宇宙原始密度揺らぎの非ガウス性

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2020/1/22 基礎物理学研究所

原始揺らぎの非ガウス性に関連した共著者(敬称略)

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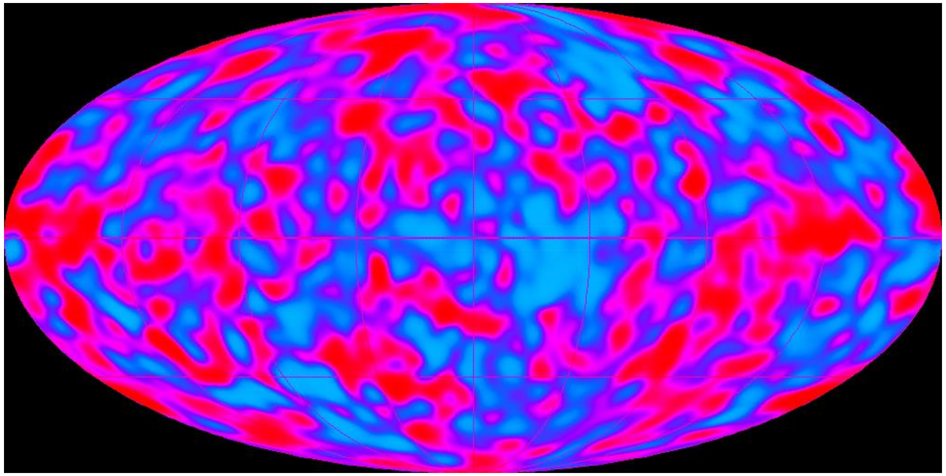
複数場インフレーション、カーバトン、非一様再加熱、等曲率揺らぎ、宇宙ひも、プレヒーティング、ungaussiton、ultra-slow rollなど

皆様、どうもありがとうございました

宇宙原始密度揺らぎとは

宇宙初期から存在した密度の**ムラムラ**



現在の宇宙にあるあらゆる構造は、原始揺らぎが
進化したもの

原始揺らぎから、宇宙の超高エネルギー時代の情報が
得られる

非ガウス性は21世紀に入って急速に活発になった研究分野

非ガウス性の研究に対する日本人の貢献は非常に大きい

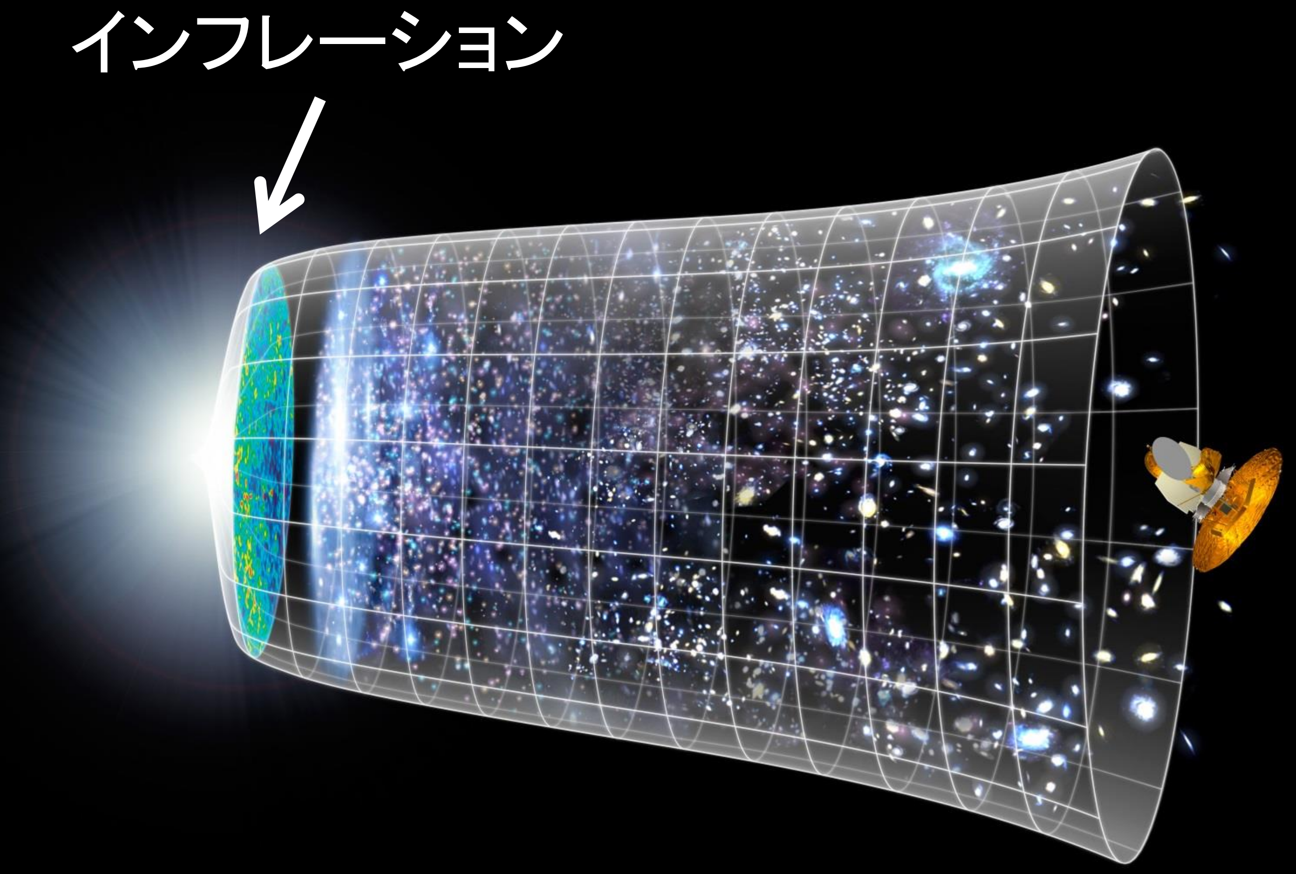
＊私の貢献はその中のごく一部です

背景

＊私のバイアスがかなり入っています

＊文献もかなり不十分です

インフレーション：宇宙の標準パラダイム



$$a(t) \approx \exp(Ht) \qquad H \approx \mathrm{const}$$

インフレーションが説明できること

- 宇宙の平坦性

- 一様・等方性

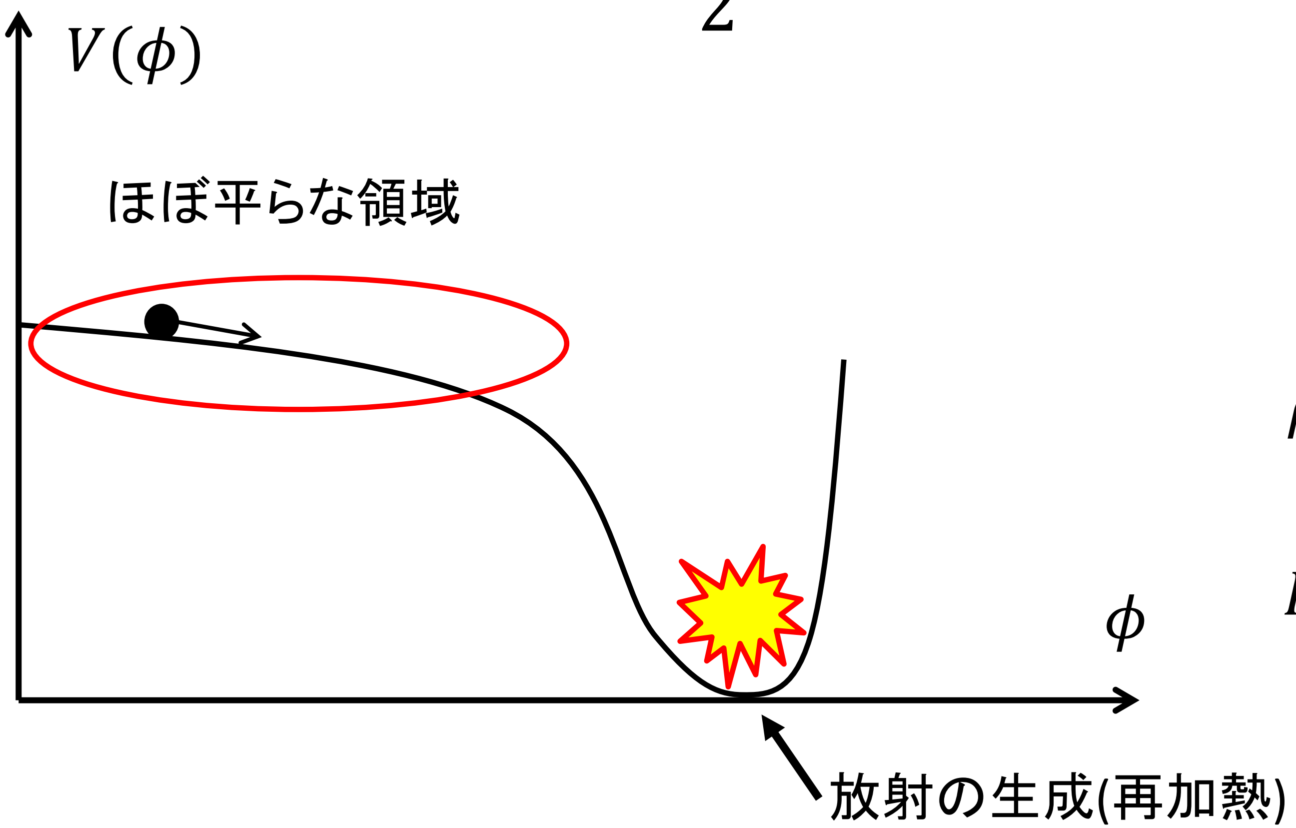
- モノポールの非検出

$$\frac{a_{end}}{a_{ini}} = e^{N} \qquad N \simeq 60$$

- 原始密度揺らぎの生成

スカラー場ϕ(インフラトン)

$$\mathcal{L} = -\frac{1}{2}(\nabla\phi)^2 - V(\phi)$$



$$\phi = \phi(t)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$\dot{\phi}$ が小さいと、近似的に宇宙項支配の宇宙となり、インフレーションが実現

スローロールパラメター

$$\varepsilon = \frac{{M_P}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta = {M_P}^2 \frac{V_{,\phi\phi}}{V}$$

$\varepsilon, |\eta| \ll 1$であれば、長い間インフレーションが持続

インフレーションが説明できること

- 宇宙の平坦性
- 一様・等方性
- モノポールの非検出

$$\frac{a_{end}}{a_{ini}} = e^N \qquad N \simeq 60$$

- **原始密度揺らぎの生成(定量比較が可能)**

原始揺らぎの生成

$$S = \frac{{M_P}^2}{2}\int d^4x\,\sqrt{-g}\;R + \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$$

量子揺らぎ

$$g_{\mu\nu} = \overline{g_{\mu\nu}} + \delta g_{\mu\nu}(t,x) \qquad\qquad \phi = \phi(t) + \delta\phi(t,x)$$

Sを揺らぎの２次まで展開

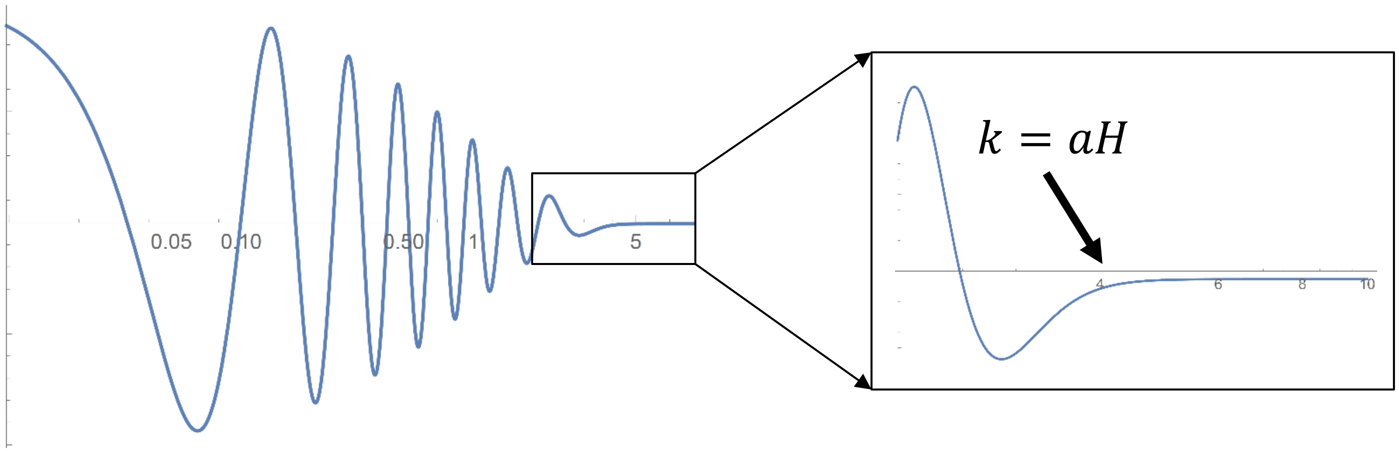
$$g_{ij} = a(t)^2 e^{2\zeta}\delta_{ij}$$

