Does the chiral magnetic effect change the dynamic universality class in QCD?

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JHEP **1811** 108 (2018)

Goals of heavy-ion collisions

BROOKHAVEN NATIONAL LABORATORY

Beam Energy Scan Theory (BEST) Collaboration

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Our Mission

The Beam Energy Scan Theory (BEST) Collaboration is a Topical Collaboration in Nuclear Theory, funded by the US Department of Energy, Office of Science, Office of Nuclear Physics for the period 2016-2020.

The BEST Collaboration, involving collaborators from two national laboratories and 11 universities, will construct and provide a theoretical framework for interpreting the results from the ongoing Beam Energy Scan program at the Relativistic Heavy Ion Collider (RHIC). The main goals of this program are to discover, or put constraints on the existence, of a critical point in the QCD phase diagram, and to locate the onset of chiral symmetry restoration by observing correlations related to anomalous hydrodynamic effects in quark gluon plasma.

For this purpose, the BEST Collaboration is developing a set of theoretical tools, including hot-dense lattice QCD, initial state models, state-of-the-art hydrodynamic codes incorporating dissipation, hydrodynamic and critical fluctuations, and the effects of the chiral anomaly, as well as hadronic models of the final state of a heavy ion collision. These tools will be used to analyze RHIC Beam Energy Scan data.

BEST COLLABORATION

Upcoming Events

- BEST Collaboration Annual Meeting, August 5-6, 2017, Stony Brook University, Stony Brook, New York, USA
- Critical Point and Onset of Deconfinement (CPOD-2017), August 7-11, 2017, Stony Brook University, Stony Brook, New York, USA
- Phases of QCD and Beam Energy Scan

Motivation

- QCD critical phenomena
  - Universality class
  - Critical behavior
- Chiral transport phenomena
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Vortical Effect (CVE)

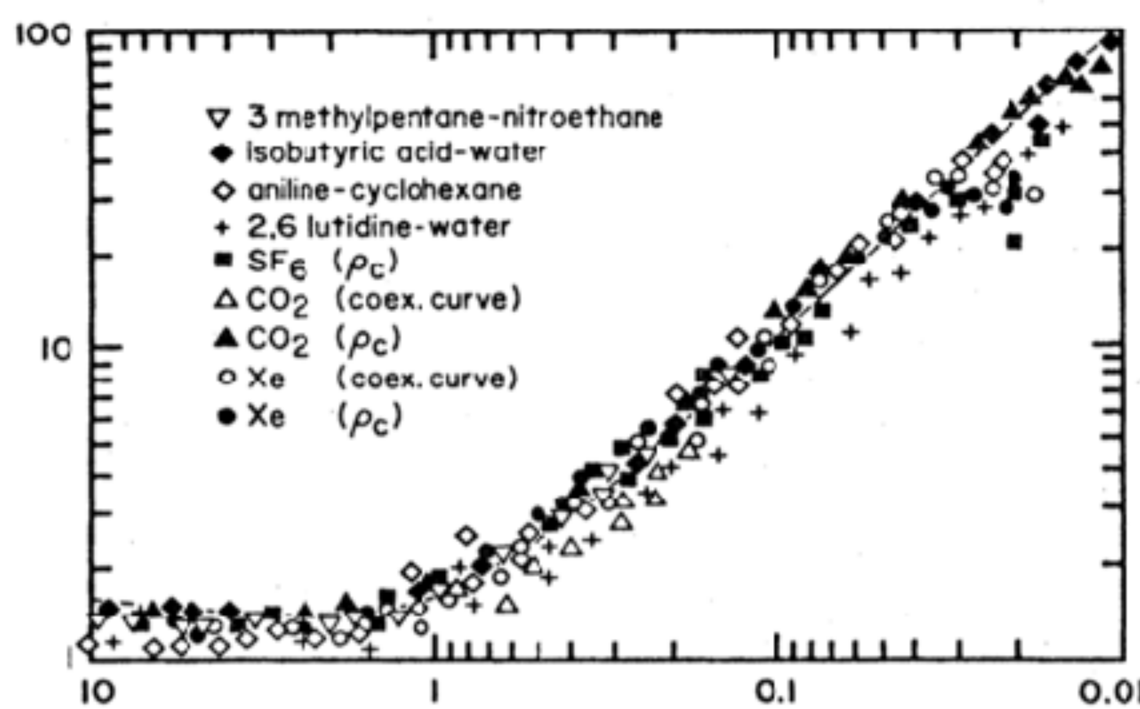
How about the interplay?

Does the CME change the dynamic critical phenomena in QCD?

