A Three Dimensional View of the SYK Model ?

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Large N and Space-time

- Generating real space from large number of internal degrees of freedom has a long history.

- Eguchi-Kawai Large-N reduction (1982)
- Matrix quantum mechanics (1990)
- BFSS Matrix Theory (1996)
- IKKT Matrix Theory (1996)
- AdS/CFT correspondence (1997)

Matrix Quantum Mechanics

- Here M is a $N \times N$ Hermitian matrix, whose dynamics is given by the Hamiltonian

$$H = \mathrm{Tr}\{-\frac{\partial^2}{\partial M \partial M} + V(M)\}$$

- At large N the singlet sector was solved by *Brezin, Itzykson, Parisi and Zuber (1980)*
- We can make a standard change of variables to the density of eigenvalues

$$M_{ij}(t) \to \rho(x,t) = \frac{1}{N}\mathrm{Tr}\delta(M - xI)$$

- The space of eigenvalues x becomes a real space.
- The theory can be written in terms of $\rho(x,t)$
- This is one of the earliest examples of holography – x is the holographic direction. *(S.R.D. & A. Jevicki, 1990)*

- In a certain limit this is in fact 1+1 dimensional string theory, whose only dynamical degree of freedom is represented by $\rho(x,t)$.
- In fact there is a detailed correspondence with usual string theory results.

(*Gross & Klebanov,; Sengupta & Wadia; Dhar, Mandal & Wadia; Polchinski and Naatsume)*

- Two dimensional string theory is interesting – this even has a black hole (*Mandal, Sengupta and Wadia; Witten).* However the black hole is not understood in the matrix model version very well. It is not in the singlet sector – and the entire theory is not solvable.

- The most interesting setup for holography is of course the AdS/CFT correspondence. (*Maldacena; Witten; Gubser, Klebanov &Polyakov*)
- E.g. a 3+1 dimensional field theory at large N becomes equivalent to a 9+1 dimensional string theory. Best understood in the supergravity limit.
- Matrix quantum mechanics in fact belongs to the same class –(*McGreevy & Verlinde*; *Klebanov, Maldacena & Seiberg*)
- However there is little hope that for e.g. N=4 one will be able to understand an explicit map from the field theory variables to the gravity variables and therefore understand how the large N theory grows these additional dimensions.
- It is clearly useful to look for solvable models of holography where we understand *how* this happens – such "recognizable carriatures" have always played key roles.
- This might also lead to a connection to more recent ideas of emergence of space-time from quantum entanglement and complexity.

Vector Models and Bilocal Fields

- Models with fields in vector representations (instead of adjoint representations) are usually solvable at large N – they have been useful to understand some aspects of the origins of holography.
- *Klebanov and Polyakov (2002)* conjectured that e.g. conformal 3d O(N) vector models are in fact dual to Higher Spin Gauge theories of Vasiliev type in one higher dimensional AdS space-time.
- *S.R.D. and A. Jevicki (2003)* proposal : this may be understood by recognizing that all invariants can be expressed in terms of Yukawa type bilocal fields.

$$\sigma(\vec{x},\vec{y}) = \frac{1}{N}\sum_i \phi^i(\vec{x})\phi^i(\vec{y})$$

- One can now express the path integral as a path integral over these bilocals. The technique for doing this was worked out by *A. Jevicki and B. Sakita (1982)*

- Now introduce center of mass and relative coordinates and express the relative coordinate in terms of the magnitude and angles, and perform an expansion

$$\vec{u} = \frac{1}{2}(\vec{x} + \vec{y}) \qquad \vec{v} = \frac{1}{2}(\vec{x} - \vec{y}) \equiv (r, \theta_1 \cdots \theta_{d-1})$$

$$\sigma(\vec{x}, \vec{y}) = \sum_{l, m_1 \cdots m_{d-2}} \varphi_{l,m_a}(\vec{u}, r) Y_{l,m_a}(\theta_a)$$

- Our first instinct was to identify r with the Poincare coordinate in AdS_{d+1}, and then the fields $\varphi_{l,m_a}(\vec{u}, r)$ would be the infinite tower of massless higher spin fields.
- If this is true, *all* the conformal transformations of $\vec{x}$ and $\vec{y}$, or equivalently $\vec{u}$, $\vec{v}$ should reproduce the Killing symmetries of AdS_{d+1} acting on higher spin fields.

