Magneto-vortical effects in strongly coupled plasma

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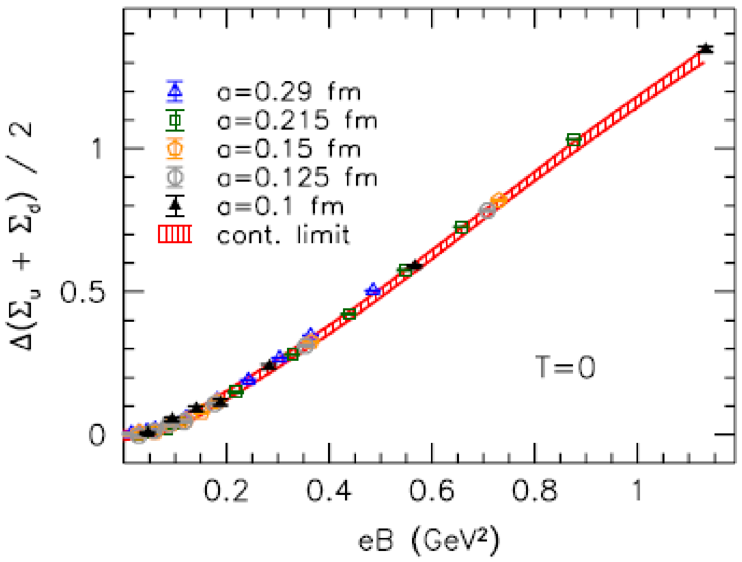
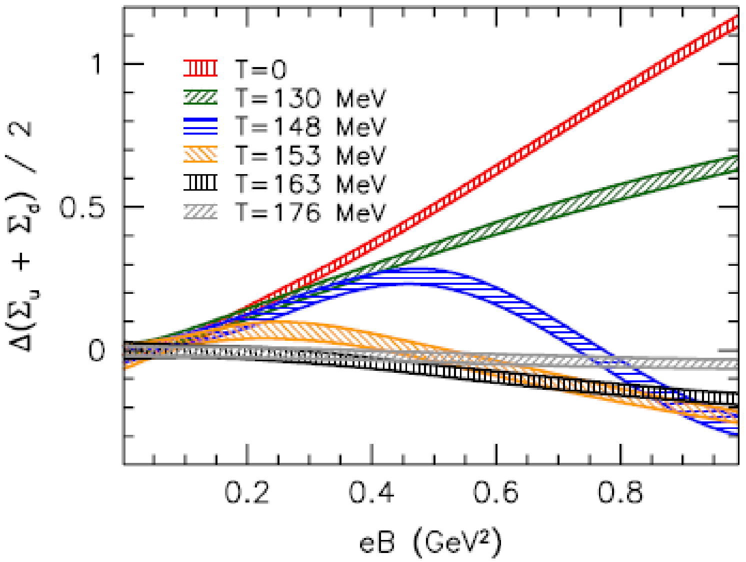
Yanyan Bu, SL, 1912.XXXXX

Outline

- Introduction to magnetic and vortical effects in QCD
- Magneto-vortical effects in free theory and general expectation
- Holographic model of magnetized plasma
- weak B results
- arbitrary B results
- Thermal Hall effect&thermal axial magnetic effect
- Conclusion&outlook