ζ:曲率揺らぎ

$$S = \int dt d^3x\,\frac{a^3\dot{\phi}^2}{2H^2}\left(\dot{\zeta}^2 - \frac{1}{a^2}\partial^i\zeta\partial_i\zeta\right)$$

摩擦ありバネ定数が時間減衰する調和振動子

フーリエ成分　$\mathcal{R}_{\vec{k}} = \int d^3x\, e^{-i\vec{k}\vec{x}}\, \mathcal{R}(t,x)$

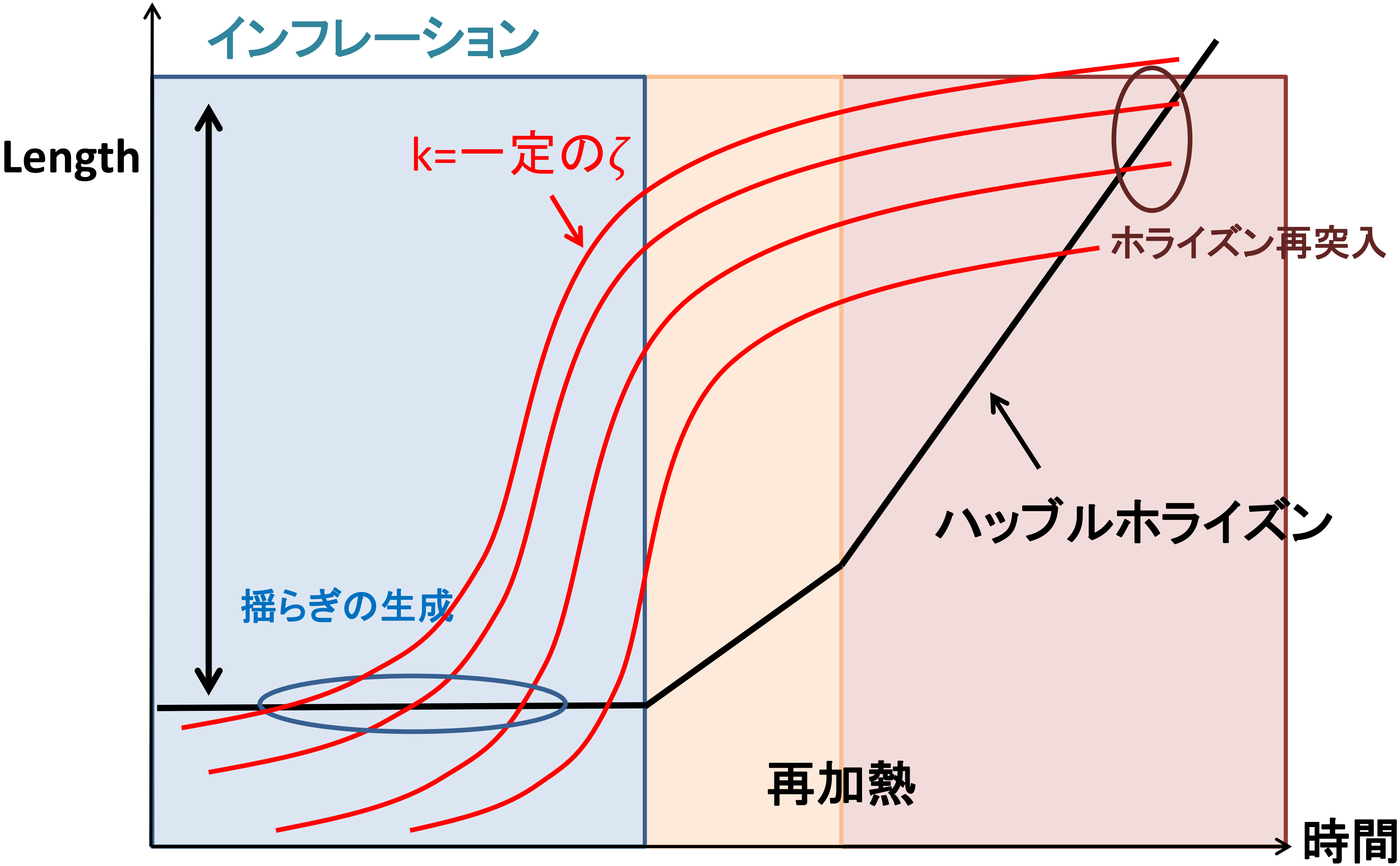


$k \lesssim aH$で揺らぎが凍結

揺らぎのパワースペクトル

$$\left\langle \mathcal{R}_{\overrightarrow{k_1}} \mathcal{R}_{\overrightarrow{k_2}} \right\rangle = (2\pi)^3 \delta(\overrightarrow{k_1} + \overrightarrow{k_2}) P_\zeta(k_1)$$

※この近似では$\mathcal{R}$の分布はガウス統計に従う



ζが観測量である

原始揺らぎのパワースペクトル

$$\boldsymbol{P_\zeta(k) = \left.\frac{V^3}{12\pi^2 {M_{pl}}^6 {V_{,\phi}}^3}\right|_{k\,=\,aH}}$$

CMBの観測から$P_\zeta \approx 2 \times 10^{-9}$

スペクトル指数 n_s

$$n_s - 1 = \frac{d\mathrm{ln}P_\zeta}{d\mathrm{ln}k} = -6\varepsilon + 2\eta \ll 1$$

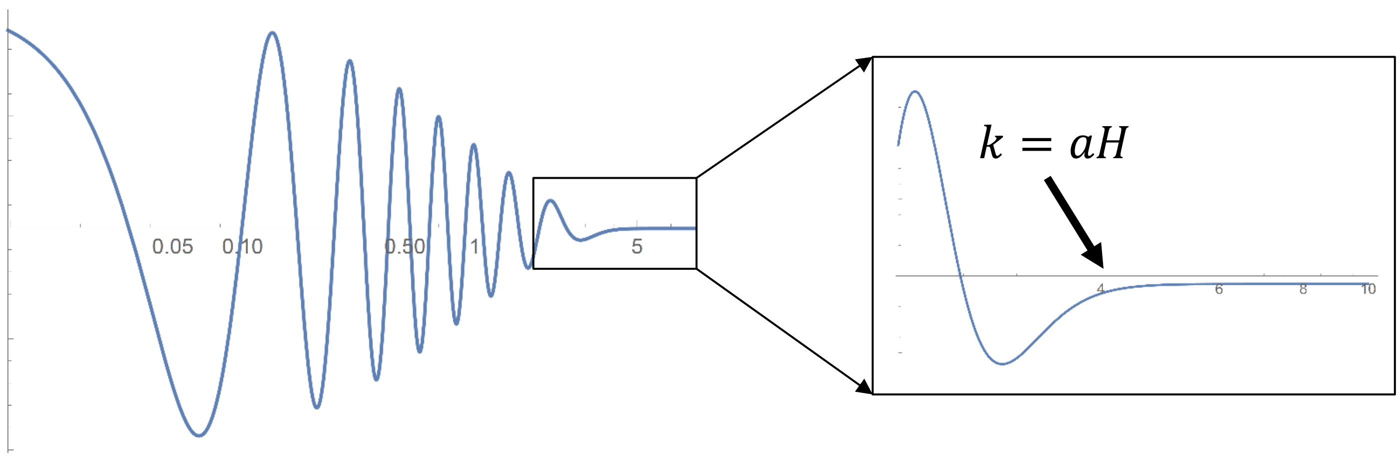
インフレーションで作られる揺らぎは、
ほぼガウシアン、ほぼスケール不変

原始揺らぎの生成

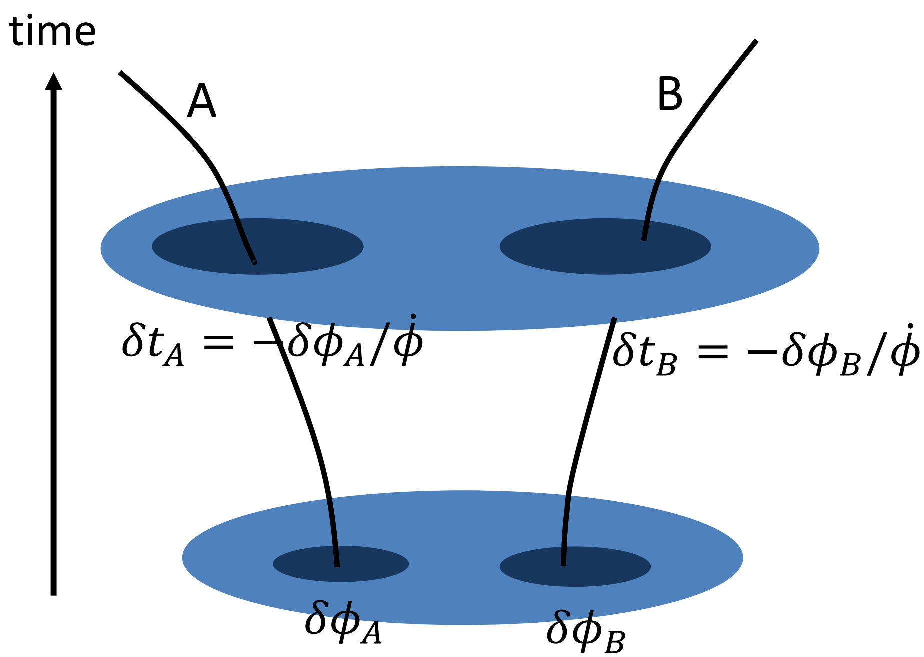
$$\phi(t,x) = \phi(t) + \delta\phi(t,x)$$

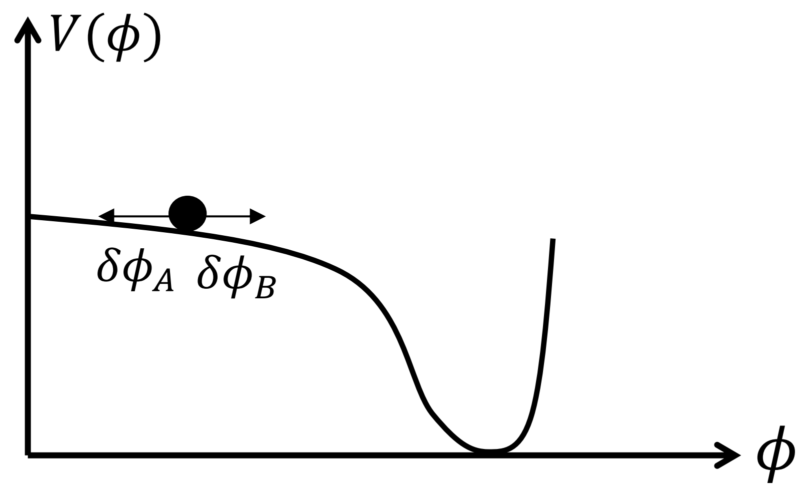
$$\delta\phi(t,x) = \int \frac{d^3k}{(2\pi)^3}\left(u_k(t)\hat{a}_k e^{ikx} + {u_k}^*(t){\hat{a}_k}^\dagger e^{-ikx}\right)$$

$$\Rightarrow \quad \ddot{u}_k + 3H\dot{u}_k + \left(\frac{k^2}{a^2} + V_{,\phi\phi}\right)u_k = 0$$



