Axion Inflation and the Lattice Weak Gravity Conjecture

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IAS

Based On

- 1409.5793/hep-th, 1503.00795/hep-th
- 1506.03447/hep-th, 1509.06374/hep-th
  - with Ben Heidenreich, Matthew Reece
- To appear
  - with Ben Heidenreich, Cody Long, Liam McAllister, Matthew Reece, John Stout

Outline

I. The Weak Gravity Conjecture

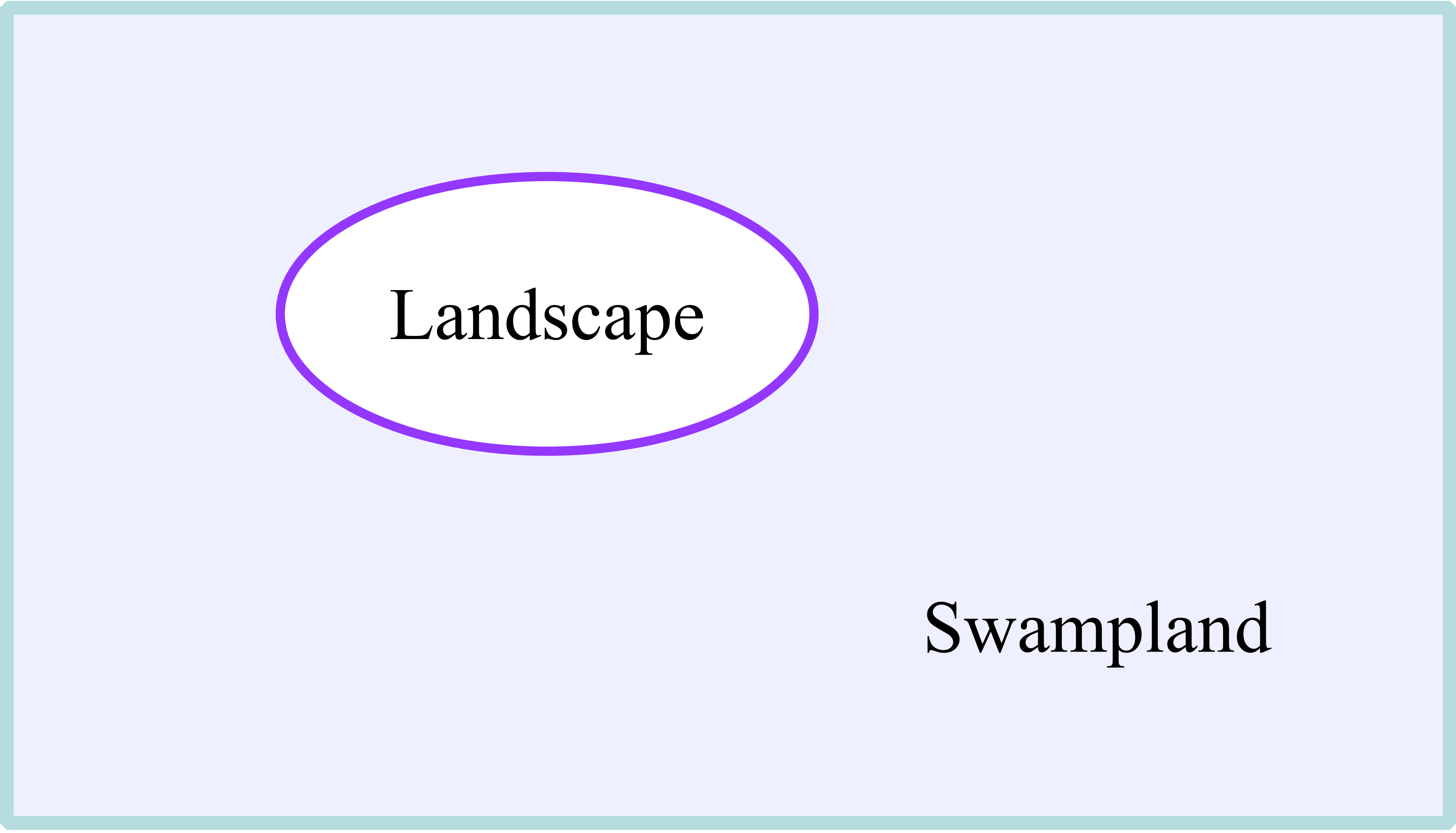
II. The WGC and Inflation

III. Strong Forms of the WGC

IV. Strong Forms of the WGC and Inflation

The Weak Gravity Conjecture

The Big Picture



Vafa ’06, Ooguri, Vafa ‘06

The Weak Gravity Conjecture

The WGC: In any $U(1)$ gauge theory that admits a UV completion with gravity, there must exist a state of charge q, mass m, such that

$$\frac{q}{m} \geq \frac{Q}{M}\Big|_{\text{extremal}}$$

Arkani-Hamed, Motl, Nicolis, Vafa, '06

The Generalized WGC

The generalized WGC: In any p-form theory in d dimensions, there must exist an electrically charged object of dimension $p-1$ and a magnetically charged object of dimension $d-p-1$ with

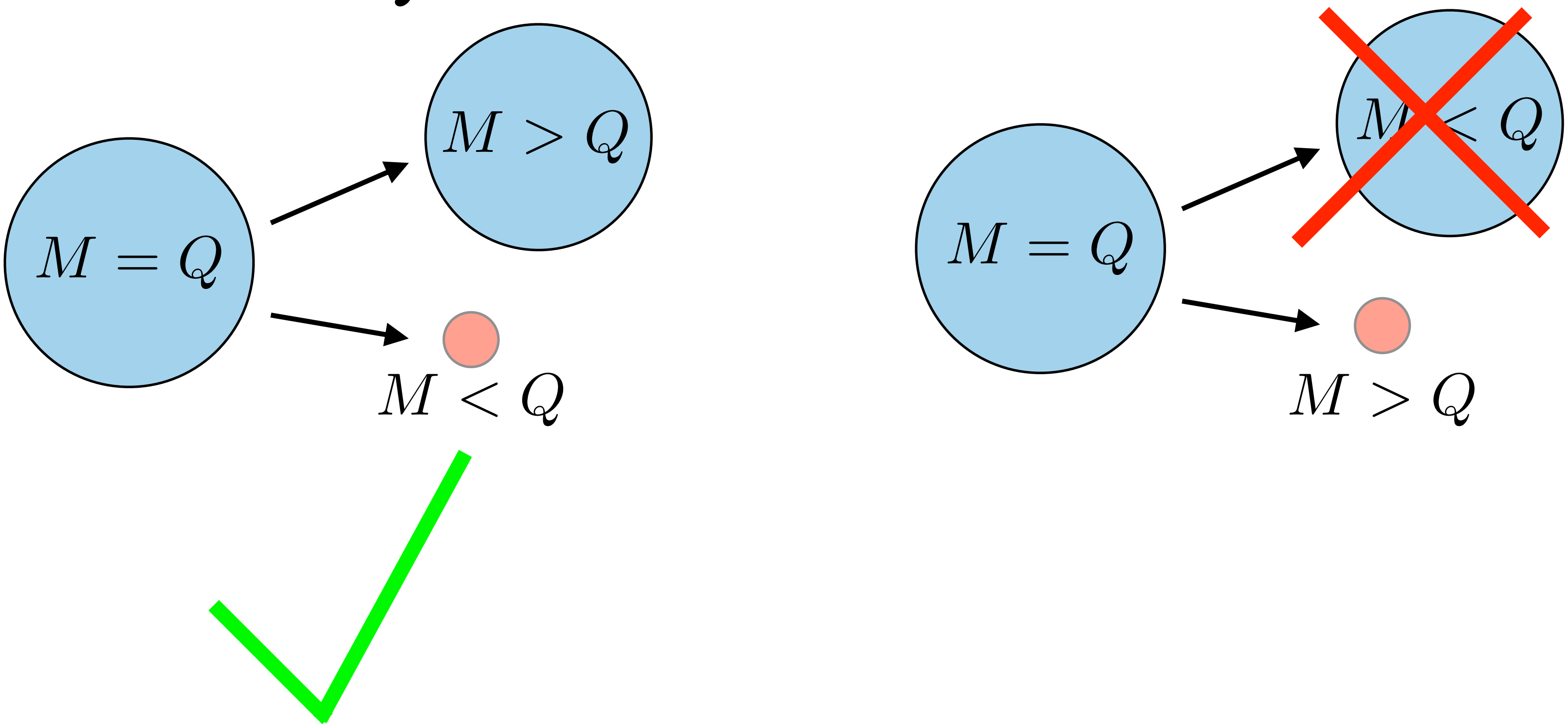
$$T_{el} \lesssim \left(\frac{g^2}{G_N}\right)^{1/2} , \quad T_{mag} \lesssim \left(\frac{1}{g^2 G_N}\right)^{1/2}$$

Strong Forms of the WGC

- The lightest charged particle must satisfy $q/m \geq Q/M|_{ext}$.
- The particle of smallest charge must satisfy $q/m \geq Q/M|_{ext}$.
- "Most" charge sites q must contain a particle with $q/m \geq Q/M|_{ext}$.

