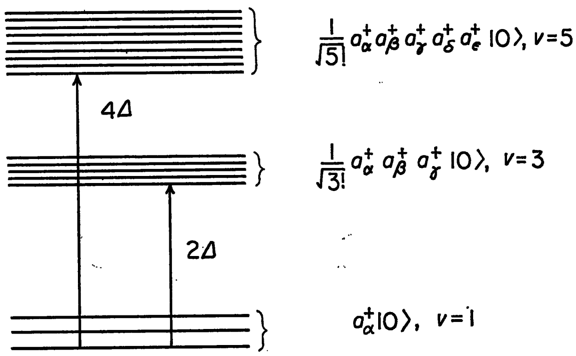

Fig. 1. Energy spectra of H_0 in odd-mass nuclei.

*) We assume that $\delta_{ab}=\delta_{j_aj_b}$ in our shell model subspace under consideration. This is satisfied in most actual cases.

the "intrinsic" excitation modes which are orthogonal to the "collective" modes, and investigations of the "collective" modes and of the interplay between the "intrinsic" and the "collective" modes will be discussed in a separate paper.

Since the quasi-particle TD space is characterized by the seniority number $v=n$ as shown in Fig. 1, it may be a better approximation to diagonalize the quasi-particle interaction $H_{\rm int}$ in (1·1) in the subspace with a fixed number of quasi-particles. This is well-known as the quasi-particle Tamm-Dancoff (TD) approximation. Among the matrix elements of $H_{\rm int}$ in the subspace with the definite quasi-particle number, the non-zero ones come from only the part H_X which conserves the quasi-particle number. Therefore, the eigenmode-creation operators $X_{s\lambda}^\dagger$ in the TD approximation (with the definite odd quasi-particle number $n=2s=\sum_a 2s_a$) are given by the linearized eigenvalue equation,

$$[H_0+H_X, X_{s\lambda}^\dagger]\cong\omega_{s\lambda}^{(0)}\cdot X_{s\lambda}^\dagger ,\qquad \omega_{s\lambda}^{(0)}>0 \tag{2·11}$$

with

$$X_{s\lambda}^\dagger=\sum_{\alpha_1,\cdots,\delta_1}\Psi_{s\lambda}^{(0)}[\alpha_1\alpha_2\cdots\alpha_{2s_a}, s_0(a)=s_a;\cdots;\delta_1\delta_2\cdots\delta_{2s_d}, s_0(d)=s_d]$$
$$\times O_s^\dagger[\alpha_1\alpha_2\cdots\alpha_{2s_a}, s_0(a)=s_a;\cdots;\delta_1\delta_2\cdots\delta_{2s_d}, s_0(d)=s_d]. \tag{2·12}$$

Here λ denotes a set of additional quantum numbers to specify the eigenmode. The operators $X_{s\lambda}^\dagger$ satisfy the anti-commutation relation in the following sense:

$$\{X_{s_>\lambda}, X_{s'_<\lambda'}^\dagger\}_+|0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|0\rangle ,$$
$$\{X_{s\lambda}^\dagger, X_{s'\lambda'}^\dagger\}_+=\{X_{s\lambda}, X_{s'\lambda'}\}_+=0 , \tag{2·13}$$

where the subscript $>$ (or $<$) of $s_>$ (or $s'_<$) denotes the relation $s\geq s'$. Thus, the set of states $X_{s\lambda}^\dagger|0\rangle$ with $2s=n$ in odd numbers provides a complete set of orthonormal bases of the quasi-particle TD space for odd-mass nuclei:

$$\langle 0|\{X_{s\lambda}, X_{s'\lambda'}^\dagger\}_+|0\rangle=\delta_{ss'}\delta_{\lambda\lambda'} . \tag{2·14}$$

Now it is clear that the quasi-particle TD space for odd-mass nuclei may also be characterized with the operators defined by

$$\boldsymbol{X}_{s\lambda}^\dagger=X_{s\lambda}^\dagger|0\rangle\langle 0| ,\quad \boldsymbol{X}_{s\lambda}=|0\rangle\langle 0|X_{s\lambda} \tag{2·15}$$

with $2s=n$ in odd numbers. By definition, the operators $\boldsymbol{X}_{s\lambda}^\dagger$ satisfy the equations

$$[H_0+H_X, \boldsymbol{X}_{s\lambda}^\dagger]=\omega_{s\lambda}^{(0)}\boldsymbol{X}_{s\lambda}^\dagger \quad (\omega_{s\lambda}^{(0)}>0) \tag{2·16}$$

and

$$\{\boldsymbol{X}_{s\lambda}, \boldsymbol{X}_{s'\lambda'}^\dagger\}_+|0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|0\rangle ,$$
$$\{\boldsymbol{X}_{s\lambda}^\dagger, \boldsymbol{X}_{s'\lambda'}^\dagger\}_+=\{\boldsymbol{X}_{s\lambda}, \boldsymbol{X}_{s'\lambda'}\}_+=0 . \tag{2·17}$$

The unit operator $\mathbf{1}$ in the quasi-particle TD space for odd-mass nuclei is given by

$$\mathbf{1}=\sum_{s\lambda}{}' \mathbf{X}_{s\lambda}^{\dagger}\mathbf{X}_{s\lambda}, \tag{2·18}$$

where $\sum'_{s\lambda}$ denotes the summation with respect to $2s$ in odd numbers. With the use of the operators $\mathbf{X}_{s\lambda}^{\dagger}$, H_0+H_X in (1·1) is now written as

$$H_0+H_X=\sum_{s\lambda}{}' \omega_{s\lambda}^{(0)}\mathbf{X}_{s\lambda}^{\dagger}\mathbf{X}_{s\lambda}\,. \tag{2·19}$$

Thus, using the "elementary excitation" operators $\mathbf{X}_{s\lambda}^{\dagger}$ (of H_0+H_X) instead of the quasi-particle operators $a_\alpha^{\dagger}$, we obtain another representation of arbitrary operator $\hat{F}$ in the quasi-particle TD space for odd-mass nuclei:

$$\boldsymbol{F}=\mathbf{1}\hat{F}\mathbf{1}=\sum_{s\lambda}{}'\sum_{s'\lambda'}{}' \langle s\lambda|\hat{F}|s'\lambda'\rangle \mathbf{X}_{s\lambda}^{\dagger}\mathbf{X}_{s'\lambda'} \tag{2·20}$$

with $|s\lambda\rangle\equiv X_{s\lambda}^{\dagger}|0\rangle$.

2·2 *The quasi-particle NTD space*

Now it is well-known that, in such a finite quantum system as nucleus, the ground-state correlation is particularly important as a collective predisposition which admits the correlated excited states to occur from the ground state. Actually, we thus have to take account of the special importance of both the seniority classification and of the ground-state correlation simultaneously, in a way that the essential physical notion obtained in the quasi-particle TD space still persists in a certain form. The guiding principle to introduce the quasi-particle NTD space is lying in the fact that, in the new-Tamm-Dancoff (NTD) method, the quasi-particle correlations which are attributed asymmetrically only to the excited states in the TD calculations are symmetrically incorporated in the ground state through the ground-state correlation. The quasi-particle NTD space for odd-mass nuclei is thus defined with a set of basis vectors,

$$Y_{s\lambda}^{\dagger}|\Phi_0\rangle \tag{2·21}$$

with $2s$ in odd numbers, where $Y_{s\lambda}^{\dagger}$ are creation operators of "*dressed*" n $(=2s)$-*quasi-particle modes* constructed within the framework of the NTD method with the ground-state correlation, and $|\Phi_0\rangle$ is the corresponding correlated ground state. Contrary to the BCS ground state $|0\rangle$, the state $|\Phi_0\rangle$ is not with a definite seniority number because of the ground-state correlation. In spite of the breakdown of the seniority number, in the quasi-particle NTD method we can still characterize the excitation modes by the amount of seniority $\varDelta v$ $(=2s=n)$ which they transfer to the ground state $|\Phi_0\rangle$.

In the completely same way as the conventional spherical tensor operator is characterized by the amount of angular momentum it transfers to the state on which it act, the quasi-spin tensor operator T_{ss_0} is characterized by the amount of the transferred seniority $\varDelta v=2s$ to the state on which it operates. Therefore, we can define the dressed n $(=2s)$-quasi-particle modes $Y_{s\lambda}^{\dagger}$ in terms of the direct products of the quasi-spin-tensor operators defined in each orbit with the total transferred seniority $\Delta v=2s=\sum_a 2s_a$:

$$Y^{\dagger}_{s\lambda}=\sum_{\alpha_i,\cdots,\delta_i}\sum_{s_0(a),\cdots,s_0(d)}\Psi_{s\lambda}[\alpha_1\alpha_2\cdots\alpha_{2s_a},s_0(a);\cdots;\delta_1\delta_2\cdots\delta_{2s_d},s_0(d)]$$
$$\times O_s^{\dagger}[\alpha_1\alpha_2\cdots\alpha_{2s_a},s_0(a);\cdots;\delta_1\delta_2\cdots\delta_{2s_d},s_0(d)],\qquad(2\cdot22)$$

where $O_s^{\dagger}[\alpha_i,s_0(a);\cdots;\delta_i,s_0(d)]$ is defined in (2·8). Within the framework of the NTD approximation, the eigenvalue equation which the amplitude $\Psi_{s\lambda}[\alpha_i,s_0(a);\cdots;\delta_i,s_0(d)]$ must satisfy is given, as usual, by

$$[H_0+H_X+H_Y,Y^{\dagger}_{s\lambda}]\cong\omega_{s\lambda}Y^{\dagger}_{s\lambda}\qquad(2\cdot23)$$

with $\omega_{s\lambda}>0$, where the part H_Y of the quasi-particle interaction $H_{\rm int}$ in (1·1) introduces the ground-state correlation.

The part H_Y is known to be essential together with H_X in constructing the collective excitation modes within the framework of the NTD method, and so we call the parts H_X and H_Y the *constructive force* (of the collective excitation modes). The part H_V in (1·1) changes the number of quasi-particles, and so has no contribution in the TD calculation with a definite number of quasi-particles. In so far as the NTD method is adopted (in describing the dressed n-quasi-particle mode) as an improvement of the TD method (for n-quasi-particles), therefore, the part H_V does not play any important role, contrary to the constructive force H_X and H_Y. The part H_V plays a decisive role as essential coupling between the various dressed n-quasi-particle modes, and so we call it the *interactive force*.*)

The dressed n-quasi-particle modes $Y^{\dagger}_{s\lambda}$ (with $2s=n$) have to satisfy the fermion-type anticommutation relation in the quasi-particle NTD space,

$$\{Y_{s>\lambda},Y^{\dagger}_{s'<\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\Phi_0\rangle,\qquad(2\cdot24)$$

just as the n-quasi-particle modes $X^{\dagger}_{s\lambda}$ (with $2s=n$) in the quasi-particle TD space satisfy (2·13). This requirement is a counterpart of the eigenvalue equation (2·23) in prescribing the elementary excitation modes in terms of the concept of transferred seniority. When (2·24) is satisfied within the framework of the NTD approximation, the set of states $Y^{\dagger}_{s\lambda}|\Phi_0\rangle$ with $2s=n$ in odd numbers becomes a complete set of orthonormal bases in the quasi-particle NTD space for odd-mass nuclei:

$$\langle\Phi_0|\{Y_{s\lambda},Y^{\dagger}_{s'\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'},\qquad(2\cdot25)$$

and, in the same way as (2·18), the unit operator in the quasi-particle NTD space for odd-mass nuclei is given by

*) It should be also noticed that the matrix elements of H_V contain the reduction (u,v)-factors which can be quite small in the middle of the shell, while the matrix elements of H_X and H_Y involve the enhancement (u,v)-factors which are close to unity for low-lying states in the middle of the shell.

$$\mathbf{1}=\sum_{s\lambda}{}' \boldsymbol{Y}_{s\lambda}^{\dagger}\boldsymbol{Y}_{s\lambda}\,, \tag{2·26}$$

where

$$\boldsymbol{Y}_{s\lambda}^{\dagger}=Y_{s\lambda}^{\dagger}|\Phi_0\rangle\langle\Phi_0|\,,\quad \boldsymbol{Y}_{s\lambda}=|\Phi_0\rangle\langle\Phi_0|Y_{s\lambda}\,. \tag{2·27}$$

In terms of the elementary excitation operators $\boldsymbol{Y}_{s\lambda}^{\dagger}$, any physical operator $\hat{F}$ is easily transcribed into the quasi-particle NTD space:

$$\hat{F}\rightarrow\boldsymbol{F}=\mathbf{1}\hat{F}\mathbf{1}=\sum_{s\lambda}{}'\sum_{s'\lambda'}{}'\langle\Phi_0|Y_{s\lambda}\hat{F}Y_{s'\lambda'}^{\dagger}|\Phi_0\rangle\boldsymbol{Y}_{s\lambda}^{\dagger}\boldsymbol{Y}_{s'\lambda'}\,. \tag{2·28}$$

Thus, the actual problem is how to estimate the matrix elements $\langle\Phi_0|Y_{s\lambda}\hat{F}Y_{s'\lambda'}^{\dagger}|\Phi_0\rangle$. As will be shown in § 5, however, a simple rule will be found when the anti-commutation relation (2·24) is satisfied.

In the following sections we study concretely the quasi-particle NTD sub-space which consists of the dressed quasi-particle modes with the transferred seniority $\Delta v(=2s)=1$ and 3, because we are considering the low-lying collective excited states in odd-mass nuclei.

§ 3. Structure of the dressed 3-quasi-particle modes

According to the definition (2·22), the eigenmode operators of the dressed 3-quasi-particle modes (which satisfy Eq. (2·23) with $2s=3$ within the NTD approximation) are written in an explicit form:

$$\begin{aligned}C_\lambda{}^{\dagger}=&\frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\cdot\boldsymbol{P}(\alpha\beta\gamma)\,a_\alpha{}^{\dagger}a_\beta{}^{\dagger}a_\gamma{}^{\dagger}\\&+\frac{1}{\sqrt{3!}}\sum_{\alpha_1\alpha_2\alpha_3}\varphi_\lambda{}^{(1)}(\alpha_1\alpha_2\alpha_3)\cdot\mathring{T}_{3/2,-1/2}(\alpha_1\alpha_2\alpha_3)\\&+\frac{1}{\sqrt{2!}}\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\varphi_\lambda{}^{(2)}(\alpha_1\alpha_2;\gamma)\cdot\mathring{T}_{10}(\alpha_1\alpha_2)\,a_{\bar{\gamma}}\\&+\sum_{\substack{(\alpha\beta)\gamma\\(a,b\neq c)}}(1+\delta_{ab})^{-1/2}\cdot\varphi_\lambda{}^{(3)}(\alpha\beta;\gamma)\cdot a_\gamma{}^{\dagger}\boldsymbol{P}(\alpha\beta)\,a_{\bar{\alpha}}a_{\bar{\beta}}\,.\end{aligned} \tag{3·1}$$

Here the symbol $\sum_{(\alpha\beta)\gamma}$ denotes the summation over the orbit-pair (ab), m_α, m_β and γ, and

$$\begin{aligned}\boldsymbol{P}(\alpha\beta\gamma)\,a_\alpha{}^{\dagger}a_\beta{}^{\dagger}a_\gamma{}^{\dagger}&\equiv\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')\,a_{\alpha'}^{\dagger}a_{\beta'}^{\dagger}a_{\gamma'}^{\dagger}\,,\\ \boldsymbol{P}(\alpha\beta)\,a_{\bar{\alpha}}a_{\bar{\beta}}&\equiv\sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')\,a_{\bar{\alpha}'}a_{\bar{\beta}'}\,,\end{aligned} \tag{3·2}$$

where the operators $\boldsymbol{P}$'s denote the projection operators by which any quasi-spin operator is removed out of the products of quasi-particles $(a_\alpha{}^{\dagger}, a_\alpha)$ on which they act, and their explicit forms are given in Appendix I. Direct calculation of Eq. (2·23) with (3·1) leads us to the following eigenvalue equation which the correlation amplitudes have to satisfy

$$\omega_\lambda\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\varphi}_\lambda\end{bmatrix}=\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A}\\ \boldsymbol{A}^T & -\boldsymbol{d}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\varphi}_\lambda\end{bmatrix}, \tag{3·3}$$

where $\boldsymbol{\psi}_\lambda$ and $\boldsymbol{\varphi}_\lambda$ denote the matrix notations symbolizing the sets of amplitudes $\psi_\lambda(\alpha\beta\gamma)$ and $\{\varphi_\lambda^{(1)}(\alpha_1\alpha_2\alpha_3), \varphi_\lambda^{(2)}(\alpha_1\alpha_2;\gamma), \varphi_\lambda^{(3)}(\alpha\beta;\gamma)\}$, respectively, and the explicit forms of matrices $\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ are given in Appendix II. The projection operators $\boldsymbol{P}$ involved in these matrices guarantee that the correlation amplitudes automatically satisfy the relations

$$\begin{aligned}\psi_\lambda(\alpha\beta\gamma) &= \sum_{\alpha'\beta'\gamma'} P(\alpha\beta\gamma|\alpha'\beta'\gamma')\psi_\lambda(\alpha'\beta'\gamma'),\\ \varphi_\lambda^{(1)}(\alpha_1\alpha_2\alpha_3) &= \sum_{\alpha_1'\alpha_2'\alpha_3'} P(\alpha_1\alpha_2\alpha_3|\alpha_1'\alpha_2'\alpha_3')\varphi_\lambda^{(1)}(\alpha_1'\alpha_2'\alpha_3'),\\ \varphi_\lambda^{(2)}(\alpha\beta;\gamma) &= \sum_{\alpha'\beta'} P(\alpha\beta|\alpha'\beta')\varphi_\lambda^{(2)}(\alpha'\beta';\gamma),\end{aligned} \tag{3·4}$$

which mean that the correlation amplitudes never contain any component due to the nucleon-number-fluctuations (i.e., due to the quasi-spin operators).

Equation (3·3) has the same formal structure as the one given in Part I for the case of the single-j-shell model, and tells us that with the definition of the metric matrix

$$\boldsymbol{\tau}=\begin{bmatrix}\boldsymbol{1} & 0\\ 0 & -\boldsymbol{1}\end{bmatrix}, \tag{3·5}$$

the correlation amplitudes satisfy the orthonormality relation in the sense

$$[\boldsymbol{\psi}_{\lambda'}^T, \boldsymbol{\varphi}_{\lambda'}^T]\boldsymbol{\tau}\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\varphi}_\lambda\end{bmatrix}=\epsilon_\lambda\delta_{\lambda\lambda'}, \tag{3·6}$$

where ϵ_λ is the sign function with $|\epsilon_\lambda|=1$ and $\boldsymbol{\psi}_\lambda^T$ denotes the transposed matrix of $\boldsymbol{\psi}_\lambda$. Due to the introduction of the backward-going components, the eigenvalue equation (3·3) has "extra" unphysical solutions which have the large amplitudes $\boldsymbol{\varphi}_{\lambda_0}$ and the small amplitudes $\boldsymbol{\psi}_{\lambda_0}$.*) As long as the eigenvalues ω_λ are real, the physical solutions have the large amplitudes $\boldsymbol{\psi}_\lambda$ and the small amplitudes $\boldsymbol{\varphi}_\lambda$. Thus the positive ϵ_λ corresponds to the physical solutions, and we can classify the eigenmode operators $C_\lambda^\dagger$ in (3·1) as follows:

$$C_\lambda^\dagger=\begin{cases}Y_\lambda^\dagger & \text{for}\quad \epsilon_\lambda=1,\\ A_{\lambda_0} & \text{for}\quad \epsilon_{\lambda_0}=-1.\end{cases} \tag{3·7}$$

The physical dressed 3-quasi-particle states are given as

$$|\lambda\rangle=Y_\lambda^\dagger|\Phi_0\rangle, \tag{3·8}$$

where $|\Phi_0\rangle$ is the correlated ground state (within the framework of the NTD approximation). The existence of the extra eigenmodes $A_{\lambda_0}^\dagger$, which have no

*) Hereafter the unphysical solutions are specified by the subscript λ_0.

physical meaning, imposes an important condition upon the state vectors in the quasi-particle NTD space: Any state vector $|\Phi\rangle$ which actually has physical meaning must satisfy the supplementary condition

$$A_{\lambda_0}|\Phi\rangle=0. \tag{3·9}$$

§ 4. Structure of the ground-state correlation

Now, it is quite important to examine the compatibility of Eqs. (3·3) and (2·24). In this section, we shall show that the requirements (2·24) is satisfied within the NTD approximation when we properly take account of characteristic of the introduced ground-state correlation.

First of all, let us investigate the characteristic of the ground-state correlation (due to the dressed 3-quasi-particle modes). The structure of the ground-state correlation should be determined *in principle* through the properties of the fundamental eigenvalue equation (3·3). As is seen from Eq. (2·23), the fundamental equation contains only the matrix elements of the constructive force, H_X and H_V. The diagrams considered in the correlated ground state $|\Phi_0\rangle$ are therefore closed diagrams which are composed by combining only the matrix elements of H_X and H_V, so that $|\Phi_0\rangle$ may be generally written as a superposition of 0-, 4-, 8-quasi-particle states:

$$|\Phi_0\rangle=C_0|0\rangle+\sum_{\alpha\beta\gamma\delta}C_1(\alpha\beta\gamma\delta)\,a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger|0\rangle$$
$$+\sum C_2(\alpha\beta\gamma\delta\varepsilon\varphi\mu\nu)\,a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger a_\varepsilon^\dagger a_\varphi^\dagger a_\mu^\dagger a_\nu^\dagger|0\rangle+\cdots, \tag{4·1}$$

where C_0 is the constant related to the normalization of $|\Phi_0\rangle$. The coefficients C's in (4·1) should be determined by the conditions $Y_\lambda|\Phi_0\rangle=0$ and $A_{\lambda_0}|\Phi_0\rangle=0$, within the framework of the NTD approximation (which we have used in obtaining the fundamental equation (3·3)). This procedure suggests that, with the basic approximation in the NTD method $n_0\ll 2\Omega$*) (i.e., $O(n_0/2\Omega)\approx 0$), the correlated ground state $|\Phi_0\rangle$ may be approximately written in a symbolized form[2])

$$|\Phi_0\rangle=C_0\exp\left[\frac{1}{\sqrt{4!}}k\sum_{\alpha\beta\gamma\delta}\chi_{J=0}(\alpha\beta\gamma\delta)\,a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger\right]|0\rangle\equiv C_0e^W|0\rangle, \tag{4·2}$$

where the constant k and $\chi_{J=0}(\alpha\beta\gamma\delta)$ are defined through the relations

$$\frac{1}{\sqrt{4!}}k\cdot\chi_{J=0}(\alpha\beta\gamma\delta)\equiv\frac{C_1(\alpha\beta\gamma\delta)}{C_0},$$
$$\sum_{\alpha\beta\gamma\delta}\chi^2_{J=0}(\alpha\beta\gamma\delta)=1. \tag{4·3}$$

*) Here n_0 and 2Ω are defined through $\langle\Phi_0|a_\alpha^\dagger a_\beta|\Phi_0\rangle\cong\delta_{\alpha\beta}\cdot n_0/2\Omega$, so that n_0 denotes the average number of quasi-particles in the ground state and 2Ω denotes the total number of single-particle states under consideration.

Needless to say, $\chi_{J=0}(\alpha\beta\gamma\delta)$ has never contain any component due to the nucleon-number-fluctuations, so that it has to satisfy

$$\chi_{J=0}(\alpha\beta\gamma\delta)=\sum_{\alpha'\beta'\gamma'\delta'}P(\alpha\beta\gamma\delta|\alpha'\beta'\gamma'\delta')\chi_{J=0}(\alpha'\beta'\gamma'\delta')\,. \tag{4·4}$$

The ground-state correlation written in the symbolized form (4·2) should be interpreted so as to be characterized by the following prescriptions:

(i) For an arbitrary operator $\hat{O}$, we have

$$\begin{aligned}\hat{O}|\Phi_0\rangle&=C_0\hat{O}e^{W}|0\rangle\\&=C_0e^{W}\left\{\hat{O}+[\hat{O},W]+\frac{1}{2!}[[\hat{O},W],W]+\cdots\right\}|0\rangle\\&\Rightarrow C_0e^{W}\{\hat{O}+[\hat{O},W]\}|0\rangle\,.\quad\text{(the NTD approximation)}\end{aligned} \tag{4·5}$$

(ii) Since the basis operators characterizing the ground-state correlation are $O^{\dagger}_{3/2}[\alpha_i,s_0(a);\beta_i,s_0(b);\gamma_i,s_0(c)]$ (Eq. (2·8) with $s=3/2$) which construct the dressed 3-quasi-particle modes and since *the operators* $O^{\dagger}_{3/2}[\alpha_i,s_0(a);\beta_i,s_0(b);\gamma_i,s_0(c)]$ *are antisymmetric with respect to the indices belonging to the same single particle orbit*, all quantities which appear in the last expression of Eq. (4·5) must keep the same property. According to the prescription (i), the supplementary condition (3·9) with (4·2) leads to a relation

$$\psi_{\lambda_0}-k\boldsymbol{C}\varphi_{\lambda_0}=0 \tag{4·6}$$

with

$$\begin{aligned}&C_{\alpha\beta\gamma,\alpha_1'\alpha_2'\alpha_3'}=3\sqrt{2}\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}_1'\tilde{\alpha}_2')\delta_{\gamma\alpha_3'}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3')\,,\\&C_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'}=6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}_2'\tilde{\gamma}')\delta_{\gamma\alpha_1'}\boldsymbol{P}^T(\alpha_1'\alpha_2')\,,\\&C_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\delta_{\gamma\gamma'}(1+\delta_{a'b'})^{-1/2}\boldsymbol{P}^T(\alpha'\beta')\,,\\&(\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\equiv(-)^{j_{a'}-m_{\alpha'}}(-)^{j_{b'}-m_{\beta'}}\chi_{J=0}(\alpha,\beta,-\alpha',-\beta')).\end{aligned}$$

For simplicity, here we have used the following abbreviations:

$$\begin{aligned}&\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma,\alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta'\gamma')\\&\quad\equiv\sum_{\mu\nu\sigma}\cdot\sum_{\mu'\nu'\sigma'}P(\alpha\beta\gamma|\mu\nu\sigma)f(\mu\nu\sigma,\mu'\nu'\sigma')P(\mu'\nu'\sigma'|\alpha'\beta'\gamma')\,,\\&\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma|\alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta')\\&\quad\equiv\sum_{\mu\nu\sigma}\cdot\sum_{\mu'\nu'}P(\alpha\beta\gamma|\mu\nu\sigma)f(\mu\nu\sigma,\mu'\nu'\gamma')P(\mu'\nu'|\alpha'\beta')\,,\end{aligned} \tag{4·7}$$

where $f(\alpha\beta\gamma,\alpha'\beta'\gamma')$ is an arbitrary function with respect to $(\alpha\beta\gamma,\alpha'\beta'\gamma')$. Combining Eqs. (4·6) and (3·6) and using the symmetry property of $\chi_{J=0}(\alpha\beta\gamma\delta)$ with respect to the permutation of $(\alpha\beta\gamma\delta)$, we obtain an equation to determine $\chi_{J=0}(\alpha\beta\gamma\delta)$ in terms of the physical amplitudes:

$$\varphi_\lambda-k\boldsymbol{C}^T\psi_\lambda=0\,. \tag{4·8}$$

Special importance of the prescription (ii) manifests itself when we evaluate, for example, the following expression:

$$\{Y_\lambda, a_\tau{}^\dagger\}_+|\Phi_0\rangle=\Big\{\frac{\sqrt{6}}{2}\sum_{\alpha'\beta'\gamma'}\delta_{\tau\tau'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}$$

$$+\frac{1}{\sqrt{2}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\tau\tau_3}\varphi_\lambda{}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\gamma}_1}^\dagger$$

$$+\sum_{\substack{\gamma_1\gamma_2\alpha\\(c\neq a)}}\delta_{\tau\tau_1}\varphi_\lambda{}^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\alpha}}{}^\dagger a_{\tilde{\gamma}_2}^\dagger$$

$$+\sum_{\substack{(\alpha'\beta')\gamma'\\(a,b\neq c)}}\delta_{\tau\tau'}\frac{1}{\sqrt{1+\delta_{a'b'}}}\varphi_\lambda{}^{(3)}(\alpha'\beta';\gamma')a_{\tilde{\beta}}^\dagger a_{\tilde{\alpha}}^\dagger\Big\}|\Phi_0\rangle\,. \tag{4·9}$$

In this case we have to evaluate the first term. With the aid of the prescription (i), we obtain first

$$\sum_{\alpha'\beta'\gamma'}\delta_{\tau\tau'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$

$$=\sqrt{6}k\sum_{\alpha'\beta'\gamma'}\cdot\sum_{\alpha\beta}\delta_{\tau\tau'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}}{}^\dagger a_{\tilde{\beta}}{}^\dagger|\Phi_0\rangle\,.$$

Then, the prescription (ii) leads the right-hand side to

$$\sqrt{6}k\sum_{\alpha'\beta'\gamma'}\cdot\sum_{\alpha\beta}\delta_{\tau\tau'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}}{}^\dagger a_{\tilde{\beta}}{}^\dagger|\Phi_0\rangle$$

$$\Rightarrow\sqrt{6}k\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\tau\tau_3}\boldsymbol{P}(\gamma_1\gamma_2\gamma_3)\cdot\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_3}\chi_{J=0}(\gamma_1\gamma_2\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$

$$+2\sqrt{6}k\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\tau\tau_1}\boldsymbol{P}(\gamma_1\gamma_2)\cdot\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_1}\chi_{J=0}(\gamma_2\alpha\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}{}^\dagger|\Phi_0\rangle$$

$$+2\sqrt{6}k\sum_{\substack{(\alpha\beta)\\(a,b\neq c)}}\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\cdot\sum_{\alpha'\beta'\gamma'}\delta_{\tau\tau'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}}^\dagger a_{\tilde{\beta}}^\dagger|\Phi_0\rangle$$

$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\tau\tau_3}\varphi_\lambda{}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$

$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\tau\tau_1}\varphi_\lambda{}^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}{}^\dagger|\Phi_0\rangle$$

$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\tau\tau'}\frac{\varphi_\lambda{}^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\tilde{\alpha}}^\dagger a_{\tilde{\beta}}^\dagger|\Phi_0\rangle\,,$$

where we have used Eq. (4·8) in the last expression. Thus, we finally obtain

$$\sum_{\alpha'\beta'\gamma'}\delta_{\tau\tau'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$

$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\tau\tau_3}\varphi_\lambda{}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$

$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\tau\tau_1}\varphi_\lambda{}^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}{}^\dagger|\Phi_0\rangle$$

$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\varphi_\lambda^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\bar\alpha}^\dagger a_{\bar\beta}^\dagger|\Phi_0\rangle\,, \tag{4·10}$$

so that (4·9) simply becomes

$$\{Y_\lambda, a_\gamma^\dagger\}_+|\Phi_0\rangle=0\,. \tag{4·11}$$

We are now in a position to show that the requirement (2·24) is satisfied. Direct calculations with the aid of Eq. (4·10) lead us to

$$\begin{aligned}
&\{Y_{\lambda'}, Y_\lambda^\dagger\}_+|\Phi_0\rangle\\
&=\Big[\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\psi_{\lambda'}(\alpha\beta\gamma)+\frac{3}{2}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\cdot\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}\\
&\quad+\frac{\sqrt{3}}{2}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\Big\{\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\varphi_{\lambda'}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\bar\gamma_2}^\dagger a_{\bar\gamma_1}^\dagger\\
&\quad+\sqrt{2}\sum_{\gamma_1\gamma_2\alpha'}\delta_{\gamma\gamma_1}\varphi_{\lambda'}^{(2)}(\gamma_1\gamma_2;\alpha')a_{\bar\alpha}^\dagger a_{\bar\gamma_2}^\dagger+\sqrt{2}\sum_{(\alpha'\beta')\gamma'}\delta_{\gamma\gamma'}\frac{\varphi_{\lambda'}^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{ab''}}}a_{\bar\beta}^\dagger a_{\bar\alpha}^\dagger\Big\}\\
&\quad+\frac{\sqrt{3}}{2}\sum_\gamma\Big\{\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\varphi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\bar\gamma_1}a_{\bar\gamma_2}+\sqrt{2}\sum_{\gamma_1\gamma_2\alpha}\delta_{\gamma\gamma_1}\varphi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\bar\gamma_2}a_{\bar\alpha}\\
&\quad+\sqrt{2}\sum_{(\alpha\beta)\gamma'}\delta_{\gamma\gamma'}\frac{\varphi_\lambda^{(3)}(\alpha\beta;\gamma')}{\sqrt{1+\delta_{ab}}}a_{\bar\alpha}a_{\bar\beta}\Big\}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}\Big]|\Phi_0\rangle\\
&=(\psi_{\lambda'}^T\psi_\lambda-\varphi_{\lambda'}^T\varphi_\lambda)|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle\,,
\end{aligned} \tag{4·12}$$

where we have dropped all terms with $O(n_0/2\Omega)\approx O(k^2/2\Omega)\approx 0$ according to the basic approximation in the NTD method, and have used Eq. (3·6) in the last relation.

The ground-state-correlation function $\chi_{J=0}(\alpha\beta\gamma\delta)$ has to satisfy Eq. (4·4), so that we obtain, with the aid of (4·5),

$$\hat{S}_-(a)|\Phi_0\rangle=0\,, \tag{4·13}$$

where $\hat{S}_-(a)$ is defined in Eq. (2·2). Equation (4·13) means that the correlated ground state has no zero-coupled quasi-particle pairs. With the aid of Eq. (4·13), we have

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=[\hat{S}_-(a), Y_\lambda^\dagger]|\Phi_0\rangle\,. \tag{4·14}$$

Since the inner product of the state vector on the right-hand side of Eq. (4·14) is of the order of $O(n_0/2\Omega)\approx 0$, we can also see that the dressed 3-quasi-particle states have no zero-coupled pairs under the basic approximation $O(n_0/2\Omega)\approx 0$, i.e.,

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=0\,. \tag{4·15}$$

It is further seen that the "one-quasi-particle" states $Y_{s=1/2,\alpha}^\dagger|\Phi_0\rangle\equiv a_\alpha^\dagger|\Phi_0\rangle$ (with $\Delta v=1$) also have no zero-coupled pairs, i.e.,

$$\hat{S}_-(a)a_\beta^\dagger|\Phi_0\rangle=0\,, \tag{4·16}$$

because we have

$$\hat{S}_-(a)a_\beta^\dagger|\Phi_0\rangle=[\hat{S}_-(a),a_\beta^\dagger]|\Phi_0\rangle=\delta_{ab}a_{\bar\beta}|\Phi_0\rangle$$

the inner product of which is of order $O(n_0/2\Omega)\approx 0$ by definition $\langle\Phi_0|a_\alpha^\dagger a_\beta|\Phi_0\rangle\cong\delta_{\alpha\beta}\cdot n_0/2\Omega$. Therefore, our quasi-particle NTD subspace, which consists of the modes with the transferred seniority $\Delta v(=2s)=1$ and 3, does not include any zero-coupled quasi-particle pair within the basic approximation $O(n_0/2\Omega)\approx 0$. It is, thus, orthogonal to any pairing-vibrational "collective" state.

§ 5. Transcription of Hamiltonian and electromagnetic multipole operators into the quasi-particle NTD subspace

The basis vectors of the quasi-particle NTD subspace under consideration are

$$\{\ Y^\dagger_{s=1/2,\alpha}|\Phi_0\rangle\equiv a_\alpha^\dagger|\Phi_0\rangle\,,\quad Y^\dagger_{s=3/2,\lambda}|\Phi_0\rangle\equiv Y_\lambda^\dagger|\Phi_0\rangle\ \}\,, \tag{5·1}$$

the orthonormality of which is satisfied (under the basic approximation $O(n_0/2\Omega)\approx 0$) because of Eqs. (4·11) and (4·12). The unit operator in this subspace is defined by

$$\mathbf{1}=\sum_\alpha \boldsymbol{a}_\alpha^\dagger\boldsymbol{a}_\alpha+\sum_\lambda \boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_\lambda\,, \tag{5·2}$$

where

$$\boldsymbol{a}_\alpha^\dagger=a_\alpha^\dagger|\Phi_0\rangle\langle\Phi_0|\,,\quad \boldsymbol{Y}_\lambda^\dagger=Y_\lambda^\dagger|\Phi_0\rangle\langle\Phi_0|\,. \tag{5·3}$$

The elementary excitation operators $(\boldsymbol{a}_\alpha^\dagger, \boldsymbol{Y}_\lambda^\dagger)$ in the quasi-particle NTD subspace satisfy the relations

$$\boldsymbol{a}_\alpha|\Phi_0\rangle=\boldsymbol{Y}_\lambda|\Phi_0\rangle=0 \tag{5·4}$$

and

$$\begin{aligned}&\{\boldsymbol{Y}_\lambda,\boldsymbol{Y}_{\lambda'}^\dagger\}_+|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle\,,\\&\{\boldsymbol{a}_\alpha,\boldsymbol{a}_\beta^\dagger\}_+|\Phi_0\rangle=\delta_{\alpha\beta}|\Phi_0\rangle\,,\\&\{\boldsymbol{Y}_\lambda,\boldsymbol{a}_\alpha^\dagger\}_+|\Phi_0\rangle=0\,.\end{aligned} \tag{5·5}$$

The non-repeatability of the excitations is a trivial result and is expressed as

$$\boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_{\lambda'}^\dagger|\Phi_0\rangle=\boldsymbol{a}_\alpha^\dagger\boldsymbol{a}_\beta^\dagger|\Phi_0\rangle=\boldsymbol{a}_\alpha^\dagger\boldsymbol{Y}_\lambda^\dagger|\Phi_0\rangle=0\,. \tag{5·6}$$

Now let us consider the transcription of a physical operator $\hat{F}$ (such as the Hamiltonian and the electromagnetic multipole operators) into the NTD subspace. According to Eq. (2·28), to do this, it is necessary to evaluate the matrix elements $\langle\Phi_0|Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle$ within the framework of the NTD approximation. For this purpose we have fully to use the properties of the eigenmode operators, such as Eqs. (4·11) and (4·12), and so rewrite the matrix element in the following two forms:

$$\langle\Phi_0|\,Y_{s>\lambda}\hat{F}Y^\dagger_{s'<\lambda'}|\Phi_0\rangle$$
$$=\begin{cases}\langle\Phi_0|\{[Y_{s>\lambda},\hat{F}],\,Y^\dagger_{s'<\lambda'}\}_+|\Phi_0\rangle+\langle\Phi_0|\hat{F}\{Y_{s>\lambda},\,Y^\dagger_{s'<\lambda'}\}_+|\Phi_0\rangle\,, & (5\cdot 7\text{a})\\ \langle\Phi_0|\{Y_{s>\lambda},\,[\hat{F},\,Y^\dagger_{s'<\lambda'}]\}_+|\Phi_0\rangle+\langle\Phi_0|\{Y_{s>\lambda},\,Y^\dagger_{s'<\lambda'}\}_+\hat{F}|\Phi_0\rangle\,. & (5\cdot 7\text{b})\end{cases}$$

The evaluation of the first terms, which include a double commutator, is easily performed. As for the second terms, it is convenient to use the form in (5·7a), because we generally obtain

$$\{Y_{s<\lambda},\,Y^\dagger_{s'>\lambda'}\}_+|\Phi_0\rangle\neq 0\quad\text{for}\quad s\neq s'\,,$$

i.e.,

$$\langle\Phi_0|\{Y_{s>\lambda},\,Y^\dagger_{s'<\lambda'}\}_+\neq 0\quad\text{for}\quad s\neq s'\,,$$

which is in contrast with the simple relation (2·24), i.e., (4·11). Therefore, we adopt the form (5·7a) and easily obtain

$$\langle\Phi_0|\,Y_{s>\lambda}\hat{F}Y^\dagger_{s'<\lambda'}|\Phi_0\rangle=\langle\Phi_0|\{[Y_{s>\lambda},\hat{F}],\,Y^\dagger_{s'<\lambda'}\}_+|\Phi_0\rangle+\delta_{ss'}\delta_{\lambda\lambda'}\langle\Phi_0|\hat{F}|\Phi_0\rangle\,.\qquad(5\cdot 8)$$

This means *a prescription rule* in evaluating the matrix elements $\langle\Phi_0|\,Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle$: *At first we perform the calculation of the commutation relation between the physical operator* $\hat{F}$ *and the eigenmode operator of the higher transferred seniority number, and then take the anti-commutation relation with the one of the lower transferred seniority number.*

Using the prescription rule, we obtain

$$\begin{aligned}\langle\Phi_0|\,Y_{\lambda'}HY_\lambda{}^\dagger|\Phi_0\rangle\,(=\langle\Phi_0|\,Y_{\lambda'}(H_0+H_X+H_V)\,Y_\lambda{}^\dagger|\Phi_0\rangle)&\\ =\{\omega_\lambda+\langle\Phi_0|H|\Phi_0\rangle\}\,\delta_{\lambda\lambda'}\,,&\\ \langle\Phi_0|a_\beta Ha_\alpha{}^\dagger|\Phi_0\rangle\,(=\langle\Phi_0|a_\beta(H_0+H_X+H_V)\,a_\alpha{}^\dagger|\Phi_0\rangle)&\\ =\{E_a+\langle\Phi_0|H|\Phi_0\rangle\}\,\delta_{\alpha\beta}&\qquad(5\cdot 9)\end{aligned}$$

and

$$\begin{aligned}&\langle\Phi_0|\,Y_\lambda Ha_\alpha{}^\dagger|\Phi_0\rangle\,(=\langle\Phi_0|\,Y_\lambda H_Ya_\alpha{}^\dagger|\Phi_0\rangle)\\ &=2\sqrt{6}\sum_{\alpha'\beta'\gamma'}(u_{a'}u_{b'}v_{c'}u_a-v_{a'}v_{b'}u_{c'}v_a)\mathcal{V}_{\alpha'\beta'\bar{\gamma}'\alpha}\psi_\lambda(\alpha'\beta'\gamma')\\ &\quad+2\sqrt{2}\sum_{\alpha_1'\alpha_2'\alpha_3'}\{(u^3_{a'}v_a-v^3_{a'}u_a)+2u_{a'}v_{a'}(u_{a'}u_a-v_{a'}v_a)\}\mathcal{V}_{\alpha_1'\alpha_2'\bar{\alpha}_3'\alpha}\varphi_\lambda{}^{(1)}(\alpha_1'\alpha_2'\alpha_3')\\ &\quad-4\sum_{\substack{\alpha_1'\alpha_2'\gamma\\(a'\neq c)}}[u_{a'}v_{a'}(u_au_c-v_av_c)\cdot\mathcal{V}_{\alpha_1'\alpha_2'\alpha\bar{\gamma}}+(u^2_{a'}-v^2_{a'})\,(u_av_c+v_au_c)\cdot\mathcal{V}_{\alpha_1'\gamma\bar{\alpha}_2'\alpha}]\\ &\qquad\qquad\times\varphi_\lambda{}^{(2)}(\alpha_1'\alpha_2';\gamma)\\ &\quad+4\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}[(u_{a'}u_{b'}u_{c'}v_a-v_{a'}v_{b'}v_{c'}u_a)\cdot\mathcal{V}_{\alpha'\beta'\bar{\gamma}'\alpha}\end{aligned}$$

$$-2(u_{a'}v_{b'}u_{c'}u_a - v_{a'}u_{b'}v_{c'}v_a)\cdot \mathcal{V}_{a\tilde{\alpha}'\beta'\gamma'}]\frac{\varphi_\lambda^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}. \tag{5·10}$$

According to Eq. (2·28), we can thus obtain the explicit form of the transcribed Hamiltonian in the quasi-particle NTD subspace:

$$\boldsymbol{H} = \mathbf{1}H\mathbf{1} = U\cdot\mathbf{1} + \boldsymbol{H}^{(0)} + \boldsymbol{H}^{(\mathrm{int})}$$

$$\equiv U\cdot\mathbf{1} + \sum_\alpha E_a \boldsymbol{a}_\alpha{}^\dagger \boldsymbol{a}_\alpha + \sum_\lambda \omega_\lambda \boldsymbol{Y}_\lambda{}^\dagger \boldsymbol{Y}_\lambda + \sum_{\alpha\lambda} V_{\mathrm{int}}(\alpha,\lambda)\cdot\{\boldsymbol{Y}_\lambda{}^\dagger \boldsymbol{a}_\alpha + \boldsymbol{a}_\alpha{}^\dagger \boldsymbol{Y}_\lambda\}, \tag{5·11}$$

where

$$V_{\mathrm{int}}(\alpha,\lambda) \equiv \langle\Phi_0|Y_\lambda H a_\alpha{}^\dagger|\Phi_0\rangle \quad (=\langle\Phi_0|a_\alpha H Y_\lambda{}^\dagger|\Phi_0\rangle)$$

and U is a constant related to the energy of the correlated ground state. As is seen from the matrix elements $V_{\mathrm{int}}(\alpha,\lambda)$ given in (5·10), the *effective interaction* $\boldsymbol{H}^{(\mathrm{int})}$ between the different types of modes results only from the interactive force H_Y of the original interaction.

The electromagnetic multipole operators are the one-body operators which are generally written as

$$\hat{Q}_{LM}^{(\pm)} = \sum_{\alpha\beta}(\alpha|Q_{LM}^{(\pm)}|\beta)c_\alpha{}^\dagger c_\beta$$

$$\equiv \sum_{\alpha\beta}\{\mathring{Q}_{LM}^{(\pm)}(\alpha\beta)(a_\alpha{}^\dagger a_\beta{}^\dagger \pm a_{\tilde{\beta}} a_{\tilde{\alpha}}) + \bar{Q}_{LM}^{(\pm)}(\alpha\beta)a_\alpha{}^\dagger a_\beta\}$$

$$+\sum_\alpha(\alpha|Q_{LM}^{(\pm)}|\alpha)v_a{}^2\cdot\frac{1\pm1}{2}, \tag{5·12}$$

where the double symbol $(\pm)$ is related to the conventional transformation property*) of the multipole operators with respect to the time reversal, and $\mathring{Q}_{LM}^{(\pm)}(\alpha\beta)$ and $\bar{Q}_{LM}^{(\pm)}(\alpha\beta)$ are defined respectively by

$$\mathring{Q}_{LM}^{(\pm)}(\alpha\beta) = -\tfrac{1}{2}(\alpha|Q_{LM}^{(\pm)}|\tilde{\beta})(u_a v_b \pm v_a u_b),$$

$$\bar{Q}_{LM}^{(\pm)}(\alpha\beta) = (\alpha|Q_{LM}^{(\pm)}|\beta)(u_a u_b \mp v_a v_b). \tag{5·13}$$

By definition, $\mathring{Q}_{LM}^{(\pm)}(\alpha\beta)$ satisfies the relation $\mathring{Q}_{LM}^{(\pm)}(\beta\alpha) = -\mathring{Q}_{LM}^{(\pm)}(\alpha\beta)$ and $\bar{Q}_{LM}^{(\pm)}(\alpha\beta)$ satisfies the relation $\bar{Q}_{LM}^{(\pm)}(\tilde{\beta}\tilde{\alpha}) = \pm\bar{Q}_{LM}^{(\pm)}(\alpha\beta)$. With the aid of the prescription rule (5·8), we now obtain the transcribed electromagnetic operators in the quasi-particle NTD subspace:

$$\hat{Q}_{LM}^{(\pm)} \to \boldsymbol{Q}_{LM}^{(\pm)} = \mathbf{1}\hat{Q}_{LM}^{(\pm)}\mathbf{1}$$

$$= C_{LM}^{(\pm)}\mathbf{1} + \sum_{\alpha\beta} Q_{LM}^{(\pm)}(\alpha\beta)\boldsymbol{a}_\alpha{}^\dagger \boldsymbol{a}_\beta + \sum_{\lambda\lambda'} Q_{LM}^{(\pm)}(\lambda\lambda')\boldsymbol{Y}_\lambda{}^\dagger \boldsymbol{Y}_{\lambda'}$$

$$+\sum_{\alpha\lambda}\{Q_{LM}^{(\pm)}(\alpha\lambda)\boldsymbol{a}_\alpha{}^\dagger \boldsymbol{Y}_\lambda + Q_{LM}^{(\pm)}(\lambda\alpha)\boldsymbol{Y}_\lambda{}^\dagger \boldsymbol{a}_\alpha\}. \tag{5·14}$$

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*) The time reverse of the electromagnetic multipole operator Q_{LM} is characterized by $TQ_{LM}T^\dagger = \tau\cdot(-)^M Q_{L-M}$, where $\tau = \pm1$. The operators $Q_{LM}^{(+)}$ and $Q_{LM}^{(-)}$ denote those with $\tau = +1$ and $\tau = -1$, respectively.

Explicit expressions of the coefficients $C_{LM}^{(\pm)}$, $Q_{LM}^{(\pm)}(\alpha\beta)$, $Q_{LM}^{(\pm)}(\lambda\lambda')$ and $Q_{LM}^{(\pm)}(\lambda\alpha)$ are given in Appendix III.

§ 6. Concluding remarks

On the basis of the quasi-particle NTD method, we have developed a systematic microscopic theory of describing the collective excitations in spherical odd-mass nuclei. The theory have led us to the concept of a new kind of fermion-type collective excitation mode, in the just same manner as the RPA for even-even nuclei leads us to the concept of "phonon" as a boson. As will be discussed in the next paper,[5] recent accumulation of experimental data seems to confirm the systematic presence of such a kind of collective excited state in many odd-mass nuclei.

So far, the collective excited states in odd-mass nuclei has been conventionally described in terms of the language of the quasi-particle-phonon-coupling theory.[7] Needless to say, the framework of our theory includes that of the phonon-quasi-particle-coupling theory as a special approximated one. In this point of view, it is quite interesting to investigate microscopic structure of breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture", in the light of our theory. This will be made in a forthcoming paper.

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Appendix I

In Eq. (3·2) we have used the projection operators, $\boldsymbol{P}(\alpha\beta\gamma)$ and $\boldsymbol{P}(\alpha\beta)$, given by

$$\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma)\equiv\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')f(\alpha'\beta'\gamma'),$$
$$\boldsymbol{P}(\alpha\beta)g(\alpha\beta)\equiv\sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')g(\alpha'\beta'), \qquad (\mathrm{AI}\cdot 1)$$

by which arbitrary functions $f(\alpha\beta\gamma)$ and $g(\alpha\beta)$ are antisymmetrized with respect to (α,β,γ) and (α,β) respectively, and any angular-momentum-zero-coupled-pair component is removed out of the antisymmetrized functions $f^A(\alpha\beta\gamma)$ and $g^A(\alpha\beta)$. Here we define their explicit expressions.

The antisymmetrization operator of three-body system is given by

$$P^A(\alpha\beta\gamma|\alpha'\beta'\gamma')=\frac{1}{3!}\sum_{P(\alpha'\beta'\gamma')}\delta_P\cdot(\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}), \qquad (\mathrm{AI}\cdot 2)$$

where $\sum_{P(\alpha'\beta'\gamma')}$ denotes the summation over all the permutations with respect to (α',β',γ') and δ_P takes the value $+1$ for even permutations and the value -1 for odd permutations. As is easily seen, this operator satisfies the relation of projection operator:

$$\sum_{\alpha''\beta''\gamma''} P^A(\alpha\beta\gamma|\alpha''\beta''\gamma'')P^A(\alpha''\beta''\gamma''|\alpha'\beta'\gamma') = P^A(\alpha\beta\gamma|\alpha'\beta'\gamma'). \qquad \text{(AI·3)}$$

In the coupled-angular-momentum representation, the antisymmetrization operator (AI·2) is represented by

$$\begin{aligned} P_I^A(ab(J)c|a'b'(J')c') &= \sum_{m_\alpha m_\beta m_\gamma} \cdot \sum_{m_\alpha' m_\beta' m_\gamma'} \cdot \sum_{MM'} (j_a j_b m_\alpha m_\beta|JM) \\ &\times (Jj_c M m_\gamma|IK)\,(j_a' j_b' m_\alpha' m_\beta'|J'M')\,(J'j_c' M' m_\gamma'|IK) \\ &\times P^A(\alpha\beta\gamma|\alpha'\beta'\gamma'). \end{aligned} \qquad \text{(AI·4)}$$

In this representation, with the aid of Eq. (AI·4) the projection operator $P_I(ab(J)c|a'b'(J')c')$, which removes out any angular-momentum-zero-coupled-pair component (from the functions on which it operates), is easily obtained by

$$P_I(ab(J)c|a'b'(J')c') = P_I^A(ab(J)c|a'b'(J')c')$$

$$- \begin{cases} 0 & \text{for } a\neq b\neq c, \\ P_I^A(ab(J)c|a'b'(J')c')\,\delta_{J0} & \text{for } a=b\neq c, \\ \dfrac{P_I^A(ab(0)c|a'b'(J')c')P_I^A(ab(J)c|a'b'(0)c')}{P_I^A(ab(0)c|a'b'(0)c')} & \text{for } a=b=c, \\ -(-)^{j_b+j_c+J}(2J+1)^{1/2}\begin{Bmatrix} j_a & j_b & J \\ I & j_c & 0 \end{Bmatrix} P_I^A(ac(0)b|a'b'(J')c') & \\ & \text{for } a=c\neq b, \\ -(-)^{j_b+j_c}(2J+1)^{1/2}\begin{Bmatrix} j_b & j_a & J \\ I & j_c & 0 \end{Bmatrix} P_I^A(bc(0)a|a'b'(J')c') & \\ & \text{for } a\neq b=c. \end{cases} \qquad \text{(AI·5)}$$

With the expression (AI·5), the projection operator $P(\alpha\beta\gamma|\alpha'\beta'\gamma')$ in (AI·1) (in the m-scheme) is defined through the relation

$$\begin{aligned} P_I(ab(J)c|a'b'(J')c') &= \sum_{m_\alpha m_\beta m_\gamma} \cdot \sum_{m_\alpha' m_\beta' m_\gamma'} \cdot \sum_{MM'} (j_a j_b m_\alpha m_\beta|JM) \\ &\times (Jj_c M m_\gamma|IK)\,(j_a' j_b' m_\alpha' m_\beta'|J'M')\,(J'j_c' M' m_\gamma'|IK)\,P(\alpha\beta\gamma|\alpha'\beta'\gamma'). \end{aligned}$$

The projection operator $\boldsymbol{P}(\alpha\beta)$ of two-body system is defined in a similar way and its explicit form is trivial.

Appendix II

Here, we write down the explicit forms for the matrix elements of $\boldsymbol{D}$, $\boldsymbol{d}$

and A in the eigenvalue equation (3·3).

With the definitions

$$V^{(F)}_{\alpha\beta\alpha'\beta'} = 2V_X^{(1)}(\alpha\beta,\alpha'\beta') + V_X^{(2)}(\alpha\beta,\alpha'\beta') - V_X^{(2)}(\beta\alpha,\alpha'\beta'), \tag{AII·1}$$

$$\begin{aligned} V^{(B)}_{\alpha\beta\alpha'\beta'} &= 2V_Y(\alpha\beta,\alpha'\beta') + 2V_Y(\alpha'\beta',\alpha\beta) - 2V_Y(\alpha\tilde{\beta}',\tilde{\beta}\alpha') \\ &\quad - 2V_Y(\tilde{\beta}\alpha',\alpha\tilde{\beta}') + 2V_Y(\beta\tilde{\beta}',\tilde{\alpha}\alpha') + 2V_Y(\tilde{\alpha}\alpha',\beta\tilde{\beta}'), \end{aligned} \tag{AII·2}$$

where $V_X^{(1)}(\alpha\beta,\alpha'\beta')$, $V_X^{(2)}(\alpha\beta,\alpha'\beta')$ and $V_Y(\alpha\beta,\alpha'\beta')$ are defined by (1·2), the matrix elements of $3\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ are given by

$$\begin{aligned}
3D_{\alpha\beta\gamma,\alpha'\beta'\gamma'} &= \boldsymbol{P}(\alpha\beta\gamma)\{(E_a+E_b+E_c)\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'} + 3V^{(F)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\}\boldsymbol{P}^T(\alpha'\beta'\gamma'),\\
d_{\alpha_1\alpha_2\alpha_3,\alpha_1'\alpha_2'\alpha_3'} &= \boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\{E_a\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\delta_{\alpha_3\alpha_3'} + V^{(F)}_{\alpha_1\alpha_2\alpha_1'\alpha_2'}\delta_{\alpha_3\alpha_3'}\}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3'),\\
d_{\alpha_1\alpha_2\gamma,\alpha_1'\alpha_2'\gamma'} &= \boldsymbol{P}(\alpha_1\alpha_2)\{E_c\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\delta_{\gamma\gamma'} + 2V^{(F)}_{\alpha_2\gamma\alpha_2'\gamma'}\delta_{\alpha_1\alpha_1'}\}\boldsymbol{P}^T(\alpha_1'\alpha_2'),\\
d_{\alpha\beta\gamma,\alpha'\beta'\gamma'} &= (E_a+E_b-E_c)\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'} + 2\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}V^{(F)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},\\
d_{\alpha_1\alpha_2\alpha_3,\alpha_1'\alpha_2'\gamma'} &= \sqrt{2}\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(F)}_{\alpha_1\alpha_2\alpha_2'\gamma'}\delta_{\alpha_3\alpha_1'}\boldsymbol{P}^T(\alpha_1'\alpha_2'),\\
d_{\alpha_1\alpha_2\alpha_3,\alpha'\beta'\gamma'} &= \sqrt{2}\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(F)}_{\alpha_1\alpha_2\alpha'\beta'}\delta_{\alpha_3\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},\\
d_{\alpha_1\alpha_2\gamma,\alpha'\beta'\gamma'} &= 2\boldsymbol{P}(\alpha_1\alpha_2)V^{(F)}_{\alpha_2\gamma\alpha'\beta'}\delta_{\alpha_1\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},\\
A_{\alpha\beta\gamma,\alpha_1'\alpha_2'\alpha_3'} &= \sqrt{3}\boldsymbol{P}(\alpha\beta\gamma)V^{(B)}_{\alpha\beta\alpha_1'\alpha_2'}\delta_{\gamma\alpha_3'}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3'),\\
A_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'} &= \sqrt{6}\boldsymbol{P}(\alpha\beta\gamma)V^{(B)}_{\alpha\beta\alpha_2'\gamma'}\delta_{\gamma\alpha_1'}\boldsymbol{P}^T(\alpha_1'\alpha_2'),\\
A_{\alpha\beta\gamma,\alpha'\beta'\gamma'} &= \sqrt{6}\boldsymbol{P}(\alpha\beta\gamma)V^{(B)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}}.
\end{aligned} \tag{AII·3}$$

Here we have used the abbreviations for the projection operators in (4·7), for simplicity.

Appendix III

Here, we write down the explicit expressions of the coefficients in Eq. (5·14):

$$C^{(\pm)}_{LM} = \sum_\alpha (\alpha|Q^{(\pm)}_{LM}|\alpha)v_a^2\cdot\frac{1\pm1}{2}, \tag{AIII·1}$$

$$\begin{aligned} Q^{(\pm)}_{LM}(\alpha\beta) &\equiv \langle\Phi_0|a_\alpha\hat{Q}^{(\pm)}_{LM}a_\beta^\dagger|\Phi_0\rangle - C^{(\pm)}_{LM}\delta_{\alpha\beta}\\ &= \langle\Phi_0|\{[a_\alpha,\hat{Q}^{(\pm)}_{LM}],a_\beta^\dagger\}_+|\Phi_0\rangle = \bar{Q}^{(\pm)}_{LM}(\alpha\beta), \end{aligned} \tag{AIII·2}$$

$$\begin{aligned} Q^{(\pm)}_{LM}(\lambda\alpha) &\equiv \langle\Phi_0|Y_\lambda\hat{Q}^{(\pm)}_{LM}a_\alpha^\dagger|\Phi_0\rangle\\ &= \langle\Phi_0|\{[Y_\lambda,\hat{Q}^{(\pm)}_{LM}],a_\alpha^\dagger\}_+|\Phi_0\rangle \end{aligned}$$

$$=\sqrt{6}\sum_{\alpha'\beta'\gamma'}\delta_{\alpha\gamma'}\mathring{Q}^{(\pm)}_{LM}(\alpha'\beta')\psi_\lambda(\alpha'\beta'\gamma')$$

$$\pm\Big\{\sqrt{2}\sum_{\alpha_1\alpha_2\alpha_3}\delta_{\alpha\alpha_3}\mathring{Q}^{(\pm)}_{LM}(\alpha_1\alpha_2)\varphi_\lambda{}^{(1)}(\alpha_1\alpha_2\alpha_3)$$

$$+2\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\delta_{\alpha\alpha_1}\mathring{Q}^{(\pm)}_{LM}(\alpha_2\gamma)\varphi_\lambda{}^{(2)}(\alpha_1\alpha_2;\gamma)$$

$$+2\sum_{\substack{(\alpha'\beta')\gamma'\\(a,b\neq c)}}\delta_{\alpha\gamma'}\mathring{Q}^{(\pm)}_{LM}(\alpha'\beta')\frac{\varphi_\lambda{}^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}\Big\},\qquad\text{(AIII}\cdot\text{3)}$$

$$Q^{(\pm)}_{LM}(\lambda\lambda')\equiv\langle\Phi_0|Y_\lambda\widehat{Q}^{(\pm)}_{LM}Y^\dagger_{\lambda'}|\Phi_0\rangle-C^{(\pm)}_{LM}\delta_{\lambda\lambda'}$$

$$=\langle\Phi_0|\{[Y_\lambda,\widehat{Q}^{(\pm)}_{LM}],Y^\dagger_{\lambda'}\}_+|\Phi_0\rangle$$

$$=3\sum\cdot\sum\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)\bar{Q}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}\boldsymbol{P}^T(\alpha'\beta'\gamma')\psi_{\lambda'}(\alpha'\beta'\gamma')$$

$$-\sum\cdot\sum\varphi_\lambda{}^{(1)}(\alpha_1\alpha_2\alpha_3)\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\{\bar{Q}^{(\pm)}_{LM}(\alpha_3\alpha_3')\mp2\bar{Q}^{(\pm)}_{LM}(\alpha_3\alpha_3')\}$$

$$\times\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3')\varphi^{(1)}_{\lambda'}(\alpha_1'\alpha_2'\alpha_3')$$

$$\pm\sqrt{2}\sum\cdot\sum\{\varphi_\lambda{}^{(2)}(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\bar{Q}^{(\pm)}_{LM}(\gamma\alpha_3')\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}$$

$$\times\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3')\varphi^{(1)}_{\lambda'}(\alpha_1'\alpha_2'\alpha_3')+\varphi_\lambda{}^{(1)}(\alpha_1'\alpha_2'\alpha_3')$$

$$\times\boldsymbol{P}(\alpha_1'\alpha_2'\alpha_3')\bar{Q}^{(\pm)}_{LM}(\alpha_3'\gamma)\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\boldsymbol{P}^T(\alpha_1\alpha_2)\varphi^{(2)}_{\lambda'}(\alpha_1\alpha_2;\gamma)\}$$

$$-\sqrt{2}\sum\cdot\sum\Big\{\varphi_\lambda{}^{(3)}(\alpha\beta;\gamma)\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\bar{Q}^{(\pm)}_{LM}(\gamma\alpha_3')\delta_{\alpha_1'\alpha}\delta_{\alpha_2'\beta}$$

$$\times\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3')\varphi^{(1)}_{\lambda'}(\alpha_1'\alpha_2'\alpha_3')+\varphi_\lambda{}^{(1)}(\alpha_1'\alpha_2'\alpha_3')$$

$$\times\boldsymbol{P}(\alpha_1'\alpha_2'\alpha_3')\bar{Q}^{(\pm)}_{LM}(\alpha_3'\gamma)\delta_{\alpha_1'\alpha}\delta_{\alpha_2'\beta}\frac{\boldsymbol{P}^T(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\varphi^{(3)}_{\lambda'}(\alpha\beta;\gamma)\Big\}$$

$$-\sum\cdot\sum\varphi_\lambda{}^{(2)}(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\{(1\mp1)\bar{Q}^{(\pm)}_{LM}(\alpha_1\alpha_1')\delta_{\alpha_2\alpha_2'}\delta_{\gamma\gamma'}$$

$$\mp\bar{Q}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}-\bar{Q}^{(\pm)}_{LM}(\alpha_1\alpha_1')\delta_{\alpha_2\gamma'}\delta_{\gamma\alpha_2'}\}$$

$$\times\boldsymbol{P}^T(\alpha_1'\alpha_2')\varphi^{(2)}_{\lambda'}(\alpha_1'\alpha_2';\gamma')$$

$$-2\sum\cdot\sum\Big[\varphi_\lambda{}^{(3)}(\alpha\beta;\gamma)\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\{\bar{Q}^{(\pm)}_{LM}(\gamma\alpha_1')\delta_{\alpha\alpha_2'}\delta_{\beta\gamma'}$$

$$\mp\bar{Q}_{LM}(\alpha\alpha_2')\delta_{\beta\gamma'}\delta_{\gamma\alpha_1'}\}\boldsymbol{P}^T(\alpha_1'\alpha_2')\varphi^{(2)}_{\lambda'}(\alpha_1'\alpha_2';\gamma')$$

$$+\varphi_\lambda{}^{(2)}(\alpha_1'\alpha_2';\gamma')\boldsymbol{P}(\alpha_1'\alpha_2')\{\bar{Q}^{(\pm)}_{LM}(\alpha_1'\gamma)\delta_{\alpha\alpha_2'}\delta_{\beta\gamma'}$$

$$\mp\bar{Q}^{(\pm)}_{LM}(\alpha_2'\alpha)\delta_{\beta\gamma'}\delta_{\gamma\alpha_1'}\}\frac{\boldsymbol{P}^T(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\varphi^{(3)}_{\lambda'}(\alpha\beta;\gamma)\Big]$$

$$-2\sum\cdot\sum\varphi_\lambda{}^{(3)}(\alpha\beta;\gamma)\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\{\bar{Q}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}$$

$$\mp2\bar{Q}^{(\pm)}_{LM}(\beta\beta')\delta_{\alpha\alpha'}\delta_{\gamma\gamma'}\}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}}\varphi^{(3)}_{\lambda'}(\alpha'\beta';\gamma').\qquad\text{(AIII}\cdot\text{4)}$$

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Microscopic Structure of a New Type of Collective Excitation in Odd-Mass Mo, Ru, I, Cs and La Isotopes

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With the aid of microscopic theory of collective excitations in spherical odd-mass nuclei proposed by Kuriyama, Marumori and Matsuyanagi, structures of low-lying collective $5/2^+$ states in odd-mass I, Cs and La isotopes and of collective $3/2^+$ states in odd-mass Mo and Ru isotopes are investigated. These collective $5/2^+$ and $3/2^+$ states, which are hard to understand within the framework of the conventional quasi-particle-phonon-coupling theory, are identified as a new kind of fermion-type collective excitation mode. The change in microscopic structure of these states depending on the mass number is also investigated in relation with the shell structure.