- This works for dilatations, translations and rotations

$$
\begin{aligned}
\delta_D\sigma &= -\left(u^i\frac{\partial}{\partial u^i} + r\frac{\partial}{\partial r} + \Delta\right)\sigma \\
\delta_T\sigma &= t^i\frac{\partial}{\partial u^i}\sigma \\
\delta_R\sigma &= \theta^{ij}\left(u^i\frac{\partial}{\partial u^j} - u^j\frac{\partial}{\partial u^i} + L_{ij}\right)\sigma
\end{aligned}
$$

- Here Δ is the scaling dimension of σ

$$L_{ij} = v_i\frac{\partial}{\partial v_j} - v_j\frac{\partial}{\partial v_i}$$

are angular momentum generators which become interpreted as spin in the d+1 dimensional theory.

- These are now isometries of AdS_{d+1} with r being identified as the Poincare coordinate.

- However this does not work for the special conformal transformations

$$\delta_S \sigma = \{[2(\epsilon \cdot u)u^i - |u|^2\epsilon^i - r^2\epsilon^i]\frac{\partial}{\partial u^i} + 2(\epsilon \cdot u)r\frac{\partial}{\partial r} + 2\Delta(\epsilon \cdot u) + (\epsilon^j u^i - u^j \epsilon^i)L_{ji}\}\sigma$$
$$+ 2(\epsilon \cdot v)v^i \frac{\partial}{\partial u^i}\sigma$$

- The last term is not present for the corresponding isometry. In fact this term mixes up fields of different spins.

EXCEPT WHEN $d = 1$

- Vasiliev theory is supposed to be dual to the gauged version of the 2+1 dimensional O(N) model, e.g. with a Chern-Simons gauge field.
- This means that we need to restrict to the singlet sector. A way to do this is to consider bilocals at equal times and use a Hamiltonian formalism.
- For d=3 O(N) *De Mello Koch, Rodrigues, Jevicki and Jin (2011)* invented a non-local transformation of the fields in light front quantization to have these symmetries realized linearly – the resulting formalism works, but rather messy.
- It is not known how to impose suitable conditions on the unequal time bilocals to obtain this.
- Recently, however, the $d = 1$ bilocals have become quite useful in understanding a toy model of holography : the Sachdev-Ye-Kitaev model.

Bilocals in d=1

- When we have one dimension, the SL(2,R) transformations are

$$\begin{aligned} \delta x &= \epsilon_1, \qquad \delta x = \epsilon_2 x \qquad \delta x = \epsilon_3 x^2 \\ \delta y &= \epsilon_1, \qquad \delta y = \epsilon_2 y \qquad \delta y = \epsilon_3 y^2 \end{aligned}$$

- Defining $z = (x - y) \qquad t = (x + y)$
- We get the following transformations

$$\begin{aligned} \delta t &= \epsilon_1 \qquad \delta z = 0 \\ \delta t &= \epsilon_2 t \qquad \delta z = \epsilon_2 z \\ \delta t &= \frac{1}{2}\epsilon_3(t^2 + z^2) \qquad \delta z = \epsilon_3 t z \end{aligned}$$

- These are exactly the Killing isometries of Lorentzian AdS_2

$$ds^2 = \frac{1}{z^2}[-dt^2 + dz^2]$$

SYK Model

- This is a quantum mechanical model of N real fermions which are all connected to each other by a random coupling. The Hamiltonian is

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i\, \chi_j\, \chi_k\, \chi_l$$

- The couplings are random with a Gaussian distribution with width J
- This model is of interest since this displays quantum chaos and thermalization.
- When N is large, one can treat this using replicas, and Kitaev showed that one can restrict to replica diagonal space –basically may forget about replicas