Critical phenomena

- Static critical phenomena, e.g., specific heat
- Dynamic critical phenomena, e.g., kinetic coefficient, speed of sound

Diffusion coefficient



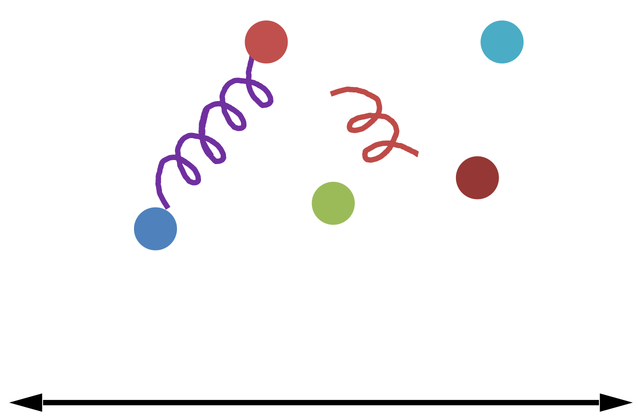
$q\xi$

Swinney and Henry (1973)

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Microscopic theory

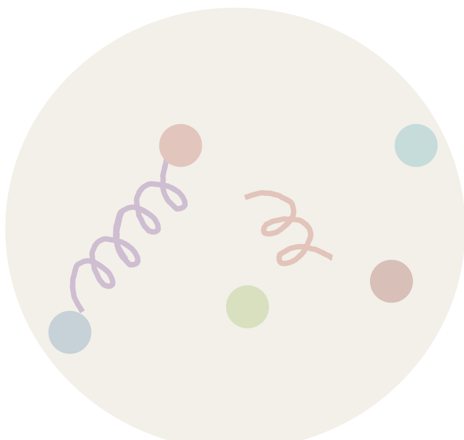


$$\xi \gg \Lambda^{-1}$$

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Microscopic theory



$\xi \gg \Lambda^{-1}$

Integrating out

Effective theory

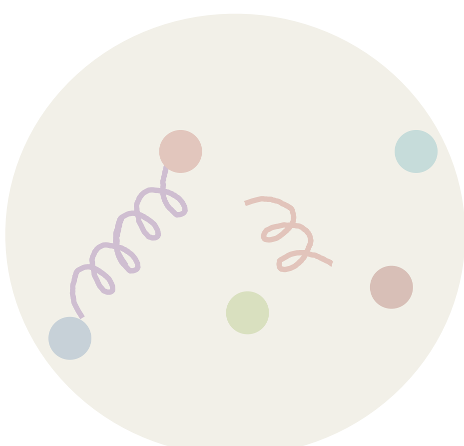
Hydrodynamic variables:

- Order parameters
- Conserved densities
- Nambu-Goldstone modes

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Microscopic theory



$\xi \gg \Lambda^{-1}$

Integrating out

Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved densities
- Nambu-Goldstone modes

Same symmetries

Dynamic universality class

P. C. Hohenberg and B. I. Halperin (1977)

Class	Symmetry	Order Parameter	Conserved Charge	System
A	$\mathbb{Z}(2)$	Nonconserved		Ising model
B	$\mathbb{Z}(2)$	Conserved		Uniaxial Ferromagnets
C	$\mathbb{Z}(2)$	Nonconserved	Energy	Uniaxial Antiferromagnets
E	$\mathrm{U}(1) \times \mathcal{C}$	Nonconserved	$\mathrm{U}(1)$	Easy-plane magnets
F	$\mathrm{U}(1)$	Nonconserved	$\mathrm{U}(1)$	Superfluid ^{4}He
G	$\mathrm{O}(N)$	Nonconserved	$\mathrm{O}(N)$	Isotropic Antiferromagnets
H	$\mathbb{Z}(2)$	Conserved	Momentum	Liquid gas
J	$\mathrm{O}(N)$	Conserved		Isotropic Ferromagnets

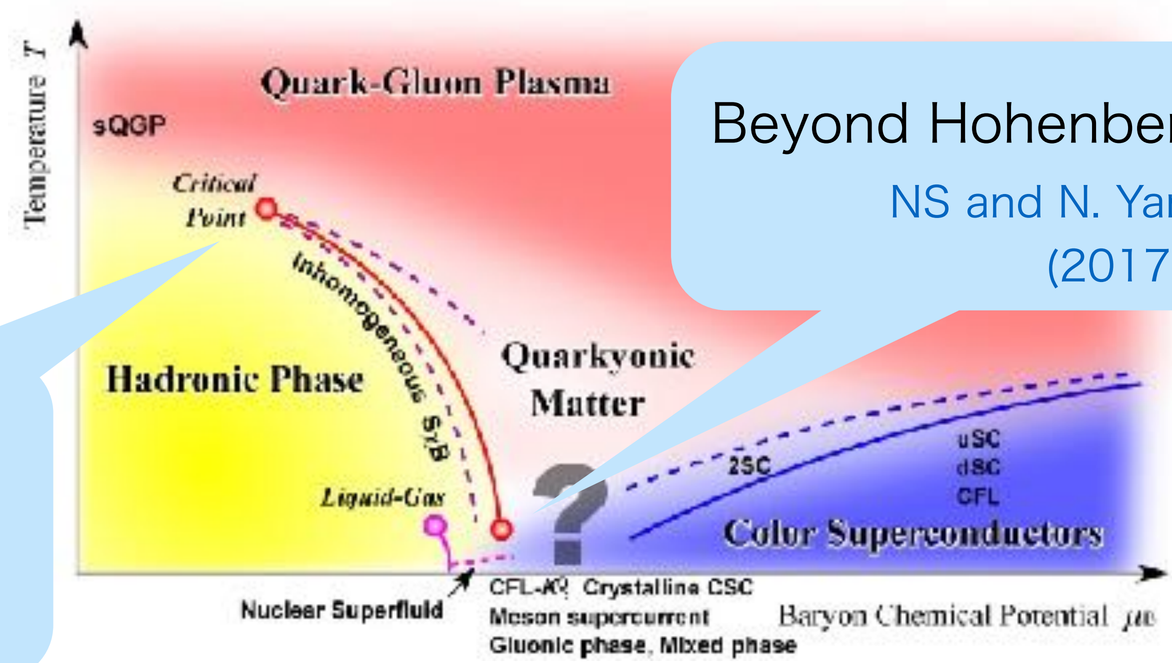
Classification based on symmetries and low-energy gapless modes