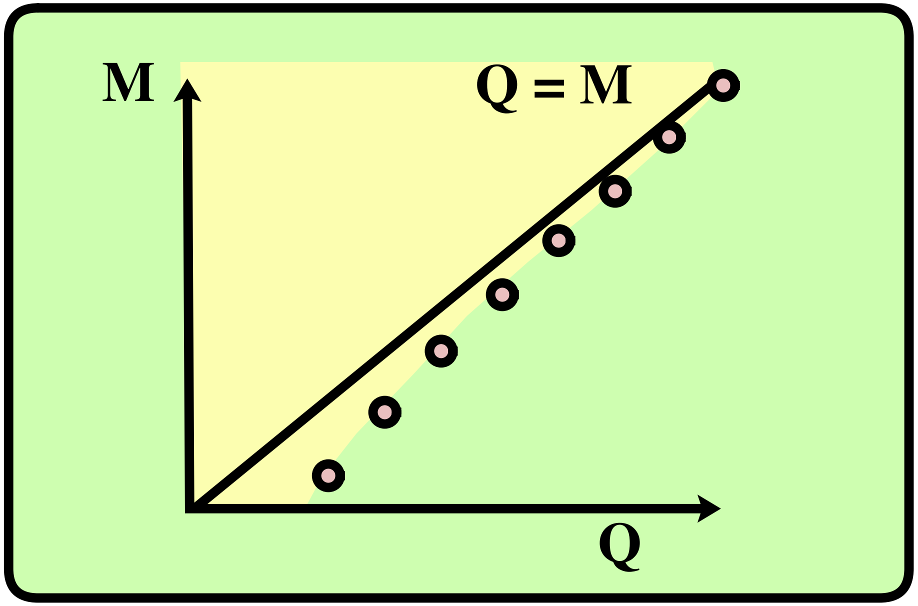
Why should the WGC be true?

- If not, extremal BH will be unable to decay.
- If not, near-extremal BH will move towards extremality



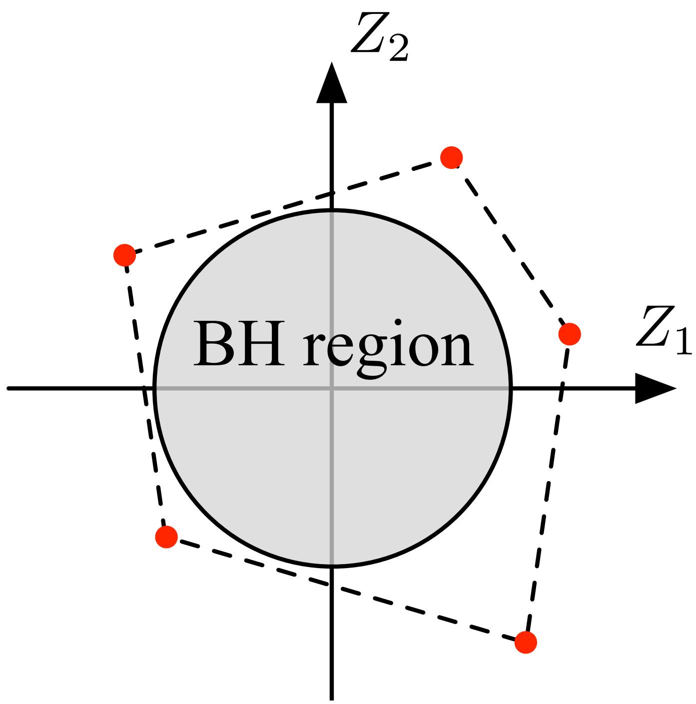
Why should the WGC be true?

- Many examples from string theory, KK theory:



The WGC for Multiple *U(1)*s

1. Take theory with N 1-form $U(1)$s.
2. Consider charge-to-mass vectors $\vec{z}_i = \frac{\vec{q}_i}{m_i} M_p$.
3. WGC: Convex Hull of $\{\vec{z}_i\}$ must contain unit ball in $\mathbb{R}^N$.



Cheung, Remmen '14

The WGC and Inflation

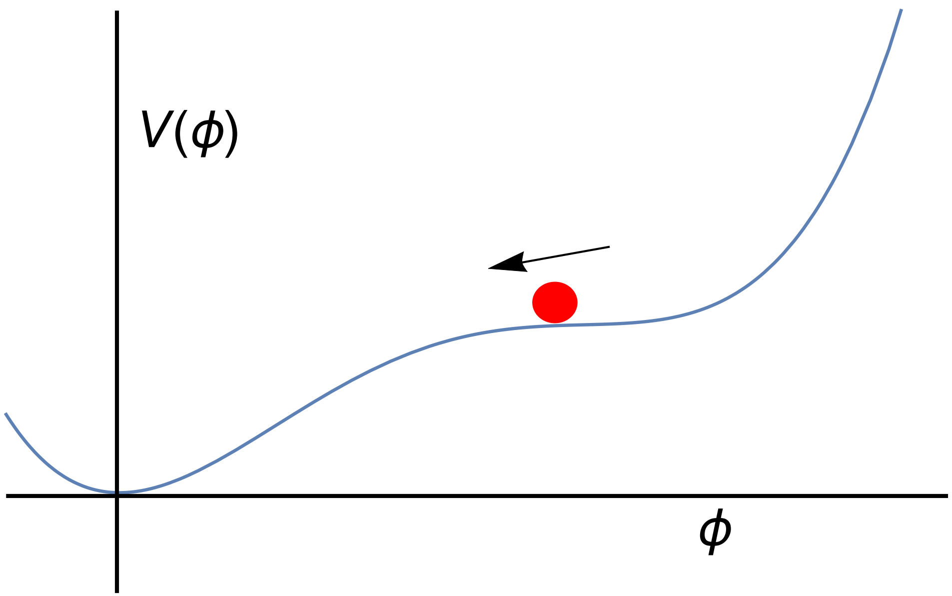
Inflation

Problem: Why is the universe so flat and so homogeneous?

Solution: Inflation—postulated period of exponential expansion in the early universe

Inflation in Field Theory

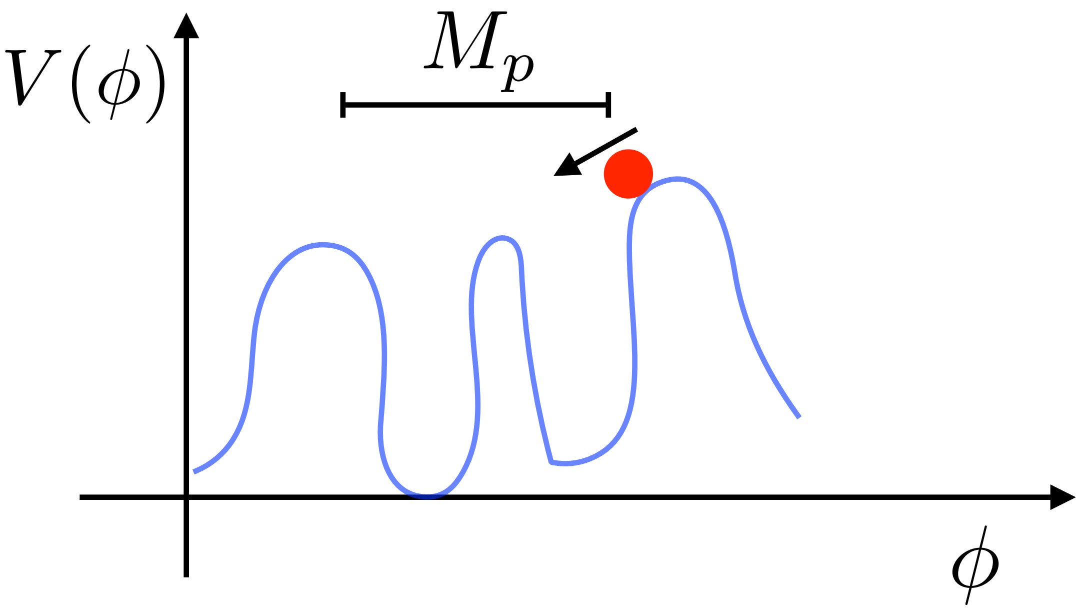
- Inflation can be thought of as the theory of a ball rolling down a hill with friction



$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \qquad \eta_V = M_p^2 \frac{V''(\phi)}{V(\phi)}.$$

Inflation in Field Theory

- Large-field inflation $(\Delta\phi \gtrsim M_p)$ runs afoul of QG:



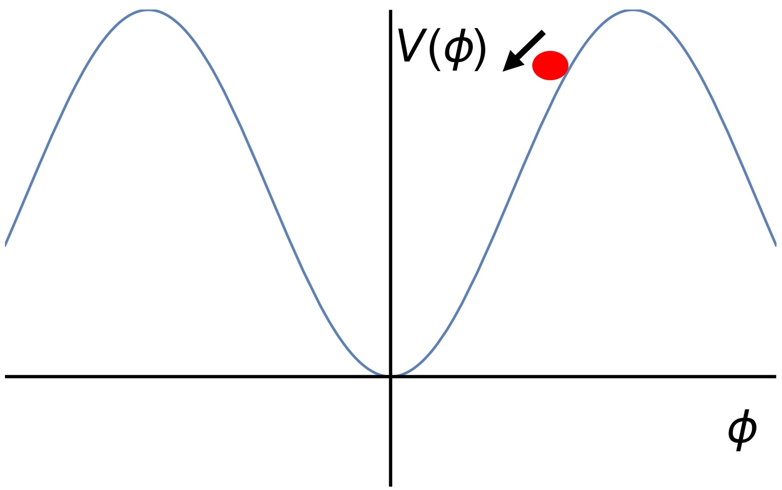
- Solution: impose shift symmetry $\phi \to \phi + c$

Axion Inflation

- Axions (scalars with perturbative shift symmetry) acquire periodic potential via instanton effects:

$$V(\phi) = \Lambda^4 (1 - \cos \frac{\phi}{f}) + \ldots$$

"axion decay constant"



- Need $f > M_p$ for inflation

WGC and Axion Inflation

Consider 0-form ϕ with decay constant f.