$k \lesssim aH$で揺らぎが生成　　$\delta\phi \approx \frac{H}{2\pi}$





$$ds^2 \approx -dt^2 + a(t)^2 e^{2\zeta(t,x)} d\vec{x}^2$$

$$\zeta(t,x) = H\delta t(t,x) = -\frac{H}{\dot{\phi}}\delta\phi$$

インフレーションの持続時間の差 ⇒ 曲率揺らぎ

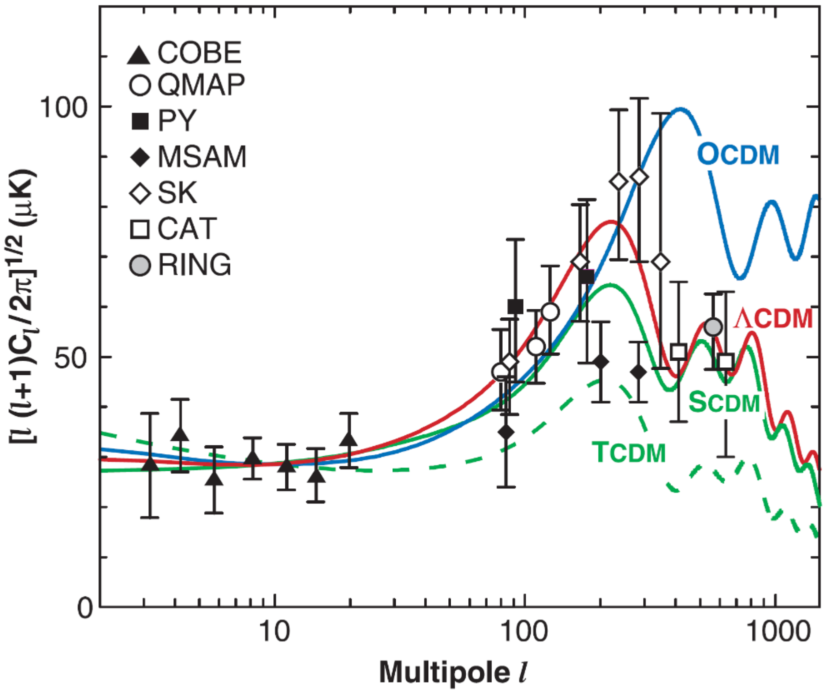
原始重力波

$$\delta g_{00} = 0,\ \delta g_{0i} = 0,\ \delta g_{ij} = h_{ij} \qquad (h_{ij,i} = \delta^{ij} h_{ij} = 0)$$

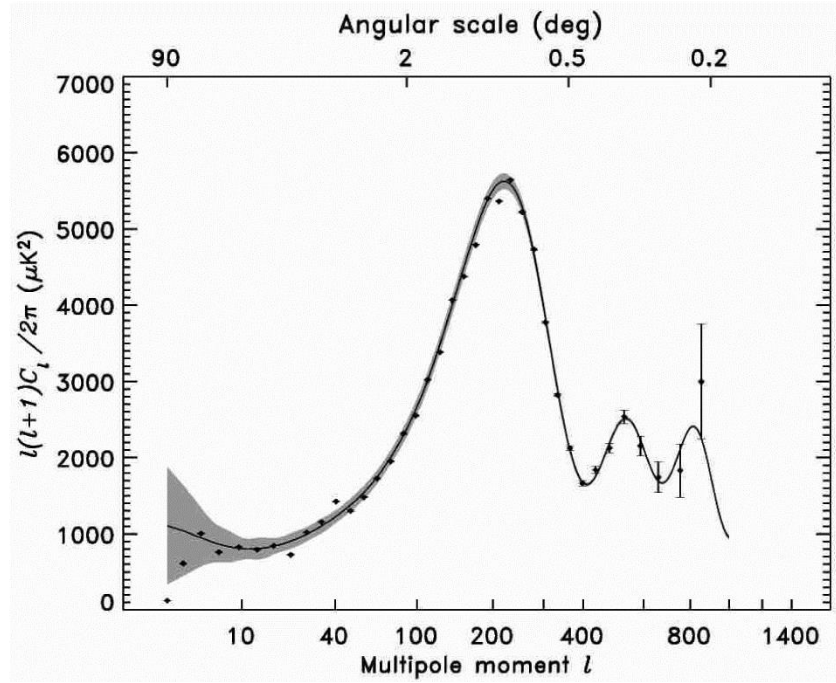
$$\boldsymbol{P_h(k) = \left.\frac{H^2}{2\pi^2 {M_{pl}}^2}\right|_{k\,=\,aH}}$$

テンソルスカラー比: r

$$\boldsymbol{r = \frac{P_h}{P_\zeta}}$$

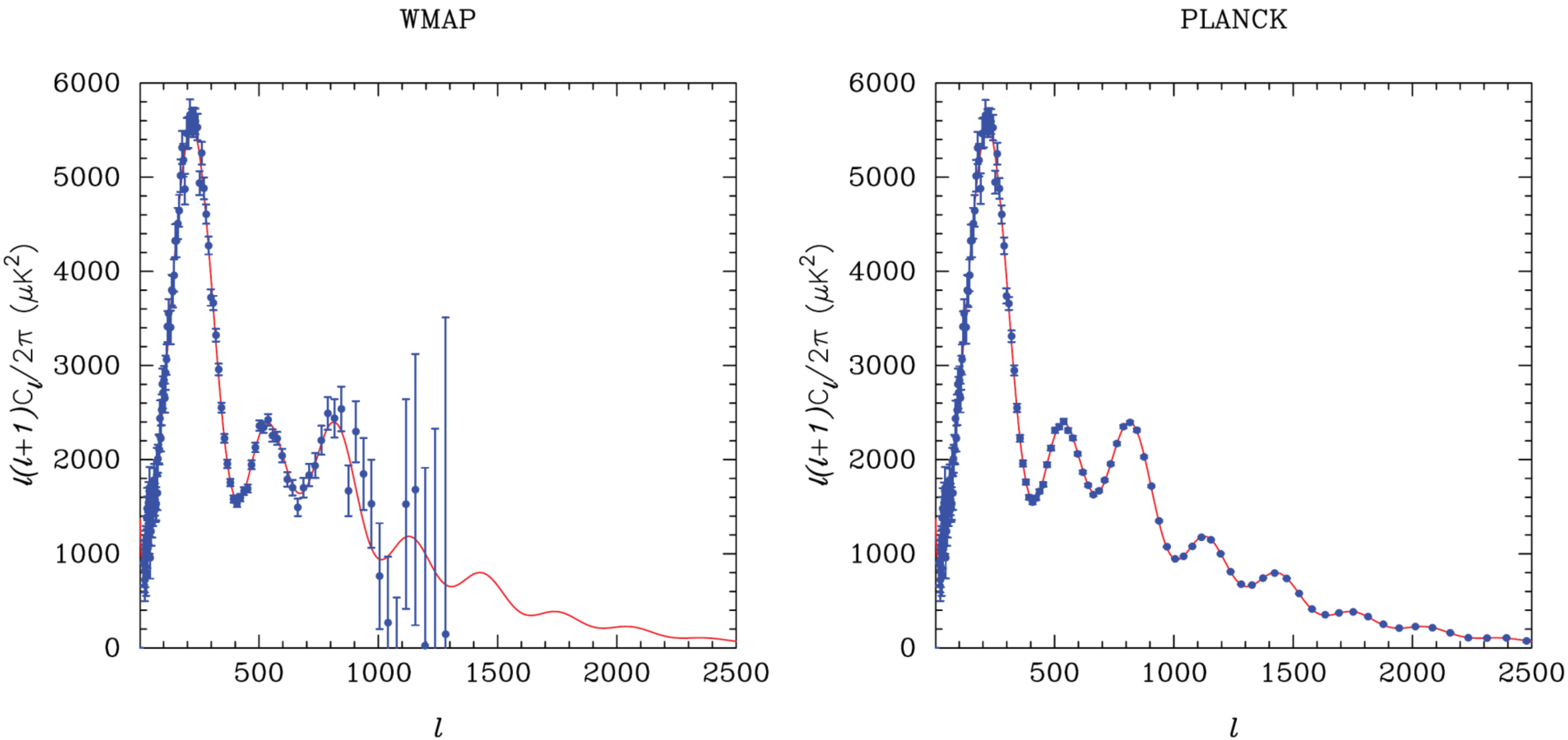


Bahcall+ 1999



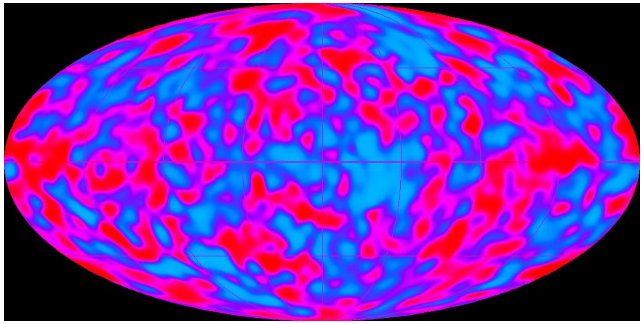
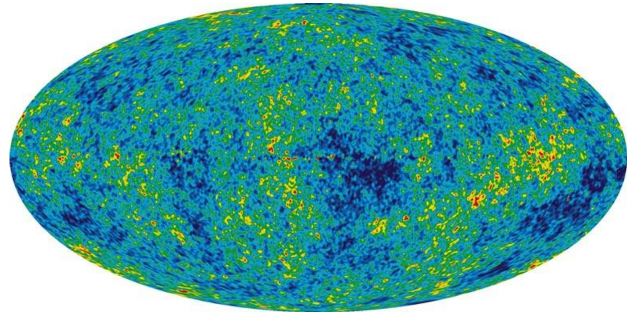
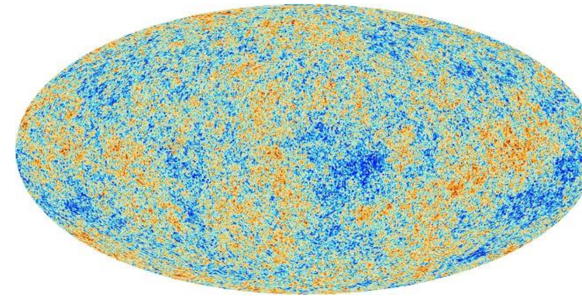
Hinshaw+ 2003

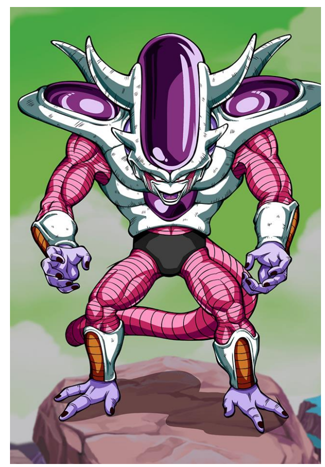
WMAPの出現で、原始揺らぎの精密検証の時代が到来した



Planck blue book 2005

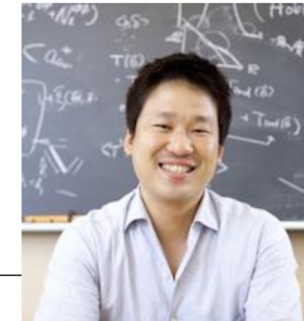
さらにより強力なPlanckが10年後くらいにやってくる




原始揺らぎの生成機構を調べられる
機運が盛り上がってきた

揺らぎの非ガウス性の先駆的論文



小松氏

Acoustic Signatures in the Primary Microwave Background Bispectrum

Eiichiro Komatsu* and David N. Spergel†
Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA.