Dynamic universality class in QCD

- Second-order chiral phase transition: O(4) antiferromagnet

K. Rajagopal and F. Wilczek (1992)

- QCD critical points:



Beyond Hohenberg & Halperin

NS and N. Yamamoto (2017)

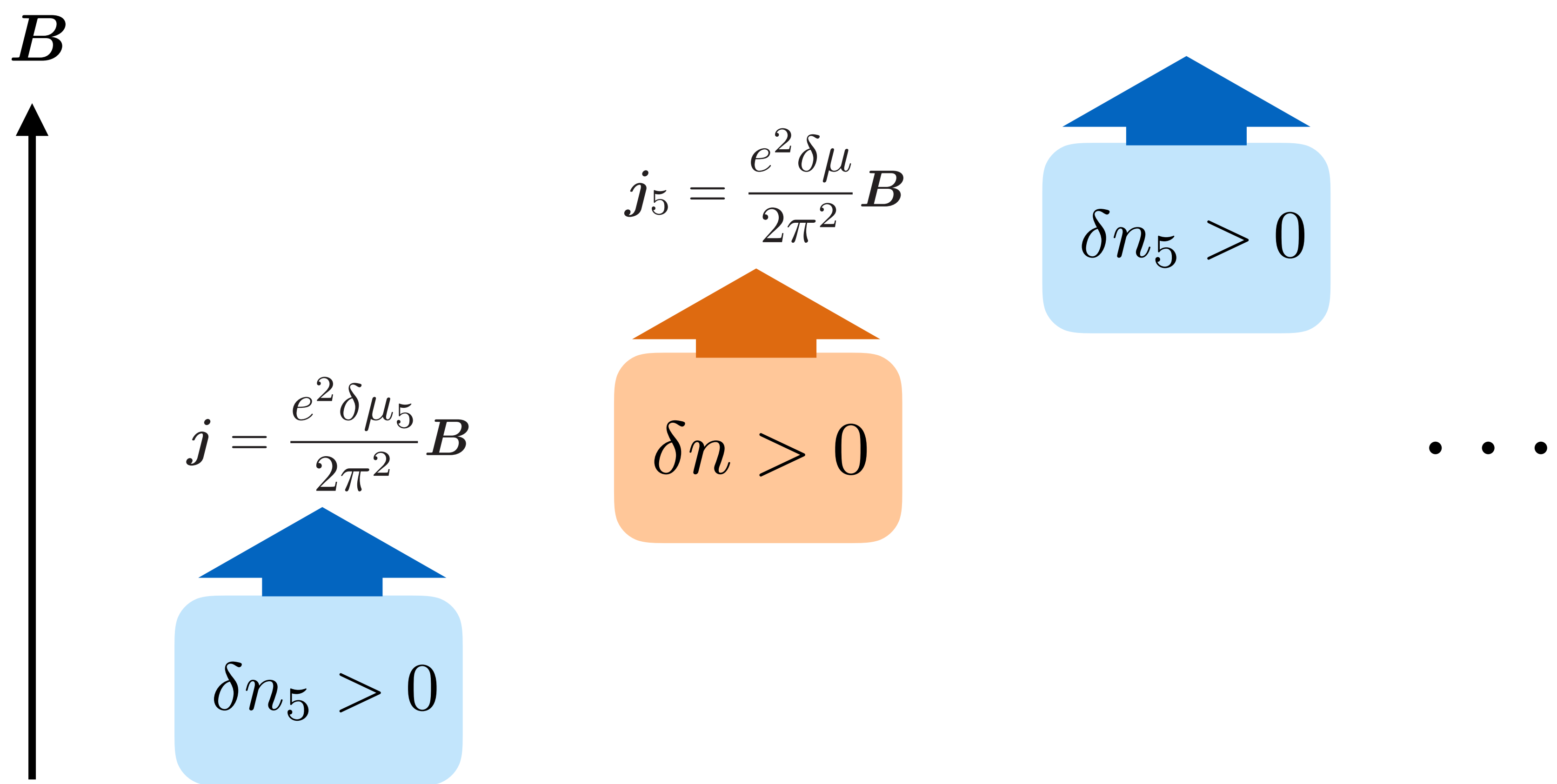
Liquid gas (model H)