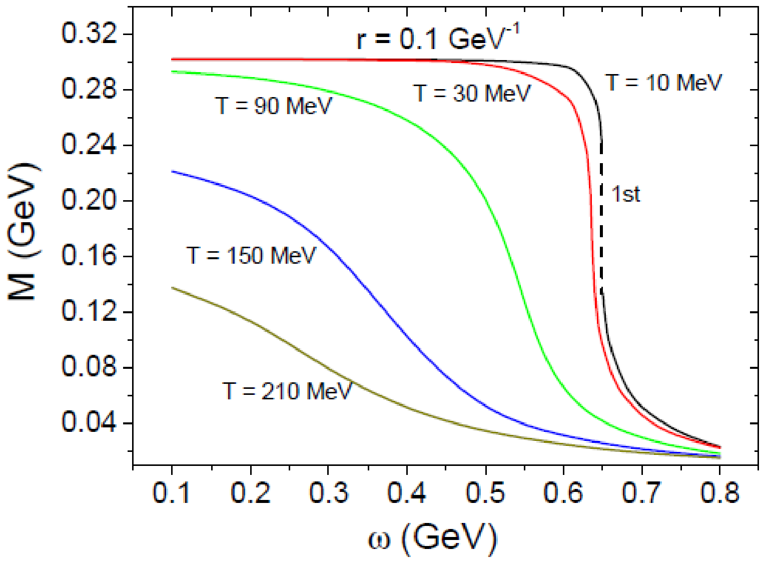
(Non-anomalous) magnetic effect in QCD





Gusynin, Miransky, Shovkovy, PRL 1994
Bali et al, PRD 2012,
Bali et al, JHEP 2012

(Non-anomalous) vortical effect in QCD



Jiang, Liao, PRL 2016
Ebihara, Fukushima, Mameda, PLB 2017

Anomalous magnetic and vortical effects

Chiral Magnetic/Separation Effect(CME/CSE)

$\boldsymbol{j} = C\mu_5 e\boldsymbol{B}$ $\qquad$ $\boldsymbol{j}_5 = C\mu e\boldsymbol{B}$

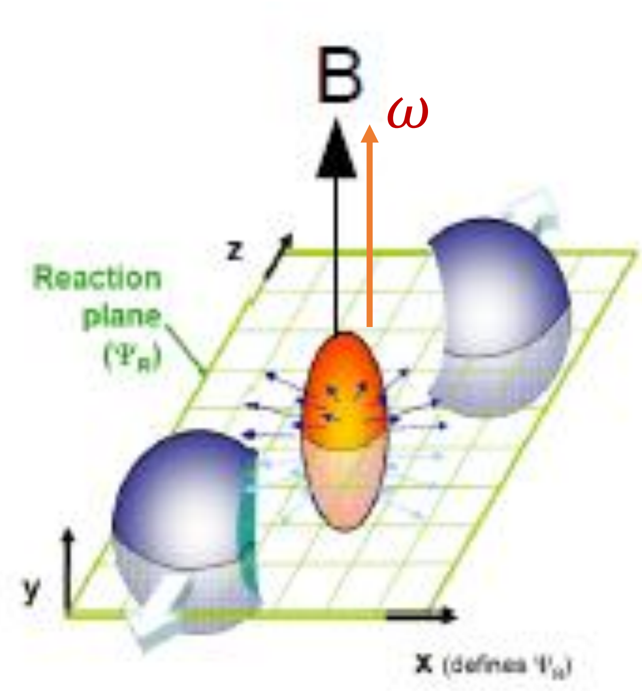
Vilenken, PRD 1980
Metlitski, Zhitnitsky, PRD 2005
Kharzeev, McLerran, Warringa, NPA 2008

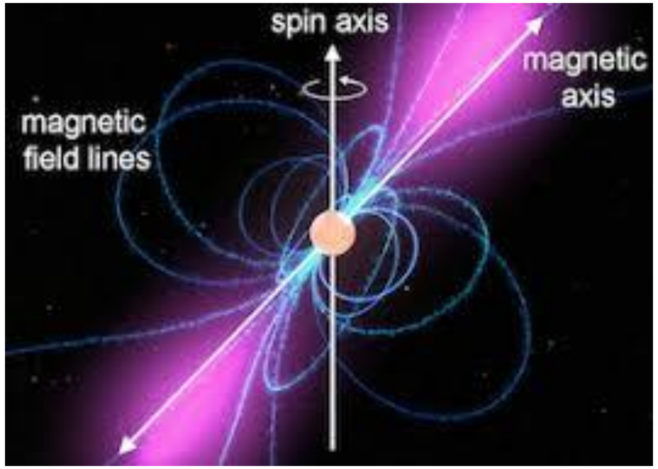
Chiral Vortical Effect(VCVE/ACVE)

$\boldsymbol{j} = C\mu_5\mu\boldsymbol{\omega}$ $\qquad$ $\boldsymbol{j}_5 = C(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3})\boldsymbol{\omega}$

Vilenken, PRD 1979
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011
Landsteiner et al, PRL 2011

Interplay of magnetic and vortical fields





Magneto-vortical effects?