- Averaging over the J_{ijkl} gives rise to the action

$$S = -\frac{1}{2}\int dt \sum_{i=1}^{N} \chi_i \partial_t \chi_i - \frac{J^2}{8N^3}\int dt_1 \int dt_2 (\sum_{i=1}^{N} \chi_i(t_1)\chi_i(t_2))^4$$

- We can now express the path integral in terms of bilocal collective field (*Jevicki, Suzuki and Yoon*)

$$\Psi(t_1, t_2) \equiv \frac{1}{N}\sum_{i=1}^{N} \chi_i(t_1)\chi_i(t_2)$$

- The path integral is now

$$\int \mathcal{D}\Psi(t_1, t_2)\ e^{-S_c[\Psi]}$$

- Where the collective action includes the jacobian for transformation from the original variables to the new bilocal fields

$$S_c[\Psi] \;=\; \frac{N}{2}\int dt_1 \partial_{t_1}\Psi(t_1,t_2)|_{t_1\to t_2} + \frac{N}{2}\,\mathrm{Tr}\log\Psi \;-\; \frac{J^2 N}{8}\int dt_1 dt_2\,\Psi^4(t_1,t_2)$$

- The equations of motion are the large N Dyson-Schwinger equations

$$\frac{\partial}{\partial t_1}\Psi(t_1,t_2) + \delta(t_1-t_2) - J^2\int dt_3[\Psi(t_3,t_1)]^3\Psi(t_3,t_2) = 0$$

- At strong coupling – which is the IR of the theory – the first term can be neglected
- The saddle point solution (*Kitaev; Polchinski and Rosenhaus*)

$$\Psi_0(t_1,t_2) \;=\; \left(\frac{1}{4\pi J^2}\right)^{\frac{1}{4}}\frac{\mathrm{sgn}(t_{12})}{\sqrt{|t_{12}|}}\;.$$

The Strong Coupling Spectrum

- Expand the bilocal action around the large N saddle point

$$\Psi(t_1, t_2) = \Psi_0(t_1, t_2) + \sqrt{\frac{2}{N}}\, \eta(t_1, t_2)$$

$$\eta(t_1, t_2) \equiv \Phi(t, z) = \sum_{\nu,\omega} \tilde{\Phi}_{\nu,\omega} u_{\nu,\omega}(t, z)$$

$$u_{\nu,\omega}(t, z) = \mathrm{sgn}(z)\, e^{i\omega t}\, Z_\nu(|\omega z|)$$

- Where $Z_\nu(x)$ denotes a complete orthonormal set of combination of Bessel functions (*Polchinski and Rosenhaus)*

$$Z_\nu(x) = J_\nu(x) + \xi_\nu\, J_{-\nu}(x)\,, \qquad \xi_\nu = \frac{\tan(\pi\nu/2) + 1}{\tan(\pi\nu/2) - 1}$$

- The order is real discrete $\nu = 3/2 + 2n$ or purely imaginary continuous $\nu = ir$

- These appear when one diagonalizes the kernel which appears in the quadratic action for $\eta(t_1, t_2)$ and are in fact eigenfunctions of the wave operator of a scalar field in AdS_2 with mass at the BF bound

$$\left[z^2(-\partial_t^2 + \partial_z^2) + \frac{1}{4}\right] e^{-i\omega t} z^{1/2} Z_\nu(\omega z) = \nu^2 \; e^{-i\omega t} z^{1/2} Z_\nu(\omega z)$$

- The orthonormality and completeness relations are

$$\int_0^\infty \frac{dx}{x} Z_\nu^*(x) \, Z_{\nu'}(x) \; = \; N_\nu \, \delta(\nu - \nu') \qquad N_\nu \; = \begin{cases} (2\nu)^{-1} & \text{for } \nu = 3/2 + 2n \\ 2\nu^{-1} \sin \pi\nu & \text{for } \nu = ir \, , \end{cases}$$

$$\int \frac{d\nu}{N_\nu} Z_\nu^*(|x|) \, Z_\nu(|x'|) \; = \; x \, \delta(x - x') \, .$$

