Holographic Charged Fluid with Chiral Electric Separation Effect

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based on JHEP 1809 (2018) 083 [arXiv:1803.08389]

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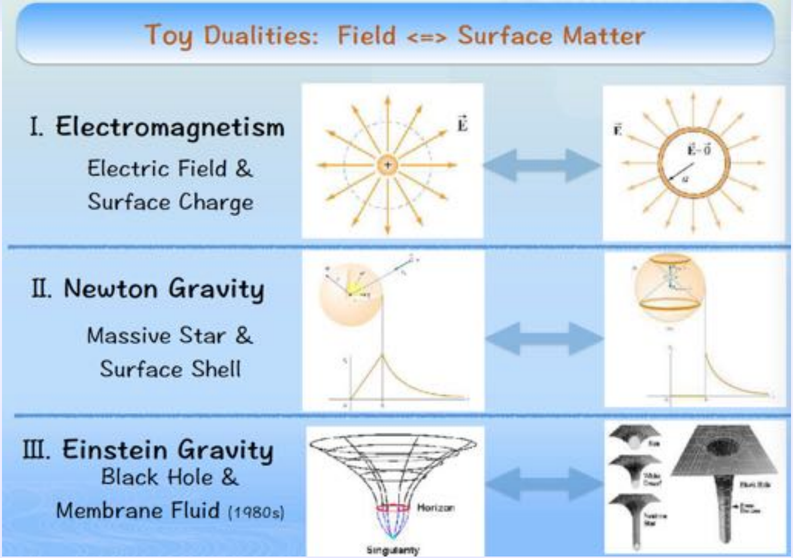
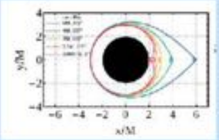
Outline I

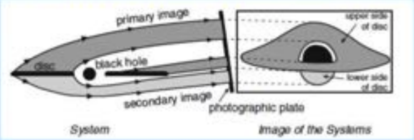

Toy Dualities: Field <=> Surface Matter
I. Electromagnetism
Electric Field &
Surface Charge
II. Newton Gravity
Massive Star &
Surface Shell
III. Einstein Gravity
Black Hole &
Membrane Fluid (1980s)
Horizon
Singularity

Membrane Paradigm: Black Hole <=> Surface Fluid

T. Doumer & K. Thorne, (1980s-)



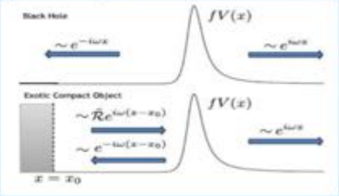
Event Horizon Telescope ('19)



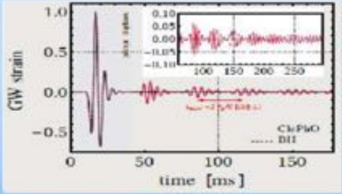
Effective Membrane on Stretched horizon?

$$\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab})$$

Viscosity & Conductivity



Echoes from Compact Objects [1706.06155 PRD'17]

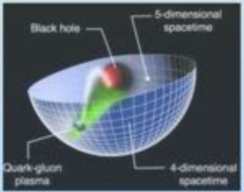


Echoes from the Abyss [1612.00266 PRD'17]

Figures: J. Luminet / M. Sevi, 1709.01525 [Nat.Astron.]

Yun-Long Zhang Holographic Dark Fluid

Successful Holographic Dualities: Geometry <=> Quantum Matters

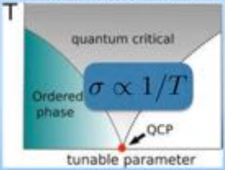


$$Z_{CFT} = \langle e^{S_{CFT}} \rangle \overset{AdS/CFT}{\simeq} e^{S_{AdS}}|_{\text{on-shell}}$$

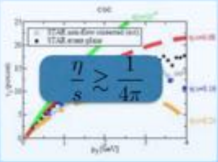
Holographic Quantum Matters
on the Conformal Boundary

$$\langle T_{\mu\nu} \rangle \sim \frac{\delta S_{AdS}}{\delta \gamma^{\mu\nu}} \quad \langle J_\mu \rangle \sim \frac{\delta S_{AdS}}{\delta A^\mu} \quad \langle \mathcal{O} \rangle \sim \frac{\delta S_{AdS}}{\delta \phi}$$

