Chiral Kinetic Theory from Landau level basis

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Dec 18, 2019, Workshop on Quantum kinetic theories in magnetic and vortical fields
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arXiv:1909.11514
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Outline

- Axial anomaly and manifestations
- CKT as ħ expansion and its difficulty at higher order
- CKT from Landau level basis
- Simple example: longitudinal conductivity
- Less obvious example: transverse conductivity of QGP
- Summary and Outlook

Axial anomaly

Chiral fermion

$\psi \rightarrow \psi e^{i\alpha}$

$\psi \rightarrow \psi e^{i\gamma^5\alpha}$

$J_R^\mu = \bar{\psi}\gamma^\mu \frac{1+\gamma^5}{2}\psi = \bar{\psi}_R\gamma^\mu \psi_R$

$J_L^\mu = \bar{\psi}\gamma^\mu \frac{1-\gamma^5}{2}\psi = \bar{\psi}_L\gamma^\mu \psi_L$

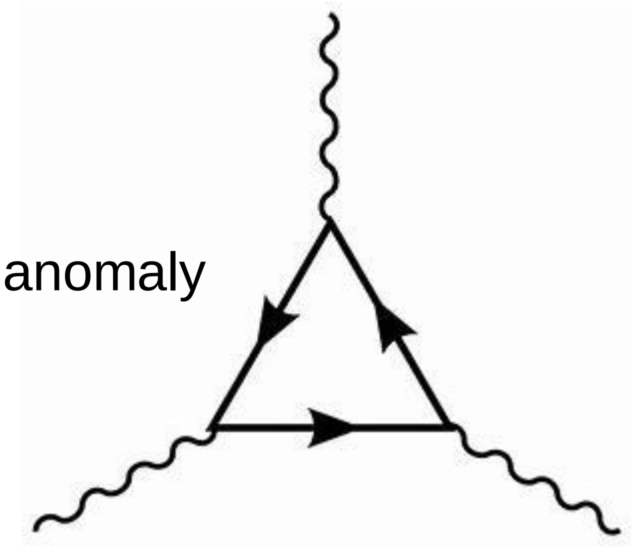
$J = J_R^\mu + J_L^\mu = \bar{\psi}\gamma^\mu\psi$

