Chirality Generation in Strong Electromagnetic Fields via the Schwinger Mechanism

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Quantum kinetic theories in magnetic and vortical fields
YITP, Dec. 12, 2019

PC, K. Fukushima, and S. Pu, *PRL*, 121, 261602 (2018).

Outline

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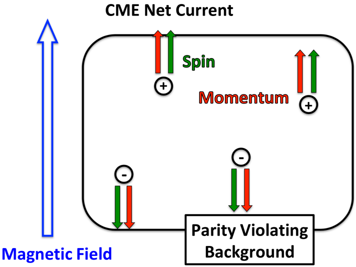
Electromagnetic current generated in the direction of magnetic field due to a net chirality.[1] - **Chiral Magnetic Effect (CME)**

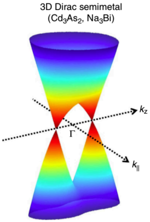
$$\mathbf{j}_{cme} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}(\mathbf{x})$$



[1] K. Fukushima, D. Kharzeev, and H. Warringa, *PRD* 78, 074033 (2008).

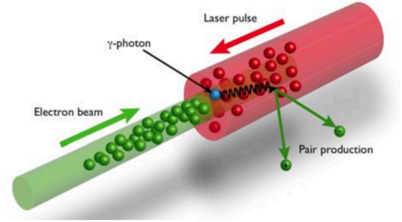
- Relativistic fermionic dispersion relation in Weyl/Dirac Semimetals.
- CME and anomaly directly observed.
- Need to probe the CME and anomaly in QED and QCD:

3-D Semimetals ✓



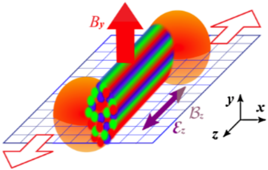
M. Neupane, et. al., *Nat. Comm.* 10.1038 (2014)

QED - High Powered Lasers ?



M. Marklund and J. Lundin, *Eur.Phys.J.D* 55, 319326 (2009)

QCD - Heavy Ion Collisions ?



K.Fukushima, D.Kharzeev, H.Warringa, *PRL* 104,212001(2010)

Dirac Semimetals

- Negative magnetoresistence observation in $(m \approx 0)$ semimetal.[2]
- **Observation of CME!**
  - μ_5, the chiral chemical potential generated through the anomaly.

Heavy Ion Collisions Fluctuations in topology and local parity violation as a means of observing the CME.

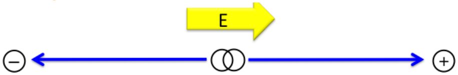
Remaining questions in need of clarification:

- How to **generate chirality?**
- What about **finite mass**?
- CME **in and out-of-equilibrium**?

The Schwinger mechanism can provide a solution.

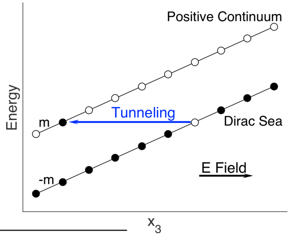
[2] Q. Li, et. al., *Nat. Physics* 10.1038 (2016).

- Instability of the QFT vacuum under an external electric field: **Schwinger pair production**[3]



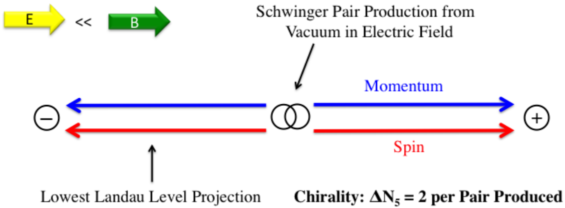
- Effective action, $\langle out|in\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{i\int \bar{\psi}(i\not{D}-m)\psi}$, imaginary part under constant electric field:

$$\text{Num. of pairs} = 1 - |\langle out|in\rangle|^2 \approx \exp\left(-\frac{\pi m^2}{eE}\right)$$



[3] J. Schwinger, *Phys. Rev.* 82, 664 (1951).

- Add in a strong magnetic field



- Increments the chirality!