Charged object is an instanton of action S.

$$\mathrm{WGC} \Rightarrow 1/(fS) \gtrsim 1/M_p .$$

$$S > 1 + \mathrm{WGC} \Rightarrow f \lesssim M_p$$

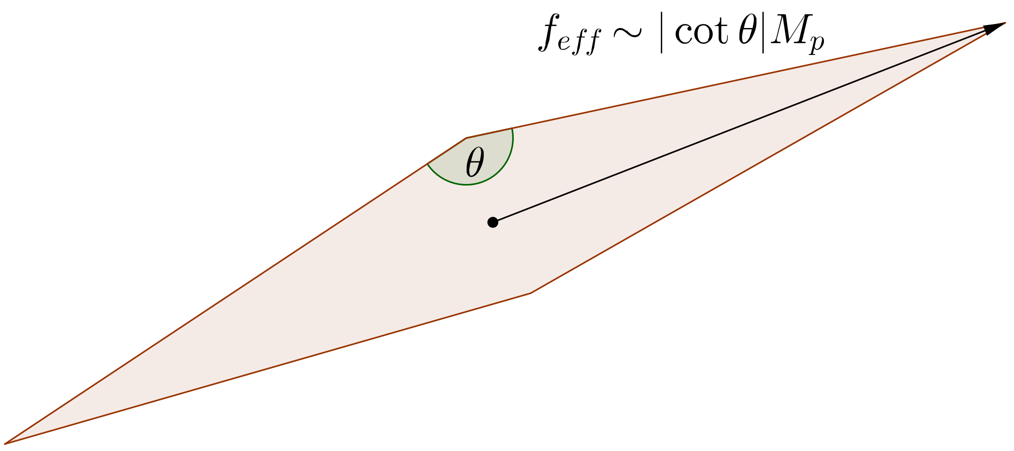
Incompatible with inflation!

Agrees with string theory

Banks, Dine, Fox, Gorbatov '03

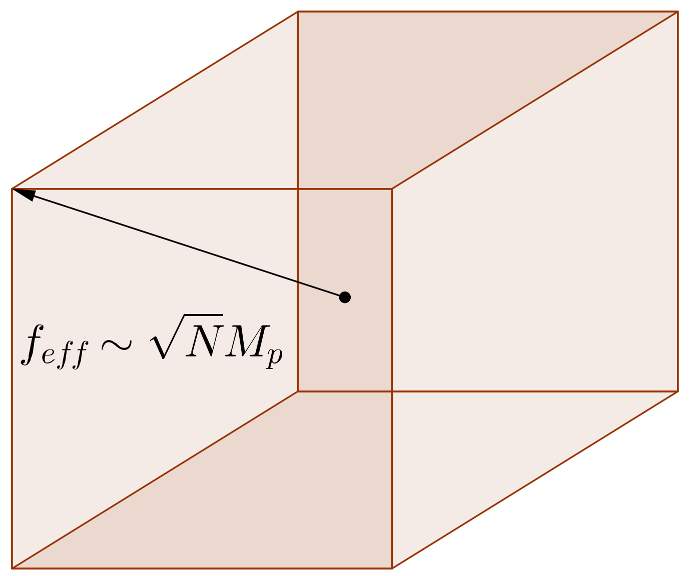
Multi-Axion Inflation

- Recruit additional axions:



Decay Constant Alignment

Kim, Nilles, Peloso '04

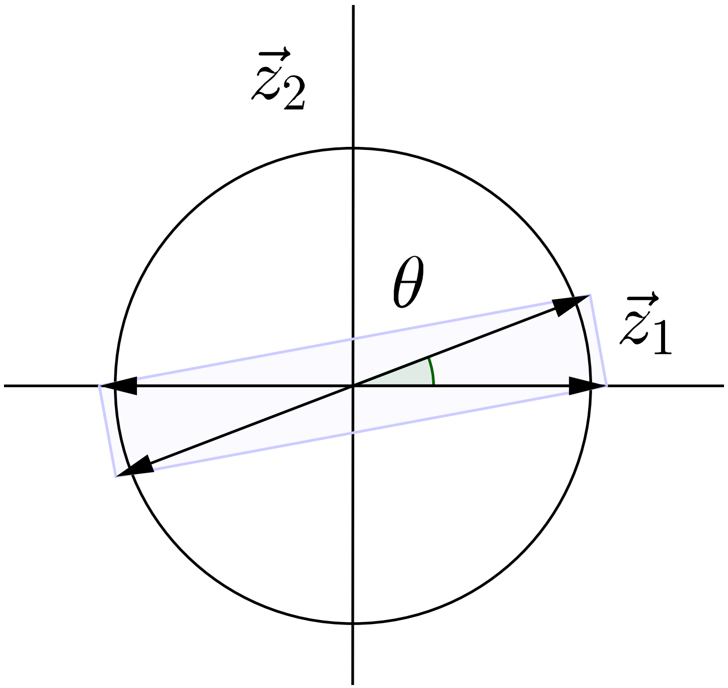


N-flation

Dimopoulos, Kachru,
McGreevy, Wacker '05

Multi-Axion Inflation

- Simplest models violate convex hull condition



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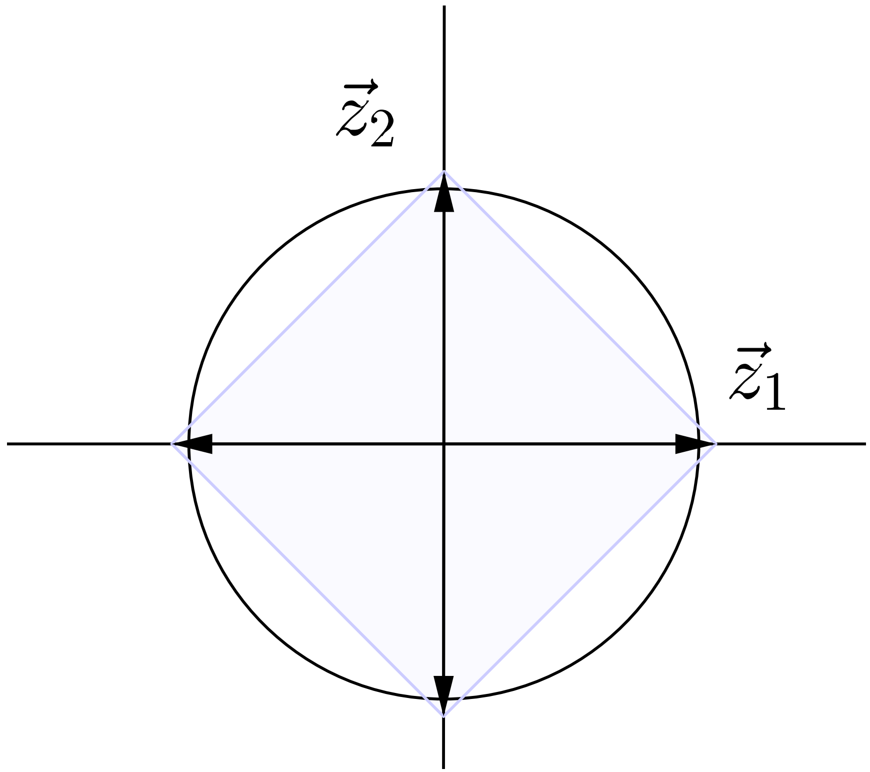
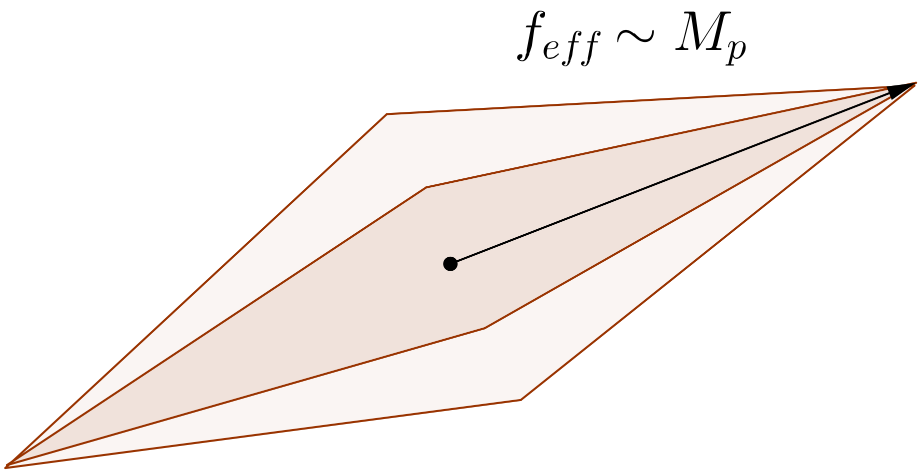