$J_5 = J_R^\mu - J_L^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$

Symmetry at classical level

$\partial_\mu J^\mu = 0$ $\partial_\mu J_5^\mu = 0$

Triangle anomaly

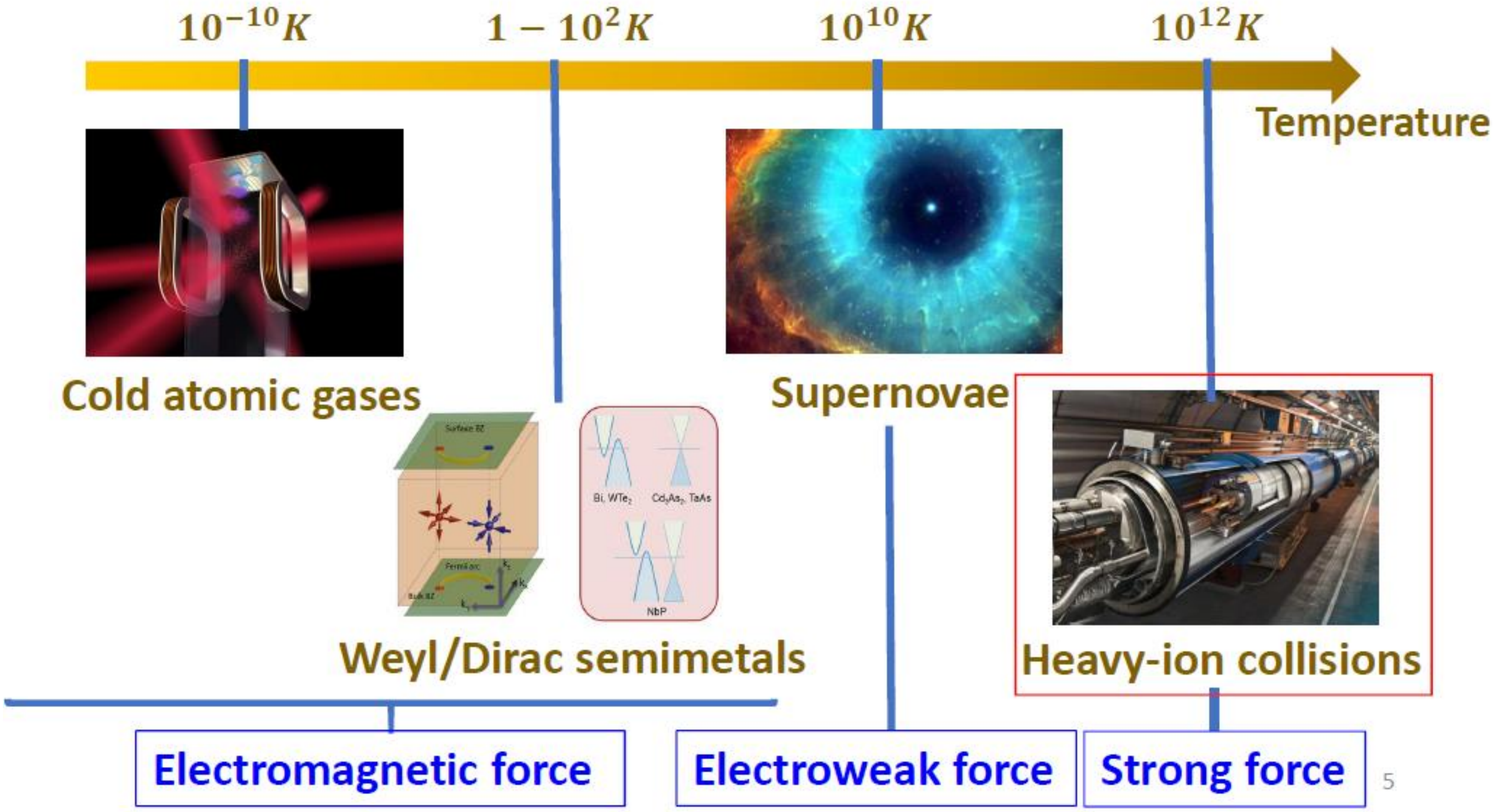


Axial symmetry breaking at quantum level

$\partial_\mu J^\mu = 0$ $\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2}F\tilde{F} - \frac{g^2 N_f}{8\pi^2}trG\tilde{G}$

Anomalous transports

may happen across a very broad hierarchy of scales



Transport phenomena in heavy ion collisions

Anomalous transports, non-dissipative

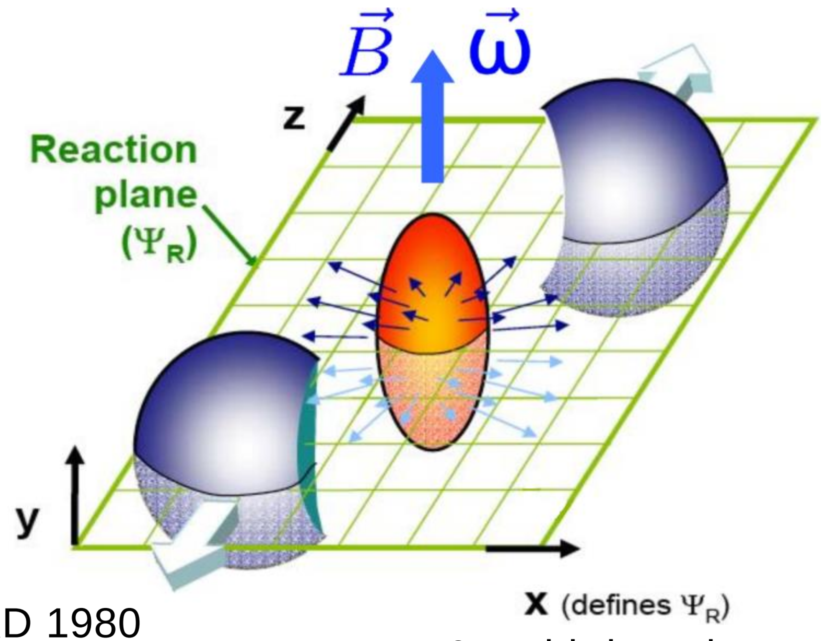
Chiral Magnetic/Separation Effect(CME/CSE)

$$\mathbf{J} = C\mu_5 e\mathbf{B} \qquad \mathbf{J}_5 = C\mu e\mathbf{B}$$

Kharzeev, Zhitnitsky, NPA 2007

Kharzeev, McLerran, Warringa, NPA 2008

Metlitski, Zhitnitsky, PRD 2005



strong & rapid decaying B

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

Chiral Vortical Effect(VCVE/ACVE)

$$\mathbf{J} = C\mu\mu_5\boldsymbol{\omega} \qquad \mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

Vilenken, PRD 1980

Erdmenger et al, JHEP 2009

Banerjee et al, JHEP 2011

Electric/Hall conductivity, dissipative

$$\mathbf{J} = \frac{\tau}{3\pi^2}\left(\mu^2 + \frac{\pi^2 T^2}{3}\right)\left(\mathbf{E} - \tau\frac{\partial \mathbf{E}}{\partial t}\right) + \frac{\tau^2\mu}{3\pi^2}\mathbf{E} \times \mathbf{B}$$

weak E & B

Gorbar, Shovkovy et al. PRD 2016

Theoretical frameworks for transport phenomena

Anomalous hydrodynamics: near equilibrium, can be strongly coupled.