H. Fujii (2003), D. T. Son and M. Stephanov (2004)

Phase diagram from K. Fukushima and T. Hatsuda (2010)

Chiral magnetic wave (CMW)

G. M. Newman (2006), D. E. Kharzeev and H. Yee (2011)



$$\frac{\partial^2 \delta n}{\partial t^2} = \frac{e^4 B^2}{4\pi^4 \chi \chi_5} \boldsymbol{\nabla}^2 \delta n$$

Susceptibilities:

$$\chi \equiv \frac{\delta n}{\delta \mu}$$

$$\chi_5 \equiv \frac{\delta n_5}{\delta \mu_5}$$

“Does the CME (or the CMW) change the dynamic universality class in QCD?”

Answer

Second-order chiral phase transition under $\boldsymbol{B}_{\mathrm{ex}}$

absence of CME	presence of CME
model E	model A

M. Hongo, NS, and N. Yamamoto (2018)

Outline

1. Setup
2. Static critical phenomena
   - Ginzburg-Landau theory
3. Dynamic critical phenomena
   - Langevin theory

Setup

- 2 flavor QCD with massless u, d quarks at finite T, $\mu_{\rm I}$, $\boldsymbol{B}$
- Chiral Symmetry

$$\begin{array}{ccc} \boldsymbol{B}=\boldsymbol{0} & & \boldsymbol{B}\neq\boldsymbol{0} \\ \mathrm{SU}(2)_{\rm L}\times\mathrm{SU}(2)_{\rm R} & \xrightarrow{\text{explicitly}} & \mathrm{U}(1)_{\rm L}^{\tau^3}\times\mathrm{U}(1)_{\rm R}^{\tau^3} \\ q_{\rm L,R}\to e^{i\theta^a_{\rm L,R}\tau^a}q_{\rm L,R} & & q_{\rm L,R}\to e^{i\theta^3_{\rm L,R}\tau^3}q_{\rm L,R} \end{array}$$

- Second-order chiral phase transition

Hydrodynamic variables

- Order parameter

$$\phi_\alpha \equiv \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$

chiral condensate: $\sigma \equiv \bar{q}q$

neutral pion: $\pi \equiv \bar{q}i\gamma_5\tau^3 q$

- Conserved densities

$$n_{\mathrm{I}} \equiv \bar{q}\gamma^0\tau^3 q \qquad n_{\mathrm{I5}} \equiv \bar{q}\gamma^0\gamma^5\tau^3 q$$

Ginzburg-Landau theory

$$F = \int \mathrm{d}\boldsymbol{r} \left[\frac{r}{2}(\phi_\alpha)^2 + \frac{1}{2}(\boldsymbol{\nabla}\phi_\alpha)^2 + u(\phi_\alpha)^2(\phi_\beta)^2 + \frac{1}{2\chi} n_{\mathrm{I}}^2 + \frac{1}{2\chi_5} n_{\mathrm{I5}}^2 + \gamma n_{\mathrm{I}} \phi_\alpha^2 \right]$$

- Near second-order phase transition $\longrightarrow$ small order parameters
- Long-range behavior $\longrightarrow$ derivative expansion
- QCD symmetries $\longrightarrow$ constraints on the expansion

chiral symmetry and CPT symmetries

Renormalization group analysis

$$F=\int \mathrm{d}\boldsymbol{r}\left[\frac{r}{2}(\phi_\alpha)^2+\frac{1}{2}(\boldsymbol{\nabla}\phi_\alpha)^2+u(\phi_\alpha)^2(\phi_\beta)^2+\frac{1}{2\chi}n_{\mathrm{I}}^2+\frac{1}{2\chi_5}n_{\mathrm{I5}}^2+\gamma n_{\mathrm{I}}\phi_\alpha^2\right]$$

- Correlation functions $\longrightarrow$ Renormalization group equation

$$\langle\mathcal{O}[\phi,n_{\mathrm{I}},n_{\mathrm{I5}}]\rangle=\frac{\int \mathcal{D}\phi\mathcal{D}n_{\mathrm{I}}\mathcal{D}n_{\mathrm{I5}}\,O[\phi,n_{\mathrm{I}},n_{\mathrm{I5}}]e^{-F}}{\int \mathcal{D}\phi\mathcal{D}n_{\mathrm{I}}\mathcal{D}n_{\mathrm{I5}}\,e^{-F}}$$

- Renormalization invariance $\longrightarrow$ Fixed-point values

Renormalization group analysis

$$F=\int \mathrm{d}\boldsymbol{r}\left[\frac{r}{2}(\phi_\alpha)^2+\frac{1}{2}(\boldsymbol{\nabla}\phi_\alpha)^2+u(\phi_\alpha)^2(\phi_\beta)^2+\frac{1}{2\chi}n_{\mathrm{I}}^2+\frac{1}{2\chi_5}n_{\mathrm{I5}}^2+\gamma n_{\mathrm{I}}\phi_\alpha^2\right]$$