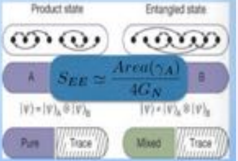
Holographic
Strange Metal (2008-)



Holographic
Hydrodynamics (2003-)



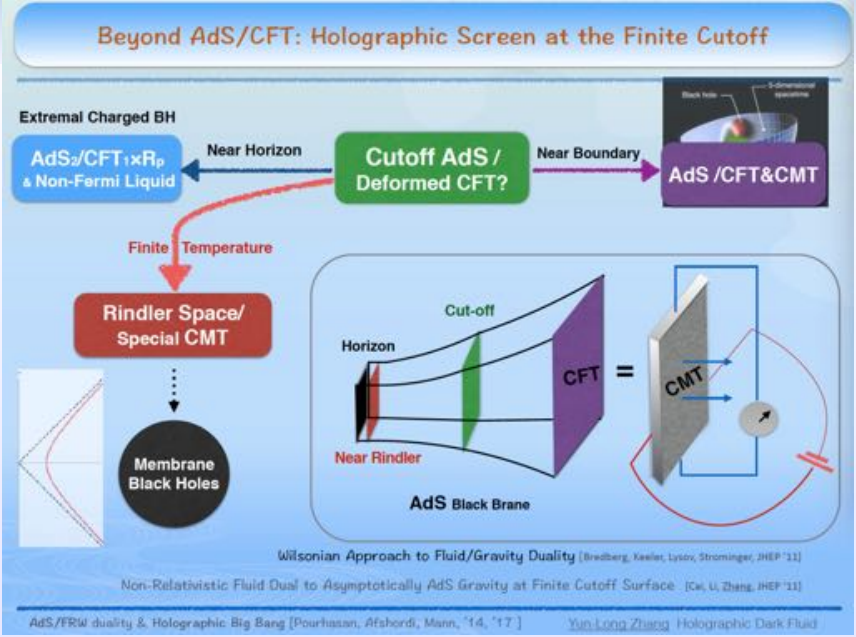
Holographic
Entanglement (2006-)



Hartnoll-Herzog-Horowitz (HHH'08) Kovtun-Son-Starinets (KSS '03) Ryu-Takayanagi (RT, '06)

S. Sachdev (Quantum & Phase) D. T. Son (Nuclear & Fluid) X.-G. Wen (Topological Order)

ICTP Dirac Medal 2018 (狄拉克奖)


Beyond AdS/CFT: Holographic Screen at the Finite Cutoff
Extremal Charged BH
AdS₂/CFT₁×Rₚ & Non-Fermi Liquid
Near Horizon
Cutoff AdS / Deformed CFT?
Near Boundary
AdS /CFT&CMT
Finite Temperature
Rindler Space/ Special CMT
Membrane Black Holes
Cut-off
Horizon
CFT
=
CMT
Near Rindler
AdS Black Brane
Wilsonian Approach to Fluid/Gravity Duality [Bredberg, Keeler, Lysov, Strominger, JHEP '11]
Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff Surface [Cai, Li, Zhang, JHEP '11]
AdS/FRW duality & Holographic Big Bang [Pourhasan, Afshordi, Mann, '14, '17]
Yun-Long Zhang Holographic Dark Fluid

Cutoff AdS Fluid: Universal Hydrodynamic Viscosities

AdS Metric $$ds^2_{p+2} = -r^2 f(r) d\tau^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 dx_i dx^i$$

Induced Metric $$ds^2_{p+1} = -r_c^2 f(r_c) d\tau^2 + r_c^2 dx_i dx^i$$

Dual Tensor $$\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab} + C\gamma_{ab})$$

Constraint equations $$2G_{\mu b} n^\mu|_{r_c} = 2\partial^a (K_{ab} - \gamma_{ab} K) = 0 \Rightarrow \partial^a T_{ab} = 0$$

Cutoff AdS Fluid: Universal holographic conductivities in first order

Holographic Cutoff AdS Fluid in Non-relativistic limit
[Cai, Li, Zhang, JHEP 1107(2011)027]

$$\partial_r \sim \epsilon^0, \quad \partial_i \sim v_i \sim \partial_i \phi \sim \epsilon^1, \quad \partial_\tau \sim P \sim \epsilon^2$$

