Chiral magnetic response with large gradients of axial imbalance

Based on arXiv:1911.00933

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in collaboration with
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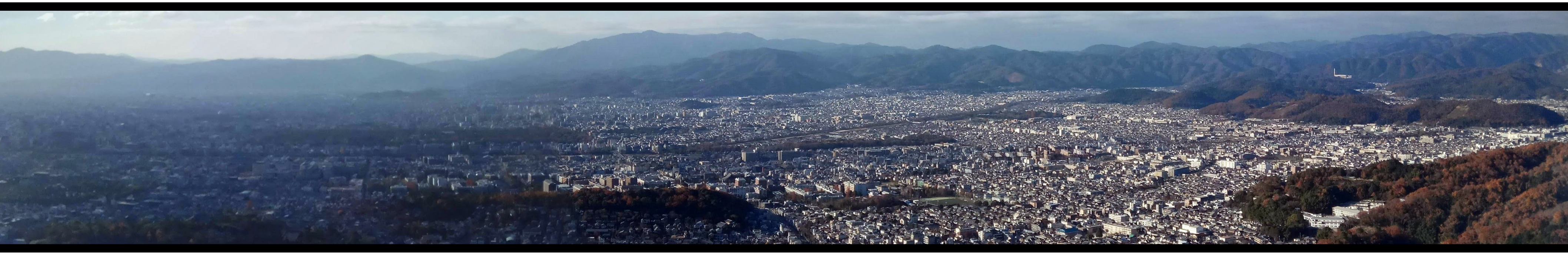

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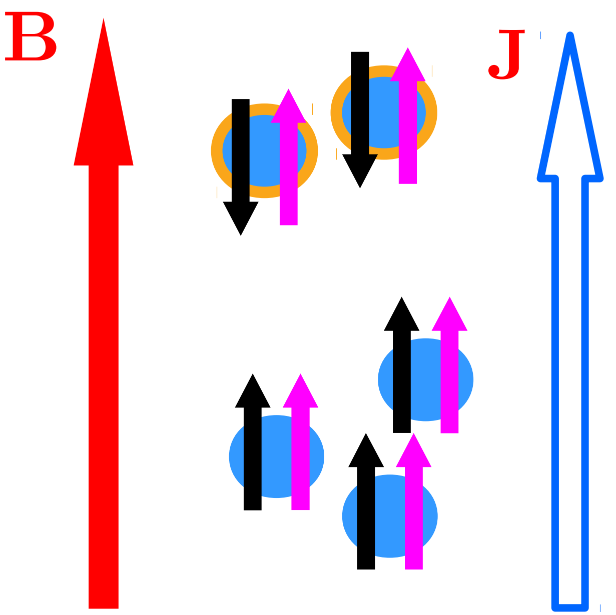
Outline

- **Phenomenology & introduction**
  HIC observability, Weyl-semimetals
  anomaly in QED

- **Linear response to EM and axial fields**
  charge conservation and axial anomaly
  subtleties of the constant field limit
  time-dependent magnetic field

- **Constant *B* and arbitrary axial field**
  spatial structure of response for point-like source
  late time behavior after quench
  charge asymmetry: interplay of scales
  electric current subleading in gradient expansion

Simple picture of CME



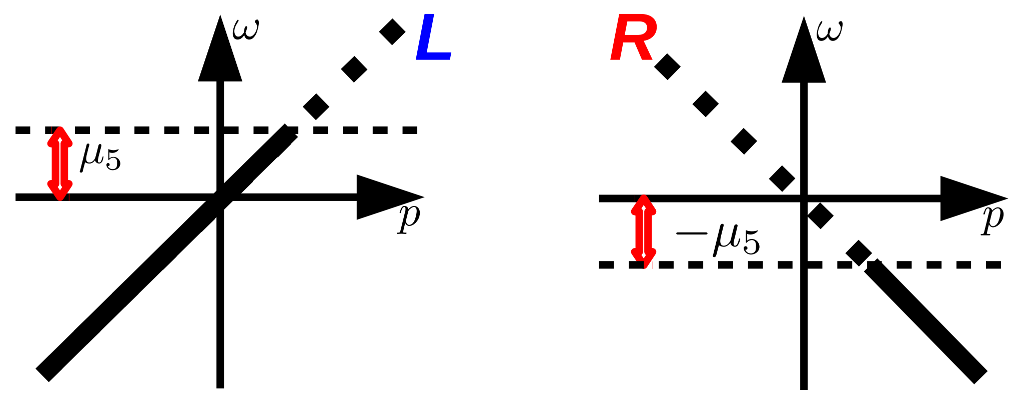
(+ opposite charge)

$$n_R \neq n_L \quad \langle \mathbf{s} \rangle \sim \mathbf{B}$$

$$\langle \mathbf{p} \rangle \sim (n_R - n_L)\langle \mathbf{s} \rangle$$

$$\mathbf{J} = \sigma \mathbf{E} + C_A \mu_5 \mathbf{B}$$
$$\mathbf{J}_5 = \# \mu \mu_5 \mathbf{E} + C_A \mu \mathbf{B}$$

chiral fermions, affected by homog. E||B fields



Simple cartoon of the CME

- chiral fermions prefer to align their spin parallel to magnetic field
- fermions move along the direction of ***B*** according to their chirality
- imbalance in the number of the two chiral species results in a charge sensitive electric current ***J***
- ***The real, dynamical origin of the CME is the change of momentum space topology in magnetic field (Berry-curvature)***

Consistent with Maxwell-Chern-Simons electrodynamics:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A^{\mu} J_{\mu} + \frac{C_A}{4} \theta \widetilde{F}^{\mu\nu} F_{\mu\nu}$$

$$\nabla \times \mathbf{B} - \dot{\mathbf{E}} = \mathbf{J} - C_A \left(\dot{\theta} \mathbf{B} - \nabla \theta \times \mathbf{E} \right)$$

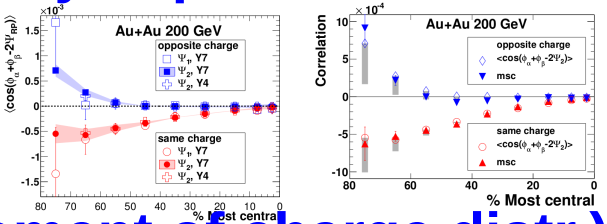
$$\nabla \cdot \mathbf{E} = \rho - C_A \nabla \theta \cdot \mathbf{B} \qquad \mathbf{J} = \frac{e^2}{2\pi^2} \dot{\theta} \mathbf{B}$$

See:
D. E. Kharzeev *etal.*, Prog. Part. Nucl. Phys **88**, 1 (2016)
Kharzeev, Stephanov, Yee, PRD **95,** 051901 (2016)
K. Landsteiner, Acta Phys. Pol. B **47**, 2617 (2016)
D. Kharzeev (edited by) *etal.*, Lec. Notes in Phys., Volume **871** (2013)
A. Bzdak *etal.*, arXiv:1906.00936

How to measure CME in HIC?

What signs to look for?

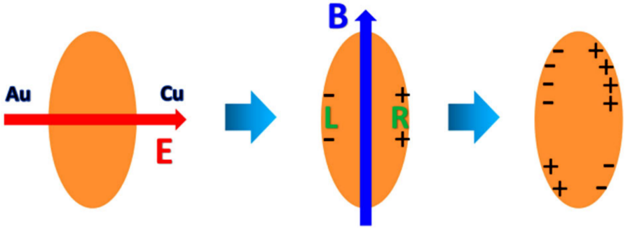
- **charge separation → dipole asymmetry in production**



- **CMW → Cu+Au coll. (quadrupole moment of charge distr.)**

*see: Burnier, Liao, Kharzeev, Yee PRL **107**, 052303 (2011),*
*Huang & Liao, PRL **110**, 232302 (2013)*

Might not exist...



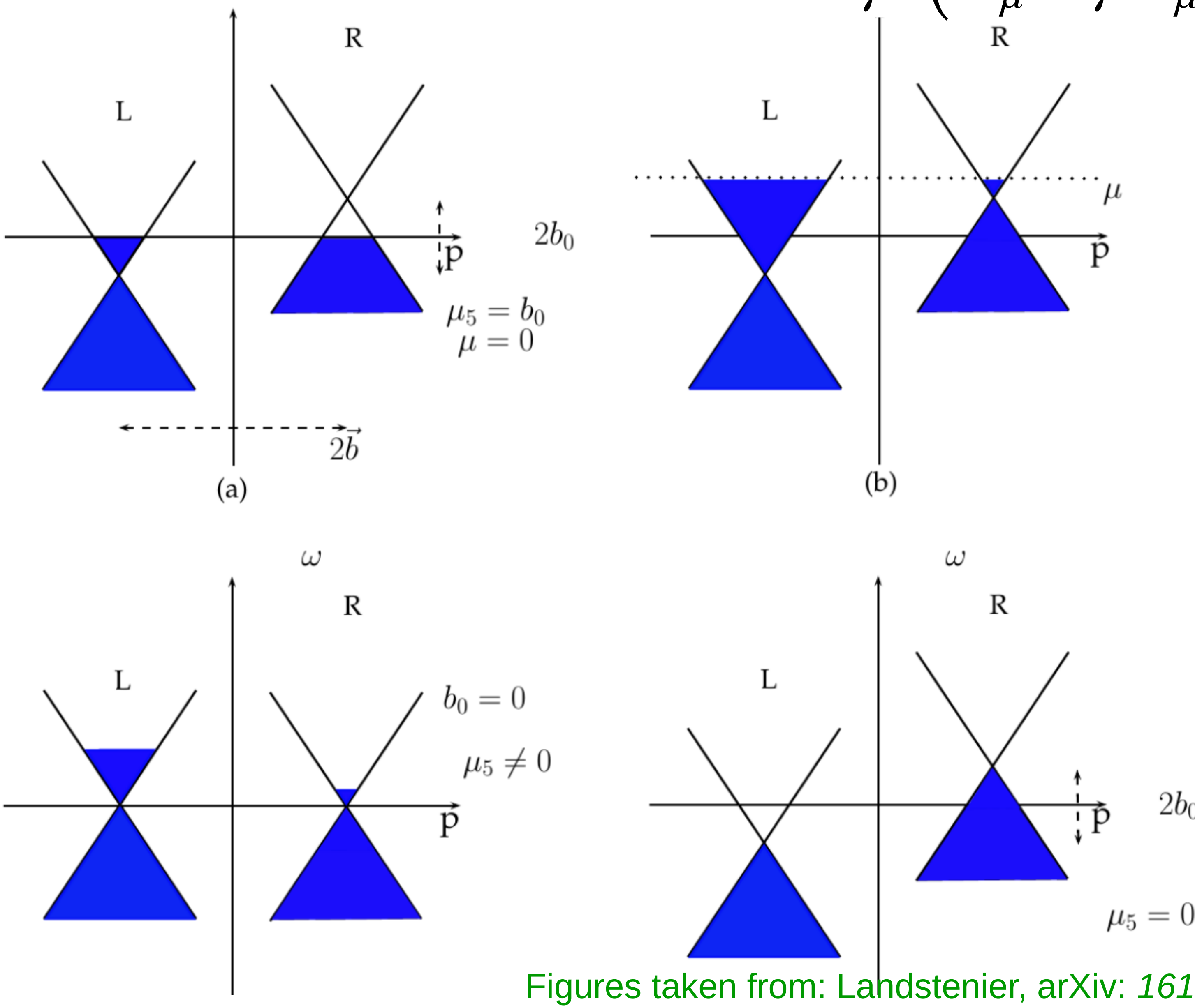
- **other things:**

CSL ("chiral soliton lattice" nonzero quark masses → anoumalous Hall current & B—Omega coupling;
K. Nishimura, aX:1711.02190
transition radiation as a probe of chiral anomaly – circularly polarized photons at given angle to the jet direction
Tuchin PRL 121, 182301 (2018)
Change in critical behaviour? (see Sogabe Noriyuki JHEP11(2018)108)

main theor. uncertainties: related to initial state & LT of sources
from experimental POV: background...

