Quantum Kinetic Equation in the Rotating Frame and Chiral Kinetic Theory

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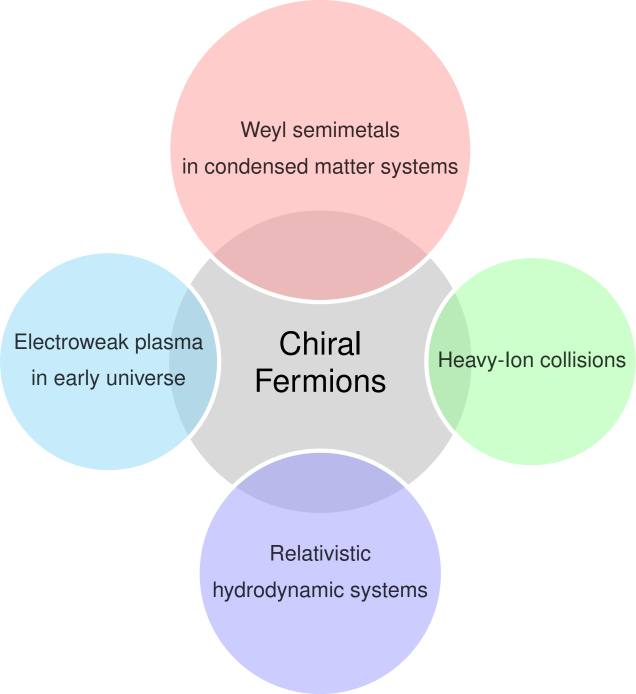

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Chiral Anomalies:

CME

$\boldsymbol{j}_{CME} = \frac{Q}{2\pi^2}\mu_5 \boldsymbol{B}$; chiral magnetic effect has been observed in heavy-ion collisions

CSE

$\boldsymbol{j}_5^{CSE} = \frac{Q}{2\pi^2}\mu \boldsymbol{B}$;
chiral separation effect

CVE

$\boldsymbol{j}_{CVE} = \frac{\mu\mu_5}{\pi^2}\boldsymbol{\omega}$; chiral vortical effect has been discovered in hydrodynamic systems

LPE

$\boldsymbol{j}_5^{LPE} = \left(\frac{T^2}{6} + \frac{\mu^2+{\mu_5}^2}{2\pi^2}\right)\boldsymbol{\omega}$;
local polarization effect

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Chiral Kinetic Theory:

- M. A. Stephanov and Y. Yin, *Chiral Kinetic Theory*, Phys. Rev. Lett. 109, 162001 (2012).
- J-W.Chen, S. Pu, Q. Wang, and X-N. Wang, *Berry Curvature and Four-Dimensional Monopoles in the Relativistic Chiral Kinetic Equation*, Phys. Rev. Lett. 110, 262301 (2013).
- C. Y. Hidaka, S. Pu, and D. L. Yang, *Relativistic chiral kinetic theory from quantum field theories,* Phys. Rev. D 95, 091901 (2017).
- Ö. F. Dayi, E. Kilinçarslan, and E. Yunt, *Semiclassical dynamics of Dirac and Weyl particles in rotating coordinates,* Phys. Rev. D 95, 085005 (2017); Ö. F. Dayi, E. Kilinçarslan, *Nonlinear chiral plasma transport in rotating coordinates,* Phys. Rev. D 96, 043514 (2017).
- Ö. F. Dayi and E. Kilinçarslan, *Quantum kinetic equation in the rotating frame and chiral kinetic theory,* Phys. Rev. D 98, 081701(R) (2018).