Magneto-vortical effects

lowest Landau level, free theory:

Hattori, Yin, PRL 2017

$$J^t = q_f \frac{C_A}{2} B \cdot \Omega$$

CSE →

$$J_5^z = C_A \mu B = C_A \frac{J^t}{\chi} B = |q_f| \frac{C_A}{2} B \cdot \Omega$$

$$\chi = C_A |q_f| B$$

Covariant anomaly:

$$\partial_\mu J^\mu = q_f^2 C_A (E_5 \cdot B + E \cdot B_5)$$

$$\partial_t J^t = q_f^2 C_A \partial_t A_5 \cdot B \quad \rightarrow \quad J^t = q_f^2 C_A A_5 \cdot B$$

$$-\Delta H = q_f A_5 \cdot j_5 = \frac{\Omega}{2} \cdot 2S$$

Ok in free theory

How about in interacting theory?

What to expect for magneto-vortical effects

$$J^t = q_f \frac{C_A}{2}(\vec{B}\cdot\vec{\Omega})$$

$$\vec{J}_5 = |q_f| \frac{C_A}{2}(\vec{B}\cdot\vec{\Omega})\hat{B}$$

more generally

$$J^t = \xi(B,T)(\vec{B}\cdot\vec{\Omega})$$

$$\vec{J}_5 = \sigma(B,T)\vec{\Omega}$$

	C	P	T
J^t	$-$	$+$	$+$
J_5^z	$+$	$+$	$-$
B	$-$	$+$	$-$
Ω	$+$	$+$	$-$

ξ, σ allowed by CPT

Holographic model for magnetized plasma

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left\{R[g] + 12 - \frac{1}{4}(F^V)^2 - \frac{1}{4}(F^a)^2 + \epsilon^{MNPQR}A_M \times\left[\frac{1}{3}\alpha(F^a)_{NP}(F^a)_{QR} + \alpha(F^V)_{NP}(F^V)_{QR} + \lambda R^Y{}_{XNP}R^X{}_{YQR}\right]\right\}$$

Dictionary:

$$g_{\mu\nu} \leftrightarrow T^{\mu\nu}$$
$$V_\mu \leftrightarrow J^\mu$$
$$A_\mu \leftrightarrow J_5^\mu$$

Dual field content:
N^2 gauge boson, neutral under $U(1)_V$
$4N^2$ Weyl fermion, charged under $U(1)_V$
$3N^2$ complex scalars, charged under $U(1)_V$

$$J^\mu \sim \sum_f q_f \bar{\psi}_f \gamma^\mu \psi_f + \cdots$$

Anomalies in the holographic model

$$0 = EV^M \equiv \nabla_N \left(F^V\right)^{NM} + 2\alpha\epsilon^{MNPQR} \left(F^a\right)_{NP} \left(F^V\right)_{QR},$$

$$0 = EA^M \equiv \nabla_N \left(F^a\right)^{NM} + \alpha\epsilon^{MNPQR} \left[\left(F^V\right)_{NP} \left(F^V\right)_{QR} + \left(F^a\right)_{NP} \left(F^a\right)_{QR}\right]$$
$$+ \lambda\epsilon^{MNPQR} R^Y{}_{XNP} R^X{}_{YQR}.$$

$M = r$ →

$$\partial_\mu J^\mu = 8\alpha(E_5 \cdot B + E \cdot B_5)$$

$$\partial_\mu J_5^\mu = 8\alpha(E \cdot B + E_5 \cdot B_5) + \lambda R \wedge R$$

Chiral + gravitational anomalies

treat $\alpha \& \lambda$ as free parameters

Neutral magnetic brane background

$$ds^2 = 2drdt - f(r)dt^2 + e^{2W_T(r)}(dx^2 + dy^2) + e^{2W_L(r)}dz^2$$

$$V = Bxdy \Rightarrow \vec{B} = B\hat{z}$$

$$f(r \simeq r_h) = 0 + f'(r_h)(r - r_h) + \cdots \qquad T = \left.\frac{\partial_r(f(r))}{4\pi}\right|_{r=r_h}$$