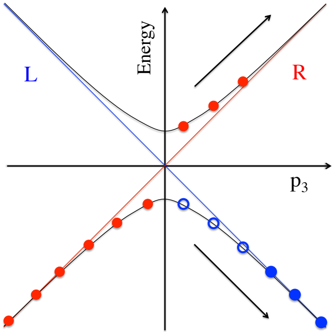
$$\text{Num. of pairs} = \frac{e^2 EB}{4\pi^2} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right)$$

$$\xrightarrow{B \gg E} \frac{e^2 EB}{4\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right)$$

$$= \frac{1}{2}\partial_t n_5$$

n_5 is the chiral density.

- Again use Dirac sea picture–but here in momentum space.
- **Lowest Landau level** due to strong magnetic field $\approx 1 + 1$.
- Turn on background electric field. Both left and right particles get their momentum shifted.
- Tunneling via Schwinger mechanism–mass gap traversible!



The **axial Ward identity**,

$$\partial_\mu j^{\mu 5} = 2im\bar{\psi}\gamma^5\psi + \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B},$$

for $j^{\mu 5} = \bar{\psi}\gamma^\mu\gamma^5\psi$ and fermion mass, m, is exact and well-known at the **operator** level.

- Yet, its **vacuum expectation value** behavior requires elucidation.
- Particularily the case when

  $$\langle in| \neq \langle out|$$

  such as for the **Schwinger mechanism** in parity violating fields!

Setup and Strategy

- Study the axial Ward identity and CME, and also applications such as fluctuations in chirality and the chiral condensate, under the simplest parity breaking background:

  Parallel electric, E, and magnetic, B, fields
  (in $\hat{x}_3$ direction)

- **Exactly solvable background!** Plus, clear results unobsecured by technicalities.
- Determine expectation values using $\langle out| \, |in\rangle$ and $\langle in| \, |in\rangle$ vacuum states using
  1. Schwinger proper time
  2. Worldline path integral techniques
  3. Real-time formalism

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Standard QFT treatment of well-known in-out propagator for homogeneous fields is

$$S^c(x,y) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, i\psi(x)\bar{\psi}(y) e^{i\int d^4x \bar{\psi}(i\not{D}-m)\psi}$$
$$= i \langle out| \, T\{\psi(x)\bar{\psi}(y)\} \, |in\rangle$$
$$= (i\not{D} + m) \int_0^\infty ds\, g(x,y,s)$$

with kernel, g, in proper time, s,

$$g(x,y,s) = i \langle x| \exp(-i(\hat{\not{D}}^2 + m^2)s) \, |y\rangle \, .$$

Kernel may be cast into worldline path integral representation.

- **Worldline Path Integral:**

$$g(x,y,s) = i\int \mathcal{D}x\,\mathcal{P}\exp\Big\{i\int_0^s d\tau[-\frac{1}{4}\dot{x}^2 - A_\mu \dot{x}^\mu - \frac{1}{2}F_{\mu\nu}\sigma^{\mu\nu} - m^2]\Big\}$$

- For our Fock-Schwinger gauge choice the spin factor is:

$$\mathcal{P}e^{-i\int_0^s d\tau \frac{1}{2} eF_{\mu\nu}\sigma^{\mu\nu}} = \begin{pmatrix} e^{-e(E-iB)s} & & & 0 \\ & e^{e(E-iB)s} & & \\ & & e^{e(E+iB)s} & \\ 0 & & & e^{-e(E+iB)s} \end{pmatrix}$$

- Spin factor and remaining Boson path integral separate, but are connected through proper time, s.

- Path integral portion reduces to one of a Boson

$$\int \mathcal{D}x \, \exp\Big\{ i \int_0^s d\tau [-\frac{1}{4}\dot{x}^2 - A_\mu \dot{x}^\mu] \Big\}$$

- Evalute through steepest decents
  - Classical solution contains all x and y information.
  - Fluctuation prefactor contains Landau level (both magnetic and electric) structure.
- Reproduce exact result:

$$g(x,y,s) = \frac{e^2 EB}{(4\pi)^2} \sinh^{-1}(eEs) \sin^{-1}(eBs) \times \exp\left[-i \Big(\frac{1}{2} eF \cdot \sigma + m^2 \Big) s + i\varphi(x,y,s) \right]$$

$$\varphi(x,y,s) = \frac{1}{2} x_\mu eF^{\mu\nu} y_\nu - \frac{1}{4} z_\mu (\coth(eFs))^{\mu\nu} eF_{\nu\sigma} z^\sigma$$

- One advantage of the proper time formalism is a natural heat kernel regularization.
- e.g. For the psuedoscalar calculation

$$\mathrm{tr}(\gamma_5 \mathcal{P} e^{-i\int_0^s d\tau \frac{1}{2} eF_{\mu\nu}\sigma^{\mu\nu}}) = 4i\sin(eBs)\sinh(eEs),$$

cancelling with the bosonic part and greatly simplifying the calculation. Unregularized, this quantity would be zero.