Holographic Forced Fluid Dynamics on finite Cutoff
[Cai, Li, Nie, Zhang, NPB 864 (2012) 260]

$$\partial_i v^i = 0, \quad \partial_\tau v_i + v^j \partial_j v_i + \partial_i P - \nu \partial^2 v_i = f_i^\phi + f_i^q$$

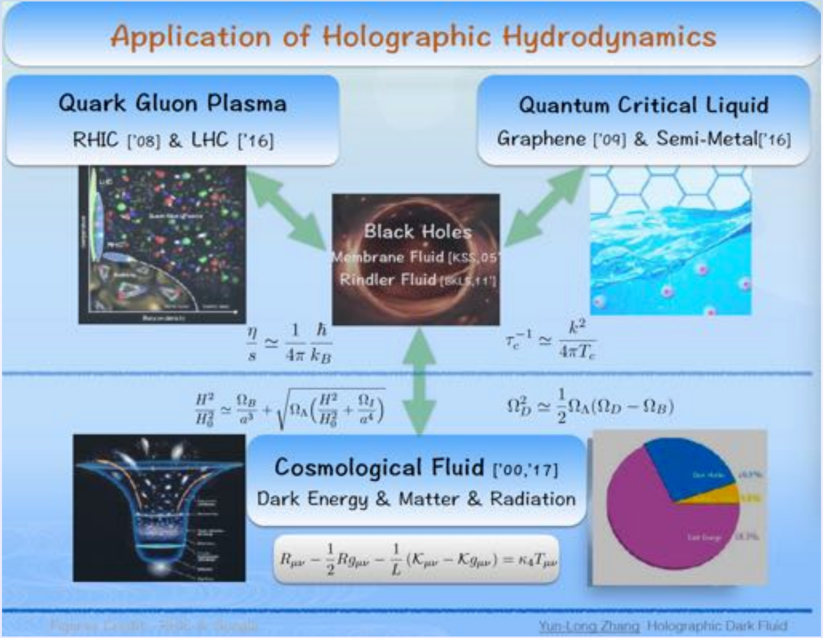
Incompressible Navier-Stokes from Chern-Simons Modified Gravity
Cai, Li, Qi, Zhang, PRD 86 (2012) 086008]

$$\partial_\tau v_i + v^j \partial_j v_i + \partial_i P - \nu \partial^2 v_i - (\tilde{\nu} \epsilon_{ij} \partial^2 v_j + \tilde{\zeta} \epsilon^{jk} \partial_i \partial_j v_k) = f_i$$

Holographic Charged Fluid with Anomalous Current at Finite Cutoff
[Bai, Hu, Lee, Zhang, JHEP 1211 (2012) 054]

$$\xi_B = c\left(\mu - \frac{1}{2}\frac{n\mu^2}{\rho + p}\right), \quad \xi_V = c\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\rho + p}\right)$$

Yun-Long Zhang Holographic Dark Fluid


Application of Holographic Hydrodynamics
Quark Gluon Plasma
RHIC ['08] & LHC ['16]
Quantum Critical Liquid
Graphene ['09] & Semi-Metal['16]
Black Holes
Membrane Fluid [KSS,05']
Rindler Fluid [BKLS,11']
$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$
$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$
$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$
$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$
Cosmological Fluid ['00,'17]
Dark Energy & Matter & Radiation
$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}(K_{\mu\nu} - Kg_{\mu\nu}) = \kappa_4 T_{\mu\nu}$
Figures Credit: RHIC & Google
Yun-Long Zhang Holographic Dark Fluid

The Electronic Currents in the Chiral Fluid

- For the fluid with conserved charges, one needs the constitutive relations for the associated currents. One example is the Ohm's law

$$\vec{J}_V \equiv \langle \bar{\psi} \vec{\gamma} \psi \rangle = \sigma \vec{E} \,. \tag{1}$$

- For a system with charged chiral fermions, chiral anomaly induces anomalous transport phenomena. e.g. Chiral Magnetic Effect (CME)

$$\vec{J}_V = \xi_B \vec{B} \,, \qquad \text{c.f. D. Kharzeev, '04} \tag{2}$$

- Via the chiral anomaly effect, an axial current is generated along an external magnetic field, which is the Chiral Separation Effect (CSE)

$$\vec{J}_A \equiv \langle \bar{\psi} \vec{\gamma}_5 \psi \rangle = \xi_{5B} \vec{B} \,, \qquad \text{c.f. D. T. Son and A. R. Zhitnitsky, '04} \tag{3}$$