Son, Surowka, PRL 2009

Neiman, Oz, JHEP 2011

Chiral kinetic theory: can be far from equilibrium, weakly coupled

talks by Shovkovy, P.F. Zhuang, X.G. Huang, Kilincarslan, Mameda

Son, Yamamoto, PRD 2012, PRL 2012

Stephanov, Yin, PRL 2012

Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRL 2012, PRD 2014

Manuel, Torres-Rincol, PRD 2014

Hidaka, Pu, Yang, PRD 2016

Huang, Shi, Jiang, Liao, Zhuang PRD 2018

Liu, Gao, Mameda, Huang, PRD 2019

CKT as ħ expansion and its difficulty at higher order

$O(\hbar^0)$: spinless particle $\partial_t f + \mathbf{v}\cdot\nabla_x f + Q(E + \mathbf{v}\times\mathbf{B})\cdot\nabla_p f = 0$ $\delta(p^2)$

$O(\hbar)$: particle with Berry curvature $\boldsymbol{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$ & magnetic moment $\frac{\mathbf{p}}{2|\mathbf{p}|^2}$ $\delta(p^2) \to \delta(p^2 + \hbar Q\mathbf{B}\cdot\mathbf{p}/p_0)$

$$(1 + \hbar Q\boldsymbol{\Omega}\cdot\mathbf{B})\partial_t f$$
$$+[\mathbf{v} + \hbar Q(\mathbf{E}\times\boldsymbol{\Omega}) + \hbar Q(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B}]\cdot\nabla_x f$$
$$+Q[\mathbf{E} + \mathbf{v}\times\mathbf{B} + \hbar Q(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}]\cdot\nabla_p f = 0$$

valid when

$\sqrt{\hbar E}, \sqrt{\hbar B}, \hbar\partial_X \ll p$

$O(\hbar^2)$: particle no longer on-shell, simple picture lost $\delta(p^2) \nrightarrow \delta((\tilde{p}^2)$

$$p_\mu G^\mu_{(\mathbf{0})}[f\delta(\tilde{p}^2)] + \frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times[\mathbf{p}f\delta(\tilde{p}^2)]\right\} + \hbar^2 C(f) = 0$$

$C(f)$: off-shell effect Gao, Liang, Q. Wang, X.N. Wang PRD 2018

CKT in two different regimes

Free particle basis: subject to weak electromagnetic field $\sqrt{\hbar B} \ll p$

with increasing B, CKT more IR singular, Landau quantization effect becomes relevant.

Landau level basis: subject to strong magnetic field $\sqrt{\hbar B} \gg p$

with increasing B, lower Landau levels truncation more accurate.

Landau level(LL) basis

Dirac equation $[i\gamma^\sigma(\partial_\sigma + ieA_\sigma) - m + \mu\gamma^0 + \mu_5\gamma^0\gamma_5]\psi(x) = 0$

Wigner function $W_{\alpha\beta}(X,P) = \int \frac{d^4X'}{(2\pi)^4} \exp(-ip_\mu x'^\mu) \left\langle \bar{\psi}_\beta\left(X + \frac{1}{2}X'\right) U\left(A, X + \frac{1}{2}X', X - \frac{1}{2}X'\right) \psi_\alpha\left(X - \frac{1}{2}X'\right)\right\rangle$

constant B, can be strong

gauge link $U\left(A, X + \frac{1}{2}X', X - \frac{1}{2}X'\right) = \exp\left(-ieByx'\right)$

LL solution

n=0 LLL

$$W(P) = \sum_{n,s}\Big\{f_{\rm FD}(E^{(n)}_{p_z s} - \mu)\delta(p_0 + \mu - E^{(n)}_{p_z s})W^{(n)}_{+,s}(\mathbf{p})$$
$$+[1 - f_{\rm FD}(E^{(n)}_{p_z,s} + \mu)]\delta(p_0 + \mu + E^{(n)}_{p_z,s})W^{(n)}_{-,s}(\mathbf{p})\Big\}$$

Sheng, Rischke, Wang, Vasak EPJA 2018

Wigner function for right-handed chiral fermion

$$W(X,P)=\int \frac{d^4X'}{(2\pi)^4}\exp(-iP\cdot X')\left\langle \psi^\dagger\left(X+\frac{1}{2}X'\right)U\left(A,X+\frac{1}{2}X',X-\frac{1}{2}X'\right)\psi\left(X-\frac{1}{2}X'\right)\right\rangle$$

Example: lowest Landau level(LLL)

$$W(P)=f(p_0)\left[\delta(p_0-|p_z|)\theta(p_z)W_+^{(0)}(\mathbf{p})+\delta(p_0+|p_z|)\theta(-p_z)W_-^{(0)}(\mathbf{p})\right]$$

$$W_\pm^{(0)}(\mathbf{p})=\frac{2}{(2\pi)^3}\exp\left(-\frac{\mathbf{p}_T^2}{eB}\right)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

LLL dispersion: 1+1D

LLL spin: aligns with B

transverse size: $\frac{1}{\sqrt{eB}}$

only LLL contributes to J_z

$$J_z = C\mu_5 eB \quad J_{5z}=C\mu eB$$

Sheng, Rischke, Wang, Vasak EPJA 2018

CKT from LL basis with perturbation

$$W(X,P)=\int\frac{d^4X'}{(2\pi)^4}\exp(-iP\cdot X')\langle\psi^\dagger(Y)U(A+a,Y,Z)\psi(Z)\rangle,\quad Y=X+\frac{1}{2}X',\ Z=X-\frac{1}{2}X'$$