CME in cond. mat. systems – WSM

$$\gamma^{\mu}\left(i\partial_{\mu} + \gamma^{5} b_{\mu}\right)\psi = 0$$



Figures taken from: Landstenier, arXiv: *1610.04413 (2016)*

CME in cond. mat. systems

- **anom. conductivity → B^2 term**

$$\dot{n}_5 = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \frac{n_5 - n_{5,0}}{\tau_5}$$

$\chi_5 \delta\mu_5$

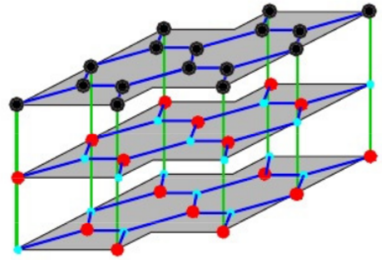
$$\delta\mu_5 = \frac{\tau_5}{\chi_5}\frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

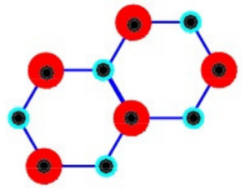
$$J^i = \sigma E^i + \frac{e^2}{2\pi^2}\delta\mu_5 B^i = \left(\delta^{ij}\sigma + \frac{\tau_5}{\chi_5}\frac{e^2}{4\pi^2}B^i B^j\right)E_j$$

- **possible pCME in graphene ?**

check out:
A. J. Mizher etal. arXiv: *1803.05794*

- Parity breaking "mass": $M = m_3\gamma_3 + m_o\gamma_3\gamma_5$.
- Place the graphene on a Boron Nitride substrate - m_3:





- PCME Lagrangian [AJM, C. Villavicencio, A. Raya, IJMP B30, 1550257 (2015)]:

$$\mathcal{L} = \bar{\Psi}[i\partial\!\!\!/ + \mu\gamma^0 + (eA_3^{\text{ext}} - m_3)\gamma^3 - m_o\gamma^3\gamma^5]\Psi.$$

Phenomenology & introduction

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^{\mu}i\partial_{\mu}\psi - eA_{\mu}\overline{\psi}\gamma^{\mu}\psi - A_{5,\mu}\overline{\psi}\gamma^{\mu}\gamma^{5}\psi$$

vector current $J^{\mu} = \overline{\psi}\gamma^{\mu}\psi$ axial-vector current $J_5^{\mu} = \overline{\psi}\gamma^{\mu}\gamma^{5}\psi$

U(1) axial anomaly:
no simultaneous conservation of vector and axial-vector charges

$$\partial_{\mu}J^{\mu} = 0$$ **(consistent anomaly!)**

$$\partial_{\mu}J_5^{\mu} = \frac{1}{2\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{1}{6\pi^2}\mathbf{E}_5\cdot\mathbf{B}_5$$

(In order to keep the vector charge conservation intact, regularization is needed.)

See for example: Landstenier, arXiv: *1610.04413 (2016)*

Introduction of axial coupling: new transport phenomena
→ fundamentally different nature compared to the usual electric transport: behave differently with respect to parity-inversion and time-reversal transformations
→ nonequ. effects can be taken into account (although axial field is an auxiliary quantity)

Anomaly in QED

See for example: Landstenier, arXiv: *1610.04413 (2016)*

U(1) vector current: $J^\mu = \overline{\Psi}\gamma^\mu\Psi$

U(1) axialvector current: $J_5^\mu = \overline{\Psi}\gamma^\mu\gamma^5\Psi$

$$\partial_\mu J^\mu = 0$$

(consistent anomaly!)

$$\partial_\mu J_5^\mu = \frac{1}{2\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{1}{6\pi^2}\mathbf{E}_5\cdot\mathbf{B}_5 \longrightarrow \frac{1}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

$$\partial_\mu J^\mu = \frac{1}{2\pi^2}(\mathbf{E}\cdot\mathbf{B}_5 + \mathbf{E}_5\cdot\mathbf{B}) \longrightarrow \frac{1}{2\pi^2}(\nabla\mu_5)\cdot\mathbf{B}$$

$$\partial_\mu J_5^\mu = \frac{1}{2\pi^2}(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5) \longrightarrow \frac{1}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$$

fermions coupled to gauge fields:

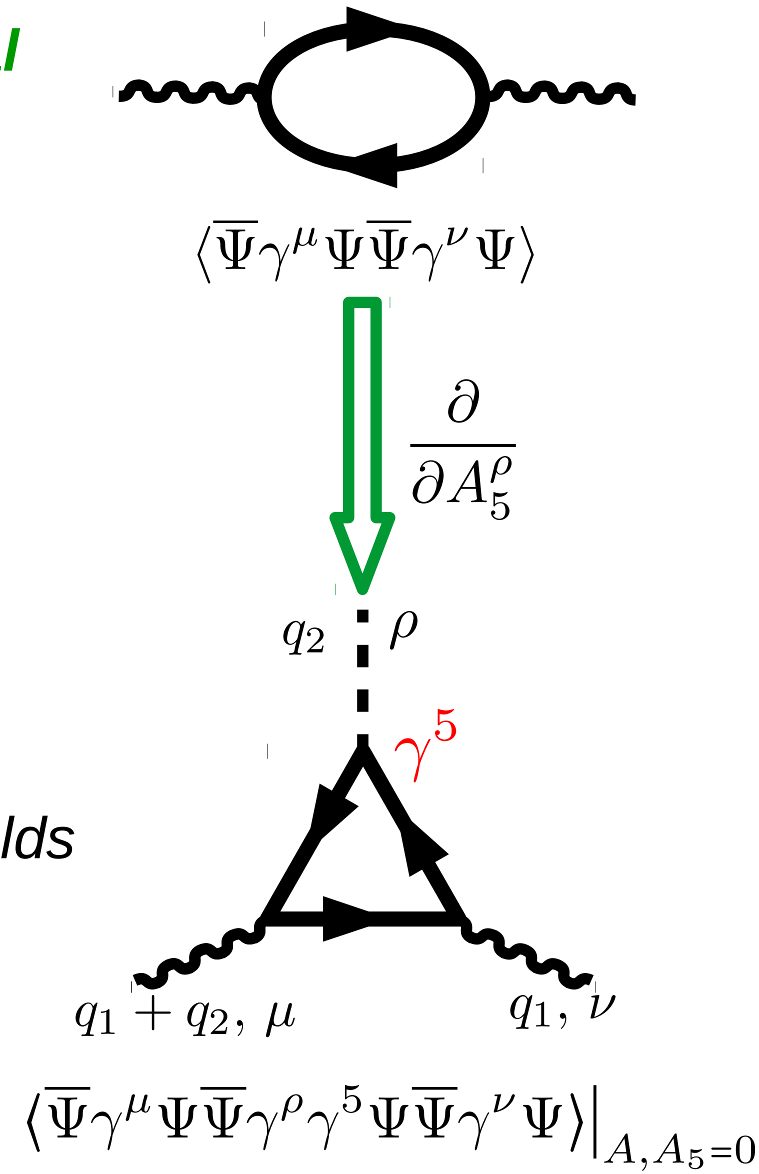
- ✔ maintaining gauge invariance
  - → *costs the anomalous divergence of the axial current*
- ✔ the anomaly comes from the UV behaviour of the fermionic propagator

Anomalous conductivities

- **static (↔ steady state) current:** ***universal***
  → *given by the anomaly (1-loop)*
  → *no further quantum corrections!*

- **BUT relaxation dynamics:**
  → ***depends on the underlying theory***

- **approximation: linear response**
  → *microscopic dynamics is not effected by the extarnal fields*
  → *gradient corrections to hydrodynamic fields*



Linear response

CME *(also anom. Hall)*

$$\langle J^\mu \rangle = \langle J^\mu J^\nu \rangle\big|_{A,A_5=0} A_\nu + \underline{\langle J^\mu J^\nu J_5^\rho \rangle\big|_{A,A_5=0} A_\nu A_{5,\rho}} + \ldots$$

electric current ~ axial imbalance × magnetic field

$$\langle J_5^\mu \rangle = \langle J_5^\mu J_5^\nu \rangle\big|_{A,A_5=0} A_{5,\nu} + \underline{\frac{1}{2} \langle J_5^\mu J^\nu J^\rho \rangle\big|_{A,A_5=0} A_\nu A_\rho} + \frac{1}{2} \langle J_5^\mu J_5^\nu J_5^\rho \rangle\big|_{A,A_5=0} A_{5,\nu} A_{5,\rho} + \ldots$$

CSE

Neglecting the electric and axial magnetic fields

axial-vector potential $A_5 = (A_{5,0}, \mathbf{0})$

vector potential $A = (0, \mathbf{A})$

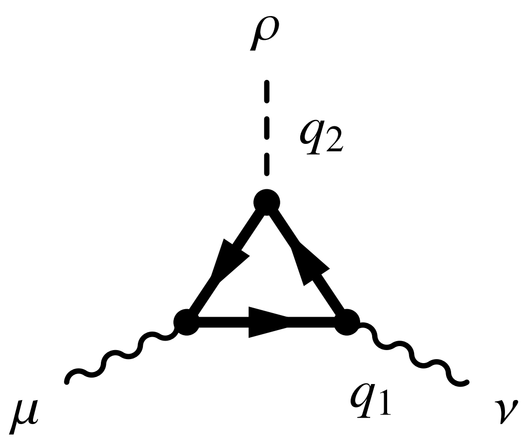
When the axial and vector fields are dynamical, the transport relations couple together leading to collective excitations like the chiral magnetic wave.