- γ=0 $\longrightarrow$ Wilson-Fisher fixed point (2 components)
- γ≠0 $\longrightarrow$ Inclusion of n_{I}

$$\bar{r}_\infty=-\frac{\epsilon}{5},\quad \bar{u}_\infty=\frac{\epsilon}{40},\quad v_\infty=\frac{\epsilon}{20},\quad \left(v\equiv\frac{\gamma^2\chi\Lambda^{-\epsilon}}{8\pi^2}\right)$$

- Static critical phenomenon:

$$\chi_{\mathrm{phys}}\sim\xi^{\frac{\alpha}{\nu}},\qquad \left(\frac{\alpha}{\nu}=\frac{\epsilon}{5}\right)$$

finite μ_{I} effect

nonlinear Langevin equation

$$\frac{\partial \phi_\alpha(\boldsymbol{r},t)}{\partial t} = -\Gamma \frac{\delta F}{\delta \phi_\alpha(\boldsymbol{r},t)} - g \int \mathrm{d}\boldsymbol{r}' \left[\phi_\alpha(\boldsymbol{r},t), n_{\mathrm{I5}}(\boldsymbol{r}',t)\right] \frac{\delta F}{\delta n_{\mathrm{I5}}(\boldsymbol{r}',t)} + \xi_\alpha(\boldsymbol{r},t)$$

$$[n_{\mathrm{I5}}(\boldsymbol{r},t), \phi_\alpha(\boldsymbol{r}',t)] = \varepsilon_{\alpha\beta}\phi_\beta \delta(\boldsymbol{r}-\boldsymbol{r}')$$

$$\langle \xi_\alpha(\boldsymbol{r},t)\xi_\beta(\boldsymbol{r}',t')\rangle = 2\Gamma\delta_{\alpha\beta}\delta^d(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')$$

Fluctuation-dissipation relation

nonlinear Langevin equation

$$\frac{\partial n_{\mathrm{I}}(\boldsymbol{r},t)}{\partial t} = \lambda \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n_{\mathrm{I}}(\boldsymbol{r},t)} - \int \mathrm{d}\boldsymbol{r}' \left[n_{\mathrm{I}}(\boldsymbol{r},t), n_{\mathrm{I5}}(\boldsymbol{r}',t) \right] \frac{\delta F}{\delta n_{\mathrm{I5}}(\boldsymbol{r}',t)} + \zeta(\boldsymbol{r},t)$$

$$\left[n_{\mathrm{I}}(\boldsymbol{r},t), n_{\mathrm{I5}}(\boldsymbol{r}',t) \right] = C\boldsymbol{B} \cdot \boldsymbol{\nabla}\delta(\boldsymbol{r}-\boldsymbol{r}')$$

anomaly coefficient

Anomalous commutation relation

R. Jackiw and K. Johnson (1969)

S. L. Adler and D. G. Boulware (1969)

Dynamic Renormalization group

1. Field theory equivalent to the Langevin theory

   P. C. Martin, E. D. Siggia, and A. Rose (1973), H-K. Janssen (1976), C. De Dominicis (1978)

2. Ordinary field theoretical methods
   (Feynman rules, Perturbation theory, Dynamic RG equations)

Dynamic Renormalization group

- Merits: nonlinear effects, noise fluctuations
- Difference from static RG:

  - Dynamic critical exponent z :

$$\boldsymbol{r} \to b^{-1}\boldsymbol{r}, \quad \omega' = b^z \omega$$

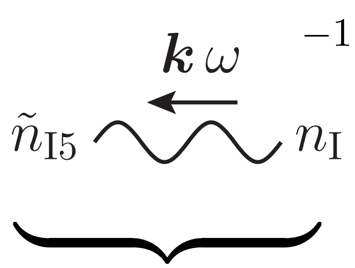
Scaling factor

  - Double degrees of freedom

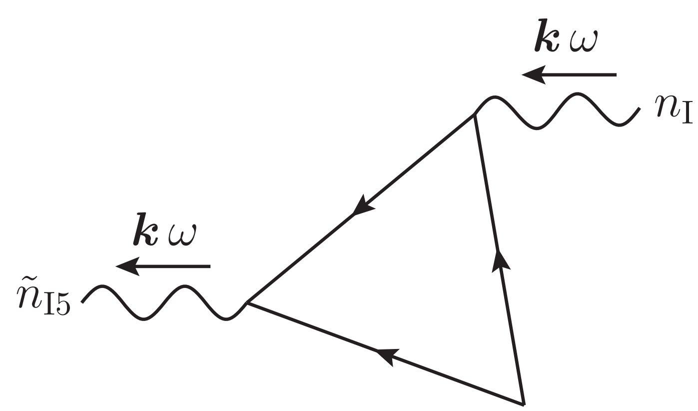
$$\psi \equiv \phi_\alpha, n_{\mathrm{I}}, n_{\mathrm{I5}}, \quad \tilde{\psi} \equiv \tilde{\phi}_\alpha, \tilde{n}_{\mathrm{I}}, \tilde{n}_{\mathrm{I5}}$$

Renormalization of C?