- The separation of chiral charge could also be induced by an external electric field, which is called Chiral Electric Separation Effect (CESE)

$$\vec{J}_A = \sigma_{5e} \vec{E}, \qquad \text{c.f. X. G. Huang and J. F. Liao, '13} \tag{4}$$

Ohmic and CESE Conductivities

Table: Ohmic and CESE Conductivities in The High Temperature Regime

Pre-factors	QED Plasma ['13]	QGP (u, d) ['14]	S-S Model ['14]	$U(1)_V \times U(1)_A$ ['18]
$\chi_V \equiv \sigma_e/T$	$\frac{15.7}{e^3 \ln(1/e)}$	$13.0 \frac{\mathrm{Tr}_f Q_e Q_V}{g_c^4 \ln(1/g_c)}$	$0.025 \frac{8N_c^2 g_{\mathrm{YM}}^2}{81}$	π/q_V^2
$\chi_A \equiv \sigma_{5e}/(T\bar{\mu}\bar{\mu}_5)$	$\frac{20.5}{e^3 \ln(1/e)}$	$14.5 \frac{\mathrm{Tr}_f Q_e Q_A}{g_c^4 \ln(1/g_c)}$	$0.002 \frac{8N_c^2 g_{\mathrm{YM}}^2}{81}$	$-\pi/q_A^2$

'13, Huang-Liao, Axial Current Generation from Electric Field: Chiral Electric Separation Effect
'14, Jiang-Huang-Liao, Chiral electric separation effect in the quark-gluon plasma
'14, Pu-Wu-Yang, Holographic Chiral Electric Separation Effect
'18, Bu-Cai-Yang-Zhang, Holographic Charged Fluid with Chiral Electric Separation Effect

- In this table, e, g_c, g_{YM} are the gauge couplings of QED, QCD, the dual $SU(N_c)$ gauge theory, respectively. For the calculations in QGP with two light quarks (u, d), $Q_e = \mathrm{Diag}(2/3, -1/3)$, Q_V and Q_A are the vector and axial charge matrices in flavor space.
- For the holographic $U(1)_V \times U(1)_A$ model, we have restored the bulk gauge couplings q_V^2 and q_A^2, which are related to parameters of boundary theory by $1/q_V^2 \sim 1/q_A^2 \propto N_c N_f/(4\pi^2)$.

Relativistic currents in the chiral fluid

- In the Landau-Lifshitz frame where $u_\mu J_V^\mu = -\rho$ and $u_\mu J_A^\mu = -\rho_5$,

$$J_V^\mu = \rho\, u^\mu + \sigma_e \left[E^\mu - T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \tilde{\sigma}_{5e} T P^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi\omega^\mu + \xi_B B^\mu\right),$$
$$J_A^\mu = \rho_5\, u^\mu + \sigma_{5e} \left[E^\mu - T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \tilde{\sigma}_e T P^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi_5\omega^\mu + \xi_{5B} B^\mu\right),$$

- ρ, ρ_5 are vector and axial charge densities. The external electromagnetic fields E^μ, B^μ and the fluid's vorticity ω^μ are

$$E^\mu \equiv F^{\mu\nu} u_\nu, \quad B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}, \quad \omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta. \tag{5}$$

- The $\tilde{\sigma}_{5e}$- and $\tilde{\sigma}_e$-terms are relevant to chiral charge diffusions, while ξ_5-term is an axial analogue of CVE. Dynamical equations

$$\partial^\mu T_{\mu\nu} = J_V^\alpha F_{\nu\alpha}, \qquad \partial_\mu J_V^\mu = 0, \qquad \partial_\mu J_A^\mu = \mathcal{C} E_\mu B^\mu, \tag{6}$$

where the axial current is not conserved due to chiral anomaly effect and $\mathcal{C}$ denotes the anomaly coefficient.

The stress-energy tensor for relativistic fluid

- For relativistic fluid, the stress-energy tensor is parameterized as

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu + \mathcal{P} P_{\mu\nu} + \pi_{\mu\nu}, \tag{7}$$

where $u_\mu, \mathcal{E}, \mathcal{P}$ are the velocity, energy density and pressure of the fluid, and $P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}$ is the projection tensor.