AVV response function

As a consequence of local vector charge conservation, the AVV vertex fulfils the following identities (Ward-Takahasi)

$$(q_1 + q_2)_\mu \widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = q_{1,\nu} \widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = 0$$

$$q_{2,\rho} \widetilde{\Gamma}^{\rho\mu\nu}_{AVV} = i\epsilon^{\mu\nu\alpha\beta} q_{1,\alpha} q_{2,\beta} \cdot \frac{e^2}{2\pi^2}$$



Third equation: anomalous nonconservation of the axial-vector charge

$$\langle J^\mu J^\nu J_5^\rho \rangle \equiv \Gamma^{\rho\mu\nu}_{AVV}: \text{ AVV vertex}$$

$$J^\mu(x) = \int_{q_1} \int_{q_2} \widetilde{A}_\nu(q_1) \widetilde{A}_{5,\rho}(q_2) \widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1, q_2) e^{ix\cdot(q_1+q_2)}$$

$$J_5^\rho(x) = \int_{q_1} \int_{q_2} \widetilde{A}_\mu(q_1) \widetilde{A}_\nu(q_2) \widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_2, -q_1 - q_2) e^{ix\cdot(q_1+q_2)}$$

AVV triangle

$$\widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1,q_2) = -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p+q_1)\gamma^\nu iG^A(p)+$$
$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p+q_1)\gamma^\nu iG^A(p)+$$
$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^R(p+q_1)\gamma^\nu iG^A(p)+$$
$$+\gamma^\mu iG^C(p+q_1+q_2)\gamma^\nu iG^A(p+q_2)\gamma^\rho\gamma^5 iG^C(p)+$$
$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$
$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

$$G^{R/A}(p) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\frac{\rho(\omega,\mathbf{p})}{p_0-\omega\pm i0^+}$$

$$iG^{12/21}(p) = \rho(p)\cdot\begin{cases} -n_{FD}(p_0/T) \\ 1-n_{FD}(p_0/T)\end{cases}$$

$$G^{11/22} = \frac{G^{12}+G^{21}}{2}\pm\left(G^R+G^A\right)$$

$$G^C = (1-2n_{FD}(p_0/T))\,\rho(p)$$

– {same terms with *m=M>> all other scales*}

AVV triangle

$$\widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1,q_2) = -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p+q_1)\gamma^\nu iG^A(p)+$$
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$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$
$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\nu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

ρ=0:
sensitivity to the $q_2 \to 0$ limit
(CME)

ρ=i:
no such sensitivity (CSE)

– {same terms with *m=M>> all other scales*}

AVV triangle

$$iG(p+q)q\cdot\Gamma_V iG(p) = G(p+q) - G(p)$$

$$q_{1\nu}\widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1,q_2) =$$

$$= -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p+q_1) - \gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p)$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p+q_1)+$$

$$-\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p)+$$

$$+\gamma^\mu G^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$-\gamma^\mu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p) - \gamma^\mu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

– {same terms with *m=M>>all other scales*}

AVV triangle

$$q_{1\nu}\widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1,q_2) =$$

$$iG(p+q)q\cdot\Gamma_V iG(p) = G(p+q) - G(p)$$

$$= -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{\gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p+q_1) - \gamma^\mu iG^C(p+q_1+q_2)\gamma^\rho\gamma^5 G^A(p)$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p+q_1)+$$

$$-\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 G^C(p)+$$

$$+\gamma^\mu G^C(p+q_1+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$-\gamma^\mu iG^C(p+q_2)\gamma^\rho\gamma^5 iG^A(p)+$$

$$+\gamma^\mu iG^R(p+q_1+q_2)\gamma^\rho\gamma^5 iG^C(p) - \gamma^\mu iG^R(p+q_2)\gamma^\rho\gamma^5 iG^C(p)\Big\}$$

– {same terms with *m=M>>all other scales*}

AVV triangle

$$G(p)\gamma^5 + \gamma^5 G(p) = \gamma^5 g(p)$$

$$\begin{aligned}
q_{2\rho}\cdot\widetilde{\Gamma}^{\rho\mu\nu}_{AVV}(q_1,q_2) = -\frac{ie^2}{2}\int_p \mathrm{tr}\Big\{ & \gamma^\mu\gamma^5 \overbrace{iG^C(p+q_1+q_2)\not{q}_2 iG^A(p+q_1)}^{G^C(p+q_1+q_2)}\gamma^\nu iG^A(p) + \\
& + \gamma^\mu\gamma^5 \overbrace{iG^R(p+q_1+q_2)\not{q}_2 iG^C(p+q_1)}^{-G^C(p+q_1)}\gamma^\nu iG^A(p) + \\
& + \gamma^\mu\gamma^5 \overbrace{iG^R(p+q_1+q_2)\not{q}_2 iG^R(p+q_1)}^{G^R(p+q_1+q_2)-G^R(p+q_1)}\gamma^\nu iG^A(p) + \\
& + \gamma^\mu\gamma^5 iG^C(p+q_1+q_2)\gamma^\nu \overbrace{iG^A(p+q_2)\not{q}_2 iG^C(p)}^{G^A(p+q_2)-G^A(p)} + \\
& + \gamma^\mu\gamma^5 iG^R(p+q_1+q_2)\gamma^\nu \overbrace{iG^C(p+q_2)\not{q}_2 iG^A(p)}^{G^C(p+q_2)} + \\
& + \gamma^\mu\gamma^5 iG^R(p+q_1+q_2)\gamma^\nu \overbrace{iG^R(p+q_2)\not{q}_2 iG^C(p)}^{-G^C(p)} \Big\}
\end{aligned}$$

$$G^{R,A}(p+q)\not{q}G^{R,A}(p) = \\ = -G^{R,A}(p+q) + G^{R,A}(p),$$

$$G^C(p+q)\not{q}G^{R,A}(p) = \\ = -G^C(p+q),$$

$$G^{R,A}(p+q)\not{q}G^C(p) = \\ = G^C(p).$$

$$\underbrace{-\{m = M \gg q_1,\, q_2;\ g(p)\neq 0\}}$$

$$@ = -i\frac{e^2}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}$$

Limiting cases of CME conductivity

$\mathbf{q}_2 \to 0$ percedes $q_{20} \to 0$

μ_5 first set to homogeneous

ANOMALY $\mathbf{J} = \frac{e^2}{2\pi^2} A_{5,0} \mathbf{B}$

$q_{20} \to 0$ percedes $\mathbf{q}_2 \to 0$

$A_{5,0}$ first set to time independent

$q_{10} \to 0$ lastly: $\frac{2}{3} \times$ **ANOMALY**

$$\frac{q_{10}}{q_{10} + i0^+} \cdot \# \longrightarrow \frac{q_{10}}{q_{10} + i\gamma} \cdot \#$$

The $q_{10} \to 0$ ambiguity was pointed out by several authors (Fukushima, Kharzeev, Satow and others, see: PRD 90, 014027) That is DIFFERENT from the ambiguity of constant AXIAL field!

See: Hou, Hui, Ren, *JHEP* **5**, 46 (2011); Wu, Hou, Ren, *Phys. Rev. D* **96**, 096015 (2017)

Limiting cases of CME conductivity

$\mathbf{q}_2 \to 0$ percedes $q_{20} \to 0$

μ_5 first set to homogeneous

ANOMALY $\mathbf{J} = \frac{e^2}{2\pi^2} A_{5,0} \mathbf{B}$

$q_{20} \to 0$ percedes $\mathbf{q}_2 \to 0$

$A_{5,0}$ first set to time independent

ZERO

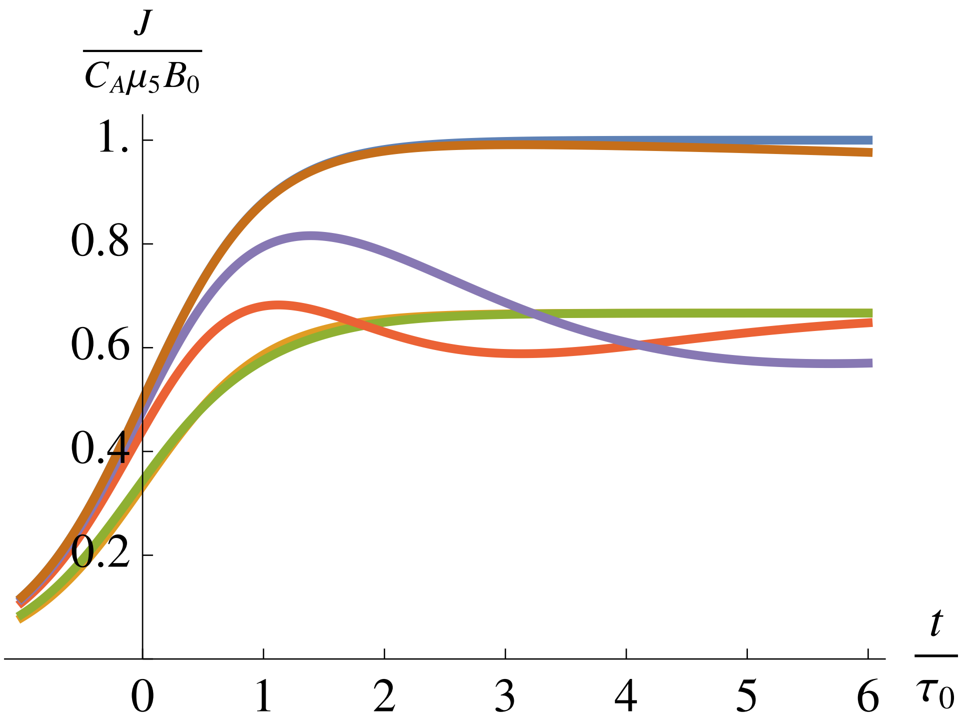
Vanishing static conductivity shows the inherently nonequilibrium nature of CME
remark: NO ambiguity for CSE conductivity

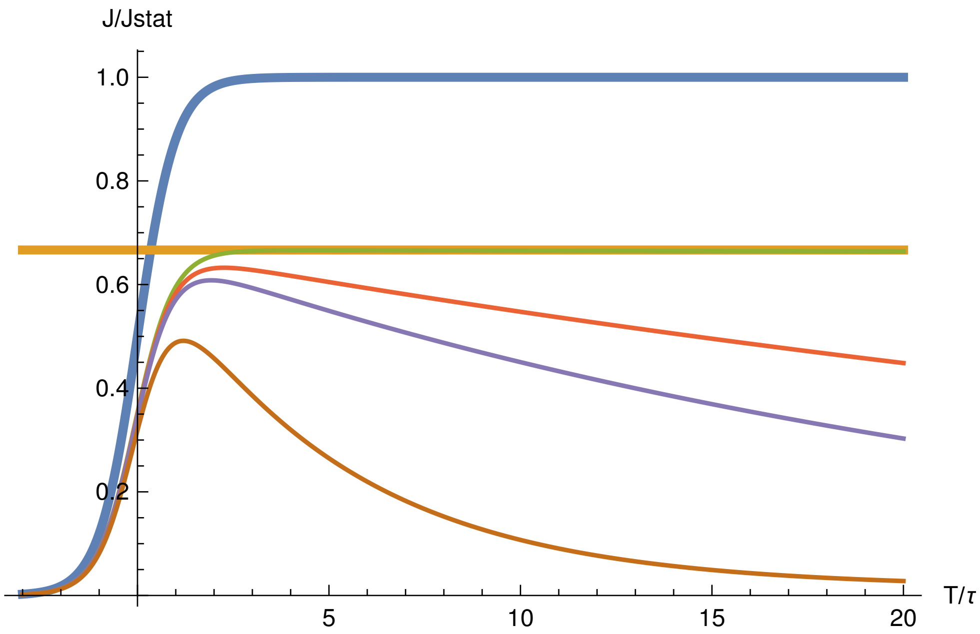
See: Hou, Hui, Ren, *JHEP* **5**, 46 (2011); Wu, Hou, Ren, *Phys. Rev. D* **96**, 096015 (2017)