Anisotropic: $T^{xx} = T^{yy} \neq T^{zz}$

strongly coupled neutral plasma subject to external B

Analytic background known for weak B
Numerical background available for arbitrary B

Introducing vorticity

on the boundary $$ds^2_{\mathrm{M}} = -dt^2 + d\vec{x}^2 + 2h_{ti}(t,\vec{x})dtdx^i$$

vorticity $$\Omega^i = \frac{1}{2}\epsilon^{ijk}\nabla_j u_k = \frac{1}{2}\epsilon^{ijk}\partial_j h_{tk}$$

$$\xi = \frac{2}{B}\lim_{q\to 0}\frac{\langle J^t T^{ty}\rangle}{iq}, \qquad \sigma = 2\lim_{q\to 0}\frac{\langle J_5^z T^{ty}\rangle}{iq}.$$

Introducing finite ω necessary to uniquely define the “neutral” state

in the bulk $$\delta(ds^2) = 2e^{W_T(r)}\left[\delta g_{ty}(r,t,x)dtdy + \delta g_{xy}(r,t,x)dxdy\right],$$

$$\delta V = \delta V_t(r,t,x)dt + \delta V_x(r,t,x)dx, \qquad \delta A = \delta A_z(r,t,x)dz,$$

adiabatic limit: $\omega \to 0$ then $q \to 0$

decoupled dynamics for steady state: $T^{ty}(x), J^t(x), J_5^z(x)$

Analytic results for weak B

$$J^t = \xi(B,T)(\vec{B}\cdot\vec{\Omega})$$

$$\xi = \frac{128}{3}(12\log 2 - 5)\alpha\lambda - 2\log r_h - 1 + \mathcal{O}(B/T^2)$$

anomalous non-anomalous

$$\vec{J}_5 = \sigma(B,T)\vec{\Omega}$$

$$r_h = 4\pi T$$

$$\sigma = r_h^2\left[64\lambda - \frac{32\lambda B}{3r_h^2} + \mathcal{O}(B^2/T^4)\right]$$

anomalous

Non-anomalous contribution agrees with MHD

$$\xi = \frac{128}{3}(12\log 2 - 5)\alpha\lambda - 2\log r_h - 1 + \mathcal{O}(B/T^2)$$

$$J^t = 2\left(M_{\Omega,\mu}\vec{B}\cdot\vec{\Omega} - 2p_{,B^2}\vec{B}\cdot\vec{\Omega}\right)$$

Kovtun, Hernandez, JHEP 2017

$$2p_{,B^2} \equiv 2\frac{\partial p}{\partial(B^2)}$$

magnetic susceptibility

$$2p_{,B^2} = \log r_h \to \log\frac{4\pi T}{M}$$

M renormalization scale

$$M_\Omega \equiv \frac{\partial p}{\partial(B\cdot\Omega)}$$

magneto-vortical susceptibility

$$M_\Omega = -\frac{1}{2}\mu$$

Scheme dependence

$$\Delta S_{\mathrm{c.t.}} = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-\gamma}\left(\frac{a}{4}\left(F^V\right)_{\mu\nu}\left(F^V\right)^{\mu\nu}\right)$$

$$\Delta J^\mu = -\frac{a}{2\kappa^2}\sqrt{-\gamma}\nabla_\nu F^{\mu\nu}$$

In the presence of $h_{ty}(x)$ $\qquad \Delta J^t \sim a\vec{B}\cdot\vec{\Omega}$

shift magnetic susceptibility $-2p_{,B^2}$

Anomalous contributions

$$\sigma = r_h^2 \left[64\lambda - \frac{32\lambda B}{3r_h^2} + \mathcal{O}(B^2/T^4) \right]$$

modified CVE, no CSE!