§ 1. Introduction

From among complicated spectra of the low-energy excitations in odd-mass nuclei with mass numbers around $A \sim 100$, recent experiments reveal noticeable "collective" behaviour of the first $3/2^+$ states in odd-neutron nuclei and that of the second $5/2^+$ states in odd-proton nuclei, which are difficult to understand within the framework of the conventional quasi-particle-phonon-coupling (QPC) theory:[1] In odd-neutron Mo and Ru isotopes with $N=53, 55$ and 57, there systematically appear "collective" $3/2^+$ states with enhanced $E2$- and hindered $M1$-transitions to the single-quasi-particle (1QP) $5/2^+$ states.[2)~5)] (See Fig. 1(b).) In odd-proton I, Cs and La isotopes, the second $5/2^+$ states display the enhanced $E2$- and retarded $M1$-transitions to the 1QP $7/2^+$ states, indicating their strong "collective" nature characteristically.[6)~9)] The excitation energies of the second $5/2^+$ states (measured from the 1QP $7/2^+$ states) decrease as the neutron number goes from the magic number $N=82$ to $N=72$. (See Fig. 1(a).) Furthermore, the first and the second $5/2^+$ states are lying close, proposing an interesting problem of clarifying the difference of their microscopic structure.[6)]

Recently we have proposed a new systematic microscopic theory of describing the collective excitations in spherical odd-mass nuclei.[10),11)] The theory can be regarded as a natural extension of the conventional quasi-particle-random-phase approximation (RPA) for even-mass nuclei into the case of spherical odd-mass nuclei. In the same manner as the conventional RPA for even-mass nuclei leads us

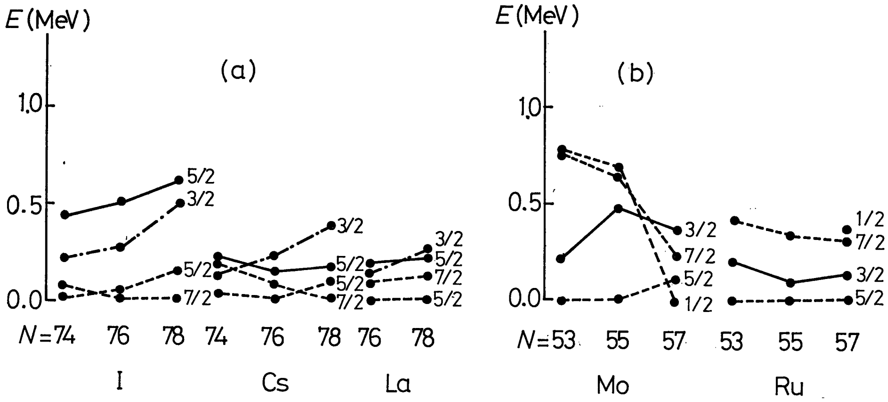

Fig. 1(a). Experimental trends of the excitation energies of the $5/2_2^+$ states (solid lines) and of the $3/2_1^+$ states (dotted lines) in odd-mass I, Cs and La isotopes.[6)~9)] The energies are presented relatively to those of the 1QP states (broken lines).

Fig. 1(b). Experimental trend of the excitation energies of the $3/2^+$ states (solid lines) in odd-mass Mo and Ru isotopes.[2)~5)] The energies are presented relatively to those of the 1QP states (broken lines).

to the concept of "phonon" as a boson, the theory necessarily leads us to the concept of a new kind of *fermion-type collective excitation mode*, i.e., the "dressed" three-quasi-particle (3QP) mode. In the light of this theory, in previous papers[12),13)] we have obtained a conclusion that the appearance of the low-lying anomalous coupling states (ACS) in odd-mass nuclei have to be regarded as a typical phenomenon in which the new kind of fermion-type collective mode manifests itself as a relatively pure eigenmode. It has also been emphasized that the physical condition of the enhancement of the three-quasi-particle (3QP) correlation (characterizing this new colletive mode) is not always specific to the ACS but also more general in odd-mass nuclei. Thus, we have suggested that the new collective mode should be expected to exist in many spherical odd-mass nuclei and to play an important role in their low-lying collective excitations.[13)]

The main purpose of this paper is to propose an interpretation to identify the above-mentioned (collective) first $3/2^+$ states (in odd-mass Mo and Ru isotopes) and the (collective) second $5/2^+$ states (in odd-mass I, Cs and La isotopes) as evidences for the presence of this new kind of collective mode. The first motive for this identification is directly obtained when we notice a similarity between the above-mentioned electromagnetic properties of the collective $3/2^+$ and $5/2^+$ states and those of the ACS with spin $I=j-1$: Characteristics of the electromagnetic properties of the ACS are 1) much enhanced $E2$-transitions to the 1QP states with spin $I=j$ which are comparable in magnitude with those of phonon transitions in neighboring even-even nuclei and 2) hindered corresponding $M1$-transitions. Of course, there is an important difference in shell structure between the collective $3/2^+$ and $5/2^+$ states and the ACS: In the case of the ACS the special situation of shell structure is the existence of a high-spin unique-parity orbit which is being filled with several nuclenos, while in the case of the collective $3/2^+$ and $5/2^+$ states many shell orbits with the same even parity are lying close and equally active for the 3QP correlation. In this point of view, it is quite interesting to see to what extent we can persist in the similarity between the ACS and the collective $3/2^+$ and $5/2^+$ states. In this paper, therefore, special interest will be taken in investigating microscopic structures of the collective $3/2^+$ and $5/2^+$ states and their change depending on the mass number.

In § 2, the dressed 3QP modes as the new type of collective modes in odd-mass Mo, Ru, I, Cs and La isotopes are presented by the use of the conventional pairing-plus-quadrupole-force model. In § 3, a criterion in investigating the similarity and difference between the 3QP correlations characterizing these new modes and those characterizing the ACS is given, and in §§ 3 and 4 discussion on the calculated results is given. In § 5, coupling effects between 1QP and the dressed 3QP modes are examined. In contrast to the case of the ACS, the dressed 3QP mode investigated in this paper is lying close, in energy, to the 1QP mode with the same spin and parity. For instance, the first $5/2^+$ states and the collective second $5/2^+$ states in I, Cs and La isotopes are lying especially close to each other.[6] At first sight, therefore, the two $5/2^+$ states seem to couple strongly with each other. However, it will be clarified that there exists an interesting mechanism to make the coupling effects weak. The concluding remark is given in § 6.

§ 2. Dressed 3QP mode as a new type of collective mode

According to the general theory,[11] the creation operator for the dressed 3QP mode is given in terms of the quasi-particle operators $(a_\alpha{}^\dagger, a_{\tilde{\alpha}} \equiv (-)^{j_a - m_\alpha} a_{-\alpha})$ *) as follows:

$$Y^\dagger_{nIK} = \frac{1}{\sqrt{3\,!}} \sum_{\alpha\beta\gamma} \psi^{(\mathrm{I})}_{nI}(\alpha\beta\gamma) \cdot \boldsymbol{P}(\alpha\beta\gamma)\, a_\alpha{}^\dagger a_\beta{}^\dagger a_\gamma{}^\dagger$$

$$+ \sum_{(\mu\nu)\gamma} \frac{1}{\sqrt{1+\delta_{mn}}} \psi^{(\mathrm{II})}_{nI}(\mu\nu;\gamma)\, \boldsymbol{P}(\mu\nu)\, a_\mu{}^\dagger a_\nu{}^\dagger a_\gamma{}^\dagger$$

$$+ \frac{1}{\sqrt{3\,!}} \sum_{\alpha_1\alpha_2\alpha_3} \varphi^{(\mathrm{I})}_{nI}(\alpha_1\alpha_2\alpha_3)\, \boldsymbol{P}(\alpha_1\alpha_2\alpha_3) \frac{1}{\sqrt{3}} :\{a^\dagger_{\alpha_1} a_{\tilde{\alpha}_2} a_{\tilde{\alpha}_3} + a_{\tilde{\alpha}_1} a^\dagger_{\alpha_2} a_{\tilde{\alpha}_3} + a_{\tilde{\alpha}_1} a_{\tilde{\alpha}_2} a^\dagger_{\alpha_3}\}:$$

*) The single-particle states are characterized by a set of quantum numbers $\alpha \equiv \{n, l, j, m,$ charge $q\}$. In association with a Greek letter α, we use a Roman letter a to denote the same set except for the magnetic quantum number m. We further use a subscript $-\alpha$, which is obtained from α by changing the sign of the magnetic quantum number.

$$+\frac{1}{\sqrt{2}}\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\varphi_{nI}^{(\mathrm{II})}(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\frac{1}{\sqrt{2}}:\{a_{\alpha_1}^{\dagger}a_{\tilde{\alpha}_2}+a_{\tilde{\alpha}_1}a_{\alpha_2}^{\dagger}\}:a_{\tilde{\gamma}}$$

$$+\sum_{\substack{(\alpha\beta)\gamma\\(a\neq c,\,b\neq c)}}\frac{1}{\sqrt{1+\delta_{ab}}}\varphi_{nI}^{(\mathrm{III})}(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)a_{\gamma}^{\dagger}a_{\tilde{\alpha}}a_{\tilde{\beta}}$$

$$+\sum_{(\mu\nu)\gamma}\frac{1}{\sqrt{1+\delta_{mn}}}\varphi_{nI}^{(\mathrm{IV})}(\mu\nu;\gamma)\boldsymbol{P}(\mu\nu)a_{\gamma}^{\dagger}a_{\tilde{\mu}}a_{\tilde{\nu}}\,. \qquad (2\cdot1)$$

Here α, β and γ denote a set of quantum numbers of the single particle states for protons (neutrons) and μ and ν those for neutrons (protons) in the case of odd proton (neutron) nuclei. The subscript i $(=1, 2, 3)$ of α is used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit a is necessary. The symbol $\sum_{(\alpha\beta)\gamma}$ represents the summation with respect to the orbit pair (ab), m_α, m_β and γ, and we have used the notation

$$\boldsymbol{P}(\alpha\beta\gamma)a_{\alpha}^{\dagger}a_{\beta}^{\dagger}a_{\gamma}^{\dagger}\equiv\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')a_{\alpha'}^{\dagger}a_{\beta'}^{\dagger}a_{\gamma'}^{\dagger},$$

$$\boldsymbol{P}(\alpha\beta)a_{\tilde{\alpha}}a_{\tilde{\beta}}\equiv\sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')a_{\tilde{\alpha}'}a_{\tilde{\beta}'}\,, \qquad (2\cdot2)$$

where the operators $\boldsymbol{P}$ denote the projection operators by which any angular-momentum-zero-coupled-pair component is removed out of $(a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger)$ and $(a_{\tilde{\alpha}}a_{\tilde{\beta}})$ respectively. The projection operators $\boldsymbol{P}$, the explicit forms of which are given in Appendix I in Ref. 11), guarantee the dressed 3QP modes to be orthogonal to both the spurious states (due to the nucleon-number non-conservation) and the pairing vibrational modes.

The collective modes given by Eq. (2·1) are characterized by the amount of seniority $\Delta v=3$ which they transfer to the correlated ground state.[11] The first two terms on the right-hand side of Eq. (2·1) represent the forward-going components and the others are the backward-going ones originated from the ground-state correlation. It is evident that the ground-state correlation is essential to bring about the collectivity of excitation modes in the doubly-open-proton-neutron systems such as nuclei under consideration. As was shown in § 3 of Ref. 11), within the framework of the new-Tamm-Dancoff approximation (i.e., the quasi-particle RPA) we obtain the eigenvalue equation which the correlation amplitudes should satisfy:

$$\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A}\\ \boldsymbol{A}^T & -\boldsymbol{d}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_{nI}\\ \boldsymbol{\varphi}_{nI}\end{bmatrix}=\omega_{nI}\begin{bmatrix}\boldsymbol{\psi}_{nI}\\ \boldsymbol{\varphi}_{nI}\end{bmatrix}, \qquad (2\cdot3)$$

where $\boldsymbol{\psi}_{nI}$ and $\boldsymbol{\varphi}_{nI}$ denote the matrix notations symbolizing the sets of the forward amplitudes $\{\psi_{nI}^{(\mathrm{I})}(\alpha\beta\gamma)$ and $\psi_{nI}^{(\mathrm{II})}(\mu\nu;\gamma)\}$ and the sets of the backward amplitudes $\{\varphi_{nI}^{(\mathrm{I})}(\alpha_1\alpha_2\alpha_3), \varphi_{nI}^{(\mathrm{II})}(\alpha_1\alpha_2;\gamma), \varphi_{nI}^{(\mathrm{III})}(\alpha\beta;\gamma)$ and $\varphi_{nI}^{(\mathrm{IV})}(\mu\nu;\gamma)\}$ respectively, and the explicit forms of matrices $\boldsymbol{D}, \boldsymbol{d}$ and $\boldsymbol{A}$ (and its transpose $\boldsymbol{A}^T$) are given in Appendix II of Ref. 11).

A program code named BARYON-1 has been constructed to solve Eq. (2·3), and in the actual calculation the conventional pairing-plus-quadrupole ($P+QQ$)-force model has been adopted. Since our aim is not to obtain a detailed quantitative fitting with experimental data but to get an essential understanding of structures of the collective $3/2^+$ and $5/2^+$ states which are hard to understand within the framework of the QPC theory, we have used the same values of the single-particle energies and of the pairing-force strength G as those adopted in the QPC theory of Kisslinger and Sorensen,[1] except for the quadrupole-force strength χ. We have also made the same truncation of shell-model space as they have made: The shell-model subspace for I, Cs and La isotopes consists of the orbits $\{\pi; 1g^+_{7/2}, 2d^+_{5/2}, 1h^-_{11/2}, 2d^+_{3/2}, 3s^+_{1/2}\}$ and $\{\nu; 2d^+_{5/2}, 1g^+_{7/2}, 3s^+_{1/2}, 1h^-_{11/2}, 2d^+_{3/2}\}$, and the subspace for Mo and Ru isotopes is composed of the orbits $\{\pi; 1f^-_{5/2}, 2p^-_{3/2}, 2p^-_{1/2}, 1g^+_{9/2}\}$ and $\{\nu; 2d^+_{5/2}, 1g^+_{7/2}, 3s^+_{1/2}, 1h^-_{11/2}, 2d^+_{3/2}\}$.

The calculation of Eq. (2·3) may be performed in two steps: In the first step the matrices $3\boldsymbol{D}$ and $\boldsymbol{d}$ are diagonalized, and then the resultant total matrix is treated as the second step. The two-step-diagonalization procedure is, of course, equivalent to the direct diagonalization. In this paper, however, we have adopted the following approximation: In the second step, the resultant matrix is truncated within 50 dimensions, i.e., 10 and 40 dimensions for the forward and backward parts respectively. Accuracy of this approximation in numerical calculations has been checked by comparing some results with the corresponding ones of full calculations, and has been assured to be satisfactory except for some special cases where the excitation energy of the dressed 3QP mode is lying extremely close to the critical point for the instability of the spherical BCS vacuum.

§ 3. Structure of collective excitations in odd-proton I, Cs and La isotopes

a) *Collective $5/2^+$ states*

In Fig. 3, the calculated excitation energies of the collective $5/2_2^+$ states as the dressed 3QP modes are shown by solid lines. The energies are measured from those of the 1QP $7/2^+$ states (denoted by broken lines). The experimental trend[6)~9)] that the excitation energies of the collective $5/2_2^+$ states (measured from those of the 1QP $7/2^+$ states) decrease as the neutron number goes away from the magic number is well realized in a rather magnified way. Such magnification is inherent to the new-Tamm-Dancoff approximation (i.e., the RPA) with $P+QQ$ force which we have adopted, as is well known in calculating the phonon energies in even-even nuclei in terms of the RPA with $P+QQ$ force. It is clear from Fig. 3 that the first $7/2_1^+$ and $5/2_1^+$ states correspond to the 1QP states (broken lines) related to the orbits $1g_{7/2}$ and $2d_{5/2}$, respectively.

Before discussing the calculated $B(E2)$-values in Table I, it may be impor-

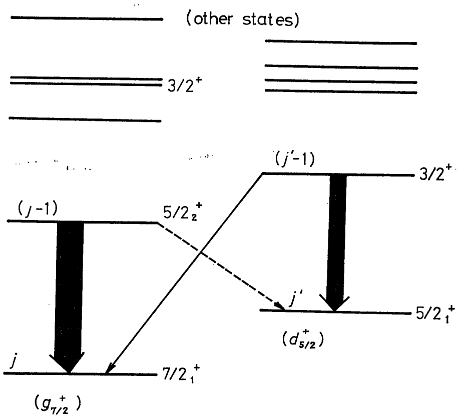

Fig. 2. Graphic explanation of the $E2$-transition properties.

tant to set up a criterion for investigating the similarity and difference between the 3QP correlations characterizing the new dressed 3QP modes and those characterizing the ACS. As has been shown in the previous papers,[12),18)] in the case of the ACS, the triggering effect of the 3QP correlations which strongly violates the concept of "phonon" in odd-mass nuclei is restricted among quasi-particles in a specific high-spin and unique-parity orbit which is being filled, because of the parity-selection property of the quadrupole force. On the contrary, in the case of the dressed 3QP mode under consideration (which has the normal positive parity), many shell orbits with the same even parity (such as $g^+_{7/2}$, $d^+_{5/2}$, $d^+_{3/2}$ and $s^+_{1/2}$) are lying close and equally active for the 3QP correlations. A criterion useful for discussing the newly produced 3QP correlations is given

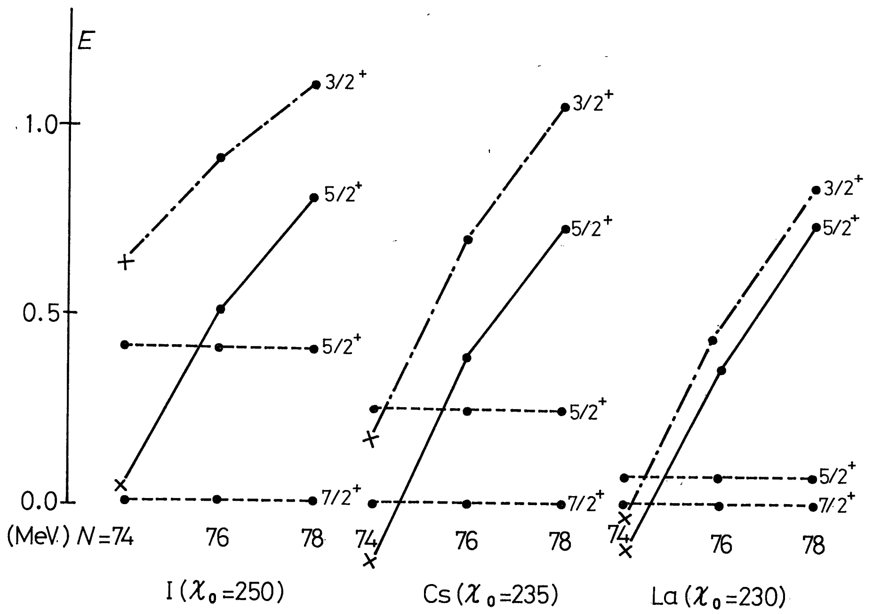

Fig. 3. Calculated excitation energies of the dressed 3QP modes with $I^\pi=5/2^+$ (solid lines) and with $I^\pi=3/2^+$ (dotted lines), in odd-mass I, Cs and La isotopes. They are measured from those of the 1QP states (broken lines). The adopted values of the quadrupole-force-strength parameter χ_0 (defined by $\chi_0\equiv\chi b^4A^{5/3}$, b^2 being the harmonic-oscillator-range parameter) are written in unit of MeV and the cross symbol × indicates a complex eigenvalue due to too strong χ_0.

as follows:[14] "If the 3QP correlations mainly come from a specific orbit, for instance, the $1g_{7/2}$ orbit when we consider the collective $5/2_2^+$ states, we may say that the structure of the $5/2_2^+$ states is similar to that of the ACS. As has been shown in Ref. 13), an important characteristic in this case is that the value of $B(E2; 5/2_2^+ \to 7/2_1^+)$ (i.e., $B(E2; I=j-1 \to j)$) is greatly enhanced compared with the values of $E2$-transitions to the other 1QP states, for instance, $B(E2; 5/2_2^+ \to 5/2_1^+)$ (i.e., $B(E2; I=j-1 \to j' \neq j)$). (See Fig. 2.) On the other hand, if the 3QP correlations among quasi-particles in the different orbits are of importance, the $E2$-transitions to the other 1QP states, for instance, $B(E2; 5/2_2^+ \to 5/2_1^+)$ have to be also strongly enhanced. In this case, therefore, we may say that the structure of $5/2_2^+$ states differs considerably from that of the ACS." Needless to say, the eigenmode operator (2·1) covers these two cases, so that we can achieve, with the aid of this criterion, an essential understanding of the microscopic structure of the new collective excited states.

Now, the calculated $B(E2)$-values in Table I(a) demonstrate that the $B(E2; 5/2_2^+ \to 7/2_1^+)$'s are stronger by about one order in magnitude than the other $B(E2; 5/2_2^+ \to 5/2_1^+)$'s. Thus we can conclude that the structure of the $5/2_2^+$

Table I. Calculated $B(E2)$-values from the $5/2_2^+$ states (a) and from the $3/2_1^+$ states (b) as the dressed 3QP modes in odd-mass I, Cs and La isotopes (in unit of $e^2 \cdot 10^{-50}\,\text{cm}^4$). The harmonic-oscillator-range parameter $b^2 = 1.0\,A^{1/3}\,\text{fm}^2$ and the effective charges $e_p^{\text{eff}} = 1.5e$ and $e_n^{\text{eff}} = 0.5e$, are used. The values of χ_0 are the same as the ones in Fig. 3.

(a)

	$5/2_2^+ \to 7/2_1^+$		$5/2_2^+ \to 5/2_1^+$	
	Cal.	Exp.	Cal.	Exp.
^{129}I	6.8	2.1±0.4[a)]	0.1	
^{131}I	4.5		0.0	
^{131}Cs	10.7	23[b)]	0.4	
^{133}Cs	8.8	10.4±1.2[a)]	0.2	
^{133}La	15.7		0.7	
^{135}La	8.2	26[c)]	0.4	1.5[c)]

(b)

	$3/2_1^+ \to 5/2_1^+$		$3/2_1^+ \to 7/2_1^+$	
	Cal.	Exp.	Cal.	Exp.
^{129}I	1.9		2.4	7.0±0.8[a)]
^{131}I	0.3		2.0	
^{131}Cs	9.0		1.7	
^{133}Cs	3.5		1.8	7.2±0.8[a)]
^{133}La	15.1		1.1	
^{135}La	10.2	>4.8[c)]	1.0	

a) Ref. 7). b) Ref. 8). c) Ref. 9).

states is similar to that of the ACS with spin $I=j-1$. (In this case, j corresponds to $1g_{7/2}$.) In fact, microscopic structure of the calculated amplitudes of

Table II. Main correlation amplitudes of the dressed 3QP modes (defined through Eq. (2·1)) in Cs, La and I. The first and second lines specify the types and the components in the coupled-angular-momentum representation, respectively. The abbreviations such as

$$\{(55)7\}=\{(d_{5/2}d_{5/2})_{I=2}g_{7/2}\},$$
$$\{(55)'7\}=\{(d_{5/2}d_{5/2})_{I=4}g_{7/2}\},$$
$$\{777\}=\{(g_{7/2})^3\},\ \cdots\cdots,\ \text{etc.},$$

are used. The values of the forward-going amplitudes are listed in the third line, while those of the backward-going in the fourth line. To specify the kind of the backward-going amplitudes, we use the same superscripts (II) and (III) that specify the kind of the amplitudes in Eq. (2·1).

(a) $5/2_2^+$ state in ^{131}Cs

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
{777}	{(55)7}	{(77)5}	{(77)'5}	{(77)3}	{(11, 11)7}	{(33)7}	{(13)7}	{(37)7}	{(11, 11)5}
0.82	0.30	−0.17	0.11	−0.10	0.62	0.31	−0.23	0.19	−0.10
0.30	0.01 (II) 0.21 (III)	−0.04 (II) −0.08 (III)	0.05	−0.04 (II) 0.00 (III)	0.45	0.22	−0.11	0.10	−0.09

(b) $5/2_2^+$ state in ^{133}La

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
{777}	{(55)7}	{(77)5}	{(77)3}	{(77)'3}	{(11, 11)7}	{(33)7}	{(13)7}	{(37)7}	{(11, 11)5}
0.81	0.43	−0.21	−0.14	−0.13	0.71	0.36	−0.27	0.21	−0.15
0.32	0.02 (II) 0.29 (III)	−0.04 (II) −0.09 (III)	−0.05 (II) −0.33 (III)	−0.06 (II)	0.52	0.25	−0.22	0.13	−0.12

(c) $3/2_1^+$ state in ^{129}I

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
{777}	{(77)5}	{(77)'5}	{(55)7}	{555}	{(11, 11)7}	{(11, 11)5}	{(33)7}	{(33)5}	{(13)7}
−0.57	0.43	0.10	0.20	0.15	0.55	0.33	0.29	0.17	−0.15
−0.08	0.02 (II) 0.16 (III)	0.03 (II)	0.01 (II) 0.08 (III)	0.06	0.22	0.19	0.11	0.10	−0.05

(d) $3/2_1^+$ state in ^{133}La

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
{(77)5}	{555}	{(55)7}	{(55)'7}	{777}	{(11, 11)5}	{(33)5}	{(13)5}	{(11, 11)7}	{(37)5}
0.64	0.59	0.21	0.16	−0.14	0.72	0.36	−0.20	0.20	0.20
0.02 (II) 0.40 (III)	0.24	0.05 (II) 0.08 (III)	0.08 (II)	−0.05	0.50	0.24	−0.22	0.14	0.14

the $5/2_2^+$ states (as the dressed 3QP modes) is very similar to that of the ACS which has been investigated in the previous papers:[12),13)] The forward-going amplitudes of $(\pi\pi\pi)$-type with the largest $\{\pi(g_{7/2})^3\}$ component and of $(\nu\nu\pi)$-type are strongly coupled with each other and the backward-going amplitudes of $(\nu\nu\pi)$-type become larger as the neutron number decreases. Some examples of the main amplitudes in Cs and La are shown in Tables II(a) and II(b).

It is rather a wonder that the overall similarity between the $5/2^+$ states and the ACS persists in spite of the different situation in their shell structure. The reason is understood as follows: In I isotopes, the chemical potential for protons lies close to the $1g_{7/2}$ orbit and the energy difference between the 1QP $1g_{7/2}$ and 1QP $2d_{5/2}$ states is relatively large (i.e., $\Delta E \sim 400$ keV), so that, realizing the similarity to the ACS, the component $\{\pi(g_{7/2})^3\}$ in the forward amplitudes $\psi^{(1)}_{nI=5/2}$ (in Eq. (2·1)) reaches the maximum. As the proton number increases, the chemical potential shifts up and the energy difference between the 1QP $1g_{7/2}$ and 1QP $2d_{5/2}$ states decreases to about $\Delta E \sim 100$ keV in La isotopes. In La isotopes, we may thus expect the components $\{\pi(g_{7/2})^2 d_{5/2}\}$ and $\{\pi\, g_{7/2}(d_{5/2})^2\}$ to grow up appreciably. However, this trend is actually not so appreciable as expected, as is seen from Tables II(a) and II(b). This is due to the following facts: i) The considerable smallness of the value of spin-flip matrix element $(2d_{5/2}\|r^2Y_2\|1g_{7/2})$ in comparison with that of non-spin-flip one $(1g_{7/2}\|r^2Y_2\|1g_{7/2})$, ii) the special favouring of the $I=(j-1)$-coupling in the $(g_{7/2})^3$-configurations and iii) the prohibition of $(d_{5/2})^3$-configurations with spin $I=5/2$.

b) *Collective $3/2^+$ states*

Here it is interesting to notice that, in addition to the collective $5/2_2^+$ states discussed above, experimental data[6)~9)] reveal the systematic presence of the $3/2_1^+$ states the excitation energies of which (measured from the 1QP $5/2_1^+$-states) decrease as the neutron number goes away from $N=82$ to $N=72$ with the same trend as in the case of the collective $5/2_2^+$ states. (See the dotted lines in Fig. 1(a).)

Regarding the $3/2_1^+$ states as the dressed 3QP modes, we have also calculated their excitation energies and $B(E2)$ values. (See the dotted lines in Fig. 3 and Table I(b).) To see the qualitative trend of the decrease in the excitation energies, we have used, in Fig. 3, the same values of the single-particle energies as adopted by Kisslinger and Sorensen[1)] in their QPC theory, although the special lowering of the excitation energies (of the $3/2_1^+$ states) sensitively depends on the adopted single-particle energy difference between the $2d_{5/2}$ and $1g_{7/2}$ orbits. If the adopted single-particle energies are modified slightly from those used by Kisslinger and Sorensen,[1)] we can easily realize the appearance of the $3/2_1^+$ states below the collective $5/2_2^+$ states, without changing the characteristic of the $5/2_2^+$ states mentioned above, in agreement with the experimental positions.

According to the criterion discussed in § 3 a) (on the structure of the dressed 3QP modes), the calculated $B(E2)$-values in Table I(b) suggest that the 3QP correlations among quasi-particles in the different orbits are rather strong in the $3/2_1^+$ states compared with the collective $5/2_2^+$ states: The values of $B(E2; 3/2_1^+\to5/2_1^+)$ and $B(E2; 3/2_1^+\to7/2_1^+)$ are both enhanced, with ratios changing from I isotopes to La isotopes. Main amplitudes of the $3/2_1^+$ states in I isotopes as the dressed 3QP modes are shown in Table II(c), from which we can easily see why the competition between the two $E2$-transitions (to the 1QP $5/2_1^+$ and 1QP $7/2_1^+$ states) is remarkable in I isotopes. As the proton number increases, the chemical potential for protons shifts up toward the $2d_{5/2}$ orbit (from the $1g_{7/2}$ orbit), so that in La isotopes the component $\{\pi(g_{7/2})^3\}$ in the forward amplitudes $\psi_{nI=3/2}^{(1)}$ (in Eq. (2·1)) is diminished and the component $\{\pi(d_{5/2})^3\}$ grows up. (See Table 2(d).) The increase in $B(E2; 3/2_1^+\to5/2_1^+)$ in La isotopes is clearly due to the growth of the 3QP correlation in the $2d_{5/2}$ orbit. In this sense, we may say that the $3/2_1^+$ states in La isotopes have a structure similar to that of the ACS with $I=j-1$ (where j corresponds to the 1QP $5/2^+$ state), although the component $\{\pi(d_{5/2})^3\}$ is not still maximum but coupled with the other components, e.g., $\{\pi(g_{7/2})^2d_{5/2}\}$, rather strongly.

§ 4. Structure of collective $3/2_1^+$ states in odd-neutron Mo and Ru isotopes

The calculated results of the collective $3/2_1^+$ states in odd-neutron Mo and Ru isotopes as the dressed 3QP modes are shown in Fig. 4 and Table III. The

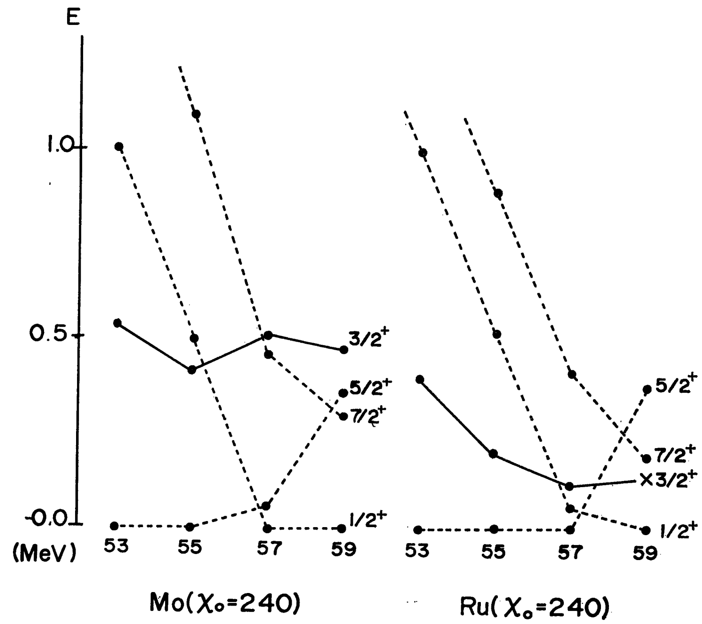

Fig. 4. Calculated excitation energies of the dressed 3QP modes with $I^\pi=3/2^+$ (solid lines) in odd-mass Mo and Ru isotopes. See the caption for Fig. 3.

Table III. Calculated $B(E2)$-values from the $3/2^+$ states as the dressed 3QP modes in odd-mass Mo and Ru isotopes (in unit of $e^2\cdot10^{-50}\,\mathrm{cm}^4$). The parenthesized numbers in the fourth column indicate the spins of the final states. The parameters used are the same as in Table I.

	$3/2_1^+\to5/2_1^+$		other transitions	
	Cal.	Exp.	Cal.	
^{95}Mo	2.0	5.3[a]	$0.3(7/2_1^+)$,	$0.3(1/2_1^+)$
^{97}Mo	4.4		$0.1(7/2_1^+)$,	$0.4(1/2_1^+)$
^{99}Mo	6.2		$0.4(3/2_1^+)$,	$0.3(1/2_1^+)$
^{97}Ru	4.9	7.4[b]	$0.2(7/2_1^+)$,	$0.8(1/2_1^+)$
^{99}Ru	11.3	13.1[c]	$1.9(7/2_1^+)$,	$2.9(1/2_1^+)$
^{101}Ru	13.9	5.7[c]	$0.7(7/2_1^+)$,	$0.8(1/2_1^+)$

a) Ref. 2). b) Ref. 3). c) Ref. 4).

excitation energies of the $3/2_1^+$ states and the values of $B(E2;3/2_1^+\to5/2_1^+)$ are reproduced very well. Thus, the $5/2_1^+$ and $3/2_1^+$ states are identified as the 1QP and dressed 3QP states, respectively. Futhermore, the special enhancement of $B(E2;3/2_1^+\to5/2_1^+)$ shown in Table III (when compared with the other $B(E2)$'s) suggests that the structure of the $3/2_1^+$ states is similar to that of the ACS with $I=j-1$. (In this case, j corresponds to the 1QP $5/2^+$ state.) From Table IV, we can easily see the similarity of the $3/2_1^+$ states to the ACS, although the fine structure is different appreciably as a result of the 3QP correlations among quasi-particles in the different orbits with the same parity ($d_{5/2}$, $s_{1/2}$, $g_{7/2}$ and $d_{3/2}$).

Table IV. Main correlation amplitudes of the dressed 3QP modes with $I=3/2^+$ in Ru isotopes. Notations are the same as in Table II.

(a) $3/2_1^+$ state in ^{97}Ru

$(\nu\nu\nu)$ type		$(\pi\pi\nu)$ type	
{555}	{(55)1}	{(99)5}	{(99)1}
0.89	−0.53	0.70	−0.15
0.32	−0.22(II) −0.27(III)	0.47	−0.30

(b) $3/2_1^+$ state in ^{101}Ru

$(\nu\nu\nu)$ type					$(\pi\pi\nu)$ type			
{(55)1}	{555}	{(77)5}	{(11, 11)5}	{(55)7}	{(99)5}	{(99)1}	{(99)7}	{(13)5}
−0.87	0.68	0.32	0.19	0.19	0.98	−0.22	0.18	0.13
−0.49(II) −0.11(III)	0.35	0.02(II) 0.32(III)	0.17(III)	0.06(II) 0.10(III)	0.86	−0.22	0.22	0.16

§ 5. Coupling between the dressed 3QP and 1QP modes

In contrast to the case of the ACS, the dressed 3QP states, especially the collective $5/2_2^+$ states in I, Cs and La isotopes, are lying close, in energy, to the 1QP states with the same spin and parity. It is, therefore, inevitable to examine their coupling effects. According to the general theory,[11] the original Hamiltonian is transcribed unambiguously into *the quasi-particle-new-Tamm-Dancoff subspace*, the basis vectors of which are $\{a_\alpha^\dagger|\Phi_0\rangle$ and $Y_{nIK}^\dagger|\Phi_0\rangle\}$. ($|\Phi_0\rangle$ donotes the correlated ground state.) The transcribed Hamiltonian is of the form[11]

$$H=\sum_\alpha E_\alpha \boldsymbol{a}_\alpha^\dagger \boldsymbol{a}_\alpha+\sum_{nIK}\omega_{nI}\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{Y}_{nIK}+\sum_{\alpha,nIK}V_{\rm int}(\alpha;nI)\cdot\{\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{a}_\alpha+\boldsymbol{a}_\alpha^\dagger \boldsymbol{Y}_{nIK}\}, \quad (5\cdot 1)$$

where

$$\boldsymbol{a}_\alpha^\dagger \equiv a_\alpha^\dagger|\Phi_0\rangle\langle\Phi_0|, \quad \boldsymbol{Y}_{nIK}^\dagger\equiv Y_{nIK}^\dagger|\Phi_0\rangle\langle\Phi_0|\,. \quad (5\cdot 2)$$

The third term of the transcribed Hamiltonian (5·1) represents the *interaction* between the dressed 3QP and 1QP modes, and comes from the H_Y-type original interaction (shown in Fig. 5), which has not played any role in constructing the dressed 3QP modes. The *effective coupling strength* $V_{\rm int}(\alpha;nI)$ is thus composed of the matrix elements of H_Y accompanied by the amplitudes of the dressed 3QP mode $Y_{nIK}^\dagger$, and its explicit expression is given in Eq. (5·10) of Ref. 11).

The results of calculations of the coupling effects due to the interaction in (5·1) are shown in Table V. It is noticeable that the coupling effects are very small even in the situation where the two $5/2^+$ states are close to each other in energy (i.e., $\Delta\omega\sim 0.01$ MeV). The mechanism to make the coupling effects (between the dressed 3QP and 1QP modes) so small has to be found in

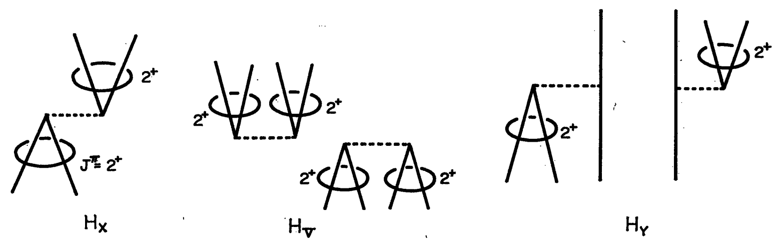

Fig. 5. Graphic representation of the matrix elements of the quadrupole force. The quadrupole force (in the quasi-particle representation) is conventionally divided as $H_{QQ}=H_X+H_V+H_Y$. The part H_X represents scattering of the pair of quasi-particles coupled to $J^\pi=2^+$, and the part H_V represents a pair-creation and a pair-annihilation of quasi-particles coupled to $J^\pi=2^+$. The parts H_X and H_V play an essential role in constructing the dressed 3QP mode and are called the *constructive force.* The part H_Y represents the coupling between a quasi-particle and the pair of quasi-particles coupled to $J^\pi=2^+$. This part plays a decisive role as essential coupling between the different types of elementary excitation modes, for instance, the interaction between the 1QP and the dressed 3QP modes, and is called the *interactive force.*[11]

Table V. Calculated results for the coupling effects. In the third column are listed the components of the 1QP modes, while in the fourth and fifth columns those of the lowest dressed 3QP modes and of the next higher ones, respectively. In the sixth column are listed the values of the energy shifts due to the coupling effects in MeV.

Nucleus	State	$a_I^\dagger\|\Phi_0\rangle$	$Y_{1,I}^\dagger\|\Phi_0\rangle$	$Y_{2,I}^\dagger\|\Phi_0\rangle$	$\Delta\omega$
^{129}I	$5/2_1^+$	0.90	−0.18	0.39	−0.23
	$5/2_2^+$	0.15	0.98	0.10	0.01
^{131}Cs	$5/2_1^+$	0.93	0.01	0.35	−0.14
	$5/2_2^+$	−0.01	1.00	−0.01	0.00
^{133}Cs	$5/2_1^+$	0.97	0.07	−0.25	−0.08
	$5/2_2^+$	−0.06	1.00	0.03	0.00
^{133}La	$5/2_1^+$	0.96	0.15	−0.26	−0.09
	$5/2_2^+$	−0.14	0.99	0.06	0.01
^{135}La	$5/2_1^+$	0.99	0.01	−0.13	−0.02
	$5/2_2^+$	−0.01	1.00	0.00	0.00
^{95}Mo	$3/2_1^+$	0.24	0.97	0.01	−0.12
^{97}Mo	$3/2_1^+$	0.10	0.99	0.01	−0.01
^{99}Mo	$3/2_1^+$	0.60	0.67	−0.33	−0.31
^{97}Ru	$3/2_1^+$	−0.17	0.98	0.00	−0.05
^{99}Ru	$3/2_1^+$	0.37	0.90	0.17	−0.25
^{101}Ru	$3/2_1^+$	−0.39	0.86	0.27	−0.12

the microscopic structure of the effective coupling strength $V_{\text{int}}(\alpha; nI)$. From the microscopic structure of the effective coupling strength between the dressed 3QP $5/2_2^+$ and 1QP $5/2_1^+$ modes in the case of the $P+QQ$ force, we can find the following origins to weaken the coupling: 1) The important matrix elements of H_Y (of the quadrupole force) in the effective coupling strength, which are accompanied by large components of the amplitudes of the dressed 3QP $5/2^+$ mode, always contain the spin-flip matrix element $\langle 2d_{5/2}\|r^2Y_2\|1g_{7/2}\rangle$ which is

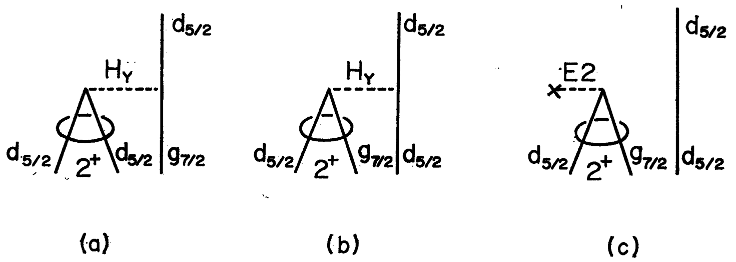

Fig. 6. Example of the important matrix element (of H_Y) which contributes to the effective coupling strength with $I^\pi=5/2^+$. Matrix elements (a) and (b) are of exchange with each other. The effects of such mutually exchanging parts to the effective coupling strength are not constructive but destructive to each other. It should be noted that, in the case of calculating the $E2$-transition (c), such destructive effects never appear because one of them (a) is forbidden.

considerably smaller compared with the diagonal matrix element $\langle 1g_{7/2}\|r^2Y_2\|1g_{7/2}\rangle$. It is interesting to remember that the considerable smallness of the ratio of $\langle 2d_{5/2}\|r^2Y_2\|1g_{7/2}\rangle$ to $\langle 1g_{7/2}\|r^2Y_2\|1g_{7/2}\rangle$ is also one of the important origins to bring about the ACS-like structure for the collective $5/2_2^+$ state. 2) The pairs of matrix elements of H_Y (such as Figs. 6(a) and (b)), which are exchangeable for each other, have to be always involved in the effective coupling strength, because the antisymmetrization among the three quasi-particles composing the dressed 3QP ($5/2^+$) mode is properly taken into account. Actual calculations tell us that the effects of such exchange parts on the effective coupling strength (between the dressed 3QP $5/2_2^+$ and 1QP $5/2_1^+$ modes) are not constructive but rather destructive to each other. It is interesting to notice that this important reduction effect of the effective coupling strength never appears in the conventional QPC theory, because the antisymmetrization between the "odd" quasi-particle and the quasi-particle pair composing the "phonon" is not taken into account in the evaluation of the coupling strength in the QPC theory. 3) In addition to these effects, it should be also pointed out that effective coupling strength depends characteristically on the reduction factors $(u_ju_{j'}-v_jv_{j'})$ which become especially small in La isotopes.

All of these effects cooperate so as to weaken the effective coupling strength between the 1QP $5/2_1^+$ state and the *lowest* dressed 3QP $5/2_2^+$ state. As a result, the 1QP $5/2_1^+$ mode becomes to couple rather with the *next higher* dressed 3QP $5/2_3^+$ mode, as shown in Table V.

§ 6. Concluding remarks

We have shown that the collective $5/2^+$ states in odd-proton I, Cs and La isotopes and the collective $3/2^+$ states in odd-neutron Mo and Ru isotopes are identified as the new collective "elementary" excitation mode, i.e., the dressed 3QP mode. We have also shown that the physical condition for the appearance of the dressed 3QP modes is not specific to the ACS but quite general in spherical odd-mass nuclei. The presence of a high-spin unique parity orbit in the major shell, such as in the case of the ACS, is not a necessary condition for the realization of the dressed 3QP modes. Rather, the important condition is the proximity of the chemical potential to the orbit of the odd quasi-particle and its relation with the neighbouring shell structure. Even if many orbits with the same parity are lying close to each other and the energy separations between the orbit of interest and the others are not so large as in the case of the ACS, one cannot expect diminution of the important role of the 3QP correlations in the specifc orbit. Furthermore, the physical condition (in the shell structure) to weaken the effective coupling strength between the 1QP mode and the "collective" dressed 3QP mode (with the same spin and parity) is common to that for the realization of the ACS-like dressed 3QP mode. Thus, the dressed

3QP modes similar to the ACS can exist as relatively purel elementary excitation modes over a wide range of spherical odd-mass nuclei.

Here it should also be emphasized that, even if the special role of the 3QP correlation in the specific orbit is relatively relaxed, this does not necessarily mean that we return to the physical situation which can be treated within the framework of the QPC theory. Rather, it may indicate the necessity of investigating the roles of the 3QP correlations newly arised in the different orbits. Importance of such effects has been briefly mentioned for the case of the $3/2_1^+$ states in I, Cs and La isotopes. This will be discussed in a forthcoming paper[14] in connection with the investigation on microscopic structure of breaking and persistency of the "phonon-plus-quasi-particle picture" based on the semi-phenomenological approaches.[1),15)~17)]

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Part I.

Introduction

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§1. Main motive

In the past decade the studies on nuclear structure have found the concept of phonon as an elementary mode of excitation in the nuclear system increasingly significant. On the other hand, the studies have suggested that the simple phonon model (based on the harmonic approximation) cannot give a satisfactory description of rather complicated anharmonicity effects, i.e., deviations from the simple phonon model are quite essential in a finite many-body quantal system such as the nucleus. Furthermore, the recent rapid accumulation of experimental data suggests the existence of certain "hidden regularities" in the complicated anharmonicity effects. Thus, one of the important subjects in current nuclear study is the sublation (aufheben) of the very concept of elementary modes of excitation in connection with the structure of the anharmonicity. Concerning such a subject, several annual research projects have been organized in Japan by the Research Institute for Fundamental Physics since 1969. Some important problems to be attacked at the first stage of the study were set up in the beginning of the research projects. One of them was to investigate the possibility of proposing an algebraic method of pair operators, which starts with the special nucleon-pair operators as the basis operators instead of the "phonon" as an ideal boson.[1] Along this line, the algebraic method is being extensively investigated by Yamamura et al.[2] Another problem was to construct a microscopic theory by which the structure of the complicated anharmonicity effects can be investigated in a simple systematic way. The essential part of our investigation concerning the present paper has been performed as a part of the research along this line.

In order to explain the situation at that time, we start with a brief survey of the results of analyses of the anharmonicity effects in even-even nuclei by Yamamura, Tokunaga and Marumori[3] in 1967 in terms of the boson expansion method.[4] They first classified the anharmonicity effects into two characteristic types; i) *kinematical effects,* i.e., effects due to the Pauli principle among the quasi-particles belonging to different (bound) quasi-particle pairs which are regarded as ideal bosons (i.e., "phonons") under the quasi-particle-random-phase approximation (RPA), and ii) *dynamical effects,* i.e., effects due to the residual interaction which has been omitted in the RPA. After calculating the kinematical and the dynamical effects (in the pairing-plus-quadrupole-force model) with the use of a perturbation theory based on the boson expansion method, they arrived at the following conclusion[3]: The simple "two-phonon" concept (as a possibility of repeating the excitation of an ideal boson twice) is actually broken in the following sense. i) Both the kinematical and the dynamical effects become unexpectedly large in the absolute values when the "phonon" energy under the RPA comes close to the actual experimental value. ii) When the energy of the "two-phonon" state under the RPA is close to those of the non-collective two-quasi-particle states, the coupling between the "two-phonon" state and the non-collective two-quasi-particle states due to the dynamical effects becomes too significant to be treated by the perturbation theory. In this case, which occurs most often in actual nuclei, we are forced to make a diagonalization of the coupling, which leads to a strong mixing of the two states and breaks the simple "two phonon" concept.

From this conclusion, one may naturally expect that the (quasi-particle-) higher-random-phase approximation (HRPA)[5] is promising in taking these significant anharmonicity effects into account, because it does not use the picture of repeating the "phonon" excitation twice. It is known that in the HRPA (the second RPA) the kinematical effects on the so-called "two-phonon" states due to the Pauli principle among the four quasi-particles are fully taken into account. Furthermore, the dynamical effects, i.e., the coupling between the two-quasi-particle excitation modes and the "two-phonon" modes are properly considered.*) Unfortunately, such a merit of the HRPA is merely one of formal logic. Actually we encounter the well-known formal difficulty which is inherently connected with the non-symmetrical form of the secular matrix coming from the linearized equation of motion for the eigenmode operator of the HRPA. The other rather serious formal difficulty in the HRPA is also known to arise from the spurious-state problem, which originates from the nucleon-number-non-conservation in the quasi-particle basis. As is well-known, it is one of the important advantages of the RPA that both the

*) Since both two-quasi-particle and four-quasi-particle amplitudes (in the sense of the new-Tamm-Dancoff (NTD) approximation with the ground-state correlation) are taken into account in the eigenmode operator of the HRPA, the excitation energies of both the first and the second excited states (which roughly correspond to the "one-phonon" and the "two-phonon" states of the RPA, respectively) are simultaneously obtained through the (linearized) equation of motion for the eigenmode operator in the NTD approximation.

"phonon" states and the (correlated) ground state are orthogonal to the spurious states within the framework of the approximation. However, the HRPA never leads us to either the "physical" excited states or the "physical" ground states which are orthogonal to the spurious states. Thus, we may conclude that, without overcoming these difficulties in essence and not in superficies, we cannot enjoy the above-mentioned essential merit of the HRPA in treating the anharmonicity effects. Nevertheless, any theories or methods overcoming the difficulties had not yet been achieved at that time. This was the reason why the authors' first task in collaboration with Kanesaki, Sakata and Takada[6)~9)] was to construct a new systematic microscopic theory which overcomes the difficulties in the HRPA and to treat both the kinematical and the dynamical anharmonicity effects in a simple systematic way.

§2. Outline of theory

In contrast with the HRPA, the underlying philosophy of our theory is not to intend a *direct, formal* diagonalization of the Hamiltonian within a subspace characterized by the eigenmode operator of the HRPA, but rather to start with an extraction of the basic physical elements from the subspace.

Our first task is to develop a method which enables us to uniquely separate the spurious components from the quasi-particle states and to precisely keep the one-to-one correspondence between the seniority number and the quasi-particle number. This problem is studied in Chap. 1 of Part II. According to the method developed in Chap. 1, we can regard the space of states described by the quasi-particles as a product space composed of "intrinsic" and "collective" spaces. The "intrinsic" space consists of the states which never involve $J=0$-coupled quasi-particle pairs, while the "collective" space consists of the states which include only $J=0$-coupled quasi-particle pairs and are always orthogonal to the "intrinsic" states. Needless to say, all of the spurious components belong to the "collective" space, and a special one of "collective" vibrations (under the RPA) with zero energy in this subspace is known to be due to the nucleon-number non-conservation.

Secondly, in the "intrinsic" space, we construct the correlated n-quasi-particle excitation modes (with $n=2, 4, 6, \cdots$ for even-even nuclei and with $n=1, 3, 5, \cdots$ for odd-mass nuclei) within the framework of the new Tamm-Dancoff (NTD) method with the ground-state correlation. The creation operators of these excitation modes consist of n-quasi-particle (creation and annihilation) operators accompanied by the correlation amplitudes involving the ground-state correlation in the NTD sense. The excitation modes are hereafter called the "dressed" n-quasi-particle (nQP) modes, and their detailed formal structure is studied in Chap. 2 for odd-mass nuclei. In order to specify the dressed nQP modes precisely in the "intrinsic" space, as is shown in Chap. 2, it is decisive to use the concept of spherical tensors in the quasi-spin space which has been introduced through the quasi-spin formalism (for the pairing correlations).[10] The dressed 2QP mode (with the lowest energy eigenvalue), which is the simplest one among the dressed nQP modes, is nothing but the "phonon" under the RPA. In this sense, we may say that our theory can be regarded as a natural extension of the RPA. The dressed 4QP states (with the lowest energy eigenvalues) correspond to the "two-phonon" states of the RPA, but the kinematical effects due to the Pauli principle among the four quasi-particles are fully taken into account in these states.

With the aid of the dressed nQP modes, we can introduce a set of orthogonal basis vectors consisting of the (correlated) ground state and the dressed nQP states. We call the space spanned by the orthogonal set the quasi-particle NTD space. Within the framework of the NTD approximation, this space is, by definition, orthogonal to the "collective" space which involves all of the spurious components. The basic physical idea underlying the introduction of the quasi-particle NTD space is as follows. Let us recall that the use of the quasi-particle basis can be regarded as an attempt to classify both the ground state and the excited states in terms of the seniority number v, keeping one-to-one correspondence between the seniority number and the quasi-particle number n. Then, the orthogonal basis vectors characterizing the quasi-particle basis are the BCS ground state (with $v=0$) and the n-quasi-particle states *with the condition* $n=v$. These orthogonal basis vectors with the definite quasi-particle numbers $n=v$ span the "quasi-particle Tamm-Dancoff (TD) space", which is merely the "intrinsic" space mentioned above. Now it is well known that, in a many-body quantal system such as the nucleus, the ground-state correlation is particularly important as a collective predisposition which allows the correlated excited states to occur from the ground state. We must therefore take account of the importance of both the seniority classification and the ground-state correlation simultaneously, in a way that the essential physical concept obtained in the quasi-particle TD space would still persist in a certain form. The guiding principle to introduce the quasi-particle NTD space lies in the fact that, in the NTD method, the quasi-particle correlations which are asymmetrically attributed to only the excited states in the TD calculations are symmetrically incorporated in the ground state through the ground-state correlation. In contrast with the BCS ground state in the quasi-particle TD space, the ground state in the quasi-particle NTD space is not with a definite seniority number because of the ground-state correlation. In spite of such a breaking of the seniority classification, in the quasi-particle NTD method we can still characterize the excitation modes, i.e., the dressed nQP modes by the amount of seniority $\Delta v=n$ which they transfer to the correlated ground state.

Our third task is to find a method of transcription of the physical operators in the quasi-particle TD space into the quasi-particle NTD space. The transcription should satisfy some self-consistency conditions within the framework of the (employed) NTD approximation under which the quasi-particle NTD space has been introduced. Details of the method of transcription is also discussed in Chap. 2. It is shown that, after the transcription into the quasi-particle NTD space, the residual interaction which has been omitted in constructing the dressed nQP modes manifests itself as a coupling between the different excitation modes. In our theory, the dynamical effects are then obtained by diagonalizing the coupling. The eigenmode creation operator, which is obtained by diagonalizing the coupling within the quasi-particle NTD subspace (composed of the dressed 2QP and dressed 4QP states), is formally of the same form as that of the HRPA, when written explicitly in terms of the quasi-particle operators. Nevertheless, in our theory, the difficulties inherent to the HRPA never appear because of our proper choice of the quasi-particle NTD space. From this point of view, the microscopic structure of the so-called "two-phonon" states is being investigated by Iwasaki, Kanesaki, Marumori, Sakata and Takada.[11)]

§3. Dressed 3QP mode as a new type of elementary mode of collective excitation

According to our theory, the simplest of the collective excitation modes in even-even nuclei is the dressed 2QP mode (with the lowest energy eigenvalue) as a "bound" state of two quasi-particles, which is nothing but the "phonon" under the RPA. In the case of odd-mass nuclei, the simplest of the collective excitation modes is the dressed 3QP mode (with the lowest energy eigenvalue). Thus, in the same manner as the RPA for even-mass nuclei leads us to the concept of "phonon" as a boson, the theory may necessarily lead us to the concept of a new kind of fermion-type collective excitation mode, i.e., *the dressed 3QP mode as a "bound" state of three quasi-particles.* So far, the collective excited states in odd-mass nuclei have conventionally been described in terms of the quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen.[12)] From this point of view, it is quite interesting to investigate whether or not this new kind of collective mode systematically exists in many spherical odd-mass nuclei, playing an important role in their low-lying collective states.

There was already a positive reason to expect the presence of the new kind of collective mode. In 1967, Bohr and Mottelson[13)] emphasized a significant effect of quasi-particle-phonon coupling, which had been completely omitted in the conventional QPC theory of Kisslinger and Sorensen. They have shown the extreme importance of this new effect in terms of the perturbation theory based on the self-consistent particle-vibration-coupling approach[14)] (i.e., the "nuclear field theory"), and have suggested that "the conventional description of collective excited states of almost all spherical odd-mass nuclei is significantly affected by the inclusion of the effect". It has also been demonstrated that the new effect essentially originates from the Pauli principle between the quasi-particles composing the phonon and the odd quasi-particle (i.e., the kinematical effect among the three quasi-particles), and brings about a significant three-quasi-particle correlation. Now, according to the nuclear field theory,[14] the strength of the particle-vibration coupling, f, is obtained by dividing a standard coupling matrix element by the phonon energy $\hbar\omega$. In situations where $f \ll 1$ (, the *weak coupling case*), we can safely treat the coupling by the perturbation theory.[15] For $f \gg 1$ (, the *strong coupling case*), the particle produces a static shape deformation, and the coupled system must be treated by a separation between rotational and intrinsic degrees of freedom. The nuclear field theory has clarified that, in contrast with the case of octupole mode where the values of $f_{\lambda=3}$ are typically about 0.1 to 0.3, the coupling strength for the quadrupole mode, $f_{\lambda=2}$, may become larger than unity. This implies that, for the quadrupole mode, the new effect bringing about the significant three-quasi-particle correlation should be taken into account not by the perturbation approximation but by diagonalizing the Hamiltonian in a suitable subspace. The dressed 3QP mode just satisfies this requirement, because it fully takes into account the kinematical effect among the three quasi-particles within the NTD approximation which is not the perturbation approximation. From this point of view, our theory includes the possibility of such an *intermediate coupling case* where the internal structure of the phonon itself is affected to form the dressed 3QP mode as a bound state. (See Fig. 1.)

Along this line of thought, investigations of microscopic structures of low-lying collective states in spherical odd-mass nuclei have been made. We have then obtained the conclusion that the appearance of the low-lying anomalous coupling (AC) states with spin $I=j-1$ can be regarded as a typical phenomena in which the new kind of collective mode (i.e., the dressed 3QP mode as a "bound" state of three quasi-particles) manifests itself as a relatively pure eigenmode.[16] It has also been shown that the physical condition of the enhancement of the three-quasi-particle correlation (characterizing this new collective mode) is not specific to the AC states but more general in odd-mass nuclei.[18] Thus, we have suggested that the new collective mode exists in

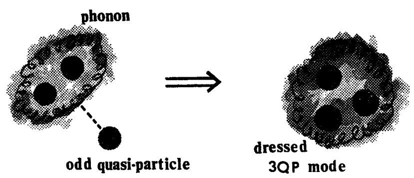

Fig. 1.

many spherical odd-mass nuclei and plays an important role in their low-lying collective states. It seems that recent experiments are revealing the systematic presence of this new kind of collective mode from among complicated spectra of the low-energy excitations in spherical odd-mass nuclei. The detailed review of these investigations[16)~19)] is the main subject in Chaps. 3 and 4 in Part III.

The framework of our theory includes the QPC theory as a special weak coupling case in which the characteristic three-quasi-particle correlation is seriously reduced by some physical conditions in shell structure. Therefore, our theory enables us to investigate the microscopic structure of the breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture". This investigation is the subject of Chap. 5.

The investigations of collective excitations in spherical odd-mass nuclei in Chaps. 3, 4 and 5 have been made with the use of the pairing-plus-quadrupole ($P+QQ$) force.[20)] Since we have widely employed the characteristic properties of the quadrupole force, it is indispensable to examine whether the conclusions obtained from Chap. 3 to Chap. 5 are specific to the $P+QQ$ force or not. This is the problem which is studied in Chap. 6.

§4. Coupling between pairing mode and dressed *n*QP mode

According to the method developed in Chap. 1, we can regard the space of states in terms of quasi-particles as a product space consisting of the "intrinsic" and "collective" spaces. In this representation, the original quasi-particle interaction is classified into three types: The first represents an interaction causing mixing among the "intrinsic" states, the second among the "collective" states and the last between "collective" and "intrinsic" states. The first-type interaction in the "intrinsic" space can furthermore be divided into two parts: One of them is the so-called *constructive force* which is responsible for constructing the dressed *n*QP modes, and the other the so-called *interactive force* which manifests itself as the coupling among the different *n*QP modes after the transcription into the quasi-particle NTD space. What we have investigated in Part III as the dynamical effect is nothing but the effect originating from this interactive force.

The other new type of dynamical effect may arise from a third-type interaction which causes the mixing between "collective" and "intrinsic" states. Since the "collective" space involves all of the quantum fluctuations of the pairing field, i.e., the excitation modes of $J=0$-coupled quasi-particle pairs, the third-type interaction can be expressed as the coupling between the pairing modes and the dressed *n*QP modes. The formal structure and the physical implication of the coupling are discussed in Chap. 7, although the detailed analysis of its effect in comparison with experiment is in the course of investigation as a next subject.

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Part II.

General Formulation of Theory

Chapter 1. Intrinsic and Collective Degrees of Freedom in Quasi-Spin Space

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§1. Introduction

It is well known that the use of the quasi-particle basis in the BCS theory can be regarded as an attempt to characterize both the ground state and the excited states in terms of the seniority number $v\equiv\sum_a v_a$*) in such a way that the number of quasi-particles $n\equiv\sum_a n_a$ is equivalent to the seniority v. This is one of the most important motives for introducing the quasi-particle basis.

In this approach, there is however a serious difficulty arising from the spurious-state problem due to the nucleon-number non-conservation. Owing to the fact that any quasi-particle basis vectors $|\phi\rangle$ are not eigenstates of the nucleon-number operator $\hat{\mathcal{N}}$, the use of the quasi-particle basis inevitably introduces the spurious states arising from the nucleon-number fluctuations such as $(\hat{\mathcal{N}}-\mathcal{N})|\phi\rangle$, and only the states orthogonal to the spurious states correspond to those of a physical nucleus.

Thus, in the use of the quasi-particle representation, it is decisive to develop a method which can uniquely separate the spurious components from the quasi-particle states $|\phi\rangle$, keeping the one-to-one correspondence between the seniority number v and the quasi-particle number n. This is the problem which is studied in this chapter.

*) In this paper we adopt the spherical j-j coupling shell model. The single-particle states are then characterized by the set of quantum numbers $\alpha=$ {the charge q, n, l, j, m}. In association with the Greek letter α, we use the Roman letter a to denote the same set except the magnetic quantum number m. We also use the notation $\bar{\alpha}$, which is obtained from α by changing the sign of the magnetic quantum number. We furthermore use the notation $f(\tilde{\alpha})=(-)^{j_a-m_\alpha}f(\bar{\alpha})$ where $f(\alpha)$ is an arbitrary function of α.

§2. Preliminaries

It is well known that the quasi-particle can be regarded as substantiation of the concept of seniority. This is easily seen with the use of the quasi-spin formalism.[1] Since this formalism plays an important role in our theory, we start the discussion with its brief recapitulation.

2-1 *Quasi-spin space*

Let us define the quasi-spin operators of the single-particle orbit a as*)

$$\hat{\mathcal{S}}_+(a)=\sqrt{\frac{\Omega_a}{2}}\sum_{m_{\alpha_1}m_{\alpha_2}}(j_aj_am_{\alpha_1}m_{\alpha_2}|00)c^\dagger_{\alpha_1}c^\dagger_{\alpha_2},$$

$$\hat{\mathcal{S}}_-(a)=\sqrt{\frac{\Omega_a}{2}}\sum_{m_{\alpha_1}m_{\alpha_2}}(j_aj_am_{\alpha_1}m_{\alpha_2}|00)c_{\alpha_2}c_{\alpha_1}, \tag{2·1}$$

$$\hat{\mathcal{S}}_0(a)=\frac{1}{2}(\sum_{m_\alpha}c^\dagger_\alpha c_\alpha-\Omega_a),\quad \Omega_a=j_a+\frac{1}{2},$$

where $c^\dagger_\alpha$ and c_α are the creation and annihilation operators of a nucleon in the single-particle state α. These operators then satisfy the same commutation relations as those of the angular-momentum operators:

$$[\hat{\mathcal{S}}_+(a),\ \hat{\mathcal{S}}_-(a)]=2\hat{\mathcal{S}}_0(a),\quad [\hat{\mathcal{S}}_0(a),\ \hat{\mathcal{S}}_\pm(a)]=\pm\hat{\mathcal{S}}_\pm(a). \tag{2·2}$$

The state vectors are specified by the quantum numbers $S(a)$ and $S_0(a)$, which are the eigenvalues of the quasi-spin $\hat{\mathcal{S}}(a)^2=\hat{\mathcal{S}}_+(a)\hat{\mathcal{S}}_-(a)+\hat{\mathcal{S}}_0(a)^2-\hat{\mathcal{S}}_0(a)$ and its projection $\hat{\mathcal{S}}_0(a)$, respectively. They span the quasi-spin subspace of the orbit a:**)

$$\{|S(a),\ S_0(a));\ S(a),\ S_0(a)=-S(a),\ -S(a)+1,\ \cdots,\ S(a)\}. \tag{2·3}$$

The physical meaning of the quantum numbers $S(a)$ and $S_0(a)$ is known to be related simply to the seniority number and the nucleon number, respectively, through the relations

$$S(a)=\frac{1}{2}(\Omega_a-v_a)\quad\text{and}\quad S_0(a)=\frac{1}{2}(\mathcal{N}_a-\Omega_a), \tag{2·4}$$

where v_a and $\mathcal{N}_a$ stand for the seniority number and the nucleon number in the orbit a, respectively.

With the quasi-spin operators (2·1) we can define irreducible tensor

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*) The subscripts $i=1, 2, 3, \cdots$ of α are used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit is necessary.