Contents

- The general gauge invariant Wigner operator[1]:

$$\hat{W}_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-\frac{i}{\hbar}p\cdot y} \bar{\psi}_\nu(x_+) U(x_+,x_-) \psi_\mu(x_-),$$

the gauge link: $U(x_+,x_-) = \mathscr{P} e^{-\frac{iQ}{\hbar} y^\mu \int_0^1 ds A_\mu(x_\pm + sy)}; x_\pm = x \pm y/2.$

$$W_{\mu\nu}(x,p) = \langle \hat{W}_{\mu\nu}(x,p) \rangle$$

[1] D. Vasak, M. Gyulassy, H-T. Elze, Annals of Physics **173**,462 (1987)

[2] H.D Sivak, Annals of Physics **159**,351 (1985)

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- The Quantum Kinetic Equation[1]

$$\gamma_\mu \left(p^\mu + \frac{i\hbar}{2} \nabla^\mu \right) W(x,p) = 0.$$

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- The Quantum Kinetic Equation[1]

$$\gamma_\mu \left(p^\mu + \frac{i\hbar}{2} \nabla^\mu \right) W(x,p) = 0.$$

- $\nabla^\mu = \partial_x^\mu - QF^{\mu\nu}\partial_\nu^p$
- $W = \frac{1}{4}(\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^\mu\gamma^5\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu})$ [2]

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- $\tilde{\nabla}^\mu \Rightarrow \nabla^\mu + ?$

[1] L. Rezzola and O. Zanotti, *Relativistic Hydrodynamics* (Oxford University Press, New York (2013))

[2] Ö. F. Dayi and E. Kilinçarslan, Phys. Rev. D **98**, 081701(R) (2018)

- $\tilde{\nabla}^\mu \Rightarrow \nabla^\mu + ?\ \ \frac{Q}{2\pi^2}\mu_5 \boldsymbol{B} \rightarrow \frac{\mu\mu_5}{\pi^2}\boldsymbol{\omega}$

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- $\tilde{\nabla}^\mu \Rightarrow \nabla^\mu + \,?\;\; \frac{Q}{2\pi^2}\mu_5 \boldsymbol{B} \rightarrow \frac{\mu\mu_5}{\pi^2}\boldsymbol{\omega}$
- Circulation tensor[1] :

$$\begin{aligned} C_{\mu\nu} &= \partial_\mu((u\cdot p)u_\nu) - \partial_\nu((u\cdot p)u_\mu) \\ &= 2h\Omega_{\mu\nu} + (\partial_\mu\; u\cdot p)u_\nu - (\partial_\nu\; u\cdot p)u_\mu, \end{aligned}$$

kinematic vorticity tensor: $\Omega_{\mu\nu} = \frac{1}{2}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right).$

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- We add $(C_{\mu\nu} - 2h\Omega_{\mu\nu})\partial^p_\nu$ to ∇_μ to take into account the noninertial forces[2] :

$$\tilde{\nabla}^\mu = \partial^\mu_x - [QF^{\mu\nu} + (\partial^\mu n^\alpha)p_\alpha n^\nu - (\partial^\nu n^\alpha)p_\alpha n^\mu]\partial^p_\nu.$$

[1] L. Rezzola and O. Zanotti, *Relativistic Hydrodynamics* (Oxford University Press, New York (2013))

[2] Ö. F. Dayi and E. Kilinçarslan, Phys. Rev. D **98**, 081701(R) (2018)

$$\hbar \varepsilon_{\mu\nu\alpha\rho} \tilde{\nabla}^{\alpha}_{(n)} \mathscr{J}^{\rho}_{\chi} = -2\chi \left(p_\mu \mathscr{J}_{\nu_\chi} - p_\nu \mathscr{J}_{\mu_\chi}\right)$$

$$p_\mu \mathscr{J}^{\mu}_{\chi} = 0 \qquad \tilde{\nabla}_\mu \mathscr{J}^{\mu}_{\chi} = 0$$

$$\mathscr{J}^{\mu}_{\chi} = \frac{1}{2}\left(\mathscr{V}^{\mu} + \chi \mathscr{A}^{\mu}\right)$$

$$\gamma_\mu \left(p^\mu + \frac{i\hbar}{2}\tilde{\nabla}^\mu\right) W = 0$$

$$W = \frac{1}{4}\left(\mathscr{F} + i\gamma^5 \mathscr{P} + \gamma^\mu \mathscr{V}_\mu + \gamma^\mu \gamma^5 \mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu} \mathscr{S}_{\mu\nu}\right)$$