Dirac equation: $\not{D}_Z\langle\psi^\dagger(Y)\psi(Z)\rangle=\langle\psi^\dagger(Y)\psi(Z)\rangle\not{D}_Y^\dagger=0,\qquad \not{D}_Z=\sigma^\mu D_{Z\mu},\ \ D_{Z\mu}=\partial_{Z\mu}+iA_\mu$

up to all order for constant $F_{\mu\nu}$ and $O(f_{\mu\nu}),O(\partial_X)$

background perturbation

$$A_\mu\rightarrow A_\mu+a_\mu$$

$$\Rightarrow\begin{cases}F_{\mu\nu}\rightarrow F_{\mu\nu}+f_{\mu\nu}\\ U(A+a,Y,Z)=\exp[-iX'\cdot(A+a)]\end{cases}$$

$$\left(\frac{1}{2}\Delta_\mu-iP_\mu\right)\sigma^\mu W(X,P)=\left(\frac{1}{2}\Delta_\mu+iP_\mu\right)W(X,P)\sigma^\mu=0,\qquad \Delta_\mu\equiv\frac{\partial}{\partial X^\mu}-\frac{\partial}{\partial P_\nu}(F_{\mu\nu}+f_{\mu\nu})$$

Example: $F_{\mu\nu}\rightarrow(0,0,B),\ \ f_{\mu\nu}\rightarrow(E_x,E_y,E_z)$

CKT from LL basis with for components

Hermitian: $2W = F\cdot 1 + j_i\sigma^i$

Currents: $J_0 = \int d^4P\, F,\qquad J_i = \int d^4P\, j_i$

EOM:

$$\left(\frac{1}{2}\Delta_\mu - iP_\mu\right)\sigma^\mu W(X,P) = 0$$
$$\left(\frac{1}{2}\Delta_\mu + iP_\mu\right)W(X,P)\sigma^\mu = 0$$

$$\Delta_0 F + \Delta_i j_i = 0,$$
$$\Delta_0 j_i + \Delta_i F - 2\epsilon^{ijk}p_j j_k = 0,$$

Transport equations

$$p_0 F - p_i j_i = 0,$$
$$-p_0 j_i + p_i F + \frac{1}{2}\epsilon^{ijk}\Delta_j j_k = 0,$$

Constraint equations

CKT at leading order satisfied by background

LLL n=0

$$F_r^{(0)} = j_{3r}^{(0)} = \frac{1}{(2\pi)^3} f_r(p_0) \frac{E_{p_z}^{(0)} + r p_z}{2E_{p_z}^{(0)}} \Lambda^{(0)}(p_T),$$

background

$$2W = F \cdot 1 + j_i \sigma^i$$

HLL n>0

$$F_{rs}^{(n)} = \frac{1}{(2\pi)^3} f_r(p_0) \frac{E_{p_z}^{(n)} + rs\sqrt{p_z^2 + 2nB}}{2E_{p_z}^{(n)}} \left(\Lambda_+^{(n)}(p_T) + \frac{s p_z}{\sqrt{p_z^2 + 2nB}} \Lambda_-^{(n)}(p_T) \right),$$

$$j_{3rs}^{(n)} = \frac{1}{(2\pi)^3} f_r(p_0) \frac{E_{p_z}^{(n)} + rs\sqrt{p_z^2 + 2nB}}{2E_{p_z}^{(n)}} \left(\frac{s p_z}{\sqrt{p_z^2 + 2nB}} \Lambda_+^{(n)}(p_T) + \Lambda_-^{(n)}(p_T) \right),$$

$$j_{irs}^{(n)} = \frac{1}{(2\pi)^3} f_r(p_0) \frac{E_{p_z}^{(n)} + rs\sqrt{p_z^2 + 2nB}}{2E_{p_z}^{(n)}} \left(\frac{2nB}{\sqrt{p_z^2 + 2nB}} \frac{s p_i}{p_T^2} \Lambda_+^{(n)}(p_T) \right),$$

Dirac equation

$$\left(\frac{1}{2}\Delta_\mu - iP_\mu\right)\sigma^\mu W(X,P) = 0$$

$$\left(\frac{1}{2}\Delta_\mu + iP_\mu\right)W(X,P)\sigma^\mu = 0$$

CKT with perturbation

$$\Delta_0 F + \Delta_i j_i = 0,$$

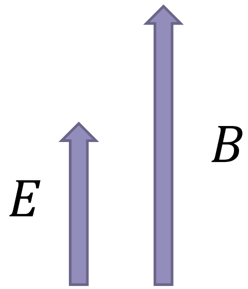
$$\Delta_0 j_i + \Delta_i F - 2\epsilon^{ijk} p_j j_k = 0,$$

$$p_0 F - p_i j_i = 0,$$

$$-p_0 j_i + p_i F + \frac{1}{2}\epsilon^{ijk}\Delta_j j_k = 0,$$

In the absence of ∂_X and $f_{\mu\nu}$, automatically satisfied by the momentum distribution $p_T \& p_z$ at individual LL independent of the energy distribution $f_r(p_0)$