N-flation

Multi-Axion Inflation

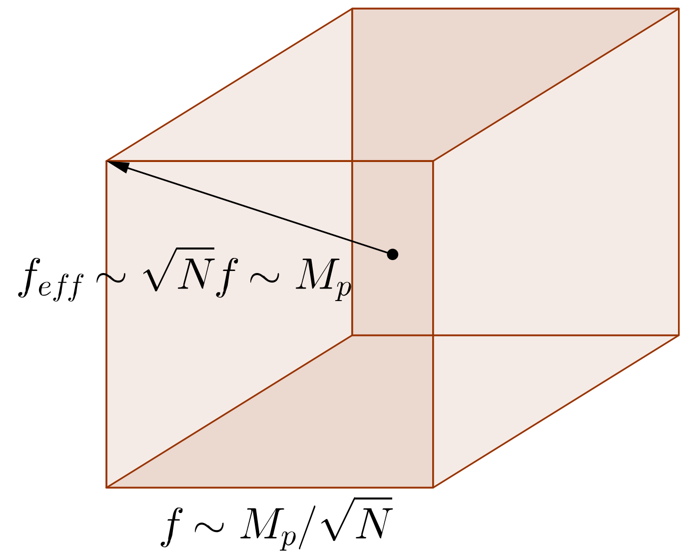
- Similar constraints apply to simplest multi-axion models

TR '14, '15
Brown, Cottrell, Shiu, Soler '15
Montero, Uranga, Valenzuela '15



Decay Constant Alignment

Kim, Nilles, Peloso '04



N-flation

Dimopoulos, Kachru, McGreevy, Wacker '05

A Loophole

- Take a model with two instantons:

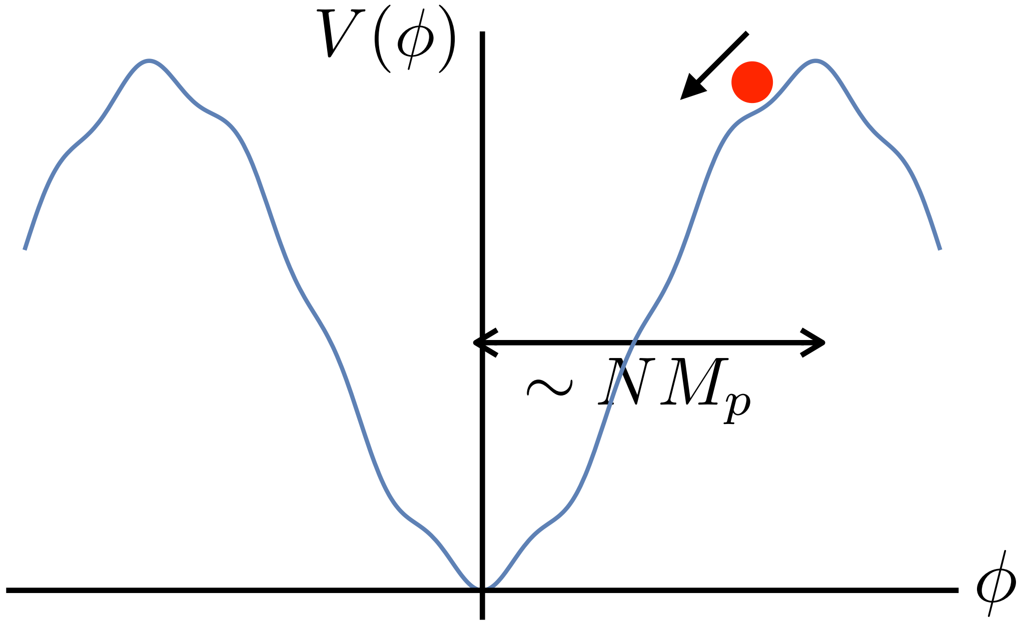
$$V \sim e^{-S_1}(1 - \cos\frac{\phi}{f_1}) + e^{-S_N}(1 - \cos\frac{\phi}{f_N})$$

Instanton charge	1	N
Decay constant	$f_1 = \frac{N}{4} M_p$	$f_N = \frac{1}{4} M_p$
Instanton Action	$S_1 = 1$	$S_N = 4$
$Z_{\mathrm{W}GC} = M_p/(fS)$	$Z_1 = 4/N$	$Z_N = 1 \gtrsim 1$

Satisfies WGC!

A Loophole

$$V \sim e^{-S_1}(1 - \cos\frac{\phi}{f_1}) + e^{-S_N}(1 - \cos\frac{\phi}{f_N})$$



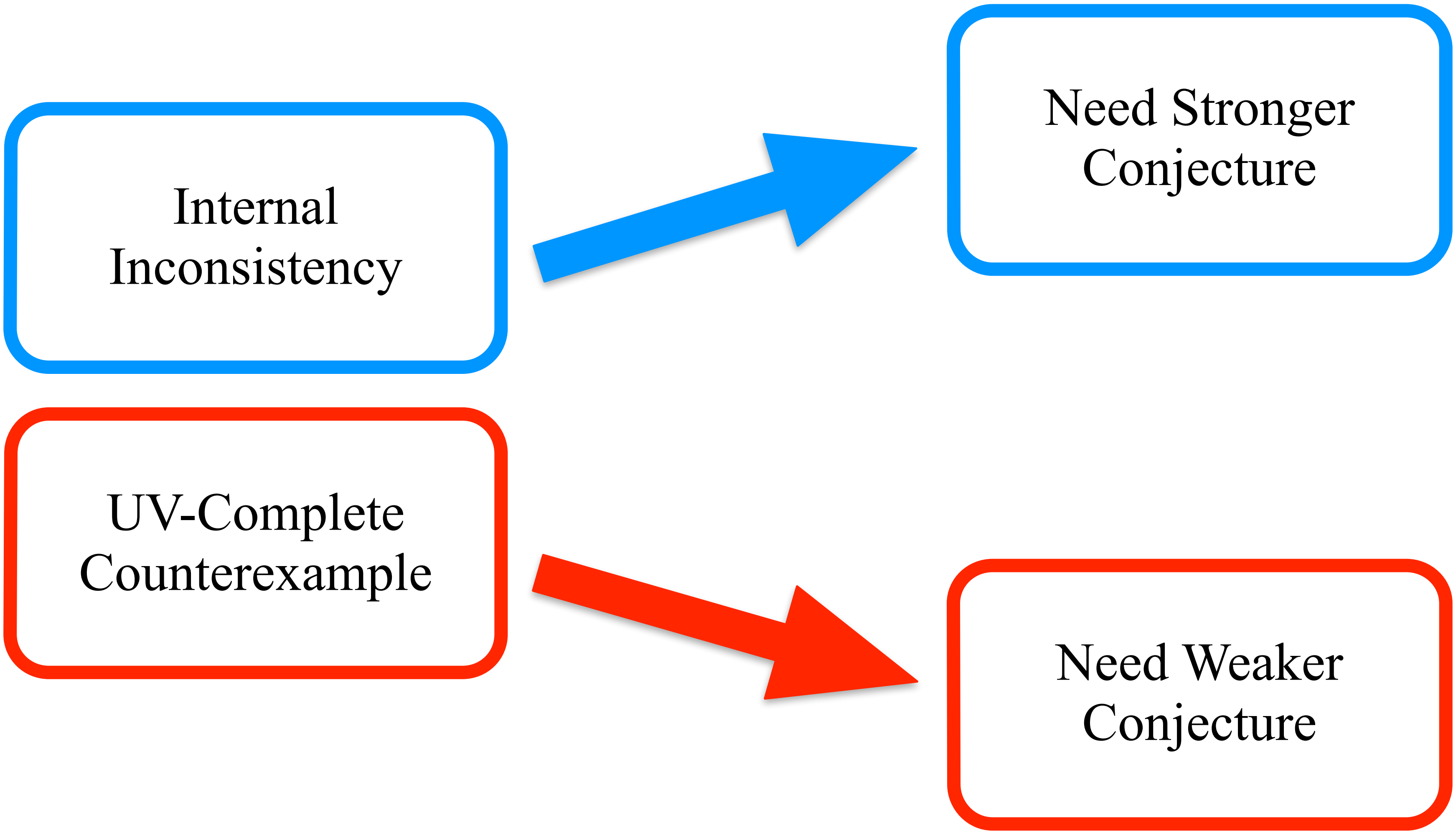
$\Rightarrow$ super-Planckian f_1 $\Rightarrow$ successful inflation!