**) The quasi-spin space for the general many j-shell case is simply expressed as the direct product composed of the quasi-spin subspace of each orbit. Therefore, for simplicity, we discuss the case of a single orbit in this section.

operators in the quasi-spin subspace of the orbit a, as usual, by the commutation relations

$$[\hat{S}_0(a), \mathcal{T}_{k\kappa}(a)]=\kappa\mathcal{T}_{k\kappa}(a),$$
$$[\hat{S}_\pm(a), \mathcal{T}_{k\kappa}(a)]=\sqrt{(k\mp\kappa)(k\pm\kappa+1)}\,\mathcal{T}_{k,\kappa\pm1}(a), \tag{2·5}$$

where $\mathcal{T}_{k\kappa}(a)$ is the κ-component of an irreducible tensor of rank k, and the indicies k and κ are analogous to the quantum numbers $S(a)$ and $S_0(a)$ for the corresponding wavefunction multiplet. The index κ takes on $2k+1$ values from $-k$ to k. The single-particle operators $c_\alpha^\dagger$ and c_α are then regarded as spinors in the quasi-spin subspace:

$$\mathcal{T}_{1/2,1/2}(\alpha)=c_\alpha^\dagger \quad \text{and} \quad \mathcal{T}_{1/2,-1/2}(\alpha)=c_{\tilde{\alpha}}\equiv(-)^{j_a-m_a}c_{\bar{\alpha}}. \tag{2·6}$$

The irreducible tensors can be obtained from the products of the spinors by the standard vector-coupling procedures. For example, we have

$$\mathcal{T}_{1,1}(\alpha_1\alpha_2)=c_{\alpha_1}^\dagger c_{\alpha_2}^\dagger,$$
$$\mathcal{T}_{1,0}(\alpha_1\alpha_2)=\frac{1}{\sqrt{2}}(c_{\alpha_1}^\dagger c_{\tilde{\alpha}_2}+c_{\tilde{\alpha}_1}c_{\alpha_2}^\dagger), \tag{2·7a}$$
$$\mathcal{T}_{1,-1}(\alpha_1\alpha_2)=c_{\tilde{\alpha}_1}c_{\tilde{\alpha}_2}$$

and

$$\mathcal{T}_{3/2,3/2}(\alpha_1\alpha_2\alpha_3)=c_{\alpha_1}^\dagger c_{\alpha_2}^\dagger c_{\alpha_3}^\dagger,$$
$$\mathcal{T}_{3/2,1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(c_{\alpha_1}^\dagger c_{\alpha_2}^\dagger c_{\tilde{\alpha}_3}+c_{\alpha_1}^\dagger c_{\tilde{\alpha}_2}c_{\alpha_3}^\dagger+c_{\tilde{\alpha}_1}c_{\alpha_2}^\dagger c_{\alpha_3}^\dagger),$$
$$\mathcal{T}_{3/2,-1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(c_{\alpha_1}^\dagger c_{\tilde{\alpha}_2}c_{\tilde{\alpha}_3}+c_{\tilde{\alpha}_1}c_{\alpha_2}^\dagger c_{\tilde{\alpha}_3}+c_{\tilde{\alpha}_1}c_{\tilde{\alpha}_2}c_{\alpha_3}^\dagger), \tag{2·7b}$$
$$\mathcal{T}_{3/2,-3/2}(\alpha_1\alpha_2\alpha_3)=c_{\tilde{\alpha}_1}c_{\tilde{\alpha}_2}c_{\tilde{\alpha}_3}.$$

Here it should be noted that there is no interference between the coupling of the quasi-spin and that of the ordinary angular momentum, since $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ commute with the angular-momentum operators $\hat{J}_\pm$ and $\hat{J}_0$.

2-2 *Rotation in quasi-spin space*[2)]

The quasi-spin operators $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ are associated with the transformation of state vectors under the rotation of the coordinate system in the quasi-spin subspace of orbit a. Let us take up a new coordinate system K' obtained from the original one K (on which the argument has so far been) by a rotation specified in terms of the Euler angle $\omega_a=(\phi_a, \theta_a, \psi_a)$. The transformed state vectors are then given by

$$|S(a), S_0(a)\rangle = R(\omega_a)|S(a), S_0(a))$$
$$= \sum_{S_0(a)'} (S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a)) \times |S(a), S_0(a)'), \tag{2·8}$$

where $R(\omega_a)$ is the unitary rotation operator in the quasi-spin subspace of orbit a.

$$R(\omega_a) = \exp[-i\phi_a \hat{\mathcal{S}}_z(a)]\exp[-i\theta_a \hat{\mathcal{S}}_y(a)]\exp[-i\psi_a \hat{\mathcal{S}}_z(a)],$$
$$\hat{\mathcal{S}}_z(a) = \hat{\mathcal{S}}_0(a), \quad \hat{\mathcal{S}}_y(a) = \frac{1}{2i}(\hat{\mathcal{S}}_+(a) - \hat{\mathcal{S}}_-(a)), \tag{2·9}$$

and the state vector $|\cdots)$ designates one in the original system K while $|\cdots\rangle$ denotes a state vector in the new coordinate system K'. It must be remembered that the quantum numbers $S(a)$ and $S_0(a)$ in the state $|S(a), S_0(a)\rangle$ are the eigenvalues of $\hat{\boldsymbol{S}}(a)^2 = R(\omega_a)\hat{\boldsymbol{\mathcal{S}}}(a)^2 R(\omega_a)^{-1} (= \hat{\boldsymbol{\mathcal{S}}}(a)^2)$ and $\hat{S}_0(a) = R(\omega_a)\hat{\mathcal{S}}_0(a) R(\omega_a)^{-1}$, respectively. Thus the state vectors defined by (2·8) also span the quasi-spin space:

$$\{|S(a), S_0(a)\rangle;\ S(a), |S_0(a)| \leqq S(a)\}. \tag{2·10}$$

The matrix element of $R(\omega_a)$ defines the conventional D-function*) in the quasi-spin subspace:

$$D^{S(a)}_{S_0(a)' S_0(a)}(\omega_a) = (S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a))^*$$
$$= \langle S(a), S_0(a)'|R(\omega_a)|S(a), S_0(a)\rangle^*. \tag{2·11}$$

With the relation (2·11), the relation (2·8) becomes

$$|S(a), S_0(a)\rangle = \sum_{S_0(a)'} D^{S(a)*}_{S_0(a)' S_0(a)}(\omega_a)|S(a), S_0(a)'). \tag{2·12}$$

Since $R(\omega_a)$ is unitary, this can be rewritten as

$$|S(a), S_0(a)) = \sum_{S_0(a)'} D^{S(a)}_{S_0(a) S_0(a)'}(\omega_a)|S(a), S_0(a)'\rangle. \tag{2·13}$$

By definition, the irreducible tensors in the new coordinate system K', $T_{k\kappa}(a)$, are related to those in the original system through

$$T_{k\kappa}(a) = R(\omega_a)\mathcal{T}_{k\kappa}(a)R(\omega_a)^{-1} = \sum_{\kappa'} D^{k*}_{\kappa'\kappa}(\omega_a)\mathcal{T}_{k\kappa'}(a),$$
$$\mathcal{T}_{k\kappa}(a) = \sum_{\kappa'} D^{k}_{\kappa\kappa'}(\omega_a) T_{k\kappa'}(a). \tag{2·14}$$

Now let us take up a new coordinate system K'_{ω_0} specified by the Euler angle $\omega_0 \equiv (\phi_a = 0,\ -\theta_a,\ \psi_a = 0)$. According to Eq. (2·14), we have the quasi-spin spinors $T_{1/2,\kappa}(a)$ in the K'_{ω_0}-system

*) We use a definition of the D-function which is adopted by Bohr and Mottelson. Therefore the D-function adopted here is the complex conjugate of that of Rose and differs from that employed by Edmonds by the factor $(-)^{S_0(a)-S_0(a)'}$.

$$\begin{bmatrix} T_{1/2,\,1/2}(\alpha) \\ T_{1/2,\,-1/2}(\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\theta_a/2) & -\sin(\theta_a/2) \\ \sin(\theta_a/2) & \cos(\theta_a/2) \end{bmatrix} \begin{bmatrix} \mathcal{T}_{1/2,\,1/2}(\alpha) \\ \mathcal{T}_{1/2,\,-1/2}(\alpha) \end{bmatrix}. \tag{2·15}$$

With the definition

$$T_{1/2,\,1/2}(\alpha)=a_\alpha^\dagger \quad \text{and} \quad T_{1/2,\,-1/2}(\alpha)=a_{\bar{\alpha}}, \tag{2·16}$$

Eq. (2·15) can be rewritten as

$$a_\alpha^\dagger = u_a c_\alpha^\dagger - v_a c_{\bar{\alpha}}, \qquad a_\alpha = u_a c_\alpha - v_a c_{\bar{\alpha}}^\dagger,$$
$$u_a = \cos(\theta_a/2), \qquad v_a = \sin(\theta_a/2). \tag{2·17}$$

This is nothing but the Bogoliubov transformation. We can therefore say that the Bogoliubov transformation merely corresponds to a special rotation ω_0 of the coordinate system in the quasi-spin space.

In this new coordinate system K'_{ω_0}, i.e., in the quasi-particle representation, the quasi-spin operators are given as

$$\begin{aligned}
\hat{S}_+(a) &= R(\omega_0)\hat{S}_+(a)R(\omega_0)^{-1} \\
&= \sqrt{\frac{\Omega_a}{2}}\,\Sigma_{m_{a_1}m_{a_2}}(j_a j_a m_{a_1} m_{a_2}|00)a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger, \\
\hat{S}_-(a) &= \sqrt{\frac{\Omega_a}{2}}\,\Sigma_{m_{a_1}m_{a_2}}(j_a j_a m_{a_1} m_{a_2}|00)a_{\alpha_2} a_{\alpha_1}, \\
\hat{S}_0(a) &= \frac{1}{2}(\Sigma_{m_a} a_\alpha^\dagger a_\alpha - \Omega_a).
\end{aligned} \tag{2·18}$$

Since $\hat{\boldsymbol{S}}(a)^2 = R(\omega_0)\hat{\boldsymbol{S}}(a)^2 R(\omega_0)^{-1} = \hat{\boldsymbol{S}}(a)^2$, the quasi-spin quantum number $S(a)$ of the state $|S(a),\ S_0(a)\rangle$ in the quasi-particle representation K'_{ω_0} has the same physical meaning as that in the original system:

$$S(a) = \frac{1}{2}(\Omega_a - v_a). \tag{2·19}$$

On the other hand, from relation (2·18) the physical meaning of the quantum number $S_0(a)$ is now related to the number of quasi-particles n_a in the orbit a:

$$S_0(a) = \frac{1}{2}(n_a - \Omega_a). \tag{2·20}$$

Needless to say, the BCS ground state $|\phi_0\rangle$ (in the orbit a) is given by

$$|\phi_0\rangle = |S(a) = \Omega_a/2,\ S_0(a) = -S(a)\rangle. \tag{2·21}$$

2-3 *Definition of collective and intrinsic states*

By the definition of the state $|S(a),\ S_0(a) = -S(a)\rangle$ of the orbit a, we obtain

$$\hat{S}_-(a)|S(a),\ S_0(a)=-S(a)\rangle=0, \tag{2·22}$$

which means that there is no $J=0$-coupled quasi-particle pair in this state. In this case, with the aid of Eqs. (2·19) and (2·20), the following relation is obtained:

$$\frac{1}{2}(n_a-\Omega_a)=-\frac{1}{2}(\Omega_a-v_a),\ \text{i.e.,}\ n_a=v_a. \tag{2·23}$$

Thus, for a class of states $|\phi_{\rm intr}\rangle$ which consists of the direct product of the states satisfying Eq. (2·22), we have

$$\hat{S}_-(a)|\phi_{\rm intr}\rangle=0 \tag{2·24}$$

for all $\hat{S}_-(a)$, so that the following well-known statement is satisfied: The use of the quasi-particle basis can be regarded as an attempt to characterize both the ground state and the excited states in terms of the seniority number $v\equiv\sum_a v_a$ in such a way that the number of quasi-particles $n\equiv\sum_a n_a$ is equivalent to the seniority v.

The condition (2·24) means that the states $|\phi_{\rm intr}\rangle$ never contain any $J=0$-coupled quasi-particle pair. In this sense, we call $|\phi_{\rm intr}\rangle$ "intrinsic states". On the other hand, the states characterized by $v_a=0$ and $n_a\neq0$ include only $J=0$-coupled quasi-particle pairs and are always orthogonal to the intrinsic states $|\phi_{\rm intr}\rangle$. Hence we call such a class of states "collective states" $|\phi_{\rm col}\rangle$. Needless to say, all spurious components due to the nucleon-number non-conservation belong to the "collective states", and a special one of "collective" vibrations (under the RPA) with zero energy in this "collective subspace" is known as due to the nucleon-number non-conservation.

§3. The Hamiltonian

The Hamiltonian under consideration is that of a spherically symmetric j-j coupling shell model with a general two-body interaction which is invariant under rotation, reflection and time reversal:

$$H=\sum_\alpha(\epsilon_a-\lambda_a)c_\alpha^\dagger c_\alpha+\sum_{\alpha\beta\gamma\delta}\mathcal{V}_{\alpha\beta\gamma\delta}c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma, \tag{3·1}$$

where ϵ_a and λ_a represent the single-particle energy and the chemical potential, respectively. The matrix elements of the interaction satisfy the relations*)

$$\mathcal{V}_{\alpha\beta\gamma\delta}=-\mathcal{V}_{\beta\alpha\gamma\delta}=-\mathcal{V}_{\alpha\beta\delta\gamma}=\mathcal{V}_{\beta\alpha\delta\gamma}=\mathcal{V}_{\gamma\delta\alpha\beta}$$
$$=\mathcal{V}_{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\tilde{\delta}}. \tag{3·2}$$

*) It is possible to treat all matrix elements of the Hamiltonian as real quantities if the phase convention is suitably chosen. In this paper we always assume this to be the case.

After the Bogoliubov transformation (2·17) and with the use of notations*),[3]

$$\begin{aligned} &\Delta_a = -2\sum_\gamma u_c v_c \mathcal{V}_{a\bar{a}\gamma\bar{\gamma}}, \\ &\mu_a = -4\sum_\gamma v_c^2 \mathcal{V}_{a\gamma a\gamma}, \\ &\eta_a = \epsilon_a - \lambda_a - \mu_a, \end{aligned} \tag{3·3}$$

the Hamiltonian is expressed in terms of the quasi-particle operators as follows:

$$H = U_0 + H_0 + H_1 + H_{\rm int}, \tag{3·4}$$

$$\begin{aligned} &U_0 = \sum_a \left[\left(\eta_a + \frac{1}{2}\mu_a\right) v_a^2 - \frac{1}{2} u_a v_a \Delta_a\right], \\ &H_0 = \sum_a [\eta_a (u_a^2 - v_a^2) + 2u_a v_a \Delta_a] a_\alpha^\dagger a_\alpha, \\ &H_1 = \sum_a \left[\eta_a u_a v_a - \frac{1}{2}(u_a^2 - v_a^2)\Delta_a\right](a_\alpha^\dagger a_{\bar{\alpha}}^\dagger + a_{\bar{\alpha}} a_\alpha), \end{aligned}$$

$$\begin{aligned} H_{\rm int} &= H_X + H_V + H_Y, \\ &H_X = \sum_{\alpha\beta\gamma\delta} V_X(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma, \\ &H_V = \sum_{\alpha\beta\gamma\delta} V_V(\alpha\beta\gamma\delta)(a_\alpha^\dagger a_\beta^\dagger a_{\bar{\delta}}^\dagger a_{\bar{\gamma}}^\dagger + a_{\bar{\gamma}} a_{\bar{\delta}} a_\beta a_\alpha), \\ &H_Y = \sum_{\alpha\beta\gamma\delta} V_Y(\alpha\beta\gamma\delta)(a_\alpha^\dagger a_\beta^\dagger a_{\bar{\delta}}^\dagger a_\gamma + a_\gamma^\dagger a_{\bar{\delta}} a_\beta a_\alpha), \end{aligned}$$

where the abbreviations

$$\begin{aligned} V_X(\alpha\beta\gamma\delta) &= V_X^{(G)}(\alpha\beta\gamma\delta) + V_X^{(F)}(\alpha\beta\gamma\delta) - V_X^{(F)}(\beta\alpha\gamma\delta) \\ &= (u_a u_b u_c u_d + v_a v_b v_c v_d)\mathcal{V}_{\alpha\beta\gamma\delta} + (u_a v_b u_c v_d + v_a u_b v_c u_d)\mathcal{V}_{\alpha\bar{\delta}\bar{\beta}\gamma} \\ &\quad - (u_b v_a u_c v_d + v_b u_a v_c u_d)\mathcal{V}_{\beta\bar{\delta}\bar{\alpha}\gamma}, \\ V_V(\alpha\beta\gamma\delta) &= u_a u_b v_c v_d \mathcal{V}_{\alpha\beta\gamma\delta}, \\ V_Y(\alpha\beta\gamma\delta) &= 2(u_a u_b u_c v_d - v_a v_b v_c u_d)\mathcal{V}_{\alpha\beta\gamma\delta} \end{aligned}$$

have been used.

The parameters u, v and the chemical potentials λ are determined as usual by the set of equations

$$\begin{aligned} &2u_a v_a = \Delta_a / E_a, \qquad u_a^2 - v_a^2 = \eta_a / E_a, \\ &E_a = \sqrt{\eta_a^2 + \Delta_a^2}, \\ &N_n = 2\sum_a \Omega_a v_a^2 = \sum_a \Omega_a (1 - \eta_a / E_a), \end{aligned} \tag{3·5}$$

where N_n is the neutron (proton) number and the summation a runs over the neutron (proton) orbits.

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*) We assume that, among the single-particle orbits a, b, ⋯, a given set of the quantum numbers {charge q, parity and j-value} occurs at once.

Then H_0 takes the form

$$H_0 = \sum_a E_a a_a^\dagger a_a, \tag{3·6}$$

where E_a denotes the quasi-particle energy.

The main part of the pairing correlations is taken into account as the (self-consistent) quasi-particle field. The eigenstates of H_0 are given by those of the quasi-spin $\hat{S}(a)^2$ and its projection $\hat{S}_0(a)$, with additional quantum numbers. The term $H_{\rm int}$ represents the residual interaction among quasi-particles. The role of residual interaction can be classified into three types: The first is the role of mixing among the states in the intrinsic subspace $\{|\phi_{\rm intr}\rangle\}$, the second among the states in the collective subspace; and the last between collective and intrinsic states. These roles may be expressed as follows:

$$H = H_{\rm col} + H_{\rm intr} + H_{\rm coupl}, \tag{3·7}$$

where $H_{\rm col}$, $H_{\rm intr}$ and $H_{\rm coupl}$ stand for the collective-, intrinsic-, and coupling-Hamiltonians, respectively.

We show, in the remaining part of this chapter, that the original Hamiltonian (3·4) can be transformed unambiguously into the form (3·7).

§4. Collective variables associated with pairing correlations

4-1 *Extension of quasi-spin space and introduction of collective variables*

The procedure of adding pairs of fermion must eventually end if the number of states available is finite, whereas there is nothing to prevent operating again and again on a state with a boson creation operator. In order to define the canonical-conjugate collective variables describing the pairing excitations in terms of boson operators, it is thus necessary to extend the quasi-spin space in such a way that the multiplet in the quasi-spin space (2·10) becomes infinite with allowed values of $S_0(a)$ going to steps of unity from $S(a)+1$ to $+\infty$:

$$\{|S(a), S_0(a)\rangle;\ S(a),\ -S(a) \leqq S_0(a) < +\infty\}. \tag{4·1}$$

With the aid of the extended quasi-spin space we introduce boson operators $b_a^\dagger$ and b_a, which satisfy the commutation relations

$$[b_a, b_c^\dagger] = \delta_{ac}, \qquad [b_a, b_c] = [b_a^\dagger, b_c^\dagger] = 0 \tag{4·2}$$

and characterize the state vectors in the extended quasi-spin space by

$$|S(a), S_0(a) = -S(a) + N_a\rangle = \frac{1}{\sqrt{N_a!}}(b_a^\dagger)^{N_a}|S(a), S_0(a) = -S(a)\rangle,$$
$$b_a|S(a), S_0(a) = -S(a)\rangle = 0. \tag{4·3}$$

Then we have

$$\begin{aligned}
b_a^\dagger|S(a), S_0(a)=-S(a)+N_a\rangle \\
&=\sqrt{N_a+1}\,|S(a), S_0(a)=-S(a)+N_a+1\rangle, \\
b_a|S(a), S_0(a)=-S(a)+N_a\rangle \\
&=\sqrt{N_a}\,|S(a), S_0(a)=-S(a)+N_a-1\rangle.
\end{aligned} \tag{4·4}$$

Explicitly, such an introduction of the boson operators is made in terms of the Holstein-Primakoff transformation[4)]

$$\begin{aligned}
&\mathring{S}_+(a)=b_a^\dagger\sqrt{2\mathring{S}(a)-\mathring{N}(a)}, \\
&\mathring{S}_-(a)=\sqrt{2\mathring{S}(a)-\mathring{N}(a)}\,b_a, \\
&\mathring{S}_0(a)=\mathring{N}(a)-\mathring{S}(a),
\end{aligned} \tag{4·5}$$

where the boson number operator $\mathring{N}(a)$ of the orbit a and the operator $\mathring{S}(a)$ are defined respectively by

$$\mathring{N}(a)=b_a^\dagger b_a, \tag{4·6}$$

$$\mathring{S}(a)\{\mathring{S}(a)+1\}=\mathring{\boldsymbol{S}}(a)^2. \tag{4·7}$$

The operators $\mathring{S}_\pm(a)$, $\mathring{S}_0(a)$ and $\mathring{\boldsymbol{S}}(a)^2$ denote the extensions of the quasi-spin operators $\hat{S}_\pm(a)$, $\hat{S}_0(a)$ and $\hat{\boldsymbol{S}}(a)^2$ into the extended quasi-spin space: In a "physical subspace" which coincides with the quasi-spin space, these extended quasi-spin operators are identical with the original quasi-spin operators. In a "unphysical subspace" which corresponds to the extended part (in the extended quasi-spin space), the operators $\mathring{S}_x(a)=\{\mathring{S}_+(a)+\mathring{S}_-(a)\}/2$ and $\mathring{S}_y(a)=\{\mathring{S}_+(a)-\mathring{S}_-(a)\}/2i$ become anti-hermitian. We may write the extended quasi-spin operators as

$$\begin{aligned}
&\mathring{S}_x(a)=i\hat{T}_x(a), \quad \mathring{S}_y(a)=i\hat{T}_y(a), \quad \mathring{S}_0(a)=\hat{T}_0(a), \\
&\hat{T}_+(a)\equiv\hat{T}_x(a)+i\hat{T}_y(a)=b_a^\dagger\sqrt{\mathring{N}(a)-2\mathring{S}(a)}, \\
&\hat{T}_-(a)\equiv\hat{T}_x(a)-i\hat{T}_y(a)=\sqrt{\mathring{N}(a)-2\mathring{S}(a)}\,b_a,
\end{aligned} \tag{4·8}$$

so that we have

$$\begin{aligned}
\mathring{\boldsymbol{S}}(a)^2&=\mathring{S}_x(a)^2+\mathring{S}_y(a)^2+\mathring{S}_0(a)^2 \\
&=\hat{T}_0(a)^2-\hat{T}_x(a)^2-\hat{T}_y(a)^2.
\end{aligned} \tag{4·9}$$

The algebra of $\hat{T}_x(a)$, $\hat{T}_y(a)$ and $\hat{T}_0(a)$,

$$\begin{aligned}
&[\hat{T}_x(a), \hat{T}_y(a)]=-i\hat{T}_0(a), \quad [\hat{T}_y(a), \hat{T}_0(a)]=i\hat{T}_x(a), \\
&[\hat{T}_0(a), \hat{T}_x(a)]=i\hat{T}_y(a),
\end{aligned} \tag{4·10}$$

characterizes transformation of the noncompact group with allowed values of $S_0(a)$ from $\{S(a)+1\}$ to $+\infty$: By definitions (4·8) and (4·5) we obtain in the unphysical subspace

$$\hat{T}_\pm(a)|S(a),\, S_0(a)\rangle=\sqrt{S_0(a)\{S_0(a)\pm1\}-S(a)\{S(a)+1\}}\,|S(a),\, S_0(a)\pm1\rangle, \tag{4·11}$$

which is consistent with the property $S_0(a)^2 \geqq S(a)\{S(a)+1\}$ derived from Eq. (4·9). Then the condition

$$\hat{T}_-(a)|S(a),\, S_0(a)_{\rm min}\rangle=0 \tag{4·12}$$

leads to the relation

$$S_0(a)_{\rm min}\{S_0(a)_{\rm min}-1\}=S(a)\{S(a)+1\}, \tag{4·13}$$

which means $S_0(a)_{\rm min}=S(a)+1$.

The last relation in Eq. (4·5) in the physical space is written as

$$\hat{N}(a)=b_a^\dagger b_a=\hat{S}(a)+\hat{S}_0(a), \tag{4·14}$$

which has the eigenvalues

$$N_a=S(a)+S_0(a)=\frac{1}{2}(n_a-v_a). \tag{4·15}$$

This means that the boson number N_a of the orbit a is merely the number of '$J=0$' coupled quasi-particle pairs in the orbit a. Therefore, the intrinsic states $|\phi_{\rm intr}\rangle$, which consist of $|S(a),\, S_0(a)=-S(a)\rangle$ and are defined by Eq. (2·24) always satisfy

$$\mathring{N}(a)|\phi_{\rm intr}\rangle=0, \tag{4·16}$$

which is consistent with Eq. (4·3). We may thus say that the intrinsic states are not affected at all by the extension of the quasi-spin space and always belong to the physical subspace. This is in marked contrast with the collective states $|\phi_{\rm col}\rangle$ in which the boson operators play an essential role.

4-2 *Canonical coordinates and canonical conjugate momenta*

With the aid of the boson operators $b_a^\dagger$ and b_a, we can define collective coordinates $\mathring{q}_a$ and their canonical conjugate momenta $\mathring{p}_a$ describing the pairing excitations by

$$\mathring{q}_a=\frac{i}{\sqrt{2}}(b_a-b_a^\dagger),\qquad \mathring{p}_a=\frac{1}{\sqrt{2}}(b_a+b_a^\dagger). \tag{4·17}$$

Of course, these operators satisfy the canonical commutation relations

$$[\mathring{q}_a, \mathring{p}_b]=i\delta_{ab},\qquad [\mathring{q}_a, \mathring{q}_b]=[\mathring{p}_a, \mathring{p}_b]=0. \tag{4·18}$$

§5. Canonical transformation into collective representation

5-1 *Introduction of auxiliary variables and supplementary condition*

We now apply the canonical transformation method with auxiliary variables[5),6)] to the system under consideration.

First, we introduce redundant canonical variables (i.e., auxiliary variables) $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$, which satisfy the canonical commutation relations

$$[\boldsymbol{q}_a, \boldsymbol{p}_b]=i\delta_{ab}, \qquad [\boldsymbol{q}_a, \boldsymbol{q}_b]=[\boldsymbol{p}_a, \boldsymbol{p}_b]=0, \tag{5·1}$$

and are independent of the quasi-particle operators $(a_\alpha^\dagger, a_\alpha)$ and the boson operators $(b_a^\dagger, b_a)$:

$$\begin{aligned}&[\boldsymbol{q}_a, a_\alpha^\dagger]=[\boldsymbol{q}_a, a_\alpha]=[\boldsymbol{q}_a, b_a^\dagger]=[\boldsymbol{q}_a, b_a]=0,\\&[\boldsymbol{p}_a, a_\alpha^\dagger]=[\boldsymbol{p}_a, a_\alpha]=[\boldsymbol{p}_a, b_a^\dagger]=[\boldsymbol{p}_a, b_a]=0.\end{aligned} \tag{5·2}$$

With the redundant variables we may define redundant bosons as

$$\boldsymbol{b}_a^\dagger=\frac{1}{\sqrt{2}}(\boldsymbol{p}_a+i\boldsymbol{q}_a), \qquad \boldsymbol{b}_a=\frac{1}{\sqrt{2}}(\boldsymbol{p}_a-i\boldsymbol{q}_a). \tag{5·3}$$

In order to compensate for the over-completeness in the degrees of freedom due to the introduction of the auxiliary variables, we impose on the state vectors a supplementary condition

$$\hat{\boldsymbol{N}}(a)|\Psi\rangle=0, \tag{5·4}$$

$$\hat{\boldsymbol{N}}(a)\equiv\boldsymbol{b}_a^\dagger\boldsymbol{b}_a, \tag{5·5}$$

which physically implies that we are only considering the subspace with no auxiliary bosons. Since the original Hamiltonian H is independent of the auxiliary variables introduced, i.e.,

$$[H, \hat{\boldsymbol{N}}(a)]=0, \tag{5·6}$$

the Schrödinger equation

$$H|\Psi\rangle=E|\Psi\rangle, \tag{5·7}$$

with the supplementary condition (5·4) is exactly equivalent to the starting Schrödinger equation without the auxiliary variables.

5-2 *Canonical transformation*

Now, let us define the following canonical transformation:

$$\begin{aligned}&U_{\mathrm{col}}=U_1\cdot U_2\cdot U_1,\\&U_1=\exp[i\Sigma_a\mathring{p}_a\boldsymbol{q}_a], \qquad U_2=\exp[-i\Sigma_a\mathring{q}_a\boldsymbol{p}_a],\end{aligned} \tag{5·8}$$

where the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ are given by Eq. (4·17). The following relations are then easily derived:

$$U_{\rm col}\mathring{q}_a U_{\rm col}^{-1}=\boldsymbol{q}_a, \qquad U_{\rm col}\mathring{p}_a U_{\rm col}^{-1}=\boldsymbol{p}_a,$$
$$U_{\rm col}\boldsymbol{q}_a U_{\rm col}^{-1}=-\mathring{q}_a, \qquad U_{\rm col}\boldsymbol{p}_a U_{\rm col}^{-1}=-\mathring{p}_a \tag{5·9}$$

and thus we have

$$U_{\rm col}\hat{N}(a)U_{\rm col}^{-1}=b_a^\dagger b_a. \tag{5·10}$$

This implies that, in the representation after the canonical transformation which we call "collective representation", the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ are completely replaced by the redundant variables $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$, respectively.

The Schrödinger equation in the collective representation is obtained from Eq. (5·7) with the condition (5·4), by regarding both the Hamiltonian H and the state vectors $|\Psi\rangle$ (defined on the physical subspace) as their extensions $\mathring{H}$ and $|\mathring{\Psi}\rangle$ into the extended quasi-spin space:*) It becomes

$$\boldsymbol{H}|\Psi\rangle\!\rangle=E|\Psi\rangle\!\rangle \tag{5·11}$$

with the supplementary condition

$$\mathring{N}(a)|\Psi\rangle\!\rangle=0, \qquad \mathring{N}(a)=b_a^\dagger b_a=\mathring{S}(a)+\mathring{S}_0(a), \tag{5·12}$$

where

$$\boldsymbol{H}=U_{\rm col}\mathring{H}U_{\rm col}^{-1}, \qquad |\Psi\rangle\!\rangle=U_{\rm col}|\mathring{\Psi}\rangle. \tag{5·13}$$

5-3 *Collective representation*

Since the original Hamiltonian $\mathring{H}$ is independent of the auxiliary variables, we have

$$[\mathring{H}, \boldsymbol{q}_a]=[\mathring{H}, \boldsymbol{p}_a]=0, \tag{5·14}$$

which is transformed into the collective representation as

$$[\boldsymbol{H}, \mathring{q}_a]=[\boldsymbol{H}, \mathring{p}_a]=0. \tag{5·15}$$

This implies that, in the collective representation, the collective variables $(\mathring{q}_a, \mathring{p}_a)$ involved implicitly in the original Hamiltonian $\mathring{H}$ are completely replaced by the auxiliary variables $(\boldsymbol{q}_a, \boldsymbol{p}_a)$, and the collective modes of the system are visualized by the explicit appearance of the auxiliary variables in the Hamiltonian $\boldsymbol{H}$. By comparing the supplementary condition (5·12) with

*) Knowledge of the explicit properties of $\mathring{H}$ in the unphysical subspace is actually not necessary at all in our discussions, provided that the commutation properties of $\mathring{H}$ with the collective variables $\mathring{q}_a$ and $\mathring{p}_a$ in the unphysical subspace are the same as those in the physical space.

Eq. (4·16), we can furthermore see that in this representation, the degrees of freedom associated with the quasi-particle operators merely describe the intrinsic motion of the system. Thus, the Hilbert space in this representation may be characterized as the direct product of a boson space (which is associated with the auxiliary bosons $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$) and the intrinsic space composed of the intrinsic states $|\phi_{\rm intr}\rangle$ (which are defined by Eq. (2·24) and always belong to the physical subspace): The basis vectors (of the orbit a) can be represented as

$$|S(a),\, N_a\rangle\!\rangle \equiv |N_a\rangle_{\rm col}\cdot|S(a),\, S_0(a)=-S(a)\rangle, \tag{5·16}$$

where $|N_a\rangle_{\rm col}$ denotes the collective state vector associated with the boson operators $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$:

$$|N_a\rangle_{\rm col} \equiv \frac{1}{\sqrt{N_a!}}(\boldsymbol{b}_a^\dagger)^{N_a}|0\rangle_B, \tag{5·17}$$

where $|0\rangle_B$ is the vacuum of the boson, i.e., $\boldsymbol{b}_a|0\rangle_B=0$, and the states $|S(a),$ $S_0(a)=-S(a)\rangle$ compose the intrinsic states $|\phi_{\rm intr}\rangle$. It should be noted that, in the collective representation, all unphysical effects as a result of the extension of the quasi-spin space arise only in the collective boson space (associated with $\boldsymbol{b}_a^\dagger$ and $\boldsymbol{b}_a$) and the intrinsic states remain unchanged in the physical subspace.

§6. Collective representation of the Hamiltonian

6-1 *Perturbative expansion of the Hamiltonian in terms of collective variables*

The collective representation of the Hamiltonian cannot be expected to take a simple and compact form. We here adopt a perturbative expansion in terms of the collective variables. In this expansion, we choose the collective variables $\boldsymbol{X}_\mu^\dagger$ and $\boldsymbol{X}_\mu$ as the basis of the expansion, which are the eigenmode creation and annihilation operators of the pairing vibration under the RPA:

$$\begin{aligned} \boldsymbol{X}_\mu^\dagger &= \textstyle\sum_a \{\psi_\mu(a)\boldsymbol{b}_a^\dagger + \phi_\mu(a)\boldsymbol{b}_a\}, \\ \boldsymbol{X}_\mu &= \textstyle\sum_a \{\psi_\mu(a)\boldsymbol{b}_a + \phi_\mu(a)\boldsymbol{b}_a^\dagger\}. \end{aligned} \tag{6·1}$$

It is well known that such pairing vibrations include a special zero-energy solution, which implies that the RPA includes enough of the residual (pairing) interaction to restore the breaking of the nucleon-number conservation by the BCS approximation. Their definitions and the details of the zero-energy solution are given in Appendix 1B.

Thus the auxiliary variables $\boldsymbol{q}_a$ and $\boldsymbol{p}_a$ in the canonical transformation (5·8) are regarded as the functions of these basis operators $\boldsymbol{X}_\mu^\dagger$ and $\boldsymbol{X}_\mu$:

$$\begin{aligned} \boldsymbol{q}_a &= \frac{i}{\sqrt{2}}(\boldsymbol{b}_a - \boldsymbol{b}_a^\dagger) = \frac{i}{\sqrt{2}}\Sigma_\mu\{\psi_\mu(a)+\phi_\mu(a)\}(\boldsymbol{X}_\mu - \boldsymbol{X}_\mu^\dagger), \\ \boldsymbol{p}_a &= \frac{1}{\sqrt{2}}(\boldsymbol{b}_a + \boldsymbol{b}_a^\dagger) = \frac{1}{\sqrt{2}}\Sigma_\mu\{\psi_\mu(a)-\phi_\mu(a)\}(\boldsymbol{X}_\mu^\dagger + \boldsymbol{X}_\mu). \end{aligned} \tag{6·2}$$

By the use of the orthonormality relation

$$\Sigma_a\{\psi_\nu(a)-\phi_\nu(a)\}\{\psi_\mu(a)+\phi_\mu(a)\} = \delta_{\mu\nu}, \tag{6·3}$$

the canonical transformation (5·8) is rewritten as

$$\begin{aligned} U_1 &= \exp[i\Sigma_a \mathring{p}_a \boldsymbol{q}_a] = \exp[i\Sigma_\mu \mathring{P}_\mu \boldsymbol{Q}_\mu], \\ U_2 &= \exp[-i\Sigma_a \mathring{q}_a \boldsymbol{p}_a] = \exp[-i\Sigma_\mu \mathring{Q}_\mu \boldsymbol{P}_\mu], \end{aligned} \tag{6·4}$$

where

$$\begin{aligned} &\mathring{Q}_\mu = \frac{i}{\sqrt{2}}(X_\mu - X_\mu^\dagger), \qquad \mathring{P}_\mu = \frac{1}{\sqrt{2}}(X_\mu + X_\mu^\dagger), \\ &X_\mu^\dagger \equiv \Sigma_a\{\psi_\mu(a)b_a^\dagger + \phi_\mu(a)b_a\}, \qquad X_\mu \equiv \Sigma_a\{\psi_\mu(a)b_a + \phi_\mu(a)b_a^\dagger\}. \end{aligned} \tag{6·5}$$

Then, with the aid of the well-known formula

$$\exp(iT)\cdot O\cdot\exp(-iT) = T + i[T, O] - \frac{1}{2}[T, [T, O]] + \cdots, \tag{6·6}$$

we obtain a perturbative expansion of the Hamiltonian in the collective representation in terms of the pairing-vibration modes $(\boldsymbol{X}_\mu^\dagger, \boldsymbol{X}_\mu)$:

$$\begin{aligned} \boldsymbol{H} &= U_{\rm col}\mathring{H}U_{\rm col}^{-1} \\ &= \hat{h}_{00} + \Sigma_\mu\{\boldsymbol{X}_\mu^\dagger \hat{h}_{10}(\mu) + \boldsymbol{X}_\mu \hat{h}_{01}(\mu)\} \\ &\quad + \frac{1}{2}\Sigma_{\mu\nu}\{\boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\nu^\dagger \hat{h}_{20}(\mu\nu) + \boldsymbol{X}_\nu \boldsymbol{X}_\mu \hat{h}_{02}(\mu\nu) + 2\boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\nu \hat{h}_{11}(\mu\nu)\} + \cdots, \end{aligned} \tag{6·7}$$

which is written in a form of the normal ordered product with respect to the creation and annihilation operators $(\boldsymbol{X}_\mu^\dagger, \boldsymbol{X}_\mu)$. The operators h_{ij} in Eq. (6·7) are given explicitly as

$$\begin{aligned} \hat{h}_{00} &= \mathring{H} - \Sigma_\mu\{X_\mu^\dagger[X_\mu, \mathring{H}] + [\mathring{H}, X_\mu^\dagger]X_\mu\} \\ &\qquad + \frac{1}{2}\Sigma_{\mu\nu}\{X_\mu^\dagger X_\nu^\dagger[X_\nu, [X_\mu, \mathring{H}]] + [[\mathring{H}, X_\mu^\dagger], X_\nu^\dagger]X_\nu X_\mu \\ &\qquad\qquad + 2X_\mu^\dagger[[X_\mu, \mathring{H}], X_\nu^\dagger]X_\nu\} + \cdots, \\ \hat{h}_{10}(\mu) &= \hat{h}_{01}(\mu)^\dagger \\ &= [X_\mu, \mathring{H}] - \Sigma_\nu\{X_\nu^\dagger[X_\nu, [X_\mu, \mathring{H}]] + [[X_\mu, \mathring{H}], X_\nu^\dagger]X_\nu\} + \cdots, \\ \hat{h}_{20}(\mu\nu) &= \hat{h}_{02}(\mu\nu)^\dagger \\ &= [X_\mu, [X_\nu, \mathring{H}]] + \cdots, \\ \hat{h}_{11}(\mu\nu) &= [[X_\mu, \mathring{H}], X_\nu^\dagger] + \cdots. \end{aligned} \tag{6·8}$$

Here we write down explicitly only the terms in $\hat{h}_{ij}$ which include single and double commutators of $\mathring{H}$.

Since $[\boldsymbol{H}, b_a^\dagger]=[\boldsymbol{H}, b_a]=0$ (from Eq. (5·15)), we obtain

$$[\hat{h}_{ij}, b_a^\dagger]=[\hat{h}_{ij}, b_a]=0, \tag{6·9}$$

which means that the operators $\hat{h}_{ij}$ only involve the intrinsic degrees of freedom represented in terms of the quasi-particle basis. Thus, $\hat{h}_{00}$ may be regarded as the intrinsic Hamiltonian, and its eigenstates are always made to satisfy the supplementary condition:

$$b_a|\phi_{\text{intr}}\rangle=0, \quad \text{i.e.,} \quad \hat{S}_-(a)|\phi_{\text{intr}}\rangle=0. \tag{6·10}$$

6-2 *Effective Hamiltonian*

Now, let us recall the well-known relation for the pairing-vibration modes:

$$[\mathring{H}, X_\mu^\dagger]=\omega_\mu X_\mu^\dagger+Z_\mu, \tag{6·11}$$

where Z means "interaction" which is neglected under the RPA. With the aid of Eq. (6·11), $\hat{h}_{ij}$ in Eq. (6·8) may be rewritten as

$$\begin{aligned}
&\hat{h}_{00}=\hat{h}_{00}^{(0)}+\hat{h}_{00}^{(1)}+\hat{h}_{00}^{(2)},\\
&\quad \hat{h}_{00}^{(0)}\equiv\mathring{H}-\textstyle\sum_\mu \omega_\mu X_\mu^\dagger X_\mu,\\
&\quad \hat{h}_{00}^{(1)}\equiv-\textstyle\sum_\mu\{X_\mu^\dagger Z_\mu^\dagger+Z_\mu X_\mu\},\\
&\quad \hat{h}_{00}^{(2)}\equiv\frac{1}{2}\textstyle\sum_{\mu\nu}\{X_\mu^\dagger X_\nu^\dagger[X_\nu, Z_\mu^\dagger]+[Z_\mu, X_\nu^\dagger]X_\nu X_\mu+2X_\mu^\dagger[Z_\mu^\dagger, X_\nu^\dagger]X_\nu\}+\cdots,\\
&\hat{h}_{10}(\mu)=\hat{h}_{10}^{(1)}(\mu)+\hat{h}_{10}^{(2)}(\mu),\\
&\quad \hat{h}_{10}^{(1)}(\mu)\equiv Z_\mu^\dagger,\\
&\quad \hat{h}_{10}^{(2)}(\mu)\equiv-\textstyle\sum_\nu\{X_\nu^\dagger[X_\nu, Z_\mu^\dagger]+[Z_\mu^\dagger, X_\nu^\dagger]X_\nu\}+\cdots,\\
&\hat{h}_{11}(\mu\nu)=\hat{h}_{11}^{(0)}(\mu\nu)+\hat{h}_{11}^{(2)}(\mu\nu),\\
&\quad \hat{h}_{11}^{(0)}(\mu\nu)\equiv\omega_\mu\delta_{\mu\nu}, \quad \hat{h}_{11}^{(2)}(\mu\nu)\equiv[Z_\mu^\dagger, X_\nu^\dagger]+\cdots,\\
&\hat{h}_{20}(\mu\nu)=\hat{h}_{20}^{(2)}(\mu\nu)\equiv[X_\nu, Z_\mu^\dagger]+\cdots.
\end{aligned}$$

At this stage, we recall that the space in which all the intrinsic operators $\hat{h}_{ij}$ act must be the intrinsic space, which obeys the supplementary condition (6·10) and consists of the state vectors $|S(a), S_0(a)=-S(a)\rangle$. Therefore, provided that the supplementary condition (6·10) is always kept to be satisfied properly, we may drop all terms in $\hat{h}_{ij}$ which *explicitly* have either the operator $b^\dagger$ on the leftmost side or the operator b on the rightmost side. For instance, we may make such reductions of $\hat{h}_{00}^{(0)}$ and $\hat{h}_{00}^{(1)}$, respectively, as

$$\hat{h}_{00}^{(0)} \Longrightarrow H + \text{const},$$
$$\text{const} \equiv -\langle\phi_0|\Sigma_\mu \omega_\mu X_\mu^\dagger X_\mu|\phi_0\rangle = -\Sigma_{\mu a}\phi_\mu(a)^2\omega_\mu, \tag{6·12}$$
$$\hat{h}_{00}^{(1)} = -\Sigma_\mu[\Sigma_a\{\psi_\mu(a)b_a^\dagger + \phi_\mu(a)b_a\}Z_\mu^\dagger + Z_\mu\Sigma_a\{\psi_\mu(a)b_a + \phi_\mu(a)b_a^\dagger\}]$$
$$= -\Sigma_\mu[\{\Sigma_a\psi_\mu(a)b_a^\dagger Z_\mu^\dagger + \Sigma_a\phi_\mu(a)[b_a, Z_\mu^\dagger] + \Sigma_a\phi_\mu(a)Z_\mu^\dagger b_a\} + \text{h.c.}]$$
$$\Longrightarrow -\Sigma_{\mu\nu}[\{\Sigma_a\phi_\mu(a)\psi_\nu(a)[X_\nu, Z_\mu^\dagger] - \Sigma_a\phi_\mu(a)\phi_\nu(a)[X_\nu^\dagger, Z_\mu^\dagger]\} + \text{h.c.}]. \tag{6·13}$$

Thus, the Hamiltonian (6·7) may be effectively written as

$$\boldsymbol{H} = \text{const} + \boldsymbol{H}_{\text{col}} + H_{\text{intr}} + H_{\text{coupl}}, \tag{6·14a}$$

$$\text{const} = -\Sigma_{\mu a}\phi_\mu(a)^2\omega_\mu,$$
$$\boldsymbol{H}_{\text{col}} = \Sigma_\mu \omega_\mu \boldsymbol{X}_\mu^\dagger \boldsymbol{X}_\mu,$$
$$H_{\text{intr}} = H - \Sigma_{\substack{\mu\nu\\a}}\phi_\mu(a)\phi_\nu(a)[X_\nu, Z_\mu] - \frac{1}{2}\Sigma_{\substack{\mu\nu\\a}}\phi_\mu(a)\psi_\nu(a)\{[X_\mu, Z_\nu^\dagger] + [Z_\mu, X_\nu^\dagger]\},$$
$$H_{\text{coupl}} = \Sigma_\mu(\boldsymbol{X}_\mu^\dagger Z_\mu^\dagger + \boldsymbol{X}_\mu Z_\mu) + \frac{1}{2}\Sigma_{\mu\nu}\{2\boldsymbol{X}_\mu^\dagger\boldsymbol{X}_\nu[X_\mu, Z_\nu]$$
$$+ \boldsymbol{X}_\mu^\dagger\boldsymbol{X}_\nu^\dagger[X_\mu, Z_\nu^\dagger] + \boldsymbol{X}_\nu\boldsymbol{X}_\mu[Z_\mu, X_\nu^\dagger]\}, \tag{6·14b}$$

where the terms involving commutators higher than double are neglected. The constant term represents the energy of ground-state correlation due to the collective pairing vibration; the terms H_{intr} and $\boldsymbol{H}_{\text{col}}$ are the intrinsic- and collective-Hamiltonians, respectively, and H_{coupl} represents the coupling between the collective and intrinsic degrees of freedom.

Now it is clear that we have achieved the aim of unambiguously writing the Hamiltonian in the form (3·7) where the roles of residual interaction are explicitly expressed.

§7. Concluding remarks

With the explicit use of the quasi-spin formalism, we have defined the collective subspace $\{|\phi_{\text{col}}\rangle\}$, which is associated with the pairing correlations and includes all the spurious components due to the nucleon-number non-conservation in the quasi-particle representation, and the intrinsic subspace $\{|\phi_{\text{intr}}\rangle\}$ which does not include any spurious components. The intrinsic states are characterized by the one-to-one correspondence between the seniority number v_a and the quasi-particle number n_a. Furthermore, we have shown that, by an introduction of canonical transformation with auxiliary variables, the collective and intrinsic degrees of freedom are represented with the auxiliary bosons and the quasi-particle operators, respectively. It has been shown that the original Hamiltonian H can be transformed into the effective Hamiltonian which is described in terms of both (collective) boson and fermion degrees of freedom.

In the next chapter we investigate the intrinsic excitation modes in further detail. In Chap. 7 we show that the coupling Hamiltonian $H_{\rm coupl}$ can be uniquely rewritten in terms of the collective modes of pairing vibration and the elementary modes of intrinsic excitation, within the NTD approximation.

Appendix 1A. Matrix elements of two-body interaction

According to Eq. (3·1), the general effective two-body interaction which is invariant under rotation, reflection and time reversal is given by

$$H_{\rm int}=\sum_{\alpha\beta\gamma\delta}\mathcal{V}_{\alpha\beta\gamma\delta}c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma \tag{1A·1}$$

with Eq. (3·2). The invariance properties of $H_{\rm int}$ under rotations and reflections are explicitly shown when it is rewritten in an invariant tensor product form with respect to the nucleon-pair operators coupled to angular momentum JM. Thus, according to Baranger's notations,[3] we are led to write the matrix elements $\mathcal{V}_{\alpha\beta\gamma\delta}$ in the form

$$\begin{aligned}\mathcal{V}_{\alpha\beta\gamma\delta}&=-\frac{1}{2}\sum_{JM}G(abcd;J)(j_aj_bm_\alpha m_\beta|JM)(j_cj_dm_\gamma m_\delta|JM)\\&=-\frac{1}{2}\sum_{JM}F(acdb;J)(-)^{j_c-m_\gamma}(j_aj_cm_\alpha\bar{m}_\gamma|JM)\\&\qquad\times(-)^{j_b-m_\beta}(j_dj_bm_\delta\bar{m}_\beta|JM),\end{aligned} \tag{1A·2}$$

upon which the parity and the charge conservations impose the conditions $(-)^{l_a+l_b}=(-)^{l_c+l_d}$ and $q_a+q_b=q_c+q_d$, respectively. When we further impose the isobaric invariance upon $H_{\rm int}$, Eq. (1A·2) is written, with the use of the isospin formalism, as

$$\begin{aligned}\mathcal{V}_{\alpha\beta\gamma\delta}&=-\frac{1}{2}\sum_{JMTM_T}G(abcd;JT)(j_aj_bm_\alpha m_\beta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\alpha\tau_\beta|TM_T\right)\\&\qquad\times(j_cj_dm_\gamma m_\delta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\gamma\tau_\delta|TM_T\right)\\&=-\frac{1}{2}\sum_{JMTM_T}F(acdb;JT)s_\gamma(j_aj_cm_\alpha\bar{m}_\gamma|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\alpha\bar{\tau}_\gamma|TM_T\right)\\&\qquad\times s_\beta(j_dj_bm_\delta\bar{m}_\beta|JM)\left(\frac{1}{2}\frac{1}{2}\tau_\delta\bar{\tau}_\beta|TM_T\right),\end{aligned} \tag{1A·3}$$

where $s_\gamma\equiv(-)^{j_c-m_\gamma}(-)^{1/2-\tau_\gamma}$ and τ denotes the z-component of the nucleon isospin. From the hermiticity of $H_{\rm int}$ and its time reversal invariance, G and F must be real. From Eq. (3·2), we also have the following properties

$$\begin{aligned} G(abcd;JT)&=G(cdab;JT)\\ &=-\theta(abJT)G(bacd;JT)=-\theta(cdJT)G(abdc;JT),\\ F(acdb;JT)&=F(dbac;JT)\\ &=\theta(abJT)\theta(cdJT)F(cabd;JT), \end{aligned} \tag{1A·4}$$

where $\theta(abJT)=(-)^{j_a+j_b-J}(-)^{1-T}$. The F and G type matrix elements are related with each other through the relation

$$F(acdb;JT)$$
$$=-\sum_{J'T'}(2J'+1)(2T'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix}G(abdc;J'T'). \tag{1A·5}$$

In the text we do not explicitly use the isospin formalism, but use the so-called m-scheme in the isospin. By specifying the proton and neutron explicitly by the letters π and ν respectively, the matrix elements in the m-scheme of isospin are given in terms of the above F and G type matrix elements as

$$\begin{aligned} &G(a_\pi b_\pi c_\pi d_\pi;J)=G(a_\nu b_\nu c_\nu d_\nu;J)=G(abcd;JT=1),\\ &G(a_\pi b_\nu c_\pi d_\nu;J)=G(a_\nu b_\pi c_\nu d_\pi;J)=\frac{1}{2}[G(abcd;JT=1)+G(abcd;JT=0)],\\ &F(a_\nu c_\pi d_\nu b_\pi;J)=F(a_\pi c_\nu d_\pi b_\nu;J)\\ &\qquad=-\frac{1}{2}\sum_{J'}(2J'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}[G(abdc;J'T=1)+G(abdc;J'T=0)],\\ &F(a_\nu c_\nu d_\pi b_\pi;J)=F(a_\pi c_\pi d_\nu b_\nu;J)\\ &\qquad=-\frac{1}{2}\sum_{J'}(2J'+1)\begin{Bmatrix} j_a & j_b & J' \\ j_d & j_c & J \end{Bmatrix}[G(abdc;J'T=1)-G(abdc;J'T=0)],\\ &F(a_\nu c_\pi d_\pi b_\nu;J)=F(a_\pi c_\nu d_\nu b_\pi;J)=0. \end{aligned} \tag{1A·6}$$

Appendix 1B. Pairing vibrational modes

In the text we use the pairing vibrational modes as the basis of the perturbative expansion of the Hamiltonian in collective representation. We here give their definitions, and discuss some related problems to the pairing vibrational modes.

1B-1 *The pairing Hamiltonian*

We start with the simplest case, i.e., with the pairing Hamiltonian

$$H^{(p)}=\sum_\alpha(\epsilon_a-\lambda)c_\alpha^\dagger c_\alpha-\frac{1}{4}G\cdot\sum_\alpha c_\alpha^\dagger c_{\bar\alpha}^\dagger\cdot\sum_\beta c_{\bar\beta}c_\beta, \tag{1B·1}$$

where $c_{\bar\alpha}^\dagger\equiv(-)^{j_a-m_a}c_{\bar\alpha}^\dagger$ and G is the strength of the pairing force. After the

Bogoliubov transformation (2·17), this is written in terms of the quasi-particle operators as

$$H^{(p)}=U_0^{(p)}+H_0^{(p)}+H_1^{(p)}+H_{\rm int}^{(p)}, \tag{1B·2}$$
$$U_0^{(p)}=\sum_a 2\Omega_a\left\{\left(\eta_a+\frac{1}{2}Gv_a^2\right)v_a^2-\frac{1}{2}u_av_a\Delta\right\},$$
$$H_0^{(p)}=\sum_a\{\eta_a(u_a^2-v_a^2)+2u_av_a\Delta\}\hat{n}_a,$$
$$H_1^{(p)}=\sum_a\{2\eta_au_av_a-(u_a^2-v_a^2)\Delta\}\{\hat{S}_+(a)+\hat{S}_-(a)\},$$
$$H_{\rm int}^{(p)}=H_X^{(p)}+H_V^{(p)}+H_Y^{(p)}+H_{\rm exch}^{(p)}, \tag{1B·3}$$
$$H_X^{(p)}=-G\sum_{ac}(u_a^2u_c^2+v_a^2v_c^2)\hat{S}_+(a)\hat{S}_-(c),$$
$$H_V^{(p)}=\frac{1}{2}G\sum_{ac}(u_a^2v_c^2+v_a^2u_c^2)\{\hat{S}_+(a)\hat{S}_+(c)+\hat{S}_-(c)\hat{S}_-(a)\},$$
$$H_Y^{(p)}=G\sum_{ac}(u_a^2-v_a^2)u_cv_c\{\hat{S}_+(a)\hat{n}_c+\hat{n}_c\hat{S}_-(a)\},$$
$$H_{\rm exch}^{(p)}=-G\sum_{ac}u_av_au_cv_c\{\hat{n}_a\hat{n}_c-\hat{n}_a\delta_{ac}\},$$

where

$$\eta_a=\epsilon_a-\lambda-Gv_a^2,\qquad \Omega_a=j_a+1/2,\qquad \Delta=G\sum_a\Omega_au_av_a,$$

$\hat{n}_a=\sum_{m_a}a_a^\dagger a_a$, and $\hat{S}_\pm(a)$ are defined in Eq. (2·18). As usual, the parameters u and v are determined so as to eliminate the "dangerous term" $H_1^{(p)}$. The quasi-particle energy term $H_0^{(p)}$ is then reduced to

$$H_0^{(p)}=\sum_aE_a\hat{n}_a,$$
$$E_a=\sqrt{\eta_a^2+\Delta^2}. \tag{1B·4}$$

Applying the canonical transformation (5·8) after the extension $H^{(p)}\rightarrow\mathring{H}^{(p)}$ (i.e., $\hat{S}_\pm(a)$, $\hat{S}_0(a)\rightarrow\mathring{S}_\pm(a)$, $\mathring{S}_0(a)$), we obtain the pairing Hamiltonian in the collective representation:

$$\boldsymbol{H}^{(p)}=U_{\rm col}\cdot\mathring{H}^{(p)}\cdot U_{\rm col}^{-1}=U_0^{(p)}+\boldsymbol{H}_0^{(p)}+\boldsymbol{H}_{\rm int}^{(p)}, \tag{1B·5}$$
$$\boldsymbol{H}_0^{(p)}=\sum_aE_a\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\},$$
$$\boldsymbol{H}_{\rm int}^{(p)}=\boldsymbol{H}_X^{(p)}+\boldsymbol{H}_V^{(p)}+\boldsymbol{H}_Y^{(p)}+\boldsymbol{H}_{\rm exch}^{(p)}, \tag{1B·6}$$
$$\boldsymbol{H}_X^{(p)}=-G\sum_{ac}(u_a^2u_c^2+v_a^2v_c^2)\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\sqrt{\Omega_c-\hat{n}_c-\hat{\boldsymbol{N}}(c)}\,\boldsymbol{b}_c,$$
$$\boldsymbol{H}_V^{(p)}=\frac{1}{2}G\sum_{ac}(u_a^2v_c^2+v_a^2u_c^2)[\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\boldsymbol{b}_c^\dagger\sqrt{\Omega_c-\hat{n}_c-\hat{\boldsymbol{N}}(c)}+\text{h.c.}],$$
$$\boldsymbol{H}_Y^{(p)}=G\sum_{ac}(u_a^2-v_a^2)u_cv_c[\boldsymbol{b}_a^\dagger\sqrt{\Omega_a-\hat{n}_a-\hat{\boldsymbol{N}}(a)}\,\{\hat{n}_c+2\hat{\boldsymbol{N}}(c)\}+\text{h.c.}],$$
$$\boldsymbol{H}_{\rm exch}^{(p)}=-G\sum_{ac}u_av_au_cv_c[\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\}\{\hat{n}_c+2\hat{\boldsymbol{N}}(c)\}-\{\hat{n}_a+2\hat{\boldsymbol{N}}(a)\}\delta_{ac}].$$

Needless to say, in this collective representation, the boson operators ($\boldsymbol{b}_a^\dagger$, $\boldsymbol{b}_a$) and the quasi-particle operators ($a_a^\dagger$, a_a) describe the collective and intrinsic degrees of freedom respectively, and therefore their mutual interweaving is clearly visualized.

1B-2 *Pairing vibrational modes under RPA*

Now let us make the expansion

$$\sqrt{\Omega_a-\hat{n}_a-\hat{N}(a)}=\sqrt{\Omega_a}\cdot\left[1-\left\{\frac{\hat{n}_a+\hat{N}(a)}{2\Omega_a}\right\}-\frac{1}{2}\left\{\frac{\hat{n}_a+\hat{N}(a)}{2\Omega_a}\right\}^2+\cdots\right], \tag{1B·7}$$

and then take up the following lowest order terms including only the collective variables from the Hamiltonian (1B·5)

$$\begin{aligned}\boldsymbol{H}_{\text{col}}=2\sum_a E_a\hat{N}(a)-G\sum_{ac}\sqrt{\Omega_a\Omega_c}\,(u_a^2u_c^2+v_a^2v_c^2)\boldsymbol{b}_a^\dagger\boldsymbol{b}_c\\ +\frac{1}{2}G\sum_{ac}\sqrt{\Omega_a\Omega_c}(u_a^2v_c^2+v_a^2u_c^2)(\boldsymbol{b}_a^\dagger\boldsymbol{b}_c^\dagger+\boldsymbol{b}_c\boldsymbol{b}_a),\end{aligned} \tag{1B·8}$$

where we have, as usual, neglected the terms originating from the exchange term $\boldsymbol{H}_{\text{exch}}^{(p)}$. This is nothing but the pairing vibrational Hamiltonian within the RPA and can be diagonalized with the pairing vibrational modes:

$$\begin{aligned}&[\boldsymbol{H}_{\text{col}},\ \boldsymbol{X}_n^\dagger]=\omega_n\boldsymbol{X}_n^\dagger;\ \omega_n>0,\\ &\boldsymbol{X}_n^\dagger=\sum_a\{\psi_n(a)\boldsymbol{b}_a^\dagger+\phi_n(a)\boldsymbol{b}_a\},\end{aligned} \tag{1B·9}$$

which satisfy the commutation relations

$$[\boldsymbol{X}_n,\ \boldsymbol{X}_m^\dagger]=\delta_{nm},\qquad [\boldsymbol{X}_n,\ \boldsymbol{X}_m]=[\boldsymbol{X}_n^\dagger,\ \boldsymbol{X}_m^\dagger]=0. \tag{1B·10}$$

The eigenvalue equation of the amplitudes takes the form

$$\omega_n\begin{bmatrix}\psi_n\\ \phi_n\end{bmatrix}=\begin{bmatrix}\boldsymbol{D}^{(p)} & -\boldsymbol{A}^{(p)}\\ \boldsymbol{A}^{(p)} & -\boldsymbol{D}^{(p)}\end{bmatrix}\begin{bmatrix}\psi_n\\ \phi_n\end{bmatrix}, \tag{1B·11}$$

where the matrix elements of $\boldsymbol{D}^{(p)}$ and $\boldsymbol{A}^{(p)}$ are given by

$$\begin{aligned}&D_{ac}^{(p)}=2E_a\delta_{ac}-G\sqrt{\Omega_a\Omega_c}\,(u_a^2u_c^2+v_a^2v_c^2),\\ &A_{ac}^{(p)}=G\sqrt{\Omega_a\Omega_c}(u_a^2v_c^2+v_a^2u_c^2).\end{aligned} \tag{1B·12}$$

The pairing vibrational modes for the general Hamiltonian given by Eq. (3·4) are also given in a similar way. In this case the matrix elements of $\boldsymbol{D}^{(p)}$ and $\boldsymbol{A}^{(p)}$ in Eq. (1B·11) are given as

$$\begin{aligned}&D_{ac}^{(p)}=2E_a\delta_{ac}+\sum_{m_\alpha m_\gamma}\frac{1}{\sqrt{\Omega_a\Omega_c}}\,V_X(\alpha\tilde{\alpha}\gamma\tilde{\gamma}),\\ &A_{ac}^{(p)}=-2\sum_{m_\alpha m_\gamma}\frac{1}{\sqrt{\Omega_a\Omega_c}}\,\{V_V(\alpha\tilde{\alpha}\gamma\tilde{\gamma})+V_V(\gamma\tilde{\gamma}\alpha\tilde{\alpha})-4V_V(\alpha\gamma\alpha\gamma)\},\end{aligned} \tag{1B·13}$$

where the definitions of $V_X(\alpha\beta\gamma\delta)$ and $V_V(\alpha\beta\gamma\delta)$ are given after Eq. (3·4) in the text.

1B-3 *Mode associated with breaking of nucleon-number conservation*

The eigenvalue equation (1B·11) is known to possess a special zero-energy solution which implies that RPA includes enough of the residual interaction to restore the nucleon-number conservation broken by the BCS solutions: The nucleon-number operator in the RPA is obtained by applying the expansion (1B·7) to the nucleon-number operator in the collective representation and by taking up the lowest order terms:

$$\begin{aligned}\mathcal{N}_{\rm RPA}&=\sum_a 2\Omega_a v_a^2+\mathcal{N}^{(0)},\\ \mathcal{N}^{(0)}&=2\sum_a\sqrt{\Omega_a}\,u_a v_a(\boldsymbol{b}_a^\dagger+\boldsymbol{b}_a).\end{aligned}\tag{1B·14}$$

This operator satisfies

$$[\boldsymbol{H}_{\rm col},\mathcal{N}_{\rm RPA}]=[\boldsymbol{H}_{\rm col},\mathcal{N}^{(0)}]=0\tag{1B·15}$$

which means that the nucleon-number operator in the RPA itself is a special solution of the RPA equation (1B·9). It is then convenient to define an operator as

$$\boldsymbol{\Phi}^{(0)}=-i\sum_a\Phi(a)(\boldsymbol{b}_a^\dagger-\boldsymbol{b}_a),\tag{1B·16}$$

which satisfies the equation

$$[\boldsymbol{\Phi}^{(0)},\mathcal{N}^{(0)}]=i,\qquad [\boldsymbol{H}_{\rm col},\boldsymbol{\Phi}^{(0)}]=-i\frac{1}{I_0}\mathcal{N}^{(0)}.\tag{1B·17}$$

Equation (1B·17) with Eq. (1B·15) is sufficient to determine $\boldsymbol{\Phi}^{(0)}$ and the (c-number) inertia parameter I_0. The canonical variables ($\mathcal{N}^{(0)}$, $\boldsymbol{\Phi}^{(0)}$) are commutable with pairing vibrational modes ($\boldsymbol{X}_n^\dagger$, $\boldsymbol{X}_n$) with non-zero eigenvalues, and therefore, with the set of operators $\boldsymbol{X}_n^\dagger$, $\boldsymbol{X}_n$, $\mathcal{N}^{(0)}$ and $\boldsymbol{\Phi}^{(0)}$, the boson operators ($\boldsymbol{b}_a^\dagger$, $\boldsymbol{b}_a$) may be expanded as

$$\begin{aligned}\boldsymbol{b}_a^\dagger&=\sum_n\{[\boldsymbol{X}_n,\boldsymbol{b}_a^\dagger]\boldsymbol{X}_n^\dagger+[\boldsymbol{b}_a^\dagger,\boldsymbol{X}_n^\dagger]\boldsymbol{X}_n\}+i[\boldsymbol{b}_a^\dagger,\boldsymbol{\Phi}^{(0)}]\mathcal{N}^{(0)}+i[\mathcal{N}^{(0)},\boldsymbol{b}_a^\dagger]\boldsymbol{\Phi}^{(0)}\\ &=\sum_n\{\psi_n(a)\boldsymbol{X}_n^\dagger-\phi_n(a)\boldsymbol{X}_n\}+\Phi(a)\mathcal{N}^{(0)}+2i\sqrt{\Omega_a}\,u_a v_a\boldsymbol{\Phi}^{(0)}.\end{aligned}\tag{1B·18}$$

The correlated ground state of $\boldsymbol{H}_{\rm col}$, denoted by $|0_B))$, is defined in part by the conventional requirement that it be the vacuum of the non-zero modes:

$$\boldsymbol{X}_n|0_B))=0.\tag{1B·19}$$

In addition to this, the zero-energy mode must also be taken into account to complete the definition of $|0_B))$. In order for the ground state to have a definite nucleon number, it is necessary to take on the condition

$$\mathcal{N}^{(0)}|0_B))=0. \tag{1B·20}$$

However, this requirement is known to be too stringent to be compatible with the basic assumptions of the RPA.[7] Therefore, we adopt the following limiting procedure: In so far as the zero-energy mode is concerned, the correlated ground state is assumed to be specified by

$$[\mathcal{N}^{(0)}-i\varepsilon_0 I_0^2 \Phi^{(0)}]|0_B))=0, \tag{1B·21}$$

where ε_0 is a real positive parameter. We can then define annihilation and creation operators

$$\begin{aligned} X_0&=\frac{1}{\sqrt{2\varepsilon_0}}\left[\frac{\mathcal{N}^{(0)}}{I_0}-i\varepsilon_0 I_0\Phi^{(0)}\right]=\Sigma_a\{\psi_0(a)b_a+\phi_0(a)b_a^\dagger\},\\ X_0^\dagger&=\frac{1}{\sqrt{2\varepsilon_0}}\left[\frac{\mathcal{N}^{(0)}}{I_0}+i\varepsilon_0 I_0\Phi^{(0)}\right]=\Sigma_a\{\psi_0(a)b_a^\dagger+\phi_0(a)b_a\}, \end{aligned} \tag{1B·22}$$

which satisfy $[X_0, X_0^\dagger]=1$. As ε_0 tends toward zero, X_0 and $X_0^\dagger$ separately tend toward infinity as $1/\sqrt{\varepsilon_0}$, while the behaviour of the corresponding term of H_{col} in this limit becomes

$$\varepsilon_0 I_0 X_0^\dagger X_0 \longrightarrow \frac{1}{2I_0}(\mathcal{N}^{(0)})^2, \tag{1B·23}$$

which is finite and is just the one expected from Eq. (1B·17).