If the primordial fluctuations are non-Gaussian, then this non-Gaussianity will be apparent in the cosmic microwave background (CMB) sky. With their sensitive all-sky observation, MAP and Planck satellites should be able to detect weak non-Gaussianity in the CMB sky. On large angular scale, there is a simple relationship between the CMB temperature and the primordial curvature perturbation: $\Delta T/T = -\Phi/3$. On smaller scales; however, the radiation transfer function becomes more complex. In this paper, we present the angular bispectrum of the primary CMB anisotropy that uses the full transfer function. We find that the bispectrum has a series of acoustic peaks that change a sign, and a period of acoustic oscillations is twice as long as that of the angular power spectrum. Using a single non-linear coupling parameter to characterize the amplitude of the bispectrum, we estimate the expected signal-to-noise ratio for COBE, MAP, and Planck experiments. In order to detect the primary CMB bispectrum by each experiment, we find that the coupling parameter should be larger than 600, 20, and 5 for COBE, MAP, and Planck experiments, respectively. Even for the ideal noise-free and infinitesimal thin-beam experiment, the parameter should be larger than 3. We have included effects from the cosmic variance, detector noise, and foreground sources in the signal-to-noise estimation. Since the simple inflationary scenarios predict that the parameter is an order of 0.01, the detection of the primary bispectrum by any kind of experiments should be problematic for those scenarios. We compare the sensitivity of the primary bispectrum to the primary skewness and conclude that when we can compute the predicted form of the bispectrum, it

astro-ph/0005036

パラメータf_{NL}の導入

perturbation and the proper transfer function.
xplore the simplest weak non-linear coupling case:

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + \boxed{f_{NL}}\left(\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2(\mathbf{x})\rangle\right),$$

$_L(\mathbf{x})$ denotes the linear gaussian part of the perturbation. $\langle\Phi(\mathbf{x})\rangle = 0$
e non-linear coupling constant. This model is based upon the slow
1] and Gangui et al. [12] found that f_{NL} is given by a certain combi

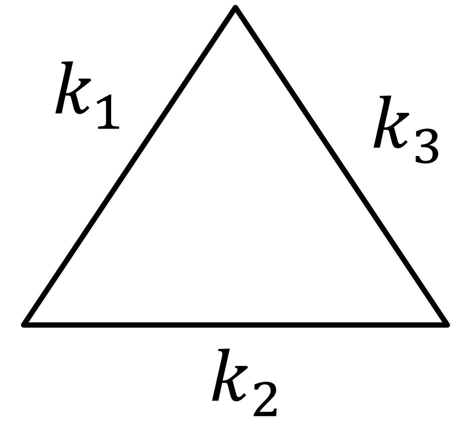
ガウシアン　　非ガウシアン

$$\left\langle \Phi_{\overrightarrow{k_1}}\Phi_{\overrightarrow{k_2}}\Phi_{\overrightarrow{k_3}} \right\rangle = 2f_{NL}(2\pi)^3\left(P_\Phi(k_1)P_\Phi(k_2) + 2\text{ perms}\right)\delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3})$$

$$\left\langle \Phi_{\overrightarrow{k_1}}\Phi_{\overrightarrow{k_2}}\Phi_{\overrightarrow{k_3}} \right\rangle = (2\pi)^3 B_\Phi(k_1, k_2, k_3)\delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3})$$

バイスペクトル

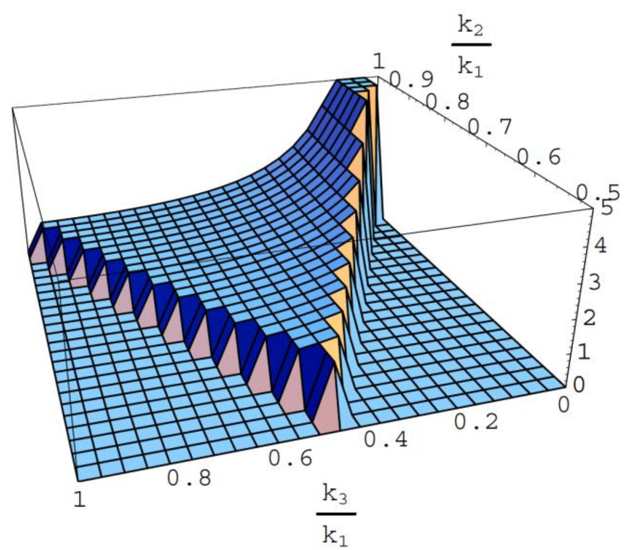
$$\left\langle \Phi_{\overrightarrow{k_1}} \Phi_{\overrightarrow{k_2}} \Phi_{\overrightarrow{k_3}} \right\rangle \propto \delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3})$$

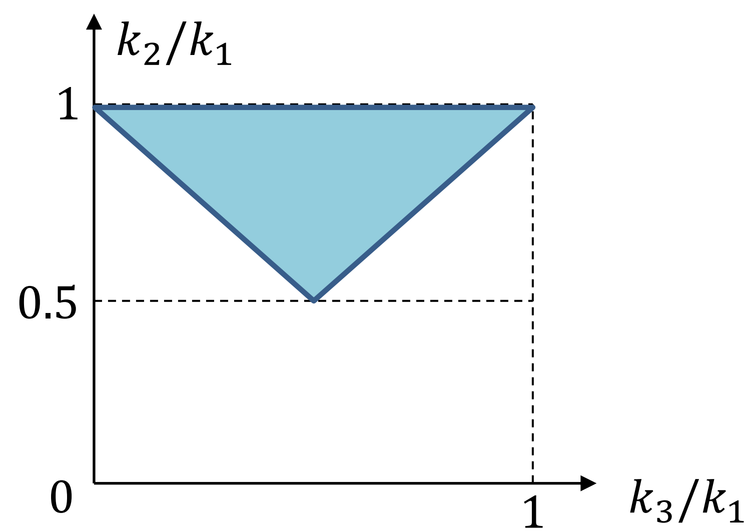


Bは、k_1, k_2, k_3 の関数

ローカル型

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}\left(\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle\right)$$





ローカル型のバイスペクトルは、squeezed limit ($k_3 \to 0$)で大きくなる

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If the primordial fluctuations are non-Gaussian, then this non-Gaussianity will be apparent in the cosmic microwave background (CMB) sky. With their sensitive all-sky observation, MAP and Planck satellites should be able to detect weak non-Gaussianity in the CMB sky. On large angular scale, there is a simple relationship between the CMB temperature and the primordial curvature perturbation: $\Delta T/T = -\Phi/3$. On smaller scales; however, the radiation transfer function becomes more complex. In this paper, we present the angular bispectrum of the primary CMB anisotropy that uses the full transfer function. We find that the bispectrum has a series of acoustic peaks that change a sign, and a period of acoustic oscillations is twice as long as that of the angular power spectrum. Using a single non-linear coupling parameter to characterize the amplitude of the bispectrum, we estimate the expected signal-to-noise ratio for COBE, MAP, and Planck experiments. In order to detect the primary CMB bispectrum by each experiment, we find that the coupling parameter should be larger than 600, 20, and 5 for COBE, MAP, and Planck experiments, respectively. Even for the ideal noise-free and infinitesimal thin-beam experiment, the parameter should be larger than 3. We have included effects from the cosmic variance,
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FIRST YEAR *WILKINSON MICROWAVE ANISOTROPY PROBE* (*WMAP*) OBSERVATIONS: TESTS OF GAUSSIANITY

E. Komatsu [2], A. Kogut [3], M. R. Nolta [4], C. L. Bennett [3], M. Halpern [5], G. Hinshaw [3], N. Jarosik [4], M. Limon [3,6], S. S. Meyer [7], L. Page [4], D. N. Spergel [2], G. S. Tucker [3,6,8], L. Verde [2,9]
E. Wollack [3], E. L. Wright [10]

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ABSTRACT

We present limits to the amplitude of non-Gaussian primordial fluctuations in the *WMAP* 1-year cosmic microwave background sky maps. A non-linear coupling parameter, $f_{\rm NL}$, characterizes the amplitude of a quadratic term in the primordial potential. We use two statistics: one is a cubic statistic which measures phase correlations of temperature fluctuations after combining all configurations of the angular bispectrum. The other uses the Minkowski functionals to measure the morphology of the sky maps. Both methods find the *WMAP* data consistent with Gaussian primordial fluctuations and establish limits, $-58 < f_{\rm NL} < 134$, at 95% confidence. There is no significant frequency or scale dependence of $f_{\rm NL}$. The *WMAP* limit is 30 times better than *COBE*,

単一場スローロールインフレーションのバイスペクトル

Maldacena 2003

$$S=\frac{{M_P}^2}{2}\int d^4x\sqrt{-g}\;R+\int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right)$$

$$B(k_1,k_2,k_3)=\frac{H^4}{16{M_P}^4\epsilon_1}\frac{1}{{k_1}^3{k_2}^3{k_3}^3}\left[\left(1+\frac{k_1}{k_T}\right)\frac{{k_2}^2{k_3}^2}{k_T}+\left(\overrightarrow{k_1}\cdot\overrightarrow{k_2}+2\text{ perms}\right)\left(-k_T+\frac{k_1k_2+k_2k_3+k_3k_1}{k_T}\right.\right.$$

$$\Rightarrow\quad \boldsymbol{\frac{B}{P^2}=O(\varepsilon,\eta)}$$

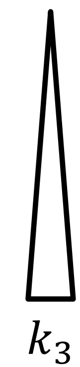
$B\sim10^{-20}$: 非ガウス性は非常に微弱

f_{NL}はスローロールパラメータの大きさ

単一場スローロールインフレーション

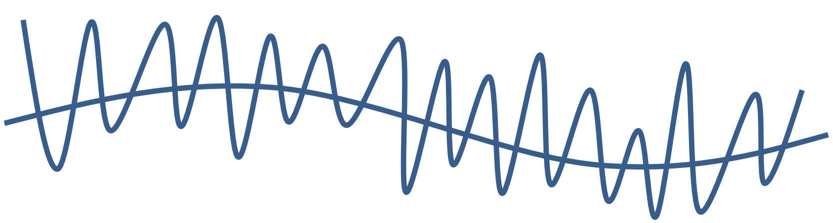
Squeezed limit

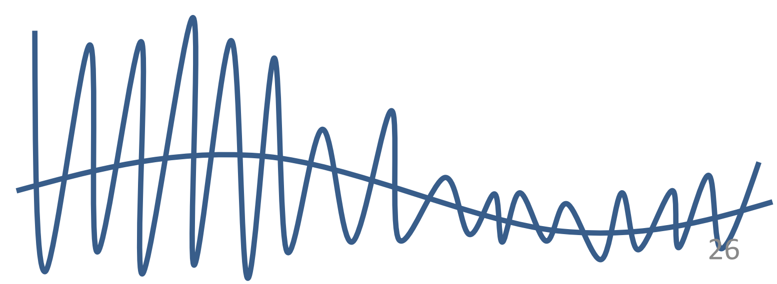
$$\lim_{k_3 \to 0} B(k, k, k_3) = B_{local}(k, k, k_3)$$



$$\Rightarrow \quad f_{NL} = \frac{5}{12}(1 - n_s)$$

ただし、観測量への焼き直しが必要



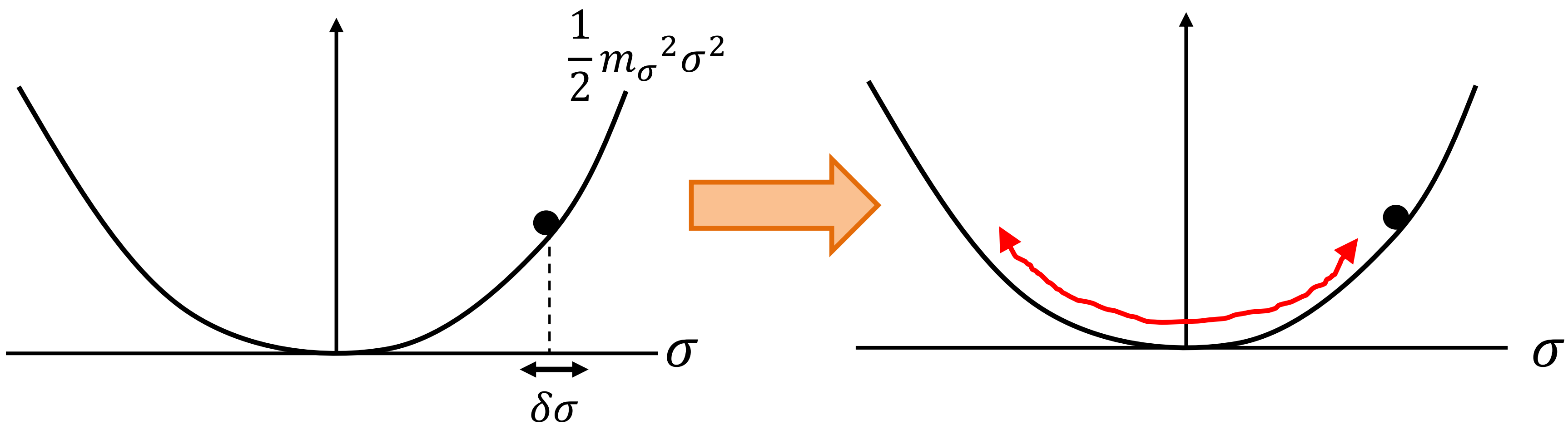


カーバトンモデル

Moroi, Takahashi 2001
Enqvist, Sloth 2002
Lyth, Wands 2002

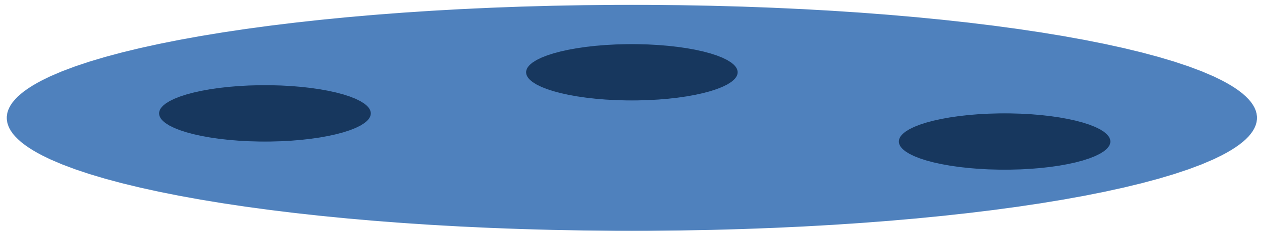
インフラトンとは別の場(カーバトン)が原始揺らぎを作るモデル

軽い場がインフレーションによって揺らぐ　$\delta\sigma \simeq \frac{H}{2\pi}$ (ガウシアン)