Invoking a Strong Form

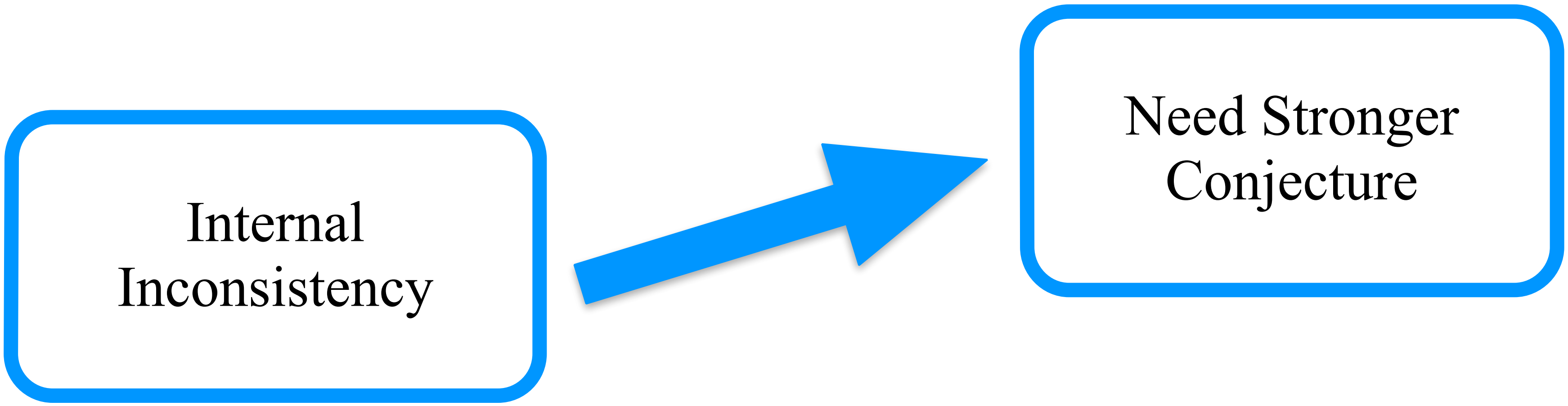
- In this model:
  - The instanton of smallest action does not satisfy the WGC bound
  - The instanton of smallest charge does not satisfy the WGC bound
  - "Most" charge sites are not occupied by instantons satisfying the WGC bound
- Similar considerations apply to multi-axion models of inflation

Strong Forms of the WGC

Methodology

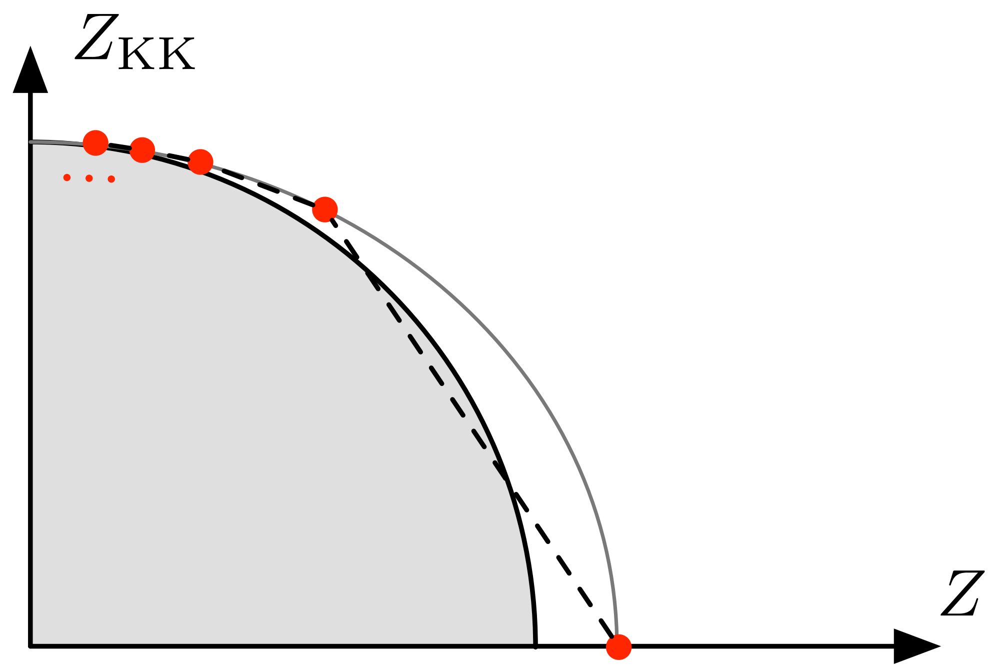


Methodology



The WGC and Dim. Reduction

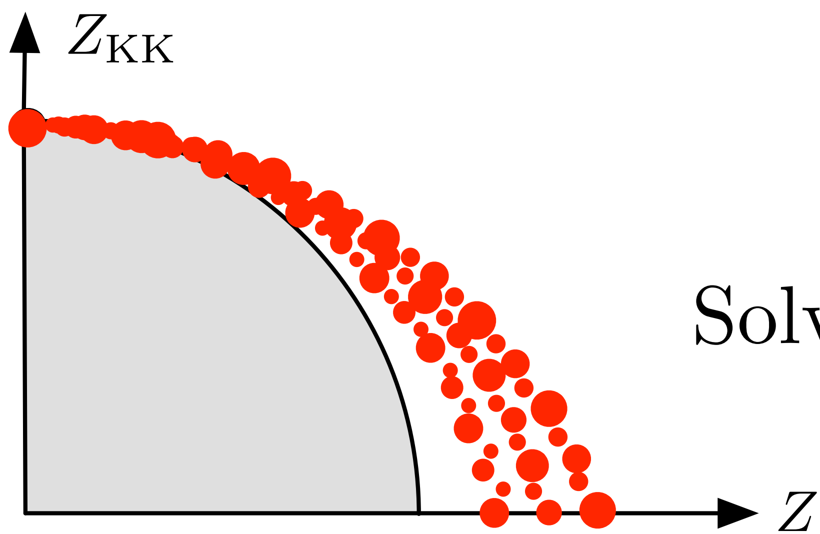
- Consider *(d+1)*-dimensional U(1) theory satisfying WGC
- KK reduce on small circle
- Find violation of WGC in *d* dimensions!



The sLWGC

- Leads us to suspect strong form:

Sublattice Weak Gravity Conjecture: There exists some sublattice of the full charge lattice with particles satisfying $|\vec{q}|/m \geq |\vec{Q}|/M|_{\text{ext}}$.



Solves inconsistency!

Top-Down Evidence for the sLWGC

- $SO(32)/E_8 \times E_8$ heterotic string states
- $T^{4,6}/\Gamma$ orbifolds of type II/heterotic string theory
- Higgs branch of $T^4/\mathbb{Z}_3$ heterotic orbifold

Top-Down Evidence for the sLWGC

- Perturbative string partition function:

$$Z(\mu, \bar{\mu}, \tau, \bar{\tau}) := \mathrm{Tr}(q^{\Delta} \bar{q}^{\tilde{\Delta}} y^{Q} \bar{y}^{\tilde{Q}})$$

$$q = e^{2\pi i \tau} \qquad\qquad y = e^{2\pi i \mu}$$

$$\Delta, \tilde{\Delta} = \text{conformal weights}$$

$$Q, \tilde{Q} = \text{left, right-moving charges}$$

Top-Down Evidence for the sLWGC

- Modular invariance implies spectrum is invariant under shifts

$$Q \to Q + \rho,\ \tilde{Q} \to \tilde{Q} + \tilde{\rho} \qquad \rho \in \Gamma_Q^*,\ \tilde{\rho} \in \tilde{\Gamma}_Q^*$$

with $\Delta - \frac{1}{2}Q^2,\ \tilde{\Delta} - \frac{1}{2}\tilde{Q}^2$ held fixed,

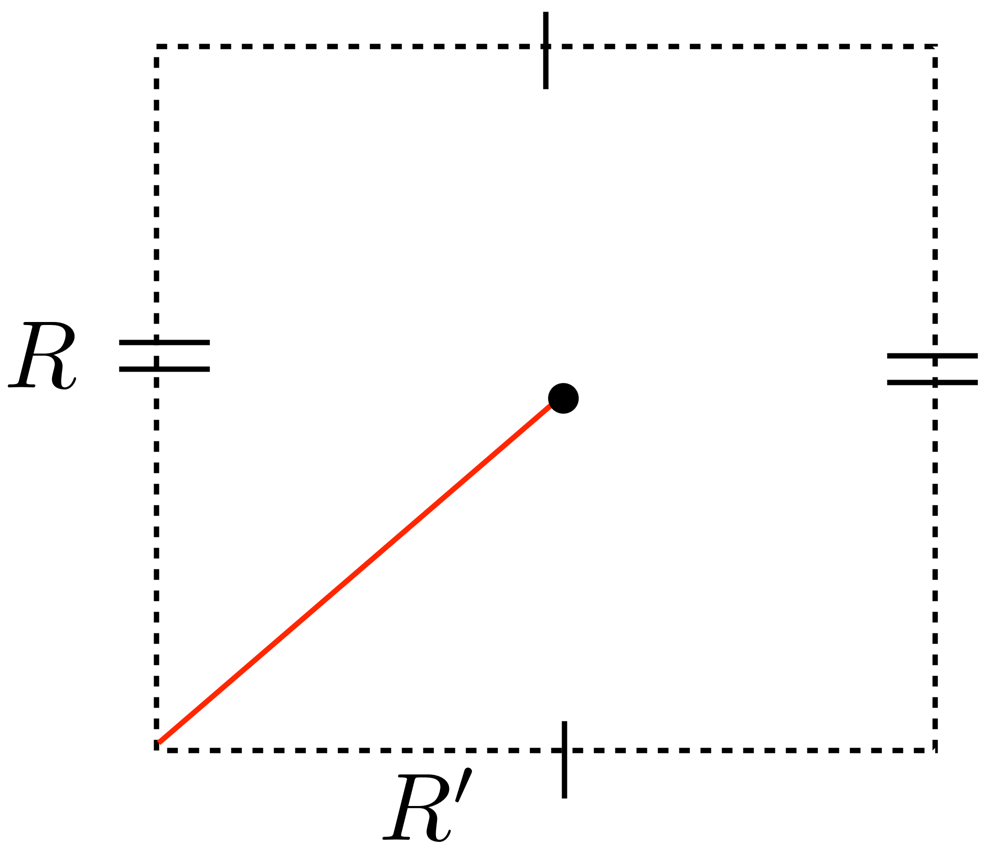
- Existence of graviton, $\Delta = \tilde{\Delta} = Q = \tilde{Q} = 0$ then implies sublattice of charged particles satisfying the WGC bound