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- $\mathscr{J}_\chi^{\mu} = \mathscr{J}_\chi^{(0)\mu} + \hbar \; \mathscr{J}_\chi^{(1)\mu}$
- $p_\mu \mathscr{J}_\chi^{\mu} = 0 \Rightarrow \mathscr{J}_\chi^{(0)\mu} = p^\mu \delta(p^2) f_\chi^0$

- $\mathscr{J}_\chi^\mu = \mathscr{J}_\chi^{(0)\mu} + \hbar\, \mathscr{J}_\chi^{(1)\mu}$
- $p_\mu \mathscr{J}_\chi^\mu = 0 \Rightarrow \mathscr{J}_\chi^{(0)\mu} = p^\mu \delta(p^2) f_\chi^0$
- $\varepsilon^{\mu\nu\alpha\rho} \tilde{\nabla}_{(n)\alpha} \mathscr{J}_{\chi\rho}^{(0)} = -2\chi [p^\mu \mathscr{J}_\chi^{(1)\nu} - p^\nu \mathscr{J}_\chi^{(1)\mu}] \Rightarrow$

$$\begin{aligned} \mathscr{J}_\chi^{(1)\mu} &= p^\mu \delta(p^2) f_\chi^1 - \frac{\delta(p^2)}{2p^2} \chi Q \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} p_\nu f_\chi^0 \\ &- \frac{\delta(p^2)}{p^2} \chi \varepsilon^{\mu\nu\alpha\rho} p_\nu (\partial_\alpha n_\beta) p^\beta n_\rho f_\chi^0 + \mathscr{K}^\mu, \end{aligned}$$

$$\mathscr{K}^\mu = S_{(n)}^{\mu\nu} (\tilde{\nabla}_{(n)\nu} f_\chi^0) \delta(p^2),$$

where

$$S_{(n)}^{\mu\nu} = \frac{\chi}{2n \cdot p} \varepsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma$$

corresponds to spin tensor.

j^{μ}

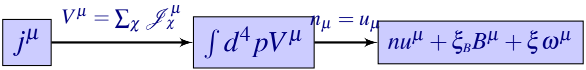
j_5^{μ}

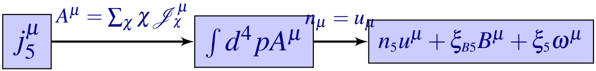
- $f_{\chi}^{FD} = \frac{2}{(2\pi)^3} \sum_{s=\pm 1} \frac{\theta(sn\cdot p)}{e^{s(u\cdot p - \mu_{\chi})/T} + 1}$

$$j^{\mu} \xrightarrow{V^{\mu} = \sum_{\chi} \mathscr{J}^{\mu}_{\chi}} \int d^4 p V^{\mu}$$

$$j_5^{\mu} \xrightarrow{A^{\mu} = \sum_{\chi} \chi \mathscr{J}^{\mu}_{\chi}} \int d^4 p A^{\mu}$$

- $n_{\mu} = u_{\mu}$





- $n = \frac{\mu}{3\pi^2}(\mu^2 + 3\mu_5^2 + \pi^2 T^2); \qquad \mu = \frac{\mu_R + \mu_L}{2}$
- $n_5 = \frac{\mu_5}{3\pi^2}(3\mu^2 + \mu_5^2 + \pi^2 T^2); \qquad \mu_5 = \frac{\mu_R - \mu_L}{2}$

$$j^\mu \xrightarrow{V^\mu = \sum_\chi \mathscr{J}^\mu_\chi} \int d^4p V^\mu \xrightarrow{n_\mu = u_\mu} nu^\mu + \xi_B B^\mu + \xi \omega^\mu$$

$$j_5^\mu \xrightarrow{A^\mu = \sum_\chi \chi \mathscr{J}^\mu_\chi} \int d^4p A^\mu \xrightarrow{n_\mu = u_\mu} n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu$$