- The integral here is a shorthand for a sum over discrete modes and an integral over imaginary values

- This leads to the quadratic action

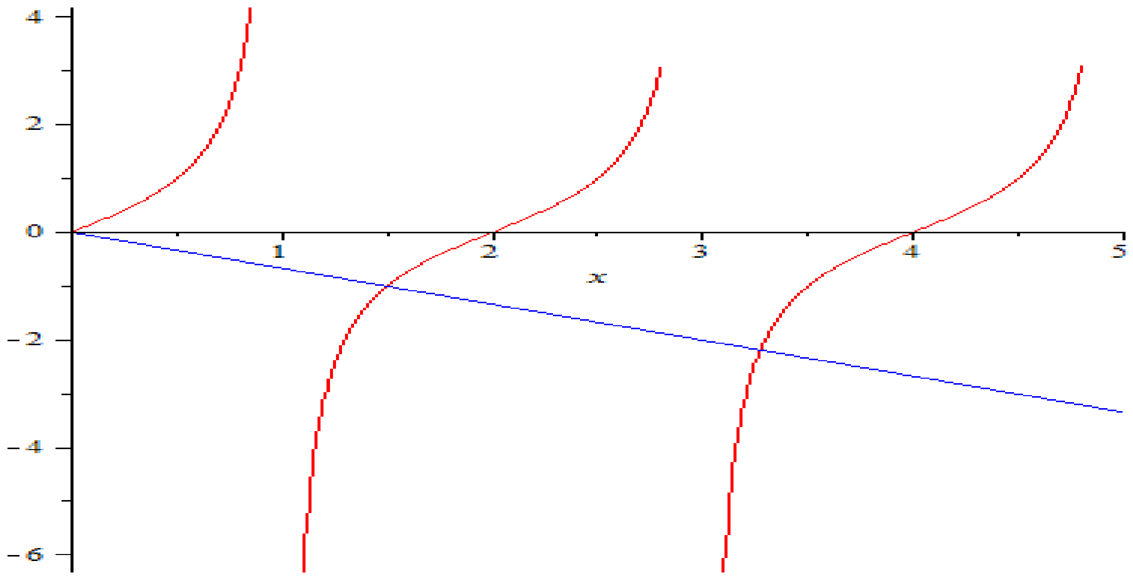
$$S_{(2)} = \frac{3J}{32\sqrt{\pi}} \sum_{\nu,\omega} N_\nu \, \tilde{\Phi}_{\nu,\omega} \big(\tilde{g}(\nu) - 1\big) \tilde{\Phi}_{\nu,\omega}$$

where

$$\boxed{\tilde{g}(\nu) = -\frac{2\nu}{3} \cot\left(\frac{\pi\nu}{2}\right)}$$

- The spectrum is therefore given by the solutions of the equation

$$\tilde{g}(\nu) = 1 \qquad\qquad \nu = p_m$$



The Bilocal Propagator

- The 4 point function of the fermions at large J has been calculated by Kitaev, *Polchinski and Rosenhaus*, *Jevicki, Suzuki and Yoon*, *Maldacena and Stanford*…..
- This is the two point function of the bilocal fluctuations. Performing the integral over ν the propagator can be expressed as a sum over poles

$$\mathcal{D}(t,z;t',z') = -\frac{32\pi^{\frac{3}{2}}}{3J}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-t')} \sum_{m=1}^{\infty} R(p_m)\, \frac{Z_{-p_m}(|\omega|z^{>})J_{p_m}(|\omega|z^{<})}{N_{p_m}}$$

- Here $z^{>}(z^{<})$ denotes the greater (smaller) of z and z', and

$$R(p_m) = \frac{3p_m^2}{[p_m^2+(3/2)^2][\pi p_m - \sin(\pi p_m)]}$$

is the residue at the pole $\nu = p_m$

- The object $\Phi(t, z)$ appeared as a field in 1+1 dimensions.
- However the field action in real space is non-polynomial in derivatives.