- Nontrivial near the second-order chiral phase transition



$\frac{C}{\chi_{\mathrm{I}}}\boldsymbol{B}\cdot\boldsymbol{k}$



one loop correction
(log divergence)

cancelled by the log divergence in $\chi_{\mathrm{I}} \sim \xi^{\frac{\alpha}{\nu}}$

No loop-corrections on the CME coefficient

New dynamic critical behavior

- Speed of the chiral magnetic wave

From statics

$$c_{\rm s}^2 \equiv \frac{C^2 B^2}{\chi \chi_5} \sim \xi^{-\frac{\alpha}{\nu}},$$

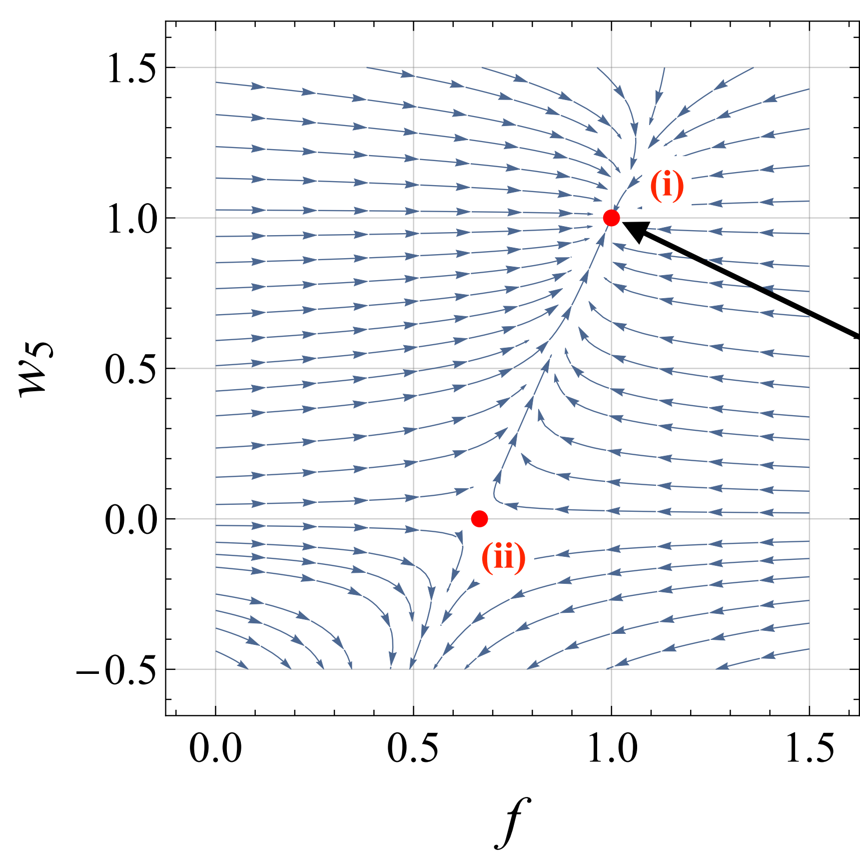
$$\chi \sim \xi^{\frac{\alpha}{\nu}}, \quad \frac{\alpha}{\nu} = \frac{\epsilon}{5}$$

Critical attenuation

Flow diagrams

RG variables： $f \equiv \dfrac{g^2\Lambda^{-\epsilon}}{8\pi^2\lambda_5\Gamma}, \quad w \equiv \dfrac{\Gamma\chi}{\lambda}, \quad w_5 \equiv \dfrac{\Gamma\chi_5}{\lambda_5}, \quad h \equiv \dfrac{CB}{\sqrt{\lambda\lambda_5}\Lambda}$