- CME: $\xi_B = \frac{Q}{2\pi^2}\mu_5$; CVE: $\xi = \frac{\mu\mu_5}{\pi^2}$

$$j^\mu \xrightarrow{V^\mu = \Sigma_\chi \mathscr{J}^\mu_\chi} \int d^4p V^\mu \xrightarrow{n_\mu = u_\mu} nu^\mu + \xi_B B^\mu + \xi \omega^\mu$$

$$j^\mu_5 \xrightarrow{A^\mu = \Sigma_\chi \chi \mathscr{J}^\mu_\chi} \int d^4p A^\mu \xrightarrow{n_\mu = u_\mu} n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu$$

- CME: $\xi_B = \frac{Q}{2\pi^2}\mu_5$; CVE: $\xi = \frac{\mu\mu_5}{\pi^2}$
- CSE: $\xi_{B5} = \frac{Q}{2\pi^2}\mu$ LPE: $\xi_5 = \frac{T^2}{6} + \frac{\mu^2+{\mu_5}^2}{2\pi^2}$

- $p_\mu \mathscr{J}_\chi^\mu = 0$
- $\varepsilon^{\mu\nu\alpha\rho} \tilde{\nabla}_{(n)\alpha} \mathscr{J}_{\chi\rho}^{(0)} = -2\chi[p^\mu \mathscr{J}_\chi^{(1)\nu} - p^\nu \mathscr{J}_\chi^{(1)\mu}]$
- $\tilde{\nabla}_\mu \mathscr{J}_\chi^\mu = 0$

- $\tilde{\nabla}_\mu \mathscr{J}^\mu_\chi = 0$
- The covariant chiral kinetic equation:

$$\tilde{\nabla}_\mu \mathscr{J}^\mu_\chi = \delta\left(p^2 + \chi\hbar Q \frac{u_\mu \tilde{F}^{\mu\nu} p_\nu}{u\cdot p}\right)\{p\cdot\tilde{\nabla}\left(1 + \hbar\chi\frac{S^{\mu\nu}\Omega_{\mu\nu}}{u\cdot p}\right) + \frac{\chi\hbar Q}{u\cdot p} S^{\mu\nu} E_\mu \tilde{\nabla}_\nu - \frac{\hbar\chi}{u\cdot p} p_\mu \tilde{\Omega}^{\mu\nu}\tilde{\nabla}_\nu + \frac{\hbar\chi}{u\cdot p}(\tilde{\Omega}^{\mu\nu} p_\mu u_\nu)\Omega^{\mu\nu} p_\nu \partial^{(p)}_\mu\} f_\chi = 0.$$

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- $f^1_\chi \Rightarrow \chi \frac{S^{\mu\nu}\Omega_{\mu\nu}}{u\cdot p} f^0_\chi + f^1_\chi$; Phys. Rev. Lett. 115, 021601 (2015).

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- Energy dispersion relation from the delta function:

$$\mathscr{E}^\chi_s = |\boldsymbol{p}|(1 - \hbar s Q \chi \boldsymbol{b}_s \cdot \boldsymbol{B}).$$

$$\delta\left(p^2+\chi\hbar Q\frac{u_\mu \tilde{F}^{\mu\nu}p_\nu}{u\cdot p}\right)\{p\cdot\tilde{\nabla}\left(1+\hbar\chi S^{\mu\nu}\Omega_{\mu\nu}\right)$$
$$+\frac{\chi\hbar Q}{u\cdot p}S^{\mu\nu}E_\mu\tilde{\nabla}_\nu-\frac{\hbar\chi}{u\cdot p}p_\mu\tilde{\Omega}^{\mu\nu}\tilde{\nabla}_\nu$$
$$+\frac{\hbar\chi}{u\cdot p}(\tilde{\Omega}^{\mu\nu}p_\mu u_\nu)\Omega^{\mu\nu}p_\nu\partial_\mu^{(p)}\}f_\chi=0$$

$$\left(\sqrt{\eta}\frac{\partial}{\partial t}+\sqrt{\eta}\dot{\boldsymbol{x}}\cdot\frac{\partial}{\partial \boldsymbol{x}}+\sqrt{\eta}\dot{\boldsymbol{p}}\cdot\frac{\partial}{\partial \boldsymbol{p}}+I_0\frac{\partial}{\partial p^0}\right)f|_{p^0=\mathscr{E}_s^\chi}=0.$$