- c.f. Fujikawa's method[4] to calculate the regulated tr γ_5 uses the following calculation (Euclidean space):

$$\mathrm{tr}(\gamma_5 e^{\frac{1}{2}eF_{\mu\nu}\sigma^{\mu\nu}/M^2}) = \frac{i}{2M^4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \quad \text{for} \quad M\to\infty$$

- Automatically treat regulated γ_5 with proper time!

[4] K. Fujikawa, *Phys. Rev. Lett.* 42, 1195 (1979).

In-out pseudoscalar[5], chiral density, and CME can be found:

$$\begin{aligned} \langle out|\, \bar{\psi} i\gamma^5 \psi \,|in\rangle &= -\operatorname{tr} \gamma^5 S^c(x,x) \\ &= -\frac{e^2 EB}{4m\pi^2} \\ \langle out|\, \partial_t n_5 \,|in\rangle &= 0 \\ \langle out|\, \bar{\psi} \gamma^\mu \psi \,|in\rangle &= 0 \end{aligned}$$

- **The anomaly has vanished!**
- Naïve treatment of massless limit would lead to an incorrect result! → Limit should be taken last.
- *Also, there appears to be an inconsistency...*

[5] J. Schwinger, *Phys. Rev.* 82, 664 (1951).

Consider the physically heuristic scenerio, there it was found that

$$\partial_t n_5 = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right) \xrightarrow{m\to 0} \frac{e^2 EB}{2\pi^2}$$

Yet here, for an in-out expectation value, we found

$$\langle out| \, \partial_t n_5 \, |in\rangle = 0$$

for any mass (even $m \to 0$).

Inconsistency?

Not quite

- **What's the problem $\rightarrow$ Inequivalent Vacuum States!**
- Need out-of-equilibrium setup to see produced pairs.

But In-Out expectation values perfectly valid calculations.

- Wick rotated time from QFT partition function to temperature $\rightarrow$ **Equilibrium VEV**.

Massless QED and QED in a massless limit can be quite different theories. e.g. Shielded charge in massless case. Care must taken in QCD too.

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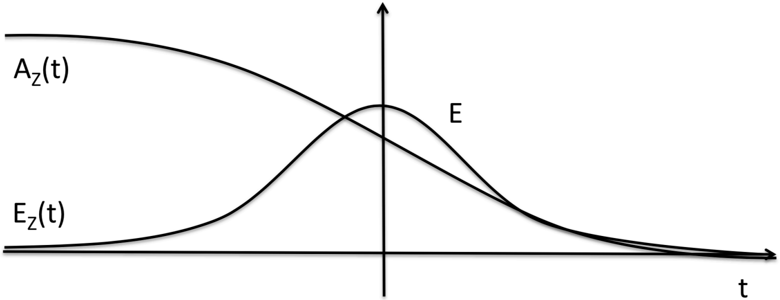
Inequivalent asymptotic $t \to \pm\infty$ **states**. Example: Sauter (pulsed) electric field.



$$E(t) = E\cosh^{-2}(t)\hat{z} \text{ and } A_z = -E\tanh(t) + E$$

$$\langle in| \neq \langle out|$$

In-out propagator, S^c, misses out-of-equilibrium phenomenon.

- **Solution is provided by in-in vacuum states!**
- An in-in precription is the same as a **Keldysh-Schwinger (KS), or real-time**, formalism.
- Any system which produces Schwinger pair production applies.
- However, direct application of the KS formalism to expectation values is challenging and few exact results are known...

Look for application of the worldline proper time formalism!