$(w, h) = (0, 0)$



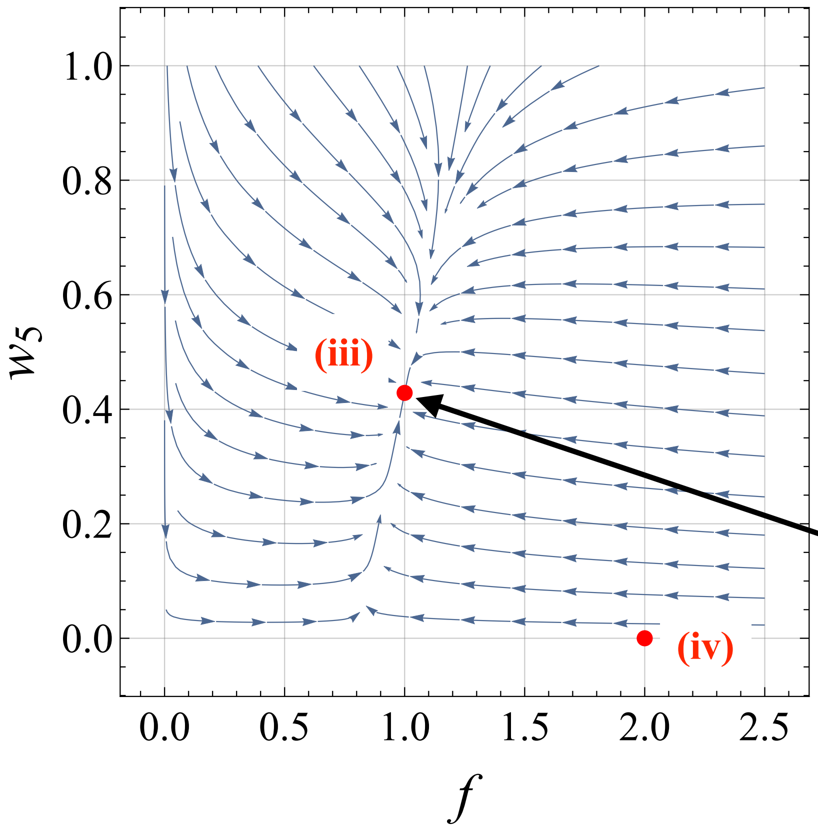
model E

$z = \dfrac{d}{2}$

Flow diagrams

RG variables : $f \equiv \dfrac{g^2\Lambda^{-\epsilon}}{8\pi^2\lambda_5\Gamma}, \quad w \equiv \dfrac{\Gamma\chi}{\lambda}, \quad w_5 \equiv \dfrac{\Gamma\chi_5}{\lambda_5}, \quad h \equiv \dfrac{CB}{\sqrt{\lambda\lambda_5}\Lambda}$