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- $\int d^4p\{\text{4D TE}\}=\int d^3p\{\text{3D TE}\}$

$$\left(\sqrt{\eta}\frac{\partial}{\partial t}+\sqrt{\eta}\dot{\boldsymbol{x}}\cdot\frac{\partial}{\partial\boldsymbol{x}}+\sqrt{\eta}\dot{\boldsymbol{p}}\cdot\frac{\partial}{\partial\boldsymbol{p}}\right)f(t,\boldsymbol{x},\boldsymbol{q})=0,$$

$$\delta\left(p^2+\chi\hbar Q\frac{u_\mu \tilde{F}^{\mu\nu}p_\nu}{u\cdot p}\right)\{p\cdot\tilde{\nabla}\left(1+\hbar\chi S^{\mu\nu}\Omega_{\mu\nu}\right)$$
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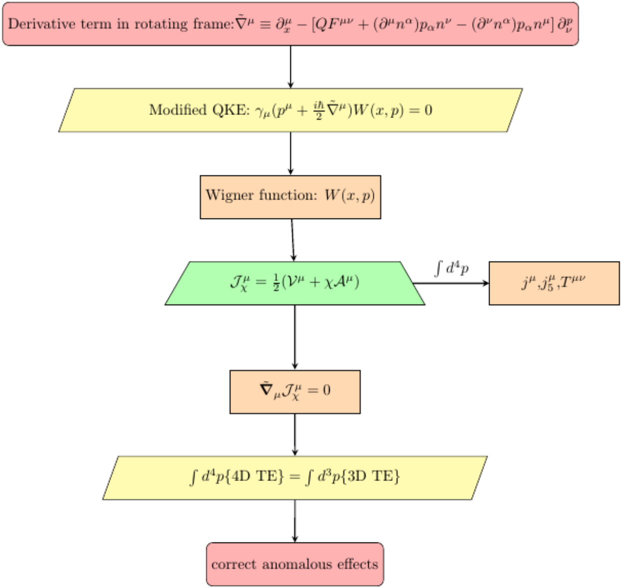
$$\dot{\boldsymbol{p}}\left[\frac{\partial f_\chi(x,p)}{\partial\boldsymbol{p}}\right]_{p_0=\mathscr{E}}+\dot{\boldsymbol{p}}\cdot\frac{\partial\mathscr{E}}{\partial\boldsymbol{p}}\left[\frac{\partial f_\chi(x,p)}{\partial p_0}\right]_{p_0=\mathscr{E}}=\dot{\boldsymbol{p}}\,\frac{\partial f_\chi(t,\boldsymbol{x},\mathscr{E},\boldsymbol{p})}{\partial\boldsymbol{p}}.$$

- $\int d^4p\{4\text{D TE}\} = \int d^3p\{3\text{D TE}\}$.
- $\left(\sqrt{\eta}{}^{\chi}_{s}\frac{\partial}{\partial t} + (\sqrt{\eta}\dot{\boldsymbol{x}})^{\chi}_{s}\cdot\frac{\partial}{\partial \boldsymbol{x}} + (\sqrt{\eta}\dot{\boldsymbol{p}})^{\chi}_{s}\cdot\frac{\partial}{\partial \boldsymbol{p}}\right) f^{eq}_{\chi,s}(t,\boldsymbol{x},\boldsymbol{p}) = 0$