- Up to the first order in the derivative expansion, the viscous component $\pi_{\mu\nu}$ takes the form,

$$\pi_{\mu\nu} = -\eta P_\mu^\alpha P_\nu^\beta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} P_{\alpha\beta} \partial_\sigma u^\sigma\right) - \zeta P_{\mu\nu} \partial_\alpha u^\alpha, \tag{8}$$

where η, ζ are the shear viscosity and bulk viscosity, respectively. We will take the Landau-Lifshitz frame so that $u^\mu \pi_{\mu\nu} = 0$.

Holographic $U(1)_V \times U(1)_A$ model

- We consider $(4+1)$-dimensional AdS bulk

$$\mathcal{S}_{\mathcal{M}} = \frac{1}{16\pi G_5}\int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-g}\,(R-2\Lambda) + \mathcal{S}_F + \mathcal{S}_{\mathrm{ct}} + \mathcal{S}_{\mathcal{K}}, \tag{9}$$

with the $U(1)_V \times U(1)_A$ gauge fields

$$\mathcal{S}_F = -\int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-g}\left(\frac{1}{4q_V^2}F_{MN}F^{MN} + \frac{1}{4q_A^2}\tilde{F}_{MN}\tilde{F}^{MN}\right), \tag{10}$$

where $F_{MN} \equiv \partial_M A_N - \partial_N A_M$ and $\tilde{F}_{MN} \equiv \partial_M \tilde{A}_N - \partial_N \tilde{A}_M$. A_M and $\tilde{A}_M$ denote the vector and axial bulk gauge fields, which are dual to vector and axial currents J_V^μ, J_A^μ of the boundary theory, respectively.

- According to AdS/CFT correspondence, the holographic stress-energy tensor and currents on the boundary theory are

$$T_{\mu\nu} \equiv \lim_{r\to\infty}\frac{-2r^2}{\sqrt{-\gamma}}\frac{\delta\mathcal{S}_{\mathcal{M}}}{\delta\gamma^{\mu\nu}}, \quad J^\mu \equiv \lim_{r\to\infty}\frac{\delta\mathcal{S}_{\mathcal{M}}}{\delta A_\mu}, \quad J_5^\mu \equiv \lim_{r\to\infty}\frac{\delta\mathcal{S}_{\mathcal{M}}}{\delta\tilde{A}_\mu}. \tag{11}$$

AdS_5 black brane with two charges

- The homogeneous solution of the bulk theory is,

$$\begin{aligned} ds^2_{(0)} &= g^{(0)}_{MN}\mathrm{d}x^M\mathrm{d}x^N = 2\mathrm{d}t\mathrm{d}r - r^2 f(r)\mathrm{d}t^2 + r^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \\ f(r) &\equiv 1 - \frac{M}{r^4} + \frac{Q^2+\tilde{Q}^2}{r^6} = \frac{\left(r^2-r_h^2\right)\left(r^2-r_-^2\right)\left(r^2+r_h^2+r_-^2\right)}{r^6}, \\ A_{(0)} &= -\frac{\sqrt{3}Q}{r^2}\mathrm{d}t, \qquad \tilde{A}_{(0)} = -\frac{\sqrt{3}\tilde{Q}}{r^2}\mathrm{d}t, \end{aligned} \tag{12}$$

  where $M, Q, \tilde{Q}$ are constant parameters of the bulk theory, r_h is the largest root for $f(r)=0$, defining the location of event horizon.

- The Hawking temperature, identified as the temperature of dual boundary field theory, is

$$T_0 = \frac{\partial_r\left(r^2 f(r)\right)}{4\pi}\bigg|_{r=r_h} = \frac{r_h}{\pi}\left(1 - \frac{Q^2+\tilde{Q}^2}{2r_h^6}\right), \tag{13}$$

The holographic hydrodynamics of chiral fluid

- The dual stress-energy tensor and currents of the boundary theory are

$$T^{\mu\nu}_{(0)} = 3M\,\delta^\mu_t\delta^\nu_t + M\,\delta^\mu_i\delta^\nu_j\,, \qquad J^\mu_{(0)} = 2\sqrt{3}Q\,\delta^\mu_t\,, \qquad J^\mu_{5(0)} = 2\sqrt{3}\tilde{Q}\,\delta^\mu_t\,, \tag{14}$$

we can read out the energy density, pressure and charge densities,

$$\mathcal{E} = 3M, \qquad \mathcal{P} = M, \qquad \rho = 2\sqrt{3}Q, \qquad \rho_5 = 2\sqrt{3}\tilde{Q}, \tag{15}$$