$J^t \neq 0, \qquad \mu = 0$ costs no energy to create vector charge

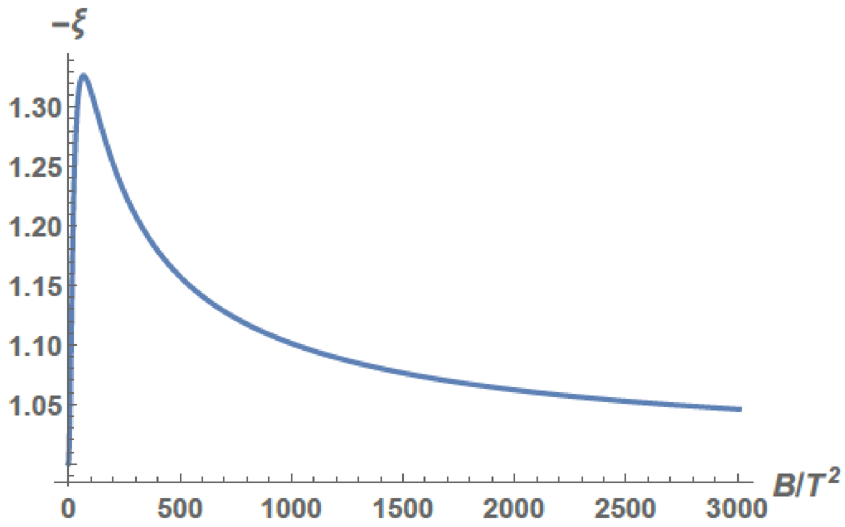
$$\mu = \delta V_t(r=\infty) - \delta V_t(r=r_h)$$

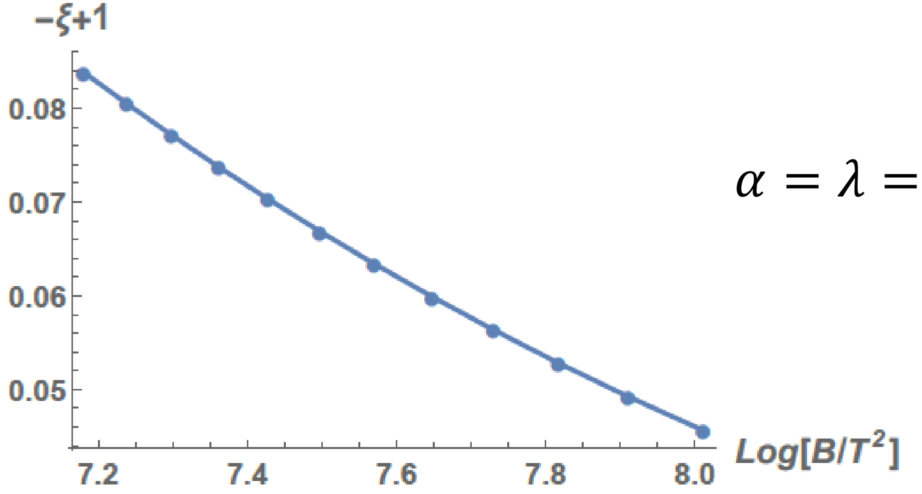
$$\xi = \frac{128}{3}(12\log 2 - 5)\alpha\lambda - 2\log r_h - 1 + \mathcal{O}(B/T^2)$$

$$\Omega \rightarrow J_5^z \sim O(\lambda) \rightarrow J^t \sim O(\lambda\alpha)$$

higher order O($\alpha^2 B^2$) possible

Numerical results for arbitrary B (non-anomalous)