カーバトンはその後($H \approx m_\sigma$)振動し、ダスト成分として進化

カーバトンはその後($H \approx \Gamma_\sigma$)崩壊し、放射優勢宇宙へ



スーパーハッブルスケールでの揺らぎの生成は、局所的に起こる

$$\zeta(x) = F(\delta\sigma(x))$$

⇒揺らぎの非ガウス性は、ローカル型

カーバトンで作られる原始揺らぎ

$$\zeta \simeq r_d \frac{\delta\sigma}{\sigma} \qquad r_d = \left.\frac{\rho_\sigma}{\rho_{rad}}\right|_{H=\Gamma_\sigma}$$

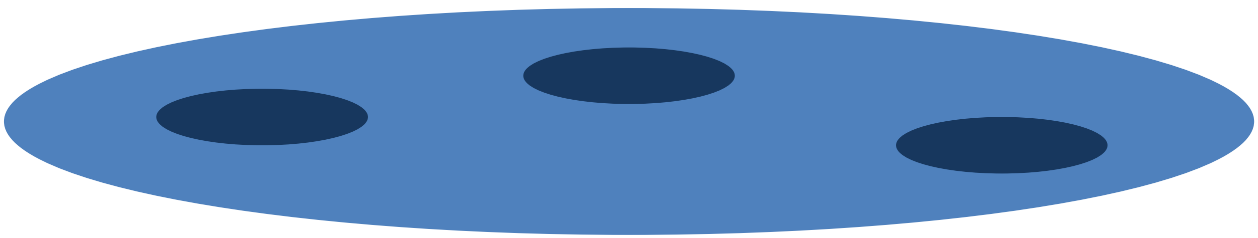
$$\boldsymbol{f_{NL} \simeq \frac{1}{r_d}}$$

$r_d \ll 1$ ならば $f_{NL} \gg 1$、$r_d \sim 1$ ならば $f_{NL} = O(1)$

非一様再加熱モデル Kofman 2003, Dvali+ 2004

インフラトンの崩壊率が場所ごとに異なるモデル

$$\Gamma_\phi = \Gamma_\phi(\sigma)$$



$\rho_\phi \propto a^{-3}$ $\rho_{rad} \propto a^{-4}$

$$\zeta = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma}$$

Γ_ϕとζの間の非線形性からの寄与

$$\boldsymbol{f_{NL} = 5}$$

一般にインフラトン以外の場が寄与すると、ローカル型の非ガウス性を持つ原始揺らぎが作られる

f_{NL}の大きさはモデル依存

複数場スローロールインフレーションでは、大抵の場合f_{NL}はスローロールパラメータの大きさ

Yokoyama+ 2007

※ただし、$f_{NL} = \frac{5}{12}(1 - n_s)$は成立しない

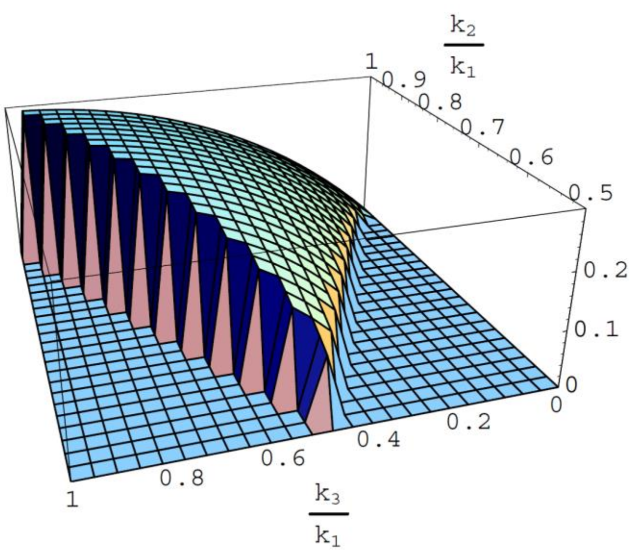
非正準項インフレーション、高階微分、揺らぎの有効理論

$$\mathcal{L} = P(X,\phi)$$

K-inflation, DBI inflation, ghost inflationなど

等辺型($k_1 \sim k_2 \sim k_3$)のバイスペクトル

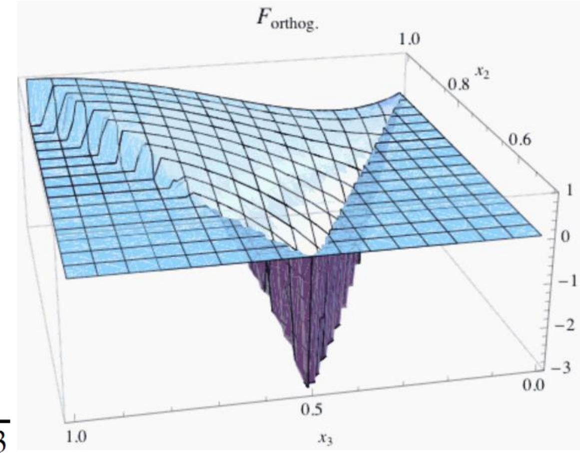
$$B_\Phi^{\rm equil}(k_1,k_2,k_3) = 6A^2 f_{\rm NL}^{\rm equil} \times \left\{ -\frac{1}{k_1^{4-n_s}k_2^{4-n_s}} - \frac{1}{k_2^{4-n_s}k_3^{4-n_s}} - \frac{1}{k_3^{4-n_s}k_1^{4-n_s}} - \frac{2}{(k_1k_2k_3)^{2(4-n_s)/3}} + \left[\frac{1}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} + (5\ \text{permutations}) \right] \right\},$$



$$f_{NL}^{\rm equil} \sim \frac{1}{{c_s}^2}$$

${c_s}^2 \ll 1$ならば$f_{NL}^{\rm equil} \gg 1$

直交型($k_1 \sim k_2 \sim k_3$)のバイスペクトル



$$B_\Phi^{\mathrm{ortho}}(k_1, k_2, k_3) = 6A^2 f_{\mathrm{NL}}^{\mathrm{ortho}}$$
$$\times \left\{ -\frac{3}{k_1^{4-n_s} k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s} k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s} k_1^{4-n_s}} - \frac{8}{(k_1 k_2 k_3)^{2(4-n_s)/3}} \right.$$
$$\left. + \left[\frac{3}{k_1^{(4-n_s)/3} k_2^{2(4-n_s)/3} k_3^{4-n_s}} + (5 \text{ perm.}) \right] \right\}.$$

バイスペクトル$F_{(1)}$と$F_{(2)}$の内積

$$F_{(1)} \cdot F_{(2)} = \sum_{k_i^{\mathrm{physical}}} F_{(1)}(k_1, k_2, k_3) F_{(2)}(k_1, k_2, k_3) / \left(P_{k_1} P_{k_2} P_{k_3}\right)$$

小まとめ

$f_{NL} \geq O(1)$の検出は、単一場正準インフレーションをただちに棄却

バイスペクトルの波数依存性から揺らぎの生成機構が分かる

非ガウス性は、インフレーションのクリーンなテスト

f_{NL} の観測的制限

$-3500 < f_{NL} < 2000$ (COBE 4-year data)

$-58 < f_{NL} < 134$ (WMAP 1st year, 95CL)

$-54 < f_{NL} < 114$ (WMAP 3 year, 95CL)

この辺から**ザワザワ**

$-9 < f_{NL} < 111$ (WMAP 5 year, 95CL)

$-10 < f_{NL} < 74$ (WMAP 7 year, 95CL)

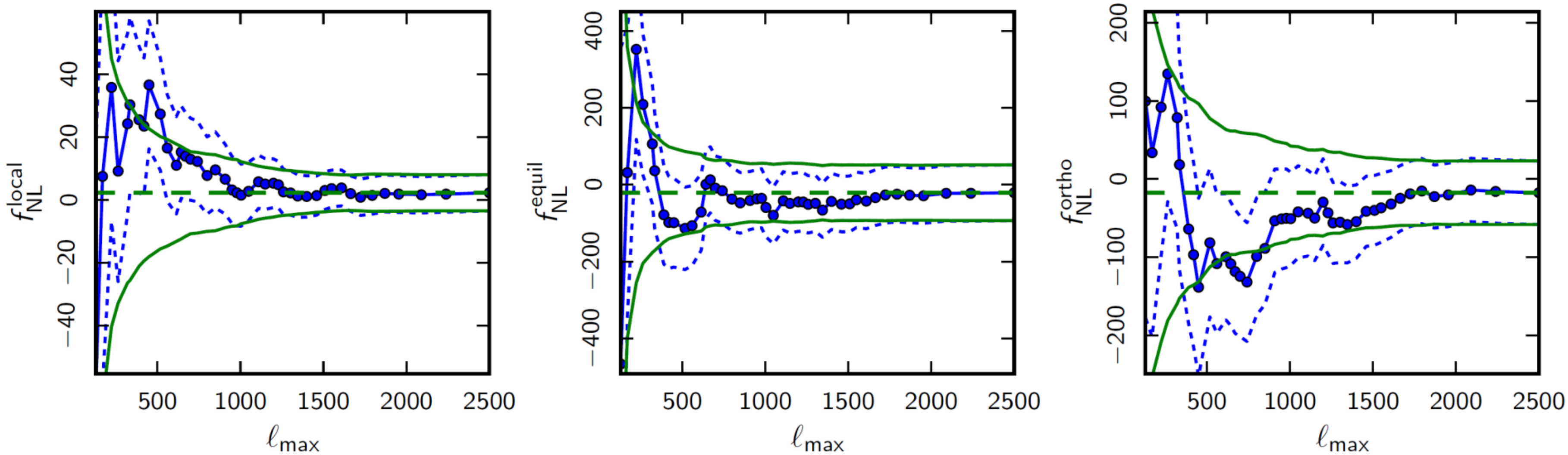
$-3 < f_{NL} < 77$ (WMAP 9 year, 95CL)

この辺で**鎮火**

$f_{NL} = 2.7 \pm 5.8$ (Planck, 2014, 68CL)

$f_{NL} = -0.9 \pm 5.1$ (Planck, 2015, 68CL)

有意な検出なし



Planckの解析結果は、WMAPのそれと無矛盾

Planckによる非ガウス性への制限(2018年)

$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1;\; f_{\rm NL}^{\rm equil} = -26 \pm 47;\text{ and } f_{\rm NL}^{\rm ortho} = -38 \pm 24\ (68\ \%\ \text{CL, statistical}).$$

４点相関関数(トリスペクトル)の観測可能性

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Angular trispectrum of CMB temperature anisotropy from primordial non-Gaussianity with the full radiation transfer function

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We calculate the cosmic microwave background (CMB) angular trispectrum, spherical harmonic transform of the four-point correlation function, from primordial non-Gaussianity in primordial curvature perturbations characterized by a constant nonlinear coupling parameter, $f_{\rm NL}$. We fully take into account the effect of the radiation transfer function, and thus provide the most accurate estimate of the signal-to-noise ratio of the angular trispectrum of CMB temperature anisotropy. We find that the predicted signal-to-noise ratio of the trispectrum summed up to a given l is approximately a power-law, $(S/N)(<l) \sim 2.2 \times 10^{-9} f_{\rm NL}^2 l^2$, up to the maximum multipole that we have reached in our numerical calculation, $l = 1200$, assuming that the error is dominated by cosmic variance. Our results indicate that the signal-to-noise ratio of the temperature trispectrum exceeds that of the bispectrum at the critical multipole, $l_c \sim 1500(50/|f_{\rm NL}|)$. Therefore, the trispectrum of the Planck data is more sensitive to primordial non-Gaussianity than the bispectrum for $|f_{\rm NL}| \gtrsim 50$. We also report the predicted constraints on the amplitude of trispectrum, which may be useful for other non-Gaussian models such as curvaton models.