Limiting cases – constant $A_{5,0}$

For finite relaxation time the asymptotic 2/3 decays away

There is still retardation effect for short times



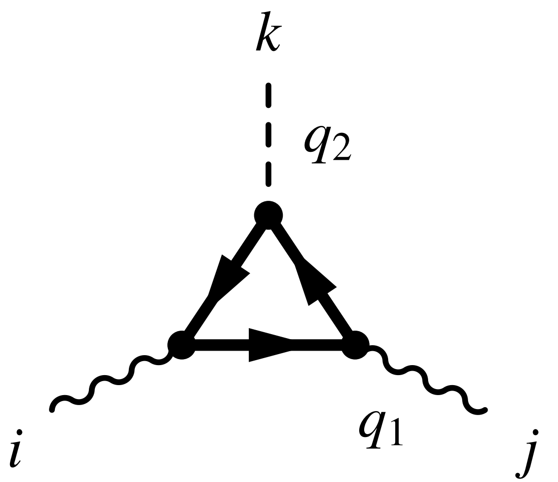


$$\frac{B(t)}{B_0} = \frac{1 + \tanh(t/\tau_0)}{2}$$

$\tau_0 T$ = 5.0, 1.0, 0.2, 0.1, 0.01

More about static conductivity

$$\mathbf{J} = \begin{cases} \frac{e^2}{2\pi^2} A_{5,0}\mathbf{B}, & \text{homog.} \\ 0, & \text{static} \end{cases}$$



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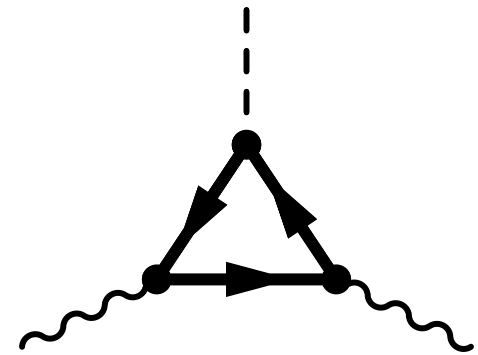


$$\int \text{đ}^4 p \, \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j \frac{\partial G^A(p)}{\partial p_k}\right\} \frac{-2n'_{FD}(p_0/T)}{T}$$

topologically protected (?)
for *m=0*

$$\epsilon^{ijk} \int \text{đ}^4 p \frac{2M^2(1-2n_{FD}(p_0/T))}{[(p_0-i0^+)^2-\mathbf{p}^2-M^2]^3}$$

PV-reg.: ~heavy fermions
beyond the scales of
any interactions

$$\frac{e^2}{2\pi^2}\epsilon^{ijk}$$

More about static conductivity

$$\lim_{\mathbf{q}_2\to 0}\lim_{q_{20}\to 0}\frac{\partial}{\partial q_{1k}}\widetilde{\Gamma}^{0ij}_{AVV}\Big|_{q_1=0} =$$

$$= -\frac{e^2}{2}\int_p \mathrm{tr}\Big\{\gamma^i\gamma^5\left(-\frac{\partial}{\partial p_0}G^A(p_0,\mathbf{p})\right)\gamma^j\frac{\partial}{\partial p_k}G^A(p_0,\mathbf{p})+$$
$$-\gamma^i\gamma^5\left(-\frac{\partial}{\partial p_0}G^R(p_0,\mathbf{p})\right)\gamma^j\frac{\partial}{\partial p_k}G^R(p_0,\mathbf{p})+$$
$$-\gamma^i\gamma^5\frac{\partial}{\partial p_k}G^A(p_0,\mathbf{p})\gamma^j\left(-\frac{\partial}{\partial p_0}G^A(p_0,\mathbf{p})\right)+$$
$$+\gamma^i\gamma^5\frac{\partial}{\partial p_k}G^R(p_0,\mathbf{p})\gamma^j\left(-\frac{\partial}{\partial p_0}G^R(p_0,\mathbf{p})\right)\Big\}(1-2n(p_0))$$
$$-\{\text{same with } m = M \gg \text{ all other scales}\}+$$

→ seems to be robust against fermionic interactions

→ Coleman-Hill-like non-renormalization theorem?

$$e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j\frac{\partial G^A(p)}{\partial p_k}\right\}(-2n'(p_0))$$

$$+\frac{e^2}{2}\int_p \mathrm{tr}\Big\{\gamma^i\gamma^5 g^A_M(p_0,\mathbf{p})\gamma^0 G^A_M(p_0,\mathbf{p})\gamma^j\frac{\partial}{\partial p_k}G^A_M(p_0,\mathbf{p})+$$
$$-\gamma^i\gamma^5 g^R_M(p_0,\mathbf{p})\gamma^0 G^R_M(p_0,\mathbf{p})\gamma^j\frac{\partial}{\partial p_k}G^R_M(p_0,\mathbf{p})+$$
$$+\gamma^i\gamma^5 g^A_M(p_0,\mathbf{p})\frac{\partial}{\partial p_k}G^A_M(p_0,\mathbf{p})\gamma^j G^A_M(p_0,\mathbf{p})\gamma^0+$$
$$-\gamma^i\gamma^5 g^R_M(p_0,\mathbf{p})\frac{\partial}{\partial p_k}G^R_M(p_0,\mathbf{p})\gamma^j G^R_M(p_0,\mathbf{p})\gamma^0\Big\}(1-2\widetilde{n}(p_0))$$

→ contribution from the regulator term only

→ fermionic interactions could not change it!

$$e^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0 G^A_M(p)\gamma^j\frac{\partial G^A_M(p)}{\partial p_k}\right\}g^A_M(p)(1-2n(p_0))$$

More about static conductivity

Staring with the weak coupling expression

$$\int \text{đ}^4 p \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j \frac{\partial G^A(p)}{\partial p_k}\right\} \frac{-2n'_{FD}(p_0/T)}{T}$$

conductivity from weak coupling
there is other contr. from vector vertex
OR is there?

Using Ward-identity to reformulate solely in terms of the propagator

$$G_0\gamma^\mu G_0 = \partial^\mu G_0$$

$$\Gamma^\mu(p,p) = G^{-1}(p)\partial^\mu G(p) G^{-1}(p)$$

$$\int \text{đ}^4 p \mathrm{tr}\left\{\gamma^5 G^{-1}(\partial^i G)G^{-1}GG^{-1}(\partial^j G)G^{-1}(\partial^k G)\right\}(-2\tilde{n}'(p_0))$$

Formula bears reparametrization invariance:
no contributions from interactions?

$$\mathbf{p} \mapsto \mathbf{p}'$$

$$G_0(0,\mathbf{p}'(\mathbf{p})) = G(0,\mathbf{p})$$

~renormalization group trf.

$$\int \mathrm{d}^3\mathbf{p} \mathrm{tr}\left\{\gamma^5 G_A^{-1}(0,\mathbf{p})\partial^i G_A(0,\mathbf{p}) G_A^{-1}\partial^j G_A G_A^{-1}\partial^k G_A\right\}$$

AVV response function – constant ***B***

AVV response function – constant *B*

MH, D. Hou, J. Liao, H. Ren: 1911.00933

$$J^i(x) = \int đ^4 q_1 \int đ^4 q_2 \widetilde{A}_j(q_1)\widetilde{A}_{5,0}(q_2)\widetilde{\Gamma}^{0ij}_{AVV}(q_1,q_2)e^{ix\cdot(q_1+q_2)} =$$

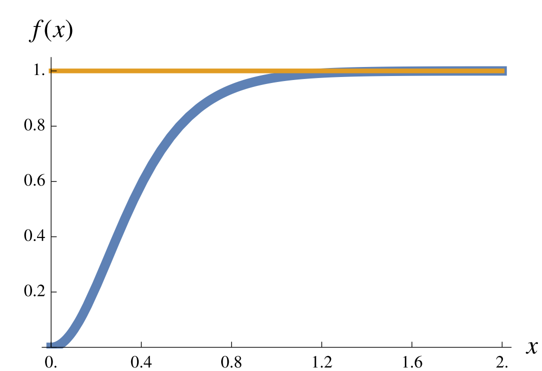
$$= \int_{-\infty}^{\infty} dt' \int đ^3\mathbf{q}\overline{A}_{5,0}(t',\mathbf{q})\overline{\sigma}^i_A(t'-t,\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$\overline{\sigma}^i_A(t,\mathbf{q}) = \int_{-\infty}^{\infty} đq_0 e^{iq_0 t}\frac{i}{2}\epsilon^{jlk}B^l \left.\frac{\partial\widetilde{\Gamma}^{0ij}_{AVV}(q_1,q_2=q)}{\partial q_{1k}}\right|_{q_1=0}$$

→ weak coupling limit: the conductivity can be given analytically.
→ finite temperature contributions are absent in the charge density
→ for T=0 there are contributions result of the retardation, but also instantaneous response

$$\overline{\sigma}^i_A(t,\mathbf{q}) = \frac{e^2}{2\pi^2}\left\{B^i\delta(t) + \frac{\theta(-t)}{2}\left[q\sin(qt)\left(B^i+\widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})\right) - \frac{\partial}{\partial t}\left(\frac{\sin(qt)}{qt}\right)f(tT)\left(B^i-\widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})\right)\right]\right\}$$

$$f(x) = 4x\int_0^{\infty} dy\, n_{FD}(y)\sin(2yx) = 1 - \frac{2\pi x}{\sinh(2\pi x)} \longrightarrow \begin{cases} 0, & x\to 0 \\ 1, & x\to\infty \end{cases}$$



AVV response function – constant *B*

MH, D. Hou, J. Liao, H. Ren: 1911.00933

$$\widetilde{J}^i(t,\mathbf{q}) = \int_{-\infty}^{\infty} \mathrm{d}\tau \widetilde{A}_{5,0}(t+\tau,\mathbf{q}) \frac{e^2}{2\pi^2} \left\{ B^i \left(\delta(\tau) + \frac{\theta(-\tau)}{2} \left[q\sin(q\tau) - \frac{\partial}{\partial\tau} \left(\frac{\sin(q\tau)}{q\tau} \right) f(\tau T) \right] + \right.\right.$$

$$\left. + (\mathbf{B}\cdot\widehat{\mathbf{q}})\widehat{q}^i \frac{\theta(-\tau)}{2} \left[q\sin(q\tau) + \frac{\partial}{\partial\tau} \left(\frac{\sin(q\tau)}{q\tau} \right) f(\tau T) \right] \right\}$$