$$\alpha = \lambda = 0$$

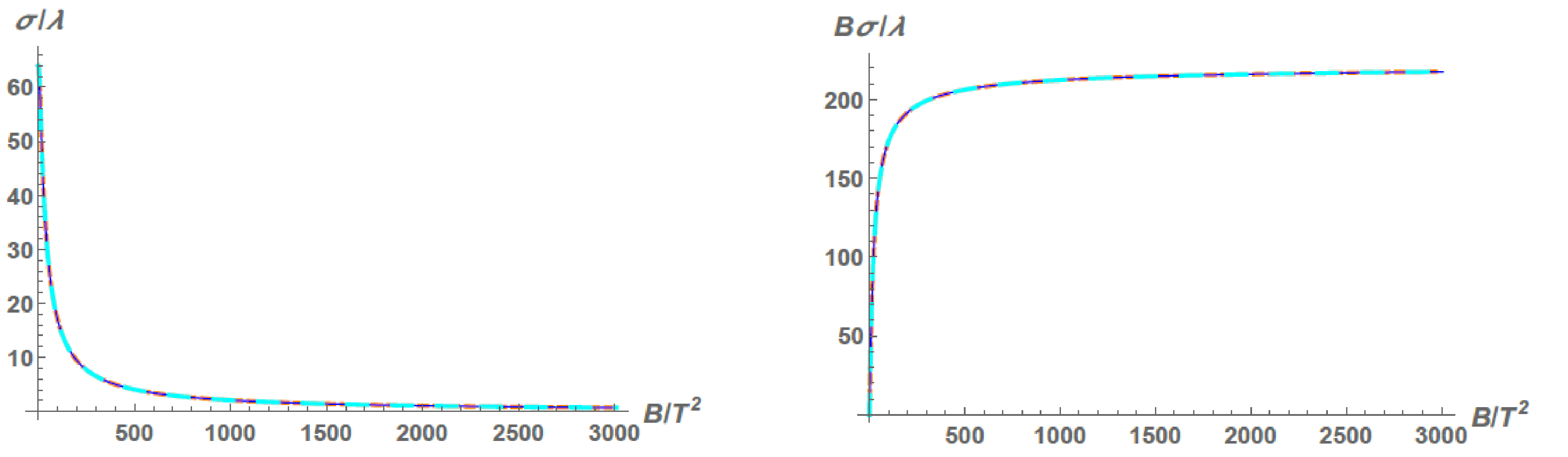
$$\xi_{non-anom} \simeq -1 + \#\log\frac{B}{T^2}$$

vacuum limit $\varepsilon(B) = \#B^2 + \#B^2\log\frac{B}{M}$

Fuini, Yaffe, JHEP 2015

Numerical results for arbitrary B (anomalous σ)