$$\Phi = \Phi_g + f_{NL}\Phi_g^2$$ （ローカル型非ガウス性）

$$\langle\Phi\Phi\Phi\Phi\rangle \simeq f_{NL}^2\langle\Phi_g\Phi_g\rangle\langle\Phi_g\Phi_g\rangle$$

トリスペクトルを調べる機運が高まった

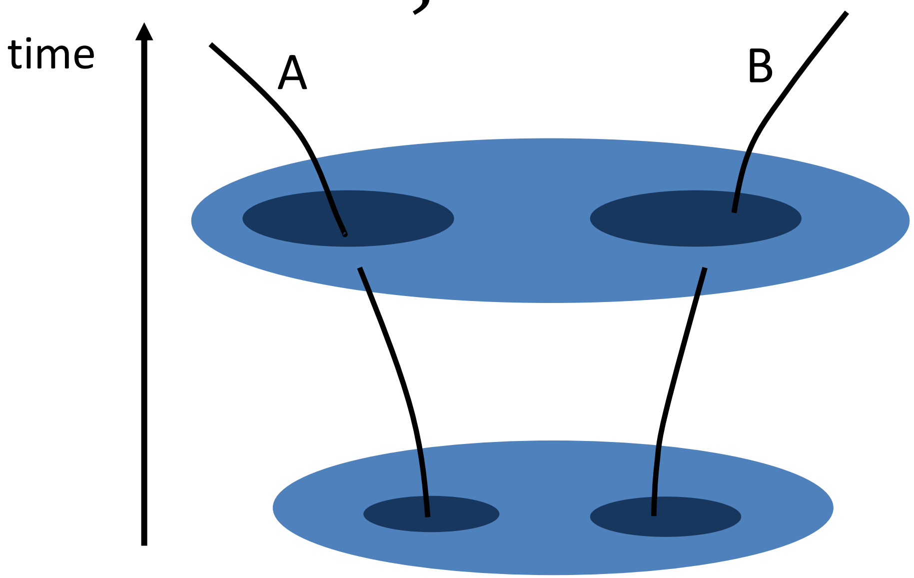
４点相関関数

$$\zeta(x) = \sum_a N_a\,\delta\phi^a(x) + \frac{1}{2}\sum_{a,b} N_{,ab}\delta\phi^a(x)\delta\phi^b(x) + \cdots$$

モデルごとに$N_a, N_{,ab},..$が異なる

δN formalismを使って、展開係数が求められる

$$\zeta = \delta N$$



トリスペクトル

$$\left\langle \zeta_{\overrightarrow{k_1}} \zeta_{\overrightarrow{k_2}} \zeta_{\overrightarrow{k_3}} \zeta_{\overrightarrow{k_4}} \right\rangle = (2\pi)^3 T_\zeta(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3}, \overrightarrow{k_4}) \delta^{(3)}(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3} + \overrightarrow{k_4})$$

$$\zeta(x) = \sum_a N_a \, \delta\phi^a(x) + \frac{1}{2} \sum_{a,b} N_{,ab} \delta\phi^a(x) \delta\phi^b(x) + \cdots$$

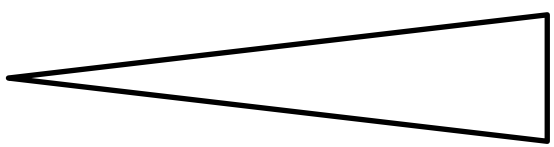
$\delta\phi^a$:ガウス揺らぎ

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

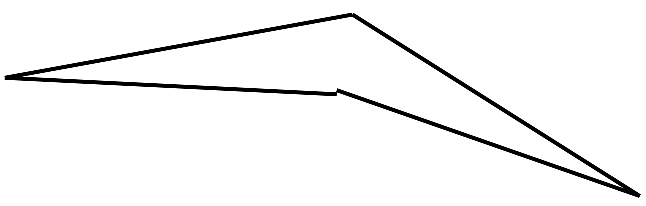
$$\begin{aligned} T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \quad &= \quad \underline{\tau_{\rm NL}} \left(P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms} \right) \\ &\quad + \frac{54}{25} \underline{g_{\rm NL}} \left(P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms} \right) \end{aligned}$$

$$\frac{6}{5} f_{\mathrm{NL}} = \frac{N_a N_b N_{ab}}{(N_c N_c)^2}$$

$$\tau_{\mathrm{NL}} = \frac{N_a N_b N_{ac} N_{bc}}{(N_d N_d)^3} \qquad \frac{54}{25} g_{\mathrm{NL}} = \frac{N_{abc} N_a N_b N_c}{(N_d N_d)^3}$$



f_{NL}



τ_{NL}

何かありそう

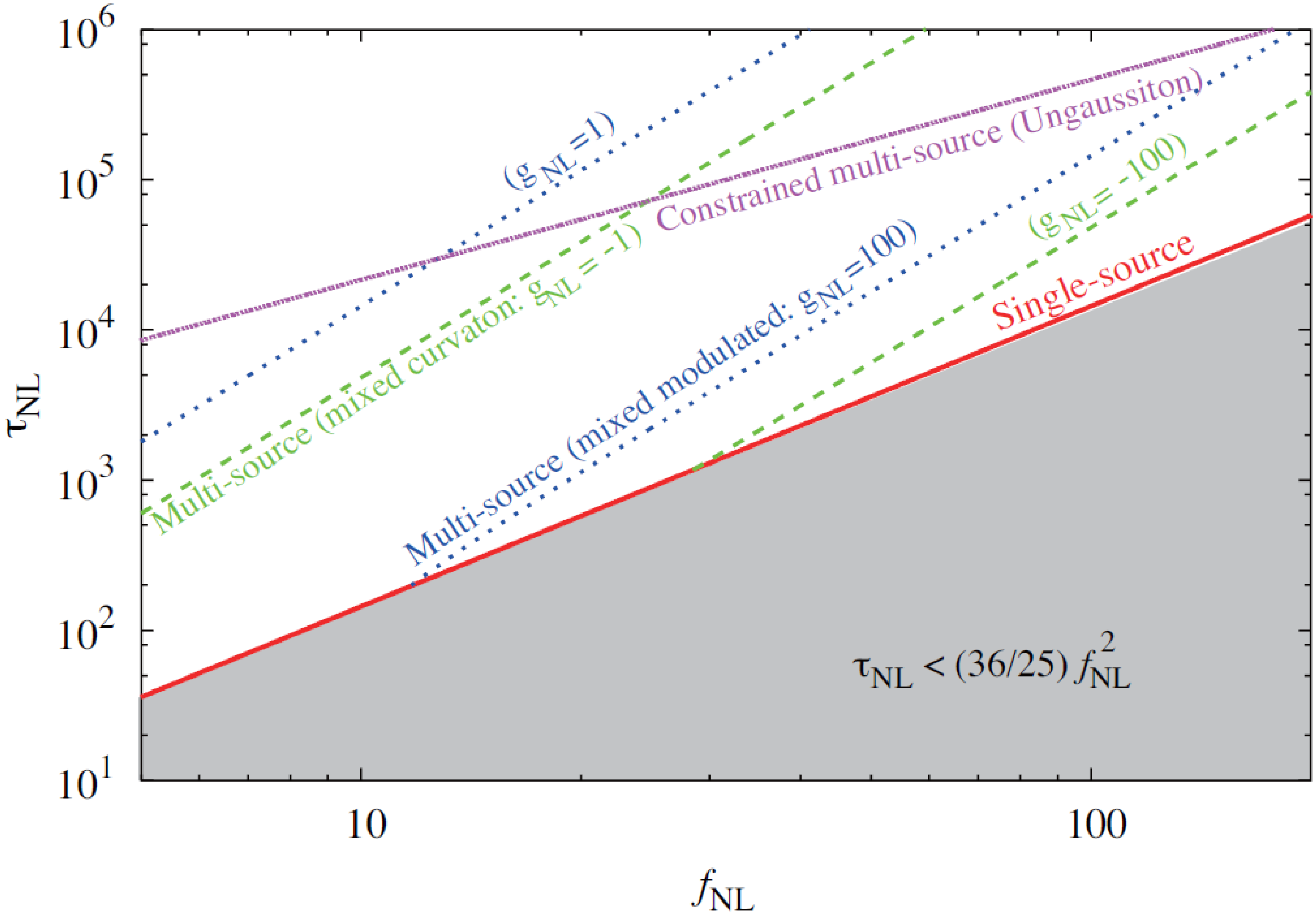
次の不等式が成り立つ

$$\tau_{NL} \geq \frac{36}{25} f_{NL}^2$$

Suyama&Yamaguchi 2007

- 寄与する揺らぎが単一の場合は等号が成立
- 寄与する揺らぎが複数の場合は不等号
- 非ガウス性はトリスペクトルが卓越する可能性

等号が成立するかどうかが測定できると、原始揺らぎに寄与した場が単数か複数か判別できる



τ_{NL}
f_{NL}
10^1
10^2
10^3
10^4
10^5
10^6
10
100
$(g_{NL}=1)$
Multi-source (mixed curvaton: $g_{NL}=-1$)
Constrained multi-source (Ungaussiton)
Multi-source (mixed modulated: $g_{NL}=100$)
$(g_{NL}=-100)$
Single-source
$\tau_{NL} < (36/25)\, f_{NL}^2$

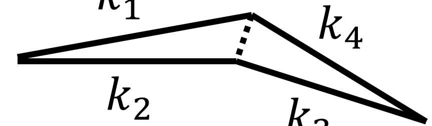
TS+ 2010

不等式の拡張

Assassi+ 2012

$$f_{NL} = \frac{5}{12} \lim_{k_3 \to 0} \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_2) P_\zeta(k_3)}$$

$$\tau_{NL} = \frac{1}{4} \lim_{k_{12} \to 0} \frac{T_\zeta(\overrightarrow{k_1}, \overrightarrow{k_2}, \overrightarrow{k_3}, \overrightarrow{k_4})}{P_\zeta(k_1) P_\zeta(k_3) P_\zeta(k_{12})}$$



$$\langle \zeta^2(\vec{x}) \zeta^2(0) \rangle = \sum_{n, \vec{q}} \langle \zeta^2(\vec{x}) | n_{\vec{q}} \rangle \langle n_{\vec{q}} | \zeta^2(0) \rangle$$

$$\int d^3x \ e^{-i\vec{k}\cdot\vec{x}} \, \langle \zeta^2(\vec{x}) \zeta^2(0) \rangle = \sum_n |\langle n_{\vec{k}} | \zeta^2(0) \rangle|^2$$

$$\int d^3x\ e^{-i\vec{k}\cdot\vec{x}} \langle \zeta^2(\vec{x})\zeta^2(0)\rangle = \frac{|\langle \zeta_{\vec{k}}|\zeta^2(0)\rangle|^2}{P_\zeta(k)} + \sum_m |\langle m_{\vec{k}}|\zeta^2(0)\rangle|^2 \geq \frac{|\langle \zeta_{\vec{k}}|\zeta^2(0)\rangle|^2}{P_\zeta(k)}$$

$$\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\ T_\zeta(\vec{q}_1,\vec{k}-\vec{q}_1,\vec{q}_2,-\vec{k}-\vec{q}_2) \geq \frac{\left|\int \frac{d^3q}{(2\pi)^3}\ B_\zeta(k,q,k-q)\right|^2}{P_\zeta(k)}$$

一般化された不等式

$$\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\left(\tau_{NL} - \frac{36}{25}f_{NL}^2\right)P_\zeta(q_1)P_\zeta(q_2) \geq 0$$