CKT in longitudinal & transverse conductivities

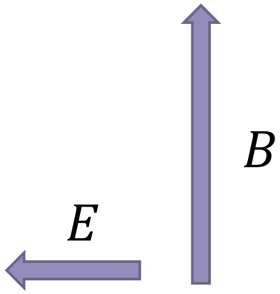


Longitudinal motion classical, system evolves within LL basis

Hattori, Satow, PRD 2016

Hattori, Li, Satow, Yee PRD 2018

Fukushima, Hidaka, PRL 2018; 1906.02683



Transverse motion quantum, system evolves beyond LL basis

CKT in longitudinal $O(E)$ perturbation

LLL state background: only time and longitudinal components nonvanishing

$$j_3 = \frac{p_z}{p_0} F \propto f(p_0)\delta(p_0 - p_z)\exp\left(-\frac{\mathbf{p}_T^2}{eB}\right), \quad j_1 = j_2 = 0$$

$$\Delta_0 F + \Delta_i j_i = 0$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + E_z \frac{\partial}{\partial p_0}\right) f_\pm = 0$$

For homogeneous $E \parallel B$, only energy distribution $f_\pm(p_0)$ changes

$$E_z = \dot{p}_z$$

$$\dot{z} = \frac{p_z}{p_0} = 1$$

$$\int d^4 P$$

$$f_+(p_0 = 0) = f_-(p_0 = 0)$$

$$\frac{\partial f_\pm}{\partial t} + \dot{z}\frac{\partial f_\pm}{\partial z} + \dot{p}_z \frac{\partial f_\pm}{\partial p_z} = C[f_\pm, f_g]$$

Hattori, Li, Satow, Yee PRD 2017)

$$\partial_\mu J_5^\mu = \frac{E_z B}{2\pi^2}$$

$$\partial_\mu J^\mu = 0$$

CKT in transverse *O*(*E*) perturbation

LLL state background: only time and longitudinal components nonvanishing

$$j_3 = \frac{p_z}{p_0} F \propto f(p_0)\delta(p_0 - p_z)\exp\left(-\frac{\mathbf{p}_T^2}{eB}\right), \quad j_1 = j_2 = 0$$

Beyond LL basis: $F \to F + \delta F, j_i \to j_i + \delta j_i, \quad \delta F \sim \delta j_i \sim O(E)$

$$\Delta_0 F + \Delta_i j_i = -\frac{\delta F}{\tau},$$

$$\Delta_0 j_i + \Delta_i F - 2\epsilon^{ijk} p_j j_k = -\frac{\delta j_i}{\tau},$$

relaxation time approximation

$$\delta j_i = E_i \delta j_{\|} + \epsilon^{ij} E_j \delta j_{\perp} \quad \text{(i=x,y)}$$

$\delta F, \delta j_z$ odd function of $\mathbf{p}_T$

Transverse components excited

$$\delta j_{\|} = \frac{4p_z \tau F - \left(2B\tau + \frac{1}{\tau}\right)\frac{\partial F}{\partial p_0}}{\left(2B\tau + \frac{1}{\tau}\right)^2 + 4p_z^2},$$

$$\delta j_{\perp} = \frac{2p_z \frac{\partial F}{\partial p_0} + 2\left(2B\tau + \frac{1}{\tau}\right)\tau F}{\left(2B\tau + \frac{1}{\tau}\right)^2 + 4p_z^2}$$

$$\frac{\partial F}{\partial p_0} \propto \delta'(p_0 - p_z)$$

Transverse conductivity

$$\delta\mathbf{J} = \sigma_\perp(B,T,\mu,\tau)\,\mathbf{E} + \sigma_H(B,T,\mu,\tau)\,\frac{\mathbf{B}\times\mathbf{E}}{B} \qquad \delta J_0 = \delta J_z = 0$$

$$\sqrt{eB} \gg T, \mu, \frac{1}{\tau} \qquad \sigma_\perp \to \frac{e^2}{8\pi^2\tau}\left(1 + \frac{1}{eB}\left(\mu^2 + \frac{\pi^2 T^2}{3} - \frac{1}{2\tau^2}\right)\right), \qquad \sigma_H \to -\frac{e^2\mu}{4\pi^2}$$

$\sigma_\perp \sim O(1/\tau_\perp)$ compare to $\sigma_\parallel \sim O(\tau_\parallel B)$ — Hattori, Li, Satow, Yee PRD 2018

$\sigma_H \sim O(\mu B^0)$ compare to weak B limit $\sigma_H \sim O(\mu\tau_0^2 B)$ — Gorbar, Shovkovy et al PRD 2016