T=0

$$J^i(t,\mathbf{r}) = \frac{e^2}{2\pi^2} \left\{ B^i A_{5,0}(t,\mathbf{r}) - \frac{2}{3} B^i A_{5,0}(t,\mathbf{r}) + \right.$$

$$+ \frac{1}{8\pi} \int \mathrm{d}^2\widehat{\mathbf{r}'} \int_0^{\infty} \mathrm{d}r' \left[\left(r'\partial_1^2 A_{5,0}(t-r',\mathbf{r}+\mathbf{r}') \right) (B^i + B^i_{\parallel,\mathbf{r}'}) + \right.$$

$$- \left(\partial_1 A_{5,0}(t-r',\mathbf{r}+\mathbf{r}') + \frac{A_{5,0}(t-r',\mathbf{r}+\mathbf{r}') - A_{5,0}(t,\mathbf{r}+\mathbf{r}')}{r'} \right) (B^i - 3B^i_{\parallel,\mathbf{r}'}) +$$

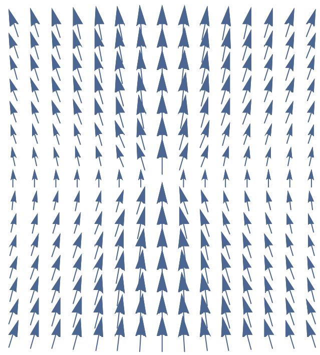
$$\left. \left. + \left(\partial_1 A_{5,0}(t-r',\mathbf{r}+\mathbf{r}') f(r'T) - A_{5,0}(t-r',\mathbf{r}+\mathbf{r}') T f'(r'T) \right) (B^i - B^i_{\parallel,\mathbf{r}'}) \right] \right\}$$

T>0

Point-like $A_{5,0}$ – constant *B*

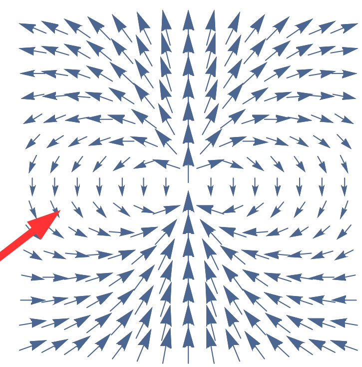
MH, D. Hou, J. Liao, H. Ren: 1911.00933

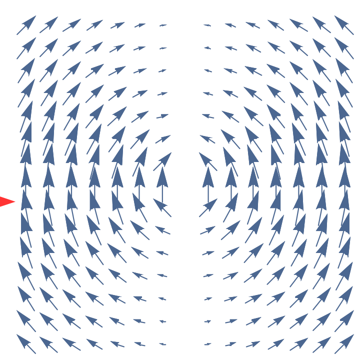
$$
\begin{aligned}
\mathbf{J}(t,\mathbf{r}) = \frac{e^2}{2\pi^2}\Bigg\{ & \frac{1}{3}\mathbf{B}A_{5,0}(t)\delta^{(3)}(\mathbf{r}) + \\
& + \frac{1}{2}\Bigg[\frac{A''_{5,0}(t-r)}{r}\boxed{(\mathbf{B}+(\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}})} + \\
& - \left(\frac{A'_{5,0}(t-r)}{r^2}+\frac{A_{5,0}(t-r)-A_{5,0}(t)}{r^3}\right)\boxed{(\mathbf{B}-3(\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}})} + \\
& + \frac{A'_{5,0}(t-r)f(rT)-A_{5,0}(t-r)Tf'(rT)}{r^2}\boxed{(\mathbf{B}-(\mathbf{B}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}})}\Bigg]\Bigg\}
\end{aligned}
$$

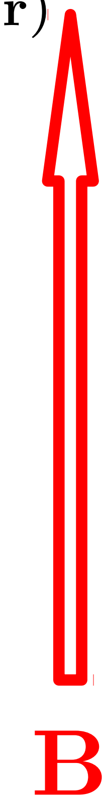


For a centered source:

$$A_{5,0}(\mathbf{r},t) = A_{5,0}(t)\delta^{(3)}(\mathbf{r})$$







Quenched $A_{5,0}$ – constant *B*

MH, D. Hou, J. Liao, H. Ren: 1911.00933

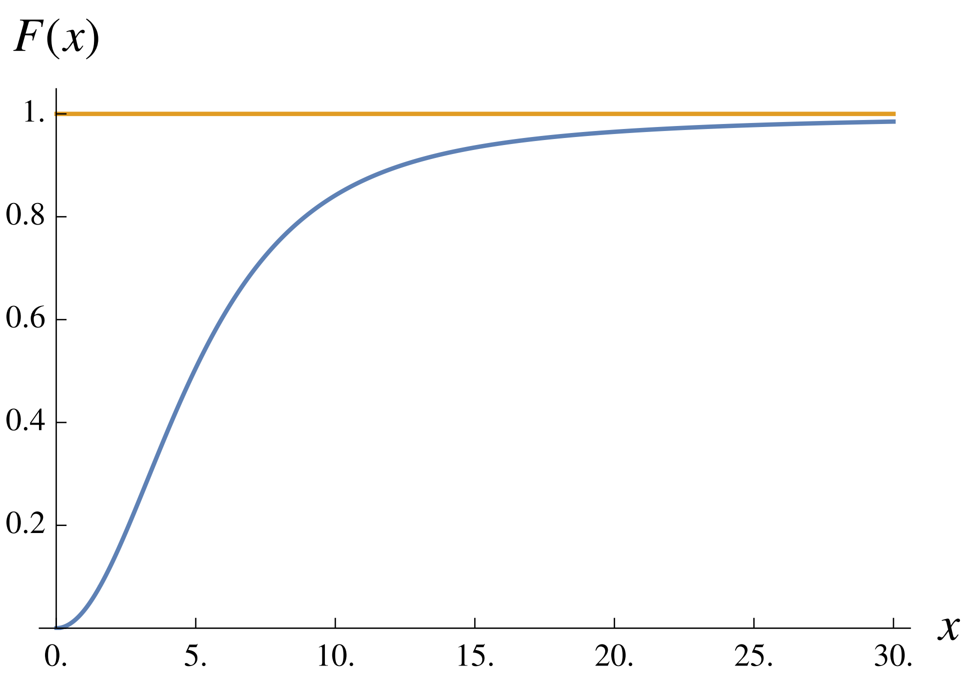
Sudden change in $A_{5,0}$ – asymptotic current?

$$A_{5,0}(t,\mathbf{r}) = \theta(t) A_{5,0}(\mathbf{r})$$

$$\widetilde{J}^i(t\to\infty,\mathbf{q}) = \frac{A_{5,0}(\mathbf{q})}{2}\left[B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}}) + (B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}}))\int_0^\infty \mathrm{d}\tau \frac{\partial}{\partial\tau}\left(\frac{\sin q\tau}{q\tau}\right) f(\tau T)\right] =$$

$$=: \frac{A_{5,0}(\mathbf{q})}{2}\left(B^i - \frac{q^i(\mathbf{B}\cdot\mathbf{q})}{q^2}\right) F(q/T)$$

Current is divergence free!



$T\to 0$: current-dipole,

$$\mathbf{J} = \frac{1}{4\pi}\frac{1}{2}\nabla_{\mathbf{r}} \times \int \mathrm{d}^3\mathbf{r}' \frac{A_{5,0}(\mathbf{r}'-\mathbf{r})\mathbf{B}\times\mathbf{r}'}{(r')^3}$$

$T\to$ larger than any scale (even that of spatial inhomogeneities: zero current

q~5T: cut-off,
spatially localized current!

Quenched $A_{5,0}$ – constant **B**

MH, D. Hou, J. Liao, H. Ren: 1911.00933

Sudden change in $A_{5,0}$ – asymptotic current?

$$A_{5,0}(t,\mathbf{r}) = \theta(t) A_{5,0}(\mathbf{r})$$

$$\widetilde{J}^i(t\to\infty,\mathbf{q}) = \frac{A_{5,0}(\mathbf{q})}{2}\left[B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}}) + \left(B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})\right)\int_0^\infty d\tau \frac{\partial}{\partial\tau}\left(\frac{\sin q\tau}{q\tau}\right) f(\tau T)\right] =$$

$$=: \frac{A_{5,0}(\mathbf{q})}{2}\left(B^i - \frac{q^i(\mathbf{B}\cdot\mathbf{q})}{q^2}\right) F(q/T)$$

Current is divergence free!

for a localized source: $A_{5,0}(\mathbf{q}) = V_D \cdot A_{5,0}$

$$\overline{\mathbf{J}} = \frac{1}{V_D}\int_D d^3\mathbf{r}\, \mathbf{J}(t\to\infty,\mathbf{r}) = \frac{e^2}{2\pi^2} A_{5,0}\mathbf{B}\,\frac{1-f(RT)}{3}$$

Vanishing long-time charge asymmetry

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$$\Delta Q = \int_{-\infty}^{\infty} \mathrm{d}t \int_S \mathrm{d}^2\mathbf{r}\widehat{\mathbf{B}} \cdot \mathbf{J}(t,\mathbf{r}) =$$

Already seen: long-time behavior of conductivity is dipolar in space

$$= \int đ^3\mathbf{q} \int_S \mathrm{d}^2\mathbf{r}\widehat{B}_i \widetilde{\sigma}^i_A(q_0 = 0, \mathbf{q}) \widetilde{A}_{5,0}(q_0 = 0, \mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}} =$$

$$= \frac{e^2 B}{4\pi^2} \int đ^3\mathbf{q} \int_S \mathrm{d}^2\mathbf{r} \left(1 - (\widehat{\mathbf{B}}\cdot\widehat{\mathbf{q}})^2\right) F(q/T) \widetilde{A}_{5,0}(0,\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$q' = (-q_0, -\mathbf{q} + q_\parallel \widehat{\mathbf{B}})$$

$$\longrightarrow 0\,!$$

$$q = (q_0, \mathbf{q})$$

Assuming that observation time is long enough (t→∞, compared to the time-scales of the sources); takeing into account all the current through a large enough surface (area of *S*→∞) !