- To construct and interpret an in-in propagator for any background field from a **canonical operator formalism**.
- Expand field operators in eigenvectors of the Dirac equation with either positive energy solutions, ϕ_{+n}, or negative energy solutions, ϕ_{-n}, with eigenvalue n.
- $a_n^{in}, b_n^{in} |\Omega_{in}\rangle = \langle\Omega_{in}| a_n^{in\,\dagger}, b_n^{in\,\dagger} = 0$ with the usual anti-commutation relations applying.

$$\psi(x) = \sum_n a_n^{in} \phi_{+n}^{in}(x) + b_n^{in\,\dagger} \phi_{-n}^{in}(x)$$

$$\psi(x) = \sum_n a_n^{out} \phi_{+n}^{out}(x) + b_n^{out\,\dagger} \phi_{-n}^{out}(x)$$

for $n \in (p, \text{Landau level, etc.})$

- Start from in-in normalization and insert complete set of out states (also SK generating functional without sources):

$$\langle in|in\rangle = 1 = \sum_{\alpha} \langle in|\alpha, out\rangle \langle \alpha, out|in\rangle$$
$$= \left|\langle out|in\rangle\right|^2 + \sum_{n,m} \langle in|\, b_n^{\dagger\, out} a_m^{\dagger\, out} \,|out\rangle \langle out|\, a_m^{out} b_n^{out} \,|in\rangle + \ldots$$

- First term: Probability that the vacuum stays the vacuum (no pairs of particles in out state).
- Second term: Probability for Schwinger pair production of single pair.
- Apply to **in-in propagator** $\rightarrow$

$$S^c_{in}(x,y) = i \langle in|\, T\{\psi(x)\bar{\psi}(y)\} \,|in\rangle$$

- Insert complete set of out states into in-in propagator (normalization factors $c_v := \langle out|in\rangle$ is explicit here):

$$
\begin{aligned}
&i\,\langle in|\,\psi(x)\bar{\psi}(y)\,|in\rangle = i\,\langle in|out\rangle\,\langle out|\,\psi(x)\bar{\psi}(y)\,|in\rangle \\
&+\sum_{n,m}\langle in|\,b_n^{\dagger\,out}a_m^{\dagger\,out}\,|out\rangle\,\langle out|\,a_m^{out}b_n^{out}\psi(x)\bar{\psi}(y)\,|in\rangle+\ldots \\
&= i\,\langle out|\,\psi(x)\bar{\psi}(y)\,|in\rangle\,\tfrac{1}{c_v} \\
&+ i\,\langle out|\,a_n^{out}\psi\,|in\rangle\,\tfrac{1}{c_v}\,\langle in|\,b_n^{\dagger\,out}a_m^{\dagger\,out}\,|out\rangle\,\tfrac{1}{c_v^*}\,\langle out|\,b_m^{out}\bar{\psi}\,|in\rangle\,\tfrac{1}{c_v}+\ldots
\end{aligned}
$$

- **In-in propagator includes in-out propagator + an additional term predicting the creation of a pairs at infinity!**
- Sum over all pairs→

$$S^c(x,y) = i\sum_{N=0}^{\infty}\frac{1}{N!^2}\sum_{\substack{m_1\ldots m_N\\ n_1\ldots n_N}}\langle out|\, b^{in\,\dagger}_{n_1}\ldots b^{in\,\dagger}_{n_N}a^{in\,\dagger}_{m_1}\ldots a^{in\,\dagger}_{m_N}\,|in\rangle$$

$$\times\,\langle in|\, a^{in}_{m_N}\ldots a^{in}_{m_1}\, b^{in}_{n_N}\ldots b^{in}_{n_1}\, T\psi(x)\bar{\psi}(y)\,|in\rangle$$

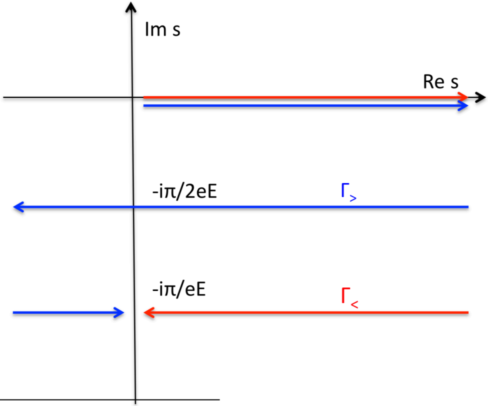
Only $N = 0, 1$ contribute; simpler calculation:

$$S^c(x,y) = S^c_{in}(x,y) + \phi^{in}_{-n}(x)\,\langle out|\, b^{in\,\dagger}_{n}a^{in\,\dagger}_{m}\,|in\rangle\,\bar{\phi}^{in}_{+m}(y)$$

- Solve Dirac equation and sum over eigenvectors.
- S^c too can be exactly constructed from an eigendecomposition.