see also Shiu et al. '16

Methodology

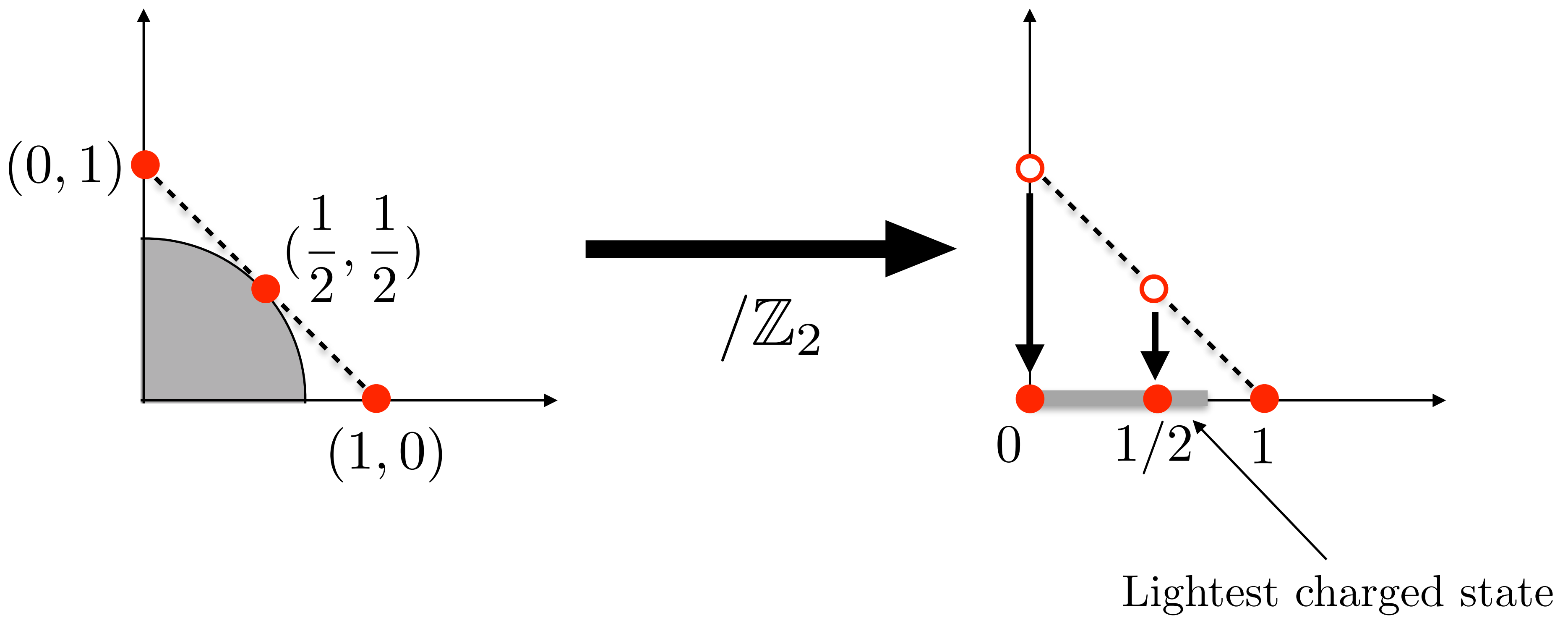
WGC Strong Form Counterexamples

- Type II $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with non-trivial space group



$$q^2 \sim {R'}^2$$
$$m^2 \sim R^2 + {R'}^2$$

WGC Strong Form Counterexamples



Strong WGCs Violated!

Strong Forms of the WGC and Inflation

(s)LWGC and Inflation

- Model with P instantons, N axions, WGC bounds assume

$$P = N$$

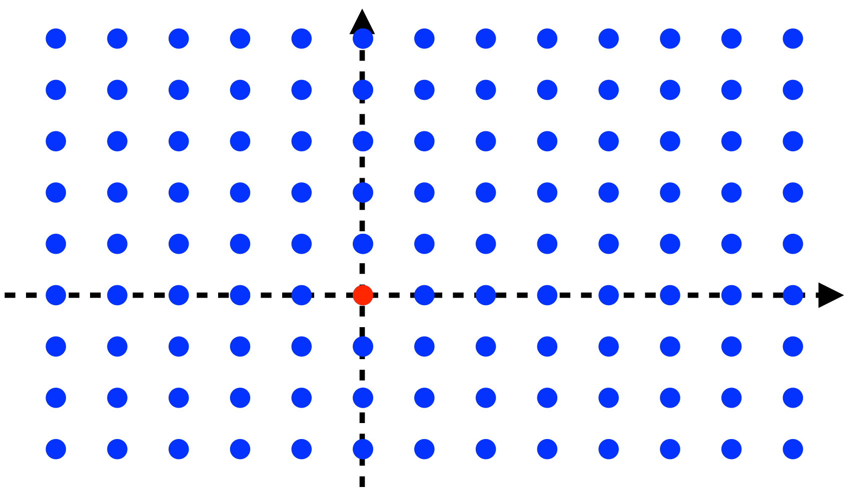
- (s)LWGC says

$$P = \infty$$

- Need to revisit WGC constraints in light of this new conjecture

The Setup

- Consider square lattice Λ of instantons saturating WGC bound:



$$|\vec{Q}| := \sqrt{\vec{Q} \cdot K^{-1} \cdot \vec{Q}}$$

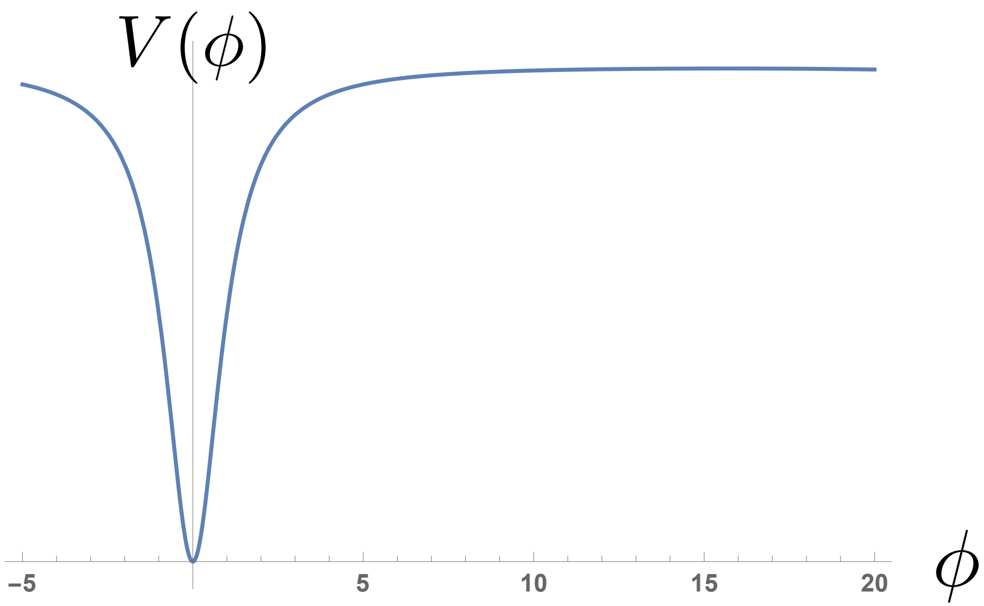
$$\mathcal{L} \supset -\frac{1}{2} K_{ab} \partial\phi^a \partial\phi^b - \sum_{\vec{Q} \in \Lambda} e^{-|\vec{Q}|} \cos(\vec{Q} \cdot \vec{\phi} + \delta_{\vec{Q}})$$