With the *long-time/large S* assumption the vanishing of the transported charge is robust (i.e. no corrections from interactions)

$$\Delta Q = \int đ^4 q \int_{-\infty}^{\infty} đq_\parallel \widehat{B}_i \widetilde{\Gamma}^{0ij}_{AVV}(q,q') \widetilde{A}_j(q) \widetilde{A}_{5,0}(q')$$

Writing ΔQ in terms of the vertex function: essentially the consequence of the local charge conservation

$$q_\parallel \widehat{B}_i \widetilde{\Gamma}^{0ij}_{AVV}(q,q') = (q+q')_\mu \widetilde{\Gamma}^{0\mu j}_{AVV}(q,q') \equiv 0$$

Charge asymmetry and the interplay of many scales

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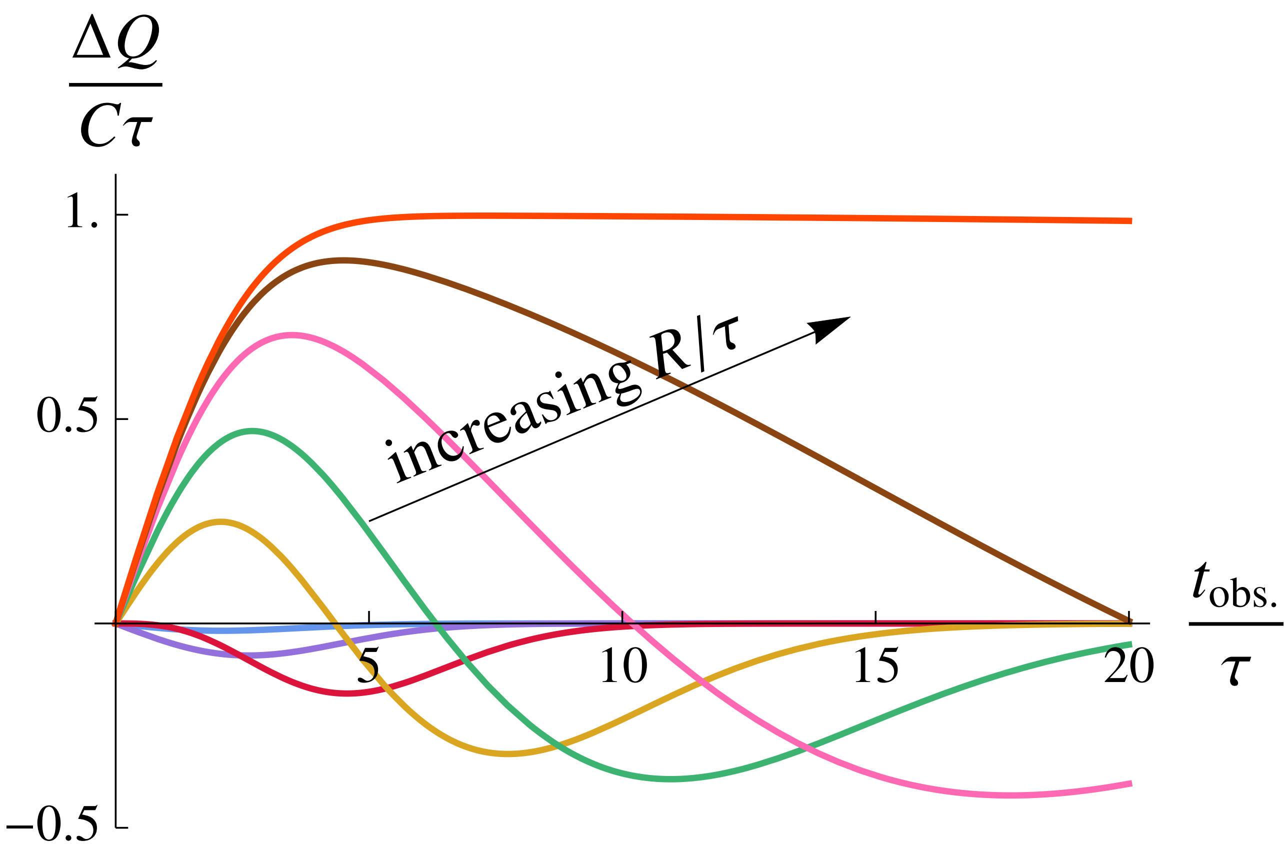
Explore the charge separation but only in a finite time window $t_{obs.}$!

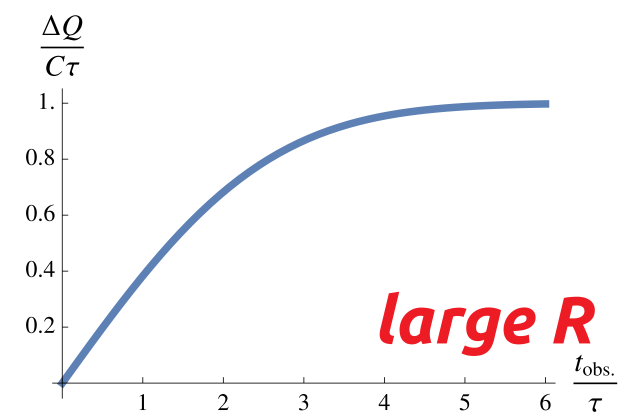
Axial imbalance is parametrized by an ***impulse-like profile***

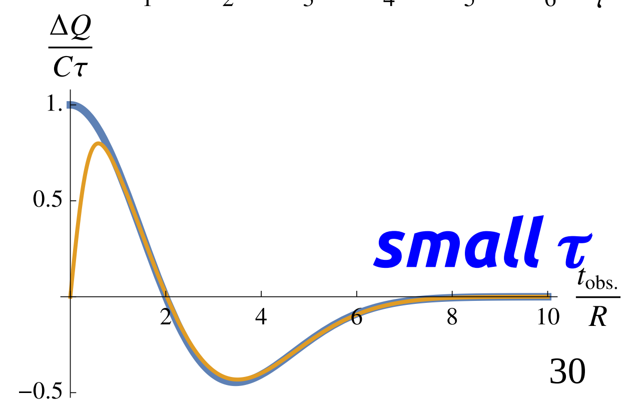
$$\overline{A}_{5,0}(t,q) \propto \exp\left(-\frac{q^2R^2}{2} - \frac{t^2}{2\tau^2}\right)$$

Interplay of many scales:

$$(R,\ \tau,\ t_{\mathrm{obs.}})$$





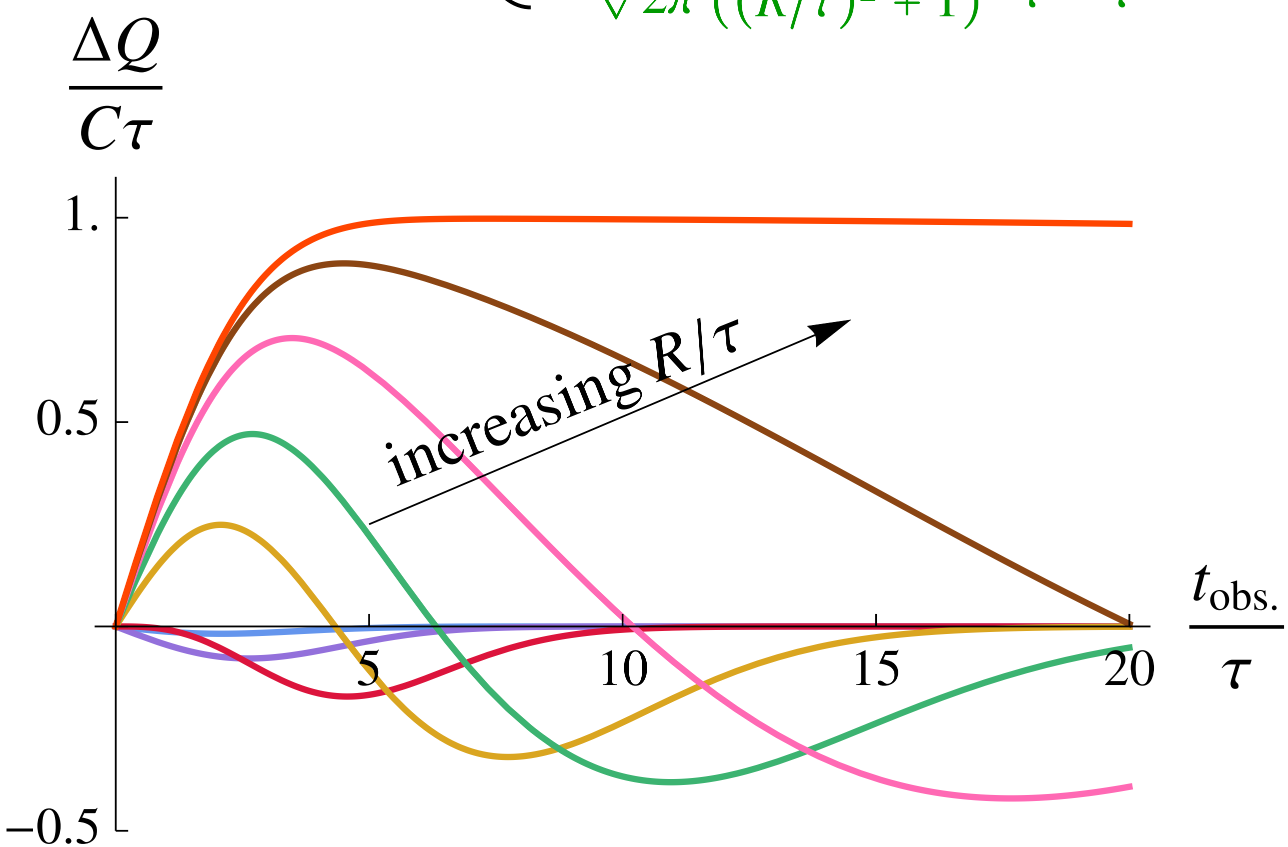


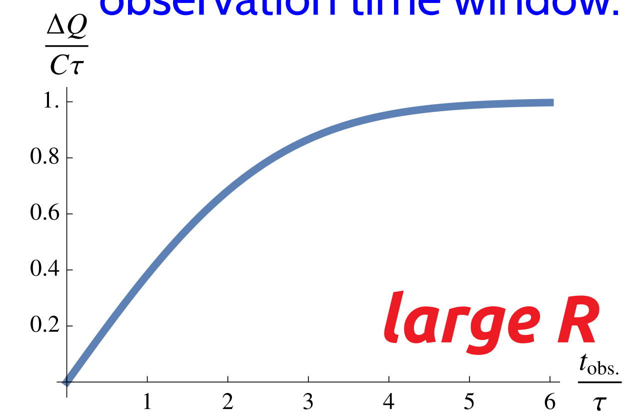
Charge asymmetry and the interplay of many scales

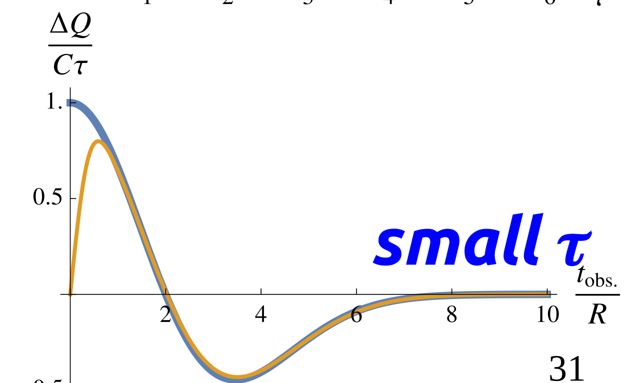
$$\frac{\Delta Q(t_{\text{obs.}},\tau,R)}{C\tau} \longrightarrow \begin{cases} \operatorname{erf}\left(\dfrac{t_{\text{obs.}}}{2\sqrt{2}\tau}\right) & \textbf{large } \boldsymbol{R} \\ e^{-\frac{t_{\text{obs.}}^2}{8R^2}}\left(1-\dfrac{t_{\text{obs.}}^2}{4R^2}\right) & \textbf{small } \boldsymbol{\tau} \\ \approx \dfrac{1}{\sqrt{2\pi}}\dfrac{(R/\tau)^2-1}{((R/\tau)^2+1)^2}\dfrac{R^2}{\tau^2}\dfrac{t_{\text{obs.}}}{\tau} + \mathcal{O}((t_{\text{obs.}}/\tau)^2) & \end{cases}$$

large enough source: behaves like naive CME for intermediate times (homogeneity limit).

smaller size or shorter pulse: system quickly reaches the vanishing of ΔQ within the observation time window.