CKT in transverse $O(E)$ & $O(\partial_t)$ perturbation

Assume: $\mathbf{E} \propto e^{-i\omega t}$

$$\frac{1}{\tau} \to \partial_t + \frac{1}{\tau} \qquad \tau \to \tau_\omega \equiv \frac{\tau}{1 - i\omega\tau}$$

effective relaxation time

$$\delta \boldsymbol{J} = \sigma_\perp(B, T, \mu, \tau_\omega)\, \boldsymbol{E} + \sigma_H(B, T, \mu, \tau_\omega)\, \frac{\mathbf{B}\times\mathbf{E}}{B}$$

$$\sqrt{eB} \gg T, \mu, \frac{1}{\tau} \qquad \sigma_\perp \to \frac{e^2}{8\pi^2\tau_\omega}\left(1 + \frac{1}{eB}\left(\mu^2 + \frac{\pi^2 T^2}{3} - \frac{1}{2\tau_\omega^2}\right)\right), \qquad \sigma_H \to -\frac{e^2\mu}{4\pi^2}$$

$\sigma_\perp \sim O(\omega)$ Enhancement of transverse conductivity

$\sigma_\perp \sim O(B^0)$ Milder B dependence at larger B

qualitative agreement with holographic study

Li, SL, Mei, PRD 2019

Limitation of CKT in transverse conductivity

Parameter range for $\mathbf{E} \propto e^{-i\omega t}$ $\quad \frac{1}{\tau_{\mathrm{Drift}}} \ll \omega \ll \frac{1}{\tau}$

No drift $\mathbf{v} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$ → plasma frame coincides with background B frame

Far from hydrodynamic limit → $\sigma_\perp$ outside the regime of $\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^x}(\omega, \mathbf{k}=0) = \omega^2 \rho_\perp \frac{w_0^2}{B_0^4}$

Hernandez, Kovtun JHEP 2017

Near equilibrium → $G_{JxJx} \propto i\omega A_x = E_x$

Transverse conductivity of QGP

Exact solution in LLL for $\sigma_\perp$

$$\sigma_\perp = 2N_c \sum_f \frac{Q_f^2 e^3 B}{(2\pi)^2}\left[\int_0^\infty \left(\frac{1}{e^{\beta(p_z-\mu_q)}+1}+\frac{1}{e^{\beta(p_z+\mu_q)}+1}\right)\frac{4\tau_\omega p_z}{\left(2|Q_f|eB\tau_\omega+\frac{1}{\tau_\omega}\right)^2+4p_z^2}dp_z+\frac{1}{2|Q_f|eB\tau_\omega+\frac{1}{\tau_\omega}}\right]$$

Estimate parameters in HIC:

$$eB \sim m_\pi^2 - 10m_\pi^2$$

$$T = 350MeV$$

$$\frac{e^2\tau}{3}\chi = 0.37e^2\sum_f Q_f^2 T$$ Ding et al, PRD 2011

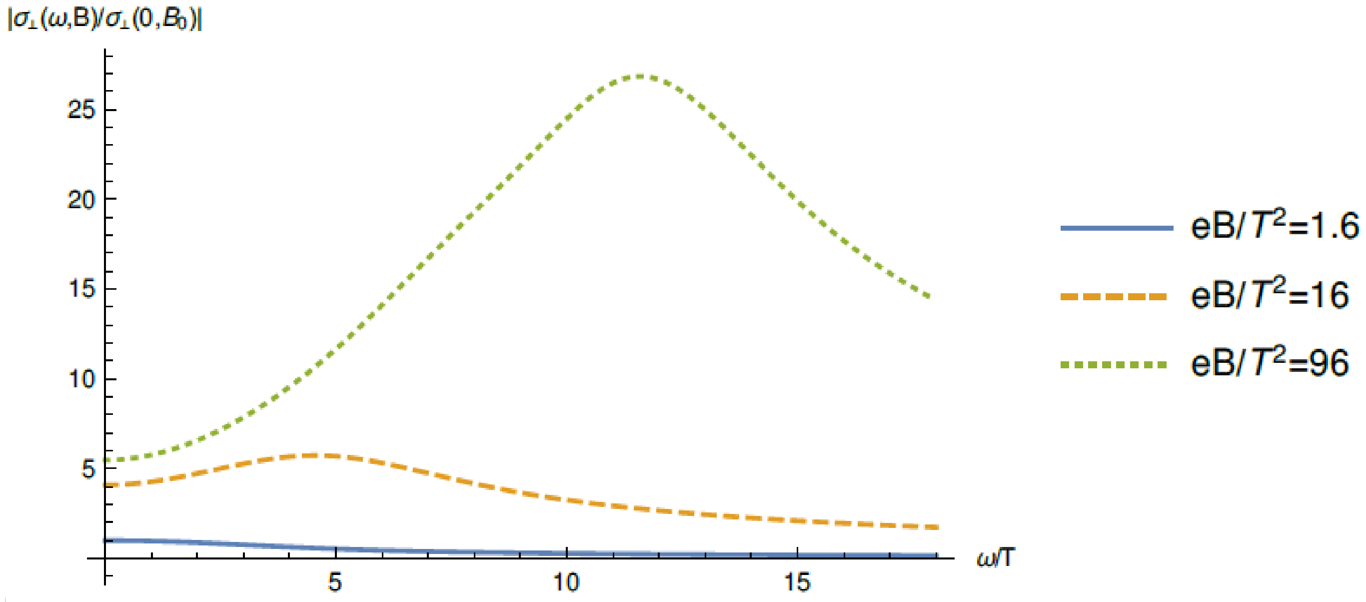
$$\frac{\omega}{T} = \frac{2\pi}{\tau_B T},\ \tau_B \sim 0.2\text{fm} - 1\text{fm}$$ lifetime of B