Fradkin, et. al.[6] demonstrated that the in-in propagator, is expressible entirely in terms of the worldline kernel, $z = x - y$:

$$S^c_{in}(x,y) = (i\not{D} + m)\left[\theta(z_3)\int_{\Gamma_>} ds + \theta(-z_3)\int_{\Gamma_<} ds\right] g(x,y,s).$$



[6] E. Fradkin, G. Gitman, and S. Shvartsman, *Quantum Electrodynamics in Unstable Vacuum* 1991.

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In-In (Real-time) Pseudoscalar and Axial Anomaly:

$$\begin{aligned}\langle in|\,\bar{\psi} i\gamma^5\psi\,|in\rangle &= -\operatorname{tr}\gamma^5 S^c_{in}(x,x)\\ &= -\frac{e^2EB}{4m\pi^2}\left[1-\exp\left(-\frac{m^2\pi}{eE}\right)\right]\end{aligned}$$

And using the axial Ward identity:

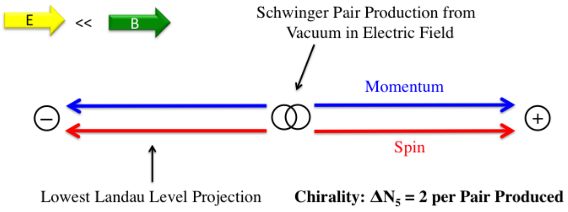
$$\langle in|\,\partial_t n_5\,|in\rangle = \frac{e^2EB}{2\pi^2}\exp\left(-\frac{m^2\pi}{eE}\right)$$

- **Chirality is spontaneously generated from the vacuum through the Schwinger mechanism!** And only through the Schwinger mechanism.
- Mass effects for the one-loop axial Ward identity.

Real-time Pseudoscalar and Axial Ward Identity

Chirality Generation from the Schwinger Mechanism

- Generation of chirality agrees with physical heuristic picture:



- **One-loop exact**–for homogeneous parallel electric and magnetic fields. All Landau levels are kept.
  But only the lowest Landau level contributes!
- We can find analogous results by calculating the chiral density directly! $\longrightarrow$

In-in chiral density can be found like before:

$$n_5 = \langle in| \bar{\psi}\gamma_0\gamma^5\psi |in\rangle = i\,\mathrm{tr}\,\gamma_0\gamma^5 S^c_{in}(x,x).$$

Though mathematically more subtle.

- Here $\not{D}$ acts on $\left[\theta(z_3)\int_{\Gamma_>} ds + \theta(-z_3)\int_{\Gamma_<} ds\right]$ as well!
- In fact time dependence emerges in a non-trivial way:

$$\lim_{x_3\to y_3}\delta(x_3-y_3) = \lim_{x_3\to y_3}\frac{1}{2\pi}\int_{-\infty}^{\infty} dp_3 \exp(ip_3(x^3-y^3))$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dp_3 := \frac{Et}{2\pi}$$

As is typically identified for constant fields, e.g. Nikishov[7].

[7] A. Nikishov, *JETP* Vol. 30, Num. 4 (1970).

- Only the $\not{D}$ part (as opposed to the m part in calculating the pseudoscalar) contributes to n_5.
- As before, even though all Landau level are taken into account only the lowest Landau level contributes.
- Only the *time* part is non-zero.
- **Agreement** with axial Ward identity and pseudoscalar calculations!

$$n_5 = \frac{e^2 EB}{2\pi^2}\, t \exp\left(-\frac{m^2\pi}{eE}\right)$$

Linearly dependent in time and with the Schwinger mechanism characteristic mass suppression. Chirality with mass effects!