- c.f. the holographic stress energy density and currents

$$T_{\mu\nu} = -\frac{1}{8\pi G_5}\lim_{r\to\infty} r^2\Big[\Big(\mathcal{K}_{\mu\nu} - \mathcal{K}\gamma_{\mu\nu} + 3\gamma_{\mu\nu} - \frac{1}{2}\mathcal{G}_{\mu\nu}\Big) + \mathcal{T}^F_{\mu\nu}\Big], \tag{16}$$

$$J^\mu = -\frac{1}{q_V^2}\lim_{r\to\infty} r^2\big(n_M F^{M\mu} + D_\nu F^{\nu\mu}\log r\big), \tag{17}$$

$$J^\mu_5 = -\frac{1}{q_A^2}\lim_{r\to\infty} r^2\big(n_M \tilde{F}^{M\mu} + D_\nu \tilde{F}^{\nu\mu}\log r\big), \tag{18}$$

Transports from Fluid/Gravity Correspondences

- The shear and bulk viscosities take universal values of holographic conformal fluids with entropy density s,

$$\eta = \frac{1}{4\pi} s, \qquad \zeta = 0, \tag{19}$$

(c.f. $T_{\mu\nu} = \mathcal{E} u_\mu u_\nu + \mathcal{P} P_{\mu\nu} - 2\eta\sigma_{\mu\nu} - \zeta\theta P_{\mu\nu}$)

- The dissipative transport coefficients in the currents

$$\sigma_e = \frac{\sigma_Q \mu^2 + \sigma_0 \mu_5^2}{\mu^2 + \mu_5^2}, \qquad \sigma_{5e} = \tilde{\sigma}_{5e} = \mu\mu_5 \frac{\sigma_Q - \sigma_0}{\mu^2 + \mu_5^2}, \qquad \tilde{\sigma}_e = \frac{\sigma_Q \mu_5^2 + \sigma_0 \mu^2}{\mu^2 + \mu_5^2}, \tag{20}$$

where σ_Q and σ_0 are given by

$$\sigma_Q \equiv \frac{\sigma_0 \left(Ts\right)^2}{\left(Ts + \mu\rho + \mu_5\rho_5\right)^2} = \frac{\sigma_0 \left(Ts\right)^2}{\left(\mathcal{E} + \mathcal{P}\right)^2}, \quad \sigma_0 \equiv \frac{\pi T}{2} \left(1 + \sqrt{1 + \frac{2}{3} \frac{\mu^2 + \mu_5^2}{\pi^2 T^2}}\right).$$

c.f.

$$J_V^\mu = \rho\, u^\mu + \sigma_e \left[E^\mu - TP^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \tilde{\sigma}_{5e} TP^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi\omega^\mu + \xi_B B^\mu\right),$$

$$J_A^\mu = \rho_5\, u^\mu + \sigma_{5e} \left[E^\mu - TP^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \tilde{\sigma}_e TP^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi_5\omega^\mu + \xi_{5B} B^\mu\right),$$

CESE conductivities from fluid/gravity duality

- From the dual fluid's point of view, it is more natural to re-express the conductivities in terms of chemical potentials and temperature.

$$\sigma_e = \pi T \frac{2\gamma + (3\gamma - 1)\bar{\mu}_5^2}{2(3\gamma - 2)^2} \equiv T\chi_V, \quad \sigma_{5e} = -\pi T \bar{\mu}\bar{\mu}_5 \frac{3\gamma - 1}{2(3\gamma - 2)^2} \equiv T\bar{\mu}\bar{\mu}_5\chi_A,$$

where

$$\gamma(\bar{\mu}, \bar{\mu}_5) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{3}(\bar{\mu}^2 + \bar{\mu}_5^2)}\right), \quad \bar{\mu} = \frac{\mu}{\pi T}, \quad \bar{\mu}_5 = \frac{\mu_5}{\pi T}. \tag{21}$$