$$\tau T \sim 0.3$$

$$\frac{eB}{T^2} = 0.2 - 1.6$$ higher LL to be included

$$\omega\tau \sim 1 - 5$$ transient effect maybe relevant

Plot transverse conductivity of QGP

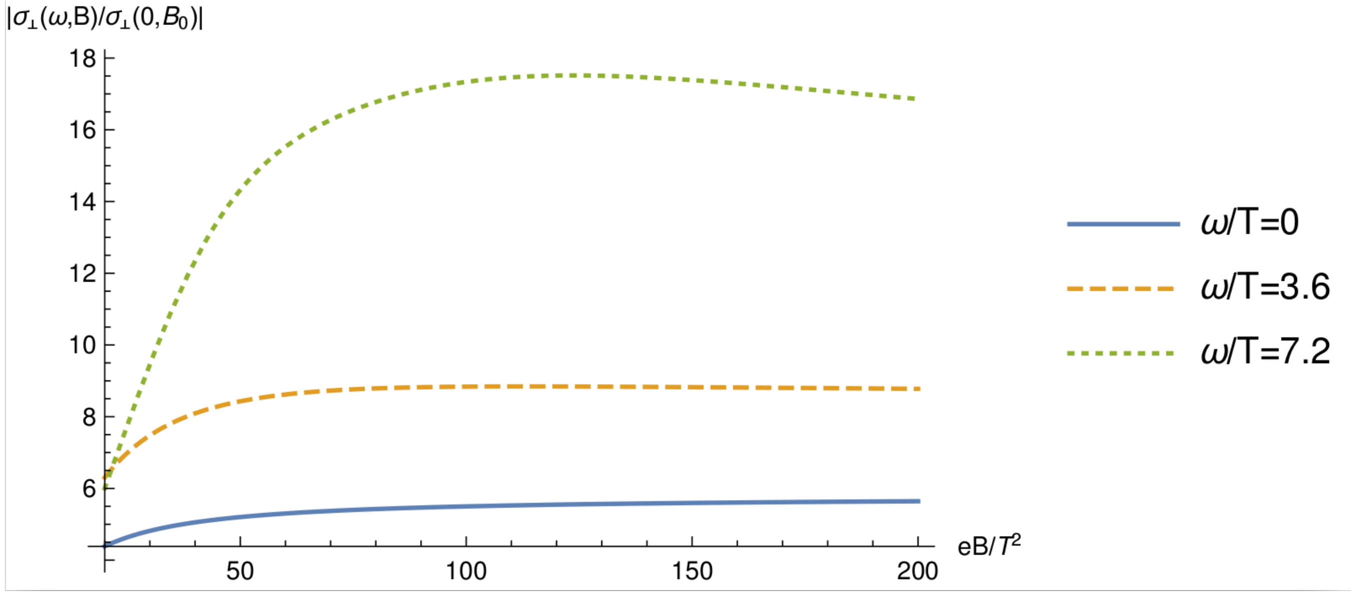


ω dependence**:**

An enhancement of conductivity

Dropping due to violation of $B \gg \omega^2$

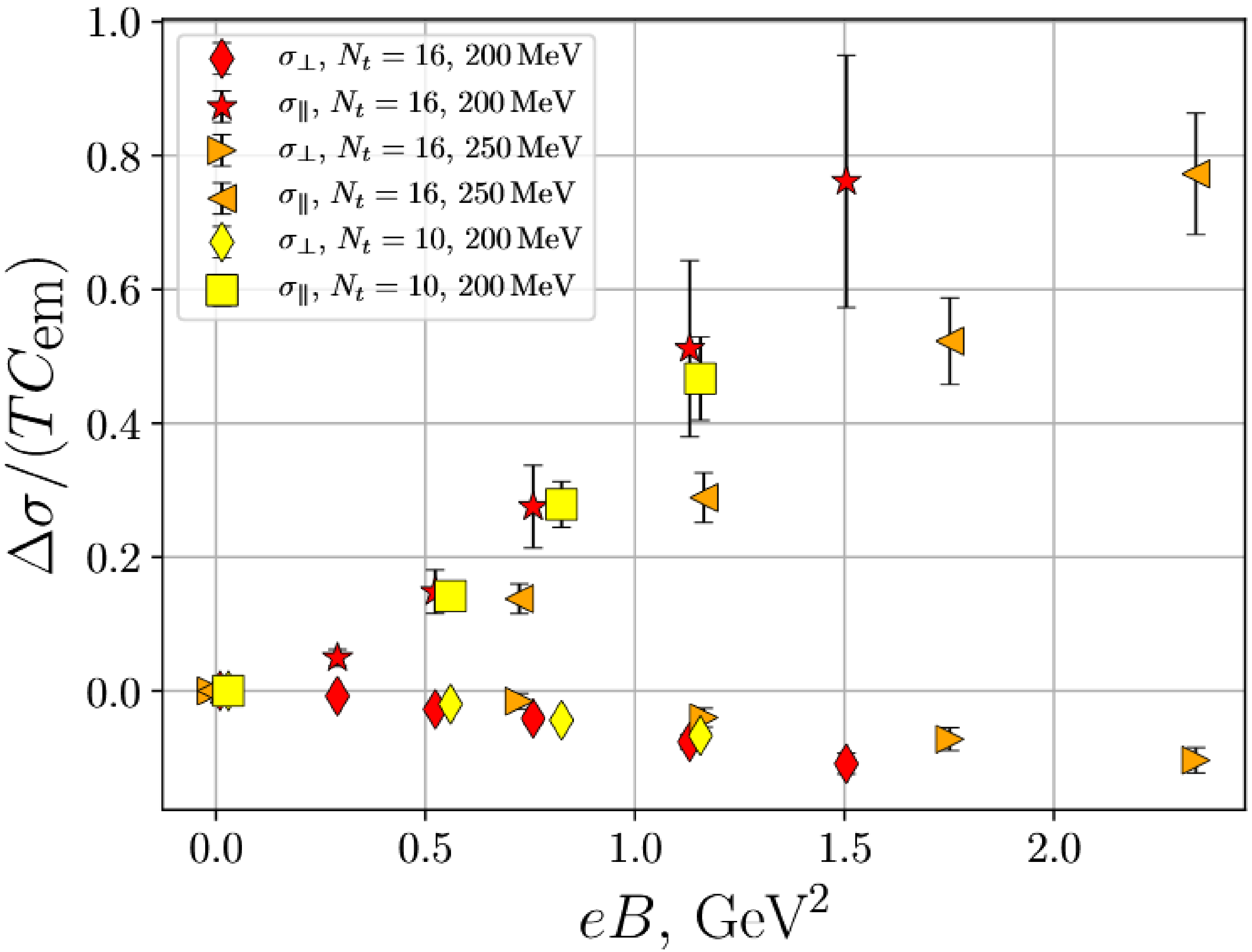
Inclusion of HLL needed for QGP



B dependence**:**

milder B dependence at larger B

B dependence compared to lattice result for $\sigma_\perp$ in QGP



Transverse conductivity decreases with increasing B suggest $\tau_\perp\ (B)$

Kotov et al, 1910.08516

$\tau_\perp \neq \tau_{||}$

field theoretic analysis $\tau_{||}$~$O(B^0)$

Hattori, S. Li, Satow, H. U. Yee, PRD 2017

need field theoretic analysis for $\tau_\perp$

Summary

- CKT from Landau level basis
- Application: CKT with homogeneous *E* field
- Transverse conductivity inversely proportional to the relaxation time
- Transverse conductivity enhancement at high frequency

Outlook

- Higher Landau levels contribution