Gradient expansion

$$\delta\mathbf{J}(t,\mathbf{q}) \equiv \mathbf{J}(t,\mathbf{q}) - \frac{e^2}{2\pi^2}\overline{A}_{5,0}(t,\mathbf{q})\mathbf{B} \approx$$

Expanding the current for $q \to 0$ up to the first non-trivial contribution:

$$\approx \frac{e^2}{4\pi^2}\int_{-\infty}^{0} \mathrm{d}\tau \overline{A}_{5,0}(t+\tau,\mathbf{q}) q^2 \tau \left[\left(1+\tfrac{1}{3}f(\tau T)\right)\mathbf{B}\left(1-\tfrac{1}{3}f(\tau T)\right)(\mathbf{B}\cdot\widehat{\mathbf{q}})\widehat{\mathbf{q}}\right] + \mathcal{O}(q^4)$$

Still too complicated. Assume clear separation between the internal timescale and temperature. With characteristic timescale of the source being τ_r: two limiting cases in τ_r/T where the deviation from the homogeneous current can be given in a differential form: (send T either to 0 or ∞)

$$\delta\mathbf{J}(t,\mathbf{q}) \approx \frac{e^2}{4\pi^2}\left(C_1 q^2 \mathbf{B} + C_2(\mathbf{B}\cdot\mathbf{q})\mathbf{q}\right)\int_{-\infty}^{0}\mathrm{d}\tau \overline{A}_{5,0}(t+\tau,\mathbf{q})\tau$$

	C_1	C_2
$\tau_r \gg T$	1	1
$T \gg \tau_r$	$\frac{4}{3}$	$\frac{2}{3}$

$$\partial_t^2 \delta\mathbf{J}(t,\mathbf{r}) = \frac{e^2}{4\pi^2}\left(C_1 \mathbf{B}\nabla_{\mathbf{r}}^2 + C_2(\mathbf{B}\cdot\nabla_{\mathbf{r}})\nabla_{\mathbf{r}}\right)A_{5,0}(t,\mathbf{r}) =$$

$$= -\frac{e^2}{4\pi^2}\left(C_1\mathbf{B}(\nabla\cdot\mathbf{E}_5) + C_2(\mathbf{B}\cdot\nabla)\mathbf{E}_5\right)$$

$$\partial_t \nabla\times\delta\mathbf{J}(t,\mathbf{r}) \neq 0$$

The gradient correction can couple to vorticity, therefore it can lead to vortex formation.

Conclusions

- *“resumming” all gradients in the axial imbalance results in a dipolar structure of the electric current*

- *because of charge conservation the overall asymmetry caused by the CME current is suppressed*

- *The long time behavior is a result of many scales: for large enough spatial size or short enough lifetime the usual CME current is dominant*

- *The subleading terms in the axial imbalance gradients can change the vorticity of the charge flow*

Take-home message

Local charge conservation greatly affects the real-time chiral magnetic response.

Charge separation can be suppressed depending on the scales of the system.

Future plans

- *full linear response analysis with axial currents*
- *small-q expansion with non-homogeneous EM & axial fields*
- ***possible implementation into simulation frameworks***
  – to describe relaxation dynamics
- *hydrodynamic simulation with gradient terms taken into account*
- *plasma modes with vorticity:*
  detailed analysis in dynamical situations

Thank you for listening!

Questions? Comments?

ありがとうございました!

check out 1911.00933

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Backup

Maxwell-Chern-Simons

Only QED + θ–term:

$$\mathcal{L}_\theta = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^\mu\left(i\partial_\mu - eA_\mu\right)\psi + \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\theta F_{\mu\nu}F_{\alpha\beta}$$

$\Rightarrow$

$$\mathcal{L}_{\mathrm{MCS}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu + \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\theta F_{\mu\nu}F_{\alpha\beta}$$

Maxwell's equations get modified:

$$\nabla\cdot\mathbf{E} = J^0 + \tfrac{e^2}{2\pi^2}\nabla\theta\cdot\mathbf{B}$$

$$\dot{\mathbf{E}} - \nabla\times\mathbf{B} = -\mathbf{J} + \frac{e^2}{2\pi^2}\left(\dot{\theta}\mathbf{B} - (\nabla\theta)\times\mathbf{E}\right)$$

imposing the anomalous Ward-identity:

$$\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \stackrel{!}{=} \partial_\mu J_5^\mu = \partial_\mu\overline{\psi}\gamma^\mu\gamma^5\psi$$

leading to:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^\mu\left(i\partial_\mu - eA_\mu - \gamma^5 A_{5,\mu}\right)\psi$$

Maxwell-Chern-Simons

QED + QCD with θ–term:

$$\mathcal{L}_{\theta,\mathrm{QCD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu}\right)\psi + \frac{g^2}{32\pi^2}\theta G^{a}_{\mu\nu}\widetilde{G}^{\mu\nu}_{a}$$

imposing the anomalous Ward-identity considering both gauge fields:

$$\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{e^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}G_{a,\mu\nu}G_{a,\alpha\beta} \stackrel{!}{=} \partial_{\mu}J^{\mu}_{5} = \partial_{\mu}\overline{\psi}\gamma^{\mu}\gamma^{5}\psi$$

leading to:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu} - \gamma^{5}A_{5,\mu}\right)\psi - \theta\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\theta\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} = \frac{e^2}{2\pi^2}\frac{\theta}{4}\partial_{\mu}J^{\mu}_{CS}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu} - \gamma^{5}A_{5,\mu}\right)\psi + \frac{e^2}{2\pi^2}A_{5,\mu}\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}A_{\nu}F_{\alpha\beta}$$

Limiting cases – absent current

$$\frac{\partial}{\partial q_{1k}}\widetilde{\Gamma}^{0ij}_{AVV} =$$

$$= \underbrace{ie^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5 G^A(p)\gamma^j\frac{\partial G^A(p)}{\partial p_k}\right\}(-2n'(p_0))}_{=:A} + \underbrace{2ie^2\int_p \mathrm{tr}\left\{\gamma^i\gamma^5\gamma^0 G^A_M(p)\gamma^j\frac{\partial G^A_M(p)}{\partial p_k}\right\}g^A_M(p)(1-2n(p_0))}_{=:B} = 0$$

$$A = \frac{ie^2\cdot 4\pi}{16\pi^4}\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}p p^2\mathrm{tr}\left\{\gamma^5\gamma^i\not{p}\gamma^j\left(\gamma^k-\frac{2p^k\not{p}}{(p_0-i0^+)^2-p^2}\right)\right\}\frac{-2n'(p_0)}{[(p_0-i0^+)^2-p^2]^2} = \frac{2e^2}{\pi^3}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}p p^2\int_{-\infty}^{\infty}\mathrm{d}p_0\frac{p_0 n'(p_0)}{[(p_0-i0^+)^2-p^2]^2} =$$

$$= -\frac{e^2}{\pi^3}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}p p^2\int_{-\infty}^{\infty}\mathrm{d}p_0 n'(p_0)\frac{\partial}{\partial p_0}\frac{1}{(p_0-i0^+)^2-p^2} = \frac{ie^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}p p^2\int_{-\infty}^{\infty}\mathrm{d}p_0 n''(p_0)\frac{\delta(p_0-p)-\delta(p_0+p)}{2p} =$$

$$= \frac{ie^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}p p n''(p) = -\frac{ie^2}{\pi^2}\epsilon^{ijk}\int_0^{\infty}\mathrm{d}p n'(p) = \frac{ie^2}{2\pi^2}\epsilon^{ijk},$$

$$B = \frac{2ie^2}{16\pi^4}4\pi\int_{-\infty}^{\infty}\mathrm{d}p_0(1-2n(p_0))\int_0^{\infty}\mathrm{d}p\mathrm{tr}\left\{\gamma^5\gamma^i\gamma^0(\not{p}+M)\gamma^j\left(\gamma^k-\frac{2p^k(\not{p}+M)}{(p_0-i0^+)^2-p^2-M^2}\right)\right\}\frac{2M}{[(p_0-i0^+)^2-p^2-M^2]^3} =$$

$$= \frac{ie^2}{\pi^3}\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}p\frac{M^2p^2(1-2n(p_0))}{[(p_0-i0^+)^2-p^2-M^2]^3}\mathrm{tr}\left\{\gamma^5\gamma^i\gamma^0\gamma^j\gamma^k\right\} = -\frac{4e^2}{\pi^3}\epsilon^{ijk}\int_{-\infty}^{\infty}\mathrm{d}p_0\int_0^{\infty}\mathrm{d}p\frac{M^2p^2(1-2n(p_0))}{[(p_0-i0^+)^2-p^2-M^2]^3} =$$

$$= -i\epsilon^{ijk}\frac{e^2}{2\pi^2}\int_0^{\infty}\mathrm{d}p\frac{M^2}{(p^2+M^2)^{3/2}} = -i\epsilon^{ijk}\frac{e^2}{2\pi^2} = -A$$

AVV triangle

$$G(p)\gamma^5 + \gamma^5 G(p) = \gamma^5 g(p)$$

$$\frac{\partial}{\partial q_{1k}}\widetilde{\Gamma}^{0ij}_{AVV} =$$

When only the Pauli-Villars term contributes:

$$= \frac{ie^2}{2} \int_p \mathrm{tr}\Big\{\gamma^\mu\gamma^5(1-2n_{FD}(p_0/T))\Big[$$
$$ig^R_M(p+q_1+q_2)\not{q}_2 iG^R_M(p+q_1)\gamma^\nu iG^R_M(p) + iG^R_M(p+q_1+q_2)\gamma^\nu iG^R_M(p+q_2)\not{q}_2 ig^R_M(p) +$$
$$ig^A_M(p+q_1+q_2)\not{q}_2 iG^A_M(p+q_1)\gamma^\nu iG^A_M(p) + iG^A_M(p+q_1+q_2)\gamma^\nu iG^A_M(p+q_2)\not{q}_2 ig^A_M(p)\Big]\Big\} =$$

$$\approx ie^2(-8iM^2)\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}\int_p(1-2n_{FD}(p_0/T))\left[\frac{1}{[(p_0+i0^+)^2-p^2-M^2]^3}-\frac{1}{[(p_0-i0^+)^2-p^2-M^2]^3}\right] =$$

$$= -\frac{4e^2}{\pi^3}\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}2\pi i\left(\frac{3}{8}\int_0^\infty \mathrm{d}y\frac{y^2}{(y^2+1)^{5/2}} - \lim_{N\to\infty}\sum_{n=0}^{N}\int_0^\infty \mathrm{d}y\frac{y^2}{\left(\frac{(2n+1)^2\pi^2}{M^2}+1+y^2\right)^3}\frac{2}{M}\right) =$$

$$\xrightarrow{M\to\infty} -i\frac{8e^2}{\pi^2}\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}\left(\frac{3}{8}\cdot\frac{1}{3}-\frac{1}{16}\right) = -i\frac{e^2}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}q_{1\sigma}q_{2\rho}$$