$$S^{(2)} = \int dt dz \{ (z^{1/2}\eta(t,z) \left[\tilde{g}(\sqrt{\mathcal{D}_B}) - 1 \right] (z^{1/2}\eta(t,z) \}$$

$$\mathcal{D}_B \equiv z^2(-\partial_t^2 + \partial_z^2) + \frac{1}{4}$$

- In fact the form of the propagator looks like a sum of contributions from an infinite number of fields in AdS
- The conformal dimensions of the corresponding operators are given by

$$h_m = \frac{1}{2} + p_m$$

- There is a special mode at $p_0 = 3/2$ which – at strong coupling – is a zero mode of the diffeomorphism invariance in the IR. We could have included this mode in the sum – this would lead to an infinite contribution.
- As is well known, the dynamics of this zero mode is given by the Schwarzian action.
- However this is all at infinite J
- For finite J, the diffeo is explicitly broken. The mode $p_0 = 3/2$ has a correction (*Maldacena and Stanford*)

$$p_0 = 3/2 - \alpha \frac{\omega}{J}$$

- This leads to the following enhanced contribution of this mode to the propagator in the zero temperature limit

$$J \int \frac{d\omega}{\omega^2} e^{i\omega(t'_+ - t_+)} \left[\frac{\sin(\omega t_-)}{\omega t_-} - \cos(\omega t_-) \right] \left[\frac{\sin(\omega t'_-)}{\omega t'_-} - \cos(\omega t'_-) \right]$$

$$t_\pm \equiv \frac{t_1 \pm t_2}{2}, \qquad t'_\pm \equiv \frac{t_3 \pm t_4}{2}$$

AdS Interpretation

- At strong coupling, it is natural to expect that this model is dual to some theory in two dimensional AdS_2
- *Maldacena, Stanford and Yan*g; *Engelsoy, Mertens and Verlinde* argued that the gravity sector should be Jackiw-Teitelboim type of dilaton-gravity in 1+1 dimensions coupled to some matter (*Alihemri and Polchinski*)

$$S = \frac{1}{16\pi G}\int \mathrm{d}^2x\sqrt{-g}\Big(\Phi^2 R - V(\Phi)\Big) + S_{\mathrm{matter}}$$

- Classical solutions have a AdS_2 metric, e.g. $ds^2 = \frac{1}{z^2}[-dt^2 + dz^2]$
- And a dilaton $\Phi^2 = 1 + \frac{a}{z}$
- A nonzero a breaks the conformal symmetry. Agrees with SYK where this breaking is in the UV (finite z). Naturally, $a \sim 1/J$

- The gravity sector of the theory is naturally thought as coming from the mode of the SYK model which is a zero mode at infinite coupling.
- In fact, the action – which comes entirely from boundary terms – is a Schwarzian action.
- *Mandal, Nayak and Wadia* has argued that the correct action is in fact Polyakov's action for 2d gravity.
- The matter part : the infinite number of poles of the SYK propagator indicates that there should be an infinite number of fields with conventional kinetics in this background.

The Matter as a KK tower

- We will now argue that this infinite tower of states is in fact the KK tower of a single scalar field in 2+1 dimensions with a Horava-Witten type compactification

$$ds^2 = \frac{1}{z^2}[-dt^2 + dz^2] + (1 - \frac{a}{z})^2 dy^2$$

- The third direction is an interval S^1/Z_2 of size 2L.
- To leading order in $a \sim 1/J$ the scalar field equation of motion

$$[\partial_t^2 - \partial_z^2 + \frac{m^2}{z^2} - \frac{1}{z^2}\partial_y^2 + V(y)]\Phi(t, z, y) = 0 \qquad V(y) = V\delta(y)$$

- We impose Dirichlet boundary condition at the end-points.
- The parameters V and L will be chosen appropriately.
- The metric above comes from near-horizon region of an extremal BTZ.