In the same way as the in-in chiral density, we can calculate the **real-time CME** current:

$$\langle in|\,\bar{\psi}\gamma^3\psi\,|in\rangle = \frac{e^2 EB\, t}{2\pi^2}\coth\left(\frac{B}{E}\pi\right)\exp\left(-\frac{\pi m^2}{eE}\right)$$

- No lowest Landau level approximation.
- Agreement with vacuum non-persistence.
- Again, here, time depedence arises from the phase space.
- **The CME is intrinsically a real-time phenomenon. Chirality is generated through the Schwinger mechanism.**
- Differences of in and out-of-equilibrium CME cannot be clarified if μ_5 is incorportated.

What are we to make of in-out expectation values then???

In-out (equilibrium) chiral density:

$$\bar{n}_5 := \langle out | \bar{\psi}\gamma_0\gamma^5\psi | in \rangle = 0$$

$$\partial_t \bar{n}_5 = 0$$

- Again, this is consistent with the previous calculation using the axial Ward identity and the In-Out pseudoscalar.
- Conservation of chirality!? For any mass.
- What are we to make of $\partial_t \bar{n}_5 = 0$?
- As alluded before this is an equilibrium, without Schwinger pair production, scenerio. Look at it from finite temperature standpoint.

- Consider a Wick rotation of time (not proper time)

$$x_0 = ix_4 = \tau \, .$$

- Vacuum states also follow rotation. Therefore the ground states are

$$\langle out| \quad \text{at}\, \tau = \beta \quad \text{and} \quad |in\rangle \quad \text{at}\, \tau = 0$$

- Then in the zero temperature limit, $\beta = \infty$, we can reproduce our in-out partition function.
- **Equilibrium scenerio**, static Euclidean expectation value. Will not predict out-of-equilbrium Schwinger pair production.
- Recall in-out matrix element predicts the vacuum will stay the vacuum, i.e., no pairs of particles in the out state.

- Confirmed on the lattice with Euclidean Monte Carlo simulation
  – **cannot pick up real time Schwinger pair production!**[8]

$\underline{m = 0 \text{ case intuitive:}}$

- Topological properties of ground state are characterized by a θ **angle.**
- However with $m = 0$ ground state must be independent of θ.
- No nonzero topological charge and no chirality flip!

$$\langle out | \, \partial_\tau j_5^\tau \, | in \rangle = 0$$

[8] A. Yamamoto, *PRL* 110, 112001 (2013).

Also the case for the in-out **equilibrium CME**:

$$\langle out|\, \bar{\psi}\gamma^3\psi \,|in\rangle = 0$$

The CME does not exist in equilibrium!

Problem lies with μ_5

- By introducing μ_5 (by hand) it appears as though the CME exists in equilibrium.
- Introduction of μ_5 makes non-equilibrium effects visible even in an equilibrium system.
- But μ_5 inherently an out-of-equilibrium quantity!

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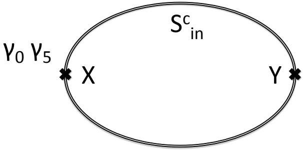
- **Heavy ion collision experiment:**
    - Global net chirality is zero.
    - **Local fluctuations might exist.**
- Parallel E and $B \rightarrow$ Glasma flux tube.
- Important to calculate fluctuations in chirality:

$$N_5 := \int d^3x \, \bar{\psi}\gamma_0\gamma_5\psi$$
$$\chi_5 := (\langle in| N_5^2 |in\rangle - \langle in| N_5 |in\rangle^2)$$
$$\bar{\chi}_5 := (\langle out| N_5^2 |in\rangle - \langle out| N_5 |in\rangle^2)$$

- As before expect very different time dependence for real-time and equilibrium values.

$$\chi_5 = \lim_{x_0 \to y_0} \int d^3x d^3y \; \mathrm{tr}[\gamma_0\gamma_5 S^c_{in}(x,y)\gamma_0\gamma_5 S^c_{in}(y,x)]$$

$=$



- Quadratic time dependence in disconnected piece.
- Expect linear time dependence in real-time chirality fluctuations.

Chirality Fluctuations

Real-time $\chi_5 \propto EBV\, t\, e^{-2\pi m^2/(eE)} + \mathrm{rest}$

Equilbrium $\bar{\chi}_5$ has no time dependence or Schwinger-like exponential mass suppression.