With the aid of the operators $(X_0, X_0^\dagger)$, Eq. (1B·18) is now rewritten as

$$b_a^\dagger=\Sigma_\mu\{\psi_\mu(a)X_\mu^\dagger-\phi_\mu(a)X_\mu\}, \qquad b_a=\Sigma_\mu\{\psi_\mu(a)X_\mu-\phi_\mu(a)X_\mu^\dagger\}. \tag{1B·24}$$

With the use of these modes of pairing vibration we also define the canonical coordinates and canonical conjugate momenta as follows:

$$P_\mu=\frac{1}{\sqrt{2}}(X_\mu+X_\mu^\dagger), \qquad Q_\mu=\frac{i}{\sqrt{2}}(X_\mu-X_\mu^\dagger) \tag{1B·25}$$

which satisfy the canonical commutation relations

$$[Q_\mu, P_\nu]=i\delta_{\mu\nu}, \qquad [Q_\mu, Q_\nu]=[P_\mu, P_\nu]=0. \tag{1B·26}$$

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Chapter 2. Theory of Intrinsic Modes of Excitation in Odd-Mass Nuclei

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§1. Introduction

In this chapter*) we develop a theory to treat elementary modes of "intrinsic" excitation in odd-mass nuclei, which should be approximate eigenmodes of the intrinsic Hamiltonian $H_{\rm intr}$ given by the definition (1·6·14)**) in the preceding chapter. We must therefore treat these eigenmodes within the "intrinsic" subspace $\{|\phi_{\rm intr}\rangle\}$ which obeys the condition $\hat{S}_-(a)|\phi_{\rm intr}\rangle=0$. These eigenmodes are constructed within the framework of the NTD approximation (with the ground-state correlation) and provide the basis vectors for the intrinsic subspace within the NTD approximation. The dressed three-quasi-particle (3QP) mode, on which emphasis is put in this paper, is regarded as one of the simplest modes of intrinsic excitation.

In formulating the theory, for simplicity, we take up only the first term, H, of $H_{\rm intr}$ in (1·6·14) as the intrinsic Hamiltonian. The inclusion of the other terms does not alter the essential ingredients of the discussion in this chapter. Hereafter (with the exception of the last chapter) we concentrate on the study of the intrinsic excitations and drop the symbols expressing "intrinsic" quantities. Furthermore we use the term "collective" in the conventional sense, i.e., for such quantities related to the modes with the ground-state correlation.

§2. Quasi-particle new-Tamm-Dancoff space

The essence of our theory of the intrinsic modes of excitation is to make an explicit use of the concept of the quasi-particle new-Tamm-Dancoff (NTD)

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**) We cite the equations in different chapters by adding the chapter number to the first place of the equation number.

space. To obtain a first understanding of the concept of the quasi-particle NTD space and of the physical operators defined in it, let us start with the conventional quasi-particle Tamm-Dancoff (TD) space characterized by the seniority number.

2-1 *Quasi-particle Tamm-Dancoff space*

As shown in §1-Chap. 1, the use of the quasi-particle representation based on the BCS theory can be regarded as an attempt to characterize the intrinsic states in terms of the seniority number $v=\sum_a v_a$, the value of which corresponds to the number of quasi-particles, $n=\sum_a n_a$. Thus the intrinsic-energy spectrum of H_0 in odd-mass nuclei has a quite characteristic structure as shown in Fig. 1 and the corresponding states with a fixed odd number of quasi-particles, $n=v(=\sum_a v_a)$, span the n-quasi-particle TD space. The quasi-particle TD space for odd-mass nuclei is therefore characterized by the orthonormal state vectors with an odd number of quasi-particles:

$$\begin{aligned}
&|v=1;\alpha\rangle=a_\alpha^\dagger|\phi_0\rangle,\\
&|v=3;\alpha\beta\gamma\rangle=\frac{1}{\sqrt{3!}}a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger|\phi_0\rangle,\\
&|v=5;\alpha\beta\gamma\delta\varepsilon\rangle=\frac{1}{\sqrt{5!}}a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger a_\varepsilon^\dagger|\phi_0\rangle,\\
&\qquad\qquad\cdots,
\end{aligned}\tag{2·1}$$

where $|\phi_0\rangle$ is the BCS ground state defined in §§1 and 2 of Chap. 1.

In order to explicitly express the requirement that any state in the quasi-particle TD space must satisfy the supplementary condition (1·6·10), it is convenient to precisely define the quasi-particle TD space by adopting the concept of the quasi-spin tensor, which has been defined in §1-Chap. 1. The quasi-spin tensor operators (in the quasi-particle representation) $T_{k\kappa}(a)$ are defined by the relation (1·2·14), i.e.,

$$T_{k\kappa}(a)=\sum_{\kappa'}D^{k*}_{\kappa'\kappa}(\omega_0)\mathcal{T}_{k\kappa'}(a).$$

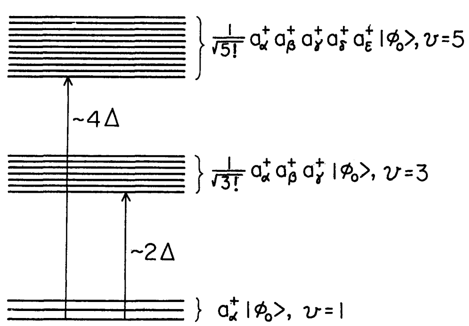

Fig. 1. Schematic energy spectra of H_0 in odd-mass nuclei.

They are constructed from the quasi-spin spinors; for example,

$$\begin{aligned}
&T_{3/2,\,3/2}(\alpha_1\alpha_2\alpha_3)=a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}a^\dagger_{\alpha_3},\\
&T_{3/2,\,1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}a_{\tilde\alpha_3}+a^\dagger_{\alpha_1}a_{\tilde\alpha_2}a^\dagger_{\alpha_3}+a_{\tilde\alpha_1}a^\dagger_{\alpha_2}a^\dagger_{\alpha_3}),\\
&T_{3/2,\,-1/2}(\alpha_1\alpha_2\alpha_3)=\frac{1}{\sqrt{3}}(a^\dagger_{\alpha_1}a_{\tilde\alpha_2}a_{\tilde\alpha_3}+a_{\tilde\alpha_1}a^\dagger_{\alpha_2}a_{\tilde\alpha_3}+a_{\tilde\alpha_1}a_{\tilde\alpha_2}a^\dagger_{\alpha_3}),\\
&T_{3/2,\,-3/2}(\alpha_1\alpha_2\alpha_3)=a_{\tilde\alpha_1}a_{\tilde\alpha_2}a_{\tilde\alpha_3}.
\end{aligned}\qquad (2\cdot2)$$

Now the quasi-particle TD space for odd-mass nuclei characterized by (2·1) is precisely defined in terms of the set of state vectors

$$\begin{aligned}
&|v=2s;\,\alpha_1\alpha_2\cdots\alpha_{2s_a},\,\beta_1\beta_2\cdots\beta_{2s_b},\,\cdots\rangle\\
&\quad=O^\dagger_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\,\kappa_a=s_a;\,\beta_1\beta_2\cdots\beta_{2s_b},\,\kappa_b=s_b;\,\cdots]|\phi_0\rangle,
\end{aligned}\qquad (2\cdot3\text{a})$$

where

$$\begin{aligned}
&O^\dagger_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\,\kappa_a;\,\beta_1\beta_2\cdots\beta_{2s_b},\,\kappa_b;\,\cdots]\\
&\quad=\frac{1}{\sqrt{(2s_a)!\,(2s_b)!\cdots}}\mathring{T}_{s_a\kappa_a}(\alpha_1\alpha_2\cdots\alpha_{2s_a})\cdot\mathring{T}_{s_b\kappa_b}(\beta_1\beta_2\cdots\beta_{2s_b})\cdots
\end{aligned}\qquad (2\cdot3\text{b})$$

with $2s=2(s_a+s_b+\cdots)=v$ in odd numbers. In Eq. (2·3b), we have used the definition

$$\begin{aligned}
\mathring{T}_{k\kappa}(\alpha_1\alpha_2\cdots\alpha_{2k})\equiv&\sum_{\alpha_1'\alpha_2'\cdots\alpha_{2k}'}P(\alpha_1\alpha_2\cdots\alpha_{2k}|\alpha_1'\alpha_2'\cdots\alpha_{2k}')\\
&\times T_{k\kappa}(\alpha_1'\alpha_2'\cdots\alpha_{2k}'),
\end{aligned}\qquad (2\cdot4)$$

where the operator $\boldsymbol{P}$ (the matrix elements of which are $P(\alpha_1\alpha_2\cdots\alpha_{2k}|\alpha_1'\alpha_2'\cdots\alpha_{2k}')$) is a projection operator by which the quasi-spin operators $\hat{S}_\pm(a)$ and $\hat{S}_0(a)$ are removed from the quasi-spin tensor $T_{k\kappa}(\alpha_1\alpha_2\cdots\alpha_{2k})$ completely. Therefore the eigenvectors of the projection operator $\boldsymbol{P}$ are closely related to the coefficients of fractional parantage (cfp) with the seniority $v_a=2k$ for $(j_a)^{2k}$-configurations, and its explicit form for $k=3/2$ is given in Appendix 2A. By the definition of the operator $O^\dagger_s$ in the relation (2·3), we obtain

$$\hat{S}_-(a)|v=2s;\,\alpha_1\alpha_2\cdots\alpha_{2s_a},\,\beta_1\beta_2\cdots\beta_{2s_b},\,\cdots\rangle=0.\qquad (2\cdot5)$$

Thus the quasi-particle TD space defined in the above manner satisfies the supplementary condition (1·6·10).

Since the quasi-particle TD space is characterized by the seniority number $v(=n)$ as shown in Fig. 1, a better approximation may be obtained by diagonalizing the quasi-particle interaction H_{int} in the Hamiltonian (1·3·4) in the subspace with a fixed number of quasi-particles. This is well known as the quasi-particle TD approximation. Among the matrix elements of H_{int} in the subspace with a definite number of quasi-particles, the non-zero ones come from only the part H_X which conserves the quasi-particle number.

Therefore the eigenmode-creation operators $Y^\dagger_{0s\lambda}$ in the TD approximation are given by the eigenvalue equation

$$[H_0+H_X,\ Y^\dagger_{0s\lambda}]\cong\omega_{0s\lambda}Y^\dagger_{0s\lambda}\quad(\omega_{0s\lambda}>0) \tag{2·6}$$

with

$$Y^\dagger_{0s\lambda}=\sum_{\substack{\alpha_i,\beta_j,\cdots\\(s=\sum_a s_a)}}\Psi_{0s\lambda}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a=s_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b=s_b;\ \cdots]$$
$$\times O^\dagger_s[\alpha_1\alpha_2\cdots\alpha_{2s_a},\ \kappa_a=s_a;\ \beta_1\beta_2\cdots\beta_{2s_b},\ \kappa_b=s_b;\ \cdots]. \tag{2·7}$$

Here λ denotes a set of additional quantum numbers to specify the eigenmodes.

The eigenmodes $Y^\dagger_{0s\lambda}$ satisfy the anti-commutation relation in the following sense:

$$\{Y_{0s_>\lambda},\ Y^\dagger_{0s'_<\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\phi_0\rangle,$$
$$\{Y^\dagger_{0s\lambda},\ Y^\dagger_{0s'\lambda'}\}_+=\{Y_{0s\lambda},\ Y_{0s'\lambda'}\}_+=0, \tag{2·8}$$

where the subscript $>$ (or$<$) of $s_>$ (or $s'_<$) denotes the relation $s\geq s'$. Thus, the set of states $Y^\dagger_{0s\lambda}|\phi_0\rangle$ with $2s=n$ in odd numbers provides a complete set of orthonormal bases of the quasi-particle TD space:

$$\langle\phi_0|\{Y_{0s\lambda},\ Y^\dagger_{0s'\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}. \tag{2·9}$$

Now it is clear that the quasi-particle TD space for odd-mass nuclei may be characterized with the operators defined by

$$\boldsymbol{Y}^\dagger_{0s\lambda}=Y^\dagger_{0s\lambda}|\phi_0\rangle\langle\phi_0|,\ \ \boldsymbol{Y}_{0s\lambda}=|\phi_0\rangle\langle\phi_0|Y_{0s\lambda}\ \text{with } 2s \text{ in odd numbers.} \tag{2·10}$$

By definition, the operators $\boldsymbol{Y}^\dagger_{0s\lambda}$ satisfy the equations

$$[H_0+H_X,\ \boldsymbol{Y}^\dagger_{0s\lambda}]=\omega_{0s\lambda}\boldsymbol{Y}^\dagger_{0s\lambda}\quad(\omega_{0s\lambda}>0) \tag{2·11}$$

and

$$\{\boldsymbol{Y}_{0s\lambda},\ \boldsymbol{Y}^\dagger_{0s'\lambda'}\}_+|\phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\phi_0\rangle,$$
$$\{\boldsymbol{Y}^\dagger_{0s\lambda},\ \boldsymbol{Y}^\dagger_{0s'\lambda'}\}_+=\{\boldsymbol{Y}_{0s\lambda},\ \boldsymbol{Y}_{0s'\lambda'}\}_+=0. \tag{2·12}$$

The unit operator $\mathbf{1}$ in the quasi-particle TD space for odd-mass nuclei is given by

$$\mathbf{1}=\Sigma'_{s\lambda}\boldsymbol{Y}^\dagger_{0s\lambda}\boldsymbol{Y}_{0s\lambda}, \tag{2·13}$$

where $\Sigma'_{s\lambda}$ denotes the summation with respect to $2s$ in odd numbers. With the use of the operators $\boldsymbol{Y}^\dagger_{0s\lambda}$, the Hamiltonian H_0+H_X in (1·3·4) is now written as

$$H_0+H_X\longrightarrow\mathbf{1}(H_0+H_X)\mathbf{1}=\Sigma'_{s\lambda}\omega_{0s\lambda}\boldsymbol{Y}^\dagger_{0s\lambda}\boldsymbol{Y}_{0s\lambda}. \tag{2·14}$$

Thus, using the elementary excitation operators $\boldsymbol{Y}^{\dagger}_{0s\lambda}$ instead of the quasi-particle operators $a^{\dagger}_{a}$, we obtain another representation of any operator $\hat{F}$ in the quasi-particle TD space for odd-mass nuclei:

$$\hat{\boldsymbol{F}}=\mathbf{1}\hat{F}\mathbf{1}=\sum'_{s\lambda}\sum'_{s'\lambda'}\langle 0s\lambda|\hat{F}|0s'\lambda'\rangle \boldsymbol{Y}^{\dagger}_{0s\lambda}\boldsymbol{Y}_{0s'\lambda'} \qquad (2\cdot15)$$

with the definition $|0s\lambda\rangle=Y^{\dagger}_{0s\lambda}|\phi_0\rangle$.

2-2 *Quasi-particle NTD space*

Now it is well known that in a finite quantal system such as nucleus, the ground-state correlation is particularly important as a collective predisposition which permits the correlated excited states to occur from the ground state. Actually, we must simultaneously take special account of both the seniority classification and the ground-state correlation, in a way that the essential physical notion obtained in the quasi-particle TD space still persists in a certain form. The guiding principle to introduce the quasi-particle NTD space lies in the fact that, in the new-Tamm-Dancoff (NTD) method, the quasi-particle correlations attributed asymmetrically to only the excited states in the TD calculations are symmetrically incorporated in the ground state through the ground-state correlation. The quasi-particle NTD space for odd-mass nuclei is thus defined with a set of basis vectors,

$$Y^{\dagger}_{s\lambda}|\Phi_0\rangle \qquad (2\cdot16)$$

with $2s$ in odd numbers, where $Y^{\dagger}_{s\lambda}$ are creation operators of "*dressed*" n $(=2s)$-quasi-particle modes constructed within the framework of the NTD method with the ground-state correlation, and $|\Phi_0\rangle$ is the corresponding ground state. Contrary to the BCS ground state $|\phi_0\rangle$, the state $|\Phi_0\rangle$ does not have a definite seniority number because of the ground-state correlation. In spite of the breakdown of the seniority number, we can still characterize the excitation modes in the quasi-particle NTD space by the amount of seniority $\Delta v(=2s=n)$ which they transfer to the ground state $|\Phi_0\rangle$.

In exactly the same way as the conventional spherical tensor operator is characterized by the amount of angular momentum it transfers to the state on which it acts, the quasi-spin-tensor operator $T_{s\kappa}$ is characterized by the amount of the transferred seniority $\Delta v=2s$ to the state on which it operates. Therefore, we can define the dressed $n(=2s)$-quasi-particle modes $Y^{\dagger}_{s\lambda}$ in terms of the direct product of the quasi-spin-tensor operators defined in each orbit with the total transferred seniority $\Delta v=2s=\sum_a 2s_a$:

$$Y^{\dagger}_{s\lambda}=\sum_{\substack{\alpha_i,\beta_j,\cdots\\(s=\sum_a s_a)}}\sum_{\kappa_a,\kappa_b,\cdots}\Psi_{s\lambda}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\kappa_a;\beta_1\beta_2\cdots\beta_{2s_b},\kappa_b;\cdots]$$
$$\times O^{\dagger}_{s}[\alpha_1\alpha_2\cdots\alpha_{2s_a},\kappa_a;\beta_1\beta_2\cdots\beta_{2s_b},\kappa_b;\cdots], \qquad (2\cdot17)$$

where $O^{\dagger}_{s}[\alpha_i,\kappa_a;\beta_j,\kappa_b;\cdots]$ is defined by (2·3b). Within the framework of the NTD approximation, the eigenvalue equation which the amplitude $\Psi_{s\lambda}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \cdots]$ must satisfy is as usual given by

$$[H_0+H_X+H_V,\ Y^\dagger_{s\lambda}]\cong\omega_{s\lambda}Y^\dagger_{s\lambda},\quad (\omega_{s\lambda}>0) \tag{2·18}$$

where the part H_V of the quasi-particle interaction $H_{\rm int}$ in (1·3·4) introduces the ground-state correlation.

The part H_V together with H_X is known to be essential in constructing the collective excitation modes within the framework of the NTD method. Hence we call the parts H_X and H_V the *constructive force* (of the collective modes). The part H_Y in (1·3·4) changes the number of quasi-particles, and has no contribution in the TD calculation with the definite number of quasi-particles. In so far as the NTD method is adopted (in describing the dressed n-quasi-particle mode) as an improvement of the TD method (for n-quasi-particles) the part H_Y does not play any important role. The part H_Y plays a decisive role as an essential coupling between the various dressed n-quasi-particle modes. Hence we call it the *interactive force*.*)

The dressed n-quasi-particle modes $Y^\dagger_{s\lambda}$ (with $2s=n$) must satisfy the fermion-type anti-commutation relation in the quasi-particle NTD space,

$$\{Y_{s>\lambda},\ Y^\dagger_{s'<\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}|\Phi_0\rangle, \tag{2·19}$$

just as the n-quasi-particle modes $Y^\dagger_{0s\lambda}$ (with $2s=n$) in the quasi-particle TD space satisfy (2·8). This requirement is necessary, together with the eigenvalue equation (2·18), for prescribing the elementary excitation modes in terms of the concept of transferred seniority. When (2·19) is satisfied within the framework of the NTD approximation, the set of states $Y^\dagger_{s\lambda}|\Phi_0\rangle$ with $2s=n$ in odd numbers becomes a complete set of orthonormal bases in the quasi-particle NTD space for odd-mass nuclei:

$$\langle\Phi_0|\{Y_{s\lambda},\ Y^\dagger_{s'\lambda'}\}_+|\Phi_0\rangle=\delta_{ss'}\delta_{\lambda\lambda'}, \tag{2·20}$$

and, in the same way as (2·13), the unit operator in the quasi-particle NTD space for odd-mass nuclei is given by

$$\mathbf{1}=\Sigma'_{s\lambda}\boldsymbol{Y}^\dagger_{s\lambda}\boldsymbol{Y}_{s\lambda}, \tag{2·21}$$

where

$$\boldsymbol{Y}^\dagger_{s\lambda}=Y^\dagger_{s\lambda}|\Phi_0\rangle\langle\Phi_0|,\qquad \boldsymbol{Y}_{s\lambda}=|\Phi_0\rangle\langle\Phi_0|Y_{s\lambda}. \tag{2·22}$$

In terms of the elementary excitation operators $\boldsymbol{Y}^\dagger_{s\lambda}$, any physical operator $\hat{F}$ is easily transcribed into the quasi-particle NTD sapce:

$$\hat{F}\longrightarrow\hat{\boldsymbol{F}}=\mathbf{1}\hat{F}\mathbf{1}=\Sigma'_{s\lambda}\Sigma'_{s'\lambda'}\langle\Phi_0|Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle\cdot\boldsymbol{Y}^\dagger_{s\lambda}\boldsymbol{Y}_{s'\lambda'}. \tag{2·23}$$

*) It should be also noted that the matrix elements of H_Y contain the reduction (u, v)-factors which can be quite small in the middle of the shell, while the matrix elements of H_X and H_V involve the enhancement (u, v)-factors which are close to unity for low-lying states in the middle of the shell.

Thus, the actual problem is how to estimate the matrix elements $\langle\Phi_0|Y_{s\lambda}\times\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle$. As shown in §5, however, a simple rule will be found when the anti-commutation relation (2·19) is satisfied.

In the following sections we concretely study the quasi-particle NTD subspace which consists of the dressed quasi-particle modes with the transferred seniority $\Delta v(=2s)=1$ and 3, because we are considering the low-lying collective excited states in odd-mass nuclei.

§3. Structure of dressed three-quasi-particle modes

According to the definition (2·17), the eigenmode operators of the dressed three-quasi-particle modes (which satisfy Eq. (2·18) with $2s=3$ within the NTD approximation) are written in the explicit form:

$$
\begin{aligned}
C^\dagger_\lambda = & \frac{1}{\sqrt{3!}}\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma \\
& +\frac{1}{\sqrt{3!}}\sum_{a_1a_2a_3}\phi^{(1)}_\lambda(a_1a_2a_3)\mathring{T}_{3/2,-1/2}(a_1a_2a_3) \\
& +\frac{1}{\sqrt{2!}}\sum_{\substack{a_1a_2\gamma\\(a\neq c)}}\phi^{(2)}_\lambda(a_1a_2;\gamma)\mathring{T}_{10}(a_1a_2)a_{\tilde{\gamma}} \\
& +\sum_{\substack{(\alpha\beta)\gamma\\(a,b\neq c)}}(1+\delta_{ab})^{-1/2}\phi^{(3)}_\lambda(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)a^\dagger_\gamma a_{\tilde{\alpha}}a_{\tilde{\beta}}\,.
\end{aligned}
\tag{3·1}
$$

Here the symbol $\sum_{(\alpha\beta)\gamma}$ denotes the summation over the orbit-pair (ab), m_α, m_β and γ, and

$$
\begin{aligned}
&\boldsymbol{P}(\alpha\beta\gamma)a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma \equiv \sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')a^\dagger_{\alpha'}a^\dagger_{\beta'}a^\dagger_{\gamma'}\,,\\
&\boldsymbol{P}(\alpha\beta)a_{\tilde{\alpha}}a_{\tilde{\beta}} \equiv \sum_{\alpha'\beta'}P(\alpha\beta|\alpha'\beta')a_{\tilde{\alpha}'}a_{\tilde{\beta}'}\,,
\end{aligned}
\tag{3·2}
$$

where the operators $\boldsymbol{P}$ stand for the projection operators by which any quasi-spin operator is removed from the products of quasi-particles $(a^\dagger_\alpha,\ a_\alpha)$. Their explicit forms are given in Appendix 2A. A direct calculation of Eq. (2·18) with (3·1) leads to the following eigenvalue equation which the correlation amplitudes should satisfy:

$$
\omega_\lambda\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\phi}_\lambda\end{bmatrix}=\begin{bmatrix}3\boldsymbol{D} & -\boldsymbol{A}\\ \boldsymbol{A}^T & -\boldsymbol{d}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\phi}_\lambda\end{bmatrix},
\tag{3·3}
$$

where $\boldsymbol{\psi}_\lambda$ and $\boldsymbol{\phi}_\lambda$ are the vector notations symbolizing the sets of amplitudes $\psi_\lambda(\alpha\beta\gamma)$ and $\{\phi^{(1)}_\lambda(a_1a_2a_3),\ \phi^{(2)}_\lambda(a_1a_2;\ \gamma),\ \phi^{(3)}_\lambda(\alpha\beta;\ \gamma)\}$, respectively, and the explicit forms of matrices $\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ are given in Appendix 2B. The projection operators $\boldsymbol{P}$ involved in these matrices guarantee that the correlation amplitudes automatically satisfy the relations

$$\begin{aligned}
&\psi_\lambda(\alpha\beta\gamma)=\sum_{\alpha'\beta'\gamma'}P(\alpha\beta\gamma|\alpha'\beta'\gamma')\psi_\lambda(\alpha'\beta'\gamma'),\\
&\phi_\lambda^{(1)}(a_1a_2a_3)=\sum_{a_1'a_2'a_3'}P(a_1a_2a_3|a_1'a_2'a_3')\phi_\lambda^{(1)}(a_1'a_2'a_3'),\\
&\phi_\lambda^{(2)}(a_1a_2;\gamma)=\sum_{a_1'a_2'}P(a_1a_2|a_1'a_2')\phi_\lambda^{(2)}(a_1'a_2';\gamma),\\
&\phi_\lambda^{(3)}(a\beta;\gamma)=\sum_{a'\beta'}P(a\beta|a'\beta')\phi_\lambda^{(3)}(a'\beta';\gamma)
\end{aligned}\tag{3·4}$$

which mean that the correlation amplitudes never contain any component due to the nucleon-number fluctuations (i.e., due to the quasi-spin operators).

Equation (3·3) tells us that with the use of the definition of the metric matrix

$$\boldsymbol{\tau}=\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & -\mathbf{1}\end{bmatrix},\tag{3·5}$$

the correlation amplitudes satisfy the orthonormality relation in the sense

$$(\boldsymbol{\psi}_{\lambda'}^T,\ \boldsymbol{\phi}_{\lambda'}^T)\boldsymbol{\tau}\begin{bmatrix}\boldsymbol{\psi}_\lambda\\ \boldsymbol{\phi}_\lambda\end{bmatrix}=\varepsilon_\lambda\delta_{\lambda\lambda'},\tag{3·6}$$

where ε_λ is the sign function with $|\varepsilon_\lambda|=1$ and $\boldsymbol{\psi}_\lambda^T$ denotes the transposed vector of $\boldsymbol{\psi}_\lambda$. Due to the introduction of the backward-going components, the eigenvalue equation (3·3) has "extra" unphysical solutions which have the large amplitudes $\boldsymbol{\phi}_{\lambda_0}$ and the small amplitudes $\boldsymbol{\psi}_{\lambda_0}$.*) As long as the eigenvalues ω_λ are real, the physical solutions have the large amplitudes $\boldsymbol{\psi}_\lambda$ and the small amplitudes $\boldsymbol{\phi}_\lambda$. Thus the positive ε_λ corresponds to the physical solutions, and we can classify the eigenmode operators $C_\lambda^\dagger$ in (3·1) as follows:

$$C_\lambda^\dagger=\begin{cases}Y_\lambda^\dagger & \text{for}\quad \varepsilon_\lambda=1,\\ A_{\lambda_0} & \text{for}\quad \varepsilon_{\lambda_0}=-1.\end{cases}\tag{3·7}$$

The physical dressed 3-quasi-particle states are given as

$$|\lambda\rangle=Y_\lambda^\dagger|\Phi_0\rangle,\tag{3·8}$$

where $|\Phi_0\rangle$ is the correlated ground state (within the framework of the NTD approximation). The existence of the extra eigenmodes $A_{\lambda_0}^\dagger$, which have no physical meaning, imposes an important condition upon the state vectors in the quasi-particle NTD space: Any state vector $|\Phi\rangle$ which actually has physical meaning must satisfy the supplementary condition

$$A_{\lambda_0}|\Phi\rangle=0.\tag{3·9}$$

§4. Structure of ground-state correlation

It is now quite important to examine the compatibility of Eqs. (3·3) and

*) Hereafter the unphysical solutions are specified by the subscript λ_0.

(2·19). In this section, we show that the requirement (2·19) is satisfied within the NTD approximation when we properly take into account the characteristics of the introduced ground-state correlation.

4-1 *Prescription of structure of ground-state correlation*

First of all, let us investigate the characteristics of the ground-state correlation (due to the dressed 3-quasi-particle modes). The structure of the ground-state correlation should be determined *in principle* through the properties of the fundamental eigenvalue equation (3·3). As is seen from Eq. (2·18), the fundamental equation contains only the matrix elements of the constructive force, H_X and H_V. The diagrams considered in the correlated ground state $|\Phi_0\rangle$ are therefore closed diagrams composed by combining only the matrix elements of H_X and H_V, so that $|\Phi_0\rangle$ may be written as a superposition of 0-, 4-, 8-quasi-particle states:

$$|\Phi_0\rangle = C_0|\phi_0\rangle + \sum_{\alpha\beta\gamma\delta} C_1(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger |\phi_0\rangle + \sum_{\alpha\beta\gamma\delta\varepsilon\xi\mu\nu} C_2(\alpha\beta\gamma\delta\varepsilon\xi\mu\nu) a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger a_\varepsilon^\dagger a_\xi^\dagger a_\mu^\dagger a_\nu^\dagger |\phi_0\rangle + \cdots, \quad (4·1)$$

where C_0 is the constant related to the normalization of $|\Phi_0\rangle$. The coefficients C in (4·1) should be determined by the conditions $Y_\lambda|\Phi_0\rangle = 0$ and $A_{\lambda_0}|\Phi_0\rangle = 0$, within the framework of the NTD approximation (which is used to obtain the fundamental equation (3·3)). This procedure suggests that, with the basic approximation in the NTD method (i.e., $O(n_0/2\Omega) \approx 0$),*) the correlated ground state $|\Phi_0\rangle$ may be written in a symbolized form

$$|\Phi_0\rangle = C_0 \exp\left[\frac{1}{\sqrt{4!}} k \sum_{\alpha\beta\gamma\delta} \chi_{J=0}(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger\right] |\phi_0\rangle \equiv C_0 \exp[W]|\phi_0\rangle, \quad (4·2)$$

where the constant k and $\chi_{J=0}(\alpha\beta\gamma\delta)$ are defined through the relations

$$\frac{1}{\sqrt{4!}} k \cdot \chi_{J=0}(\alpha\beta\gamma\delta) \equiv C_1(\alpha\beta\gamma\delta)/C_0, \qquad \sum_{\alpha\beta\gamma\delta} \chi_{J=0}(\alpha\beta\gamma\delta)^2 = 1. \quad (4·3)$$

Needless to say, $\chi_{J=0}(\alpha\beta\gamma\delta)$ never contain any component of the $J=0$-coupled quasi-particle pair, so that it should satisfy

$$\chi_{J=0}(\alpha\beta\gamma\delta) = \sum_{\alpha'\beta'\gamma'\delta'} P(\alpha\beta\gamma\delta|\alpha'\beta'\gamma'\delta') \chi_{J=0}(\alpha'\beta'\gamma'\delta'). \quad (4·4)$$

The ground-state correlation written in the symbolized form (4·2) should be interpreted as to be characterized by the following prescriptions:

(1) For an arbitrary operator $\hat{O}$, we have

*) Here n_0 and 2Ω are defined through $\langle\Phi_0|a_\alpha^\dagger a_\beta|\Phi_0\rangle \cong \delta_{\alpha\beta} \cdot n_0/2\Omega$, so that n_0 denotes the average number of quasi-particles in the ground state and 2Ω the total number of single-particle states under consideration.

$$\begin{aligned}\hat{O}|\Phi_0\rangle &= C_0\hat{O}\exp[W]|\phi_0\rangle \\ &= C_0\exp[W]\{\hat{O}+[\hat{O}, W]+\frac{1}{2}[[\hat{O}, W], W]+\cdots\}|\phi_0\rangle \\ &\Rightarrow C_0\exp[W]\{\hat{O}+[\hat{O}, W]\}|\phi_0\rangle. \quad \text{(the NTD approximation)}\end{aligned} \tag{4·5}$$

(2) Since the basis operators characterizing the ground-state correlation are $O^{\dagger}_{3/2}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \gamma_k, \kappa_c]$ (Eq. (2·3b) with $s=3/2$) which construct the dressed 3-quasi-particle modes and since *the operators* $O^{\dagger}_{3/2}[\alpha_i, \kappa_a; \beta_j, \kappa_b; \gamma_k, \kappa_c]$ *are antisymmetric with respect to the indices belonging to the same single-particle orbit,* all quantities which appear in the last expression of Eq. (4·5) must maintain the same property.

According to prescription (1), the supplementary condition (3·9) with (4·2) leads to the relation

$$\boldsymbol{\phi}_{\lambda_0}-k\boldsymbol{C}\boldsymbol{\phi}_{\lambda_0}=0 \tag{4·6}$$

with

$$\begin{aligned}C_{\alpha\beta\gamma,\alpha_1'\alpha_2'\alpha_3'} &\equiv 3\sqrt{2}\,\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}_1'\tilde{\alpha}_2')\delta_{\gamma\alpha_3'}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3'), \\ C_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'} &\equiv 6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}_2'\tilde{\gamma}')\delta_{\gamma\alpha_1'}\boldsymbol{P}^T(\alpha_1'\alpha_2'), \\ C_{\alpha\beta\gamma,\alpha'\beta'\gamma'} &\equiv 6\boldsymbol{P}(\alpha\beta\gamma)\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\delta_{\gamma\gamma'}(1+\delta_{a'b'})^{-1/2}\boldsymbol{P}^T(\alpha'\beta'). \\ (\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}') &\equiv (-)^{j_{a'}-m_{\alpha'}}(-)^{j_{b'}-m_{\beta'}}\chi_{J=0}(\alpha\beta\bar{\alpha}'\bar{\beta}'))\end{aligned}$$

For simplicity, the following abbreviations are used:

$$\begin{aligned}&\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma, \alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta'\gamma') \\ &\quad\equiv \textstyle\sum_{\sigma\mu\nu}\sum_{\sigma'\mu'\nu'}P(\alpha\beta\gamma|\sigma\mu\nu)f(\sigma\mu\nu, \sigma'\mu'\nu')P(\sigma'\mu'\nu'|\alpha'\beta'\gamma'), \\ &\boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma, \alpha'\beta'\gamma')\boldsymbol{P}^T(\alpha'\beta') \\ &\quad\equiv \textstyle\sum_{\sigma\mu\nu}\sum_{\mu'\nu'}P(\alpha\beta\gamma|\sigma\mu\nu)f(\sigma\mu\nu, \mu'\nu'\gamma')P(\mu'\nu'|\alpha'\beta'),\end{aligned} \tag{4·7}$$

where $f(\alpha\beta\gamma, \alpha'\beta'\gamma')$ is an arbitrary function with respect to $(\alpha\beta\gamma, \alpha'\beta'\gamma')$. Combining Eqs. (4·6) and (3·6) and using the symmetry property of $\chi_{J=0}(\alpha\beta\gamma\delta)$ with respect to the permutation of $(\alpha\beta\gamma\delta)$, we obtain an equation to determine $\chi_{J=0}(\alpha\beta\gamma\delta)$ in terms of the physical amplitudes:

$$\boldsymbol{\phi}_{\lambda}-k\boldsymbol{C}^T\boldsymbol{\phi}_{\lambda}=0. \tag{4·8}$$

4-2 *Orthonormality relations of intrinsic modes of excitation*

The particular importance of prescription (2) manifests itself when we evaluate the following expression:

$$\{Y_\lambda, a_\gamma^\dagger\}_+|\Phi_0\rangle = \Big\{\frac{\sqrt{6}}{2}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}$$
$$+\frac{1}{\sqrt{2}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\gamma}_1}^\dagger$$
$$+\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\gamma\gamma_1}\phi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\alpha}}^\dagger a_{\tilde{\gamma}_2}^\dagger$$
$$+\sum_{\substack{(\alpha'\beta')\gamma'\\(a,b\neq c)}}\delta_{\gamma\gamma'}(1+\delta_{a'b'})^{-1/2}\phi_\lambda^{(3)}(\alpha'\beta';\gamma')a_{\tilde{\beta}'}^\dagger a_{\tilde{\alpha}'}^\dagger\Big\}|\Phi_0\rangle. \qquad (4\cdot9)$$

In this case we must evaluate the first term. With the aid of prescription (1), we first obtain

$$\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$
$$=\sqrt{6}\,k\sum_{\alpha'\beta'\gamma'}\sum_{\alpha\beta}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}}^\dagger a_{\tilde{\beta}}^\dagger|\Phi_0\rangle.$$

Prescription (2) then leads the right-hand side to

$$6k\sum_{\alpha'\beta'\gamma'}\sum_{\alpha\beta}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}}^\dagger a_{\tilde{\beta}}^\dagger|\Phi_0\rangle$$
$$\Rightarrow\sqrt{6}\,k\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\boldsymbol{P}(\gamma_1\gamma_2\gamma_3)\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_3}\chi_{J=0}(\gamma_1\gamma_2\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$
$$+2\sqrt{6}\,k\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\gamma\gamma_1}\boldsymbol{P}(\gamma_1\gamma_2)\sum_{\alpha'\beta'\gamma'}\delta_{\gamma'\gamma_1}\chi_{J=0}(\gamma_2\alpha\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}^\dagger|\Phi_0\rangle$$
$$+2\sqrt{6}\,k\sum_{\substack{(\alpha\beta)\\(a,b\neq c)}}\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\chi_{J=0}(\alpha\beta\tilde{\alpha}'\tilde{\beta}')\psi_\lambda(\alpha'\beta'\gamma')a_{\tilde{\alpha}'}^\dagger a_{\tilde{\beta}'}^\dagger|\Phi_0\rangle$$
$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\gamma\gamma_1}\phi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}^\dagger|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_\lambda^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\tilde{\alpha}'}^\dagger a_{\tilde{\beta}'}^\dagger|\Phi_0\rangle,$$

where Eq. (4·8) is used in the last expression. Thus, we finally obtain

$$\sum_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_\lambda(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}|\Phi_0\rangle$$
$$=\frac{1}{\sqrt{3}}\sum_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}^\dagger a_{\tilde{\gamma}_2}^\dagger|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{\gamma_1\gamma_2\alpha\\(a\neq c)}}\delta_{\gamma\gamma_1}\phi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\alpha}}^\dagger|\Phi_0\rangle$$
$$+\frac{2}{\sqrt{6}}\sum_{\substack{(\alpha'\beta')\gamma'\\(a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_\lambda^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\tilde{\alpha}'}^\dagger a_{\tilde{\beta}'}^\dagger|\Phi_0\rangle, \qquad (4\cdot10)$$

so that Eq. (4·9) simply becomes

$$\{Y_\lambda, a_\gamma^\dagger\}_+|\Phi_0\rangle=0. \tag{4·11}$$

We are now in a position to show that requirement (2·19) is satisfied. A direct calculation with the aid of Eq. (4·10) leads us to the relation

$$\begin{aligned}
&\{Y_{\lambda'}, Y_\lambda^\dagger\}_+|\Phi_0\rangle \\
&=\Big[\Sigma_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\psi_{\lambda'}(\alpha\beta\gamma)+\frac{3}{2}\Sigma_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\,\Sigma_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'} \\
&\quad+\frac{\sqrt{3}}{2}\Sigma_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)a_\alpha^\dagger a_\beta^\dagger\Big\{\Sigma_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_{\lambda'}^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_2}^\dagger a_{\tilde{\gamma}_1}^\dagger \\
&\quad+\sqrt{2}\,\Sigma_{\substack{\gamma_1\gamma_2\alpha' \\ (a'\neq c)}}\delta_{\gamma\gamma_1}\phi_{\lambda'}^{(2)}(\gamma_1\gamma_2;\alpha')a_{\tilde{\alpha}'}^\dagger a_{\tilde{\gamma}_2}^\dagger \\
&\quad+\sqrt{2}\,\Sigma_{\substack{(\alpha'\beta')\gamma' \\ (a',b'\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_{\lambda'}^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}a_{\tilde{\beta}'}^\dagger a_{\tilde{\alpha}'}^\dagger\Big\} \\
&\quad+\frac{\sqrt{3}}{2}\Sigma_\gamma\Big\{\Sigma_{\gamma_1\gamma_2\gamma_3}\delta_{\gamma\gamma_3}\phi_\lambda^{(1)}(\gamma_1\gamma_2\gamma_3)a_{\tilde{\gamma}_1}a_{\tilde{\gamma}_2}+\sqrt{2}\,\Sigma_{\substack{\gamma_1\gamma_2\alpha \\ (a\neq c)}}\delta_{\gamma\gamma_1}\phi_\lambda^{(2)}(\gamma_1\gamma_2;\alpha)a_{\tilde{\gamma}_2}a_{\tilde{\alpha}} \\
&\quad+\sqrt{2}\,\Sigma_{\substack{(\alpha\beta)\gamma' \\ (a,b\neq c')}}\delta_{\gamma\gamma'}\frac{\phi_\lambda^{(3)}(\alpha\beta;\gamma')}{\sqrt{1+\delta_{ab}}}a_{\tilde{\alpha}}a_{\tilde{\beta}}\Big\}\Sigma_{\alpha'\beta'\gamma'}\delta_{\gamma\gamma'}\psi_{\lambda'}(\alpha'\beta'\gamma')a_{\beta'}a_{\alpha'}\Big]|\Phi_0\rangle \\
&=(\psi_{\lambda'}^T\psi_\lambda-\phi_{\lambda'}^T\phi_\lambda)|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle,
\end{aligned} \tag{4·12}$$

where all the terms with $O(n_0/2\Omega)\approx O(k^2/2\Omega)\approx 0$ have been dropped according to the basic approximation in the NTD method, and Eq. (3·6) has been used in the last relation.

4-3 *Orthogonality to collective degrees of freedom*

The ground-state correlation function $\chi_{J=0}(\alpha\beta\gamma\delta)$ should satisfy Eq. (4·4). Hence, with the aid of (4·5), we obtain

$$\hat{S}_-(a)|\Phi_0\rangle=0, \tag{4·13}$$

where $\hat{S}_-(a)$ is defined in Eq. (1·2·18). Equation (4·13) shows that the correlated ground state has no zero-coupled quasi-particle pairs. With the aid of Eq. (4·13), we have

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=[\hat{S}_-(a), Y_\lambda^\dagger]|\Phi_0\rangle. \tag{4·14}$$

Since the inner product of the state vectors on the right-hand side of Eq. (4·14) is of the order $O(n_0/2\Omega)\approx 0$, we can also see that the dressed 3-quasi-particle states have no zero-coupled pairs under the basic approximation $O(n_0/2\Omega)\approx 0$, i.e.,

$$\hat{S}_-(a)Y_\lambda^\dagger|\Phi_0\rangle=0. \tag{4·15}$$

Furthermore, we can see that the "one-quasi-particle" states $Y_{s=1/2,\alpha}^\dagger|\Phi_0\rangle \equiv a_\alpha^\dagger|\Phi_0\rangle$ (with $\Delta v=1$) also have no zero-coupled pairs, i.e.,

$$\hat{S}_{-}(a)a_{\alpha}^{\dagger}|\Phi_0\rangle=0, \tag{4·16}$$

because we have

$$\hat{S}_{-}(a)a_{\beta}^{\dagger}|\Phi_0\rangle=[\hat{S}_{-}(a),\ a_{\beta}^{\dagger}]|\Phi_0\rangle=\delta_{ab}a_{\bar{\beta}}|\Phi_0\rangle,$$

the inner product of which is of the order $O(n_0/2\Omega)\approx 0$ by the definition $\langle\Phi_0|a_{\alpha}^{\dagger}a_{\beta}|\Phi_0\rangle\cong\delta_{\alpha\beta}n_0/2\Omega$. Therefore, our quasi-particle NTD subspace, consisting of the modes with the transferred seniority $\Delta v(=2s)=1$ and 3, does not include any zero-coupled quasi-particle pairs within the basic approximation $O(n_0/2\Omega)\approx 0$. Thus, the subspace is orthogonal to any pairing-vibrational "collective" state.

§ 5. Transcription of Hamiltonian and electromagnetic multipole operators into quasi-particle NTD subspace

5-1 *Quasi-particle NTD subspace*

The basis vectors of the quasi-particle NTD subspace under consideration are

$$\{Y_{s=1/2,\alpha}^{\dagger}|\Phi_0\rangle\equiv a_{\alpha}^{\dagger}|\Phi_0\rangle, \quad Y_{s=3/2,\lambda}^{\dagger}|\Phi_0\rangle\equiv Y_{\lambda}^{\dagger}|\Phi_0\rangle\}, \tag{5·1}$$

the orthonormality of which is satisfied (under the basic approximation $O(n_0/2\Omega)\approx 0$) because of Eqs. (4·11) and (4·12). The unit operator in this subspace is defined by

$$\mathbf{1}=\sum_{\alpha}\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\alpha}+\sum_{\lambda}\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda}, \tag{5·2}$$

where

$$\boldsymbol{a}_{\alpha}^{\dagger}=a_{\alpha}^{\dagger}|\Phi_0\rangle\langle\Phi_0|, \qquad \boldsymbol{Y}_{\lambda}^{\dagger}=Y_{\lambda}^{\dagger}|\Phi_0\rangle\langle\Phi_0|. \tag{5·3}$$

The elementary excitation operators $(\boldsymbol{a}_{\alpha}^{\dagger},\ \boldsymbol{Y}_{\lambda}^{\dagger})$ in the quasi-particle NTD subspace satisfy the relations

$$\boldsymbol{a}_{\alpha}|\Phi_0\rangle=\boldsymbol{Y}_{\lambda}|\Phi_0\rangle=0, \tag{5·4}$$

and

$$\begin{aligned}&\{\boldsymbol{Y}_{\lambda},\ \boldsymbol{Y}_{\lambda'}^{\dagger}\}_{+}|\Phi_0\rangle=\delta_{\lambda\lambda'}|\Phi_0\rangle,\\&\{\boldsymbol{a}_{\alpha},\ \boldsymbol{a}_{\beta}^{\dagger}\}_{+}|\Phi_0\rangle=\delta_{\alpha\beta}|\Phi_0\rangle,\\&\{\boldsymbol{Y}_{\lambda},\ \boldsymbol{a}_{\alpha}^{\dagger}\}_{+}|\Phi_0\rangle=0.\end{aligned} \tag{5·5}$$

The non-repeatability of excitations is a trivial result and is expressed as

$$\boldsymbol{Y}_{\lambda}^{\dagger}\boldsymbol{Y}_{\lambda'}^{\dagger}|\Phi_0\rangle=\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{a}_{\beta}^{\dagger}|\Phi_0\rangle=\boldsymbol{a}_{\alpha}^{\dagger}\boldsymbol{Y}_{\lambda}^{\dagger}|\Phi_0\rangle=0. \tag{5·6}$$

5-2 *Transcription rule into quasi-particle NTD subspace*

Now let us consider the transcription of a physical operator $\hat{F}$ (such as the Hamiltonian and the electromagnetic multipole operators) into the NTD subspace. According to Eq. (2·23), it is necessary to evaluate the matrix elements $\langle\Phi_0|Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle$ within the framework of the NTD approximation. For this purpose we must make full use of the properties of the eigenmode operators, such as relations (4·11) and (4·12). Hence, we rewrite the matrix element in the following two forms:

$$\langle\Phi_0|Y_{s_>\lambda}\hat{F}Y^\dagger_{s'_<\lambda'}|\Phi_0\rangle$$
$$=\begin{cases}\langle\Phi_0|\{[Y_{s_>\lambda},\hat{F}],\ Y^\dagger_{s'_<\lambda'}\}_+|\Phi_0\rangle+\langle\Phi_0|\hat{F}\{Y_{s_>\lambda},\ Y^\dagger_{s'_<\lambda'}\}_+|\Phi_0\rangle, & (5\cdot7\text{a})\\ \langle\Phi_0|\{Y_{s_>\lambda},[\hat{F},\ Y^\dagger_{s'_<\lambda'}]\}_+|\Phi_0\rangle+\langle\Phi_0|\{Y_{s_>\lambda},\ Y^\dagger_{s'_<\lambda'}\}_+\hat{F}|\Phi_0\rangle. & (5\cdot7\text{b})\end{cases}$$

The evaluation of the first terms, which include a double commutator, is easily made. For the second terms, it is convenient to use the form (5·7a), because in general we obtain

$$\{Y_{s_<\lambda},\ Y^\dagger_{s'_>\lambda'}\}_+|\Phi_0\rangle\neq0\quad\text{for}\quad s\neq s',$$

i.e.,

$$\langle\Phi_0|\{Y_{s_>\lambda},\ Y^\dagger_{s'_<\lambda'}\}_+\neq0\quad\text{for}\quad s\neq s',$$

which is in contrast with the simple relation (2·19), i.e., relation (4·11). Therefore, we adopt the form (5·7a) and easily obtain

$$\langle\Phi_0|Y_{s_>\lambda}\hat{F}Y^\dagger_{s'_<\lambda'}|\Phi_0\rangle=\langle\Phi_0|\{[Y_{s_>\lambda},\hat{F}],\ Y^\dagger_{s'_<\lambda'}\}_+|\Phi_0\rangle$$
$$+\delta_{ss'}\delta_{\lambda\lambda'}\langle\Phi_0|\hat{F}|\Phi_0\rangle.\qquad(5\cdot8)$$

This means the following transcription rule for evaluating the matrix elements $\langle\Phi_0|Y_{s\lambda}\hat{F}Y^\dagger_{s'\lambda'}|\Phi_0\rangle$: *First, we calculate the commutation relation between the physical operator $\hat{F}$ and the eigenmode operator of the higher transferred seniority number, and afterwards take the anti-commutation relation with the eigenmode operator of the lower transferred seniority number.*

5-3 *Transcribed operators*

Using the transcription rule, we obtain

$$\begin{aligned}&\langle\Phi_0|Y_{\lambda'}HY^\dagger_\lambda|\Phi_0\rangle=\{\omega_\lambda+\langle\Phi_0|H|\Phi_0\rangle\}\delta_{\lambda\lambda'},\\&\langle\Phi_0|a_\alpha Ha^\dagger_\beta|\Phi_0\rangle=\{E_\alpha+\langle\Phi_0|H|\Phi_0\rangle\}\delta_{\alpha\beta}\end{aligned}\qquad(5\cdot9)$$

and

$$\langle\Phi_0|Y_\lambda Ha^\dagger_\alpha|\Phi_0\rangle(=\langle\Phi_0|Y_\lambda H_Y a^\dagger_\alpha|\Phi_0\rangle)$$
$$=-\sqrt{6}\sum_{\alpha'\beta'\gamma'}V_Y(\alpha'\beta'\alpha\tilde{\gamma}')\psi_\lambda(\alpha'\beta'\gamma')$$
$$+\sqrt{2}\sum_{\alpha_1'\alpha_2'\alpha_3'}\{V_Y(\alpha_1'\alpha_2'\tilde{\alpha}_3'\alpha)-2V_Y(\alpha_1'\alpha_2'\alpha\tilde{\alpha}_3')\}\phi^{(1)}_\lambda(\alpha_1'\alpha_2'\alpha_3')$$

$$-2\sum_{\substack{\alpha_1'\alpha_2'\gamma' \\ (a'\neq c')}} \{V_Y(\alpha\tilde{\gamma}'\alpha_1'\alpha_2')+V_Y(\tilde{\alpha}_2'\alpha\alpha_1'\gamma')+V_Y(\alpha_1'\gamma'\tilde{\alpha}_2'\alpha)\}\phi_\lambda^{(2)}(\alpha_1'\alpha_2';\gamma')$$
$$+2\sum_{\substack{(\alpha'\beta')\gamma' \\ (a',b'\neq c')}} \{V_Y(\alpha'\beta'\tilde{\gamma}'\alpha)+2V_Y(\alpha\tilde{\alpha}'\gamma'\beta')\}\frac{\phi_\lambda^{(3)}(\alpha'\beta';\gamma')}{\sqrt{1+\delta_{a'b'}}}. \qquad (5\cdot10)$$

According to Eq. (2·23), we can obtain the explicit form of the transcribed Hamiltonian in the quasi-particle NTD subspace:

$$\boldsymbol{H}=\mathbf{1}H\mathbf{1}=U\cdot\mathbf{1}+\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})}$$
$$\equiv U\cdot\mathbf{1}+\sum_\alpha E_a\boldsymbol{a}_\alpha^\dagger\boldsymbol{a}_\alpha+\sum_\lambda\omega_\lambda\boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_\lambda+\sum_{\alpha\lambda}V_{\mathrm{int}}(\alpha,\lambda)\cdot(\boldsymbol{Y}_\lambda^\dagger\boldsymbol{a}_\alpha+\boldsymbol{a}_\alpha^\dagger\boldsymbol{Y}_\lambda), \qquad (5\cdot11)$$

where $V_{\mathrm{int}}(\alpha,\lambda)\equiv\langle\Phi_0|Y_\lambda H a_\alpha^\dagger|\Phi_0\rangle$, and U is a constant related to the correlation energy of the ground state due to the dressed 3-quasi-particle modes. As seen from the matrix elements $V_{\mathrm{int}}(\alpha,\lambda)$ given in Eq. (5·10), the *effective interaction* $\boldsymbol{H}^{(\mathrm{int})}$ between the different types of modes results from only the interactive force H_Y of the original interaction.

The electromagnetic multipole operators are the one-body operators written in general as

$$\hat{O}_{LM}^{(\pm)}=\sum_{\alpha\beta}(\alpha|O_{LM}^{(\pm)}|\beta)c_\alpha^\dagger c_\beta$$
$$\equiv\sum_{\alpha\beta}\{\mathring{O}_{LM}^{(\pm)}(\alpha\beta)(a_\alpha^\dagger a_\beta^\dagger\pm a_{\tilde{\beta}}a_{\tilde{\alpha}})+\bar{O}_{LM}^{(\pm)}(\alpha\beta)a_\alpha^\dagger a_\beta\}$$
$$+\sum_\alpha(\alpha|O_{LM}^{(\pm)}|\alpha)v_a^2\cdot\frac{1\pm1}{2}, \qquad (5\cdot12)$$

where the double symbol $(\pm)$ is related to the conventional transformation property*) of the multipole operators with respect to the time reversal, and $\mathring{O}_{LM}^{(\pm)}$ and $\bar{O}_{LM}^{(\pm)}$ are defined respectively by

$$\mathring{O}_{LM}^{(\pm)}(\alpha\beta)=-\frac{1}{2}(\alpha|O_{LM}^{(\pm)}|\tilde{\beta})(u_av_b\pm v_au_b),$$
$$\bar{O}_{LM}^{(\pm)}(\alpha\beta)=(\alpha|O_{LM}^{(\pm)}|\beta)(u_au_b\mp v_av_b). \qquad (5\cdot13)$$

By definition, $\mathring{O}_{LM}^{(\pm)}(\alpha\beta)$ satisfies the relation $\mathring{O}_{LM}^{(\pm)}(\alpha\beta)=-\mathring{O}_{LM}^{(\pm)}(\beta\alpha)$, and $\bar{O}_{LM}^{(\pm)}(\alpha\beta)$ the relation $\bar{O}_{LM}^{(\pm)}(\alpha\beta)=\pm\bar{O}_{LM}^{(\pm)}(\tilde{\beta}\tilde{\alpha})$. With the aid of the transcription rule (5·8), we now obtain the transcribed electromagnetic multipole operators in the quasi-particle NTD subspace:

$$\hat{O}_{LM}^{(\pm)}\longrightarrow\hat{\boldsymbol{O}}_{LM}^{(\pm)}=\mathbf{1}\hat{O}_{LM}^{(\pm)}\mathbf{1}$$
$$=C_{LM}^{(\pm)}\cdot\mathbf{1}+\sum_{\alpha\beta}O_{LM}^{(\pm)}(\alpha\beta)\boldsymbol{a}_\alpha^\dagger\boldsymbol{a}_\beta+\sum_{\lambda\lambda'}O_{LM}^{(\pm)}(\lambda\lambda')\boldsymbol{Y}_\lambda^\dagger\boldsymbol{Y}_{\lambda'}$$
$$+\sum_{\alpha\lambda}\{O_{LM}^{(\pm)}(\alpha\lambda)\boldsymbol{a}_\alpha^\dagger\boldsymbol{Y}_\lambda+O_{LM}^{(\pm)}(\lambda\alpha)\boldsymbol{Y}_\lambda^\dagger\boldsymbol{a}_\alpha\}. \qquad (5\cdot14)$$

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*) The time reversal property of the electromagnetic multipole operator $\hat{O}_{LM}$ is characterized by $T\hat{O}_{LM}T^\dagger=\tau(-)^M\hat{O}_{L\bar{M}}$, where $\tau=\pm1$. The operators $\hat{O}_{ML}^{(+)}$ and $\hat{O}_{LM}^{(-)}$ denote those with $\tau=+1$ and $\tau=-1$, respectively.

Full expressions of the coefficients $C^{(\pm)}_{LM}$, $O^{(\pm)}_{LM}(\alpha\beta)$, $O^{(\pm)}_{LM}(\lambda\lambda')$ and $O^{(\pm)}_{LM}(\alpha\lambda)$ are given in Appendix 2D.

Any physical operator can be transcribed in the same way into the quasi-particle NTD subspace.

§6. Concluding remarks

On the basis of the quasi-particle NTD method, we have developed a systematic microscopic theory describing the collective excitations in spherical odd-mass nuclei. The theory has led us to a concept of a new kind of fermion-type collective excitation modes, in exactly the same manner as the RPA for even-even nuclei leads us to the concept of "phonon" as a boson. Needless to say, the framework of our theory includes that of the quasi-particle-phonon-coupling theory as a specially approximated version. The dressed 3-quasi-particle mode involves the phonon-like collective correlation and the three-quasi-particle correlation in a unified manner.

In Part III we apply this theory to describe the low-lying collective excited states in spherical odd-mass nuclei, and show that this unified picture plays an important role in clarifying the structure of low-lying states.

Appendix 2A. Projection operators

In Eq. (3·2) we have used the projection operators, $\boldsymbol{P}(\alpha\beta\gamma)$ and $\boldsymbol{P}(\alpha\beta)$, defined by

$$\begin{aligned} \boldsymbol{P}(\alpha\beta\gamma)f(\alpha\beta\gamma) &\equiv \sum_{\alpha'\beta'\gamma'} P(\alpha\beta\gamma|\alpha'\beta'\gamma')f(\alpha'\beta'\gamma'), \\ \boldsymbol{P}(\alpha\beta)g(\alpha\beta) &\equiv \sum_{\alpha'\beta'} P(\alpha\beta|\alpha'\beta')g(\alpha'\beta'), \end{aligned} \tag{2A·1}$$

by which arbitrary functions $f(\alpha\beta\gamma)$ and $g(\alpha\beta)$ are antisymmetrized with respect to (α, β, γ) and (α, β) respectively, and any angular-momentum-zero-coupled-pair component is removed from the antisymmetrized functions $f^A(\alpha\beta\gamma)$ and $g^A(\alpha\beta)$. Here we give their explicit definitions.

The antisymmetrization operator of three-body system is given by

$$P^A(\alpha\beta\gamma|\alpha'\beta'\gamma') = \frac{1}{3!}\sum_{\mathcal{P}(\alpha'\beta'\gamma')}\delta_{\mathcal{P}}\cdot\mathcal{P}(\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}), \tag{2A·2}$$

where $\sum_{\mathcal{P}(\alpha'\beta'\gamma')}$ denotes the summation over all the permutation with respect to $(\alpha', \beta', \gamma')$ and $\delta_{\mathcal{P}}$ takes the value $+1$ for even permutations and the value -1 for odd permutations. As is easily seen, this operator satisfies the relation of projection operator:

$$\sum_{\alpha''\beta''\gamma''} P^A(\alpha\beta\gamma|\alpha''\beta''\gamma'')P^A(\alpha''\beta''\gamma''|\alpha'\beta'\gamma') = P^A(\alpha\beta\gamma|\alpha'\beta'\gamma'). \tag{2A·3}$$

In the coupled-angular-momentum representation, the antisymmetrization operator (2A·2) is represented by

$$P_I^A(ab(J)c|a'b'(J')c')=\sum_{m_\alpha m_\beta m_\gamma}\sum_{m_{\alpha'}m_{\beta'}m_{\gamma'}}\sum_{MM'}(j_a j_b m_\alpha m_\beta|JM)$$
$$\times(Jj_c M m_\gamma|IK)(j_{a'}j_{b'}m_{\alpha'}m_{\beta'}|J'M')(J'j_{c'}M'm_{\gamma'}|IK)P^A(\alpha\beta\gamma|\alpha'\beta'\gamma')$$
$$=\frac{1}{3!}(1+\hat{\mathcal{P}}_{abJ})\Bigg[\delta_{aa'}\delta_{bb'}\delta_{cc'}\delta_{JJ'}$$
$$+\sqrt{(2J+1)(2J'+1)}\begin{Bmatrix} j_c & j_b & J' \\ j_a & I & J\end{Bmatrix}(1+\hat{\mathcal{P}}_{bcJ'})\delta_{ac'}\delta_{bb'}\delta_{ca'}\Bigg], \tag{2A·4}$$

where the operation of $\hat{\mathcal{P}}$ on any function f is defined by

$$\hat{\mathcal{P}}_{abJ}f_J(ab)=-(-)^{j_a+j_b-J}f_J(ba). \tag{2A·5}$$

With the aid of Eq. (2A·4), the projection operator $P_I(ab(J)c|a'b'(J')c')$, which removes any angular-momentum-zero-coupled-pair component (from the functions on which it operates), is easily obtained as follows:

$$P_I(ab(J)c|a'b'(J')c')=P_I^A(ab(J)c|a'b'(J')c')$$
$$\begin{cases} +0 & \text{for } a\neq b\neq c\neq a,\\ -P_I^A(ab(J)c|a'b'(J')c')\delta_{J0} & \text{for } a=b\neq c,\\ -\dfrac{P_I^A(ab(0)c|a'b'(J')c')P_I^A(ab(J)c|a'b'(0)c')}{P_I^A(ab(0)c|a'b'(0)c')} & \text{for } a=b=c,\\ +(-)^{j_a+j_c+J}(2J+1)^{1/2}\begin{Bmatrix} j_a & j_b & J\\ I & j_c & 0\end{Bmatrix}P_I^A(ac(0)b|a'b'(J')c') & \text{for } a=c\neq b,\\ +(-)^{j_b+j_c}(2J+1)^{1/2}\begin{Bmatrix} j_b & j_a & J\\ I & j_c & 0\end{Bmatrix}P_I^A(bc(0)a|a'b'(J')c') & \text{for } a\neq b=c.\end{cases} \tag{2A·6}$$

The projection operators P_I thus defined satisfy the following properties:

i) $$\sum_{a''b''c''J''}P_I(ab(J)c|a''b''(J'')c'')P_I(a''b''(J'')c''|a'b'(J')c')=P_I(ab(J)c|a'b'(J')c'), \tag{2A·7a}$$

ii) $$P_I(ab(J)c|a'b'(J')c')=P_I(a'b'(J')c'|ab(J)c), \tag{2A·7b}$$

iii) $$P_I(ab(0)c|a'b'(J')c')=P_I(ab(J)c|a'b'(0)c')=0. \tag{2A·7c}$$

With the expression (2A·6), the projection operator $P(\alpha\beta\gamma|\alpha'\beta'\gamma')$ in (2A·1) (in the m-scheme) is given through the relation

$$P_I(ab(J)c|a'b'(J')c')=\sum_{m_\alpha m_\beta m_\gamma}\sum_{m_{\alpha'}m_{\beta'}m_{\gamma'}}\sum_{MM'}(j_a j_b m_\alpha m_\beta|JM)$$
$$\times(Jj_c M m_\gamma|IK)(j_{a'}j_{b'}m_{\alpha'}m_{\beta'}|J'M')(J'j_{c'}M'm_{\gamma'}|IK)P(\alpha\beta\gamma|\alpha'\beta'\gamma'). \tag{2A·8}$$

The projection operator $P(\alpha\beta|\alpha'\beta')$ of two-body system is defined in a similar way and its explicit form in the coupled-angular-momentum representation is trivially given by

$$P_J(ab|a'b')=\frac{1}{2}(\delta_{aa'}\delta_{bb'}-(-)^{j_a+j_b-J}\delta_{ab'}\delta_{ba'})(1-\delta_{J0}). \qquad (2A\cdot 9)$$

Appendix 2B. Matrix elements of the secular equation for the dressed 3-quasi-particle modes

Here, we give the explicit forms for the matrix elements of $\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ in the eigenvalue equation (3·3).

With the definitions

$$V^{(fi)}_{\alpha\beta\alpha'\beta'}\equiv 2[V^{(G)}_{Xi}(\alpha\beta\alpha'\beta')+V^{(F)}_{Xi}(\alpha\beta\alpha'\beta')-V^{(F)}_{Xi}(\beta\alpha\alpha'\beta')], \qquad (2B\cdot 1)$$

$$V^{(bi)}_{\alpha\beta\alpha'\beta'}\equiv 2[V_{Vi}(\alpha\beta\alpha'\beta')+V_{Vi}(\alpha'\beta'\alpha\beta)-V_{Vi}(\alpha\tilde{\beta}'\tilde{\beta}\alpha')-V_{Vi}(\tilde{\beta}\alpha'\alpha\tilde{\beta}')$$
$$+V_{Vi}(\beta\tilde{\beta}'\tilde{\alpha}\alpha')+V_{Vi}(\tilde{\alpha}\alpha'\beta\tilde{\beta}')], \qquad (2B\cdot 2)$$

we first introduce the matrices $\boldsymbol{3D}^i$, $\boldsymbol{d}^i$ and $\boldsymbol{A}^i$, the elements of which are given as follows:

$$3D^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\boldsymbol{P}(\alpha\beta\gamma)[(E^i_a+E^i_b+E^i_c)\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}+3V^{(fi)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}]\boldsymbol{P}^T(\alpha'\beta'\gamma'), \qquad (2B\cdot 3a)$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha_1'\alpha_2'\alpha_3'}=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)[E^i_a\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\delta_{\alpha_3\alpha_3'}+V^{(fi)}_{\alpha_1'\alpha_2'\alpha_1\alpha_2}\delta_{\alpha_3'\alpha_3}]\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3'),$$

$$d^i_{\alpha_1\alpha_2\gamma,\alpha_1'\alpha_2'\gamma'}=\boldsymbol{P}(\alpha_1\alpha_2)[E^i_c\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}\delta_{\gamma\gamma'}+2V^{(fi)}_{\alpha_2'\gamma'\alpha_2\gamma}\delta_{\alpha_1'\alpha_1}]\boldsymbol{P}^T(\alpha_1'\alpha_2'),$$

$$d^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}[(E^i_a+E^i_b-E^i_c)\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}+2V^{(fi)}_{\alpha'\beta'\alpha\beta}\delta_{\gamma'\gamma}]\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}}, \qquad (2B\cdot 3b)$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha_1'\alpha_2'\gamma'}=\sqrt{2}\,\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(fi)}_{\alpha_2'\gamma'\alpha_1\alpha_2}\delta_{\alpha_1'\alpha_3}\boldsymbol{P}^T(\alpha_1'\alpha_2'),$$

$$d^i_{\alpha_1\alpha_2\alpha_3,\alpha'\beta'\gamma'}=\sqrt{2}\,\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)V^{(fi)}_{\alpha'\beta'\alpha_1\alpha_2}\delta_{\gamma'\alpha_3}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

$$d^i_{\alpha_1\alpha_2\gamma,\alpha'\beta'\gamma'}=2\boldsymbol{P}(\alpha_1\alpha_2)V^{(fi)}_{\alpha'\beta'\alpha_2\gamma}\delta_{\gamma'\alpha_1}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

$$A^i_{\alpha\beta\gamma,\alpha_1'\alpha_2'\alpha_3'}=\sqrt{3}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha_1'\alpha_2'}\delta_{\gamma\alpha_3'}\boldsymbol{P}^T(\alpha_1'\alpha_2'\alpha_3'),$$

$$A^i_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'}=\sqrt{6}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha_2'\gamma'}\delta_{\gamma\alpha_1'}\boldsymbol{P}^T(\alpha_1'\alpha_2'), \qquad (2B\cdot 3c)$$

$$A^i_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\sqrt{6}\,\boldsymbol{P}(\alpha\beta\gamma)V^{(bi)}_{\alpha\beta\alpha'\beta'}\delta_{\gamma\gamma'}\frac{\boldsymbol{P}^T(\alpha'\beta')}{\sqrt{1+\delta_{a'b'}}},$$

where we have used the abbreviations for the projection operators in (3·2), for simplicity. The matrix elements of **3*D***, ***d*** and ***A*** are then given by the following replacements in (2B·3):

$$V_{Xi}^{(G)}(\alpha\beta\gamma\delta) \Rightarrow V_X^{(G)}(\alpha\beta\gamma\delta), \qquad V_{Xi}^{(F)}(\alpha\beta\gamma\delta) \Rightarrow V_X^{(F)}(\alpha\beta\gamma\delta),$$
$$V_{Vi}(\alpha\beta\gamma\delta) \Rightarrow V_V(\alpha\beta\gamma\delta), \qquad E_a^i \Rightarrow E_a, \tag{2B·4}$$

where $V_X^{(G)}(\alpha\beta\gamma\delta)$, $V_X^{(F)}(\alpha\beta\gamma\delta)$ and $V_Y(\alpha\beta\gamma\delta)$ are given after Eq. (3·4) in Chap. 1.