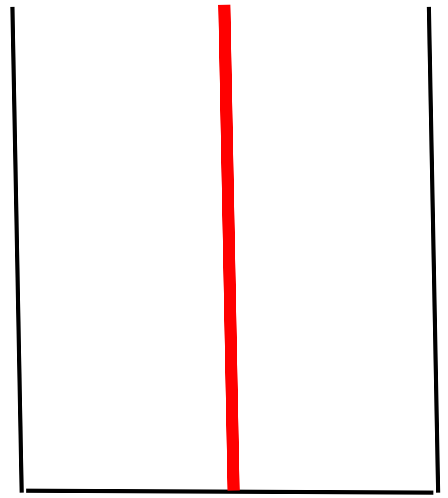
- Decompose the field

$$\Phi(t,z,y) = \int \frac{d\nu}{N_\nu} \int d\omega \sum_k \; e^{-i\omega t} z^{1/2} Z_\nu(|\omega| z) \; f_k(y) \; \chi(\omega,\nu,k)$$

- Where the $f_k(y)$ are eigenfunctions of the Schrodinger operator

$$\Big[-\partial_y^2 + V\delta(y) \Big] f(y) \; = \; k^2 \, f(y)$$

- This is a standard problem in quantum mechanics



- The odd parity solutions are

$$f(y) = \begin{cases} A\sin(k(y-L)) & (0<y<L) \\ A\sin(k(y+L)) & (-L<y<0) \end{cases} \qquad k = \frac{\pi n}{L}$$

- While the even parity solutions are

$$f(y) = \begin{cases} B\sin(k(y-L)) & (0<y<L) \\ -B\sin(k(y+L)) & (-L<y<0) \end{cases} \qquad B = \sqrt{\frac{2k}{2kL-\sin(2kL)}}$$

$$\boxed{-\frac{2}{V}k = \tan(kL)}$$

- If we choose $V=3$ $\quad L=\pi/2$ this is exactly the equation which determines the SYK spectrum.

- Substituting the expansion of the field in the action we get

$$S=\int\frac{d\omega}{2\pi}\int\frac{d\nu}{N_\nu}\sum_{p_m}(\nu^2-\nu_0^2)\chi^\star(\omega,p_m,\nu)\chi(\omega,p_m,\nu)$$

- where

$$\nu_0^2=m_0^2+\frac{1}{4}+p_m^2$$

- And p_m are the solutions of the equation

$$\frac{3}{2}\tan(\frac{\pi p_m}{2})=-p_m$$

- If we now choose $m_0^2=-1/4$ the poles of the propagator in (ω,p_m,ν) space are exactly the poles of SYK
- Note that $m_0^2=-1/4$ is the BF bound of AdS_2

The Green's function

wave function in 3rd direction

- The position space propagator is now given by

$$G^{(0)}(t,z,y;t',z',y') = -|zz'|^{\frac{1}{2}} \sum_{m=0}^{\infty} f_{p_m}(y) f_{p_m}(y') \int \frac{d\omega}{2\pi}\, e^{-i\omega(t-t')} \int \frac{d\nu}{N_\nu} \frac{Z_\nu^*(|\omega z|)\, Z_\nu(|\omega z'|)}{\nu^2 - p_m^2}\,.$$

- Once again the integral over ν stands for a sum over the discrete set and an integral over the imaginary axis.
- This integral can be performed with the result

$$G(t,z,y;t',z',y') = -\frac{1}{4}\int_{-\infty}^{\infty} d\omega \sum_{p_m} e^{-i\omega(t-t')} \frac{f_{p_m}(y) f_{p_m}(y')}{\sin(\pi p_m)} Z_{-p_m}(|\omega z^>|) J_{p_m}(|\omega z^<|)$$

- If we evaluate this at $y = y' = 0$ we get

$$G^{(0)}(t, z, 0; t', z', 0) = \frac{1}{3}|zz'|^{\frac{1}{2}} \sum_{m=0}^{\infty} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-t')}\, R(p_m)\, \frac{Z_{-p_m}(|\omega|z^{>}) J_{p_m}(|\omega|z^{<})}{N_{p_m}}\,.$$

$$R(p_m) = \frac{3p_m^2}{[p_m^2 + (3/2)^2][\pi p_m - \sin(\pi p_m)]}$$

- The nontrivial factor come from the wavefunctions evaluated at y = 0 and from other normalization factors.
- These factors exactly agree with the SYK result. There these factors come from the residues at poles apart from an overall number.