A Loophole

- Set $\delta_{\vec{Q}} = \delta = \text{const.}, N = 1,$

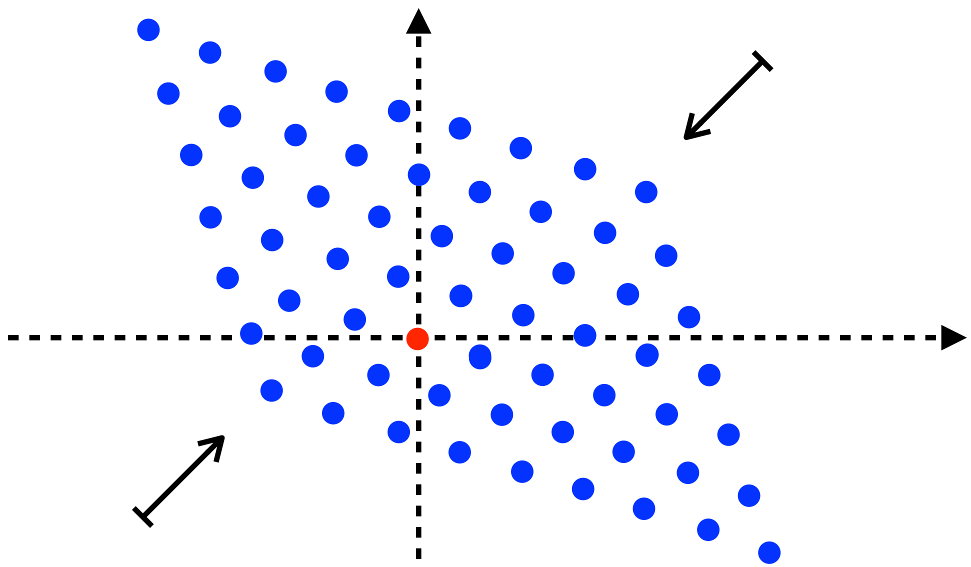
$$V(\phi) = \sum_{n\in\mathbb{Z}} e^{-n/f_0} \cos\frac{n\phi}{f_0} + \delta$$

$$= V_1 + V_0 \frac{\cos(\phi/f_0 + \delta) - e^{1/f_0}\cos\delta}{\cosh 1/f_0 - \cos\phi/f_0}.$$



A Second Loophole

- Scrunch lattice along one diagonal:

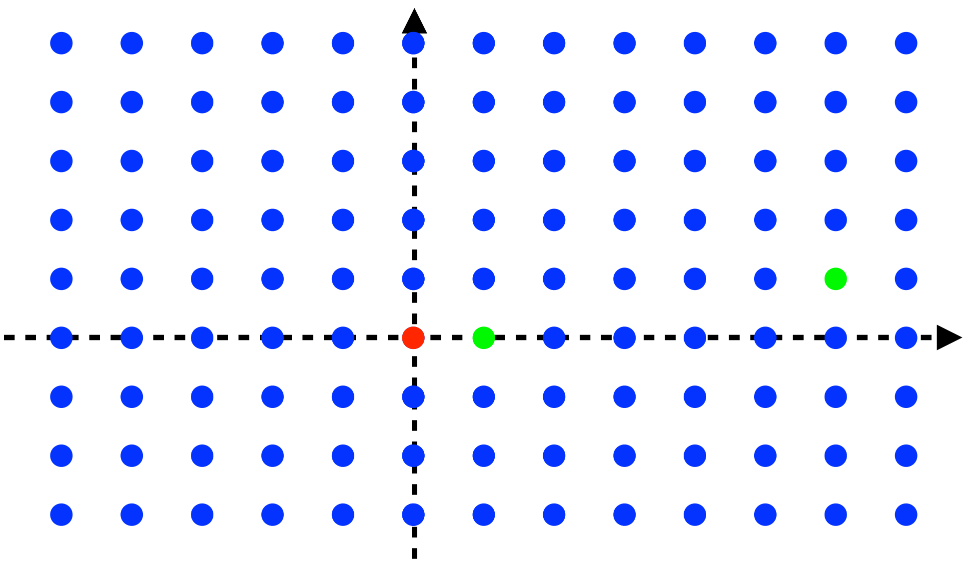


- Effect on potential is subleading in N^{-1} in scrunched direction, find

$$f_{\text{eff}} \sim \sqrt{N} M_p$$

A Third Loophole

- Set $N = 2$, take two aligned instantons to have $S_{\vec{Q}} \ll |\vec{Q}|$ and dominate potential

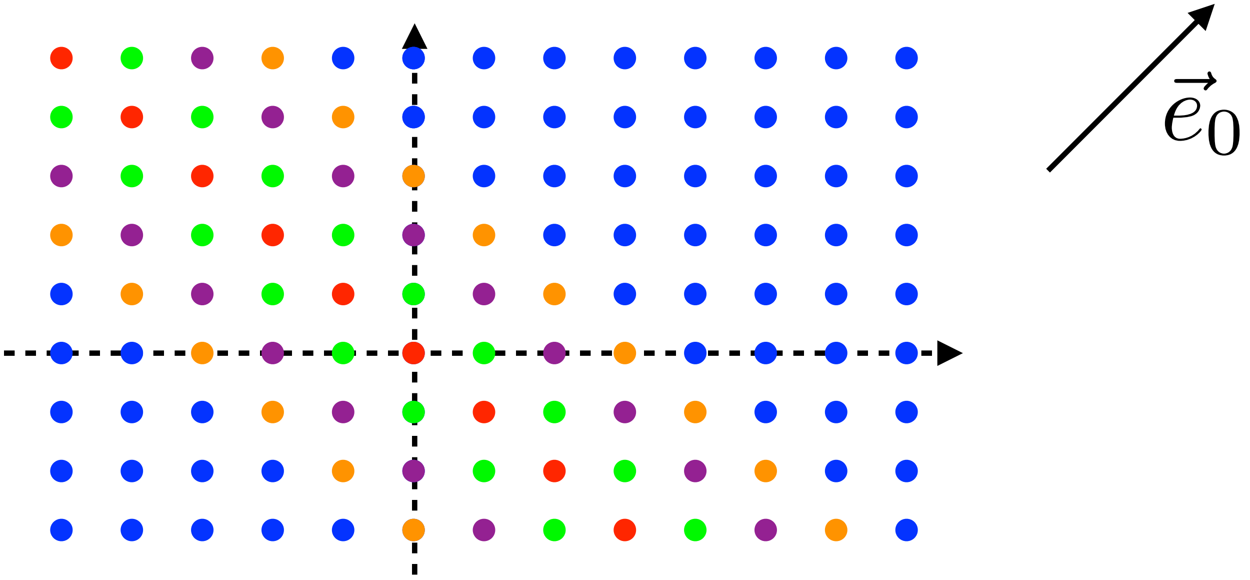


- Gives rise to standard KNP alignment

Kim, Nilles, Peloso '04

Generic Constraints

- More generally, take random phases $\delta_{\vec{Q}}$
- Given direction $\vec{e}_0$ in charge lattice, decompose potential into harmonics



Phasor Notation

- Introduce phasor notation,

$$\mathcal{L} \supset -\frac{1}{2} K_{ab} \partial\phi^a \partial\phi^b - \sum_{\vec{Q} \in \Lambda} Z_{\vec{Q}} e^{i\vec{Q}\cdot\vec{\phi}}$$

$$Z_{\vec{Q}} = e^{-|\vec{Q}| + i\delta_{\vec{Q}}}$$

- Total contribution to nth harmonic is then

$$Z_n = \sum_{\vec{Q}, \sum_i Q_i = |n|} Z_{\vec{Q}}$$

Gaussian Approximation

- For large number of instantons, can use CLT to write:

$$Z_n \sim \mathcal{N}(0, \sigma_n^2)$$

$$\sigma_n^2 = \sum_{\vec{Q}, \sum_i Q_i = |n|} e^{-2|\vec{Q}|}$$

$$\text{Higher harmonics suppressed} \Leftrightarrow \sigma_n^2 \ll \sigma_1^2$$

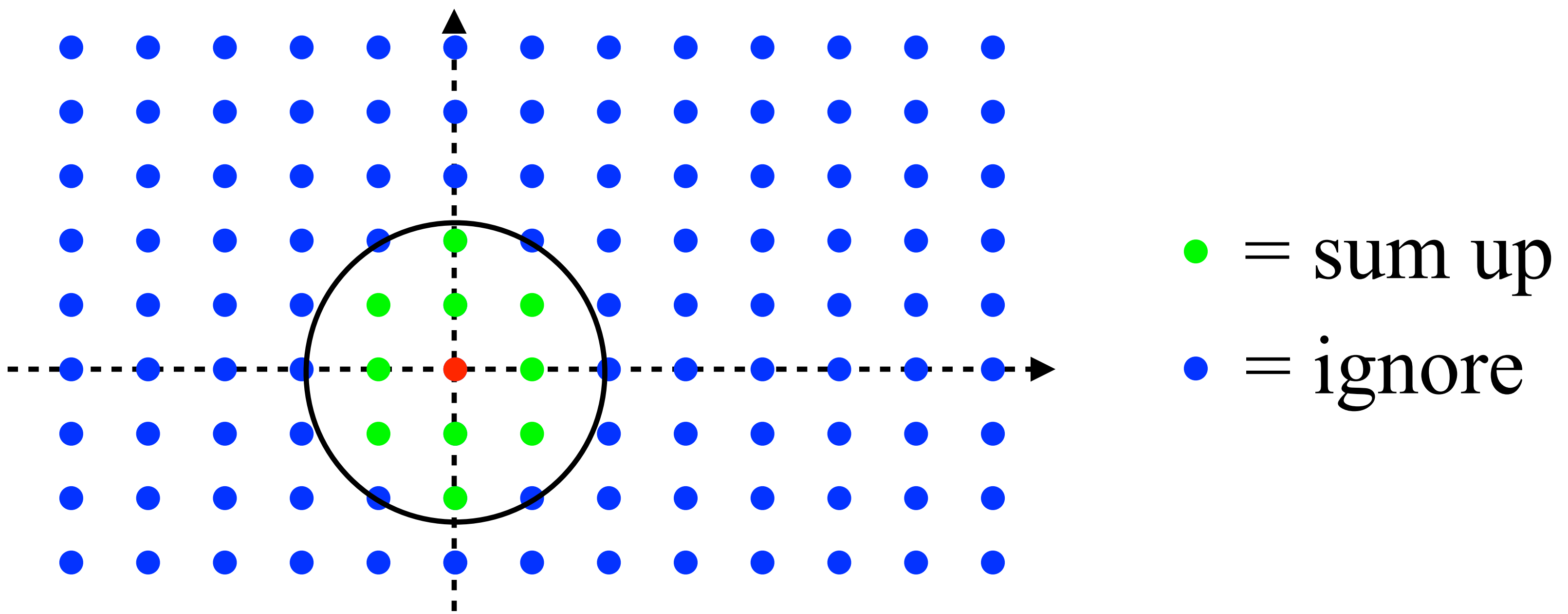
Estimating σ_n^2

Want to estimate sum:

$$\sigma_n^2 = \sum_{\vec{Q}, \sum_i Q_i = |n|} e^{-2|\vec{Q}|}$$

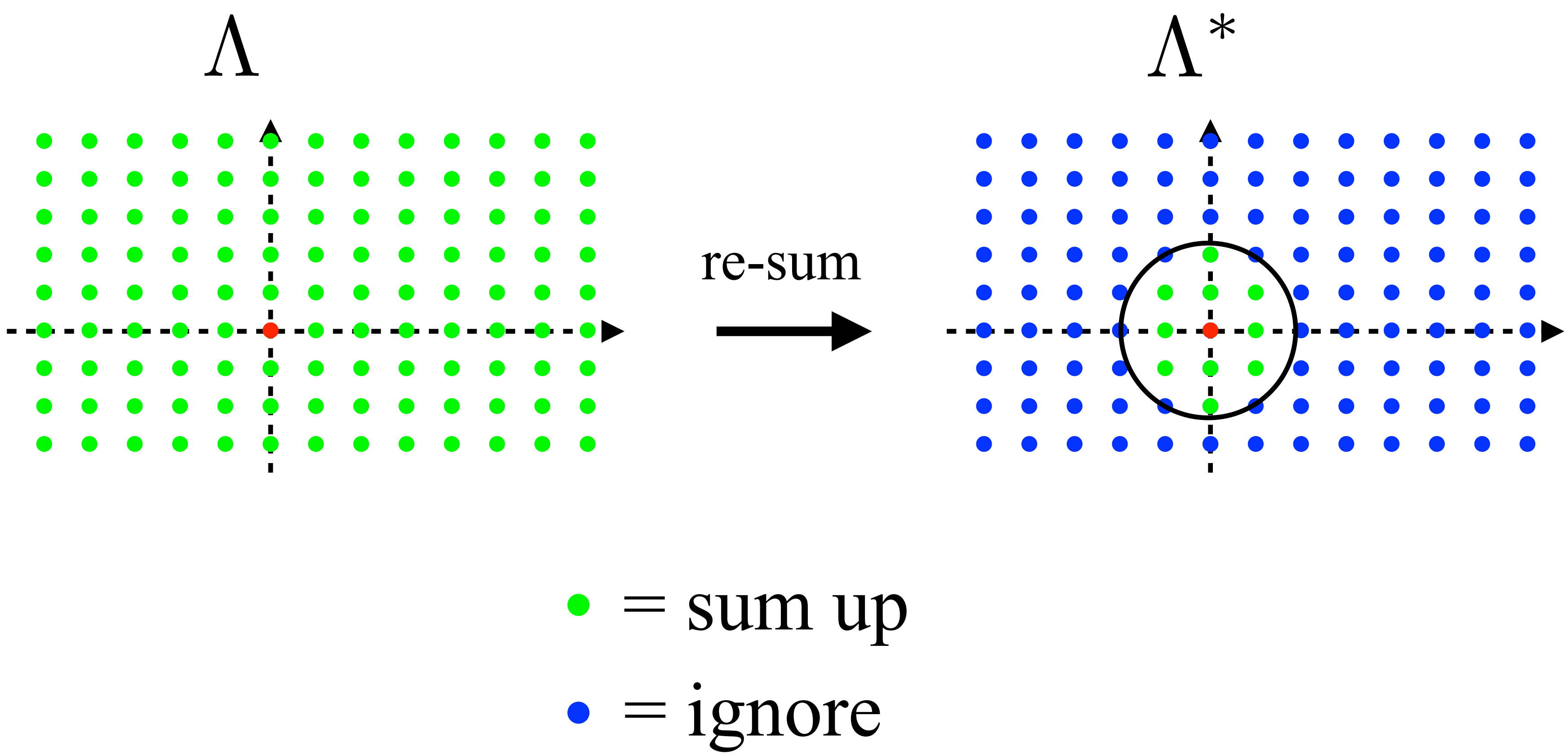
Estimating σ_n^2

Method 1: use instantons of small charges and ignore the rest (valid for small decay constants)



Estimating σ_n^2

Method 2: use Poisson resummation (valid for large decay constants)

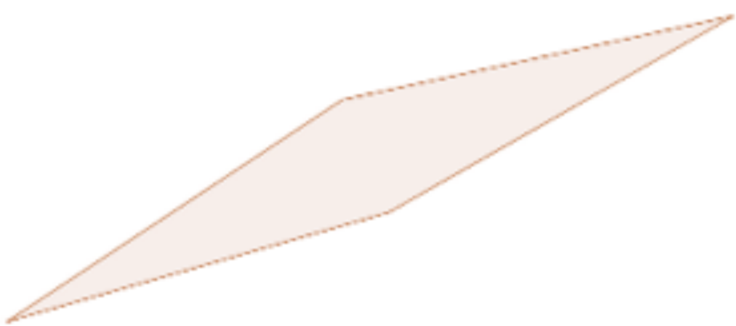


Volume Bound

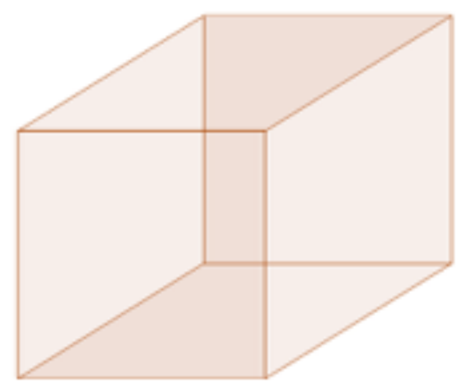
- For general K_{ab}, define

$$\text{vol } \Phi = \frac{M_p^N (2\pi)^N}{|\Lambda|}$$

with $|\Lambda|$ the volume of the unit cell, Φ the fundamental domain of axion moduli space:



Decay Constant Alignment



N-flation

Volume Bound (cont.)

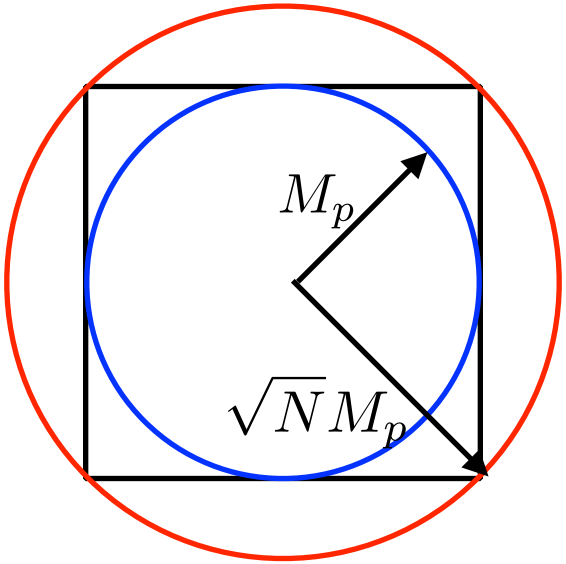
- Demanding $\sigma_n^2 \ll 1$ and approximating, get

$$\log[\text{vol }\Phi] \lesssim \log[\text{vol } D_N(2M_p)] + O(\log N)$$

with $D_N(R)$ the N-ball of radius R.

Volume Bound and *N*-flation

- Isotropic *N*-flation ($K_{ab} \propto \mathbb{I}$) violates the volume bound



- But, decay constant alignment survives

Volume Bound and Alignment

- Set $K_{ab} \in \mathrm{Wish}_N(\sigma^2, N)$

$$\Rightarrow f_{\mathrm{eff}} \lesssim O(1 M_p)$$

- Set $K^{-1}_{ab} \in \mathrm{Wish}_N(\sigma^2, N)$

$$\Rightarrow f_{\mathrm{eff}} \lesssim O(N M_p)$$

Summary

- The (s)LWGC emerges naturally from the WGC via dimensional reduction
- The WGC is incompatible with some models of axion inflation
- If the (s)LWGC is true:
  - Isotropic N-flation would be ruled out
  - Some decay constant alignment models would be ruled out
  - But, some would be allowed

Open Questions

- Can one prove the WGC? The sLWGC?

  c.f. Harlow '15, Shiu et al. '16, Hod '17, Fisher, Mogni '17
- If not, can one find a counterexample?
- Can one close the aforementioned loopholes?
- If not, can one exploit one of these opportunities in a stringy model of inflation?
- What more can the WGC tell us about low-energy physics?