It is obvious that the suffix i of V and E is completely superfluous in the above equations. The suffix i has been used here merely from formal point of view and its usefulness will become clear in Appendix 7A.

Appendix 2C. Interaction between the dressed 3-quasi-particle mode and the single-quasi-particle mode

Here, we give the explicit form of the matrix element $V_{\rm int}(a, \lambda)$ in Eq. (5·11) in a formally convenient way.

Let us first evaluate the matrix element of the following operators:

$$K_Y = \sum_{\alpha\beta\gamma\delta} \{ V_{Y1}(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_{\tilde{\delta}}^\dagger a_\gamma + V_{Y2}(\alpha\beta\gamma\delta) a_\gamma^\dagger a_{\tilde{\delta}} a_\beta a_\alpha \}. \tag{2C·1}$$

With the aid of the transcription rule given in §5-2, we obtain

$$\langle \Phi_0 | Y_\lambda K_Y a_a^\dagger | \Phi_0 \rangle = \langle \Phi_0 | \{ [Y_\lambda, K_Y], a_a^\dagger \}_+ | \Phi_0 \rangle$$
$$= (\boldsymbol{\psi}_\lambda^T, \boldsymbol{\phi}_\lambda^T) \cdot \boldsymbol{B}(a), \tag{2C·2}$$

where

$$\boldsymbol{B}(a') = \begin{pmatrix} \boldsymbol{B}^1(a') \\ \boldsymbol{B}^2(a') \end{pmatrix} \tag{2C·3}$$

the elements of which are defined by

$$B_{\alpha\beta\gamma}^1(a') = -\boldsymbol{P}(\alpha\beta\gamma)\sqrt{6}\, V_{Y1}(\alpha\beta a'\tilde{\gamma}),$$
$$B_{a_1a_2a_3}^2(a') = -\boldsymbol{P}(a_1a_2a_3)\sqrt{2}\, \{ V_{Y2}(\tilde{a}_1\tilde{a}_2a_3\tilde{a}') - 2V_{Y2}(a'\tilde{a}_1a_3a_2) \},$$
$$B_{a_1a_2\gamma}^2(a') = -\boldsymbol{P}(a_1a_2) 2 \{ V_{Y2}(\tilde{a}_2\tilde{\gamma}a_1\tilde{a}') - V_{Y2}(a'\tilde{a}_2a_1\gamma) + V_{Y2}(a'\tilde{\gamma}a_1a_2) \},$$
$$B_{a\beta\gamma}^2(a') = -\frac{\boldsymbol{P}(a\beta)}{\sqrt{1+\delta_{ab}}} 2 \{ V_{Y2}(\tilde{a}\tilde{\beta}\gamma\tilde{a}') - V_{Y2}(a'\tilde{a}\gamma\beta) + V_{Y2}(a'\tilde{\beta}\gamma a) \}. \tag{2C·4}$$

The matrix element of the interaction, $V_{\rm int}(a, \lambda)$ in Eq. (5·11), is then compactly given by

$$V_{\rm int}(a, \lambda) = (\boldsymbol{\psi}_\lambda^T, \boldsymbol{\phi}_\lambda^T) \cdot \boldsymbol{B}(a) \tag{2C·5}$$

with the following replacements in (2C·4):

$$V_{Y1}(\alpha\beta\gamma\delta)=V_{Y2}(\alpha\beta\gamma\delta)\Rightarrow V_Y(\alpha\beta\gamma\delta), \tag{2C·6}$$

where $V_Y(\alpha\beta\gamma\delta)$ is given after Eq. (3·4) in Chap. 1.

The operator K_Y has been used here merely from formal point of view and its usefulness will become clear in Appendix 7A.

Appendix 2D. Matrix elements of electromagnetic multipole operators

Here, we give the explicit forms of the coefficients in Eq. (5·14).

Let us first evaluate the matrix elements of the following operators:

$$\mathring{F}^{(\pm)}_{LM}=\sum_{\alpha\beta}\{\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)a^\dagger_\alpha a^\dagger_\beta\pm\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)a_{\bar\beta}a_{\bar\alpha}\}, \tag{2D·1}$$

$$\bar{F}^{(\pm)}_{LM}=\sum_{\alpha\beta}\bar{F}^{(\pm)}_{LM}(\alpha\beta)a^\dagger_\alpha a_\beta. \tag{2D·2}$$

With the aid of the transcription rule given in §5-2, we obtain

$$\begin{aligned}\langle\Phi_0|\,Y_\lambda\mathring{F}^{(\pm)}_{LM}a^\dagger_{\alpha'}|\Phi_0\rangle&=\sqrt{6}\,\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)\\&\pm\{\sqrt{2}\,\sum_{\alpha_1\alpha_2\alpha_3}\phi^{(1)}_\lambda(\alpha_1\alpha_2\alpha_3)\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\delta_{\alpha_3\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha_1\alpha_2)\\&+2\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\phi^{(2)}_\lambda(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\delta_{\alpha_1\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha_2\gamma)\\&+2\sum_{\substack{(\alpha\beta)\gamma\\(a\neq c,b\neq c)}}\phi^{(3)}_\lambda(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)/\sqrt{1+\delta_{ab}}\,\},\end{aligned} \tag{2D·3a}$$

$$\begin{aligned}\langle\Phi_0|a_{\alpha'}\mathring{F}^{(\pm)}_{LM}Y^\dagger_\lambda|\Phi_0\rangle&=\pm\sqrt{6}\,\sum_{\alpha\beta\gamma}\psi_\lambda(\alpha\beta\gamma)\boldsymbol{P}(\alpha\beta\gamma)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{2LM}(\tilde\alpha\tilde\beta)\\&+\sqrt{2}\,\sum_{\alpha_1\alpha_2\alpha_3}\phi^{(1)}_\lambda(\alpha_1\alpha_2\alpha_3)\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\delta_{\alpha_3\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha_1\tilde\alpha_2)\\&+2\sum_{\substack{\alpha_1\alpha_2\gamma\\(a\neq c)}}\phi^{(2)}_\lambda(\alpha_1\alpha_2;\gamma)\boldsymbol{P}(\alpha_1\alpha_2)\delta_{\alpha_1\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha_2\tilde\gamma)\\&+2\sum_{\substack{(\alpha\beta)\gamma\\(a\neq c,b\neq c)}}\phi^{(3)}_\lambda(\alpha\beta;\gamma)\boldsymbol{P}(\alpha\beta)\delta_{\gamma\alpha'}\mathring{F}^{(\pm)}_{1LM}(\tilde\alpha\tilde\beta)/\sqrt{1+\delta_{ab}}\end{aligned} \tag{2D·3b}$$

and

$$\langle\Phi_0|\,Y_\lambda\bar{F}^{(\pm)}_{LM}Y^\dagger_{\lambda'}|\Phi_0\rangle=(\boldsymbol{\psi}^T_\lambda,\boldsymbol{\phi}^T_\lambda)\begin{bmatrix}\boldsymbol{F}&0\\0&\boldsymbol{f}\end{bmatrix}\begin{bmatrix}\boldsymbol{\psi}_{\lambda'}\\\boldsymbol{\phi}_{\lambda'}\end{bmatrix}, \tag{2D·4}$$

where the matrix elements of $\boldsymbol{F}$ and $\boldsymbol{f}$ are respectively defined by

$$F_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=3\boldsymbol{P}(\alpha\beta\gamma)\bar{F}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}\boldsymbol{P}^T(\alpha'\beta'\gamma'), \tag{2D·5a}$$

$$\begin{aligned}f_{\alpha_1\alpha_2\alpha_3,\alpha'_1\alpha'_2\alpha'_3}&=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\{2\bar{F}^{(\pm)}_{LM}(\tilde\alpha'_3\tilde\alpha_3)-\bar{F}^{(\pm)}_{LM}(\alpha_3\alpha'_3)\}\delta_{\alpha_1\alpha'_1}\delta_{\alpha_2\alpha'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),\\f_{\alpha_1\alpha_2\alpha_3,\alpha'_1\alpha'_2\gamma'}&=\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\sqrt{2}\,\bar{F}^{(\pm)}_{LM}(\tilde\gamma'\tilde\alpha_3)\delta_{\alpha_1\alpha'_1}\delta_{\alpha_2\alpha'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2),\\f_{\alpha_1\alpha_2\gamma,\alpha'_1\alpha'_2\alpha'_3}&=\boldsymbol{P}(\alpha_1\alpha_2)\sqrt{2}\,\bar{F}^{(\pm)}_{LM}(\tilde\alpha'_3\tilde\gamma)\delta_{\alpha_1\alpha'_1}\delta_{\alpha_2\alpha'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),\\f_{\alpha_1\alpha_2\alpha_3,\alpha'\beta'\gamma'}&=-\boldsymbol{P}(\alpha_1\alpha_2\alpha_3)\sqrt{2}\,\bar{F}^{(\pm)}_{LM}(\alpha_3\gamma')\delta_{\alpha_1\alpha'}\delta_{\alpha_2\beta'}\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{a'b'}},\\f_{\alpha\beta\gamma,\alpha'_1\alpha'_2\alpha'_3}&=-\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}\sqrt{2}\,\bar{F}^{(\pm)}_{LM}(\gamma\alpha'_3)\delta_{\alpha\alpha'_1}\delta_{\beta\alpha'_2}\boldsymbol{P}^T(\alpha'_1\alpha'_2\alpha'_3),\end{aligned}$$

$$f_{\alpha_1\alpha_2\gamma,\alpha_1'\alpha_2'\gamma'}=\boldsymbol{P}(\alpha_1\alpha_2)[\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}_1'\tilde{\alpha}_1)-\bar{F}^{(\pm)}_{LM}(\alpha_1\alpha_1')\}\delta_{\alpha_2\alpha_2'}\delta_{\gamma\gamma'}$$
$$+\bar{F}^{(\pm)}_{LM}(\tilde{\gamma}'\tilde{\gamma})\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}+\bar{F}^{(\pm)}_{LM}(\alpha_1\alpha_1')\delta_{\alpha_2\gamma'}\delta_{\gamma\alpha_2'}]\boldsymbol{P}^T(\alpha_1'\alpha_2'),$$

$$f_{\alpha_1\alpha_2\gamma,\alpha'\beta'\gamma'}=\boldsymbol{P}(\alpha_1\alpha_2)2\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}'\tilde{\alpha}_2)\delta_{\alpha_1\gamma'}\delta_{\gamma\beta'}$$
$$-\bar{F}^{(\pm)}_{LM}(\alpha_1\gamma')\delta_{\alpha_2\alpha'}\delta_{\gamma\beta'}\}\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{a'b'}},$$

$$f_{\alpha\beta\gamma,\alpha_1'\alpha_2'\gamma'}=\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}2\{\bar{F}^{(\pm)}_{LM}(\tilde{\alpha}_2'\tilde{\alpha})\delta_{\gamma\alpha_1'}\delta_{\beta\gamma'}-\bar{F}^{(\pm)}_{LM}(\gamma\alpha_1')\delta_{\alpha\alpha_2'}\delta_{\beta\gamma'}\}\boldsymbol{P}^T(\alpha_1'\alpha_2'),$$

$$f_{\alpha\beta\gamma,\alpha'\beta'\gamma'}=\frac{\boldsymbol{P}(\alpha\beta)}{\sqrt{1+\delta_{ab}}}2\{2\bar{F}^{(\pm)}_{LM}(\tilde{\beta}'\tilde{\beta})\delta_{\alpha\alpha'}\delta_{\gamma\gamma'}$$
$$-\bar{F}^{(\pm)}_{LM}(\gamma\gamma')\delta_{\alpha\alpha'}\delta_{\beta\beta'}\}\boldsymbol{P}^T(\alpha'\beta')/\sqrt{1+\delta_{a'b'}}. \qquad (2D\cdot5b)$$

The explicit forms of the coefficients in Eq. (5·14) are then given by

$$C^{(\pm)}_{LM}\equiv\sum_a(a|O^{(\pm)}_{LM}|a)v_a^2\cdot\frac{1\pm1}{2},$$
$$O^{(\pm)}_{LM}(\alpha\beta)\equiv\langle\Phi_0|a_\alpha\hat{O}^{(\pm)}_{LM}a_\beta^\dagger|\Phi_0\rangle-C^{(\pm)}_{LM}\delta_{\alpha\beta}=\bar{O}^{(\pm)}_{LM}(\alpha\beta),$$
$$O^{(\pm)}_{LM}(\lambda\alpha)\equiv\langle\Phi_0|Y_\lambda\hat{O}^{(\pm)}_{LM}a_\alpha^\dagger|\Phi_0\rangle=\langle\Phi_0|Y_\lambda\mathring{F}^{(\pm)}_{LM}a_\alpha^\dagger|\Phi_0\rangle,$$
$$O^{(\pm)}_{LM}(\alpha\lambda)\equiv\langle\Phi_0|a_\alpha\hat{O}^{(\pm)}_{LM}Y_\lambda^\dagger|\Phi_0\rangle=\langle\Phi_0|a_\alpha\mathring{F}^{(\pm)}_{LM}Y_\lambda^\dagger|\Phi_0\rangle,$$
$$O^{(\pm)}_{LM}(\lambda\lambda')\equiv\langle\Phi_0|Y_\lambda\hat{O}^{(\pm)}_{LM}Y_{\lambda'}^\dagger|\Phi_0\rangle-C^{(\pm)}_{LM}\delta_{\lambda\lambda'}=\langle\Phi_0|Y_\lambda\bar{F}^{(\pm)}_{LM}Y_{\lambda'}^\dagger|\Phi_0\rangle \qquad (2D\cdot6)$$

with the following replacements in (2D·3) and (2D·4):

$$\mathring{F}^{(\pm)}_{1LM}(\alpha\beta)=\mathring{F}^{(\pm)}_{2LM}(\alpha\beta)\Rightarrow\mathring{O}^{(\pm)}_{LM}(\alpha\beta),$$
$$\bar{F}^{(\pm)}_{LM}(\alpha\beta)\Rightarrow\bar{O}^{(\pm)}_{LM}(\alpha\beta), \qquad (2D\cdot7)$$

where $\mathring{O}^{(\pm)}_{LM}(\alpha\beta)$ and $\bar{O}^{(\pm)}_{LM}(\alpha\beta)$ are defined by (5·13).

The operators, $\mathring{F}$ and $\bar{F}$, have been used here merely from formal point of view and its usefulness will become clear in Appendix 7B.

Part III.
Analysis of Low-Lying States in Spherical Odd-Mass Nuclei
Chapter 3. Structure of the Anomalous Coupling States with Spin $I=(j-1)$

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§1. Introduction

1-1 *Outline*

According to the *j-j* coupling shell model, a high-*j* orbit having parity opposite to that of the other orbits appears systematically in each major shell. (See Fig. 1.) When this unique-parity orbit, such as $1f^-_{7/2}$ and $1g^+_{9/2}$, is filled with nucleons in odd numbers, a competition between a spin *j*- and a spin $(j-1)$-state for the ground state occurs quite regularly. Such extra low-lying states with spin $I=(j-1)$ and with unique parity have been called the anomalous coupling (AC) states.

The AC states are well known as the typical phenomena which cannot be interpreted within the framework of the conventional quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen.[1)]

The main purpose of this chapter*) is to introduce a new microscopic model of the AC states in the light of recent experimental developments illuminating the structure of the AC states. In the microscopic model proposed here, the AC states are regarded as *typical manifestations* of the dressed three-quasi-particle (3QP) modes which have

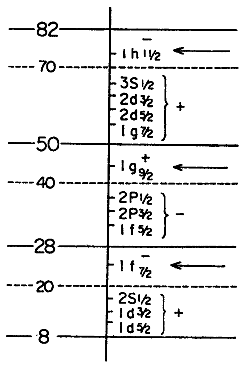


Fig. 1. Schematic representation of shell structure. The arrow denotes the high-spin, unique-parity orbit in each major shell.

*) The content of this chapter was already published by the present authors in Prog. Theor. Phys. **47** (1972), 498; **51** (1974), 779.

been introduced in Chap. 2. It is shown that, under the special condition of shell structure for the appearance of the AC states, the dressed 3QP modes manifest themselves as relatively pure eigenmodes without coupling to the single-quasi-particle (1QP) modes. Then, in the same manner as the 2^+ phonon modes (the dressed two-quasi-particle modes) are regarded as elementary excitations in spherical even-even nuclei, the AC states are regarded as typical phenomena which exhibit the elementary modes of low-energy collective excitations in spherical odd-mass nuclei. From this point of view, the mechanism of appearance of the collective 3QP correlation, which is responsible for the particular favouring of the spin $(j-1)$ states, and the process of its growth are clarified.

After a short summary of the recent experimental evidences showing collective character of the AC states, the motive for introducing the new microscopic model is discussed in §2 in connection with the picture of phonon-quasi-particle coupling. In §3, starting with the j-j coupling shell model with the pairing-plus-quadrupole (P+QQ) force,[3] we formulate the microscopic model of the AC states in a concrete form by using the general theory developed in Chap. 2.*) It is shown that the model introduced involves two essential characteristics of the AC states in a unified manner: One characteristic aspect represented by Kisslinger's 3QP "intruder state"[4] and the other characteristic aspect of strong collectiveness underlying the quasi-particle-phonon-coupling state.[1],[2] Furthermore, by investigating the stability of the spherical BCS vacuum against the collective 3QP correlation, we point out an interesting relation between our new viewpoint and the Bohr-Mottelson's old suggestion[5] concerning the possible connection between the appearance of the spin $(j-1)$ state as the ground state and the onset of quadrupole deformation. In the course of these, the relations between our microscopic model and the recent works based on the semi-microscopic models[6],[7] (which start from the particle-vibration coupling Hamiltonian[8]) are also discussed by putting special emphasis on their underlying picture for the AC states.

It is shown in §4 that the model introduced can give us an intuitive and perspective understanding of the characteristics of electromagnetic properties of the AC states. The theoretical predictions given there are examined in §5 by comparing results of numerical calculations with available experimental data. Here, special attention is paid to the systematical agreement with the common properties of the AC states observed in the experiments over a wide range of spherical odd-mass nuclei rather than numerical agreement with the experimental value at a specific nucleus. In the theoretical calculation, the coupling effect coming from the 1QP mode in the next upper

*) Throughout Part III, we take up the first term, H, of $H_{\rm intr}$ (1·6·14) as the intrinsic Hamiltonian, in the same way as in Chap. 2.

major shell, such as $1g^{+}_{7/2}$ and $1h^{-}_{9/2}$, on the dressed 3QP mode with $I=(j-1)$ is also taken into account, and, therewith, the smallness of its mixing effect on the properties of the AC states is examined. The results clearly show how we can understand the various properties of the AC states in a unified manner within the framework of the microscopic model proposed on the basis of the theory developed in Chap. 2. Thus we conclude in §6 that, in the first order approximation, the AC states with $I=(j-1)$ can very well be recognized as the typical manifestations of the dressed 3QP modes.

1-2 *Finding of collective nature of AC states*

Since the special lowering of the spin $(j-1)$ state is in clear contrast to the simple pairing-coupling scheme which favours the spin j state characterized by the seniority $v=1$, the phenomena showing the competition between the spin j and $(j-1)$ states have been discussed with special interest from the viewpoint of the nuclear coupling scheme. It has been known from the very beginning of the proposal of the j-j coupling shell model, that one of the possibilities of reproducing the extremely low-lying $(j-1)$ state is to introduce a sufficiently long-range effective force in multi-nucleon configurations j^n.[9)~15)] It was shown that, within the j^3 configurations, the spin $(j-1)$ state characterized by the seniority $v=3$ is especially lowered in energy as the range of the effective force becomes larger.[10)~15)] Kisslinger's interpretation of the $(j-1)$ state as the 3QP "intruder" state[4)] may be regarded as a model in terms of the P+QQ force, elaborated along this line of development.

Another possibility of explaining the AC states is to introduce a possibility of quadrupole deformations in nuclei: In view of the fact that the lowest state of j^3 configurations in the oblately deformed potential has the spin $I\doteq K=j-1$ (the aligned coupling scheme), Bohr and Mottelson suggested[5)] a possible connection between the appearance of the $(j-1)$ state as the ground state and the onset of the quadrupole deformation. From this point of view, the competition between the j and $(j-1)$ states is considered as a phenomenon reflecting directly the growth of quadrupole instability.

Now let us first make a systematics of the excitation energies of the $(j-1)$ states on the basis of recent accumulation of experimental data. In Fig. 2 are presented the excitation energies of the $7/2^+$ states measured from those of the 1QP $9/2^+$ states, as a function of the neutron number N. These $7/2^+$ states in the odd-proton Tc, Rh and Ag isotopes are the most well-known examples of the AC states with spin $I=(j-1)$. We can then notice a striking similarity between the behaviour of the excitation energies of the 2^+ phonon states in the sequence of even-even nuclei and that of the $(j-1)$ states in the sequence of odd-mass nuclei: As a function of N, the excitation energies of the $7/2^+$ states change in quite a parallel way to those of the 2^+

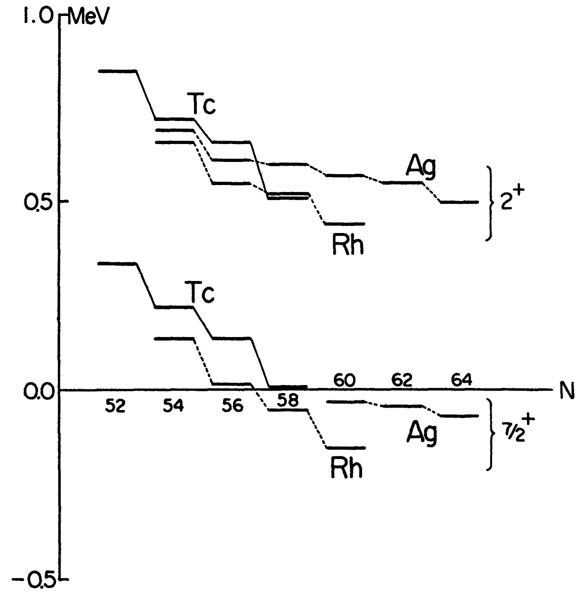


Fig. 2. Comparison between the excitation-energy systematics of the $7/2^+$ states and those of 2^+ phonon states. The phonon-energies presented are the average values between the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(N, Z) = 1/2\{\omega_{2^+}(N, Z-1) + \omega_{2^+}(N, Z+1)\}$. The energies of the $7/2^+$ states are those measured from the 1QP $9/2^+$ states.

phonon states, aside from the fact that they are shifted down about 0.5 MeV compared to those of 2^+ phonons. This similarity indicates a collective character of the $7/2^+$ states which is difficult to understand with either of the two interpretations mentioned above.

In addition to the well-known $7/2^+$ states in nuclei belonging to the $1g^+_{9/2}$-region, recent experimental works have revealed a number of new examples of the AC states with spin $9/2^-$ in odd-neutron Cd, Te and Xe isotopes belonging to the $1h^-_{11/2}$-region. The excitation energies of the $9/2^-$ states measured from the 1QP $11/2^-$ states show also the trend similar to that of the 2^+ phonon states in the neighbouring even-even nuclei.

Furthermore, recent measurements on various electromagnetic properties of the AC states have been providing us important information, directly showing their collective character. One of the most important findings is that the $E2$ transitions from the $(j-1)$ states to the 1QP states with spin j are strongly enhanced while the corresponding $M1$ transitions are moderately hindered. The amount of enhancement of the $E2$ transitions is comparable (or somewhat larger) to that of the $E2$ transitions from the 2^+ phonon states to the ground states in the adjacent even-even nuclei. Thus, the strongly collective nature of the AC states has been clearly exhibited.

A possible origin of the striking $E2$ enhancements from the $(j-1)$ states may be ascribable to the quasi-particle-phonon-coupling nature based on the theory of Kisslinger and Sorensen.[1] However, the special energy-lowering of the AC states with spin $I=(j-1)$ has not at all been accounted for within the framework of the conventional QPC theory of Kisslinger and Sorensen. Considering the striking $E2$ enhancement as an essential characteristic of the AC states, Sano and Ikegami[16),17] carried out the calculation based on the conventional QPC theory by enlarging the shell model space to several major shells. An extension of the conventional QPC theory to another direction has also been attempted by different authors.[18)~20] However, it turnd out later,[21),22] that it is difficult to ensure the conditions for eliminating the spurious states and for satisfying the Pauli principle between the quasi-particles composing the phonon and the odd quasi-particle, within a mere formal extension of the framework of the conventional QPC theory.

The three different kinds of approaches mentioned above have succeeded in explaining partial aspects of the $(j-1)$ state. That is, for the special favouring of the $I=(j-1)$ coupling in j^3 configuration, the spherical shell model with a long-range effective force, for the possible appearance of the $(j-1)$ state as the ground state, the aligned coupling scheme in the deformed model, and for the strong collectiveness exhibited by the enhancement of $B(E2;\ j-1\to j)$, the quasi-particle-phonon-coupling model. However, their mutual relationships have not yet been clarified and, therewith, the essential understanding of the structure of the AC states has not been achieved.

In the following, we first investigate the missing effect of the conventional QPC theory of Kisslinger and Sorensen, which is the main cause for the special favouring of the $(j-1)$ state.

§2. A new type of quasi-particle-phonon-coupling giving rise to collective 3QP correlations

An important effect of the quasi-particle-phonon coupling, which has been neglected for a long time, was emphasized by Bohr and Mottelson[23] in 1967: "In the *phenomenological* phonon-quasi-particle coupling model, the lowest-order-perturbation effects which contribute to all the energies of the different states of the multiplet composed of the odd quasi-particle and the one-phonon, are shown in Figs. 3A and 3B. The diagrams of the type A are nothing but the conventional ones which have so far been treated as 'phonon-quasi-particle coupling' in the QPC theory of Kisslinger and Sorensen, while the diagrams of the type B never appear in this QPC theory. The physical effect underlying the diagrams B is that *the phonon disassociates into a pair of quasi-particles, one of which reassociates with the odd quasi-*

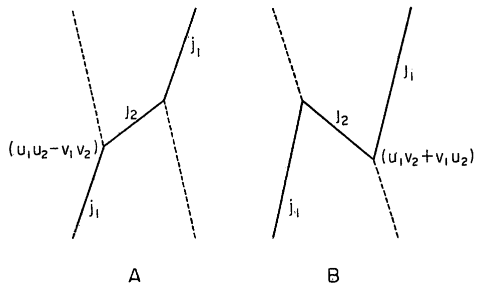


Fig. 3A. Contribution of lowest order. The solid and broken lines represent the quasi-particle and the phonon, respectively. This type of particle-vibration coupling is accompanied by the reduction factor $(u_1u_2-v_1v_2)$.
Fig. 3B. Contribution of lowest order. This type of particle-vibration coupling is accompanied by the enhancement factor $(u_1v_2+v_1u_2)$. Both (A) and (B) are taken from reference 23).

particle while the remaining quasi-particle is now the odd quasi-particle. This effect is essentially based on the Pauli principle between the quasi-particle composing the phonon and the odd quasi-particle. The extreme importance of the diagrams of type B can be recognized as follows. The diagrams of type A consist of the factor $(u_1u_2-v_1v_2)$ which can be quite small, while the diagrams of type B involve the coupling with the factor $(u_1v_2+v_1u_2)$ which is close to unity for low-lying states in the middle of the shell. Thus it is likely that the description of collective excited states of almost all spherical odd-mass nuclei is significantly effected by the inclusion of the effect."

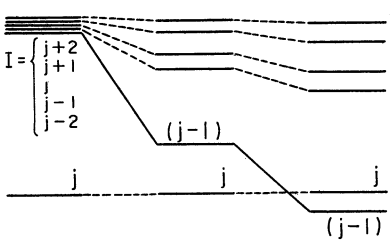
In the conventional QPC theory, the phonon is regarded as the ideal boson described by the random-phase approximation (RPA) and is commutable with the odd quasi-particle. Therefore, the effect which underlies the diagrams of type B and is based on the Pauli principle between odd quasi-particle and quasi-particles composing the phonon is *in principle* not taken into account within the framework of the theory. From the viewpoint of boson expansion methods in odd-mass nuclei, such effect is called "kinematical anharmonicity effect" based on a new type of quasi-particle-phonon coupling which is derived from two types of the original interaction, H_X and H_V in Eq. (1·3·4), represented in Fig. 7 in §3. The new type of coupling is in clear contrast to the coupling in the conventional QPC theory, which is derived from the original interaction, H_Y in Eq. (1·3·4), represented in Fig. 7 and causes "dynamical anharmonicity effect." The significance of the new type of coupling has also been emphasized in the course of investigating the "anharmonicity effects" in terms of the boson expansion method.

Of course, as the kinematical anharmonicity effect becomes more significant, the higher order diagrams of the type B must be taken into account.

Therefore, in such a situation, it is required to take the essential effect into account not by the perturbation approximation but by diagonalizing the Hamiltonian in a "certain subspace," in such a way that we adopt the new Tamm-Dancoff approximation (i.e., the RPA) when constructing the phonon modes in even-even nuclei.

It is now easy to recognize that the dominant cause which brings about the special favouring of the spin $(j-1)$ state is nothing but the effect of type B: Let us operate the effect on the degenerate multiplet composed of the odd quasi-particle and the one-phonon. Then, as is shown in §3, the $(j-1)$ state in the multiplet is affected most strongly and its excitation energy is extremely lowered as the effect becomes stronger. The AC states with $I=(j-1)$ are regarded as the phenomena in which the effect grows extremely. In fact we see the experimental data clearly exhibiting this process, for example, in the cases of Nb-Tc-Rh isotopes in §5.

In order to investigate the effect of type B on the basis of the microscopic theory, let us replace the phonon line in the diagrams B in Fig. 3 with the conventional correlated-two-quasi-particle line shown in Fig. 5, by taking account of the composite structure of the phonon. Then the diagrams B can be decomposed into the corresponding microscopic diagrams in Fig. 6.



Eig. 4.

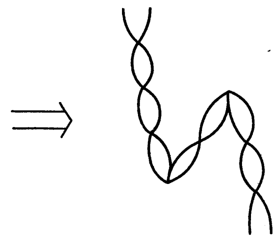


Fig. 5.

Fig. 4. Schematic illustration showing the relation between the process of growth of the 3QP correlation and the increase of the splitting of the multiplet composed of phonon plus odd quasi-particle.

Fig. 5. Representation of the phonon as a correlated two quasi-particles.

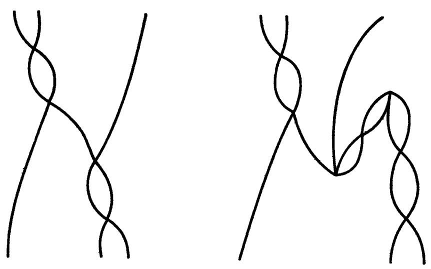


Fig. 6. Microscopic structure of diagram 3B.

The structure of the diagrams in Fig. 6 shows that they are composed of only two types of the quasi-particle interaction, H_X and H_V, which are also well known to be responsible for the phonon mode in even-even nuclei. The situation never changes even when we take account of any higher order diagram of type B, and the excited state corresponding to any such diagram is always represented as a superposition of the particle states with 3, 7, 11, 15, ... quasi-particles. These considerations lead us to the conclusion that if we succeed in constructing the correlated three-quasi-particle mode, including the corresponding ground-state correlation in the framework of the new-Tamm-Dancoff (NTD) approximation (by using the two types of the quasi-particle interaction H_X and H_V), the above mentioned requirement that the effect of type B should be taken into account not by the perturbation but by diagonalizing the Hamiltonian in a "certain subspace" is satisfied in a very suitable way. Thus we are now at a position to introduce a new microscopic model of the AC states with spin $I=(j-1)$ as typical manifestations of the dressed 3QP modes formulated in Chap. 2.

§3. Microscopic model of AC states as dressed 3QP modes

3–1 *The Hamiltonian*

Let us start with the spherically symmetric *j-j* coupling shell-model Hamiltonian with the pairing-plus-quadrupole (P+QQ) force in the quasi-particle representation:

$$
\begin{aligned}
H&=H_0+:H_{QQ}:\\
&=\sum_\alpha E_a a_\alpha^\dagger a_\alpha-\frac{1}{2}\chi\sum_M:\hat{Q}_{2M}^\dagger\hat{Q}_{2M}:,
\end{aligned}\tag{3·1}
$$

where χ is the strength of the quadrupole force, and E_a is the quasi-particle energy, determined as usual together with the parameters u_a and v_a of the Bogoliubov transformation. The symbol : : denotes the normal product with respect to the quasi-particle operators $a_\alpha^\dagger$ and a_α, and the quantity $\hat{Q}_{2M}$ is the mass-quadrupole-moment operator in terms of quasi-particles,

$$
\begin{aligned}
\hat{Q}_{2M}=\frac{1}{2}\sum_{ab}q(ab)[\xi(ab)\{A_{2M}^\dagger(ab)+A_{\widetilde{2M}}(ab)\}\\
+\eta(ab)\{B_{2M}^\dagger(ab)+B_{\widetilde{2M}}(ab)\}],
\end{aligned}\tag{3·2}
$$

where

$$
q(ab)\equiv\frac{1}{\sqrt{5}}(a\|r^2Y_2\|b)\tag{3·3}
$$

and

$$\left.\begin{aligned}\xi(ab)&\equiv(u_a v_b+v_a u_b),\\ \eta(ab)&\equiv(u_a u_b-v_a v_b).\end{aligned}\right\} \tag{3·4}$$

The operators $A^\dagger_{JM}(ab)$, $A_{\widetilde{JM}}(ab)$, $B^\dagger_{JM}(ab)$ and $B_{\widetilde{JM}}(ab)$ are the conventional pair operators defined by

$$\left.\begin{aligned}A^\dagger_{JM}(ab)&\equiv\sum_{m_\alpha m_\beta}(j_a j_b m_\alpha m_\beta|JM)a^\dagger_\alpha a^\dagger_\beta,\\ B^\dagger_{JM}(ab)&\equiv-\sum_{m_\alpha m_\beta}(j_a j_b m_\alpha m_\beta|JM)a^\dagger_\alpha a_{\tilde\beta},\\ A_{\widetilde{JM}}(ab)&\equiv(-)^{J-M}A_{J,\bar M}(ab),\\ B_{\widetilde{JM}}(ab)&\equiv(-)^{J-M}B_{J,\bar M}(ab),\end{aligned}\right\} \tag{3·5}$$

where

$$a_{\tilde\beta}\equiv s_\beta a_{\bar\beta}\equiv(-)^{j_b-m_\beta}a_{\bar\beta}. \tag{3·6}$$

The quadrupole force $:H_{QQ}:$ acting among quasi-particles can be divided into following parts according to the roles they play in constructing the elementary excitation modes:

$$\left.\begin{aligned}:H_{QQ}:&=H^{(0)}_{QQ}+H_Y+H_{EX},\\ H^{(0)}_{QQ}&=H_X+H_V,\end{aligned}\right\} \tag{3·7}$$

where

$$H_X=-\frac{\chi}{4}\sum_M\sum_{abcd}Q(ab)Q(cd)A^\dagger_{2M}(ab)A_{2M}(cd), \tag{3·7a}$$

$$H_V=-\frac{\chi}{8}\sum_M\sum_{abcd}Q(ab)Q(cd)\{A^\dagger_{2M}(ab)A^\dagger_{\widetilde{2M}}(cd)+\text{h.c.}\}, \tag{3·7b}$$

$$H_Y=-\frac{\chi}{2}\sum_M\sum_{abcd}Q(ab)R(cd)\{A^\dagger_{2M}(ab)B_{2M}(cd)+\text{h.c.}\}, \tag{3·7c}$$

$$\begin{aligned}H_{EX}&=-\frac{\chi}{2}\sum_M\sum_{abcd}R(ab)R(cd):B^\dagger_{2M}(ab)B_{2M}(cd):\\ &=\frac{5}{2}\chi\sum_{JM}\sum_{abcd}R(ab)R(cd)\begin{Bmatrix}j_a & j_b & 2\\ j_c & j_d & J\end{Bmatrix}A^\dagger_{JM}(ad)A_{JM}(cb)\end{aligned} \tag{3·7d}$$

with

$$Q(ab)\equiv q(ab)\xi(ab), \tag{3·8}$$

$$R(ab)\equiv q(ab)\eta(ab). \tag{3·9}$$

According to the inherent assumption underlying the P+QQ force model,[3),24)] we hereafter neglect the exchange term H_{EX} in the quadrupole force H_{QQ}. Then each matrix element of H_{QQ} is represented by one of

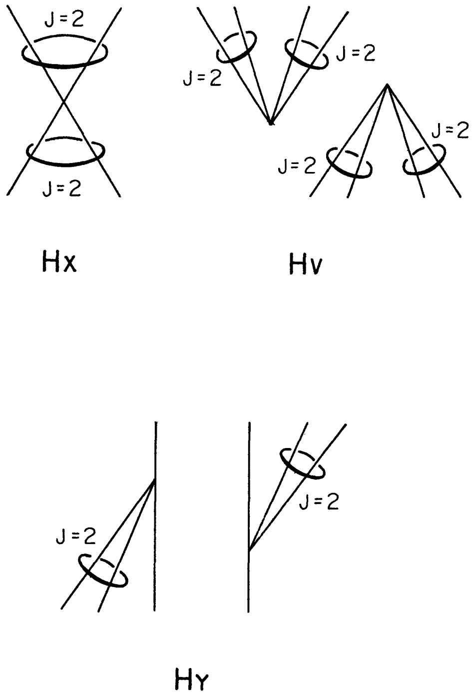


Fig. 7. Graphic representation of the matrix elements of the quadrupole force.

the diagrams in Fig. 7. The part H_X represents a scattering of the pair of quasi-particles coupled to $J^\pi=2^+$. The part H_V represents a pair-creation (or a pair-annihilation) of the quasi-particle pair coupled to $J^\pi=2^+$, so that it introduces the ground-state correlation. The part H_Y denotes a creation (or an annihilation) of the pair of quasi-particles coupled to $J^\pi=2^+$, accompanied by the scattering of a single quasi-particle. As was discussed in Part II, the parts, H_X and H_V, play an essential role in constructing the dressed 3QP modes as elementary excitations, while the part H_Y gives rise to couplings between the different types of elementary excitation modes, for instance, a coupling between the 1QP mode and the dressed 3QP mode.

3–2 *Formulation of model*

Let us now consider the systems of odd-mass nuclei in the truncated shell-model space which consists of one major harmonic-oscillator shell (for both protons and neutrons) and a high-j orbit with unique-parity entering into the major shell, and suppose the unique-parity orbit being filled with protons (or neutrons) in odd numbers. To explicitly specify the unique-parity orbit j, we use the Roman letter $p=(nlj)$*) and the corresponding Greek letters $\pi_1=(p,m_1)$, $\pi_2=(p,m_2)$,... are used to specify the single-particle

*) We will often omit the suffix p of j_p hereafter.

states in the unique-parity orbit. The Roman letters $b,c,d,\ldots$ and the corresponding Greek letters $\beta=(b,m_\beta)$, $\gamma=(c,m_\gamma),\ldots$ are used for the single-particle orbits *with the exception of the unique-parity orbit* and for the corresponding single-particle states, respectively.

In this special situation of shell structure for the appearance of the AC states, the dressed 3QP mode with parity opposite to that of major shell takes on an especially simple form due to the parity-selection property of the quadrupole force: The eigenmode operators for the dressed 3QP modes defined by (2·3·1) are simply reduced to

$$\begin{aligned} C^\dagger_{nIK}=&\frac{1}{\sqrt{3!}}\sum_{m_1m_2m_3}\{\psi^p_{nIK}(\pi_1\pi_2\pi_3)\boldsymbol{P}(\pi_1\pi_2\pi_3)T_{3/2,3/2}(\pi_1\pi_2\pi_3)\\ &\qquad+\varphi^p_{nIK}(\pi_1\pi_2\pi_3)\boldsymbol{P}(\pi_1\pi_2\pi_3)T_{3/2,-1/2}(\pi_1\pi_2\pi_3)\}\\ &+\frac{1}{\sqrt{2}}\sum_{bc}{}'\sum_{mm_\beta m_\gamma}\{\psi^c_{nIK}(\beta\gamma;\pi)\boldsymbol{P}(\beta\gamma)a^\dagger_\pi a^\dagger_\beta a^\dagger_\gamma\\ &\qquad+\varphi^c_{nIK}(\beta\gamma;\pi)\boldsymbol{P}(\beta\gamma)a^\dagger_\pi a_{\bar\beta}a_{\bar\gamma}\}, \end{aligned} \tag{3·10}$$

where I and K are the angular momentum and its projection and n denotes a set of additional quantum numbers to specify the eigenmode. The prime in the second term is used to emphasize that summation with respect to b and c should be taken by excluding the unique-parity orbit p. The operator $T_{3/2s_0}(\pi_1\pi_2\pi_3)$ in the first term is the quasi-spin tensor of rank $s=3/2$ and its projection s_0 at the unique-parity orbit p, the explicit form of which is given by (2·2·2); for example,

$$\begin{aligned} &T_{3/2,3/2}(\pi_1\pi_2\pi_3)=a^\dagger_{\pi_1}a^\dagger_{\pi_2}a^\dagger_{\pi_3},\\ &T_{3/2,-1/2}(\pi_1\pi_2\pi_3)=\frac{1}{\sqrt{3}}\{a^\dagger_{\pi_1}a_{\bar\pi_2}a_{\bar\pi_3}+a_{\bar\pi_1}a^\dagger_{\pi_2}a_{\bar\pi_3}+a_{\bar\pi_1}a_{\bar\pi_2}a^\dagger_{\pi_3}\}. \end{aligned}$$

The projection operators $\boldsymbol{P}$ in (3·10), the full definitions of which are given in Appendix 2A, guarantee the three-body-correlation amplitudes, ψ and φ, to simultaneously satisfy the anti-symmetry relation and the condition requiring that the eigenmode operator must not include any $J=0$-coupled quasi-particle pair:

anti-symmetry relations

$$\begin{aligned} &\mathcal{P}\psi^p_{nIK}(\pi_1\pi_2\pi_3)=\delta_\mathcal{P}\psi^p_{nIK}(\pi_1\pi_2\pi_3),\quad \mathcal{P}\varphi^p_{nIK}(\pi_1\pi_2\pi_3)=\delta_\mathcal{P}\varphi^p_{nIK}(\pi_1\pi_2\pi_3),\\ &\psi^c_{nIK}(\gamma\beta;\pi)=-\psi^c_{nIK}(\beta\gamma;\pi),\quad \varphi^c_{nIK}(\gamma\beta;\pi)=-\varphi^c_{nIK}(\beta\gamma;\pi) \end{aligned} \tag{3·11}$$

with $\mathcal{P}$ being the permutation operator with respect to (π_1,π_2,π_3) and $\delta_\mathcal{P}$ being defined by

$$\delta_\mathcal{P}=\begin{cases} 1 & \text{for even permutations,}\\ -1 & \text{for odd permutations.}\end{cases} \tag{3·12}$$

conditions eliminating zero-coupled pairs

$$\begin{aligned}&\sum_{m_2} \psi^p_{nIK}(\pi_1\pi_2\tilde{\pi}_2)=\sum_{m_2} \varphi^p_{nIK}(\pi_1\pi_2\tilde{\pi}_2)=0\,,\\&\sum_{m_\beta} \psi^c_{nIK}(\beta\tilde{\beta}\,;\,\pi)=\sum_{m_\beta} \varphi^c_{nIK}(\beta\tilde{\beta}\,;\,\pi)=0\,.\end{aligned} \tag{3·13}$$

The expression (3·10) for the eigenmode operators satisfying the conditions (3·11) and (3·13) implies that the dressed 3QP modes are not accompanied by the pairing "collective" components and are characterized by the amount of transferred seniority $\Delta v=3$ to the state on which they operate.

The eigenvalue equation for the three-body-correlation amplitudes ψ and φ should be obtained so that $C^\dagger_{nIK}$ becomes an eigenmode in good approximation satisfying

$$[H_0+H^{(0)}_{QQ},\ C^\dagger_{nIK}]=\omega_{nI}C^\dagger_{nIK}-Z_{nIK}, \tag{3·14}$$

where "interaction" Z_{nIK} is generally composed of the normal product of quasi-spin tensors with $s=1/2$, i.e. $\Delta v=1$, and of the higher fifth-order normal products. This is neglected in the first step which determines the dressed 3QP eigenmodes $C^\dagger_{nIK}$ (with $\Delta v=3$).

The "physical" eigenmode operators creating the dressed 3QP states, $Y^\dagger_{nIK}$ in (2·3·7), are the ones which have large amplitudes ψ and small amplitudes φ, and the other ones (with φ larger than ψ), A_{n_0IK} in (2·3·7), are the "special" operators which have no physical meanings. In Chap. 2, it has been shown that the correlated ground state $|\Phi_0\rangle$ satisfies the supplementary conditions under the basic approximation of the NTD method:

$$Y_{nIK}|\Phi_0\rangle=0, \quad A_{n_0IK}|\Phi_0\rangle=0. \tag{3·15}$$

It has also been shown that, within the same approximation, the dressed 3QP modes satisfy the fermion-like commutation relation,

$$\{Y_{n'I'K'},\ Y^\dagger_{nIK}\}|\Phi_0\rangle=\delta_{nn'}\delta_{II'}\delta_{KK'}|\Phi_0\rangle, \tag{3·16}$$

by the use of the orthonormality relation

$$\begin{aligned}&(\boldsymbol{\Psi}_{n'I'K'}\cdot\boldsymbol{\tau}\boldsymbol{\Psi}_{nIK})\\&\equiv\sum_{m_1m_2m_3}\{\psi^p_{n'I'K'}(\pi_1\pi_2\pi_3)\psi^p_{nIK}(\pi_1\pi_2\pi_3)-\varphi^p_{n'I'K'}(\pi_1\pi_2\pi_3)\varphi^p_{nIK}(\pi_1\pi_2\pi_3)\}\\&\quad+\sum_{bc}{}'\sum_{mm_\beta m_\gamma}\{\psi^c_{n'I'K'}(\beta\gamma\,;\,\pi)\psi^c_{nIK}(\beta\gamma\,;\,\pi)-\varphi^c_{n'I'K'}(\beta\gamma\,;\,\pi)\varphi^c_{nIK}(\beta\gamma\,;\,\pi)\}\\&=\epsilon_n\delta_{nn'}\delta_{II'}\delta_{KK'}\,.\end{aligned} \tag{3·17}$$

Let us introduce the coupled angular-momentum representation through the relations

$$\left.\begin{aligned}\psi^{p}_{nIK}(\pi_1\pi_2\pi_3)&=\sum_{J=\text{even}}\psi_{nI}(pp(J)p)(Jj_pMm_3|IK)(j_pj_pm_1m_2|JM),\\ \psi^{c}_{nIK}(\beta\gamma;\pi)&=\sum_{J}\psi_{nI}(bc(J)p)(Jj_pMm_\pi|IK)(j_bj_cm_\beta m_\gamma|JM),\\ &\cdots\cdots\cdots,\end{aligned}\right\} \quad (3\cdot18)$$

and define the basic amplitudes (the meaning of which is precisely given in Appendix 4A) by

$$\begin{aligned}\psi_{nI}(p^3)&\equiv\sqrt{\frac{3}{2}}N_I(p^3)^{-1}\psi_{nI}(pp(2)p),\\ \psi_{nI}(bc;p)&\equiv\sqrt{2}\,N(bc)\psi_{nI}(bc(2)p),\\ &\cdots\end{aligned} \quad (3\cdot19)$$

with

$$N(bc)\equiv\{1+\delta_{bc}\}^{-1/2}, \quad (3\cdot20)$$

$$N_I(p^3)\equiv\{C_I/2\}^{1/2}, \quad (3\cdot21)$$

$$C_I\equiv1+10\begin{Bmatrix}j & j & 2\\ j & I & 2\end{Bmatrix}-\delta_{Ij}\frac{20}{(2j)^2-1}. \quad (3\cdot22)$$

Then, the eigenvalue equation for the correlation amplitudes is simply expressed in terms of only the basic amplitudes (with the intermediate angular momentum $J=2$) as follows:

$$\{2E_p-\omega'_{nI}\}\psi_{nI}(p^3)=\chi Q(pp)N_I(p^3)\{A_{nI}+B_{nI}\}, \quad (3\cdot23a)$$

$$\{2E_p+\omega'_{nI}\}\varphi_{nI}(p^3)=\frac{1}{\sqrt{3}}\chi Q(pp)N_I(p^3)\{A_{nI}+B_{nI}\}, \quad (3\cdot23b)$$

$$\{(E_b+E_c)-\omega'_{nI}\}\psi_{nI}(bc;p)=\chi Q(bc)N(bc)\{A_{nI}+B_{nI}\}, \quad (3\cdot23c)$$

$$\{(E_b+E_c)+\omega'_{nI}\}\varphi_{nI}(bc;p)=\chi Q(bc)N(bc)\{A_{nI}+B_{nI}\}, \quad (3\cdot23d)$$

where

$$\left.\begin{aligned}A_{nI}&\equiv\sum_{(bc)}{}'\,Q(bc)N(bc)\{\psi_{nI}(bc;p)+\varphi_{nI}(bc;p)\},\\ B_{nI}&\equiv Q(pp)N_I(p^3)\left\{\psi_{nI}(p^3)+\frac{1}{\sqrt{3}}\varphi_{nI}(p^3)\right\}\end{aligned}\right\} \quad (3\cdot24)$$

and

$$\omega'_{nI}\equiv\omega_{nI}-E_p. \quad (3\cdot25)$$

Here the symbol $\sum'_{(bc)}$ denotes the summation over the orbit-pair (bc) excluding the unique-parity orbit p. In deriving Eq. (3·23), we have dropped the terms which come from the recoupling of the quadrupole force in order to keep consistency to the inherent assumption of the P+QQ force model. Formal structure of Eq. (3·23) is as simple as that of the RPA-eigenvalue equation for the phonon modes in even-even nuclei. Therefore, we can easily obtain the eigenvalues of Eq. (3·23) by using the simple dispersion equation (which is presented as Eq. (3·37)).

The normalization of the basic amplitudes (3·19) for the physical solutions becomes

$$\psi_{nI}(p^3)^2+\sum_{(bc)}{}'\psi_{nI}(bc;p)^2-\varphi_{nI}(p^3)^2-\sum_{(bc)}{}'\varphi_{nI}(bc;p)^2=1. \tag{3·26}$$

Combining Eqs. (3·23) and (3·26), we obtain the explicit expressions for the basic amplitudes:

$$\left.\begin{aligned}
&\psi_{nI}(p^3)=M_\omega Q(pp)N_I(p^3)/\{2E_p-\omega'_{nI}\},\\
&\varphi_{nI}(p^3)=\sqrt{\frac{1}{3}}M_\omega Q(pp)N_I(p^3)/\{2E_p+\omega'_{nI}\},\\
&\psi_{nI}(bc;p)=M_\omega Q(bc)N(bc)/\{(E_b+E_c)-\omega'_{nI}\},\\
&\varphi_{nI}(bc;p)=M_\omega Q(bc)N(bc)/\{(E_b+E_c)+\omega'_{nI}\}.
\end{aligned}\right\} \tag{3·27}$$

Here the normalization factor M_ω is given by

$$\begin{aligned}
M_\omega&\equiv\chi\cdot\{A_{nI}+B_{nI}\}\\
&=\left\{\frac{\partial}{\partial\omega}(S_p+S_c)\right\}^{-1/2}\\
&=\left[\frac{1}{3}Q(pp)^2C_I\frac{(2E_p)^2+8E_p\omega'_{nI}+(\omega'_{nI})^2}{\{(2E_p)^2-(\omega'_{nI})^2\}^2}\right.\\
&\qquad\left.+2\omega'_{nI}\sum_{bc}{}'\frac{Q(bc)^2(E_b+E_c)}{\{(E_b+E_c)^2-(\omega'_{nI})^2\}^2}\right]^{-1/2}
\end{aligned} \tag{3·28}$$

with

$$S_p\equiv\frac{1}{3}\frac{Q(pp)^2C_I\{4E_p+\omega'_{nI}\}}{(2E_p)^2-(\omega'_{nI})^2}, \tag{3·29}$$

$$S_c\equiv\sum_{bc}{}'\frac{Q(bc)^2\cdot(E_b+E_c)}{(E_b+E_c)^2-(\omega'_{nI})^2}. \tag{3·30}$$

3–3 *Mechanism of growth of 3QP correlation and relations to other approaches*

In order to see the microscopic structure of the dressed 3QP mode formulated in the preceding subsection and to discuss the relations to other approaches[6),7)] by paying attention to their underlying pictures for the AC states, let us decompose the eigenvalue equation (3·23) in the following way. Combining Eqs. (3·23a) and (3·23b), and also combining Eqs. (3·23c) and (3·23d), we obtain

$$\left.\begin{aligned}
&\{\chi_pS_p-1\}B_{nI}+\chi_{pc}S_pA_{nI}=0,\\
&\{\chi_cS_c-1\}A_{nI}+\chi_{pc}S_cB_{nI}=0,
\end{aligned}\right\} \tag{3·31}$$

with $\chi_p=\chi_c=\chi_{pc}=\chi$. Since this equation is linear and homogeneous with respect to A_{nI} and B_{nI}, we find that the eigenvalues ω_{nI} are the solutions of

$$(\chi_pS_p-1)(\chi_cS_c-1)-\chi_{pc}^2S_pS_c=0. \tag{3·32}$$

The physical meaning of this equation is easily understood as follows.

If χ_{pc} were zero, we would have solutions when either $\chi_p S_p=1$ or $\chi_c S_c=1$. The former is merely the dispersion equation for the dressed 3QP mode in the case of restricting our shell-model space within the unique-parity orbit p. The eigenvalue of $\chi_p S_p=\chi S_p=1$ is

$$\omega_I=E_I^*+\sqrt{(2E_I^*)^2-\left\{\frac{1}{2\sqrt{3}}\chi_I^* Q(pp)^2\right\}^2} \tag{3·33}$$

with

$$E_I^*\equiv E_p-\frac{1}{6}\chi_I^* Q(pp)^2, \tag{3·34}$$

$$\chi_I^*\equiv\chi C_I. \tag{3·35}$$

As was pointed out by Kisslinger,[4] there exists an interesting property of $6j$-symbols, that is,

$$\left.\begin{aligned}&\begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix}>0 \quad \text{for} \quad I=j-1,\\ &\begin{Bmatrix} j & j & 2 \\ j & I & 2 \end{Bmatrix}<0 \quad \text{for} \quad I\neq j-1.\end{aligned}\right\} \tag{3·36}$$

Hence, recalling the definition of C_I, (3·22), we can easily see that $\chi_I^*>\chi$ only when $I=j_p-1$ and $\chi_I^*<\chi$ for $I\neq j_p-1$. Thus the (j_p-1) state is especially lowered in energy by the quadrupole force in contrast to the other states with $I\neq j_p-1$. It is now clear that the dressed 3QP state with spin $(j-1)$ in the unique-parity orbit p is reduced to Kisslinger's 3QP "intruder" state[4] when we neglect the ground-state correlation. On the other hand, the latter equation $\chi_c S_c=1$ is exactly the same form as the well-known dispersion equation for phonon modes. Notice, however, that the "phonon" mode in this case implies the "core-excitation" which is composed of the neutron and proton quasi-particles in the truncated major shells *with the exclusion of the valence orbit p*, i.e., the unique-parity orbit. Thus we have *two* low-energy collective states (composed of the quasi-particles in the orbit p and in the core, respectively), *if the "coupling" χ_{pc} is zero.*

Now let us consider the effect on these states due to the change of χ_{pc} from zero.[1] In this case, the product $(\chi_p S_p-1)\cdot(\chi_c S_c-1)$ has to be positive so that the lower level of the two $\chi_{pc}=0$ states must be lowered in order to make each factor of the product negative while the higher level is raised making each factor positive. For sufficiently large χ_{pc}, as is the actual case of $\chi_{pc}=\chi_p=\chi_c=\chi$, there is essentially only one extremely lowered ω_{nI} left in the energy region satisfying

$$\omega_{nI}'\equiv(\omega_{nI}-E)<\text{the minimum value of } (E_b+E_c).$$

In this actual case of $\chi_p=\chi_c=\chi_{pc}=\chi$, Eq. (3·32) is simply reduced to

$$\chi^{-1}=S_p+S_c$$
$$=\frac{1}{3}\frac{Q(pp)^2C_I\{4E_p+\omega'_{nI}\}}{(2E_p)^2-(\omega'_{nI})^2}+\sum_{bc}{}'\frac{Q(bc)^2(E_b+E_c)}{(E_b+E_c)^2-(\omega'_{nI})^2}. \quad (3\cdot37)$$

The above consideration tells us that, in the special situation in shell structure for the appearance of the AC states, the dressed 3QP mode may be decomposed into the "valence-shell cluster"[6] and the "phonon" modes of the core. The "valence-shell cluster" now means the correlated three-quasi-particles in the unique-parity orbit p and reduces to Kisslinger's (3QP) "intruder state" in the Tamm-Dancoff limit. In this respect, our underlying picture for the AC states is similar with that of the semi-microscopic model of Alaga,[6),7)] which explicitly introduces the freedom of the valence-shell cluster coupled to the quadrupole vibration of the core. According to our picture for the AC states, however, the coupling between the valence-shell cluster and the phonon of the core is so strong (because of $\chi_{pc}=\chi_p=\chi_c=\chi$) that they form a new type of collective mode, i.e., the dressed 3QP mode as a bound state.

It is now clear that the introduced model of the AC states unifies the characteristics of both the "intruder states" and the conventional QPC theory, which have been considered as distinctly different from each other in the history of investigating the AC states.

Let us look further into the lowering effect (on the excitation energies of the AC states) due to the core, by adopting the adiabatic approximation:

$$\omega'_{nI}\equiv(\omega_{nI}-E_p)\ll \text{the minimum value of } (E_b+E_c). \quad (3\cdot38)$$

Many of the AC states satisfy this condition and in this case we may write

$$\left.\begin{aligned}S_p&\equiv\mathcal{A}_p+\mathcal{B}_p(\omega_{nI}-E_p)+C_p(\omega_{nI}-E_p)^2,\\ S_c&\equiv\mathcal{A}_c+C_c(\omega_{nI}-E_p)^2,\end{aligned}\right\} \quad (3\cdot39)$$

where

$$\mathcal{A}_p\equiv\frac{2}{3}\frac{Q(pp)^2C_I}{(2E_p)}>0,\quad \mathcal{B}_p\equiv\frac{1}{3}\frac{Q(pp)^2C_I}{(2E_p)^2}>0,$$
$$C_p\equiv\frac{2}{3}\frac{Q(pp)^2C_I}{(2E_p)^3}>0, \quad (3\cdot40)$$
$$\mathcal{A}_c\equiv\sum_{bc}{}'\frac{Q(bc)^2}{E_b+E_c}>0,\quad C_c\equiv\sum_{bc}{}'\frac{Q(bc)^2}{(E_b+E_c)^3}>0.$$

As a result we have from Eq. (3·37)

$$(\omega_{nI}-E_p)=-\frac{\mathcal{B}_p}{2(C_p+C_c)}+\sqrt{\frac{\mathcal{B}_p^2}{4(C_p+C_c)^2}+\frac{\chi^{-1}-(\mathcal{A}_p+\mathcal{A}_c)}{(C_p+C_c)}}. \quad (3\cdot41)$$

Comparing this to the adiabatic solution of $\chi S_p=1$, which is obtained by setting $\mathcal{A}_c=C_c=0$, we can easily see the lowering effects due to the phonon excitations of the "core." Since both $\mathcal{A}_c$ and C_c contain the factors $\xi(bc)\equiv(u_bv_c+v_bu_c)$ through the quantity $Q(bc)$, the larger the $\xi(bc)$ of the core, the smaller the ω_{nI} becomes. Thus the problem of whether the dressed 3QP modes appear extremely low in energy will be determined by two important factors:

i) The enhancement factor $\xi(pp)\equiv 2u_pv_p$ in the unique-parity orbit p,

ii) the enhancement factor $\xi(bc)$ in the core.

3-4 *Stability of spherical BCS vacuum against 3QP correlation*

As is well known in the case of even-even nuclei, when the enhancement factors $\xi(ab)$ become large and the excitation energy of the 2^+ phonon (the dressed 2QP mode) becomes zero, the instability of the spherical BCS vacuum occurs toward quadrupole deformation. In an analogous way, we expect in odd-mass nuclei that, when the enhancement factors $\xi(pp)$ and $\xi(bc)$ become large and the characteristic 3QP correlation grows so that the excitation energy of the dressed 3QP mode with $I=(j-1)$ is extremely lowered, a new type of instability may occur toward deformation.

Figure 8 represents schematically the dispersion equation (3·37) from which the excitation energies ω_{nI} of the dressed 3QP modes are determined. Its gross structure resembles that of the RPA with the P+QQ force in even-even nuclei. It should be noted, however, that the characteristics of the solution of collective type in the vicinity of $\omega_{nI}\approx E_p$ differ considerably from those obtained by simply replacing ω_{2^+} in the conventional dispersion equation of the RPA (in even-even nuclei) with $\omega'_{nI}=(\omega_{nI}-E_p)$. This difference obviously comes

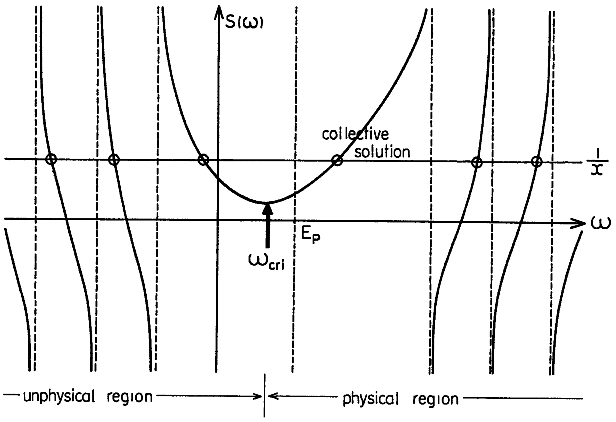


Fig. 8. Schematic representation of the dispersion equation (3·37) from which the eigenvalues of the dressed 3QP modes are determined. The arrow denotes the critical point and the right-hand side of which implies the physical region.

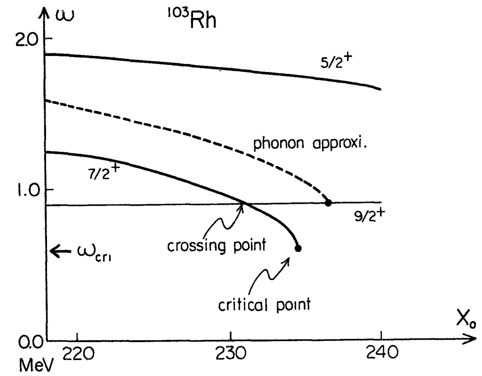


Fig. 9. Behaviour of the solutions of the dressed 3QP modes in ^{103}Rh a functions of the quadrupole-force parameter $\chi_0 \equiv \chi b^4 A^{5/3}$. The horizontal line denotes the energy of the 1QP $9/2^+$ state. For the sake of comparison, the energies of the 1QP-plus-one-phonon states are also written by broken curves, where phonon energies are calculated by the RPA.

from the characteristic structure of the first term on the right-hand side of Eq. (3·37), S_p, which directly reflects the 3QP correlation at the unique-parity orbit p. Figure 9 shows the behaviour of the excitation energy ω_{nI} in the vicinity of $\omega_{nI} \approx E_p$ in relation to that of $(\omega_{2^+}+E_p)$ (ω_{2^+} being the energy of 2^+-phonon given by the RPA). The critical energy $\omega'_{\rm cri} \equiv \omega_{\rm cri}-E_p$ may be defined at the point from which there appear a complex eigenvalue of the dressed 3QP mode with $I=(j-1)$ and is given as the solution of

$$\frac{\partial}{\partial\omega}(S_p+S_c)=0. \tag{3·42}$$

Apparently the three-body correlation amplitudes given by (3·27) diverge at the critical point.

If we neglect the part S_p in Eq. (3·42) which reflects the 3QP correlation, we have $\omega^{(c)}_{\rm cri}=E_p$. This is merely the critical energy expected from the conventional QPC theory. On the other hand, if we neglect the core contribution S_c, we have $\omega^{(p)}_{\rm cri}=(2\sqrt{3}-3)E_p<E_p$. In the actual situation, in which neither S_p nor S_c is zero, $\omega_{\rm cri}$ takes on the value between those of $\omega^{(c)}_{\rm cri}$ and $\omega^{(p)}_{\rm cri}$. The value of $\omega_{\rm cri}$ depends on the details of shell structure and on the relative magnitude of the enhancement factors i) and ii) mentioned in the preceding subsection. It is thus interesting to see that the stability of the spherical BCS vacuum is still maintained in the region $E_p>\omega_{nI}>\omega_{\rm cri}$, in which the crossing of the spin $(j-1)$- and the spin j-state has already occurred. As shown in § 5, the excitation energies of the AC states in many odd-mass nuclei fall within this region. This implies that these nuclei are lying in the "transition region," i.e., they are just before the "phase transition" from the spherical to the deformed, possessing a strong tendency toward quadrupole deformation.