- The contribution from the $p_m = 3/2$ mode is divergent.
- This happens because the propagator contains $Z_{-p_m}(|\omega z^>|)$ which includes $\xi_{-3/2}$ - this is infinite
- This is exactly as in the SYK model if we had worked at infinite coupling.

- Note that the odd parity modes did not play any role. It is in fact natural to consider the 3rd direction as an interval [0,L] with Dirichlet boundary conditions at one end and a specified value of the derivative at the other end *(Witten)*

$$\frac{df(y)}{dy}|_{y=0} = \frac{V}{2} f(0)$$

Eigenvalue shift

- The metric along the 3rd direction contains the parameter $a \sim 1/J$

$$ds^2 = \frac{1}{z^2}[-dt^2 + dz^2] + (1 - \frac{a}{z})^2 dy^2$$

- We now calculate the change of the spectrum perturbatively in this parameter.

$$\boxed{S = \frac{1}{2}\int dzdy \int \frac{d\omega}{2\pi}\, \chi_{-\omega}\left(\mathcal{D}_0 + \mathcal{D}_1\right)\chi_\omega}$$

- where

$$\mathcal{D}_0 = \partial_z^2 + \omega^2 - \frac{m_0^2}{z^2} + \frac{1}{z^2}\left(\partial_y^2 - V(y)\right),$$

$$\mathcal{D}_1 = \frac{a}{z}\left[\partial_z^2 - \frac{1}{z}\partial_z + \omega^2 - \frac{m_0^2}{z^2} - \frac{1}{z^2}\left(\partial_y^2 + V(y)\right)\right]$$

- We already solved the eigenvalue problem for $\mathcal{D}_0$. The eigenfunctions are

$$z^{1/2} Z_\nu(|\omega| z) \; f_{p_m}(y)$$

- With the eigenvalues

$$(\nu^2 - p_m^2)$$

- The first order shift of the eigenvalues are given by $< \nu, p_m | \mathcal{D}_1 | \nu, p_m >$
- The result for the shift of the "zero mode" is

$$< \nu, p_0 | \mathcal{D}_1 | \nu, p_0 >= \frac{a|\omega|}{2\pi} (2 + q_0^2)$$

- Where

$$q_0 = \int dy f_{p_m}(y) [-\partial_y^2 - V\delta(y)] f_{p_m}(y)$$

- Recalling that $a \sim 1/J$ this is exactly of the same form as the answer obtained by Maldacena and Stanford in the SYK model

The Enhanced Propagator

- We can now use this to calculate the contribution of this mode to the propagator. In the expression for the infinite coupling propagator

$$G^{(0)}(t,z,0;t',z',0) = \frac{1}{3}|zz'|^{\frac{1}{2}} \sum_{m=0}^{\infty} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-t')}\, R(p_m)\, \frac{Z_{-p_m}(|\omega|z^>)J_{p_m}(|\omega|z^<)}{N_{p_m}} \,.$$

- The term with m=0 has $p_m = 3/2$ . . The divergence comes because

$$Z_{-p_m} = J_{-p_m}(\omega z) + \frac{\tan(\pi p_m/2) - 1}{\tan(\pi p_m/2) + 1} J_{p_m}(\omega z)$$

- The coefficient of J_{p_m} is divergent at $p_m = 3/2$

- To calculate the leading effect of a finite J we need to substitute the shift of the eigenvalues which we calculated

$$p_0 = 3/2 + a\omega$$

- This leads to the "enhanced" contribution

$$G^{(0)}_{\text{zero-mode}}(t, z, 0; t', z', 0) = -\frac{9\pi}{4a}\frac{B_0^2}{(2+q_0^2)}|zz'|^{\frac{1}{2}}\int_{-\infty}^{\infty}\frac{d\omega}{|\omega|}e^{-i\omega(t-t')}J_{\frac{3}{2}}(|\omega z|)J_{\frac{3}{2}}(|\omega z'|)\,.$$