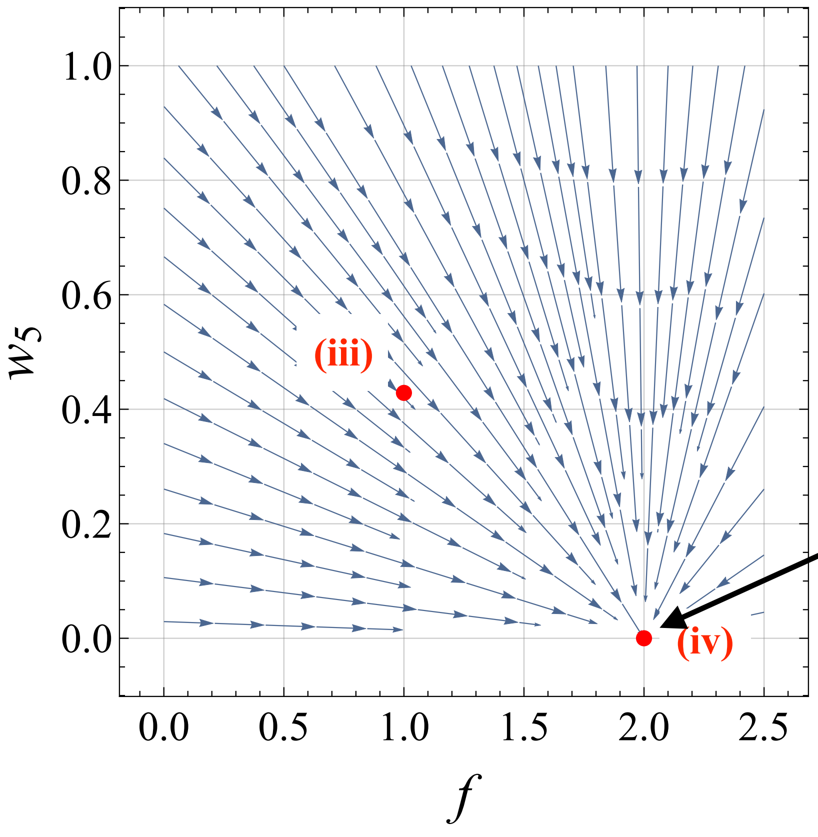
$(w, h) = (10^4, 10^0)$



model E

$z = \dfrac{d}{2}$

$(10^4, 10^6)$



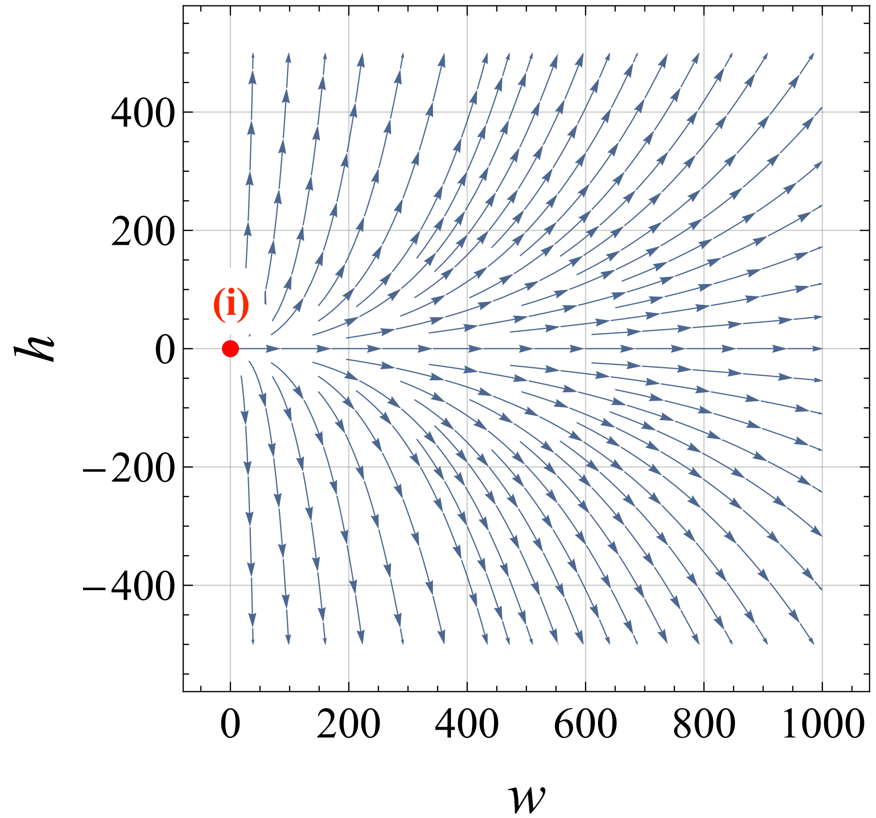
model A

$z = 2$

Flow diagrams

RG variables : $f \equiv \dfrac{g^2\Lambda^{-\epsilon}}{8\pi^2\lambda_5\Gamma}, \quad w \equiv \dfrac{\Gamma\chi}{\lambda}, \quad w_5 \equiv \dfrac{\Gamma\chi_5}{\lambda_5}, \quad h \equiv \dfrac{CB}{\sqrt{\lambda\lambda_5}\Lambda}$

$(f, w_5) = (1, 1)$



$h \gg w$ for whole region $\longrightarrow$ fixed point (iv)

model A

Dynamic universality class

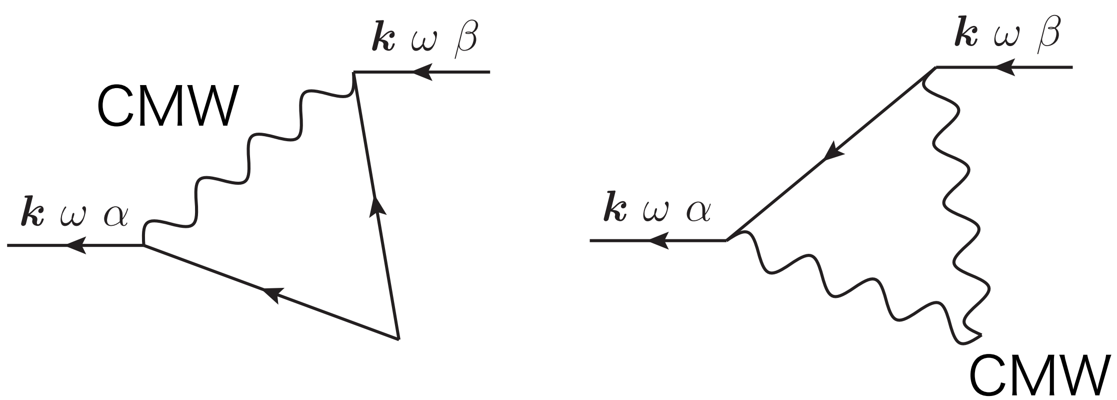
$C=0$	$C\neq 0$
model E fixed point (i)	model A fixed point (iv)

M. Hongo, NS, N. Yamamoto (2018)

CME changes the dynamic universality class.

Chiral magnetic wave

- Critical fluctuation via CMW



$$\propto \frac{1}{1+h}$$

- Suppressed at fixed point (iv)

  = Decoupling of conserved densities $\longrightarrow$ model A

Conclusion

- Critical phenomena of the second-order chiral phase transition under external magnetic field

- Interplay between the CME and the dynamic critical phenomena

C=0	C≠0
model E fixed point (i)	model A fixed point (iv)

M. Hongo, NS, N. Yamamoto (2018)

CME changes the dynamic universality class.

Outlook

- Dynamical electromagnetic field

  - $T=0, \boldsymbol{B}\neq 0, m_{\rm q}=0$ NS and N Yamamoto (2019)

    quantum anomaly $\longrightarrow$ nonrelativistic photon

  - chiral phase transition

    New dynamic university class beyond Hohenberg & Halperin

    NS and N Yamamoto (to appear)

- CVE, QCD critical point under rotation
- Nonequilibrium universality classes (KPZ, DP)