- $\int d^4p\{4\text{D TE}\} = \int d^3p\{3\text{D TE}\}.$
- $\left((\sqrt{\eta})^\chi_s \frac{\partial}{\partial t} + (\sqrt{\eta}\dot{\boldsymbol{x}})^\chi_s \cdot \frac{\partial}{\partial \boldsymbol{x}} + (\sqrt{\eta}\dot{\boldsymbol{p}})^\chi_s \cdot \frac{\partial}{\partial \boldsymbol{p}}\right) f^{eq}_{\chi,s}(t,\boldsymbol{x},\boldsymbol{p}) = 0$
- 3D chiral transport equations:

$$\begin{aligned}
\sqrt{\eta}^\chi_s &= 1 + \hbar s Q \chi \boldsymbol{b}_s \cdot \boldsymbol{B} \\
(\sqrt{\eta}\dot{\boldsymbol{x}})^\chi_s &= \boldsymbol{v}^\chi_s + \hbar\chi(\hat{\boldsymbol{p}} \cdot \boldsymbol{b}_s)(sQ\boldsymbol{B} + 2\mathcal{E}^\chi_s \boldsymbol{\omega}) \\
&\quad + \hbar s Q \chi \boldsymbol{E} \times \boldsymbol{b}_s - 2\hbar\chi(\boldsymbol{\omega} \cdot \boldsymbol{b}_s)\boldsymbol{p} \\
(\sqrt{\eta}\dot{\boldsymbol{p}})^\chi_s &= sQ\boldsymbol{E} + sQ\boldsymbol{v}^\chi_s \times \boldsymbol{B} + \mathcal{E}^\chi_s\, \boldsymbol{v}^\chi_s \times \boldsymbol{\omega} \\
&\quad + \hbar\chi Q^2 \boldsymbol{b}_s(\boldsymbol{E} \cdot \boldsymbol{B}) - 2\hbar\chi(\boldsymbol{\omega} \cdot \boldsymbol{b}_s)\boldsymbol{p} \times \mathcal{E}^\chi_s \boldsymbol{\omega} \\
&\quad - 2\hbar s Q \chi(\boldsymbol{\omega} \cdot \boldsymbol{b}_s)\boldsymbol{p} \times \boldsymbol{B}.
\end{aligned}$$

- $\boldsymbol{j}^\chi_s = \int [dp](\sqrt{\eta}\dot{\boldsymbol{x}})^\chi_s f^{eq,s}_\chi \rightarrow \boldsymbol{j}^{CME}_V, \boldsymbol{j}^{CSE}_A, \boldsymbol{j}^{CVE}_V, \boldsymbol{j}^{LPE}_A$


Derivative term in rotating frame: $\tilde{\nabla}^\mu \equiv \partial_x^\mu - [QF^{\mu\nu} + (\partial^\mu n^\alpha)p_\alpha n^\nu - (\partial^\nu n^\alpha)p_\alpha n^\mu]\,\partial_\nu^p$
Modified QKE: $\gamma_\mu(p^\mu + \frac{i\hbar}{2}\tilde{\nabla}^\mu)W(x,p) = 0$
Wigner function: $W(x,p)$
$\mathcal{J}_\chi^\mu = \frac{1}{2}(\mathcal{V}^\mu + \chi\mathcal{A}^\mu)$
$\int d^4p$
$j^\mu, j_5^\mu, T^{\mu\nu}$
$\tilde{\nabla}_\mu \mathcal{J}_\chi^\mu = 0$
$\int d^4p\{4D\ TE\} = \int d^3p\{3D\ TE\}$
correct anomalous effects

- This modified covariant kinetic theory extended to curved spacetime:

  Ö. F. Dayi and Eda Kilinçarslan, *Some features of semiclassical chiral transport in rotating frames*, Phys. Rev. D 100, 045012 (2019).

- Covariant chiral kinetic theory with collisions :

  Yoshimasa Hidaka and Di-Lun Yang, *Nonequilibrium chiral magnetic/vortical effects in viscous fluids,* Phys. Rev. D 98, 016012 (2018).

- CCKT for massive fermions:

  Jian-Hua Gao and Zuo-Tang Liang, *Relativistic Quantum Kinetic Theory for Massive Fermions and Spin Effects,* Phys. Rev. D 100, 056021 (2019).

Thank You!