- Using

$$J_{3/2} = \sqrt{\frac{2}{\pi}}z^{-3/2}(\sin z - z\cos z)$$

- Recalling that $a \sim 1/J$, this is in agreement with the SYK result of *Maldacena and Stanford*

-

q- Generalizations

- *Maldacena and Stanford* have studied generalizations of the SYK model with a q - fermion interaction.
- After averaging over the original random couplings,

$$S = -\frac{1}{2}\int dt \sum_{i=1}^{N} \chi_i \partial_t \chi_i - \frac{J^2}{8N^3}\int dt_1 \int dt_2 (\sum_{i=1}^{N} \chi_i(t_1)\chi_i(t_2))^q$$

- One can proceed to the bilocal theory in a similar manner and obtain the quadratic action for fluctuations around the saddle

$$S^{(2)} \sim \int d\nu \int d\omega \tilde{\Phi}^{\star}_{\nu,\omega}[\tilde{\kappa}(\nu) - 1]\tilde{\Phi}_{\nu,\omega}$$

- The function which appears is

$$\tilde{\kappa(\nu)} = -\frac{1}{(q-1)}\frac{\Gamma(\frac{1}{2}+\frac{1}{q})\Gamma(\frac{1}{q})}{\Gamma(\frac{3}{2}-\frac{1}{q})\Gamma(1-\frac{1}{q})}\frac{\Gamma(\frac{5}{4}-\frac{1}{q}+\frac{\nu}{2})\Gamma(\frac{5}{4}-\frac{1}{q}-\frac{\nu}{2})}{\Gamma(\frac{1}{4}+\frac{1}{q}+\frac{\nu}{2})\Gamma(\frac{1}{4}+\frac{1}{q}+\frac{\nu}{2})}$$

- The spectrum is then determined by solving

$$\tilde{\kappa(\nu)} = 1$$

- Does this follow from a 3 dimensional model ?

- Indeed it does
- The three dimensional background is now conformal to $AdS_2 \times [I]$
- The scalar field action now involves a non-trivial potential as well.
- We have now shown that the spectrum can be exactly reproduced in such a 3d model.
- We have not established a detailed correspondence with the SYK propagator as yet.
- In the large q limit, only one of these modes survives, and the effective theory is in fact a Liouville theory.

Comments

- It is important to note that the propagator we have used to identify with the fermion four point function in the SYK model is a non-standard propagator.
- This uses the modes $Z_\nu(x)$ rather than the modes $J_\nu(x)$
- The standard AdS bulk propagator for $p_m = 3/2$ is finite *(Maldacena)*.
- In this sense this is somewhat different from standard AdS/CFT.

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- In this sense this is somewhat different from standard AdS/CFT.
- The space in which the bilocals live is now-a-days called a Kinematic space – which is sometimes a de Sitter space.

Comments

- It is important to note that the propagator we have used to identify with the fermion four point function in the SYK model is a non-standard propagator.
- This uses the modes $Z_\nu(x)$ rather than the modes $J_\nu(x)$
- The standard AdS bulk propagator for $p_m = 3/2$ is finite *(Maldacena)*.
- In this sense this is somewhat different from standard AdS/CFT.
- The space in which the bilocals live is now-a-days called a Kinematic space – which is sometimes a de Sitter space.
- In fact AdS_2 is not very different from dS_2 : the propagator may be more easily interpretable in de Sitter *(Maldacena).*

- There are other puzzles as well – our *"phenomenological"* model the gravity background is fixed – in fact the $p_m = 3/2$ mode, which is supposed to be the gravity mode is already contained in our 3d scalar field.
- A lot remains to be understood.
- However we believe that the 3 dimensional nature of the dual theory is here to stay.
- Unpacking a theory with all powers of derivatives to a theory in one additional dimension is rather rare – the fact that this works nicely for all values of q is a strong indication that there is something in here…..

THANK YOU