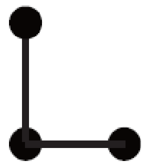
※単一場なら等号成立

f_{NL}, τ_{NL}が定数ならば　$\tau_{NL} \geq \frac{36}{25}f_{NL}^2$

より高次相関関数の間の不等式

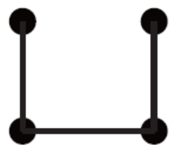
TS+ 2011

３点相関

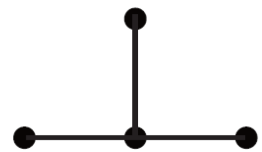


f_{NL}

４点相関

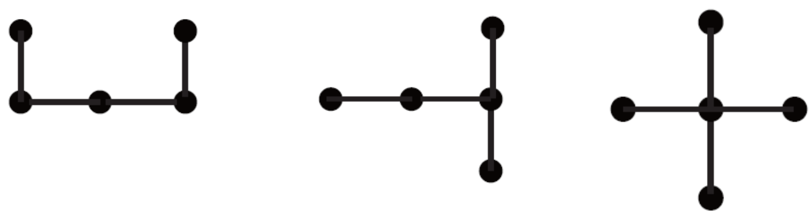


τ_{NL}

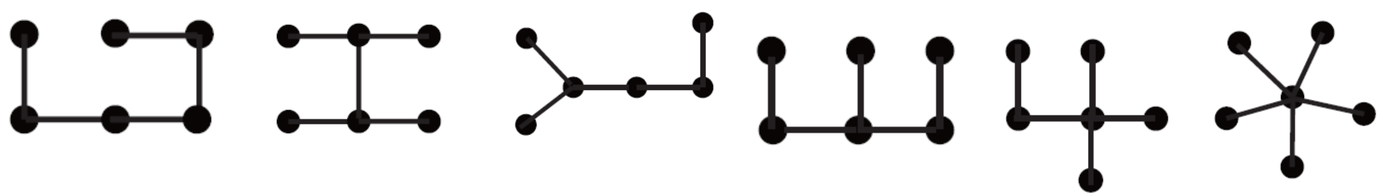


g_{NL}

５点相関



６点相関



例

$$\tau_6^{(1)}\tau_6^{(2)} \geq \left(g_6^{(1)}\right)^2 \qquad \tau_6^{(1)} \geq \tau_4^2, \qquad \tau_6^{(2)} \geq g_4^2$$

$$\tau_6^{(1)}\tau_4 \geq \left(f_5^{(1)}\right)^2, \qquad \tau_6^{(2)}\tau_4 \geq \left(f_5^{(2)}\right)^2$$

g_{NL}について

$$\frac{54}{25}g_{\rm NL} = \frac{N_{abc}N_aN_bN_c}{(N_dN_d)^3}$$

f_{NL}、τ_{NL}とは直接関係ない。個別の模型に強く依存する。

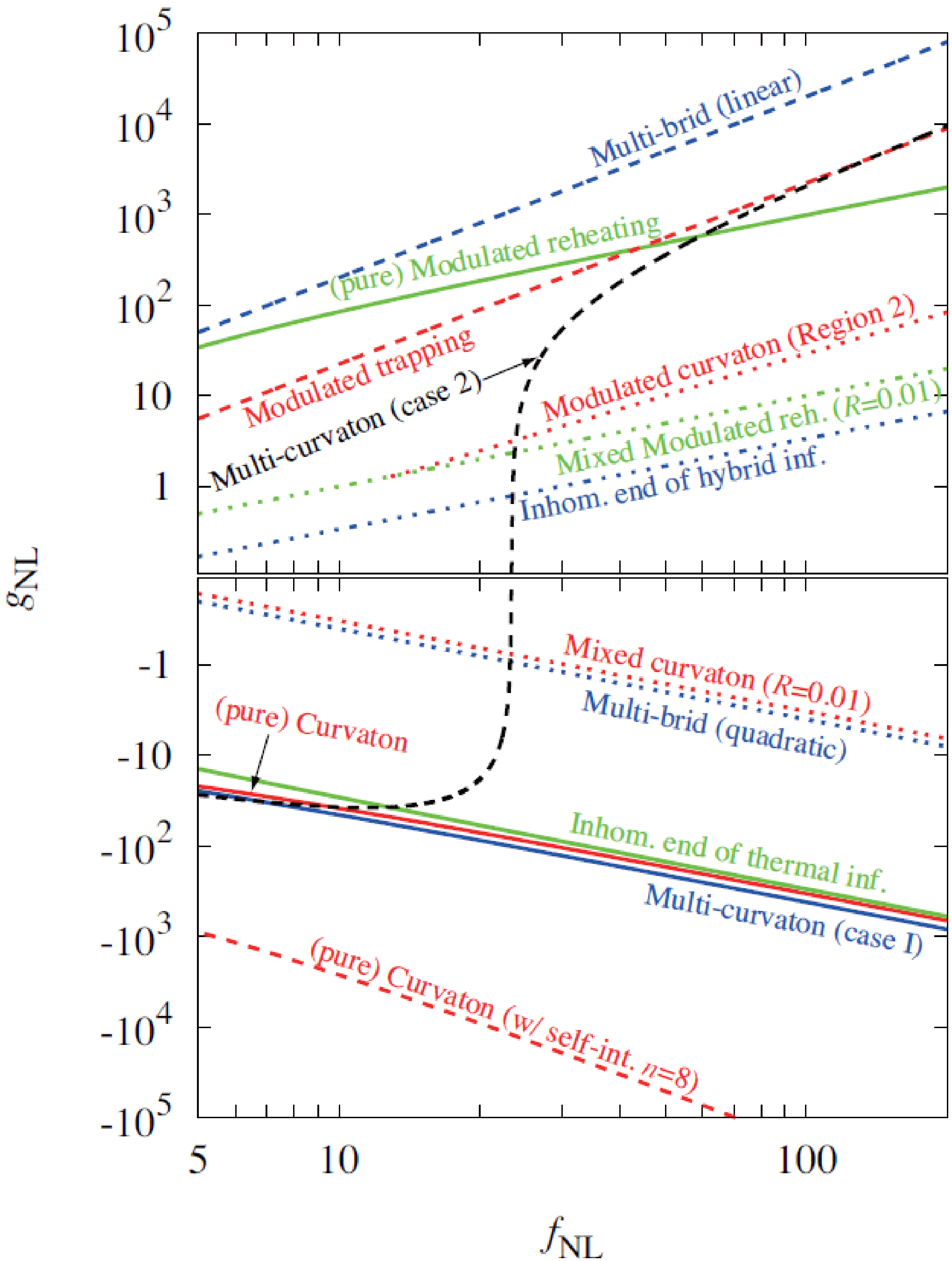
例: 自己相互作用ありカーバトン　(e.g. Engvist&Nurmi 2005)

$$V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda m_\sigma^4\left(\frac{\sigma}{m_\sigma}\right)^n$$

$$g_{\rm NL} = -\frac{10}{3}f_{\rm NL} - \frac{575}{108}$$　自己相互作用なし

$$g_{\rm NL} \simeq \underline{A_{\rm NQ}f_{\rm NL}^2} + B_{\rm NQ}f_{\rm NL} + C_{\rm NQ}$$　自己相互作用あり

g_{NL}は、ポテンシャルの形に強く依存する


Multi-brid (linear)
(pure) Modulated reheating
Modulated trapping
Multi-curvaton (case 2)
Modulated curvaton (Region 2)
Mixed Modulated reh. (R=0.01)
Inhom. end of hybrid inf.
Mixed curvaton (R=0.01)
Multi-brid (quadratic)
(pure) Curvaton
Inhom. end of thermal inf.
Multi-curvaton (case I)
(pure) Curvaton (w/ self-int. n=8)
g_{NL}
f_{NL}
10^5
10^4
10^3
10^2
10
1
-1
-10
-10^2
-10^3
-10^4
-10^5
5
10
100

TS+ 2010

$f_{NL}, \tau_{NL}, g_{NL}$の間の整合性関係式

Category	$f_{\rm NL}$–$\tau_{\rm NL}$ relation	Examples and $f_{\rm NL}$–$g_{\rm NL}$ relation
Single-source	$\tau_{\rm NL} = (6f_{\rm NL}/5)^2$	(pure) curvaton (w/o self-interaction) $[g_{\rm NL} = -(10/3)f_{\rm NL} - (575/108)]^{(a)}$
		(pure) curvaton (w/ self-interaction) $[g_{\rm NL} = A_{\rm NQ}f_{\rm NL}^2 + B_{\rm NQ}f_{\rm NL} + C_{\rm NQ}]^{(b)}$
		(pure) modulated reheating $[g_{\rm NL} = 10f_{\rm NL} - (50/3)]^{(c)}$
		modulated-curvaton scenario $\left[g_{\rm NL} = 3r_{\rm dec}^{1/2}f_{\rm NL}^{3/2}\right]^{(d)}$
		Inhomogeneous end of hybrid inflation $[g_{\rm NL} = (10/3)\eta_{\rm cr}f_{\rm NL}]$
		Inhomogeneous end of thermal inflation $[g_{\rm NL} = -(10/3)f_{\rm NL} - (50/27)]^{(e)}$
		Modulated trapping $[g_{\rm NL} = (2/9)f_{\rm NL}^2]^{(f)}$
Multi-source	$\tau_{\rm NL} > (6f_{\rm NL}/5)^2$	mixed curvaton and inflaton $[g_{\rm NL} = -(10/3)(R/(1+R))f_{\rm NL} - (575/108)(R/(1+R))^3]^{(g)}$
		mixed modulated and inflaton $[g_{\rm NL} = 10(R/(1+R))f_{\rm NL} - (50/3)(R/(1+R))^3]^{(h)}$
		mixed modulated trapping and inflaton $[g_{\rm NL} = (2/9)((1+R)/R)f_{\rm NL}^2 = (25/162)\tau_{\rm NL}]^{(i)}$
		multi-curvaton $[g_{\rm NL} = C_{\rm mc}f_{\rm NL},\ \ g_{\rm NL} = (4/15)f_{\rm NL}^2]^{(j)}$
		Multi-brid inflation (quadratic potential) $[g_{\rm NL} = -(10/3)\eta f_{\rm NL},\ \ g_{\rm NL} = 2f_{\rm NL}^2]^{(k)}$
		Multi-brid inflation (linear potential) $[g_{\rm NL} = 2f_{\rm NL}^2]^{(l)}$
Constrained multi-source	$\tau_{\rm NL} = Cf_{\rm NL}^n$	ungaussiton ($C \simeq 10^3,\ n = 4/3$)

TS+ 2010

４点相関の観測制限

$$\tau_{NL} < 2800$$ (Planck 2013, 95CL)

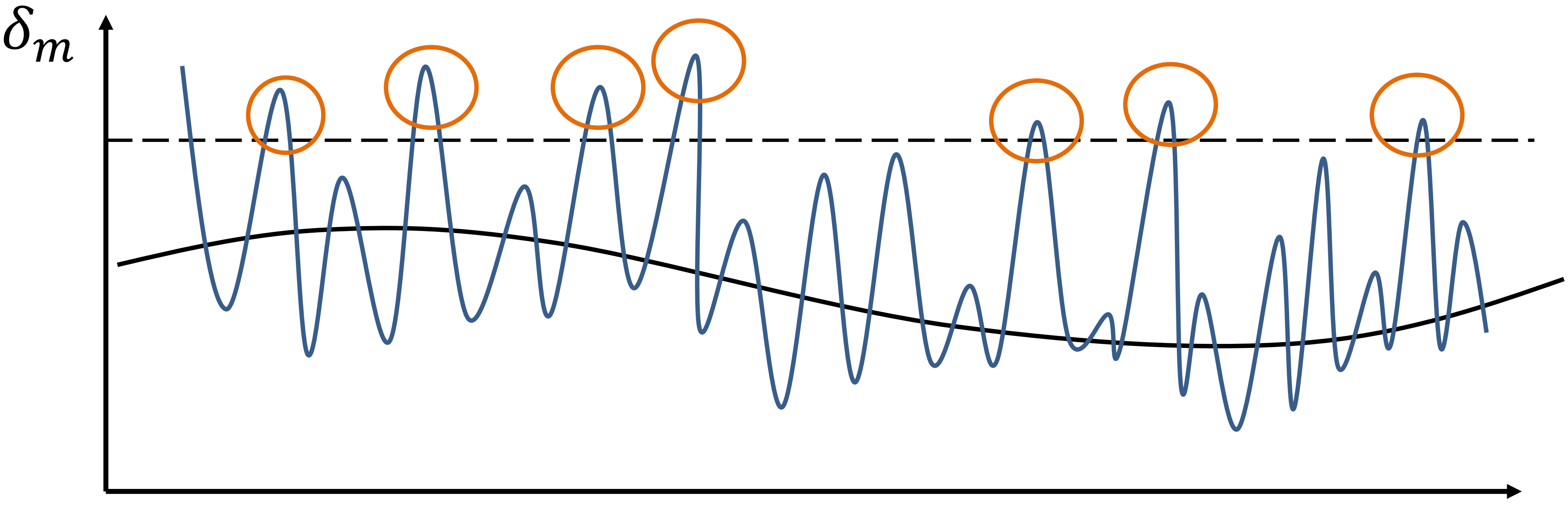
$$g_{NL} = (-5.8 \pm 6.5) \times 10^4$$ (Planck 2018, 68CL)