- Take $E \to 0$ for either equilibrium or real-time chiral condensate:[9] **Magnetic Catalysis**

$$\langle out|\,\bar{\psi}\psi\,|in\rangle\,|_{E=0} = -\frac{eB}{4\pi^2}\,m\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}e^{-im^2 s}\cot(eBs)$$
$$\simeq -\frac{eB}{4\pi^2}\,m\,\Gamma[0, m^2/\Lambda^2]\,,$$

- The lowest Landau level approximation has been used.
- With $is \to s$ produce familar magnetic catalysis integral.
- UV divergence, cutoff at $1/\Lambda^2$

[9] $\langle in|\,\bar{\psi}\psi\,|in\rangle\,|_{E\to 0} = \langle out|\,\bar{\psi}\psi\,|in\rangle\,|_{E\to 0}$

Equilibrium Chiral Condensate

$$\langle out|\, \bar{\psi}\psi \,|in\rangle \simeq -\frac{e^2 EB}{4\pi^2}\, m \int_{1/\Lambda^2}^{\infty} ds\, e^{-im^2 s} \cot(eBs) \coth(eEs)$$

$$\simeq -\frac{eB}{4\pi^2}\, m \left[\ln \frac{\Lambda^2\, e^{-\gamma_{\mathrm{E}}}}{2eE} - \mathrm{Re}\psi\left(\frac{im^2}{2eE}\right) - \frac{i\pi}{e^{\pi m^2/(eE)} - 1} \right]$$

- Again LLL approximation, also only keep terms present for large Λ.
- Imaginary piece $\rightarrow$ Signal of instability.
    - Chiral condensate at a finite θ angle resemblance.
    - Fermi-Dirac-like distribution, energy over temperature $\rightarrow \pi m^2/eE$!
- With $E \rightarrow 0$ recovery to magnetic catalysis.

Real-time Chiral Condensate

$$\langle in|\,\bar{\psi}\psi\,|in\rangle \simeq -\frac{e^2 EB}{4\pi^2}\,m\int_{1/\Lambda^2}^{\pi/eE-1/\Lambda^2} ds\, e^{-m^2 s}\coth(eBs)\cot(eEs)$$

$$\simeq \left[1 - e^{-\pi m^2/(eE)}\right] \mathrm{Re}\,\langle out|\,\bar{\psi}\psi\,|in\rangle$$

- Real observables guaranted by In-In construction.
- Again, LLL approximation, already taken Wick rotation in proper time.
- Divergences arise from both proper time limits, but both are UV divergences.
- Constituent mass is reduced with Schwinger pair production!
- Landau-Zener effect in condensed matter!

Challenges:

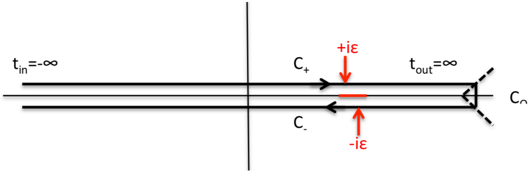
1. Schwinger pair production **non-perturbative** phenomenon:

$$\exp(-\frac{\pi m^2 c^3}{eE\hbar})$$

2. Gauge link (operator) between fermion operators.
3. $\hbar$ expansion for chiral kinetic theory.
4. Collisions.

Schwinger pair production effects particularly important for weak electric field or large mass–*enormous axial charge suppression*.

Schwinger Keldysh formalism:



$$S^{c}_{in}(x,y) = i \langle in | \theta(x_0 - y_0)\psi(x)\bar{\psi}(y) - \theta(y_0 - x_0)\bar{\psi}(y)\psi(x) | in \rangle$$
$$S^{\bar{c}}_{in}(x,y) = i \langle in | \theta(y_0 - x_0)\psi(x)\bar{\psi}(y) - \theta(x_0 - y_0)\bar{\psi}(y)\psi(x) | in \rangle$$
$$S^{>}_{in}(x,y) = i \langle in | \psi(x)\bar{\psi}(y) | in \rangle$$
$$S^{<}_{in}(x,y) = i \langle in | \bar{\psi}(y)\psi(x) | in \rangle$$

For Kadanoff Baym equations, spectral and statistical propagators:

$$F(x,y) = \frac{i}{2} \langle in | [\psi(x), \bar{\psi}(y)] | in \rangle$$
$$\rho(x,y) = \langle in | \{\psi(x), \bar{\psi}(y)\} | in \rangle$$