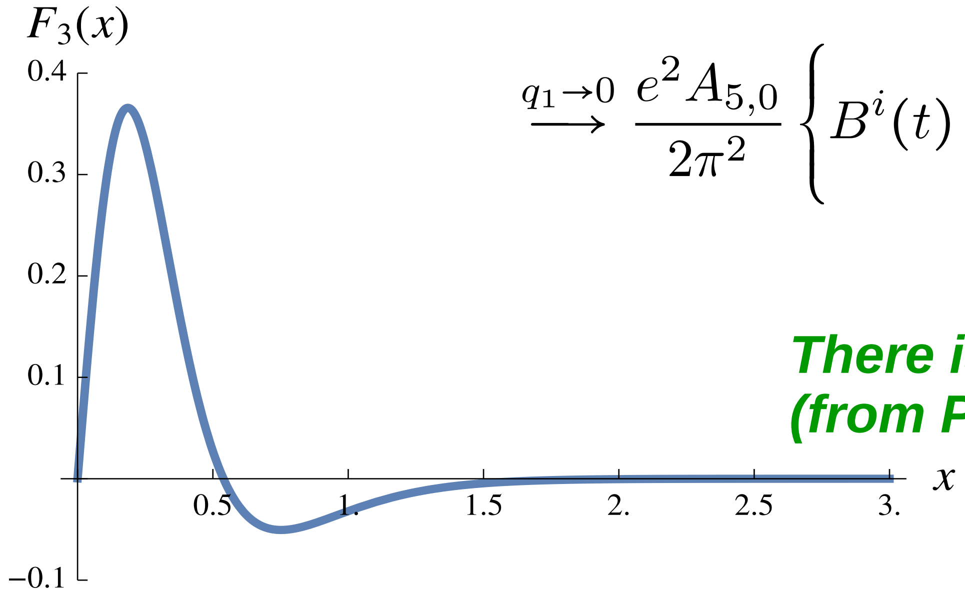
Limiting cases – constant $A_{5,0}$

$A_{5,0}$ is set first constant then homogeneous

B can be set to homogeneous with still non-trivial time-dependence

$$\widetilde{J}^i = A_{5,0}\int_{-\infty}^{\infty} \mathrm{d}\tau \widetilde{B}^i(t+\tau,\mathbf{q}_1)\frac{e^2}{2\pi^2}\left\{\delta(\tau) + \right.$$

$$\left. -4\theta(-\tau)\left[\frac{\partial}{\partial\tau}\left(\frac{\partial}{\partial\tau}\left(\frac{\sin(q_1\tau)}{q_1\tau}\right)\frac{TF_1(\tau T)}{q_1^2}\right) - \frac{\sin(q_1\tau)}{q_1\tau}TF_2(\tau T)\right]\right\}$$



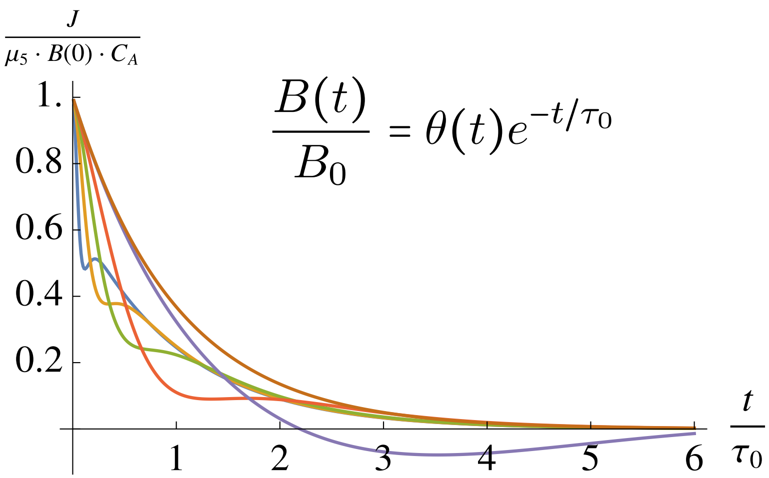
$$\xrightarrow{q_1\to 0} \frac{e^2 A_{5,0}}{2\pi^2}\left\{B^i(t) - 4\int_0^{\infty}\mathrm{d}\tau B^i(t+\tau)TF_3(\tau T)\right\}$$

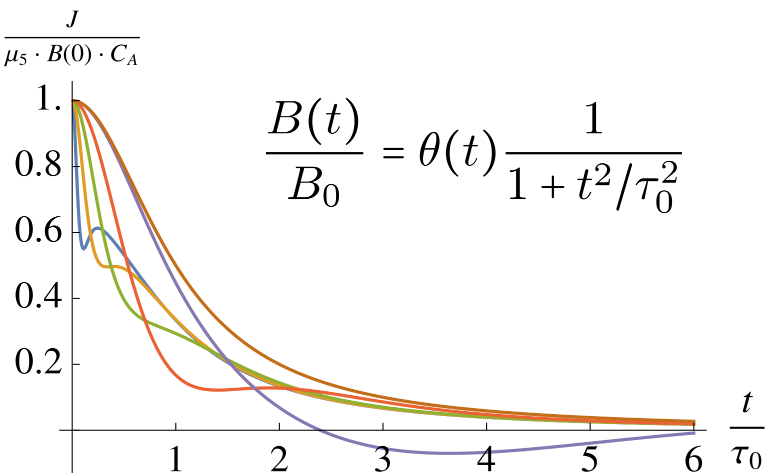
There is an instantaneous response (from PV regulator term!)

Limiting cases – constant $A_{5,0}$

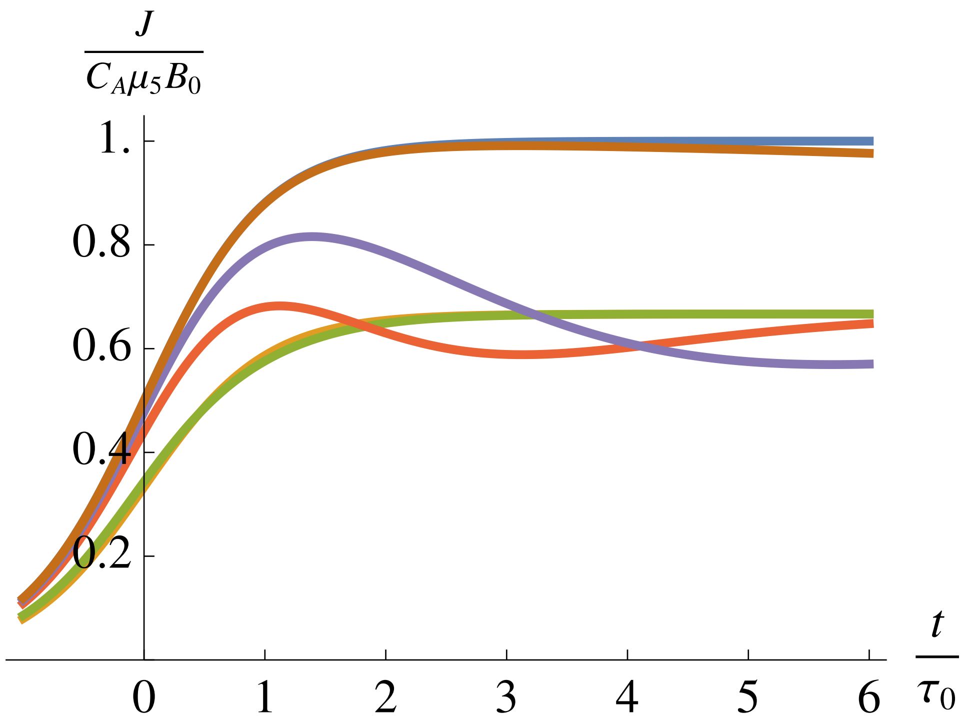
Magnetic field is homogeneous but time-dependent

Retardation is more pronounced for smaller temperatures





$\tau_0 T = 5.0,\ 2.0,\ 1.0,\ 0.5,\ 0.1$



$$\frac{B(t)}{B_0} = \frac{1 + \tanh(t/\tau_0)}{2}$$

$\tau_0 T = 5.0,\ 1.0,\ 0.2,\ 0.1,\ 0.01$

Limiting cases – constant *B*

$$\frac{\partial \mathfrak{g}^{ij}}{\partial q_{1k}}\epsilon^{ljk}B_l = -\frac{e^2}{\pi^2}\int\limits_{-\infty}^{\infty}\mathrm{d}p_0 \mathrm{sgn}(p_0)(1-2n(p_0))\int\limits_{0}^{\infty}\mathrm{d}p\delta\left(p_0^2-p^2-m^2\right)\int\limits_{-1}^{1}\mathrm{d}x$$

$$\left\{\frac{\partial}{\partial x}\frac{B_\parallel^i x(p_0 q_{20}+q_2 p x+(x^2+1)p^2)+B_\perp^i x\left(p_0 q_{20}+\left(\frac{1-x^2}{2}+1\right)p^2\right)}{(p_0+q_{20}+i0^+)^2-p^2-q_2^2-2q_2 p x-m^2}+\right.$$

$$\left.+B_\perp^i\frac{p^2}{(p_0+q_{20}+i0^+)^2-p^2-q_2^2-2q_2 p x-m^2}\right\}$$

$$J^i = \int_{q_2} e^{iq_2\cdot x}\widetilde{\mu_5}(q_{20},\mathbf{q}_2)\frac{1}{2}\left(\left.\frac{\partial \mathfrak{g}^{ij}}{\partial q_{1k}}\right|_{q_1=0,m\to 0}-\left.\frac{\partial \mathfrak{g}^{ij}}{\partial q_{1k}}\right|_{q_1=0,m=\infty}\right)\epsilon^{ljk}B_l$$

AVV response function

Backup on omega-q expression

$$\widetilde{J}^i(\omega, \mathbf{q}) = \widetilde{\sigma}^i_A(\omega, \mathbf{q})\widetilde{A}_{5,0}(\omega, \mathbf{q})$$

$$\widetilde{\sigma}^i_A(\omega, \mathbf{q}) = \int_{-\infty}^{\infty} \mathrm{d}t e^{-i\omega t}\overline{\sigma}^i_A(t, \mathbf{q}) = \frac{e^2}{4\pi^2}\Bigg\{2B^i + \frac{q^2}{\omega^2 - q^2}[B^i + \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})]$$

$$+ \int_0^{\infty} \mathrm{d}p\, n_{FD}(p/T)\left[\frac{1}{q}\ln\frac{\omega^2 - (2p-q)^2}{\omega^2 - (2p+q)^2} - 2\frac{2p-q}{\omega^2 - (2p-q)^2} - 2\frac{2p+q}{\omega^2 - (2p+q)^2}\right][B^i - \widehat{q}^i(\mathbf{B}\cdot\widehat{\mathbf{q}})]\Bigg\}$$

$$\widetilde{n}(\omega, \mathbf{q}) = \frac{\mathbf{q}\cdot\widetilde{\mathbf{J}}(\omega, \mathbf{q})}{\omega} = \frac{e^2}{2\pi^2}\widetilde{A}_{5,0}\frac{\omega}{\omega^2 - q^2}\mathbf{q}\cdot\mathbf{B}$$