- Self-energy of photon in magnetized QGP

Thank you!

Backup

right-handed chiral fermion $\bar{\psi}\psi = (\psi_L^\dagger \quad \psi_R^\dagger)\gamma^0 \otimes \begin{pmatrix}\psi_{\rm L} \\ \psi_{\rm R}\end{pmatrix} = \begin{pmatrix}\psi_R^\dagger\psi_L & \psi_L^\dagger\psi_L \\ \psi_R^\dagger\psi_R & \psi_L^\dagger\psi_R\end{pmatrix}$

$\sqrt{eB} \gg T, \mu_q, \frac{1}{\tau}$:

$$\sigma_\perp \to 2\sum_f N_c Q_f^2 \frac{e^2}{8\pi^2\tau_\omega}\left(1 + \frac{1}{|Q_f|eB}\left(\mu_q^2 + \frac{\pi^2 T^2}{3} - \frac{1}{2\tau_\omega^2}\right)\right)$$

$$\sigma_H \to -2\sum_f N_c Q_f^2 \frac{e^2\,\mu_q}{4\pi^2}$$

Exact solution in LLL for σ_H:

$$\sigma_H = -4N_c \sum_f \frac{Q_f^2 e^3 B}{(2\pi)^2}\int_0^\infty \left(\frac{1}{e^{\beta(p_z-\mu_q)}+1} - \frac{1}{e^{\beta(p_z+\mu_q)}+1}\right)\frac{2|Q_f|eB\tau_\omega^2 + 1}{\left(2|Q_f|eB\tau_\omega + \frac{1}{\tau_\omega}\right)^2 + 4p_z^2}\,dp_z$$

LLLA $\sqrt{eB} \gg T, \mu, \frac{1}{\tau}$: $\sigma_H \sim \mu$ non-dissipative, $\sqrt{eB} \ll T, \mu, \frac{1}{\tau}$: $\sigma_H \sim \mu\tau^2 B$ dissipative