- Use statistical propagator (could use $S_{in}^{>}$ or $S_{in}^{<}$).

$$F(x,y) = (i\not{D}_x + m)\left[\theta(z_3)\int_{\Gamma_>} + \theta(-z_3)\int_{\Gamma_<} - \frac{1}{2}\int_{\Gamma_0}\right] ds\, g(x,y,s)$$

- Wigner transform:

$$\tilde{F}(k,x) = \int d^4 s e^{ik\cdot x} U\left(x, x+\frac{s}{2}\right) F\left(x+\frac{s}{2}, x-\frac{s}{2}\right) U\left(x-\frac{s}{2}, x\right)$$

- Wilson loop, U, kills gauge factor in kernel, g!
  Fully gauge invariant expression.

Collisionless chiral kinetic theory to $\hbar$

$$\left\{[k_\mu + i\tfrac{1}{2}(\partial_\mu - eF_{\mu\nu}\partial_k^\nu)]\gamma^\mu - m\right\} \tilde{F}(k,x) = 0$$

Anticipate exponential suppression due to Schwinger mechanism.

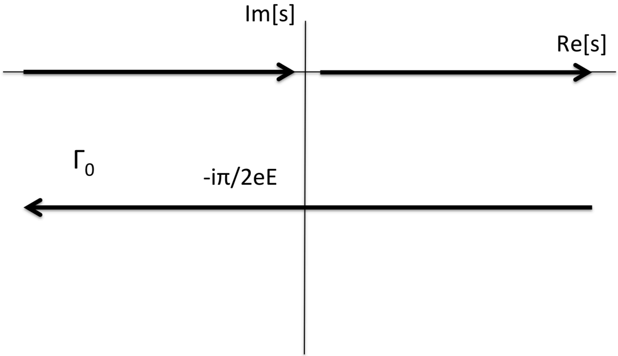
Conclusions

- How to generate chirality? $\rightarrow$ **The Schwinger mechanism.**
- But what is the **axial Ward identity VEV**, since $\langle in| \neq \langle out|$?
  - Equilibrium: $\langle out| \partial_\mu j_5^\mu |in\rangle = 0$.
  - Real time: $\langle in| \partial_\mu j_5^\mu |in\rangle = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{m^2\pi}{eE}\right)$.
- Enables the CME!

Thank you for your time and attention!

Fluctuations real-time and equilibrium in weak field large mass approximation, $eE \ll m^2$:

$$\chi_5/V = \frac{e^2 EB\, t}{2\pi^2} e^{-\frac{2m^2\pi}{eE}} + \frac{e^4 E^2 B^2}{\pi eB}\Bigg\{(57.924) e^{-\frac{2m^2\pi}{eE}} (eE)^{-\frac{3}{2}}\Big]$$
$$+\left(1 - e^{-\frac{m^2\pi}{eE}}\right)^2 \Big[-\frac{45\, e^2 E^2}{128^2\, m^7} - \frac{1}{512 m^3} + \frac{\sqrt{2\pi}\, m}{32\, 2e^2 E^2 \pi^2}\left(\Gamma[-\frac{1}{4}, \frac{m^2}{\Lambda^2}]\right)^2\Big]\Bigg\}$$
$$+\frac{e^4 E^2 B^2}{\pi eB}\left\{(2.827) e^{-\frac{2m^2\pi}{eE}} m^2 (eE)^{-\frac{7}{2}} + \left(1 - e^{-\frac{m^2\pi}{eE}}\right)^2 \left(\frac{m}{2\, e^2 E^2} + \frac{3}{64\, m^3}\right)\right\}$$
$$\bar{\chi}_5/V = \frac{e^4 E^2 B^2}{\pi eB}\left(\frac{m}{2\, e^2 E^2} + \frac{3}{64\, m^3}\right)$$
$$+\frac{e^4 E^2 B^2}{\pi eB}\left\{-\frac{45\, e^2 E^2}{128^2\, m^7} - \frac{1}{512 m^3} + \frac{\sqrt{2\pi}\, m}{32\, 2e^2 E^2 \pi^2}\left(\Gamma[-\frac{1}{4}, \frac{m^2}{\Lambda^2}]\right)^2\right\}$$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