大きな非ガウス性を生み出すモデルは棄却された

マイルドな非ガウス性($f_{NL} = O(1)$)のモデルは生き残っている

CMBでこれ以上制限が大幅に強まることはなさそう

大規模構造と非ガウス性



$$\delta_{\mathrm{gal}} = b\delta_m$$

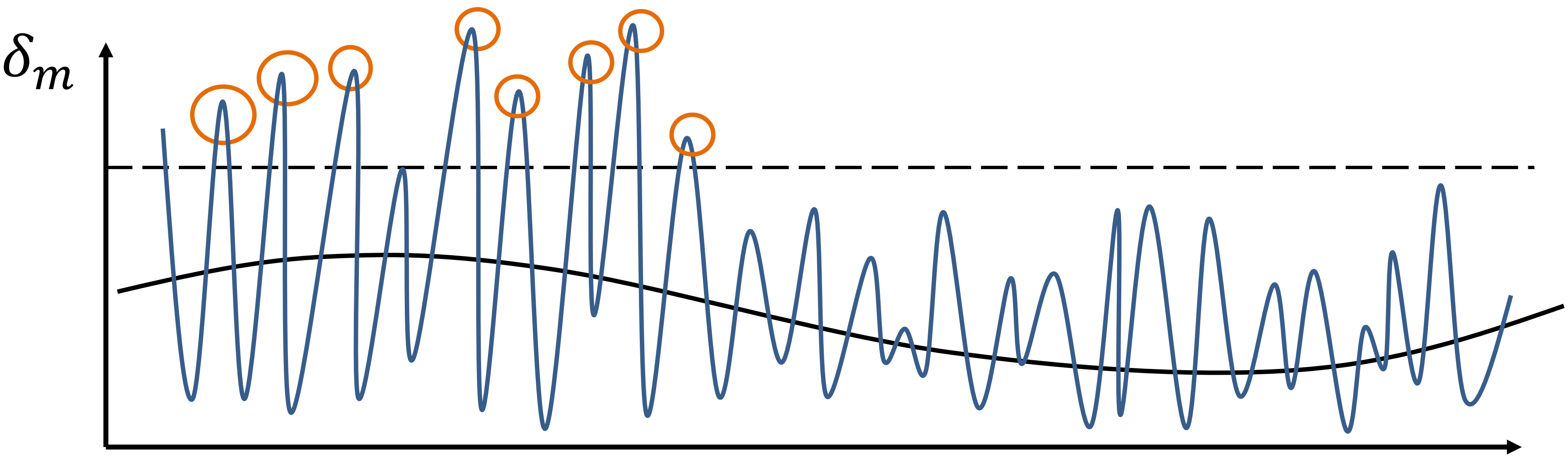
構造物(銀河など)は、物質揺らぎの情報を持つ

ローカル型非ガウス性

$$\Phi = \Phi_g + f_{NL}\Phi_g^2$$

Φ_g : ガウシアン

$$\Phi_g = \Phi_{g,s} + \Phi_{g,l} \Rightarrow \Phi_s \simeq \left(1 + 2f_{NL}\Phi_{g,L}\right)\Phi_{g,s}$$



大スケールで大きくなるバイアス $\Delta b \propto \dfrac{f_{NL}}{k^2}$

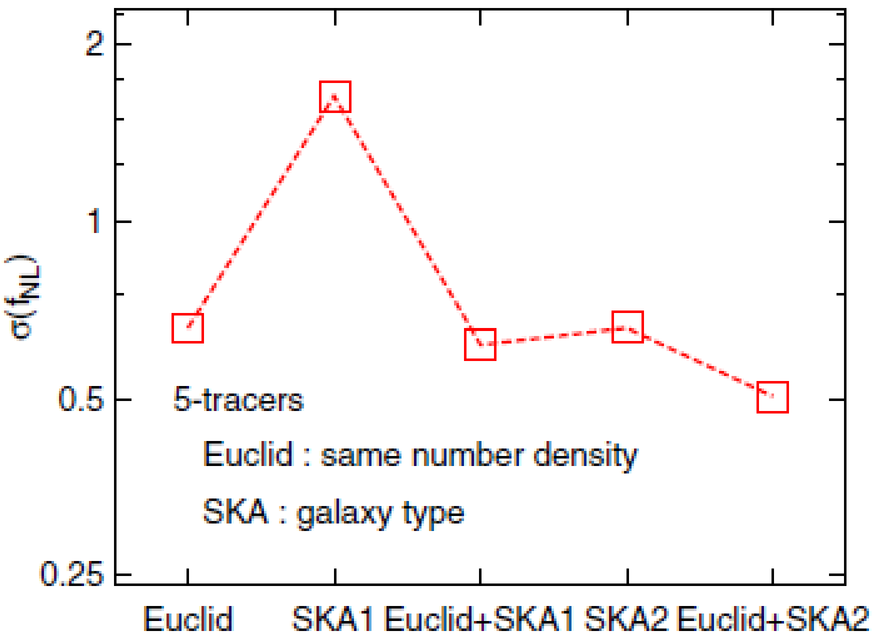
Redshift-weighted constraints on primordial non-Gaussianity from the clustering of the eBOSS DR14 quasars in Fourier space

Emanuele Castorina,[a,c] Nick Hand,[b] Uroš Seljak,[a,b,c] Florian Beutler,[d] Chia-Hsun Chuang,[e,f] Cheng Zhao,[g] Héctor Gil-Marín,[h] Will J. Percival,[i,j] Ashley J. Ross,[k] Peter Doohyun Choi,[l] Kyle Dawson,[m] Axel de la Macorra,[n] Graziano Rossi,[l] Rossana Ruggeri,[d] Donald Schneider,[o] Gong-Bo Zhao[p]

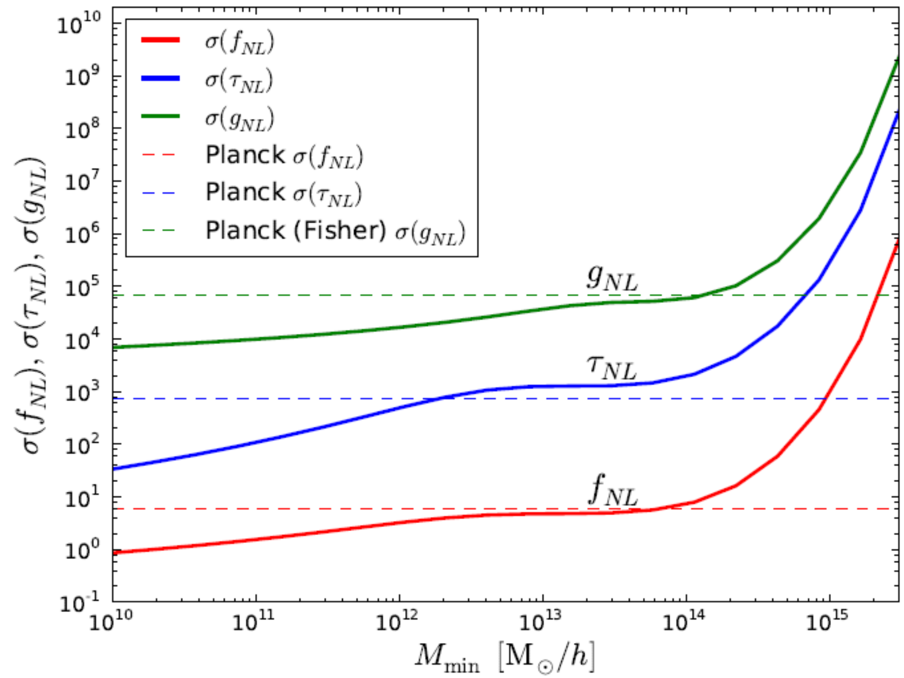
15万個ほどのクエーサーの観測

$$-51 < f_{NL} < 21$$

将来制限

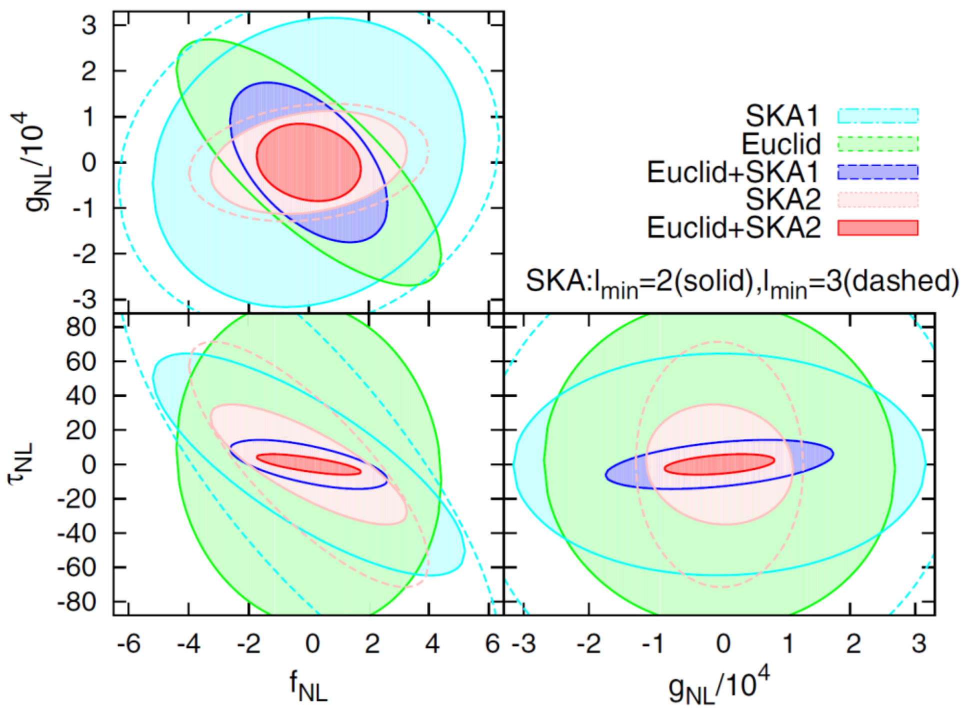


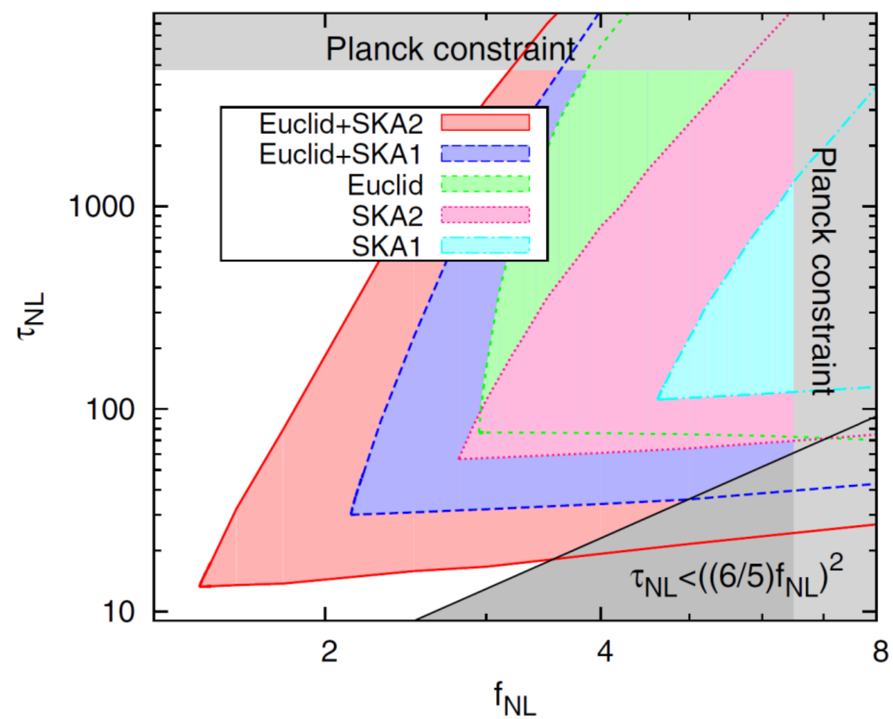
Yamauchi+ 2014



Ferraro+ 2014

将来制限 Yamauchi+ 2015





$\Delta f_{NL} \lesssim 1.5, \Delta\tau_{NL} \lesssim 17$ くらいまでいけそう

まとめ

揺らぎの非ガウス性は、インフレーション模型や揺らぎの起源を明らかにするうえで重要な観測量となった。

非ガウス性の制限の重要な局面に入りつつある

今後は、大規模構造観測から非ガウス性をより精密に探査できると期待される。