$$(\alpha, \lambda) = (0, 1/50),\ (1/20, 1/50),\ (1/20, 1/20).$$

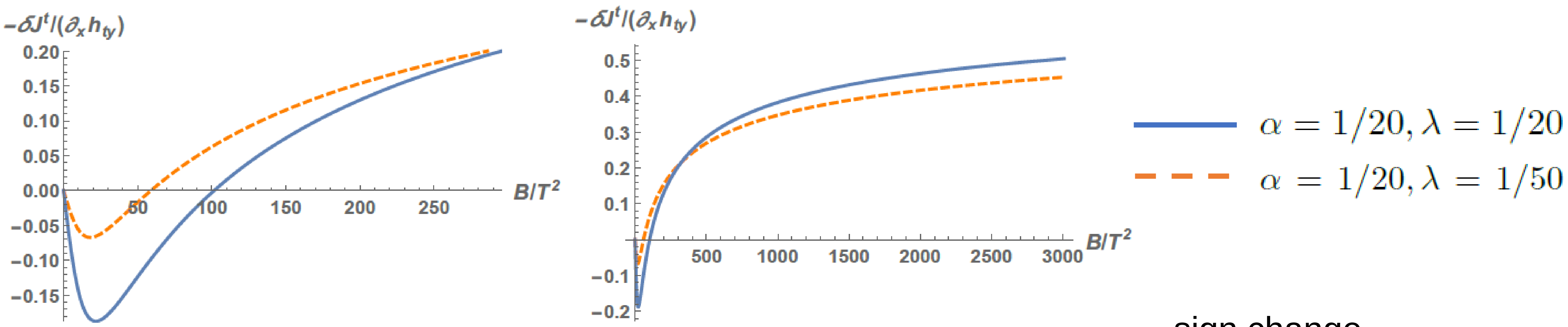


sensitive to gravitational anomaly only $\sigma \sim B^{-1}$

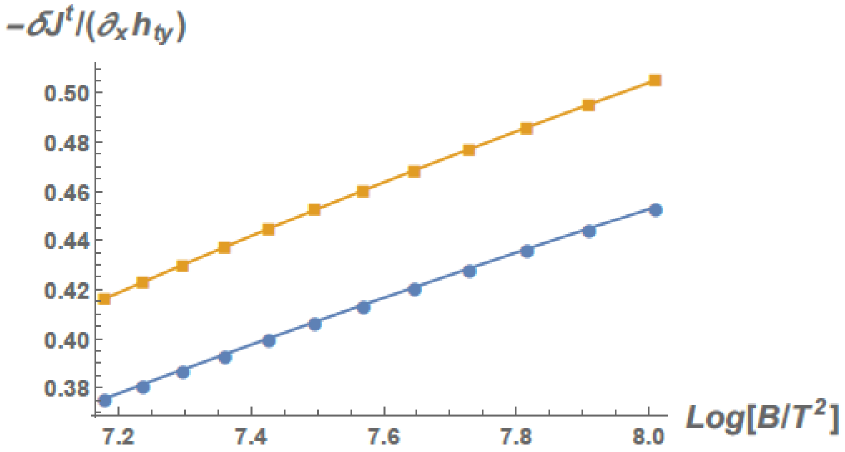
Chiral vortical conductivity suppression by B, in contrast to free theory $\sigma \sim B$

Hattori, Yin, PRL 2017

Numerical results for arbitrary B (anomalous ξ)



sign change



$J^t_{anom} \sim \log B$

$\xi_{anom} \sim \dfrac{\log B}{B}$

$\xi_{non-anom} \sim \log B$ dominates

Thermal Hall effect and thermal axial magnetic effect

$$J^t = \xi(B,T)(\vec{B}\cdot\vec{\Omega}),$$

$$J_5^z = \sigma(B,T)\Omega.$$

$$\langle J^t(q)T^{ty}(-q)\rangle = \frac{iq\xi B}{2},$$

$$\langle J_5^z(q)T^{ty}(-q)\rangle = \frac{iq\sigma}{2},$$

$$\langle T^{ty}(-q)J^t(q)\rangle_{-B} = -\frac{1}{2}iq\xi(B)B,$$

$$\langle T^{ty}(-q)J_5^z(q)\rangle_{-B} = -\frac{1}{2}iq\sigma(B).$$

$$\langle O_a(\omega,q)O_b(-\omega,-q)\rangle_B = \gamma_a\gamma_b\langle O_b(\omega,-q)O_a(-\omega,q)\rangle_{-B}$$

$$\langle T^{ty}(-q)\rangle_{-B} = \frac{1}{2}\xi(B)BE_x(-q),$$ Thermal Hall effect

$$\langle T^{ty}(-q)\rangle_{-B} = \frac{1}{2}\sigma(B)B_{5y}(-q).$$ Thermal axial magnetic effect

Anomalous modification to MHD

$$J^t = \xi(B,T)(\vec{B}\cdot\vec{\Omega}),$$

$$J_5^z = \sigma(B,T)\Omega.$$

$$T^{ty} = \frac{1}{2}\xi(-B,T)E_x B,$$

$$T^{ty} = \frac{1}{2}\sigma(-B,T)B_{5y}.$$

$$\xi = \xi_{non-anom} + \xi_{anom} = 2(2p_{,B^2} - M_{\Omega,\mu}) + \xi_{anom}$$

$$T^{ty} = -E_x B\left(2p_{,B^2} - M_{\Omega,\mu} + \frac{\xi_{anom}}{2}\right)$$

$$T^{ty} = -B_{5y}\frac{\sigma}{2}$$

Anomalous contributions to constitutive equations

Conclusion

- Magneto-vortical effect in strongly coupled plasma
- Vector charge generation: non-anomalous and anomalous parts. Non-anomalous part dominates $\sim \log B$ at large B
- Axial current generation (CVE) suppressed $\sim B^{-1}$ at large B
- Thermal Hall effect and thermal axial magnetic effect

Outlook

- Perpendicular B & Ω?
- Derive non-dissipative transport from generating functional
- Dissipative transport