Concerning the situation occuring after the "phase transition," it is interesting to recall that the aligned coupling model also gives the $(j-1)$ state as the lowest state among j^3 configurations in the oblately deformed potential.[5] However, the interrelation between the new type of instability and the onset of quadrupole deformation in odd-mass nuclei remains unclarified. Although there is no systematic evidence for the stable (quadrupole) deformation in the odd-mass nuclei with the spin $(j-1)$ ground state, it should be noted that the adjacent even-even nuclei exhibit the quasi-rotational spectra.[26]

3–5 *Mixing effects of 1QP modes on AC states*

Now let us consider the mixing effect of the 1QP mode, which lies in the next major shell, on the AC states. Following the general theory developed in Chap. 2, the effective Hamiltonian describing the system composed of the dressed 3QP modes and the 1QP modes, that is, the transcribed Hamiltonian in the quasi-particle NTD subspace

$$\{|\Phi_\alpha^{(1)}\rangle \equiv a_\alpha^\dagger|\Phi_0\rangle,\quad |\Phi_{nIK}^{(3)}\rangle \equiv Y_{nIK}^\dagger|\Phi_0\rangle\}, \tag{3·43}$$

is derived from the original P+QQ Hamiltonian as follows:

$$\left.\begin{aligned}&\boldsymbol{H}=\boldsymbol{H}^{(0)}+\boldsymbol{H}^{(\mathrm{int})},\\&\boldsymbol{H}^{(0)}=\sum_\alpha E_a\boldsymbol{a}_\alpha^\dagger\boldsymbol{a}_\alpha+\sum_{nIK}\omega_{nI}\boldsymbol{Y}_{nIK}^\dagger\boldsymbol{Y}_{nIK},\\&\boldsymbol{H}^{(\mathrm{int})}=\sum_{\substack{nIK,\alpha\\(K=m_\alpha)}}\chi_{\mathrm{int}}(a,nI)\{\boldsymbol{Y}_{nIK}^\dagger\boldsymbol{a}_\alpha+\boldsymbol{a}_\alpha^\dagger\boldsymbol{Y}_{nIK}\}.\end{aligned}\right\} \tag{3·44}$$

Here the operators $\boldsymbol{a}_\alpha^\dagger$ and $\boldsymbol{Y}_{nIK}^\dagger$ are defined by

$$\boldsymbol{a}_\alpha^\dagger \equiv a_\alpha^\dagger|\Phi_0\rangle\langle\Phi_0|, \tag{3·45}$$

$$\boldsymbol{Y}_{nIK}^\dagger \equiv Y_{nIK}^\dagger|\Phi_0\rangle\langle\Phi_0| \tag{3·46}$$

and satisfy the condition

$$\hat{S}_-(a)\boldsymbol{a}_\alpha^\dagger=\hat{S}_-(a)\boldsymbol{Y}_{nIK}^\dagger=0 \tag{3·47}$$

for all $\hat{S}_-(a)$, where $\hat{S}_-(a)$ denotes the quasi-spin operator of orbit a defined by (1·2·18). The condition (3·47) implies that $\boldsymbol{a}_\alpha^\dagger$ and $\boldsymbol{Y}_{nIK}^\dagger$ are the operators of the "intrinsic space" defined in §2-Chap. 1. The part $\boldsymbol{H}^{(\mathrm{int})}$ in (3·44) comes from the original H_Y-type interaction (3·7c) and represents the effective coupling Hamiltonian which is of interest. In the special situation of shell structure under consideration, the effective coupling strength $\chi_{\mathrm{int}}(p', nI)$ takes especially simple form:

$$\chi_{\rm int}(p', nI) = -\chi\delta_{Ij_{p'}}\frac{(p\|r^2Y_2\|p')}{\sqrt{2I+1}}(u_pu_{p'}-v_pv_{p'})$$
$$\times\left[Q(pp)N_I(p^3)\left\{\psi_{nI}(p^3)+\frac{1}{\sqrt{3}}\varphi_{nI}(p^3)\right\}\right.$$
$$\left.+\sum_{(bc)}{}' Q(bc)N(bc)\{\psi_{nI}(bc;p)+\varphi_{nI}(bc;p)\}\right]$$
$$=-\delta_{Ij_{p'}}\sqrt{\frac{5}{2I+1}}R(pp')M_\omega, \tag{3·48}$$

where p' denotes a single-particle orbit in the next major shell, and M_ω and $R(pp')$ are defined by (3·28) and (3·9), respectively. When we take account of the mixing effect of the 1QP mode on the AC states, the state vector of interest changes to*)

$$|\Phi^{(3)}_{IK}\rangle\equiv Y^\dagger_{IK}|\Phi_0\rangle \longrightarrow |\Psi^{(3)}_{IK}\rangle\equiv\sqrt{1-\zeta_I^2}\; Y^\dagger_{IK}|\Phi_0\rangle+\zeta_I a^\dagger_{\pi'}|\Phi_0\rangle, \tag{3·49}$$

where $\pi'\equiv(p', m_{\pi'})\equiv(n_{p'}l_{p'}j_{p'}, m_{\pi'})$ denotes a single-particle state with $j_{p'}=I$ and ζ_I denotes the mixing amplitude. The magnitude of the mixing amplitude ζ_I with $I=(j_p-1)$ is expected to be extremely small since the single-particle orbit p' (which has the same parity with the unique-parity orbit p and has $j_{p'}=I=j_p-1$) lies in the next upper major shell, and also since the effective coupling strength $\chi_{\rm int}$ involves, in this case, the spin-flip type matrix element $(p\|r^2Y_2\|p')$ which is small when compared with the spin-non-flip ones. Thus the special physical condition of the appearance of the AC states is just the same condition as that in which the dressed 3QP modes with $I=(j_p-1)$ manifest themselves as relatively pure eigenmodes with negligible mixings of the 1QP modes. We show quantitatively in §5 that the mixing amplitude ζ_I is indeed small.

Next, let us consider the effect of the coupling Hamiltonian on the 1QP state with spin j_p. The state vector of the 1QP state changes, due to the mixing of the dressed 3QP mode with $I=j_p$, to

$$|\Phi^{(1)}_\pi\rangle\equiv a^\dagger_\pi|\Phi_0\rangle \longrightarrow |\Psi^{(1)}_\pi\rangle\equiv\sqrt{1-\zeta_j^2}\,a^\dagger_\pi|\Phi_0\rangle+\zeta_j Y^\dagger_{jm_\pi}|\Phi_0\rangle. \tag{3·50}$$

The mixing amplitude ζ_j is, as is shown in §5, rather small. This is partly because of the smallness of the effective coupling strength $\chi_{\rm int}$ with $p'=p$ and $I=j_p$, and partly because of the relatively high excitation energy, ω_j, of the dressed 3QP mode with $I=j_p$; both of which are due to the extreme smallness of the 3QP correlation factor C_I, defined by (3·22), with $I=j_p$. In this way the mixing effect on the 1QP state with spin j_p is considerably reduced when

*) Hereafter we only consider the lowest energy solution of the dressed 3QP mode, which is of a collective type, and omit the suffix n. Needless to say, the solutions of non-collective types do not play any significant role in the following discussions.

compared to that which is given by the conventional QPC theory. The characteristic dependence of ζ_j on the $(u_p^2-v_p^2)$ factor is, however, the same as that of conventional QPC theory.

We investigate the mixing effects, both on the AC states with $I=(j-1)$ and on the 1QP state with spin j_p, paying special attention to the $M1$-transition property of the AC state which is very sensitive to these mixings in subsequent sections.

§4. Electromagnetic properties of the AC states with $I=(j-1)$

In this section, we present the qualitative predictions of the introduced model for the electromagnetic properties of the AC states with $I=(j-1)$. Experimental data revealing various aspects on the structure of the AC states have now been accumulated. The main characteristics which they exhibit are as follows.

1) Strongly enhanced $E2$ transition from the $(j-1)$ state to the 1QP state with spin j. The magnitude of the enhancement is comparable (or somewhat larger) to that from the 2^+ phonon states to the ground states in the adjacent even-even nuclei.

2) Moderately hindered $M1$ transition between $(j-1)$ state and j state.

3) The g factor of the $(j-1)$ state is approximately equal to that of the j state.

Formulae on these quantities can be obtained unambiguously by using the method developed in Chap. 2. The electromagnetic multipole operators $\hat{O}^{(\pm)}_{LK}$ defined by (2·5·12) are transcribed into the quasi-particle NTD subspace under consideration as

$$\hat{O}^{(\pm)}_{LM} \longrightarrow \hat{\boldsymbol{O}}^{(\pm)}_{LM}=\sum_{\alpha\beta}\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(1)}_\beta\rangle \boldsymbol{a}^\dagger_\alpha \boldsymbol{a}_\beta + \sum_{IK,I'K'}\langle\Phi^{(3)}_{IK}|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{I'K'}\rangle \boldsymbol{Y}^\dagger_{IK}\boldsymbol{Y}_{I'K'} + \sum_{\alpha,IK}\{\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{IK}\rangle \boldsymbol{a}^\dagger_\alpha \boldsymbol{Y}_{IK}+\text{h.c.}\} \quad (L\neq 0) \tag{4·1}$$

with the transcription coefficients

$$\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(1)}_\beta\rangle=\langle\Phi_0|\{a_\alpha,[\hat{O}^{(\pm)}_{LM},a^\dagger_\beta]_-\}_+|\Phi_0\rangle, \tag{4·2}$$

$$\langle\Phi^{(3)}_{IK}|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{I'K'}\rangle=\langle\Phi_0|\{Y_{IK},[\hat{O}^{(\pm)}_{LM},Y^\dagger_{I'K'}]_-\}_+|\Phi_0\rangle, \tag{4·3}$$

$$\langle\Phi^{(1)}_\alpha|\hat{O}^{(\pm)}_{LM}|\Phi^{(3)}_{IK}\rangle=\langle\Phi_0|\{a_\alpha,[\hat{O}^{(\pm)}_{LM},Y^\dagger_{IK}]_-\}_+|\Phi_0\rangle. \tag{4·4}$$

Thus we obtain the reduced matrix elements of interest as follows:

$$\langle\Psi^{(1)}_p\|\hat{\boldsymbol{O}}^{(\pm)}_L\|\Psi^{(3)}_I\rangle=\sqrt{1-\zeta_I^2}\sqrt{1-\zeta_j^2}\langle\Phi^{(1)}_p\|\hat{O}^{(\pm)}_L\|\Phi^{(3)}_I\rangle + \zeta_I\sqrt{1-\zeta_j^2}\langle\Phi^{(1)}_p\|\hat{O}^{(\pm)}_L\|\Phi^{(1)}_{p'}\rangle+\zeta_j\sqrt{1-\zeta_I^2}\langle\Phi^{(3)}_{I'=j}\|\hat{O}^{(\pm)}_L\|\Phi^{(3)}_I\rangle + \zeta_j\zeta_I\langle\Phi^{(3)}_{I'=j}\|\hat{O}^{(\pm)}_L\|\Phi^{(1)}_{p'}\rangle, \tag{4·5}$$

$$\langle\Psi_I^{(3)}\|\hat{O}_L^{(\pm)}\|\Psi_I^{(3)}\rangle=(1-\zeta_I^2)\langle\Phi_I^{(3)}\|\hat{O}_L^{(\pm)}\|\Phi_I^{(3)}\rangle$$
$$+2\zeta_I\sqrt{1-\zeta_I^2}\langle\Phi_I^{(3)}\|\hat{O}_L^{(\pm)}\|\Phi_{p'}^{(1)}\rangle+\zeta_I^2\langle\Phi_{p'}^{(1)}\|\hat{O}_L^{(\pm)}\|\Phi_{p'}^{(1)}\rangle,\ \cdots \tag{4·6}$$

with the definitions

$$\langle\Psi_\pi^{(1)}|\hat{O}_{LM}^{(\pm)}|\Psi_{IK}^{(3)}\rangle=\frac{(ILKM|j_p m_\pi)}{\sqrt{2j_p+1}}\langle\Psi_p^{(1)}\|\hat{O}_L^{(\pm)}\|\Psi_I^{(3)}\rangle, \tag{4·7}$$

$$\langle\Phi_\pi^{(1)}|\hat{O}_{LM}^{(\pm)}|\Phi_{IK}^{(3)}\rangle=\frac{(ILKM|j_p m_\pi)}{\sqrt{2j_p+1}}\langle\Phi_p^{(1)}\|\hat{O}_L^{(\pm)}\|\Phi_I^{(3)}\rangle,\ \text{etc.}$$

Needless to say, since the mixing amplitudes, ζ_I and ζ_j, are expected to be small, the first terms in Eqs. (4·5) and (4·6) should yield the main contribution to these reduced matrix elements unless they are strongly hindered (or forbidden). Now let us show how the experimental characteristics mentioned above can be recognized in a unified manner.

4–1 *E2 transitions between AC states and 1QP states*

As has been stressed, the collective nature of the AC state with $I=(j-1)$ has been recognized through the recent observation of strongly enhanced $E2$ transition from the AC state to the 1QP state with spin j. In the microscopic model under consideration, the reduced transition probabilities are given by

$$B(E2;I\to j_p)=\frac{1}{2I+1}|\langle\Psi_p^{(1)}\|\hat{O}_2^{(+)}\|\Psi_I^{(3)}\rangle|^2, \tag{4·8}$$

where $\hat{O}_2^{(+)}$ denotes the electric quadrupole operator in the quasi-particle NTD subspace. Since the mixing effect is expected to be negligibly small, we obtain

$$B(E2;I\to j_p)\approx\frac{1}{2I+1}|\langle\Phi_p^{(1)}\|\hat{O}_2^{(+)}\|\Phi_I^{(3)}\rangle|^2$$
$$=\Big|e_{\tau(p)}Q(pp)N_I(p^3)\Big\{\psi_I(p^3)+\sqrt{\frac{1}{3}}\varphi_I(p^3)\Big\}$$
$$+\sum_{(bc)}{}'e_{\tau(bc)}Q(bc)N(bc)\{\psi_I(bc;p)+\varphi_I(bc;p)\}\Big|^2. \tag{4·9}$$

Inserting the explicit expressions of the three-body correlation amplitudes (3·27) into (4·9), we finally obtain

$$B(E2;I\to j_p)\approx M_\omega^2|e_\tau S_p+e_{-1/2}S_c(\text{proton})+e_{1/2}S_c(\text{neutron})|^2, \tag{4·10}$$

where M_ω, S_p and S_c are defined by (3·28), (3·29) and (3·30), respectively. Here the contributions from proton-quasi-particles and from neutron-quasi-particles are written distinctively. The effective charges e_τ for neutrons ($\tau=1/2$) and protons ($\tau=-1/2$) are

$$e_{1/2}=\delta e \quad \text{and} \quad e_{-1/2}=e+\delta e. \tag{4·11}$$

For simplicity, we have adopted the same polarization charge δe for protons and neutrons. It is noteworthy that Eq. (4·9) has a structure formally similar to the corresponding expression given by the RPA in even-even nuclei, in spite of the difference due to the incorporation of the 3QP correlation. For the $E2$ transition between the AC state and the 1QP state, we can, therefore, expect the well-known enhancement associated with the structure of Eq. (4·9). In particular, we have the usual relation; the lower the excitation energy of the AC state, the larger the $B(E2)$ value becomes. Such an enhancement is a direct and natural consequence of the present model in which the motions of quasi-particles at the unique-parity orbit p and of quasi-particles excited from the "core" (the orbits b, c,..., etc.) are strongly coupled with each other to form a new type of collective excited state, i.e., the dressed 3QP state. It must also be emphasized that the cooperation of these two motions are strengthened further by the collective ground-state correlation which is enlarged as the excitation energy of the dressed 3QP mode, ω_{nI}, is lowered. Of course, the model of the AC state as the 3QP "intruder" state (given in the TD approximation within the unique-parity orbit p) cannot yield such a mechanism of striking $E2$ enhancement.

4–2 *Magnetic dipole moments of AC states*

The magnetic dipole moment of the AC state is given by

$$\mu=\sqrt{\frac{4\pi}{3}}\langle\Psi_{IK}^{(3)}|\hat{O}_{10}^{(-)}|\Psi_{IK}^{(3)}\rangle\equiv g_I I \quad \text{with} \quad K=I, \tag{4·12}$$

where $\hat{O}_{10}^{(-)}$ denotes the magnetic dipole operator in the quasi-particle NTD subspace. The g factor of the AC state, g_I, is expressed in our model as follows:

$$g_I=g_p^{(0)}+\frac{I(I+1)+j_p(j_p+1)-6}{2I(I+1)}g_p^{(1)}+\frac{I(I+1)+6-j_p(j_p+1)}{2I(I+1)}g_c, \tag{4·13}$$

where

$$g_p^{(0)}=g_p\{\psi_I(p^3)^2-\varphi_I(p^3)^2\}, \tag{4·14}$$

$$g_p^{(1)}=g_p\sum_{(bc)}{}'\{\psi_I(bc;p)^2-\varphi_I(bc;p)^2\} \tag{4·15}$$

and

$$g_c=\sqrt{\frac{5}{2}}\sum_{bcd}{}' M(bc)N(cd)^{-1}N(db)^{-1}\begin{Bmatrix} 2 & j_c & j_d \\ j_b & 2 & 1 \end{Bmatrix}$$
$$\times[\psi_I(cd;p)\psi_I(db;p)-\varphi_I(cd;p)\varphi_I(db;p)]. \tag{4·16}$$

Here g_p denotes the single-particle g factor of the unique-parity orbit p and

$$M(bc)\equiv\frac{1}{\sqrt{3}}(b\|\boldsymbol{\mu}\|c)\cdot(u_b u_c+v_b v_c) \tag{4·17}$$

with $\boldsymbol{\mu}=g_l\boldsymbol{l}+g_s\boldsymbol{s}$. In obtaining (4·13), we omitted the mixing effect of the 1QP mode with spin $(j-1)$, which lies in the next upper major shell, since the effect on the g factor of the AC state with $I=(j-1)$ is negligibly small. The physical meaning of each term in Eq. (4·13) is clear. The first term, $g_p^{(0)}$, comes from the "cluster" of quasi-particles at the "valence shell" orbit p. If we restrict our shell-model space to only the unique-parity orbit p, which is being filled, then $g_p^{(0)}$ becomes equal to g_p (because in this case $\psi_I(p^3)^2-\varphi_I(p^3)^2=1$). The second and third terms are of the same form as the Lande formula: The second term comes from the odd quasi-particle at the orbit p, i.e., p in $\psi_I(bc;p)$ and $\varphi_I(bc;p)$, while the third term comes from the quasi-particles excited from the "core," i.e., b and c in $\psi_I(bc;p)$ and $\varphi_I(bc;p)$.

It is interesting to note that the geometrical factors involved in the second and third terms in (4·13) possess characteristic dependences on the value of spin I; in the case of $I=(j_p-1)$ with high-j_p,

$$\left.\begin{aligned}\frac{I(I+1)+j_p(j_p+1)-6}{2I(I+1)}&\approx 1+O\left(\frac{1}{j_p}\right),\\ \frac{I(I+1)+6-j_p(j_p+1)}{2I(I+1)}&\approx -O\left(\frac{1}{j_p}\right).\end{aligned}\right\} \tag{4·18}$$

If we neglect the quantity of order $O(1/j_p)$, and making use of the normalization condition (3·26), we have

$$g_I\approx g_p^{(0)}+g_p^{(1)}=g_p. \tag{4·19}$$

In this way, although the g factor of the "phonon" (composed of the quasi-particles in the orbits b, c,...), i.e., g_c, may be of the order Z/A (in unit of nuclear magneton $e\hbar/2Mc$), its contribution to the g factor of the $(j-1)$ state is especially reduced. The experimental fact that $g_{I=j-1}\approx g_p$ has been often interpreted as an evidence of the simple $(j^n)_I$ configuration[10)~15)] for the structure of the AC states with $I=(j-1)$. However, as we have seen, the mechanism of obtaining the value of g_I nearly equal to g_p is distinctly different from this interpretation; in other words, in the shell model of j^n-configurations, we have $g_I=g_p^{(0)}=g_p$ for arbitrary values of I, while in the microscopic model under consideration, we have $g_I=g_p^{(0)}+g_p^{(1)}=g_p$ as a good approximation for the special case of $I=(j-1)$. It is shown in §5 that the magnitudes of $g_p^{(0)}$ and $g_p^{(1)}$ are approximately in the ratio of one to one.

4–3 *M1 transitions between AC states and 1QP states*

In contrast to the properties of $E2$ transition and of $M1$ moment described above, the $M1$-transition property of the AC state is very sensitive to the mixing effect resulting from the coupling term $\boldsymbol{H}^{(\mathrm{int})}$ in the transcribed Hamiltonian (3·44), as shown below. As usual, the reduced $M1$-transition probabilities from the AC state with $I=(j-1)$ to the 1QP state with spin j is given by

$$B(M1;\, I\to j_p)=\frac{1}{2I+1}|\langle\Psi_p^{(1)}\|\hat{\boldsymbol{O}}_1^{(-)}\|\Psi_I^{(3)}\rangle|^2 \qquad (4\cdot20)$$

with $I=j_p-1$. Since the creation operator of the dressed 3QP mode under consideration, (3·10), does not involve any components of quasi-particle pair with spin and parity 1^+, we have

$$\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle=0. \qquad (4\cdot21)$$

Namely, in the first-order approximation in which the AC state with $I=(j-1)$ is regarded as a pure dressed 3QP mode, the $M1$ transition between the $(j-1)$ state and the j state is forbidden. In fact the attenuation of the $M1$ transition has been observed in experiments. Then the retarded $M1$ transition must take place only through the mixing effects as follows.

$$B(M1;\, I\to j_p)=\frac{1}{2I+1}\Big|\zeta_I\sqrt{1-\zeta_j^2}\,\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_{p'}^{(1)}\rangle$$
$$+\zeta_j\sqrt{1-\zeta_I^2}\,\langle\Phi_{I'}^{(3)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle\Big|^2, \qquad (4\cdot22)$$

where

$$\langle\Phi_p^{(1)}\|\hat{O}_1^{(-)}\|\Phi_{p'}^{(1)}\rangle=\sqrt{\frac{3}{4\pi}}(p\|\boldsymbol{\mu}\|p')\cdot(u_pu_{p'}+v_pv_{p'})=\frac{3}{\sqrt{4\pi}}M(pp') \qquad (4\cdot23)$$

and

$$\langle\Phi_{I'}^{(3)}\|\hat{O}_1^{(-)}\|\Phi_I^{(3)}\rangle$$
$$=3\sqrt{\frac{(2I+1)(2I'+1)}{4\pi}}\Big[M(pp)\begin{Bmatrix} I & j_p & 2\\ j_p & I' & 1\end{Bmatrix}\sum_{(bc)}{}'\{\psi_{I'}(bc;p)\psi_I(bc;p)$$
$$-\varphi_{I'}(bc;p)\varphi_I(bc;p)\}+5\begin{Bmatrix} 2 & I' & j_p\\ I & 2 & 1\end{Bmatrix}\sum_{bcd}{}'M(bc)N(bd)^{-1}N(cd)^{-1}$$
$$\times\begin{Bmatrix} 2 & j_b & j_d\\ j_c & 2 & 1\end{Bmatrix}\{\psi_{I'}(cd;p)\psi_I(db;p)-\varphi_{I'}(cd;p)\varphi_I(db;p)\}\Big] \qquad (4\cdot24)$$

with $I=(j_p-1)=j_{p'}$ and $I'=j_p$.

The first term in Eq. (4·22) represents the contribution from the mixing of the 1QP state with spin $j_{p'}=j_p-1$, lying in the next upper major shell, in the AC state with $I=j_p-1$. The second term comes from the mixing of the dressed 3QP state (with $I'=j_p$) in the 1QP state (with spin j_p). Since the second term involves the $(u_p^2-v_p^2)$ factor through the mixing amplitude ζ_j, the value depends sensitively on the nucleon-occupation probability of the unique-parity orbit p and changes its sign on both sides of the half-shell, while the first term in Eq. (4·22) preserves its sign through the whole range. As a consequence of the interference effect between the two, the magnitude of the $M1$ transition becomes greatly sensitive to details of these mixing effects. The magnitude itself should of course be small compared with that of a single-particle transition (because ζ_I and ζ_j are both small), unless one moves away from the particular physical situation for the appearance of the AC states.

4–4 *Additional remarks*

1) The mixing of the 1QP state from the next upper major shell in the AC state may be directly checked with the spectroscopic factor of the single-nucleon-transfer reaction. For example, in the case of (d, p) reaction leading to the AC state, the direct transfer of single-neutron can only take place through the mixing effect. Therefore, the spectroscopic factor, which is given by $S_I=(\zeta_I u_{p'})^2 \approx \zeta_I^2$ in the NTD approximation, must be very small as long as the AC state can be regarded as a relatively pure dressed 3QP state. In fact, the spectroscopic factor of the $(j-1)$ state has been known to be extremely small, being consistent with the theoretical prediction.

2) In a similar way as has been discussed so far, we can evaluate the other properties of the AC states on the basis of the method developed in Chap. 2. In the case of evaluating the quadrupole moment, however, we should carefully examine whether we should extend our quasi-particle NTD subspace to include the dressed 5QP modes or not, because even a small mixing of such higher collective states may yield a large effect on such a quantity.

§5. Comparison of calculated results with experimental data

In this section, let us make a comparison between the calculated results and available experimental data, in order to examine quantitatively the theoretical predictions stated above.

5–1 *Procedure of numerical calculations*

The parameters entering into the solutions of Eq. (3·23) are the quadrupole-force strength χ and the quantities related to the pairing correlations (i.e., the parameters u_a and v_a of the Bogoliubov transformation and the single-quasi-particle energies E_a), which are determined from the single-particle energies ϵ_a and the pairing-force strength G.

In order to see the essential effects of the 3QP correlation originated from the quadrupole force and to fix the parameters as much as possible, we use the same values for the pairing-force strength G and for the single-particle energies ϵ_a as those adopted in the work of Kisslinger and Sorensen,[1] and also make the same truncation of shell-model space as they have made. On the other hand, the quadrupole-force strength χ is regarded as a free parameter which should be determined phenomenologically except for its usual mass-number dependence;[3]

$$\chi=\chi_0 b^{-4}A^{-5/3} \quad \mathrm{MeV}\cdot\mathrm{fm}^{-4},$$

where b^2 is the harmonic-oscillator range parameter. As is usual in the P+QQ force model, the reduced matrix element of the single-particle quadrupole moment $q(ab)$ is calculated with the harmonic-oscillator-shell-model wave functions. Then, since $q(ab)$ is proportional to b^2, the factor b^{-4} does not explicitly appear in the reduced matrix element of the quadrupole force, $1/2\cdot\chi q(ab)q(cd)$, and only χ_0 is regarded as a parameter.

Numerical calculations have been performed for the three shell regions, i.e., $1h^-_{11/2}$-odd-neutron region, $1g^+_{9/2}$-odd-proton region and $1g^+_{9/2}$-odd-neutron region. To see change of the relative excitation-energies of the dressed 3QP modes, $\omega'_I\equiv\omega_I-E_p$, over a wide sequence of spherical odd-mass nuclei, we have first used a constant value of χ_0 in each shell region. Secondly, in the evaluations of the mixing effects and of various electromagnetic quantities, we have chosen the value of χ_0 in each nucleus, so that the calculated value of ω'_I just reproduces the experimental excitation energy of the $(j-1)$ state measured from the 1QP state with spin j. In this step, the mixing effects have been calculated by taking the same value of the single-particle energy of the orbit p' (in the next upper major shell) as that adopted in the work of Uher and Sorensen[25] and also by putting $v_{p'}\approx 0$. The electromagnetic quantities have been calculated by using the polarization charge $\delta e=0.5\,e$ (for both proton and neutron) and the effective spin g factor $g_s^{\mathrm{eff}}=0.55\ g_s$, since we have adopted the P+QQ force model in the truncated shell-model space consisting of one major shell (for both protons and neutrons).

Thus it is evident that our choice of the parameters is merely the conventional one without any modifications.

5-2 *Region of $h^-_{11/2}$-odd-neutron nuclei*

This is the region in which the unique-parity orbit $h^-_{11/2}$ is being filled with neutrons. In Cd, Te and Xe isotopes, the $9/2^-$ states have been found in recent experiments at a few hundred keV in energy above the 1QP $11/2^-$ states.

In Fig. 11 are shown the calculated energy levels ω'_I for the sequences of odd-mass Cd, Sn, Te, Xe and Ba isotopes. The adopted value of χ_0 is the same as has been derived by Baranger and Kumar[24] within a few percent and also as is expected from conventional arguments in the P+QQ force model.[3] It is predicted from the result of the theoretical calculations that the excitation energies, $\omega'_I=\omega_I-E_p$, of the $9/2^-$ states are decreasing as one moves from the single-closed shell Sn isotopes to the heavier Te, Xe and Ba isotopes, and in each sequence of the isotopes, they are decreasing as the

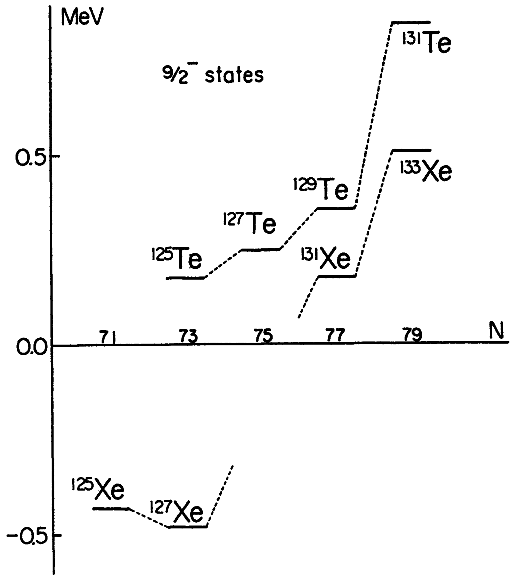


Fig. 10. Experimental trend of energy levels of the $9/2^-$ states in the $h_{11/2}^-$-odd-neutron region. The level energies are those measured from the 1QP $11/2^-$ states.
^{125}Te; Ref. 35), ^{127}Te; Ref. 29), ^{129}Te; Ref. 30), ^{131}Te; Ref. 31), ^{125,127}Xe; Ref. 34), ^{131}Xe; Ref. 32), ^{133}Xe; Ref. 33).

neutrons fill the unique-parity orbit $1h_{11/2}^-$ toward its middle. This calculated trend is naturally understood when we recall the enhancement factors of the 3QP correlation discussed at the end of §3–3: The decrease of the $9/2^-$ energy from Sn to Ba isotopes can be well understood as due to the increase of the factor $\xi(bc)$ of the core, and in each sequence of the isotopes the decrease is due to the increase of the factor $\xi(pp)$ at the unique-parity orbit $1h_{11/2}^-$.

So far, none of the low-lying $9/2^-$ states is experimentally observed in Sn isotopes.[36),78)] The reason can be explained when we consider the $9/2^-$ states as the dressed 3QP states, because in such single-closed-shell nuclei the enhancement factors $\xi(bc)$ of the core become so small that, in the theoretical calculations, the $9/2^-$ states are forced to lie at about 1 MeV above the 1QP $11/2^-$ states. In Te and Cd isotopes (in which two protons and two proton-holes are added to the proton-closed shell Sn isotopes, respectively), the $9/2^-$ states found in the experiments are well reproduced by theoretical calculations with the reasonable value of χ_0. When we regard the $9/2^-$ states as the 3QP "intruder states" of Kisslinger composed of the neutrons in $(1h_{11/2}^-)^n$-configuration, it is hard to understand the (above mentioned) different experimental situations between Sn isotopes and Te and Cd isotopes. Furthermore, according to the discussion made in §3–3, the fact that there are no near-lying $9/2^-$ states other than the first $9/2^-$ states under consideration also indicates

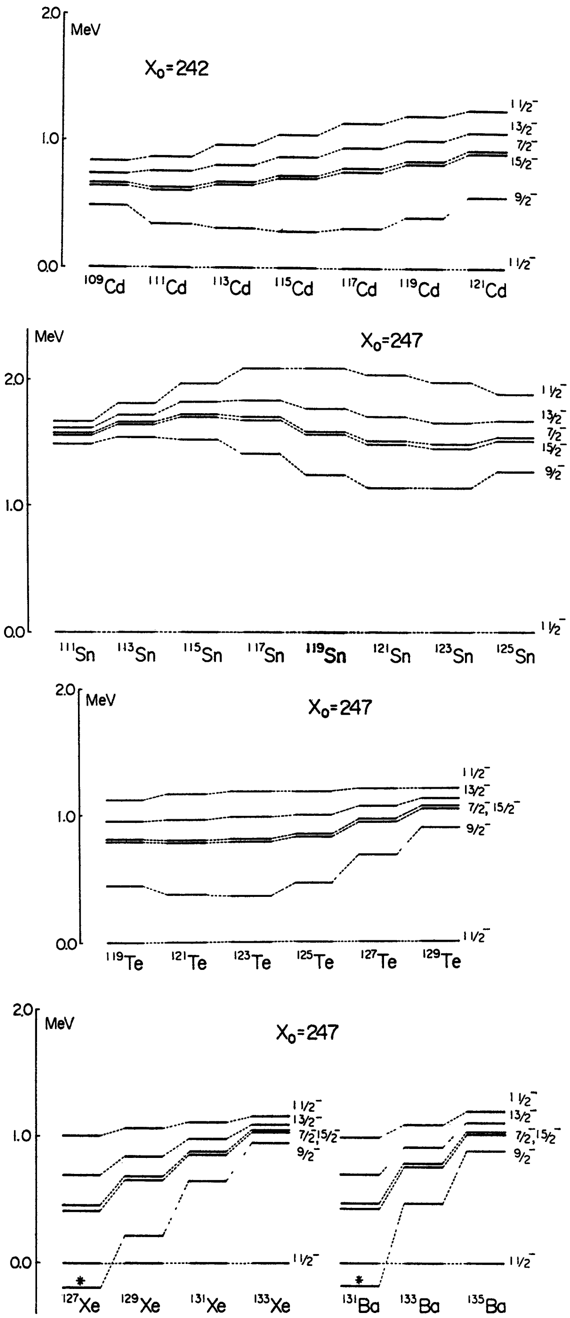


Fig. 11. Calculated excitation-energy systematics of the dressed 3QP states in the $h^{-}_{11/2}$-odd-neutron region. The level energies are those measured from the 1QP $11/2^-$ states. The quadrupole-force parameter χ_0 is fixed in each region of isotopes. The asterisk denotes the occurence of instability for this choice of the parameter χ_0.

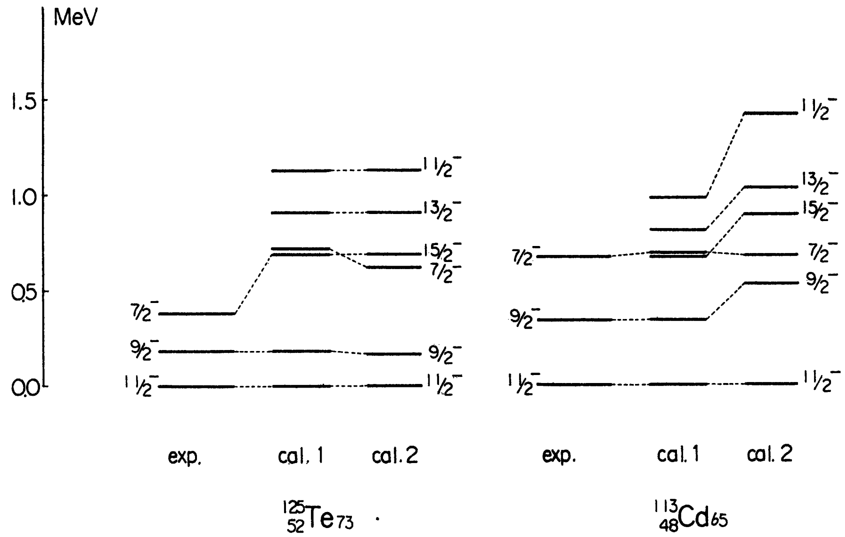


Fig. 12. Comparison between the experimental energy levels and the theoretical calculations for both cases, i.e., without the coupling effects (cal. 1) and taking account of the coupling effects (cal. 2). The energies are those measured from the 1QP $11/2^-$ states. Only the lowest-lying collective states in each spin are written in the figure. Experimental data are taken from; ^{115}Cd, Ref. 28), ^{125}Te, Ref. 35).

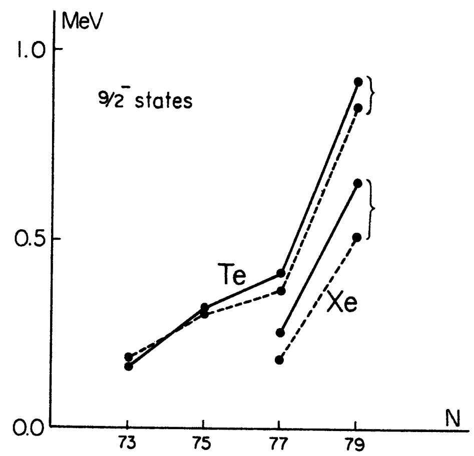


Fig. 13. Energy shifts due to the coupling effects of the dressed 3QP $9/2^-$ modes with the 1QP $h_{9/2}^-$ modes. The energies of the $9/2^-$ states in the absence of the coupling effects are connected by broken lines, while those in the presence of the coupling effects are connected by solid lines. All energies are those measured from the 1QP $11/2^-$ states.

the strong coupling between the 3QP "intruder" state and the "phonon" excited from the core.

In contrast to Te isotopes, the experimental energy change of the $9/2^-$ states in the sequence of Xe isotopes is rapid, and at the neutron deficient ^{127}Xe the $9/2^-$ state becomes lower than the $11/2^-$ state. According to our point of view, this fact indicates the growth of instability toward quadrupole deformation as one moves toward ^{127}Xe. These experimental facts are indeed those which are expected from the theoretical calculations, and the situation remains unchanged for a rather wide range of parameter χ_0. From the theoretical calculations, similar experimental aspects can also be expected in Ba isotopes. So far there is no systematic experimental evidence that the neutron-deficient odd-mass Xe isotopes, in which the $9/2^-$ states are lower than the 1QP $11/2^-$ states, have stable deformations. It is interesting to note, however, that the adjacent even-even nuclei clearly display the quasi-rotational spectra.

The magnetic moment and electromagnetic transition rates of the $9/2^-$ state have been measured in ^{125}Te and ^{127}Te. The collective structure of the $9/2^-$ states exhibited in the excitation-energy systematics mentioned above is directly demonstrated in the striking enhancement of the $B(E2)$ value between the $9/2^-$ state and the $11/2^-$ state. The esssential role of the 3QP correlation (in characterizing the $9/2^-$ state) at the unique-parity orbit $1h_{11/2}^-$ is directly reflected in its g factor which is approximately equal to that of the 1QP $11/2^-$ state. Here, in contrast to the case of the $B(E2)$ values, the quasi-particles

Table I. The correlation amplitudes of the dressed 3QP $9/2^-$ mode in ^{125}Te. The adopted value of χ_0 is 260 (MeV) and the calculated excitation energy, $\omega'=\omega-E_p$, is 0.08 MeV. The values of forward amplitudes $\psi(bc;p)$ are written in the second column, while the values of backward amplitudes $\varphi(bc;p)$ are written in the third column. In this state, the unique-parity orbit p is specified by the set of quantum numbers (neutron; $h_{11/2}^-$), and therefore only the orbital pairs, (bc)'s, are written in the first column. For convenience, the notations $\psi(pp;p)$ and $\varphi(pp;p)$ are used here to denote $\psi(p^3)$ and $\varphi(p^3)$ defined by (3·19), respectively. These amplitudes are normalized to one according to Eq. (3·26) in the text.

neutron

bc	$(h_{11/2})^2$	$(g_{7/2})^2$	$(d_{5/2})^2$	$(d_{3/2})^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi(bc;p)$	0.97	0.15	0.07	0.35	−0.03	0.27	0.11	0.11	−0.25
$\varphi(bc;p)$	0.48	0.13	0.07	0.30	−0.03	0.24	0.10	0.10	−0.22

proton

bc	$(h_{11/2})^2$	$(g_{7/2})^2$	$(d_{5/2})^2$	$(d_{3/2})^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi(bc;p)$	0.07	0.64	0.21	0.02	−0.11	0.11	0.03	0.06	−0.02
$\varphi(bc;p)$	0.07	0.51	0.18	0.02	−0.09	0.10	0.03	0.05	−0.01

excited from the core give rise to only slight difference between their g factors. The essential part of the values of the $B(E2)$ and g factor can be determined without introducing the mixing effects. On the other hand, the mixing effects coming from the coupling between the dressed 3QP mode and the 1QP mode are sensitively reflected in the occurence of the retarded $M1$ transition from the

Table II. $B(E2; 9/2^- \to 11/2^-)$ values in the $h_{11/2}^{-}$-odd-neutron region. The values are written in unit of $e^2 \cdot 10^{-50}$ cm^4. The excitation energies of the $9/2^-$ states measured from the $11/2^-$ states are listed in the second column (in unit of MeV). The $B(E2)$ values calculated by neglecting the coupling effects are listed in the third column, while those calculated by taking account of the coupling effects in the fourth column. They are compared with experimental data $B(E2)^{\rm exp}$ listed in the fifth column. The harmonic-oscillator-range parameter $b^2 = 1.0A^{1/3}$ and the polarization charge $\delta e = 0.5\,e$ are used.

nucleus	ω'_{j-1}	$B(E2)^{1)}$	$B(E2)^{2)}$	$B(E2)^{\rm exp}$
^{113}Cd	0.34	9.8	8.3	
^{115}Cd	0.33	9.4	8.7	
^{125}Te	0.18	10.7	10.6	11.5±0.5 a)
^{127}Te	0.25	8.3	8.1	9.2±1.3 b)
^{129}Te	0.36	6.2	5.8	
^{131}Te	0.85	2.8	2.6	
^{131}Xe	0.18	15.8	14.6	
^{133}Xe	0.51	7.5	6.6	

a) Ref. 37), b) Ref. 38).

Table III. Gyromagnetic ratio for the $9/2^-$ states in the $h_{11/2}^{-}$-odd-neutron region in unit of nuclear magneton ($e\hbar/2Mc$). The values calculated by neglecting the coupling effects are listed in the second column, while the values calculated by taking account of the coupling effects are listed in the third column. The experimental data for the $9/2^-$ states are listed in the fourth column, while those for the $11/2^-$ states in the fifth column. The effective spin g factor $g_s^{\rm eff} = 0.55 g_s$ is used. The g factor of the 1QP $11/2^-$ state is, therefore, assumed to be -0.19.

nucleus	$g_{j-1}^{1)}$	$g_{j-1}^{2)}$	$g_{j-1}^{\rm exp}$	$g_j^{\rm exp}$
^{113}Cd	−0.26	−0.25		−0.20 c)
^{115}Cd	−0.25	−0.25		
^{125}Te	−0.21	−0.21	−0.204±0.007 a)	−0.169±0.009 d)
^{127}Te	−0.22	−0.22	−0.214±0.014 b)	−0.165±0.009 e)
^{129}Te	−0.23	−0.22		−0.209±0.009 e)
^{131}Te	−0.25	−0.25		
^{131}Xe	−0.22	−0.22		
^{133}Xe	−0.25	−0.25		

a) Ref. 39), b) Ref. 38), c) Ref. 59), d) Ref. 41), e) Ref. 42).

Table IV. $B(M1;\ 9/2^- \rightarrow 11/2^-)$ values in the $h^{-}_{11/2}$-odd-neutron region. The values are written in unit of $(e\hbar/2Mc)^2$. The mixing amplitudes ζ_j and $\zeta_{I=j-1}$, defined by (3·50) and (3·49), are listed in the second and third columns, respectively. The contributions from the first and second terms in (4·22) are shown separately in the fourth and fifth columns, respectively. The calculated values $B(M1)^{\rm cal}$ are listed in the sixth column and are compared with the experimental data $B(M1)^{\rm exp}$ listed in the seventh column. In this calculation, the effective spin g factor $g_s^{\rm eff}=0.55\ g_s$ is used.

nucleus	ζ_j	ζ_{j-1}	M_{11}	M_{33}	$B(M1)^{\rm cal}$	$B(M1)^{\rm exp}$
^{113}Cd	−0.39	−0.09	−0.36	−2.17	1.5×10^{-1}	
^{115}Cd	−0.28	−0.09	−0.35	−1.51	8.2×10^{-2}	
^{125}Te	0.03	−0.08	−0.25	0.11	4.8×10^{-4}	$(6.5\pm0.3)\times10^{-3}$ a)
^{127}Te	0.13	−0.07	−0.19	0.45	1.6×10^{-3}	$(1.6\pm0.6)\times10^{-3}$ b)
^{129}Te	0.24	−0.06	−0.13	0.84	1.2×10^{-2}	
^{131}Te	0.25	−0.03	−0.06	1.05	2.3×10^{-2}	
^{131}Xe	0.27	−0.06	−0.15	1.13	2.3×10^{-2}	
^{133}Xe	0.36	−0.04	−0.08	1.53	5.0×10^{-2}	

a) Ref. 37), b) Ref. 38).

Table V. Electromagnetic properties of the dressed 3QP states with negative parity in ^{125}Te. The calculated values for two alternative approximations, i.e., without taking account of the coupling effects (cal. 1) and taking account of the coupling effects (cal. 2), are listed in the third and fourth columns, respectively. The experimental data are listed in the fifth column. The units are $e^2\cdot10^{-50}$ cm^4 for $B(E2)$, $e\hbar/2Mc$ for g factors and $(e\hbar/2Mc)^2$ for $B(M1)$. The polarization charge $\delta e=0.5e$ and $g_s^{\rm eff}=0.55\ g_s$ are used. The procedure of the calculation is the same as in Tables II~IV, except that the value of g_p is directly taken from the experimental value of the 1QP $11/2^-$ state ($g_p=-0.17$). The spectroscopic factors for (d,p) reaction are calculated by the approximation $S_I\approx(\zeta_I)^2$.

observable	spin	cal. 1	cal. 2	exp
	$7/2^- \rightarrow 11/2^-$	4.5	4.6	
	$9/2^- \rightarrow 11/2^-$	10.7	10.6	11.5 ± 0.5 a)
$B(E2)$	$11/2'^- \rightarrow 11/2^-$	2.5	2.5	
	$13/2^- \rightarrow 11/2^-$	3.5	3.5	
	$15/2^- \rightarrow 11/2^-$	4.6	4.6	
	$7/2^-$	−0.38	−0.38	
	$9/2^-$	−0.19	−0.19	-0.204 ± 0.007 b)
g	$11/2'^-$	−0.11	−0.11	
	$13/2^-$	−0.06	−0.06	
	$15/2^-$	−0.05	−0.05	
$B(M1)$	$9/2^- \rightarrow 11/2^-$	0.0	4.8×10^{-4}	$(6.5\pm0.3)\times10^{-3}$ a)
S	$7/2^-$	0.0	0.026	
	$9/2^-$	0.0	0.006	

a) Ref. 37), b) Ref. 39).

9/2⁻ state to the 11/2⁻ state. The actual value of $B(M1)$ is determined by the interference between the first and the second term in Eq. (4·23).

As shown in Table V, these characteristic properties of the $9/2^-$ states have been reproduced very well in the theoretical calculations without making any arbitrary alternation of the values of conventional parameters in the P+QQ force model. Thus the introduced model of the $9/2^-$ state as a typical manifestation of the dressed 3QP mode has been verified by the numerical results. Of course, similar characteristics are also expected in other nuclei and, therefore, systematic measurements of these electromagnetic quantities are expected to reveal further details of the structure of the $9/2^-$ states; in particular, it is interesting to focus our attention on the changes of their properties, from the nuclei with positive ω_I' toward the nuclei with negative ω_I'.

5-3 *Region of $g^+_{9/2}$-odd-proton nuclei*

In this region, the unique-parity orbit $1g^+_{9/2}$ is being filled with protons. In the experiments, the rapid drop in energy of the $7/2^+$ state is observed as one moves from Nb to Ag. And, as is well known, the $7/2^+$ states appear below the $9/2^+$ states in Rh isotopes heavier than ^{103}Rh and in all Ag isotopes, ^{103}Ag$\sim$^{113}Ag. In the theoretical calculations with a constant value of χ_0, the energies of $7/2^+$ states, from ^{93}Nb to ^{107}Ag and also for each isotope, go down as functions of nucleon numbers Z and N, and are in good agreement with the experimental trend. (See Figs. 14 and 15.) Here, the decrease of ω_I with $I=7/2$ can be understood as a result of the fact that two enhancement factors, $\xi(pp)$ and $\xi(bc)$, act coherently as one moves from Nb to the heavier odd-proton nuclei in this region.

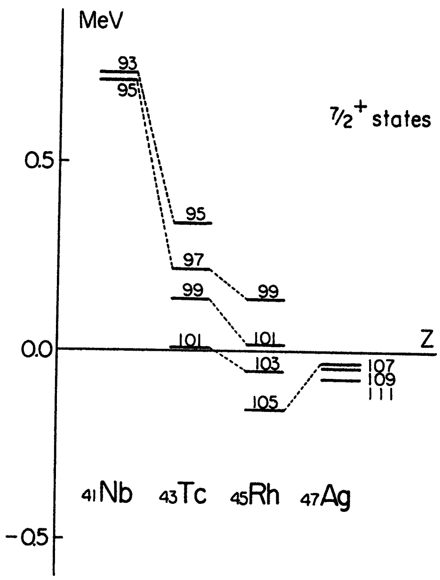


Fig. 14. Experimental trend of energy levels of the $7/2^+$ states in the $g^+_{9/2}$-odd-proton region. The level energies are those measured from the 1QP $9/2^+$ states.
^{93}Nb; Ref. 50), ^{95}Tc; Ref. 52), ^{97}Tc; Ref. 44), ^{99}Tc; Ref. 45), ^{101}Tc; Ref. 56), ^{99,101}Rh; Ref. 46), ^{103}Rh; Ref. 47), ^{105}Rh; Ref. 48), ^{109,111}Ag; Ref. 49).

The process of the rapid enhancement of the 3QP correlation can be explicitly seen when we compare the spectrum of ^{93}Nb with that of ^{95}Tc. In ^{93}Nb we see a quintet with spins from $5/2^+$ to $13/2^+$, which is interpreted as consisting of one phonon and the odd quasi-particle. The energy splitting of the multiplet may be treated as a result of relatively small perturbations due to the 3QP correlation and the mixing effects (coming from the H_Y-type interaction). However, in ^{95}Tc in which only two protons are added to ^{93}Nb, we see striking enhancement of the 3QP correlation. There, the energy splitting of the quintet amounts to the unperturbed energy of the 2^+ phonon itself and, therefore, the splitting is beyond the limit of perturbational treatment. This enhancement is obviously caused by the increment of the factor $\xi(pp)$ in ^{95}Tc when compared with that of ^{93}Nb. As is shown in Fig. 16 and Table X, the theoretical calculation reproduces these changes very well, not only in the excitation energies but also in the electromagnetic properties. The appearance of different nature among the members of the quintet essentially comes from the spin dependence of the 3QP correlation, except for the $5/2^+$ state where the coupling of the 1QP mode from the next upper major shell affects its level position non-negligibly.

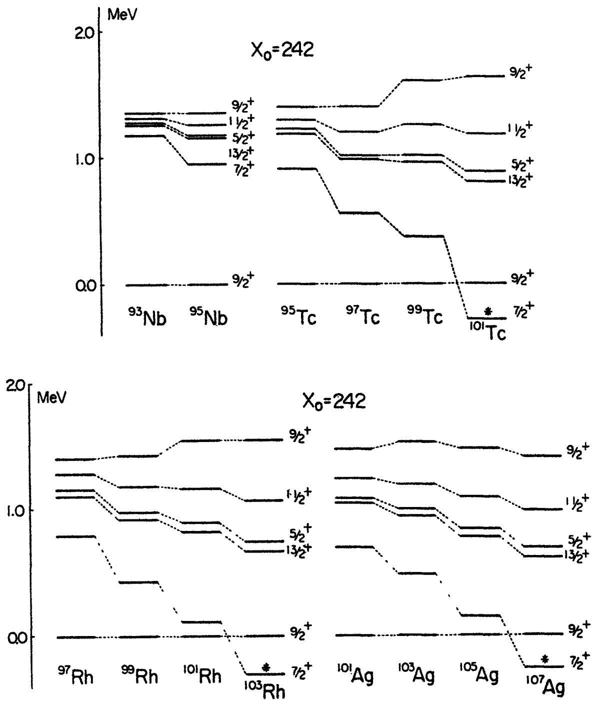


Fig. 15. Calculated excitation energy systematics of the dressed 3QP states in the $g_{9/2}^{+}$-odd-proton region. Notations are the same as in Fig. 11.

Let us further increase the enhancement factors, $\xi(pp)$ by adding protons and also $\xi(bc)$ of the core by adding neutrons. Then we arrive at such a quite different situation that only the $7/2^+$ states are extremely lowered in energy. For nuclei in which the anomalous coupling $7/2^+$ states appear below the 1QP $9/2^+$ states, we expect the growth of instability of the spherical BCS vacuum toward quadrupole deformation. Although, in the vicinity of the critical point $\omega_{\rm cri}$, we cannot expect the quantitative validity of the NTD

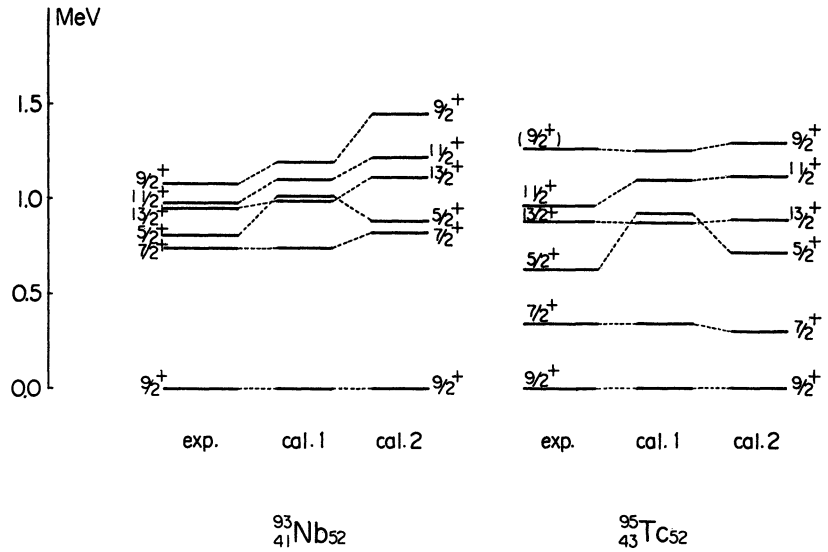


Fig. 16(a)

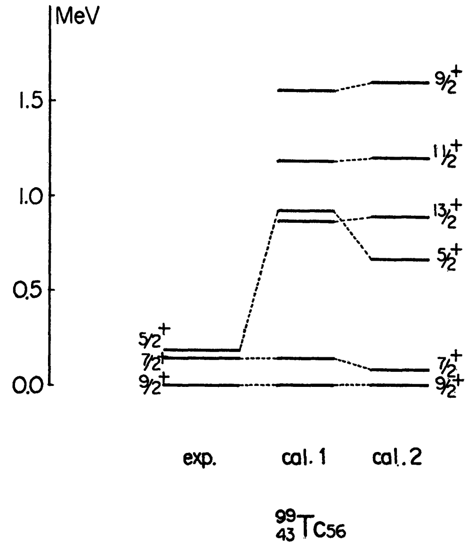


Fig. 16(b).

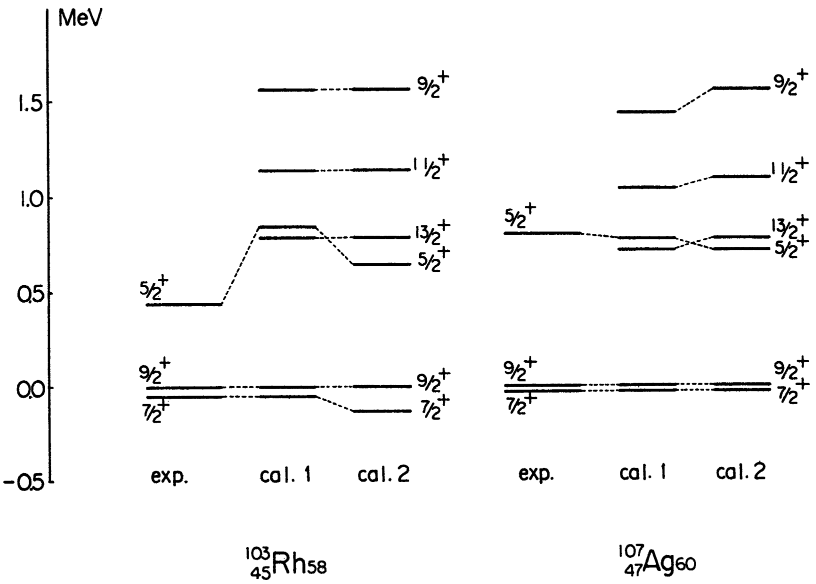


Fig. 16(c).

Fig. 16. Comparison between the experimental energy levels and the theoretical calculations in the $g^+_{9/2}$-odd-proton region. Notations are the same as in Fig. 12. For the experimental data, refer to the caption of Fig. 14.

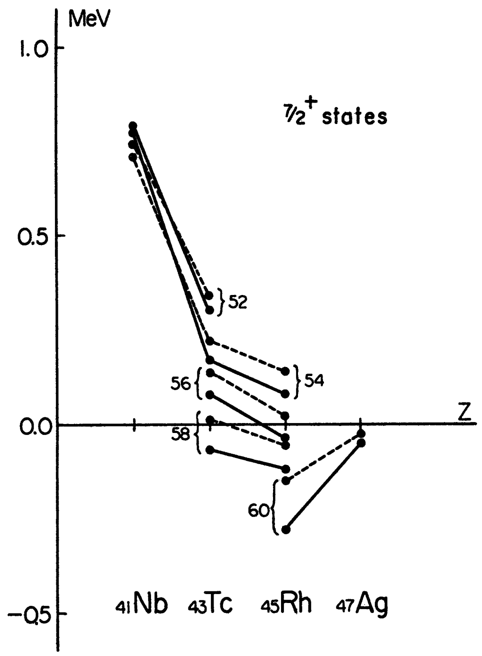


Fig. 17. Energy shifts due to the coupling effects of the dressed 3QP $7/2^+$ modes with the 1QP $g^+_{7/2}$ modes in the $g^+_{9/2}$-odd-proton region. Notations are the same as in Fig. 13.

approximation to be satisfactory, it is surprising that the experimental behaviours including the critical region, $E_p > \omega_I > \omega_{\rm cri}$, have been well reproduced in the theoretical calculations, with the value of $\chi_0 = 242$ (MeV) which is just the conventional value derived from the classical method.[3] For instance, the calculated values of $B(E2; 7/2^+ \to 9/2^+)$ with polarization charge $\delta e = 0.5\,e$ are in good agreement with available experimental data even for Ag isotopes where the $7/2^+$ states appear below the $9/2^+$ states. (See Table VII.)

5–4 *Region of $g^+_{9/2}$-odd-neutron nuclei*

In this region the unique-parity orbit $1g^+_{9/2}$ is being filled with neutrons.

Table VI. The correlation amplitudes of the dressed 3QP $7/2^+$ mode in ^{99}Tc. The adopted value of χ_0 is 248(MeV) and the calculated excitation energy, $\omega' = \omega - E_p$, is 0.14 MeV. Notations are the same as in Table I. The unique-parity orbit p denotes (proton; $1g^+_{9/2}$).

proton

bc	$(g_{9/2})^2$	$(f_{5/2})^2$	$(p_{3/2})^2$	$f_{5/2}p_{3/2}$	$f_{5/2}p_{1/2}$	$p_{3/2}p_{1/2}$
$\psi(bc;p)$	0.97	0.05	0.06	−0.03	0.08	0.10
$\varphi(bc;p)$	0.45	0.04	0.05	−0.02	0.07	0.09

neutron

bc	$(h_{11/2})^2$	$(g_{7/2})^2$	$(d_{5/2})^2$	$(d_{3/2})^2$	$g_{7/2}d_{5/2}$	$g_{7/2}d_{3/2}$	$d_{5/2}d_{3/2}$	$d_{5/2}s_{1/2}$	$d_{3/2}s_{1/2}$
$\psi(bc;p)$	0.11	0.13	0.68	0.04	−0.10	0.07	0.11	0.36	−0.07
$\varphi(bc;p)$	0.11	0.11	0.53	0.03	−0.09	0.06	0.10	0.30	−0.06

Table VII. $B(E2; 7/2^+ \to 9/2^+)$ values in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table II.

nucleus	ω'_{j-1}	$B(E2)^{1)}$	$B(E2)^{2)}$	$B(E2)^{\rm exp}$
^{93}Nb	0.74	2.4	2.3	2.25±0.16 a)
^{95}Nb	0.72	3.5	3.3	
^{95}Tc	0.34	5.2	5.2	
^{97}Tc	0.22	8.1	8.0	
^{99}Tc	0.14	11.4	11.2	13.5±1.5 b)
^{101}Tc	0.01	18.7	18.1	≈30 d)
^{99}Rh	0.14	9.3	9.2	
^{101}Rh	0.02	14.4	14.2	
^{103}Rh	−0.05	21.0	20.6	9.5 c)
^{105}Rh	−0.15	39.2	37.7	>31 c)
^{107}Ag	−0.03	20.1	19.1	
^{109}Ag	−0.04	23.6	22.3	27.4 c)
^{111}Ag	−0.07	29.2	27.5	20.1 c)

a) Ref. 51), b) Ref. 54), c) Ref. 57), d) Ref. 56).

Experimental data on the $N=47$ isotones show that the excitation energies of the $7/2^+$ states measured from those of the 1QP $9/2^+$ states become small with decreasing proton number, i.e., from $^{85}_{38}$Sr to $^{81}_{34}$Se. The same behaviour is also found in the $N=45$ isotones, i.e., from $^{83}_{38}$Sr to $^{77}_{32}$Ge. (See Fig. 18.) These behaviours are in correspondence with those of the 2^+ phonon states in the sequences of neighbouring even-even nuclei. Then the appearance of the $7/2^+$ states below the 1QP $9/2^+$ states in many nuclei belonging to this region

Table VIII. Gyromagnetic ratio for the $7/2^+$ states in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table III.

nucleus	$g^{1)}_{j-1}$	$g^{2)}_{j-1}$	g^{exp}_{j-1}	g^{exp}_{j}
^{93}Nb	1.32	1.31		1.37 a)
^{95}Nb	1.31	1.30		
^{95}Tc	1.27	1.26		
^{97}Tc	1.27	1.25		
^{99}Tc	1.25	1.23	0.75±0.26 a)	1.26 a)
^{101}Tc	1.23	1.21		
^{99}Rh	1.25	1.24		
^{101}Rh	1.23	1.22		
^{103}Rh	1.22	1.20		
^{105}Rh	1.18	1.16		
^{107}Ag	1.22	1.21		
^{109}Ag	1.22	1.21	1.22±0.037 b)	
^{111}Ag	1.21	1.20		

a) Ref. 59), b) Ref. 58).

Table IX. $B(M1;\ 7/2^+ \to 9/2^+)$ values in the $g^+_{9/2}$-odd-proton region. Notations and parameters are the same as in Table IV.

nucleus	ζ_j	ζ_{j-1}	M_{11}	M_{33}	$B(M1)^{cal}$	$B(M1)^{exp}$
^{93}Nb	−0.29	−0.18	0.68	1.86	1.9×10^{-1}	$\sim1.6\times10^{-1}$ d)
^{95}Nb	−0.29	−0.17	0.66	1.76	1.8×10^{-1}	
^{95}Tc	−0.12	−0.19	0.66	0.62	4.9×10^{-2}	
^{97}Tc	−0.13	−0.19	0.65	0.69	5.3×10^{-2}	
^{99}Tc	−0.12	−0.19	0.67	0.55	4.4×10^{-2}	$(7.6\pm0.9)\times10^{-2}$ a) $(9.44\pm0.1)\times10^{-2}$ b)
^{101}Tc	−0.11	−0.21	0.72	0.43	3.9×10^{-2}	$\sim1\times10^{-1}$ c)
^{99}Rh	0.03	−0.18	0.54	−0.15	4.7×10^{-3}	
^{101}Rh	0.03	−0.19	0.57	−0.13	5.7×10^{-3}	
^{103}Rh	0.03	−0.20	0.59	−0.10	7.1×10^{-3}	$(9.3\pm0.06)\times10^{-2}$ b)
^{105}Rh	0.04	−0.23	0.67	−0.13	8.8×10^{-3}	$<3.1\times10^{-2}$ b)
^{107}Ag	0.19	−0.18	0.40	−0.64	1.8×10^{-3}	$(4.19\pm0.4)\times10^{-2}$ b)
^{109}Ag	0.20	−0.17	0.38	−0.70	3.0×10^{-3}	3.8×10^{-2} b)
^{111}Ag	0.20	−0.17	0.38	−0.82	5.6×10^{-3}	6.9×10^{-2} b)

a) Ref. 55), b) Ref. 57), c) Ref. 56), d) Ref. 50).

Table X(a). Electromagnetic properties of the "core excited" states with positive parity in ^{93}Nb. Notations and parameters are the same as in Table V. The value of g_p is taken from the experimental value of the 1QP 9/2$^+$ state (g_p=1.37).

observable	spin	cal. 1	cal. 2	exp
	$5/2^+ \to 9/2^+$	1.1	1.6	2.8±0.2 [a]
	$7/2^+ \to 9/2^+$	2.4	2.3	2.25±0.16 [a]
$B(E2)$	$9/2^{+\prime} \to 9/2^+$	0.4	0.1	0.219±0.026 [a]
	$11/2^+ \to 9/2^+$	0.7	0.6	1.06±0.09 [a]
	$13/2^+ \to 9/2^+$	1.1	1.1	1.76±0.12 [a]
	$5/2^+$	2.16	2.11	
	$7/2^+$	1.47	1.45	
g	$9/2'^+$	1.17	1.18	
	$11/2^+$	1.01	1.01	
	$13/2^+$	0.95	0.95	
$B(M1)$	$7/2^+ \to 9/2^+$	0.0	1.93×10^{-1}	$\sim1.6\times10^{-1}$ [b]
S	$5/2^+$	0.0	0.10	
	$7/2^+$	0.0	0.03	

a) Ref. 51), b) Ref. 50).

Table X(b). Electromagnetic properties of the dressed 3QP states with positive parity in ^{99}Tc. Notations and parameters are the same as in Table V. The value of g_p is taken from the experimental value of the 1QP 9/2$^+$ state (g_p=1.26).

observable	spin	cal. 1	cal. 2	exp
	$5/2^+ \to 9/2^+$	3.2	4.0	4.5±0.5 [a]
	$7/2^+ \to 9/2^+$	11.4	11.2	13.5±1.5 [a]
$B(E2)$	$9/2'^+ \to 9/2^+$	1.0	0.9	
	$11/2^+ \to 9/2^+$	2.1	2.1	
	$13/2^+ \to 9/2^+$	3.5	3.5	
	$5/2^+$	1.67	1.65	1.44±0.12 [b]
	$7/2^+$	1.28	1.27	0.75±0.26 [b]
g	$9/2'^+$	1.10	1.10	
	$11/2^+$	1.05	1.05	
	$13/2^+$	1.05	1.05	
$B(M1)$	$7/2^+ \to 9/2^+$	0.0	4.4×10^{-2}	$(7.6\pm0.9)\times10^{-2}$ [c]
S	$5/2^+$	0.0	0.10	
	$7/2^+$	0.0	0.04	

a) Ref. 54), b) Ref. 59), c) Ref. 55).

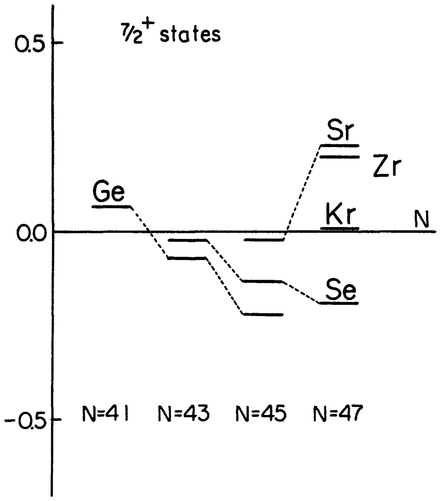


Fig. 18.

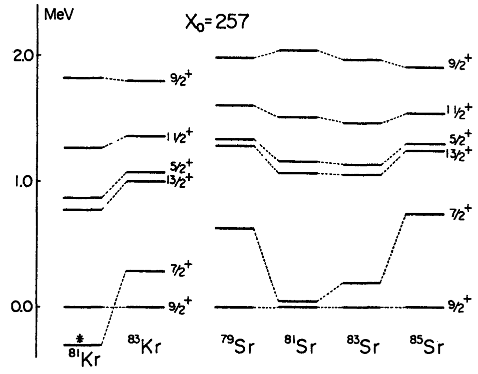


Fig. 19.

Fig. 18. Experimental trend of energy levels of the $7/2^+$ states in the $g^+_{9/2}$-odd-neutron region. The level energies are those measured from the 1QP $9/2^+$ states.

^{73}Ge; Ref. 60), ^{75,77}Ge; Ref. 79), ^{77,79}Se; Ref. 61), ^{83}Sr; Ref. 63), ^{85}Sr; Ref. 62), ^{87}Zr; Ref. 64).

Fig. 19. Calculated excitation-energy systematics of the dressed 3QP states in the $g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 11.

clearly has a close relationship with the fact that the neighbouring even-even nuclei possess a strong tendency of displaying the quasi-rotational spectra. In the case where the neutron number is in the vicinity of $N=40$ (the beginning of the $g^+_{9/2}$-orbit), in contrast to the cases of isotones with $N=46$ and 48, the neighbouring even-even nuclei seem to become increasingly unstable toward quadrupole deformation as the proton number increases toward $Z=38$ or 40.[69] Therefore, it is interesting to observe experimentarily, whether the correspondence between the behaviour of the excitation energies of the $7/2^+$ states and that of the 2^+ states also holds in this case. Another marked phenomenon in this region, which is possibly in an intimate relation to the formation of the (static) quadrupole deformed field, is the appearance of the $5/2^+$ states with decreasing energy toward the nuclei with $N=41$ (which are shown in Fig. 23 and briefly discussed below).

5–5 *On AC states with spin $I=(j-2)$*

So far we have restricted our discussions on the AC states with spin $I=(j-1)$. Now let us briefly discuss the AC states with $I=(j-2)$.

In the experiments, special lowering of the anomalous spin $(j-2)$ states has been observed in some cases among the nuclei displaying the low-lying $(j-1)$ states. We can cite as examples the $5/2^+$ states in Tc isotopes ($Z=43$) with $N\gtrsim56$ and the $5/2^+$ states in odd-neutron nuclei in the vicinity of $N=41$.

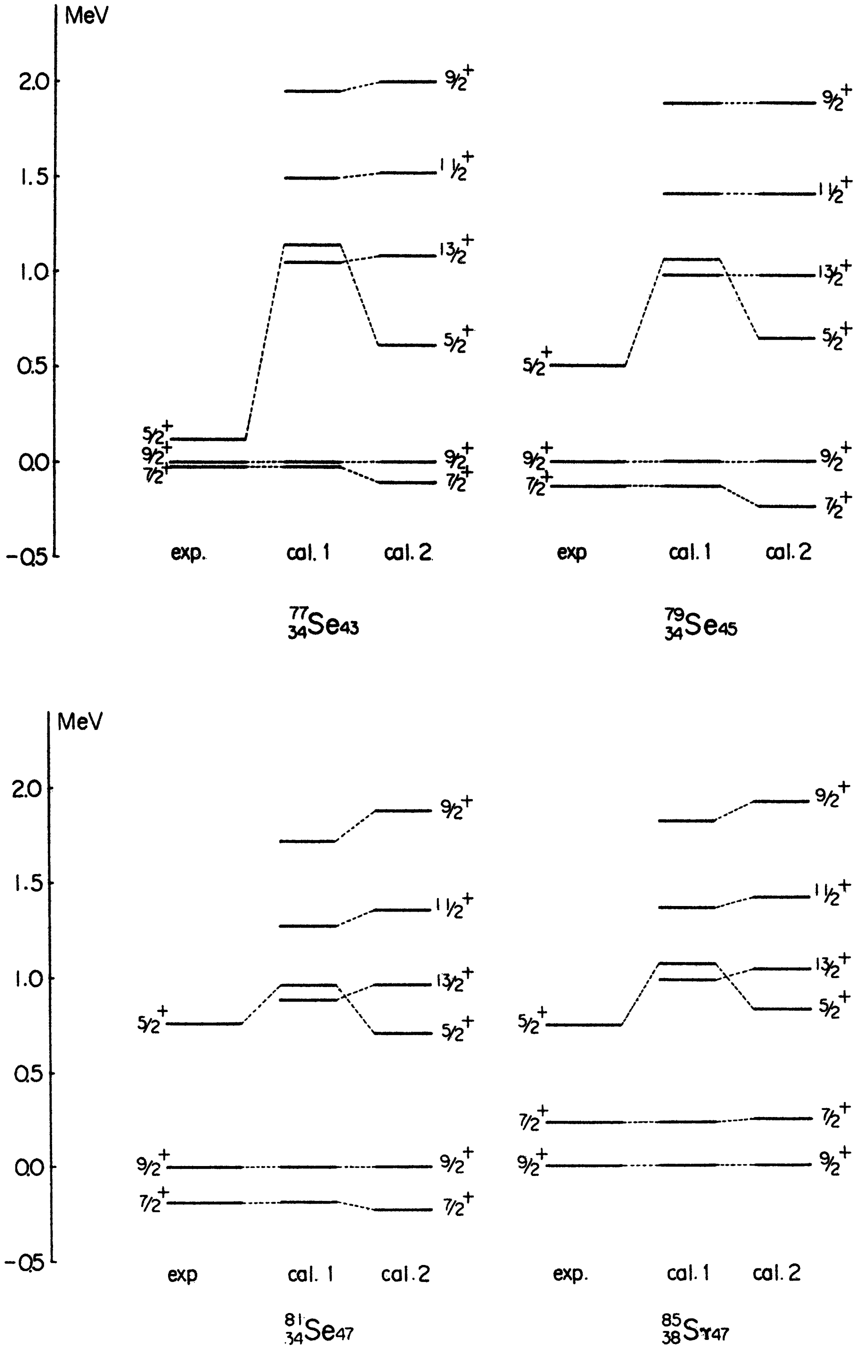


Fig. 20. Comparison between the experimental energy levels and the theoretical calculations in the $g_{9/2}^{+}$-odd-neutron region. Notations are the same as in Fig. 12. For the experimental data, refer to the caption of Fig. 18.

From the theoretical point of view based on the P+QQ force model, it is clear from the characteristic of the 3QP-correlation factor C_I defined by (3·22) that, in contrast to the case of $I=(j-1)$, the excitation energy of the dressed 3QP mode having $I=(j-2)$ cannot be lowered by the action of the 3QP correlation. Therefore, as long as we stand on the P+QQ force model, the observed $(j-2)$ states cannot be regarded as the appearances of the dressed 3QP modes with $I=(j-2)$ in their pure forms. Hence, from our point of view, the energy-lowering of the $(j-2)$ states should be attributed to the effects of couplings among the modes with different transferred seniority quantum numbers.

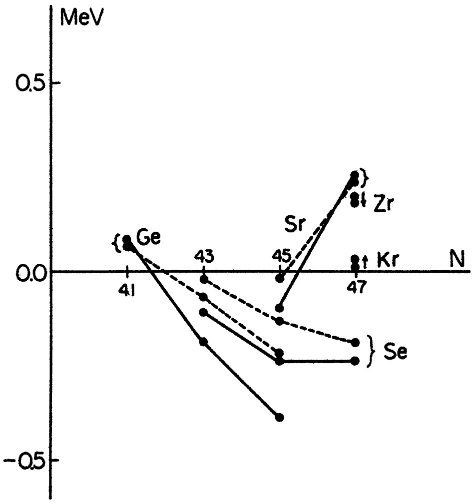


Fig. 21. Energy shifts due to the coupling effects of the dressed 3QP 7/2+ modes with the 1QP $g^+_{7/2}$ modes in the $g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 13.

Table XI. The correlation amplitudes of the dressed 3QP 7/2+ mode in ^{85}Sr. The adopted value of χ_0 is 295 (MeV) and the calculated excitation energy, $\omega'=\omega-E_p$, is 0.23 MeV. Notations are the same as in Table I. The unique-parity orbit p denotes (neutron; $1g^+_{9/2}$).

neutron

bc	$(g_{9/2})^2$	$(f_{7/2})^2$	$(f_{5/2})^2$	$(p_{3/2})^2$	$f_{7/2}f_{5/2}$	$f_{7/2}p_{3/2}$	$f_{5/2}p_{3/2}$	$f_{5/2}p_{1/2}$	$p_{3/2}p_{1/2}$
$\psi(bc;p)$	1.05	0.01	0.02	0.02	0.01	0.02	−0.01	0.04	0.05
$\varphi(bc;p)$	0.48	0.01	0.02	0 02	0.01	0.01	−0.01	0.04	0.04

proton

bc	$(g_{9/2})^2$	$(f_{7/2})^2$	$(f_{5/2})^2$	$(p_{3/2})^2$	$f_{7/2}f_{5/2}$	$f_{7/2}p_{3/2}$	$f_{5/2}p_{3/2}$	$f_{5/2}p_{1/2}$	$p_{3/2}p_{1/2}$
$\psi(bc;p)$	0.24	0.02	0.20	0.18	0.02	0.07	−0.09	0.29	0.31
$\varphi(bc;p)$	0.21	0.02	0.17	0.16	0.02	0.06	−0.08	0.25	0.26

Table XII. $B(E2; 7/2^+ \rightarrow 9/2^+)$ values in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table II.

nucleus	ω'_{j-1}	$B(E2)^{1)}$	$B(E2)^{2)}$	$B(E2)^{exp}$
^{73}Ge	0.07	19.8	17.4	9.1±0.9 [a]
^{75}Ge	−0.07	19.0	18.3	
^{77}Ge	−0.22	31.6	30.3	
^{77}Se	−0.02	18.6	18.0	
^{79}Se	−0.13	23.1	22.5	
^{81}Se	−0.19	37.1	34.3	
^{83}Kr	0.01	13.5	12.8	{5.8±1.3 [b] {2.6±1.5 [c]
^{83}Sr	−0.02	11.1	10.9	
^{85}Sr	0.23	6.0	5.9	
^{87}Zr	0.20	5.2	5.1	

a) Ref. 65), b) Ref. 66), c) Ref. 68).

Table XIII. Gyromagnetic ratio for the $7/2^+$ states in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table III.

nucleus	$g^{1)}_{j-1}$	$g^{2)}_{j-1}$	g^{exp}_{j-1}	g^{exp}_{j}
^{73}Ge	−0.25	−0.22		−0.20 [c]
^{75}Ge	−0.22	−0.20		
^{77}Ge	−0.17	−0.15		
^{77}Se	−0.23	−0.21		
^{79}Se	−0.21	−0.19		
^{81}Se	−0.16	−0.14		
^{83}Kr	−0.24	−0.23	{−0.268±0.001 [a] {−0.271±0.016 [b]	−0.215 [a]
^{83}Sr	−0.23	−0.22		
^{85}Sr	−0.25	−0.24		
^{87}Zr	−0.24	−0.24		

a) Ref. 67), b) Ref. 68), c) Ref. 59).

Table XIV. $B(M1; 7/2^+ \rightarrow 9/2^+)$ values in the $g^+_{9/2}$-odd-neutron region. Notations and parameters are the same as in Table IV.

nucleus	ζ_j	ζ_{j-1}	M_{11}	M_3	$B(M1)^{cal}$	$B(M1)^{exp}$
^{73}Ge	−0.30	−0.22	−0.84	−1.64	1.8×10^{-1}	
^{75}Ge	−0.11	−0.20	−0.70	−0.51	4.3×10^{-2}	
^{77}Ge	0.04	−0.22	−0.66	0.22	5.8×10^{-3}	
^{77}Se	−0.11	−0.19	−0.65	−0.51	4.0×10^{-2}	
^{79}Se	0.04	−0.19	−0.55	0.19	4.0×10^{-3}	
^{81}Se	0.21	−0.20	−0.46	1.20	1.6×10^{-2}	
^{83}Kr	0.19	−0.14	−0.32	0.71	4.5×10^{-3}	$(2.04\pm0.5)\times10^{-2}$ [a]
^{83}Sr	0.03	−0.15	−0.46	0.08	4.2×10^{-3}	
^{85}Sr	0.15	−0.12	−0.27	0.37	2.8×10^{-4}	
^{87}Zr	0.10	−0.12	−0.27	0.36	2.2×10^{-4}	

a) Ref. 68).

Now it is noteworthy that the excitation-energy systematics of the $(j-2)$ states and of the $(j-1)$ states are distinctly different from each other. In the examples of $1g^{+}_{9/2}$-odd-neutron nuclei, the energy of the $5/2^{+}$ state decreases as the neutron number approaches to $N=41$ (the beginning of the $g^{+}_{9/2}$-orbit), while the energy of the $7/2^{+}$ state decreases as the neutrons fill the $g^{+}_{9/2}$-orbit

Table XV. Electromagnetic properties of the dressed 3QP states with positive parity in ^{83}Kr. Notations and parameters are the same as in Table V. The value of g_p is taken from the experimental value of the 1QP $9/2^{+}$ state ($g_p=-0.22$).

observable	spin	cal. 1	cal. 2	exp
$B(E2)$	$5/2^{+}\to 9/2^{+}$	3.7	3.2	
	$7/2^{+}\to 9/2^{+}$	13.5	12.8	$\{5.8\pm1.3$ [a] ; 2.6 ± 1.5 [b]
	$9/2'^{+}\to 9/2^{+}$	2.6	2.1	
	$11/2^{+}\to 9/2^{+}$	3.0	2.9	
	$13/2^{+}\to 9/2^{+}$	3.9	3.7	
g	$5/2^{+}$	−0.46	−0.44	
	$7/2^{+}$	−0.22	−0.22	-0.268 ± 0.001 [c]
	$9/2^{+}$	−0.14	−0.14	
	$11/2^{+}$	−0.10	−0.10	
	$13/2^{+}$	−0.09	−0.09	
$B(M1)$	$7/2^{+}\to 9/2^{+}$	0.0	4.5×10^{-3}	$(2.04\pm0.5)\times10^{-2}$ [b]
S	$5/2^{+}$	0.0	0.20	
	$7/2^{+}$	0.0	0.02	

a) Ref. 66), b) Ref. 68), c) Ref. 67).

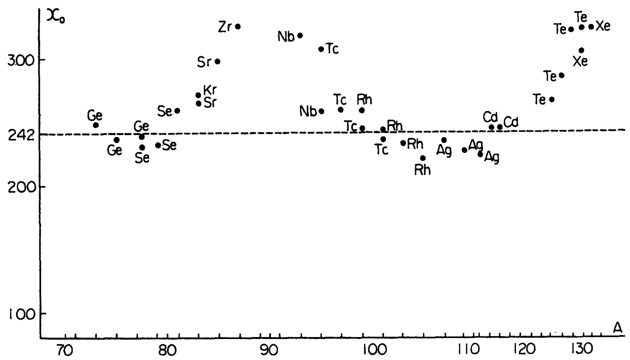


Fig. 22. Values of parameter χ_0 chosen to bring the energies of the AC state with spin $(j-1)$ into agreement with the experimental data. The parameter χ_0 is related to the quadrupole-force strength χ through $\chi=\chi_0 b^{-4}A^{-5/3}$, where b^2 is the harmonic-oscillator-range parameter and is taken to be $1.0A^{1/3}$. The broken line shows the value expected by the classical arguments.

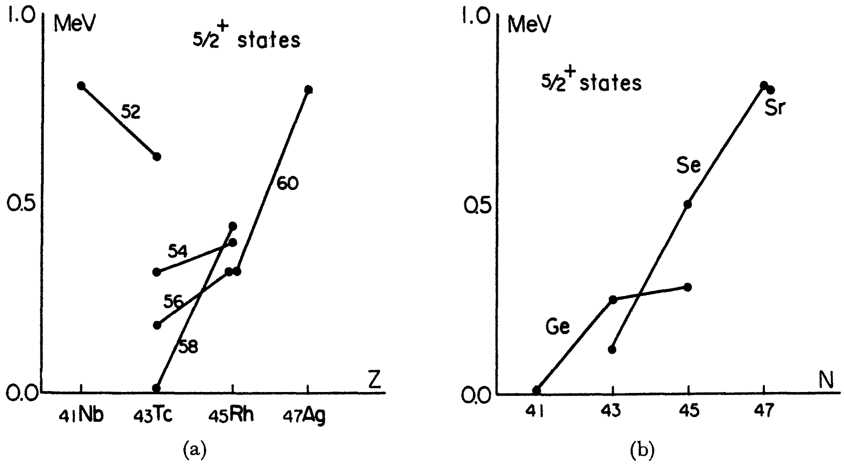


Fig. 23(a). Experimental trend of energy levels of the $5/2^+$ states in the region of $g^+_{9/2}$-odd-proton nuclei. The level energies are those measured from the 1QP $9/2^+$ states. For the experimental data, refer to the caption of Fig. 14.

Fig. 23(b). Experimental trend of energy levels of the $5/2^+$ states in the region of $g^+_{9/2}$-odd-neutron nuclei. The level energies are those measured from the 1QP $9/2^+$ states. For the experimental data, refer to the caption of Fig. 18.

toward its middle. (Compare Fig. 18 with Fig. 23.) The nuclei having extremely low-lying $(j-2)$ states seem to possess such a common feature that their Fermi surfaces (the chemical potentials) lie below the unique-parity level p and, at the same time, the nuclei have relatively large enhancement factors of the "core," $\xi(bc)$. The former situation is just the one in which the effect of the H_Y-interaction becomes strong, because of its well-known dependence on the $\eta(ab)$ factor. (See Eqs. (3·7c) and (3·9).) The latter corresponds to the situation which is responsible for the lowering of the 2^+ phonon states in the adjacent even-even nuclei. They are merely the conditions in which the effect of the coupling Hamiltonian $\boldsymbol{H}^{(\mathrm{int})}$ can become strong. Thus the special situation for the appearance of the extremely low-lying $(j-2)$ states seems to just correspond to the situations in which we can expect relatively strong couplings among the modes having different transferred seniority quantum numbers.

In Fig. 24 is shown the effect of coupling of the 1QP $2d^+_{5/2}$ mode (which lies in the next upper major shell) on the dressed 3QP mode with $5/2^+$. The calculated energy shift due to this type of coupling is rather large; in particular, remarkable energy-lowering of the $(j-2)$ states has been obtained in the numerical calculations for the nuclei in the vicinity of $N=41$. The magnitude of the energy shift is still insufficient to yield full explanation of the extreme lowering of $(j-2)$ states as observed in the experiments. This is a matter of course since we have neglected the other types of coupling, for example, that of

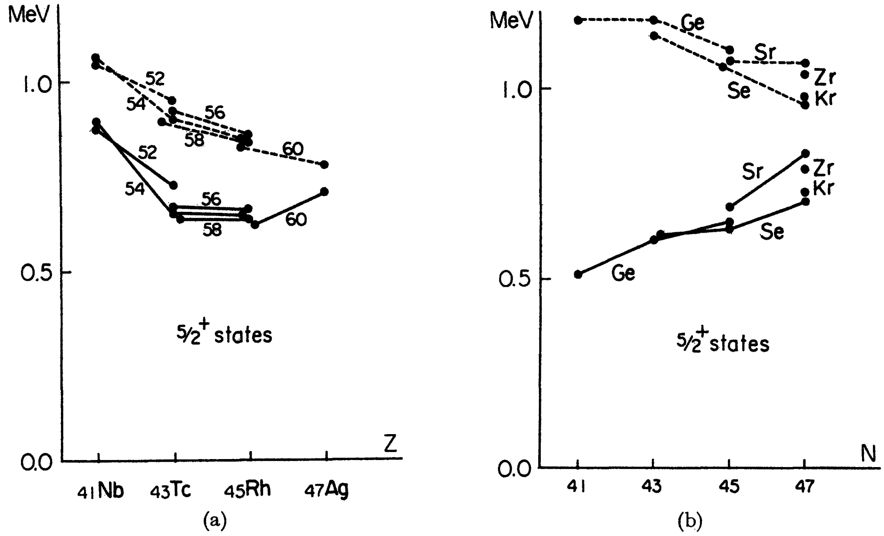


Fig. 24(a). Energy shifts due to the coupling effects of the dressed 3QP $5/2^+$ modes with the 1QP $d^+_{5/2}$ modes in the $g^+_{9/2}$-odd-proton region. Notations are the same as in Fig. 13.

Fig. 24(b). Energy shifts due to the coupling effects of the dressed 3QP $5/2^+$ modes with the 1QP $d^+_{5/2}$ modes in the $g^+_{9/2}$-odd-neutron region. Notations are the same as in Fig. 13.

the dressed 5QP mode. Nevertheless, it is interesting to note that the energy shifts considered here bring the theoretical trend in the excitation-energy systematics of the $5/2^+$ states toward agreement with the experimental one (in which the energy of the $5/2^+$ state decreases toward the nuclei with $N=41$). Furthermore, the following data seem to be consistent with the theoretical prediction on the $(j-2)$ states mentioned above: In low-energy excitations through the (d, p) reactions on nuclei in Ge-Se region, several states with anomalous spin $5/2^+$ have been observed in each nucleus, with spectroscopic factors being fragmented over these states.[70),71)]

We can easily find one of the reasons for the difference between the coupling effects for the $(j-1)$ states and for the $(j-2)$ states as follows: Let us note the matrix element $(p'\|r^2Y_2\|p)$ comprized in the effective coupling strength $\chi_{\rm int}(p', I)$. In the case of $I=(j_p-1)$, i.e., $j_{p'}=j_p-1$, the matrix element should be of spin-flip type, while in the case of $I=(j_p-2)$, i.e., $j_{p'}=j_p-2$, it is of spin-non-flip type. Since the latter is considerably larger than the former, the effective coupling strength $\chi_{\rm int}(p', I)$ for the mode with $I=(j-2)$ is larger than that for the mode with $I=(j-1)$.

In the semi-phenomenological models in which the stable quadrupole deformation is assumed, it has been known that the $(j-2)$ states as well as the $(j-1)$ states can be lowered in energy if the Coriolis interaction is suitably taken into account.[72)~74)] However, it seems difficult, to reproduce correctly the (above mentioned) different trends in the systematics between the $5/2^+$ and $7/2^+$ states (as a function of N) without any ambiguity in fixing the sign of deformation parameter β_0.[72] Nevertheless, since almost all nuclei under consideration are regarded as lying just before the critical point of phase transition (from spherical to deformed), it may be very interesting to investigate a possible "unknown effect" which presumably corresponds to the Coriolis force and persists through the transition region. Thus the characteristic difference between the nature of the $(j-1)$ and $(j-2)$ states must be of great significance in further clarifying the mechanism of growth of the quadrupole instability. In this connection, the following experimental fact may be noteworthy: The extremely low-lying $(j-2)$ states appear in odd-mass nuclei, with N or Z being about 40 or 42, whose even-even neighbours exhibit the striking dip in energy of the first excited 0^+ states.[75)~77)]

§6. Concluding remarks

Starting from the new type of quasi-particle-phonon coupling (producing the 3QP correlation), we have investigated the mechanism of forming a new type of collective excitation mode (the dressed 3QP mode) in the special condition of shell structure for the appearance of the AC states with spin $I=(j-1)$. As was emphasized by Bohr and Mottelson, the new type of quasi-particle-phonon coupling originates from the composite nature of the phonon mode and plays an increasingly important role as the phonon energy decreases. Therefore, in the situation of odd-mass nuclei in which the new type of coupling is highly developed, the phonon mode can no longer be regarded as an elementary mode. Namely, the phonon mode is strongly coupled with the 3QP "intruder" state to form the dressed 3QP mode as a new kind of elementary excitation mode in spherical odd-mass nuclei. We can then point out the existence of an intimate relationship between the process of extreme energy-lowering of the dressed 3QP mode with spin $(j-1)$ and that of developing the quasi-rotational spectra in the neighbouring even-even nuclei, in connection with the growth of instability of the spherical BCS vacuum toward quadrupole deformation.

The microscopic model of the AC states with spin $(j-1)$ as the dressed 3QP modes has been shown to yield a consistent explanation of their various characteristics; i.e., the excitation-energy systematics, the $E2$- and $M1$-transition properties, the g factor, the S factor in (d, p) reaction, etc. On the basis of the P+QQ force model, the predictions of the theory have also been examined with the numerical calculations. Thus we have arrived at the conclusion that the AC states with spin $(j-1)$ are nothing but the typical phenomena in which the dressed 3QP modes manifest themselves in their relatively pure and simple forms. In the succeeding chapters, the implication of this conclusion is investigated under general situations of shell structure, by putting special attention to the physical condition of shell structure for the striking enhancement of the 3QP correlation.

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Chapter 4. Persistency of AC State-Like Structure in Collective Excitations

—*Odd-Mass Mo, Ru, I, Cs and La Isotopes*—

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§ 1. Introduction

From among the complicated spectra of the low-lying excitations in odd-mass nuclei with mass numbers around $A\approx 100$, recent experiments reveal noticeable collective behaviour of the first $3/2^+$ states in odd-neutron nuclei and that of the second $5/2^+$ states in odd-proton nuclei, which are difficult to understand within the framework of the conventional quasi-particle-phonon-coupling (QPC) theory of Kisslinger and Sorensen:[1] In odd-neutron Mo and Ru isotopes with $N=53$, 55 and 57, there systematically appear collective $3/2^+$ states with the enhanced $E2$- and hindered $M1$-transitions to the single-quasi-particle (1QP) $5/2^+$ states. (See Fig. 1.) In odd-proton I, Cs and La isotopes, the second $5/2^+$ states display the enhanced $E2$- and retarded $M1$-transitions to the 1QP $7/2^+$ states, characteristically indicating their strong collective nature. The excitation energies of the second $5/2^+$ states measured from the 1QP $7/2^+$ states decrease as the neutron number goes from the magic number $N=82$ to $N=72$. (See Fig. 2.) Furthermore, the first and the second $5/2^+$ states lie close to each other, proposing the interesting problem of clarifying the difference of their microscopic structures.

The main purpose of this chapter*) is to propose an interpretation which identifies the first $3/2^+$ states (in odd-neutron Mo and Ru isotopes) and the second $5/2^+$ states (in odd-proton I, Cs and La isotopes) as evidences for the fact that the appearance of the dressed 3QP mode is not specific to the anomalous coupling (AC) states but more general in odd-mass nuclei. The first motive for this identification is directly obtained when we notice a similarity between the above-mentioned electromagnetic properties of the $3/2^+$

*) A preliminary version of this chapter was published by the present authors in Prog. Theor. Phys. **53** (1975), 489.

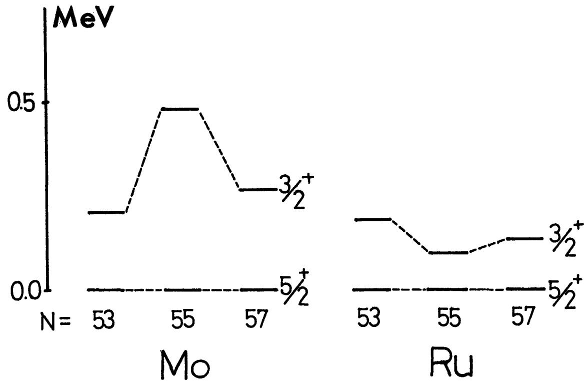


Fig. 1. Experimental trend of the excitation energies of the $3/2_1^+$ states in odd-mass Mo and Ru isotopes.
^{95}Mo; Ref. 2), ^{97}Mo; Ref. 2), ^{99}Mo; Ref. 5), ^{97}Ru; Ref. 6), ^{99}Ru; Ref. 12), ^{101}Ru; Ref. 12).

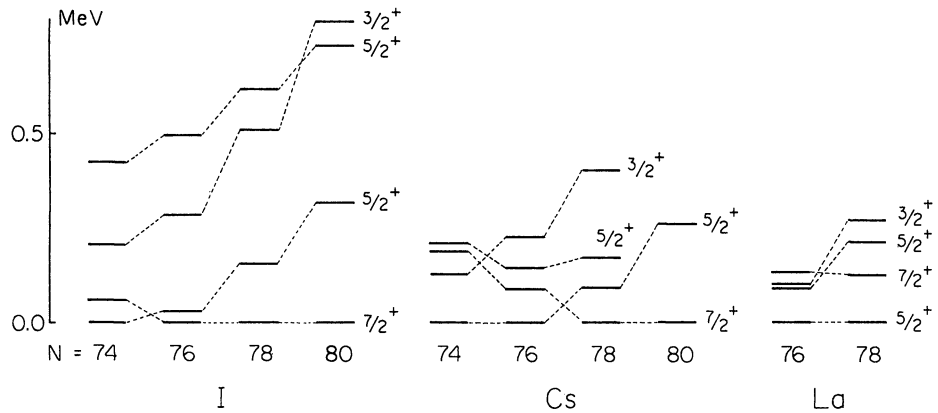


Fig. 2. Experimental trends of the excitation energies of the $5/2_2^+$ states and of the $3/2_1^+$ states in odd-mass I, Cs and La isotopes.
^{127}I; Ref. 13), ^{129}I; Ref. 19), ^{131}I; Ref. 14), ^{133}I; Ref. 15), ^{129}Cs; Ref. 22), ^{131}Cs; Ref. 23), ^{133}Cs; Ref. 27), ^{133} La; Ref. 24), ^{135}La; Ref. 28).

and $5/2^+$ states and those of the AC states with spin $I=(j-1)$. As was mentioned in the preceding chapter, the main characteristics of the electromagnetic properties of the AC states are 1) strikingly enhanced $E2$ transitions to the 1QP states with spin j, which are comparable in magnitude with those of 2^+-phonon transitions in neighboring even-even nuclei and 2) hindered corresponding $M1$ transitions. Of course, there is an important difference in shell structure between the collective $3/2^+$ and $5/2^+$ states under consideration and the AC states: In the case of the AC state the special situation of shell structure is the existence of a high-spin, unique-parity orbit which is being filled with several nucleons, while in the case of the collective $3/2^+$ and $5/2^+$ states many shell orbits with the same even parity are lying close to one another.

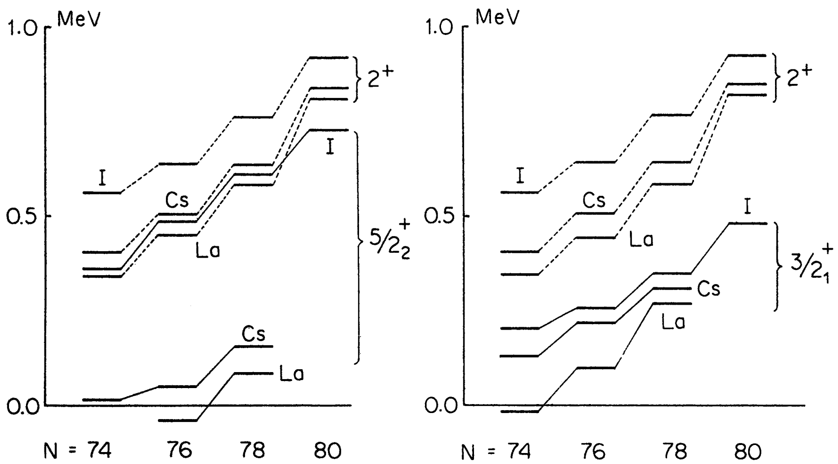


Fig. 3. Comparison between the experimental excitation-energy systematics of the 2^+ phonon states in even-even nuclei and of the $5/2_2^+$ and $3/2_1^+$ states measured from the 1QP $7/2_1^+$ and $5/2_1^+$ states, respectively. The excitation energies of the 2^+ phonons are those averaged over the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(Z,N)=1/2\{\omega_{2^+}(Z-1,N)+\omega_{2^+}(Z+1,N)\}$.

Since these orbits with the same even parity are expected to actively participate in the 3QP correlation, it is quite interesting to investigate to what extent the similarity between the AC states and the collective $3/2^+$ and $5/2^+$ states can persist. In this chapter, therefore, the mechanism of the formation of AC state-like structure in the collective excitations of Mo, Ru, I, Cs and La isotopes will be clarified, by paying special attention to its relation with the conditions of shell structure.

In § 2, the formulation of the dressed 3QP mode as the new type of collective mode in these nuclei is presented by the use of the conventional pairing-plus-quadrupole (P+QQ) force model. In § 3 a criterion to investigate the similarity and difference between the 3QP correlations characterizing these states and those characterizing the AC states is given. With the aid of this criterion, the microscopic structure of the collective $3/2^+$ and $5/2^+$ states is discussed on the basis of calculated results. Here, the change of microscopic structure of these states depending on the mass number is also investigated in relation to the shell structure. In § 4, the coupling effects between the 1QP modes and the dressed 3QP modes are examined. In contrast to the case of the AC states, the dressed 3QP mode investigated in this chapter lies close, in energy, to the 1QP mode with the same spin and parity. For instance, the first $5/2^+$ states and the collective second $5/2^+$ states in I, Cs and La isotopes lie especially close to each other. At first sight, therefore, these two $5/2^+$ states seem to couple strongly with each other. However, it will be clarified that there exists an interesting mechanism to make the coupling effects weak. Taking into account the coupling effects, an analysis of the various properties is made for the second $5/2^+$ states of I, Cs and La isotopes in § 5-a) and for the first $3/2^+$ states of Mo and Ru isotopes in § 5-b). The concluding remark is given in § 6.

§ 2. Preliminaries

In order to explicitly specify the freedom of protons and neutrons, we use Greek letters (α, β, γ) and (ρ, σ): For the odd-proton (neutron) nuclei, α, β and γ are respectively used to denote a set of quantum numbers of the single-particle states for protons (neutrons) and ρ and σ are respectively used for neutrons (protons). Then, according to the general theory developed in Chap. 2, the creation operator of the dressed 3QP mode in the P+QQ force model is given in terms of the quasi-particle operators as follows:

$$
\begin{aligned}
Y^\dagger_{nIK} = & \frac{1}{\sqrt{3!}} \sum_{\alpha\beta\gamma} \psi_{nI}(\alpha\beta\gamma) \boldsymbol{P}(\alpha\beta\gamma) a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma \\
& + \sum_{(\rho\sigma)\gamma} \{1+\delta_{rs}\}^{-1/2} \psi_{nI}(\rho\sigma;\gamma) \boldsymbol{P}(\rho\sigma) a^\dagger_\rho a^\dagger_\sigma a^\dagger_\gamma \\
& + \frac{1}{\sqrt{3!}} \sum_{\alpha_1\alpha_2\alpha_3} \varphi^{(1)}_{nI}(\alpha_1\alpha_2\alpha_3) \boldsymbol{P}(\alpha_1\alpha_2\alpha_3) T_{3/2,-1/2}(\alpha_1\alpha_2\alpha_3) \\
& + \frac{1}{\sqrt{2}} \sum_{\substack{\alpha_1\alpha_2\gamma \\ (a\neq c)}} \varphi^{(2)}_{nI}(\alpha_1\alpha_2;\gamma) \boldsymbol{P}(\alpha_1\alpha_2) T_{10}(\alpha_1\alpha_2) a_{\tilde{\gamma}} \\
& + \sum_{\substack{(\alpha\beta)\gamma \\ (a\neq c,\, b\neq c)}} \{1+\delta_{ab}\}^{-1/2} \varphi^{(3)}_{nI}(\alpha\beta;\gamma) \boldsymbol{P}(\alpha\beta) a^\dagger_\gamma a_{\tilde{\alpha}} a_{\tilde{\beta}} \\
& + \sum_{(\rho\sigma)\gamma} \{1+\delta_{rs}\}^{-1/2} \varphi^{(3)}_{nI}(\rho\sigma;\gamma) \boldsymbol{P}(\rho\sigma) a^\dagger_\gamma a_{\tilde{\rho}} a_{\tilde{\sigma}} .
\end{aligned}
\tag{2·1}
$$

Here, the subscript $i(=1,2,3)$ of α is used when the specification of the single-particle states with different magnetic quantum numbers in the same orbit a is necessary. The symbol $\sum_{(\alpha\beta)\gamma}$ represents the summation with respect to the orbital pair (ab), m_α, m_β and γ, and the operators $\boldsymbol{P}$'s denote the projection operators by which any angular-momentum-zero-coupled-pair component is removed from $(a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma)$ and $(a_{\tilde{\alpha}} a_{\tilde{\beta}})$. The projection operators $\boldsymbol{P}$, the explicit form of which are given in Appendix 2A, guarantee the dressed 3QP modes to be orthogonal to both the spurious states (due to the nucleon-number non-conservation) and the pairing vibrational modes.

The collective mode given by Eq. (2·1) is characterized by the amount of seniority $\Delta v=3$ which it transfers to the correlated ground state. The first two terms on the right-hand side of Eq. (2·1) represent the forward-going components and the others are the backward-going components which originate from the ground-state correlation. It is evident that the ground-state correlation is essential to bring about the collectivity of excitation modes in the doubly-open-proton-neutron system such as the nuclei under consideration. As was shown in § 3 of Chap. 2, within the framework of the new-Tamm-Dancoff approximation we obtain the eigenvalue equation which the correlation amplitudes should satisfy:

$$\begin{bmatrix} 3\boldsymbol{D} & -\boldsymbol{A} \\ \boldsymbol{A}^T & -\boldsymbol{d} \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}_{nI} \\ \boldsymbol{\varphi}_{nI} \end{bmatrix} = \omega_{nI} \begin{bmatrix} \boldsymbol{\psi}_{nI} \\ \boldsymbol{\varphi}_{nI} \end{bmatrix}, \tag{2·2}$$

where $\boldsymbol{\psi}_{nI}$ and $\boldsymbol{\varphi}_{nI}$ denote the matrix notations symbolizing the sets of the forward amplitudes $\{\psi_{nI}(\alpha\beta\gamma)$ and $\psi_{nI}(\rho\sigma;\gamma)\}$ and the sets of the backward amplitudes $\{\varphi^{(1)}_{nI}(\alpha_1\alpha_2\alpha_3)$, $\varphi^{(2)}_{nI}(\alpha_1\alpha_2;\gamma)$, $\varphi^{(3)}_{nI}(\alpha\beta;\gamma)$ and $\varphi^{(3)}_{nI}(\rho\sigma;\gamma)\}$, respectively. The explicit forms of matrices $3\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ (and its transpose $\boldsymbol{A}^T$) are given in Appendices 2B and 6A. When compared with the eigenvalue equation for the AC state given by (3·23) of the preceding chapter, the main complexity of Eq. (2·2) comes from the amplitudes of the type $\psi_{nI}(\alpha\beta\gamma)$: In the case of the AC states, because of the parity selection property of the quadrupole force the amplitudes $\psi_{nI}(\alpha\beta\gamma)$ are reduced to particularly simple forms, while in the case of the dressed 3QP mode under consideration (which has the same parity as that of the major shell) such a parity selection rule does not work for the main amplitudes. Of course the corresponding backward amplitudes also become relatively complicated. Therefore it is evident that we cannot expect, from the outset, the formation of such simple structure as the AC states, in the dressed 3QP mode given by (2·1).

As usual, the reduced $E2$-transition probability from the dressed 3QP state $|\Phi^{(3)}_{nIK}\rangle = Y^\dagger_{nIK}|\Phi_0\rangle$ with spin I to the 1QP state $|\Phi^{(1)}_{\delta}\rangle = a^\dagger_{\delta}|\Phi_0\rangle$ with spin j_d is defined by

$$B(E2; I\to j_d) = \frac{1}{2I+1}|\langle\Phi^{(1)}_d\|\hat{O}^{(+)}_2\|\Phi^{(3)}_{nI}\rangle|^2. \tag{2·3}$$

Following the general method developed in § 5 of Chap. 2, the reduced matrix element of the electric quadrupole operator $\hat{O}^{(+)}_2$ in Eq. (2·3) is given as follows:

$$\begin{aligned}
&\langle\Phi^{(1)}_d\|\hat{O}^{(+)}_2\|\Phi^{(3)}_{nI}\rangle \\
&= \sqrt{\frac{3}{2}} \sum_{\substack{abc \\ a'b'c'J'}} e_\tau Q(ab)\delta_{cd} P_I(ab(2)c|a'b'(J')c')\ \psi_{nI}[a'b'(J')c'] \\
&+ \sum_{(rs)c} e_\tau Q(rs)\delta_{cd}\{1+\delta_{rs}\}^{-1/2}\psi_{nI}[rs(2)c] \\
&+ \frac{1}{\sqrt{2}}\sum_{aJ} e_\tau Q(aa)\delta_{ad} P_I(aa(2)a|aa(J)a)\varphi^{(1)}_{nI}[aa(J)a] \\
&+ \sum_{\substack{ac,\,J=\text{even} \\ (a\neq c,\,J\neq 0)}} e_\tau Q(ca)\delta_{ad}\sqrt{5(2J+1)}\begin{Bmatrix} j_a & j_a & J \\ j_c & I & 2\end{Bmatrix}\varphi^{(2)}_{nI}[aa(J)c] \\
&+ \sum_{\substack{(ab)c \\ (a\neq c,\,b\neq c)}} e_\tau Q(ab)\delta_{cd}\{1+\delta_{ab}\}^{-1/2}\varphi^{(3)}_{nI}[ab(2)c] \\
&+ \sum_{(rs)c} e_\tau Q(rs)\delta_{cd}\{1+\delta_{rs}\}^{-1/2}\varphi^{(3)}_{nI}[rs(2)c],
\end{aligned} \tag{2·4}$$

where

$$Q(ab)=\frac{1}{\sqrt{5}}(a\|r^2 Y_2\|b)\cdot(u_a v_b+v_a u_b) \tag{2·5}$$

and P's are the projection operators defined in Appendix 2A. In Eq. (2·4), ψ and φ are the coupled-angular-momentum representation of the forward amplitudes and backward amplitudes, respectively, which are straightforwardly obtained through the conventional procedure of angular-momentum coupling and defined in Appendix 6A; for instance,

$$\psi_{nI}(\alpha\beta\gamma)=\sum_{JI}(j_a j_b m_\alpha m_\beta|JM)(Jj_c M m_\gamma|IK)\psi_{nI}[ab(J)c]. \tag{2·6}$$

Needless to say, the parts in (2·4) involving the backward amplitudes φ represent the effect of collective enhancement originating from the ground-state correlation. The characteristic property of the $E2$-transition matrix element given by (2·4) will be discussed in § 3 as a measure of clarifying the microscopic structure of the collective $5/2_2^+$ and $3/2_1^+$ states of interest.

A computer program named BARYON-1 was constructed to solve Eq. (2·2) and to calculate various electromagnetic properties of the dressed 3QP modes. Since our aim is not to obtain a detailed quantitative fitting with experimental data but to get an essential understanding of structures of collective $3/2_1^+$ and $5/2_2^+$ states which are difficult to understand within the framework of the conventional QPC theory, we have used exactly the same values for the single-particle energies and for the pairing-force strength G in the numerical calculations as those adopted in the calculations of Kisslinger and Sorensen.[1] We have also made the same truncation of shell-model space as Kisslinger and Sorensen have made: The shell-model subspace for I, Cs and La isotopes consists of the orbits

$$\{\pi;\, 1g_{7/2}^+,\, 2d_{5/2}^+,\, 1h_{11/2}^-,\, 2d_{3/2}^+,\, 3s_{1/2}^+\},$$
$$\{\nu;\, 2d_{5/2}^+,\, 1g_{7/2}^+,\, 3s_{1/2}^+,\, 1h_{11/2}^-,\, 2d_{3/2}^+\},$$

and the subspace for Mo and Ru isotopes is composed of the orbits

$$\{\pi;\, 1f_{5/2}^-,\, 2p_{3/2}^-,\, 2p_{1/2}^-,\, 1g_{9/2}^+\},$$
$$\{\nu;\, 2d_{5/2}^+,\, 1g_{7/2}^+,\, 3s_{1/2}^+,\, 1h_{11/2}^-,\, 2d_{3/2}^+\}.$$

The quadrupole-force strengths χ have been fixed at the values which reproduce the average energies of the 2^+ phonon states in the adjacent even-even nuclei, e.g., $\bar{\omega}_{2^+}(Z,N)\equiv 1/2\{\omega_{2^+}(Z-1,N)+\omega_{2^+}(Z+1,N)\}$ for odd-Z nuclei. (For the sake of comparison, the excitation spectra calculated with a constant value of $\chi_0\equiv\chi b^4 A^{5/3}$ for each isotopes are presented in Figs. 4(a) and 6(a).) Thus no adjustment of parameters has been attempted in the course of the calculations.

As is described in Appendix 4A, the method of solving Eq. (2·2) consists of the following steps. First, the components of the 3QP correlation amplitudes ψ and φ (in the coupled-angular-momentum representation) are orthonormalized by diagonalizing the projection operators $\boldsymbol{P}$'s entering into the eigenvalue equation (2·2). By this diagonalization, the submatrices $3\boldsymbol{D}$, $\boldsymbol{d}$ and $\boldsymbol{A}$ in Eq. (2·2) are transformed into new submatrices $3\bar{\boldsymbol{D}}$, $\bar{\boldsymbol{d}}$ and $\bar{\boldsymbol{A}}$ (the elements of which are defined between the orthonormalized components). The calculation of thus obtained eigenvalue equation (4A·5) may be further performed in two steps: In the first step, the submatrices $3\bar{\boldsymbol{D}}$ and $\bar{\boldsymbol{d}}$ are diagonalized respectively. Needless to say, the eigenvalues of $3\bar{\boldsymbol{D}}$ represent the excitation energies of "bare" 3QP states in the quasi-particle Tamm-Dancoff approximation. Then, in the second step, the resulting total matrix given by (4A·8) is diagonalized to obtain the excitation energies and correlation amplitudes of the dressed 3QP modes. In the calculation of this chapter, we have adopted the following approximation: In the second step, the diagonal matrices, $\boldsymbol{\omega}^f$ and $\boldsymbol{\omega}^b$ (which are obtained from $3\bar{\boldsymbol{D}}$ and $-\bar{\boldsymbol{d}}$ respectively in the first step) have been truncated within 10 and 40 dimensions, respectively, with the former being of increasing order and the latter being of decreasing order in energy. Accuracy of this approximation has been checked by comparing some results with the corresponding results of full calculations, and has been satisfactory except for the special case where the excitation energy of the dressed 3QP mode lies extremely close to the critical point for the instability of the spherical BCS vacuum.

§ 3. Microscopic structure of collective excitations in odd-mass Mo, Ru, I, Cs and La isotopes

3–1 *Collective* $5/2^+$ *states in I, Cs and La*

In Figs. 4(a) and 4(b) are shown the calculated results for the excitation energies of the collective $5/2_2^+$ states as the dressed 3QP modes. In these figures, the first $7/2^+$ and $5/2^+$ states correspond to the 1QP states related to the orbits $1g_{7/2}^+$ and $2d_{5/2}^+$, respectively. All the energies are measured from those of the 1QP $7/2^+$ states. The systematic appearance of the low-lying collective $5/2_2^+$ states in I, Cs and La isotopes is reproduced very well. The experimental trend that the excitation energies of the collective $5/2_2^+$ states (measured from those of the 1QP $7/2^+$ states) decrease as the neutron number goes away from the magic number is also well reproduced. This trend is seen in a rather magnified way in Fig. 4(a) in which a constant value of the quadrupole-force parameter χ_0 is used for each isotopes. Such magnification is the same as that well known in the conventional RPA with the P+QQ force in even-even nuclei, i.e., the 2^+ phonon energies calculated with the constant value of χ_0 decrease more rapidly than the experimental trend as the nucleon numbers go away from the magic number.[1)]

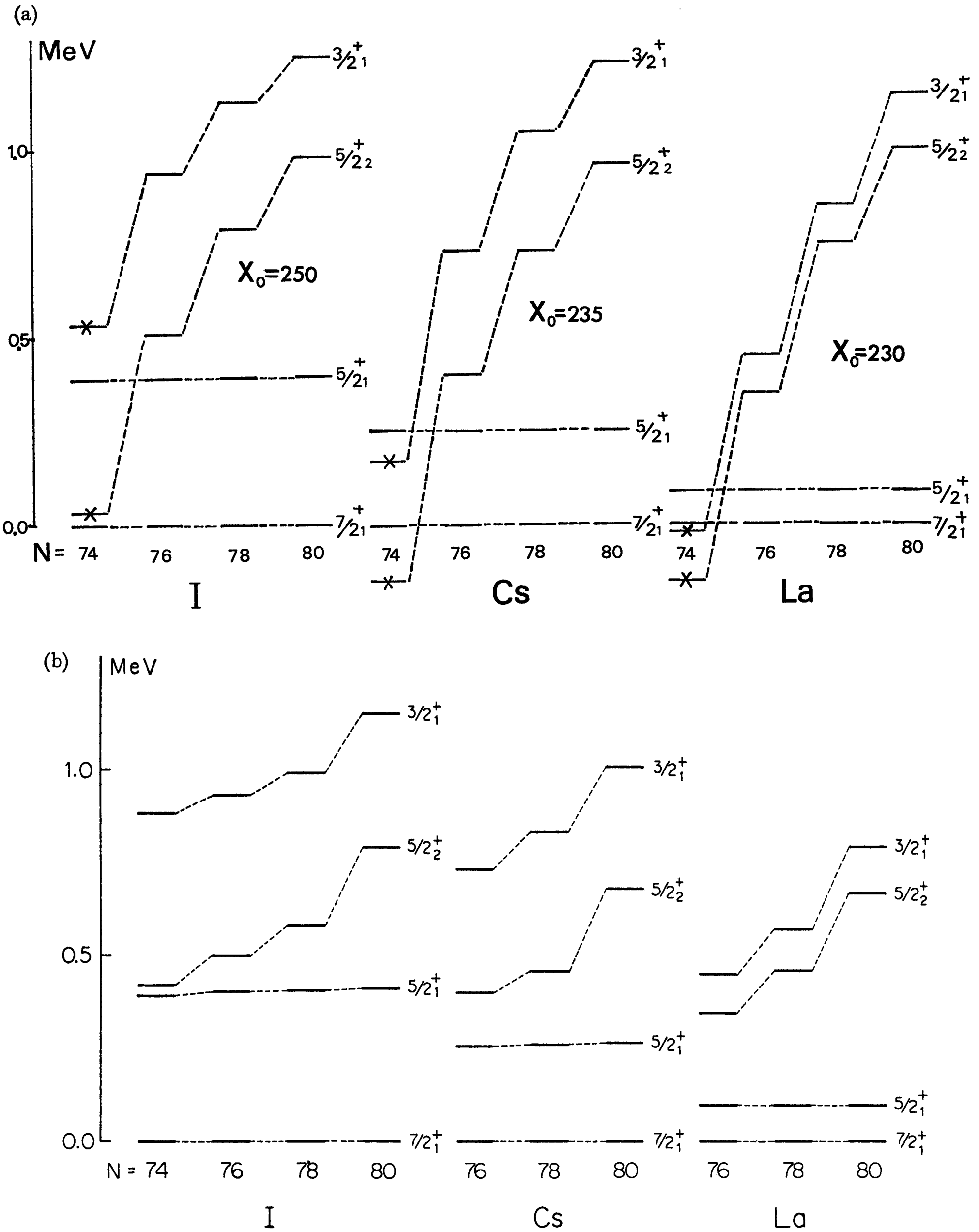


Fig. 4. Calculated excitation energies of the dressed 3QP states with $I^\pi=5/2^+$ and with $I^\pi=3/2^+$ in odd-mass I, Cs and La isotopes. They are measured from those of the lowest 1QP states. (a) The constant values of the quadrupole-force-strength parameter χ_0(defined by $\chi_0=\chi b^4 A^{5/3}$, b^2 being the harmonic-oscillator-range parameter) are used for each isotopes. (b) The values of χ_0 are chosen to reproduce the average energies of the 2^+ phonons in the adjacent even-even nuclei, i.e., $\bar{\omega}_{2^+}(Z,N)=1/2\{\omega_{2^+}(Z-1,N)+\omega_{2^+}(Z+1,N)\}$.

Before discussing the calculated $B(E2)$ values in Table I, it may be important to set up a criterion for investigating the similarity and difference between the 3QP correlations characterizing the new dressed 3QP modes and those characterizing the AC states. As was shown in the preceding chapter, in the case of the AC states, the triggering effect of the 3QP correlations which strongly violates the concept of phonon in odd-mass nuclei is restricted among quasi-particles in a specific unique-parity orbit, because of the parity-selection property of the quadrupole force. On the contrary, in the case of the dressed 3QP mode under consideration (which has the same parity as that of the major shell), many shell orbits having the same even parity (such as $g^+_{7/2}$, $d^+_{5/2}$, $d^+_{3/2}$, and $s^+_{1/2}$) lie close and are expected to be equally active for the 3QP correlations. A criterion useful for discussing the new 3QP correlations occuring under such a condition of shell structure is given as follows: If the 3QP correlations mainly come from a specific orbit, for instance, the $1g_{7/2}$ orbit when we consider the collective $5/2^+_2$ states, we may say that the structure of the $5/2^+_2$ states is similar to that of the AC states. As was shown in the preceding chapter, an important characteristic in this case is that the value of $B(E2;\ 5/2^+_2\to 7/2^+_1)$ belonging to the class of $B(E2;\ I=j-1\to j)$ is greatly enhanced compared to the ones of $E2$ transitions to the other 1QP states, for instance, $B(E2;\ 5/2^+_2\to 5/2^+_1)$ belonging to the class of $B(E2;\ I=j-1\to j'\neq j)$. In this case, a picture of the low-energy excitation structure may be given as illustrated in Fig. 5. On the other hand, if the 3QP correlations among quasi-particles in different orbits are of importance, the $E2$ transitions to the other 1QP states, for instance, the $B(E2;\ 5/2^+_2\to 5/2^+_1)$ must also be strongly enhanced. Therefore, in this case, we may say that the structure of $5/2^+_2$ states differs considerably from that of the AC states. Needless to say, the form of the eigenmode operator given by (2·1) allows us to treat these two cases on an equal footing, so that with the aid of this criterion we can achieve an essential understanding of the microscopic structure of the new collective excited states.

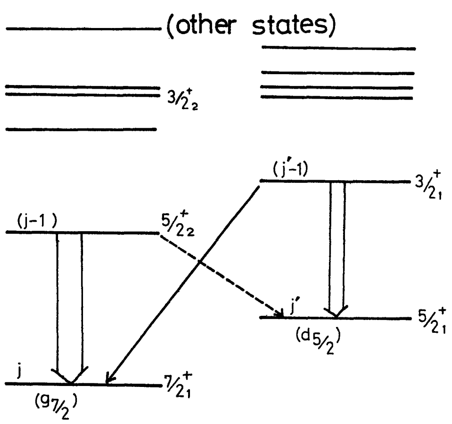


Fig. 5. Graphic explanation of the $E2$-transition properties.

Table I. Calculated $B(E2)$ values from the dressed 3QP $5/2_2^+$ states in odd-mass I, Cs and La isotopes (in unit of $e^2\cdot10^{-50}\,\mathrm{cm}^4$). The results of exact calculation are given in the second and fourth columns denoted "exact", while those calculated by adopting the ACS approximation are listed in the third and fifth columns denoted "ACS". The harmonic-oscillator-range parameter $b^2=1.0A^{1/3}\,\mathrm{fm}^2$ and the effective charges, $e_p^{\mathrm{eff}}=1.5e$ and $e_n^{\mathrm{eff}}=0.5e$, are used. The adopted values of χ_0 are the same as in Fig. 4(b) and are listed in the fifth column.

	$5/2_2^+\longrightarrow7/2_1^+$		$5/2_2^+\longrightarrow5/2_1^+$	χ_0
	exact	ACS	exact	
^{127}I	14.3	13.5	0.3	234.2
^{129}I	6.9	9.6	0.1	251.5
^{131}I	5.2	6.8	0.1	280.6
^{133}I	3.8	4.7	0.0	315.0
^{131}Cs	10.7	20.0	0.4	235.0
^{133}Cs	8.8	12.9	0.3	264.7
^{135}Cs	6.9	8.4	0.1	307.0
^{133}La	17.0	48.1	0.7	230.2
^{135}La	9.7	19.7	0.4	259.0
^{137}La	7.0	11.3	0.2	300.0

Now, the calculated $B(E2)$ values in Table I demonstrate that the $B(E2;\,5/2_2^+\rightarrow7/2_1^+)$'s are stronger by about one order in magnitude than the other $B(E2;\,5/2_2^+\rightarrow5/2_1^+)$'s. Thus we can conclude that the structure of the $5/2_2^+$ states is similar to that of the AC states with spin $I=j-1$. (In the present case, j corresponds to the orbit $1g_{7/2}$.) In fact, microscopic structure of the calculated amplitudes of the $5/2_2^+$ states (as the dressed 3QP modes) is very similar to that of the AC states which was investigated in Chap. 3: The forward-going amplitudes of $(\pi\pi\pi)$-type with the largest $\{(\pi g_{7/2})^3\}$ component and of $(\nu\nu\pi)$-type are strongly coupled with each other. The backward-going amplitudes of $(\nu\nu\pi)$-type become larger as the neutron number decreases. Some examples of the main amplitudes in I, Cs and La are shown in Table II. The amplitudes in this Table are those which are orthonormalized by diagonalizing the projection operators, $\boldsymbol{P}$'s, entering into the eigenvalue equation (2·2) in the coupled-angular-momentum representation. (See Appendices 4A and 6A.)

It is rather a wonder that the overall similarity between the $5/2_2^+$ states and the AC states persists in spite of their different situation in shell structures. To investigate the reason for this, let us look into the characteristics of the calculated amplitudes in more detail. In I isotopes, the chemical potential for protons lies close to the $1g_{7/2}$ orbit and the energy difference between the 1QP $1g_{7/2}$ and the 1QP $2d_{5/2}$ states is relatively large (i.e. $\Delta E\approx400\,\mathrm{keV}$), so that the component $\{(\pi g_{7/2})^3\}$ in the forward amplitudes $\psi_{nI=5/2}$ reaches the maximum. As the proton number increases, the chemical potential shifts

Table II. Main correlation amplitudes of the dressed 3QP $5/2_2^+$ modes in ^{127}I(a), ^{133}Cs (b) and ^{135}La(c). They are those orthonormalized according to the procedure described in Appendix 4A. The first and second rows specify the types and the components in the coupled-angular-momentum representation, respectively. Abbreviations such as

$$\{(d_{5/2})^2 g_{7/2}\} = \{d_{5/2} d_{5/2}\,(J=2) g_{7/2}\},$$
$$\{(d_{5/2})^2_4 g_{7/2}\} = \{d_{5/2} d_{5/2}\,(J=4) g_{7/2}\},$$

etc., are used. The values of the forward-going amplitudes are listed in the third row, while those of the backward-going amplitudes are listed in the fourth row. To distinguish the different kinds of backward-going amplitudes, we use the superscripts, (2) and (3), that specify the kinds of amplitudes in Eq. (2·1) when necessary.

(a) $5/2_2^+$ state in ^{127}I

$(\pi\pi\pi)$ type					$(\nu\nu\pi)$ type				
$(g_{7/2})^3$	$(d_{5/2})^2 g_{7/2}$	$(g_{7/2})^2 d_{5/2}$	$(g_{7/2})^2_4 d_{5/2}$	$(g_{7/2})^2 d_{3/2}$	$(h_{11/2})^2 g_{7/2}$	$(d_{3/2})^2 g_{7/2}$	$(s_{1/2} d_{3/2}) g_{7/2}$	$(d_{3/2} g_{7/2}) g_{7/2}$	$(h_{11/2})^2 d_{5/2}$
0.89	0.23	−0.14	0.09	−0.08	0.67	0.30	−0.09	0.19	−0.08
0.31	$\{0.01^{(2)}$ $\;\,0.17^{(3)}$	$\{-0.03^{(2)}$ $\;\,-0.07^{(3)}$	0.04	$\{-0.03^{(2)}$ $\;\,-0.01^{(3)}$	0.48	0.21	−0.14	0.10	0.08

(b) $5/2_2^+$ state in ^{133}Cs

$(\pi\pi\pi)$ type					$(\nu\nu\pi)$ type				
$(g_{7/2})^3$	$(d_{5/2})^2 g_{7/2}$	$(g_{7/2})^2 d_{5/2}$	$(g_{7/2})^2_4 d_{5/2}$	$(g_{7/2})^2 d_{3/2}$	$(h_{11/2})^2 g_{7/2}$	$(d_{3/2})^2 g_{7/2}$	$(s_{1/2} d_{3/2}) g_{7/2}$	$(d_{3/2} g_{7/2}) g_{7/2}$	$(h_{11/2})^2 d_{5/2}$
0.82	0.28	−0.17	0.11	−0.10	0.57	0.32	−0.06	0.11	−0.09
0.27	$\{0.01^{(2)}$ $\;\,0.18^{(3)}$	$\{-0.03^{(2)}$ $\;\,-0.07^{(3)}$	0.05	$\{-0.03^{(2)}$ $\;\,0.01^{(3)}$	0.36	0.19	−0.14	0.05	−0.07

(c) $5/2_2^+$ state in ^{135}La

$(\pi\pi\pi)$ type					$(\nu\nu\pi)$ type				
$(g_{7/2})^3$	$(d_{5/2})^2 g_{7/2}$	$(g_{7/2})^2 d_{5/2}$	$(g_{7/2})^2 d_{3/2}$	$(g_{7/2})^2_4 d_{3/2}$	$(h_{11/2})^2 g_{7/2}$	$(d_{3/2})^2 g_{7/2}$	$(s_{1/2} d_{3/2}) g_{7/2}$	$(d_{3/2} g_{7/2}) g_{7/2}$	$(h_{11/2})^2 d_{5/2}$
0.76	0.40	−0.20	−0.13	−0.13	0.59	0.33	−0.20	0.15	−0.12
0.26	$\{0.01^{(2)}$ $\;\,0.24^{(3)}$	$\{-0.04^{(2)}$ $\;\,-0.08^{(3)}$	$\{-0.04^{(2)}$ $\;\,0.11^{(3)}$	−0.04	0.38	0.20	−0.16	0.08	−0.09

up and the energy difference between the 1QP $1g_{7/2}$- and 1QP $2d_{5/2}$-states decreases till about $\Delta E \approx 100$keV in La isotopes. In La isotopes, therefore, we may expect that the components $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ and $\{(\pi d_{5/2})^2 \pi g_{7/2}\}$ grow up appreciably to break the AC state-like structure of the dressed 3QP mode. However, this trend is actually not as appreciable as has been expected. As is seen from Table II-(c), in particular, the component $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ which is directly connected with the largest component $\{(\pi g_{7/2})^3\}$ through the 3QP correlation among different orbits, $1g_{7/2}$ and $2d_{5/2}$, does not become extremely large. On the other hand, although the component $\{(\pi d_{5/2})^2 \pi g_{7/2})\}$ increases non-negligibly, it does not bring about the breaking of the similarity with the AC states. Here it should be pointed out that, while the component $\{(\pi g_{7/2})^2 \pi d_{5/2}\}$ contributes directly to the $B(E2;\ 5/2_2^+ \to 5/2_1^+)$, the component $\{(\pi d_{5/2})^2 \pi g_{7/2}\}$ can contribute only through angular-momentum recoupling. (Such a classification of the amplitudes is obtained by interpreting them in connection with the concept of phonon-band, as will be described in the succeeding chapter.) Associated with these characteristics in the $(\pi\pi\pi)$-type, it is also seen that the components containing the $g_{7/2}$-proton-quasi-particle play a dominant role among those of $(\nu\nu\pi)$-type. In this way, contrary to the $B(E2;\ 5/2_2^+ \to 7/2_1^+)$, the $B(E2;\ 5/2_2^+ \to 5/2_1^+)$ cannot be enhanced. Now, the origins of these characteristics in the microscopic structure of the collective $5/2^+$ state (as the dressed 3QP mode) may be found in the following facts:

1) The 3QP correlation among quasi-particles in different orbits, $1g_{7/2}$ and $2d_{5/2}$, involves the spin-flip matrix element $(2d_{5/2}\|r^2 Y_2\|1g_{7/2})$ which is considerably small, compared to the spin-non-flip one $(1g_{7/2}\|r^2 Y_2\|1g_{7/2})$ contributing the 3QP correlation at the specific orbit $1g_{7/2}$.

2) The component $\{(\pi g_{7/2})^3\}$ is increasing due to the special favouring of the $I=(j-1)$-coupling, while the corresponding component $\{(\pi d_{5/2})^3\}$ is forbidden for spin $I=5/2$.

It is interesting to note that the above-mentioned condition of shell structure (for the realization of AC state-like structure) is common to almost all major shells. Thus we can expect the picture (for low-lying excited states) illustrated in Fig. 5 to hold over several regions of spherical odd-mass nuclei. To examine the theoretical prediction, the experimental data on the electromagnetic transition rates, especially the ones between excited states, are highly desired. For the nuclei under consideration, more systematic measurement on the values of $B(E2;\ 5/2_2^+ \to 5/2_1^+)$ should confirm more definitely the conclusion given here.

3-2 *Collective* $3/2^+$ *states in I, Cs and La*

Here it is interesting to note that, in addition to the collective $5/2_2^+$ states discussed above, experimental data reveal the systematic presence of the $3/2_1^+$ states in I, Cs and La isotopes. With the same trend as in the case of collective

Table III. Calculated $B(E2)$ values from the dressed 3QP $3/2_1^+$ states in odd-mass I, Cs and La isotopes (in unit of $e^2 \cdot 10^{-50}$ cm^4). The notations and parameters adopted are the same as in Table I.

	$3/2_1^+ \longrightarrow 5/2_1^+$		$3/2_1^+ \longrightarrow 7/2_1^+$
	exact	ACS	exact
^{127}I	3.1	10.4	2.6
^{129}I	2.0	7.8	2.4
^{131}I	0.1	5.6	1.0
^{133}I	0.2	3.6	1.4
^{131}Cs	9.8	16.0	1.7
^{133}Cs	10.4	11.1	2.4
^{135}Cs	4.2	7.2	1.7
^{133}La	13.2	67.7	0.9
^{135}La	9.3	20.5	0.7
^{137}La	6.5	11.4	0.6

$5/2_2^+$ states, the excitation energies of the $3/2_1^+$ states measured from the 1QP $5/2_1^+$ states decrease as the neutron number changes from $N=82$ to $N=74$. (See Figs. 2 and 3.)

Regarding the $3/2_1^+$ states to be the dressed 3QP states, we have also calculated their excitation energies and $B(E2)$ values. The results are shown in Fig. 4 and Table III. In Fig. 4, we see that the above-mentioned experimental characteristics of the $3/2_1^+$ states are well reproduced in the calculation. The main components of the $3/2_1^+$ states are therefore identified as the dressed 3QP states. It is noted, however, that the relative level positions between the $3/2_1^+$ and $5/2_2^+$ states are not well reproduced in the calculation, especially for I isotopes and lighter Cs and La isotopes. This may indicate the necessity of modifying the adopted single-particle energies (of Kisslinger and Sorensen) and of taking the coupling effect of the 1QP $2d_{3/2}$ mode into account. However, since this disagreement does not essentially affect the discussion below, we do not attempt such improvements of the calculation here.

According to the criterion given in the preceding subsection (on the structure of the dressed 3QP modes), the calculated $B(E2)$ values in Table III suggest that the 3QP correlations among quasi-particles in different orbits are rather strong in the $3/2_1^+$ states compared to the collective $5/2_2^+$ states: The values of $B(E2;\ 3/2_1^+ \rightarrow 5/2_1^+)$ and of $B(E2;\ 3/2_1^+ \rightarrow 7/2_1^+)$ are both enhanced, with ratios changing from I isotopes to La isotopes. Main amplitudes of the $3/2_1^+$ states in I isotopes as the dressed 3QP modes are shown in Table IV-(a), from which we can easily see why the competition between the two $E2$ transitions (to the 1QP $5/2^+$ and 1QP $7/2^+$ states) is remarkable in I isotopes. Namely, among the components of both $(\pi\pi\pi)$- and $(\nu\nu\pi)$-types, the two sets of components containing the $g_{7/2}$- and $d_{5/2}$-proton-quasi-particle, respectively,

Table IV. Main correlation amplitudes of the dressed 3QP $3/2_1^+$ modes in ^{131}I(a), ^{133}Cs(b) and ^{135}La(c). Notations are the same as in Table II.

(a) $3/2_1^+$ state in ^{131}I

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
$(g_{7/2})^3$	$(g_{7/2})^2 d_{5/2}$	$(g_{7/2})^2_4 d_{5/2}$	$(d_{5/2})^2 g_{7/2}$	$(d_{5/2})^3$	$(h_{11/2})^2 g_{7/2}$	$(h_{11/2})^2 d_{5/2}$	$(d_{3/2})^2 g_{7/2}$	$(d_{3/2})^2 d_{5/2}$	$(s_{1/2} d_{3/2}) g_{7/2}$
−0.65	0.35	0.10	0.19	0.11	0.02	0.22	0.34	0.13	−0.13
−0.07	{0.02(2) {0.11(3)	0.02	{0.01(2) {0.06(3)	0.04	0.18	0.12	0.10	0.06	−0.08

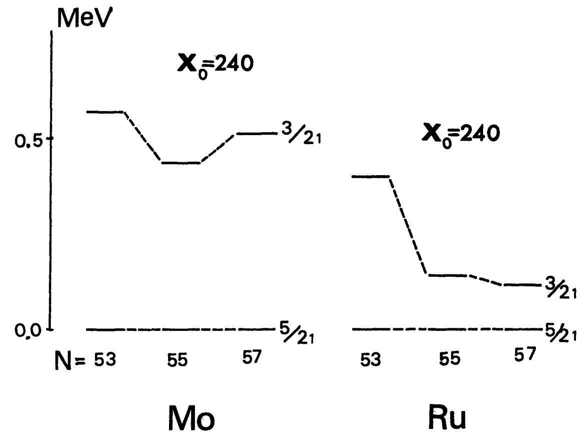
(b) $3/2_1^+$ state in ^{133}Cs

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
$(g_{7/2})^3$	$(g_{7/2})^2 d_{5/2}$	$(g_{7/2})^2_4 d_{5/2}$	$(d_{5/2})^2 g_{7/2}$	$(d_{5/2})^3$	$(h_{11/2})^2 g_{7/2}$	$(d_{3/2})^2 g_{7/2}$	$(h_{11/2})^2 d_{5/2}$	$(d_{3/2} g_{7/2}) d_{5/2}$	$(d_{3/2})^2 d_{5/2}$
−0.40	0.84	0.10	0.28	0.46	0.39	0.23	0.65	0.16	0.37
−0.08	{0.03(2) {0.07(3)	0.03	{0.04(2) {0.09(3)	0.16	0.17	0.09	0.37	0.09	0.19

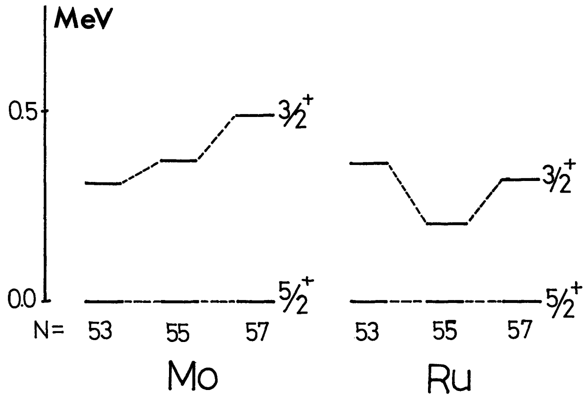
(c) $3/2_1^+$ state in ^{135}La

($\pi\pi\pi$) type					($\nu\nu\pi$) type				
$(g_{7/2})^2 d_{5/2}$	$(d_{5/2})^3$	$(d_{5/2})^2 g_{7/2}$	$(d_{5/2})^2_4 g_{7/2}$	$(g_{7/2})^3$	$(h_{11/2})^2 d_{5/2}$	$(d_{3/2})^2 d_{5/2}$	$(s_{1/2} d_{3/2}) d_{5/2}$	$(h_{11/2})^2 g_{7/2}$	$(d_{3/2} g_{7/2}) d_{5/2}$
0.62	0.56	0.21	0.15	−0.15	0.60	0.33	−0.15	0.19	0.15
{0.02(2) {0.34(3)	0.20	{0.04(2) {0.02(3)	0.06	−0.04	0.37	0.19	−0.15	0.11	0.09

play equally important roles in I isotopes. As the proton number increases, the chemical potential for protons shifts up toward the $2d_{5/2}$ orbit (from the $1g_{7/2}$ orbit), so that in La isotopes the component $\{(\pi g_{7/2})^3\}$ in the forward amplitudes $\psi_{nI=3/2}$ is diminished and the component $\{(\pi d_{5/2})^3\}$ is enlarged. (See Table IV-(c).) The increase in $B(E2; 3/2_1^+ \to 5/2_1^+)$ in La isotopes is clearly due to the growth of the 3QP correlation in the $2d_{5/2}$ orbit. Although the component $\{(\pi d_{5/2})^3\}$ is still not the largest one, the components containing the single $d_{5/2}$-proton-quasi-particle become the dominant ones among the other components of both $(\pi\pi\pi)$- and $(\nu\nu\pi)$-types. Associated with this, the value of $B(E2; 3/2_1^+ \to 7/2_1^+)$ becomes smaller in clear contrast to the increasing $B(E2; 3/2_1^+ \to 5/2_1^+)$. In this sense, we may say that the $3/2_1^+$ states in La



(a)



(b)

Fig. 6. Calculated excitation energies of the dressed 3QP $3/2_1^+$ states in odd-mass Mo and Ru isotopes. They are measured from those of the lowest 1QP states.
(a) The constant values of χ_0 are used for each isotopes.
(b) The values of χ_0 are chosen to reproduce the average energies of the 2^+ phonons in the adjacent even-even nuclei.

isotopes have a structure similar to that of the AC states with $I=j-1$ (where j corresponds to the 1QP $5/2^+$ state). Since quantitative prediction depends rather sensitively on the single-particle energies adopted, the numerical values presented in Table III should not be taken too strictly. Nevertheless, we can expect such a structure change of the $3/2_1^+$ states (from I isotopes to La isotopes) in a qualitative sense.

3-3 *Collective $3/2^+$ states in Mo and Ru*

The calculated results of the collective $3/2_1^+$ states (in odd-neutron Mo and Ru isotopes) as the dressed 3QP modes are shown in Fig. 6 and Table V. The systematic appearance of the low-lying collective $3/2_1^+$ states in the isotopes with $N=53$, 55 and 57 are reproduced very well, together with the values of $B(E2; 3/2_1^+\to5/2_1^+)$. Thus, the $5/2_1^+$ and $3/2_1^+$ states are identified as the 1QP and dressed 3QP states, respectively. Furthermore, the special enhancement of $B(E2; 3/2_1^+\to5/2_1^+)$ shown in Table V (when compared to the other $B(E2)$'s) suggests that the structure of the $3/2_1^+$ states is similar to that of the AC states with $I=j-1$. (In this case, j corresponds to the 1QP $5/2^+$ states.) From Table VI, we can easily see the similarity of the $3/2_1^+$ states to the AC states, although the fine structure is appreciably different as a result of the 3QP correlations among quasi-particles in different orbits with the same parity ($d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$). The gradual change of the microscopic structure of the $3/2_1^+$ states with increasing neutron number will be further discussed in § 5.

§ 4. New reduction effect of couplings between dressed 3QP and 1QP modes

In contrast to the case of the AC states, the dressed 3QP states, especially the collective $5/2^+$ states in I, Cs and La isotopes, lie close, in energy, to the 1QP states with the same spin and parity. It is, therefore, necessary to examine

Table V. Calculated $B(E2)$ values from the dressed 3QP $3/2_1^+$ states in odd-mass Mo and Ru isotopes (in unit of $e^2\cdot10^{-50}$ cm^4). The adopted values of χ_0 are the same as in Fig. 6(b) and are listed in the sixth column. The notations and the other parameters are the same as in Table I.

	$3/2_1^+\longrightarrow5/2_1^+$		$3/2_1^+\longrightarrow7/2_1^+$	$3/2_1^+\longrightarrow1/2_1^+$	χ_0
	exact	ACS	exact	exact	
^{95}Mo	3.1	5.2	0.3	0.3	263.0
^{97}Mo	10.1	8.0	0.6	0.8	244.3
^{99}Mo	10.3	8.6	0.1	0.5	237.2
^{97}Ru	5.6	5.7	0.2	0.8	245.0
^{99}Ru	8.6	15.4	1.9	2.9	236.0
^{101}Ru	12.8	20.3	0.7	0.8	231.0

Table VI. Main correlation amplitudes of the dressed 3QP $3/2_1^+$ modes in ^{97}Ru(a), ^{99}Ru(b) and ^{101}Ru(c). Notations are the same as in Table II.

(a) $3/2_1^+$ state in ^{97}Ru

($\nu\nu\nu$) type		($\pi\pi\nu$) type	
$(d_{5/2})^3$	$(d_{5/2})^2 s_{1/2}$	$(g_{9/2})^2 d_{5/2}$	$(g_{9/2})^2 s_{1/2}$
0.94	−0.60	0.75	−0.20
0.38	{−0.25$^{(2)}$ −0.38$^{(3)}$	0.55	−0.42

(b) $3/2_1^+$ state in ^{99}Ru

($\nu\nu\nu$) type					($\pi\pi\nu$) type			
$(d_{5/2})^3$	$(d_{5/2})^2 g_{7/2}$	$(d_{5/2})^2_4 g_{7/2}$	$(d_{5/2})^2 s_{1/2}$	$(g_{7/2})^2 d_{5/2}$	$(g_{9/2})^2 d_{5/2}$	$(p_{3/2} p_{1/2}) d_{5/2}$	$(g_{9/2})^2 s_{1/2}$	$(g_{9/2})^2 g_{7/2}$
0.86	0.22	0.12	−0.77	0.17	0.85	−0.10	−0.18	0.21
0.41	{0.05$^{(2)}$ 0.28$^{(3)}$	0.08	{−0.39$^{(2)}$ −0.19$^{(3)}$	{0.04$^{(2)}$ 0.17$^{(3)}$	0.69	−0.12	−0.27	0.42

(c) $3/2_1^+$ state in ^{101}Ru

($\nu\nu\nu$) type					($\pi\pi\nu$) type			
$(d_{5/2})^2 s_{1/2}$	$(d_{5/2})^3$	$(g_{7/2})^2 d_{5/2}$	$(h_{11/2})^2 d_{5/2}$	$(d_{5/2})^2 g_{7/2}$	$(g_{9/2})^2 d_{5/2}$	$(g_{9/2})^2 s_{1/2}$	$(g_{9/2})^2 g_{7/2}$	$(p_{1/2} p_{3/2}) d_{5/2}$
−0.98	0.74	0.33	0.21	0.19	1.13	−0.34	0.18	−0.13
{−0.43$^{(2)}$ −0.12$^{(3)}$	0.31	{0.02$^{(2)}$ 0.28$^{(3)}$	0.05	{0.05$^{(2)}$ 0.07$^{(3)}$	0.73	−0.24	0.16	−0.15

their coupling effects. According to the general formulation given in § 5-Chap. 2, the original Hamiltonian is transcribed unambiguously into the quasi-particle new-Tamm-Dancoff subspace, the basis vectors of which are $\{|\Phi_\delta^{(1)}\rangle \equiv a_\delta^\dagger|\Phi_0\rangle,\ |\Phi_{nIK}^{(3)}\rangle \equiv Y_{nIK}^\dagger|\Phi_0\rangle\}$. The transcribed Hamiltonian is of the form

$$\boldsymbol{H} = \sum_\delta E_d \boldsymbol{a}_\delta^\dagger \boldsymbol{a}_\delta + \sum_{nIK} \omega_{nI} \boldsymbol{Y}_{nIK}^\dagger \boldsymbol{Y}_{nIK} + \sum_{\substack{\delta, nIK \\ (K=m_\delta)}} V_{\text{int}}(d, nI)\{\boldsymbol{Y}_{nIK}^\dagger \boldsymbol{a}_\delta + \boldsymbol{a}_\delta^\dagger \boldsymbol{Y}_{nIK}\}, \tag{4·1}$$

where

$$\boldsymbol{a}_\delta^\dagger \equiv a_\delta^\dagger|\Phi_0\rangle\langle\Phi_0|, \quad \boldsymbol{Y}_{nIK}^\dagger \equiv Y_{nIK}^\dagger|\Phi_0\rangle\langle\Phi_0|. \tag{4·2}$$

The third term of the transcribed Hamiltonian (4·1) represents the interaction between the dressed 3QP and 1QP modes, and comes from the H_Y-type original interaction (shown in Fig. 7 of Chap. 3) which has not played any role in constructing the dressed 3QP modes. The effective coupling strength $V_{\text{int}}(d, nI)$ is thus composed of the matrix elements of H_Y accompanied by the amplitudes of the dressed 3QP mode $Y_{nIK}^\dagger$. In the P+QQ force model, the explicit form is given as follows:

$$\begin{aligned} V_{\text{int}}(d, nI) = -\chi\delta_{Ij_d}\sqrt{\frac{10}{2I+1}} &\times\Bigg[\sqrt{\frac{3}{2}} \sum_{\substack{abc \\ a'b'c'J'}} Q(ab)R(cd)P_I(ab(2)c|a'b'(J')c')\ \psi_{nI}[a'b'(J')c'] \\ &+ \sum_{(rs)c} Q(rs)R(cd)\{1+\delta_{rs}\}^{-1/2}\ \psi_{nI}[rs(2)c] \\ &+ \frac{1}{\sqrt{2}} \sum_{aJ} Q(aa)R(ad)P_I(aa(2)a|aa(J)a)\ \varphi_{nI}^{(1)}[aa(J)a] \\ &+ \sum_{\substack{ac, J=\text{even} \\ (a\neq c, J\neq 0)}} Q(ca)R(ad)\sqrt{5(2J+1)}\begin{Bmatrix} j_a & j_a & J \\ j_c & I & 2 \end{Bmatrix}\ \varphi_{nI}^{(2)}[aa(J)c] \\ &+ \sum_{\substack{(ab)c \\ (a\neq c, b\neq c)}} Q(ab)R(cd)\{1+\delta_{ab}\}^{-1/2}\ \varphi_{nI}^{(3)}[ab(2)c] \\ &+ \sum_{(rs)c} Q(rs)R(cd)\{1+\delta_{rs}\}^{-1/2}\ \varphi_{nI}^{(3)}[rs(2)c]\Bigg], \end{aligned} \tag{4·3}$$

where

$$R(cd) = \frac{1}{\sqrt{5}}(c\|r^2Y_2\|d)\cdot(u_cu_d - v_cv_d) \tag{4·4}$$

and $Q(ab)$ is defined by (2·5). The formal structure of (4·3) is very similar to that of the reduced matrix element for $E2$ transition given by (2·4). In fact, Eq. (4·3) can be obtained from (2·4) by the following replacement:

$$e_\tau Q(ab)\delta_{cd} \Rightarrow -\chi\delta_{IJd}\cdot\sqrt{\frac{10}{2I+1}}\;Q(ab)R(cd).$$

Such a formal similarity is a characteristic of the P+QQ force model adopted.

The result of calculations of the coupling effects due to the interaction

Table VII. Calculated results for the coupling effects. The mixing amplitude of the 1QP modes are given in the third column, while those of the lowest dressed 3QP modes and of the next higher ones are given in the fourth and fifth columns, respectively. In the sixth column are listed the values of the energy shifts due to the coupling effects in unit of MeV.
(a) The $5/2_1^+$ and $5/2_2^+$ states in odd-mass I, Cs and La isotopes.
(b) The $3/2_1^+$ states in odd-mass Mo and Ru isotopes.

(a)

nucleus	state	$\zeta^{(1)}(d)$	$\zeta^{(3)}(n=1,I)$	$\zeta^{(3)}(n=2,I)$	$\Delta\omega$
^{127}I	$5/2_1^+$	0.87	−0.28	0.39	−0.35
	$5/2_2^+$	0.24	0.96	0.15	0.02
^{129}I	$5/2_1^+$	0.90	−0.18	0.39	−0.23
	$5/2_2^+$	0.15	0.98	0.10	0.01
^{131}I	$5/2_1^+$	0.92	−0.20	−0.34	−0.16
	$5/2_2^+$	0.18	0.98	−0.10	0.01
^{133}I	$5/2_1^+$	0.96	−0.15	0.24	−0.08
	$5/2_2^+$	0.14	0.99	0.06	0.01
^{131}Cs	$5/2_1^+$	0.93	−0.01	−0.35	−0.15
	$5/2_2^+$	0.01	1.00	−0.01	0.00
^{133}Cs	$5/2_1^+$	0.95	−0.03	0.33	−0.13
	$5/2_2^+$	0.02	1.00	0.01	0.00
^{135}Cs	$5/2_1^+$	0.97	0.00	−0.23	−0.06
	$5/2_2^+$	0.00	1.00	0.00	0.00
^{133}La	$5/2_1^+$	0.95	0.16	−0.26	−0.15
	$5/2_2^+$	−0.15	0.99	0.06	0.01
^{135}La	$5/2_1^+$	0.96	0.07	−0.27	−0.08
	$5/2_2^+$	−0.07	1.00	0.03	0.00
^{137}La	$5/2_1^+$	1.00	0.02	−0.10	−0.01
	$5/2_2^+$	−0.02	1.00	0.00	0.00

(b)

nucleus	state	$\zeta^{(1)}(d)$	$\zeta^{(3)}(n=1,I)$	$\zeta^{(3)}(n=2,I)$	$\Delta\omega$
^{95}Mo	$3/2_1^+$	0.05	1.00	0.00	−0.01
^{97}Mo	$3/2_1^+$	0.16	0.98	0.03	−0.01
^{99}Mo	$3/2_1^+$	−0.51	0.77	−0.30	−0.17
^{97}Ru	$3/2_1^+$	−0.09	1.00	−0.00	−0.02
^{99}Ru	$3/2_1^+$	−0.25	0.96	−0.10	−0.10
^{101}Ru	$3/2_1^+$	0.44	0.83	0.30	−0.19

term in (4·1) are shown in Table VII. It is noticeable that the coupling effects are very small even in situation where the two $5/2^+$ states are close to each other in energy (i.e., $\Delta\omega \approx 0.01$ MeV). The mechanism to make the coupling effects (between the dressed 3QP and 1QP modes) so small must be found in the microscopic structure of the effective coupling strength $V_{\rm int}(d, nI)$. From the microscopic structure of the effective coupling strength between the dressed 3QP $5/2_2^+$ and 1QP $5/2_1^+$ modes in the case of the P+QQ force model, we can find the following origins to weaken the coupling:

1) The important matrix elements of H_Y (of the quadrupole force) in the effective coupling strength, which are accompanied by large components of the amplitudes of the dressed 3QP $5/2_2^+$ mode, always contain the spin-flip matrix element $(2d_{5/2}\|r^2Y_2\|1g_{7/2})$, which is considerably smaller compared to the diagonal matrix element $(1g_{7/2}\|r^2Y_2\|1g_{7/2})$. In this connection, it is interesting to recall that the considerable smallness of the ratio of $(2d_{5/2}\|r^2Y_2\|1g_{7/2})$ to $(1g_{7/2}\|r^2Y_2\|1g_{7/2})$ is also one of the important origins to bring about the AC state-like structure for the collective $5/2_2^+$ state.

2) The pairs of matrix elements of H_Y (such as Figs. 7(a) and 7(b)), which are in the relation of exchange diagrams with each other, must always be involved in the effective coupling strength, because the antisymmetrization among the three quasi-particles composing the dressed 3QP $5/2^+$ mode is properly taken into account. (See the projection operator P_I entering into $V_{\rm int}(d, nI)$.) Actual calculations tell us that the effects of such exchange parts on the effective coupling strength between the $5/2_1^+$ and $5/2_2^+$ states are not constructive but rather destructive to each other.

3) The effective coupling strength is determined by the contributions of many components of the amplitudes. For example, the components $\{(\pi d_{5/2})^2\pi g_{7/2}\}$ and $\{(\pi g_{7/2})^2\pi d_{5/2}\}$ represented by Figs. 8(a) and 8(b) both contribute to $V_{\rm int}(d, nI)$. (Notice that the summation in the first term of Eq. (4·3) should be taken with respect to the orbital triad (a, b, c).) The calculated result shows that such different components of the amplitudes generally contribute in random phase, namely, they cancel one another. For the collective $5/2_2^+$ states under consideration, this effect is extremely strong since the contributions from Figs. 8(a) and 8(b) are of approximately equal magnitudes with different signs.

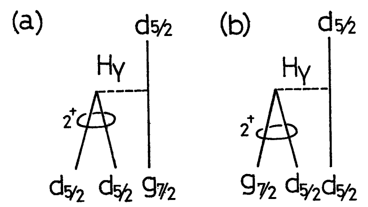


Fig. 7. Example of exchange diagrams both of which contribute to the effective coupling strength with $I^\pi = 5/2^+$.

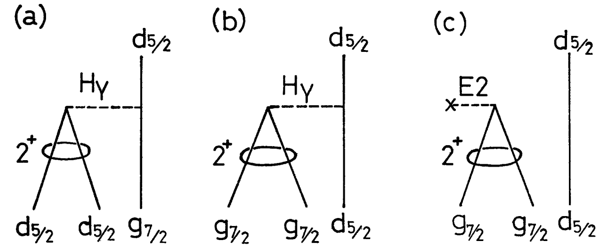


Fig. 8. Graphic representation of two kinds of diagrams both of which contribute to the effective coupling strength with $I^\pi=5/2^+$. The two kinds of diagrams, (a) and (b), contribute destructively to the effective coupling strengths of the lowest dressed 3QP $5/2_2^+$ states in I, Cs and La isotopes. It should be noted that, in the case of calculating the $E2$ transition (c), such a destructive effect never appear because one of them (a) is forbidden.

It should be pointed out, here, that Fig. 8(a) involves the small matrix element $R(g_{7/2}\ d_{5/2})$ accompanied with the large component $\{(\pi d_{5/2})^2\ \pi g_{7/2}\}$, while Fig. 8(b) involves the large matrix element $R(d_{5/2}\ d_{5/2})$ accompanied with the small component $\{(\pi g_{7/2})^2\ \pi d_{5/2}\}$. We should also note that, in the case of calculating the $E2$ transitions, such a destructive effect never appear because the matrix element corresponding to Fig. 8(a) is forbidden. (See the δ_{cd}-factor in Eq. (2·4).)

4) In addition to these effects, it should also be pointed out that the effective coupling strength depends characteristically on the reduction factors $(u_c u_d - v_c v_d)$ entering into the $R(cd)$ in $V_{\rm int}(d, nI)$. The reduction factor of the orbital pair, $g_{7/2}$ and $d_{5/2}$, becomes particularly small in La isotopes.

All of these effects cooperate in weakening the effective coupling strength between the 1QP $5/2_1^+$ state and the *lowest* dressed 3QP $5/2_2^+$ state. As a result, the 1QP $5/2_1^+$ mode couples rather with the *next higher* dressed 3QP $5/2_3^+$ mode, as shown in Table VII.

It is worthy to emphasize that the reduction effects 2) and 3) of the effective coupling strength never appear in the conventional QPC theory, because the antisymmetrization between the odd quasi-particle and the quasi-particle-pair composing the phonon is not at all taken into account in the QPC theory. The physical significance of these new reduction effects will be further discussed in Chap. 5 (where the necessity of alteration of the conventional picture on the low-energy excitations in odd-mass nuclei will be discussed *en bloc*).

§ 5. Analysis of electromagnetic properties of AC state-like collective excited states

In this section, we investigate the electromagnetic properties of the AC state-like collective excited states (the collective $5/2_2^+$ states in I, Cs and La isotopes and the collective $3/2_1^+$ states in Mo and Ru isotopes) taking the coupling effect between the dressed 3QP and 1QP modes into account. We first evaluate the extent to which the $B(E2)$ values presented in § 3 are affected by the coupling effect. Since the couplings between the dressed 3QP $5/2_2^+$ states and the 1QP $5/2_1^+$ states are very weak, it is expected that the coupling effect does not cause a large deviation from the $E2$-transition property of the $5/2_2^+$ states in the absence of the coupling effect. Secondly, we examine whether the similarity between the collective $5/2_2^+$ and $3/2_1^+$ states and the AC states holds also in their magnetic properties. If the structures of the collective $5/2_2^+$ and $3/2_1^+$ states are similar to the AC states, we expect, according to the discussion in § 4 of Chap. 3, that 1) the values of $B(M1; 5/2_2^+ \to 7/2_1^+)$ and $B(M1; 3/2_1^+ \to 5/2_1^+)$ are nearly zero and 2) $g(5/2_2^+) \approx g(7/2_1^+)$ and $g(3/2_1^+) \approx g(5/2_1^+)$. In the case of applying these criteria for the magnetic properties to the collective states under consideration, we should be careful in that the properties of $M1$-transitions and -moments are quite sensitive to the coupling effects. On the other hand, the magnitude of the coupling effect may be examined more carefully by comparing the calculated $B(M1)$ and g factors with the experimental data.

When the transcribed Hamiltonian (4·1) containing the coupling term $\boldsymbol{H}^{(\mathrm{int})}$ is diagonalized, any eigenstate in the quasi-particle NTD subspace under consideration takes the following form as a superposition of the dressed 3QP and 1QP states:

$$|IK;\nu\rangle = \zeta_\nu^{(1)}(d)\delta_{Ij_d}\delta_{Km_\delta}a_\delta^\dagger|\Phi_0\rangle + \sum_n \zeta_\nu^{(3)}(nI)\,Y_{nIK}^\dagger|\Phi_0\rangle \tag{5·1}$$

with the normalization

$$\{\zeta_\nu^{(1)}(d)\}^2 + \sum_n \{\zeta_\nu^{(3)}(nI)\}^2 = 1. \tag{5·2}$$

Here the index ν is used to label the eigenstates having the same values of angular momentum I and its projection K. The summation with respect to n, in Eqs. (5·1) and (5·2), should be taken, in principle, over all (physical) eigensolutions of the dressed 3QP modes with definite I and K (, several of which have collective nature while the others are of non-collective nature). In practice, of course, the mixing amplitudes $\zeta_\nu^{(3)}(nI)$ with $n>1$ are negligibly small in the low-lying collective $3/2_1^+$ and $5/2_2^+$ states under consideration.

The matrix elements of any electromagnetic multipole operators between the eigenstates (5·1) are given as follows:

$$\begin{aligned}\langle IK;\nu|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|I'K';\nu'\rangle &= \zeta_\nu^{(1)}(d)\zeta_{\nu'}^{(1)}(d')\delta_{Ij_d}\delta_{I'j_{d'}}\langle\Phi_\delta^{(1)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{\delta'}^{(1)}\rangle \\ &+ \zeta_\nu^{(1)}(d)\delta_{Ij_d}\sum_{n'}\zeta_{\nu'}^{(3)}(n'I')\,\langle\Phi_\delta^{(1)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{n'I'K'}^{(3)}\rangle \\ &+ \zeta_{\nu'}^{(1)}(d')\delta_{I'j_{d'}}\sum_n \zeta_\nu^{(3)}(nI)\,\langle\Phi_{nIK}^{(3)}|\hat{\boldsymbol{O}}_{LM}^{(\pm)}|\Phi_{\delta'}^{(1)}\rangle\end{aligned} \tag{5·3}$$

$$+\sum_{nn'}\zeta_{\nu}^{(3)}(nI)\zeta_{\nu'}^{(3)}(n'I')\ \langle\Phi_{nIK}^{(3)}|\hat{O}_{LM}^{(\pm)}|\Phi_{n'I'K'}^{(3)}\rangle,$$

where $\hat{O}_{LM}^{(\pm)}$ denotes the transcribed electromagnetic operator in the quasi-particle NTD subspace defined by Eq. (5·14) of Chap. 2, and its matrix elements appearing in (5·3) are given in Appendix 2D. We use the symbolical notations, such as E_{11}, E_{13}, E_{31} and E_{33}, for the first, second, third, and fourth terms on the right-hand side of (5·3), respectively. Using Eq. (5·3), we obtain for example,

$$\begin{aligned}B(E2;I_\nu\to I'_{\nu'})&=\frac{1}{2I+1}|\langle I;\nu\|\hat{O}_2^{(+)}\|I';\nu'\rangle|^2\\&=\frac{1}{2I+1}|E_{11}+E_{13}+E_{31}+E_{33}|^2,\end{aligned}\tag{5·4}$$

$$\begin{aligned}B(M1;I_\nu\to I'_{\nu'})&=\frac{1}{2I+1}|\langle I;\nu\|\hat{O}_1^{(-)}\|I';\nu'\rangle|^2\\&=\frac{1}{2I+1}|M_{11}+M_{13}+M_{31}+M_{33}|^2,\end{aligned}\tag{5·5}$$

$$\begin{aligned}\mu(I_\nu)=g(I_\nu)I&=\sqrt{\frac{4\pi}{3}}\langle IK;\nu|\hat{O}_{10}^{(-)}|IK;\nu\rangle\quad(\text{with }K=I)\\&=(g_{11}+2g_{13}+g_{33})\cdot I,\end{aligned}\tag{5·6}$$

where $\hat{O}_{2M}^{(+)}$ and $\hat{O}_{1M}^{(-)}$ are the electric quadrupole and magnetic dipole operators, respectively, and their reduced matrix elements are defined by

$$\langle IK;\nu|\hat{O}_{LM}^{(\pm)}|I'K';\nu'\rangle=\frac{(I'LK'M|IK)}{\sqrt{2I+1}}\langle I;\nu\|\hat{O}_L^{(\pm)}\|I';\nu'\rangle.\tag{5·7}$$

Following these general formulae, the calculations of the $B(E2)$, $B(M1)$ and magnetic g factors of the collective $5/2_2^+$ and $3/2_1^+$ states have been performed. In this numerical work, any modification of the Kisslinger and Sorensen's parameters which were adopted in §3 has not been attempted. We

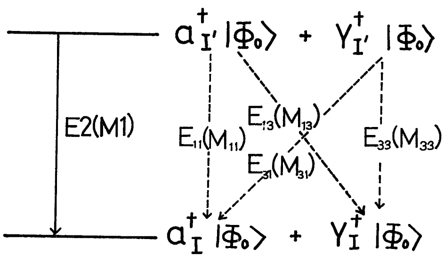


Fig. 9. Graphic explanation of the contributions from each term in Eqs. (5·4) and (5·5) to the reduced matrix element of electromagnetic operator,

have also adopted a simple choice of the effective charges and the effective spin g factors:

$$e_p = e + \delta e, \quad e_n = \delta e = 0.5e,$$

$$g_s^{\mathrm{eff}} = \begin{cases} 0.55\, g_s, & \text{(for I, Cs, La)} \\ 0.50\, g_s. & \text{(for Mo, Ru)} \end{cases}$$

5–*a*) *The region of I, Cs and La isotopes*

The results of calculations in which the coupling effects are taken into account are tabulated from Table VIII to XI. Table VIII shows that the $E2$-transition properties discussed in §3 (of the collective $5/2_2^+$ states) are well conserved even when the coupling effects are taken into account. The reason is understood as follows: Since the mixing amplitude, $\zeta_{\nu=2}^{(1)}(5/2^+)$ defined by (5·1), in the collective $5/2_2^+$ state is small, the main contributions to the $B(E2; 5/2_2^+ \to 7/2_1^+)$ are expected to come from the third and the fourth terms on the right-hand side of Eq. (5·3). Furthermore, since the value of $\langle \Phi_{nIK}^{(3)} | \hat{O}_{2M}^{(+)} | \Phi_{n'I'K'}^{(3)} \rangle$ is not large, the contribution of the E_{33} part is also small. Then the coupling effect is expected to enter mainly through the factor $\zeta_{\nu'=1}^{(1)}(7/2^+)$ in the E_{31} part. (Here, it should be noted that the $\zeta_{\nu=2}^{(3)}(n\ 5/2^+)$ factors approximately take the values $\zeta_{\nu=2}^{(3)}(n\ 5/2^+) \approx \delta_{n1}$.) The change in the value of $\zeta_{\nu'=1}^{(1)}(7/2^+)$ from unity, in turn, comes from the admixtures of the dressed 3QP modes with $I = 7/2^+$ in the 1QP $7/2_1^+$ state. Since such coupling effects on the 1QP $7/2_1^+$ state are also reduced when compared to the estimation of the QPC theory, no essential change in the enhancement property of the $B(E2; 5/2_2^+ \to 7/2_1^+)$ occurs. Following similar arguments, we can see that the hindrance property of the

Table VIII. $B(E2)$ values of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effects. The unit is $e^2\ 10^{-50}\ \mathrm{cm}^4$. The adopted parameters are the same as those in Table I.

	$5/2_2^+ \to 7/2_1^+$		$5/2_2^+ \to 5/2_1^+$	
	$B(E2)_{\mathrm{cal}}$	$B(E2)_{\mathrm{exp}}$	$B(E2)_{\mathrm{cal}}$	$B(E2)_{\mathrm{exp}}$
^{127}I	7.6		0.6	0.7±0.1[a)]
^{129}I	6.6	2.1±0.4[b)]	0.1	
^{131}I	5.1		0.2	
^{133}I	3.8		0.1	
^{131}Cs	10.2	23.6±2.5[c)]	0.4	0.95[c)]
^{133}Cs	8.0	10.4±1.2[b)]	0.2	3.5[d)]
^{135}Cs	6.7		0.1	
^{133}La	13.4		0.2	
^{135}La	9.6	22[e)]	0.2	1.7[e)]
^{137}La	7.0		0.2	

[a)] Ref. 18), [b)] Ref. 19), [c)] Ref. 25), [d)] Ref. 27), [e)] Ref. 28).

Table IX. Calculated g factors of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes. The calculated values of $g(5/2_2^+)$ listed in the second column are compared to the experimental values given in the third column. In the fourth column are listed the values calculated by adopting the ACS approximation. The calculated and experimental g factors of the 1QP $7/2_1^+$ states are given in the fifth and sixth columns, respectively, for the sake of comparison. In these calculations, effective spin g factor $g_s^{eff}=0.55\,g_s$ is used. The unit is nuclear magneton $e\hbar/2Mc$.

	$g(5/2_2^+)_{cal}$	$g(5/2_2^+)_{exp}$	$g(5/2_2^+)_{ACS}$	$g(7/2_1^+)_{cal}$	$g(7/2_1^+)_{exp}$
^{127}I	0.80		0.78	0.77	0.73 [a]
^{129}I	0.80		0.78	0.77	0.747 [b]
^{131}I	0.81		0.78	0.77	0.751 [c]
^{133}I	0.80		0.78	0.77	
^{131}Cs	0.85	{0.79±0.05 [d]; 0.74±0.03 [e]}	0.78	0.77	
^{133}Cs	0.74		0.78	0.77	
^{135}Cs	0.86		0.78	0.73	
^{133}La	0.83		0.77	0.74	
^{135}La	0.78		0.77	0.77	
^{137}La	0.78		0.78	0.77	

[a] Ref. 20), [b] Ref. 29), [c] Ref. 30), [d] Ref. 25), [e] Ref. 26).

$B(E2; 5/2_2^+ \to 5/2_1^+)$ is also conserved. Of course, their magnitudes change appreciably in a quantitative sense, since the E_{31} part itself is a small quantity in this transition.

The AC state-like structure of the collective $5/2_2^+$ states is clearly exhibited in the calculated g factors in Table IX which shows the property $g(5/2_2^+) \approx g(7/2_1^+)$. In these calculated values for $g(5/2_2^+)$, the coupling effect is negligibly small and they are determined essentially from the fourth term in Eq. (5·3). Namely, the g factors in Table IX represent those of the $5/2_2^+$ states as the pure dressed 3QP states. Now, the experimental value of the $g(5/2_2^+)$ in ^{131}Cs is, in fact, nearly equal to those of the $g(7/2_1^+)$ available in neighbouring nuclei. This fact is in excellent agreement with the theoretical prediction and, therefore, is regarded as a further evidence for the AC state-like structure of the collective $5/2_2^+$ states.

In Table IX, for the sake of comparison, are also presented the g factors calculated by completely neglecting the 3QP correlations among different orbits. In this approximation (called the ACS approximation hereafter) in which the 3QP correlation is taken into account only within the specific orbit p, the formula for the g factor is simply reduced to Eq. (4·13) of Chap. 3. Of course, for the collective $5/2_2^+$ states, the specific orbit p denotes the $1g_{7/2}$ orbit. By comparing the results of the ACS approximation to those of the exact calculation, we see that the former give qualitatively the same characteristics for the values of $g(5/2_2^+)$ as the latter. This fact implies that the 3QP correlation in the specific orbit $1g_{7/2}$ plays a decisive role in the collective $5/2_2^+$ states, compared to the 3QP correlations among different orbits.

Table X. $B(M1)$ values of the $5/2_2^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effect. The unit is $(e\hbar/2Mc)^2$. The calculated results for transitions to the 1QP $7/2_1^+$ and $5/2_1^+$ states are listed in the second and sixth columns, respectively. The corresponding experimental values are given in the third and seventh columns. The contributions from the M_{11} and M_{33} parts in Eq. (5·5) are explicitly shown in the fourth and fifth columns, respectively, since both parts frequently become the same order of magnitudes. In these calculations, $g_s^{\rm eff}=0.55\,g_s$ is used.

	$5/2_2^+\longrightarrow 7/2_1^+$		$5/2_2^+\longrightarrow 5/2_1^+$			
	$B(M1)_{\rm cal}$	$B(M1)_{\rm exp}$	M_{11}	M_{33}	$B(M1)_{\rm cal}$	$B(M1)_{\rm exp}$
^{127}I	1.4×10^{-2}	3.0×10^{-2} a)	-0.28	0.04	9.5×10^{-3}	1.4×10^{-1} a)
^{129}I	1.0×10^{-2}		-0.23	0.003	8.6×10^{-3}	
^{131}I	4.9×10^{-3}		-0.83	0.41	2.9×10^{-2}	
^{133}I	9.9×10^{-4}		-0.68	0.35	1.9×10^{-2}	
^{131}Cs	1.7×10^{-3}	4.28×10^{-3} b)	-0.17	0.07	1.7×10^{-3}	6.08×10^{-4} b)
^{133}Cs	1.0×10^{-3}	5.28×10^{-3} c)	-0.12	0.04	1.1×10^{-3}	3.52×10^{-1} c)
^{135}Cs	9.5×10^{-4}		0.01	0.02	1.3×10^{-4}	
^{133}La	5.3×10^{-4}		-0.63	0.39	1.0×10^{-2}	
^{135}La	2.1×10^{-3}	1.4×10^{-3} d)	-0.31	0.19	2.6×10^{-3}	1.77×10^{-3} d)
^{137}La	8.1×10^{-5}		-0.11	0.07	2.4×10^{-3}	

a) Ref. 17), b) Ref. 25), c) Ref. 27), d) Ref. 28).

Table X shows the calculated values of $B(M1)$. As for the $M1$ transitions between the $5/2_2^+$ and 1QP $7/2_1^+$ states, the first term in (5·3) vanishes since it is the l-forbidden quasi-particle-transition matrix element. Furthermore, the matrix element $\langle\Phi^{(3)}_{nIK}|\hat{O}^{(-)}_{1M}|\Phi^{(1)}_{\delta'}\rangle$ is very small since the dressed 3QP mode in the P+QQ force model does not contain the "$J^\pi=1^+$" quasi-particle pairs except for their appearance through angular-momentum recoupling (due to the Pauli principle among constituting three quasi-paritcles). Therefore, the values of $B(M1;\ 5/2_2^+\rightarrow 7/2_1^+)$ are mainly determined by the fourth term in (5·3), i.e., the M_{33} part. By comparing these to the available experimental data for ^{131,133}Cs and ^{135}La, we see that the calculated results reproduce the retarded $M1$ transition very well. On the other hand, the values of $B(M1;\ 5/2_2^+\rightarrow 5/2_1^+)$ are determined by the competition between the M_{11} and M_{33} parts, both of which are small quantities. The hindrance of this transition observed in some experiments may also be considered as an additional evidence for the AC state-like structure of the $5/2_2^+$ states. Although complete agreement between theoretical and experimental values should not be expected for such small quantities, we may thus assert that the $M1$-transition probabilities are of significance in examining the magnitude of the mixing amplitudes under investigation.

As we have seen, almost all characteristics of the collective $5/2_2^+$ states can be reproduced by the calculations based on the proposed theory. However,

Table XI. $B(E2)$ values of the $3/2_1^+$ states in odd-mass I, Cs and La isotopes, calculated by taking account of the coupling effects. The unit is $e^2\ 10^{-50}\,\mathrm{cm}^4$. The adopted parameters are the same as in Table III.

	$3/2_1^+ \longrightarrow 5/2_1^+$		$3/2_1^+ \longrightarrow 7/2_1^+$	
	$B(E2)_{\mathrm{cal}}$	$B(E2)_{\mathrm{exp}}$	$B(E2)_{\mathrm{cal}}$	$B(E2)_{\mathrm{exp}}$
^{127}I	0.04	6.5±0.8 [a]	1.12	11.2±0.2 [b]
^{129}I	0.05		0.65	7.0±0.8 [a]
^{131}I	0.60		0.25	
^{133}I	0.10		1.91	
^{131}Cs	2.06		0.04	
^{133}Cs	0.90		1.05	7.2±0.8 [a]
^{135}Cs	3.44		1.29	
^{133}La	11.27		0.17	
^{135}La	8.40	≥4.8 [c]	0.32	≥0.1 [c]
^{137}La	6.46		0.43	

[a] Ref. 19), [b] Ref. 16), [c] Ref. 28).

it should also be pointed out that the following experimental facts still need investigation: 1) In the (^{3}He, d) reactions performed by Auble et al.,[21] the spectroscopic factors of the collective $5/2_2^+$ states in I isotopes were found to vary from 0.12 in ^{127}I to 0.47 in ^{131}I, which are both significantly larger than the calculated values. 2) The observed values of $B(E2; 5/2_1^+ \rightarrow 7/2_1^+)$ are very large in ^{127}I and ^{129}I.[16),19)] In our calculation, since this $E2$ transition is largely of 1QP transition, such a strong enhancement has not been obtained.

5–*b*) *The region of Mo and Ru isotopes*

In contrast to the case of collective $5/2_2^+$ states discussed above, the coupling effect of the 1QP mode is not negligible for the collective $3/2_1^+$ states in Mo and Ru isotopes. (See Table VII.) One of the reasons is that the new reduction effect coming from the exchange effects originated from the Pauli principle (discussed in §4) is not so drastic in the case of collective $3/2_1^+$ states. Another reason is that, since the 1QP $d_{3/2}^+$ mode which couples to the low-lying dressed 3QP $3/2_1^+$ mode lies *higher* in energy, the reduction $(u_c u_d - v_c v_d)$ factors appearing in the main matrix elements in $V_{\mathrm{int}}(d, nI)$ are not as small as in the case of collective $5/2_2^+$ states in I, Cs and La isotopes (in which the 1QP $d_{5/2}^+$ mode lies *lower* in energy). The calculated trend that the mixing amplitude of the 1QP $d_{3/2}^+$ mode in the collective $3/2_1^+$ state becomes larger as N increases seems to be in good agreement with the experimental trend that the spectroscopic factor of the (d, p) reaction leading to the $3/2_1^+$ state changes from 0.019 in ^{95}Mo (N=53) to 0.11 in ^{99}Mo (N=57).[5]

The result of calculations for the collective $3/2_1^+$ states are tabulated in Tables XII, XIII and XIV. By comparing the calculated $B(E2; 3/2_1^+ \rightarrow$

Table XII. $B(E2;\ 3/2_1^+\to 5/2_1^+)$ values in odd-mass Mo and Ru isotopes, calculated by taking account of the coupling effects. The unit is $e^2\ 10^{-50}\ \mathrm{cm}^4$. The adopted parameters are the same as in Table V.

$3/2_1^+ \longrightarrow 5/2_1^+$

	$B(E2)_{\mathrm{cal}}$	$B(E2)_{\mathrm{exp}}$
^{95}Mo	2.9	5.70 ± 0.36 [a]
^{97}Mo	4.5	3.07 ± 0.17 [a]
^{99}Mo	2.5	
^{97}Ru	5.5	7^{+12}_{-7} [b]
^{99}Ru	7.5	13.05 [c]
^{101}Ru	8.8	5.7 [c]

[a] Ref. 2), [b] Ref. 6), [c] Ref. 7).

$5/2_1^+$) values (in Table XII) to those in Table V, we see the extent to which the coupling effects (between the dressed 3QP and 1QP modes) reduce their enhancements. Corresponding to the increasing coupling effect with N, the reduction of the $B(E2)$ values from those in the absence of the coupling effect also becomes appreciable in ^{99}Mo and ^{101}Ru with $N=57$. Here, of course, the coupling effect is not so strong as to break the zeroth-order picture of the collective $3/2_1^+$ states as the dressed 3QP states.

The magnetic dipole moments of the $3/2_1^+$ states have been known in ^{95}Mo, ^{99}Ru and ^{101}Ru. The observed values of $g(3/2_1^+)$ show a small deviation from the property $g(3/2_1^+)=g(5/2_1^+)$. Concerning the origins of this deviation, we can point out the following: 1) The deviation of about 10% (from the property $g_{J-1}=g_J$) should be expected even if the collective $3/2_1^+$ states under consideration have exactly the same structure as the AC states. (See the geometrical factors in Eq. (4·13) of Chap. 3.) 2) The mixing of the 1QP $d_{3/2}$ state in the collective $3/2_1^+$ state brings about a destructive effect on the g factor, since the values of $g(d_{3/2})$ and $g(d_{5/2})$ are of opposite signs. 3) The growth of the 3QP correlations among different orbits bring about the deviation from the property $g_{J-1}=g_J$. In the calculated g factors shown in Table XIII, the $3s_{1/2}$ quasi-particle participating in the dressed 3QP $3/2_1^+$ mode plays an important role in bringing about the deviation.

The magnitude of the difference between the values of $g(3/2_1^+)$ and $g(5/2_1^+)$ observed in experiments seems to be consistently accounted for by the effects 1) and 2). However, as shown in Table XIII, the calculated results show that the values of $g(3/2_1^+)$ are considerably affected by the effect 3). Due to the fact that the effect 3) depends quite sensitively on the single-particle energies adopted in the calculation, the calculated results are not in good agreement with the experimental data. A more accurate evaluation on the contribution of the

Table XIII. Calculated g factors of the $3/2_1^+$ states in odd-mass Mo and Ru isotopes. The calculated values of $g(3/2_1^+)$ listed in the fourth column are compared to the experimental values given in the fifth column. In the sixth column are listed the values calculated by adopting the ACS approximation. The calculated and experimental g factors of the 1QP $5/2_1^+$ states are given in the seventh and eighth columns, respectively, for the sake of comparison. The contributions from the g_{11} and g_{33} parts in Eq. (5·6) are explicity given in the second and third columns, respectively, with the aim of showing the coupling effects on the calculated $g(3/2_1^+)$ values listed in the fourth column. In these calculations, $g_s^{eff}=0.50\,g_s$ is adopted. The unit is $e\hbar/2Mc$.

	g_{11}	g_{33}	$g(3/2_1^+)_{cal}$	$g(3/2_1^+)_{exp}$	$g(3/2_1^+)_{ACS}$	$g(5/2_1^+)_{cal}$	$g(5/2_1^+)_{exp}$
^{95}Mo	0.001	−0.62	−0.62	−0.26±0.02 [a)] −0.24±0.03 [b)]	−0.35	−0.37	−0.365 [c)]
^{97}Mo	0.01	−0.26	−0.25		−0.33	−0.37	−0.373 [a)]
^{99}Mo	0.10	−0.10	−0.01		−0.28	−0.36	
^{97}Ru	0.003	−0.60	−0.60		−0.32	−0.37	
^{99}Ru	0.02	−0.31	−0.29	−0.189±0.004 [d)] −0.20 ±0.02 [e)]	−0.34	−0.36	−0.25 [f)]
^{101}Ru	0.08	−0.59	−0.48	−0.207±0.017 [g)]	−0.29	−0.35	−0.28 [c)]

[a)] Ref. 3), [b)] Ref. 4), [c)] Ref. 31), [d)] Ref. 9), [e)] Ref. 8), [f)] Ref. 11), [g)] Ref. 10).

$3s_{1/2}$ quasi-particle seems necessary.

As for the systematics of $B(M1;\ 3/2_1^+\to 5/2_1^+)$, a curious trend in the sequence of odd-mass Ru isotopes has been observed: The value decreases from ^{97}Ru ($N=53$) to ^{99}Ru ($N=55$) and then increases from ^{99}Ru to ^{101}Ru ($N=57$). (See Table XIV.) The origin of such a curious trend may be understood as follows: Since the transition matrix element of the type $\langle\Phi_\delta^{(1)}|\hat{O}_{1M}^{(-)}|\Phi_{n'I'K'}^{(3)}\rangle$ is extremely small, the M_{31} and M_{13} parts in (5·3) are negligibly small. Then, the $M1$ transition of interest can take place mainly through the M_{11}

Table XIV. $B(M1;\ 3/2_1^+\to 5/2_1^+)$ values in odd-mass Mo and Ru isotopes, calculated by taking account of the coupling effect. The unit is $(e\hbar/2Mc)^2$. The contributions from the M_{11} and M_{33} parts in Eq. (5·5) are explicitly given in the second and third columns, with the aim of showing that the $B(M1)$ values depend sensitively on the relative signs of them. In these calculations, $g_s^{eff}=0.50\,g_s$ is used.

$3/2_1^+\longrightarrow 5/2_1^+$

	M_{11}	M_{33}	$B(M1)_{cal}$	$B(M1)_{exp}$
^{95}Mo	−0.06	−0.05	3.0×10^{-3}	$(4.21\pm0.02)\times10^{-3}$ [a)]
^{97}Mo	−0.17	0.23	7.3×10^{-4}	$(2.2\pm0.4)\times10^{-2}$ [a)]
^{99}Mo	−0.47	0.32	4.2×10^{-3}	
^{97}Ru	−0.11	0.01	2.4×10^{-3}	2.20×10^{-2} [b)]
^{99}Ru	−0.26	0.43	6.9×10^{-3}	3.33×10^{-4} [c)]
^{101}Ru	0.42	0.70	2.8×10^{-1}	2.78×10^{-1} [c)]

[a)] Ref. 2), [b)] Ref. 6), [c)] Ref. 7).

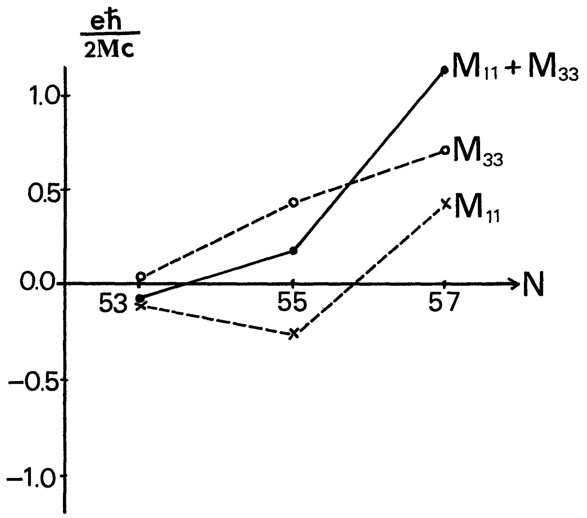


Fig. 10. The contributions from the M_{11} and M_{33} parts to the $B(M1;\, 3/2_1^+ \to 5/2_1^+)$ in Ru isotopes. (See also Table XIV.)

and M_{33} parts. The M_{11} part, which comes from the mixing effect of the 1QP $3/2^+$ state in the dressed 3QP $3/2_1^+$ state, represents the 1QP transition between the spin-flip single-particle orbits, $2d_{3/2}$ and $2d_{5/2}$. The M_{33} part, which comes from the mixing effect of the dressed 3QP $5/2^+$ states in the 1QP $5/2_1^+$ state, represents the transition between the dressed 3QP $3/2^+$ and $5/2^+$ states. The sign of the mixing amplitude $\zeta_{\nu=1}^{(1)}(3/2^+)$ involved in the M_{11} part depends essentially on the sign of the effective coupling strength $V_{\rm int}(a,\, nI)$ with $a=(2d_{3/2})$, $n=1$ and $I=3/2$, while the sign of the mixing amplitude $\zeta_{\nu=1}^{(3)}(5/2^+)$ involved in the M_{33} part depends on that of $V_{\rm int}(a,\, nI)$ with $a=(2d_{5/2})$, $n=1$ and $I=5/2$. Now, from the microscopic structure of the $V_{\rm int}(a,\, nI)$ given by Eq. (4·3) we can observe the following: The value of $V_{\rm int}(d_{3/2},\, 3/2^+)$ changes sign as one moves from ^{97}Ru to ^{101}Ru, while the value of $V_{\rm int}(d_{5/2},\, 5/2^+)$ conserves sign. The sign change in the former comes from the increase of the component $\{(\nu d_{5/2})^2 \nu s_{1/2}\}$ in the dressed 3QP $3/2_1^+$ mode, which has a sign opposite to the sign of the main component $\{(\nu d_{5/2})^3\}$. Thus, as is seen from Table XIV and Fig. 10, the phase relation between the M_{11} and M_{33} parts changes from destructive to constructive as one moves from ^{97}Ru to ^{101}Ru. Here the increase of the absolute magnitudes of the M_{11} and M_{33} parts represents the increasingly important role of the coupling effects as the neutron number goes from $N=53$ to $N=57$. In conclusion, we can say that the curious trend in $B(M1;\, 3/2_1^+ \to 5/2_1^+)$ comes from the change in the coherent property between the M_{11} and M_{33} parts, which is essentially determined by the structure change of the dressed 3QP $3/2_1^+$ mode. Since the values of the M_{11} and M_{33} parts depend rather sensitively on the adopted single-particle energies, a better agreement between theoretical and experimental $B(M1)$ values is obtainable within the framework of the introduced model if the adopted parameters are changed slightly.

For ^{95}Mo with $N=53$, it has been known that the shell-model calculation with the subspace consisting of the orbit $1g_{9/2}$ for protons and the orbit $2d_{5/2}$ for neutrons yields the low-lying $3/2_1^+$ state.[32),33)] Of course, the collective nature of the $3/2_1^+$ state cannot be fully accounted for in the Tamm-Dancoff approximation with restricted shell-model subspace. Recently, the collective structure of the $3/2_1^+$ state in ^{95}Mo has been investigated in terms of the semi-microscopic model in which the three-neutron valence-shell cluster is interacting with the quadrupole vibration of the "core".[38),39)] Similar investigations have also been done for odd-proton I isotopes with $Z=53$.[34)~37)] The results of these investigations indicate the remarkable improvements over the conventional particle-vibration-coupling model; namely, the appearance of the low-lying $3/2_1^+$ state (in ^{95}Mo) and $5/2_2^+$ states (in I isotopes) is well reproduced in these calculations, together with their enhanced $E2$-transition properties. This fact implies the importance of explicitly taking into account the three-particle correlations in the valence-shell orbits. As was discussed in § 3–3 of Chap. 3, the dressed 3QP modes under consideration are capable of decomposing into the form in which the direct relation with this semi-microscopic model is visualized. However, it should be emphasized that the essential role of the 3QP correlations (characterizing the collective excitations in spherical odd-mass nuclei) is not by any means specific to the single-closed-shell plus three-nucleon system such as ^{95}Mo and I isotopes. In fact, as we have seen, the collective $3/2_1^+$ and $5/2_2^+$ states appear quite regularly in nuclei with N or Z being 53, 55 and 57. The following fact should also be noted: In our model, for example, the collective $E2$ enhancements of the $3/2_1^+$ states (in odd-neutron Mo and Ru isotopes) are caused not only by the forward-going amplitudes of $(\pi\pi\nu)$-type but also by the backward-going amplitudes of $(\pi\pi\nu)$-type which represent the ground-state correlation. (See Table VI.) In particular, the ground-state correlation originating from the quadrupole force acting between proton- and neutron-quasi-particle-pairs plays an important role in enlarging the backward-going amplitudes of $(\pi\pi\nu)$-type. This implies that the internal structure of the quadrupole vibration (phonon) of the core is considerably affected by the interaction between the quasi-particles in the valence-shell orbits and the quasi-particles excited from the core. It seems difficult to take such an effect into account within the semi-microscopic model mentioned above.

§ 6. Concluding remarks

We have shown that the collective $5/2_2^+$ states in odd-proton I, Cs and La isotopes and the collective $3/2_1^+$ states in odd-neutron Mo and Ru isotopes are identified as the new elementary mode of collective excitation, i.e., the dressed

3QP mode. We have also shown that the physical condition for the appearance of the dressed 3QP modes is not specific to the AC states but quite general in spherical odd-mass nuclei. The presence of a high-spin orbit having parity opposite to that of the major shell, such as in the case of the AC states, is not a necessary condition for the realization of the dressed 3QP modes. Rather, the important condition is found in the shell structure of the orbits lying in the neighbourhood of the chemical potential. Even if many orbits having the same parity lie close to one another and the energy spacings between the orbit of interest and the others (with the same parity) are not so large as in the case of the AC states, one cannot expect a less dominant role of the 3QP correlation at the specific orbit lying in the vicinity of the chemical potential. Furthermore, the physical condition (in shell structure) weakening the effective coupling strength between the 1QP mode and the collective dressed 3QP mode (with the same spin and parity) is common to the condition for the realization of the AC state-like dressed 3QP mode. Thus, the dressed 3QP modes similar to the AC states can exist as relatively pure elementary excitation modes over a wide region of spherical odd-mass nuclei.

The essential roles of the 3QP correlation will not be restricted to characterize the AC state-like collective excitation modes having spin $(j-1)$ which have been investigated thus far. Rather, we should expect various roles of the 3QP correlation of which we know little at present. For example, the role of the 3QP correlation among quasi-particles in different orbits should be investigated further. In this chapter, the importance of such effects has only been briefly mentioned for the case of the $3/2_1^+$ states. In the succeeding chapter, standing on the new point of view acquired here, we will investigate microscopic structure of breaking and persistency of the conventional "phonon-plus-odd-quasi-particle picture." Our discussion will be extended to all low-lying collective excited states, including those having spins other than $(j-1)$. The present status of our picture of the low-energy excitations in spherical odd-mass nuclei will then be summarized.

Appendix 4A. Procedure of numerical calculation

Here, we describe a calculational method in solving the eigenvalue equation of the dressed 3QP mode, (3·3) of Chap. 2, full expression of which is given in Appendix 6A.

4A-1 *Orthonormal basis vectors*

In solving Eq. (3·3) of Chap. 2, we should first prepare the orthonormal basis vectors in the coupled-angular-momentum representation defined in Appendix 6A. Such a requirement is easily achieved by diagonalizing the projection operator $\boldsymbol{P}_I$, the matrix elements of which are $P_I(ab(J)c|a'b'(J')c')$ explicitly defined by (2A·6). From the property of the projection operator,

$P_I^2 = P_I$, it is clear that the eigenvalues take only the value $+1$ or 0:

$$U_I P_I U_I^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{4A·1}$$

where U_I denotes the unitary transformation which diagonalize P_I. The eigenvectors belonging to the eigenvalue 1 just coincide with the coefficients of fractional parentage (cfp) for $(j_a j_b j_c)$-configurations with seniority $v=3$ and total angular momentum I. The orthonormalized basis vectors are then obtained as

$$\bar{\psi}_{iI}[abc] = U_I(i)\psi_I[abc], \tag{4A·2}$$

where the vector $\psi_I[abc]$ is constructed from the elements

$$\{\psi_I[ab(J)c]\,;\, J,\ \mathscr{P}(abc)\}, \tag{4A·3}$$

with $\mathscr{P}(abc)$ denoting all the permutation with respect to (abc). In (4A·2), $U_I(i)$ denotes a row vector of the matrix U_I and the letter i labels the independent basis vectors. Needless to say, the projection operator P_I and the matrix U_I are both diagonal with respect to different sets of the orbital triad (abc).

4A–2 *Eigenvalue equation in terms of orthonormal vectors*

When the projection operator P_I is diagonalized, the matrix elements of the eigenvalue equation are reduced to the following form:

$$\begin{aligned} &\bar{D}_I[abc|a'b'c'] \\ &\quad \equiv U_I[abc]\cdot D_I[abc|a'b'c']\cdot U_I[a'b'c']^{-1} \\ &\quad = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} U_I[abc]\cdot M_I[abc|a'b'c']\cdot U_I[a'b'c']^{-1} \begin{bmatrix} 1' & 0' \\ 0' & 0' \end{bmatrix}, \end{aligned} \tag{4A·4}$$

where $D_I[abc|a'b'c']$ denotes the matrix composed of the elements $D_I[ab(J)c|a'b'(J')c']$ and $M_I[abc|a'b'c']$ the corresponding matrix excluding the projection operator P_I. In (4A·4), the matrix U_I which corresponds to a particular set of orbital triad (abc) is explicitly denoted as $U_I[abc]$. Thus, the vectors belonging to the zero-eigenvalue of the projection operator do not couple to the physical vectors having eigenvalue 1. We can also obtain the matrices $\bar{A}_I$ and $\bar{d}_I$ from A_I and d_I by the same procedure as above. In this way we obtain the eigenvalue equation written in terms of the orthonormal basis vectors as follows:

$$\omega_{nI}\begin{bmatrix} \bar{\phi}_{nI} \\ \bar{\varphi}_{nI} \end{bmatrix} = \begin{bmatrix} 3\bar{D}_I & -\bar{A}_I \\ \bar{A}_I^T & -\bar{d}_I \end{bmatrix} \begin{bmatrix} \bar{\phi}_{nI} \\ \bar{\varphi}_{nI} \end{bmatrix}. \tag{4A·5}$$

4A–3 *Two-step diagonalization of secular matrix*

We now diagonalize the secular matrix (4A·5) by the following two-step procedure. First we independently diagonalize the forward and backward matrices, $\bar{\boldsymbol{D}}_I$ and $\bar{\boldsymbol{d}}_I$:

$$V_I^{(f)}\cdot(3\bar{\boldsymbol{D}}_I)\cdot V_I^{(f)-1}=\begin{pmatrix}\omega_{1I}^f & & 0\\ & \omega_{2I}^f & \\ 0 & & \ddots\end{pmatrix}\equiv\boldsymbol{\omega}_I^f, \tag{4A·6}$$

$$V_I^{(b)}\cdot(-\bar{\boldsymbol{d}}_I)\cdot V_I^{(b)-1}=\begin{pmatrix}\omega_{1I}^b & & \\ & \omega_{2I}^b & \\ 0 & & \ddots\end{pmatrix}\equiv\boldsymbol{\omega}_I^b. \tag{4A·7}$$

The new secular matrix thus obtained, i.e.,

$$\begin{bmatrix}\boldsymbol{\omega}_I^f & -V_I^{(f)}\bar{\boldsymbol{A}}_I V_I^{(b)-1}\\ V_I^{(b)}\bar{\boldsymbol{A}}_I^T V_I^{(f)-1} & \boldsymbol{\omega}_I^b\end{bmatrix}, \tag{4A·8}$$

is then diagonalized as the second step. The two-step-diagonalization procedure is, of course, equivalent to the direct diagonalization. This method, however, possesses the following merits:

(1) The diagonal matrix $\boldsymbol{\omega}_I^f$ obtained in the first step gives us the solutions of the corresponding Tamm-Dancoff approximation, i.e., the solutions of the "bare" 3-quasi-particle states.

(2) In the second step, the secular matrix (4A·8) can be truncated in such a way that some restricted eigenvectors of $\bar{\boldsymbol{D}}_I$ and $\bar{\boldsymbol{d}}_I$ are sufficient to yield a good approximation for the full calculation. It should be noted here that the normalization of the correlation amplitudes given by (3·6) of Chap. 2 does not change at all by this truncation.

4A–4 *Another method for providing orthonormal basis vectors*

We can adopt an alternative method for providing the orthonormal basis vectors by rewriting the forward-going components of the eigenmode operator (3·1) of Chap. 2 as follows:

$$\begin{aligned}C_{nI}^\dagger=&\frac{1}{\sqrt{3!}}\sum_{aJ}\psi_{nI}[aa(J)a]\sum_{m_{\alpha_1}m_{\alpha_2}m_{\alpha_3}M}(j_aj_am_{\alpha_1}m_{\alpha_2}|JM)(Jj_aMm_{\alpha_3}|IK)a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger a_{\alpha_3}^\dagger\\ &+\frac{1}{\sqrt{2!}}\sum_{\substack{acJ\\(a\neq c)}}\psi_{nI}[aa(J)c]\sum_{m_{\alpha_1}m_{\alpha_2}m_\gamma M}(j_aj_am_{\alpha_1}m_{\alpha_2}|JM)(Jj_cMm_\gamma|IK)a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger a_\gamma^\dagger\\ &+\sum_{\substack{abcJ\\(a<b<c)}}\psi_{nI}[ab(J)c]\sum_{m_\alpha m_\beta m_\gamma M}(j_aj_bm_\alpha m_\beta|JM)(Jj_cMm_\gamma|IK)a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger\\ &+\sum_{(rs)cJ}\frac{\psi_{nI}[rs(J)c]}{\sqrt{1+\delta_{rs}}}\sum_{m_\rho m_\sigma m_\gamma M}(j_rj_sm_\rho m_\sigma|JM)(Jj_cMm_\gamma|IK)a_\rho^\dagger a_\sigma^\dagger a_\gamma^\dagger\\ &+\{\text{backward components being similar to the above definitions}\}.\end{aligned} \tag{4A·9}$$

In this method, the matrix of the secular equation becomes more complicated than that in the preceding subsections, hence we do not give its explict form. In the calculations of Chaps. 4, 5 and 6, we have independently adopted both methods for providing the basis vectors. They have been used to check the numerical calculations done there.

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