- In the high temperature (small chemical potential) limit or low temperature (large chemical potential) limit, the conductivities

$$\begin{aligned} \sigma_e &\simeq \pi T, \qquad \sigma_{5e} \simeq -\pi T\bar{\mu}\bar{\mu}_5, \qquad \bar{\mu}, \bar{\mu}_5 \ll 1, \\ \sigma_e &\simeq \frac{\pi T \bar{\mu}_5^2}{\sqrt{6(\bar{\mu}^2 + \bar{\mu}_5^2)}}, \quad \sigma_{5e} \simeq -\frac{\pi T \bar{\mu}\bar{\mu}_5}{\sqrt{6(\bar{\mu}^2 + \bar{\mu}_5^2)}}, \quad \bar{\mu}, \bar{\mu}_5 \gg 1. \end{aligned} \tag{22}$$

Temperature Dependence of the conductivities

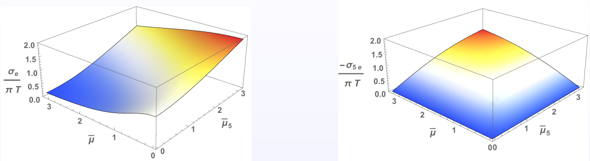


Figure: The conductivities $\sigma_e/(\pi T)$ and $-\sigma_{5e}/(\pi T)$ as functions of $\bar{\mu}$ and $\bar{\mu}_5$.

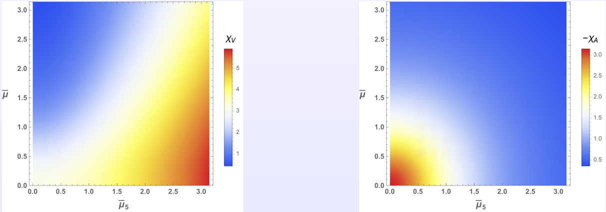


Figure: Density plots for χ_V and χ_A as functions of $\bar{\mu}$ and $\bar{\mu}_5$.

Currents in the linear response form

- Below we would like to rewrite the currents in a linear response form, in which the electric field E_i and thermal gradient $\nabla_i T$ are taken as external sources.

$$J^\mu_V = \rho\, u^\mu + \sigma_e\Big[E^\mu - TP^{\mu\nu}\partial_\nu\Big(\frac{\mu}{T}\Big)\Big] - \tilde{\sigma}_{5e}\, TP^{\mu\nu}\partial_\nu\Big(\frac{\mu_5}{T}\Big) + (\xi\omega^\mu + \xi_B B^\mu)\,,$$

$$J^\mu_A = \rho_5\, u^\mu + \sigma_{5e}\Big[E^\mu - TP^{\mu\nu}\partial_\nu\Big(\frac{\mu}{T}\Big)\Big] - \tilde{\sigma}_e\, TP^{\mu\nu}\partial_\nu\Big(\frac{\mu_5}{T}\Big) + (\xi_5\omega^\mu + \xi_{5B} B^\mu)\,,$$

- In other words, the fluid velocity u_i will be eliminated using the current conservation law. Here, the chemical potentials μ, μ_5 will be taken as constant. Consequently,

$$u^i = u_i = \frac{1}{i\omega}\Big(s\nabla_i T - \frac{\rho}{\mathcal{E}+\mathcal{P}}E_i\Big), \tag{23}$$

where $\partial_t \to -i\omega$ is used. Then, the currents turn into

$$J^t_V = \rho, \qquad J^i_V = \sigma_{(\omega)}E_i - \alpha_{(\omega)}\nabla_i T, \tag{24}$$

$$J^t_A = \rho_5, \qquad J^i_A = \sigma_{5(\omega)}E_i - \alpha_{5(\omega)}\nabla_i T, \tag{25}$$

Kubo formulas and hydrodynamic constitutive relations

- $\sigma_{(\omega)}$ is the low-frequency limit of the Ohmic electrical conductivity, and $\sigma_{5(\omega)}$ measures the CESE. $\alpha_{(\omega)}$ and $\alpha_{5(\omega)}$ are the thermoelectric conductivities for vector and axial currents.

$$\sigma_{(\omega)} = \frac{i}{\omega}\frac{\rho^2}{\mathcal{E}+\mathcal{P}} + \sigma_e, \qquad \sigma_{5(\omega)} = \frac{i}{\omega}\frac{\rho\rho_5}{\mathcal{E}+\mathcal{P}} + \sigma_{5e}, \tag{28}$$

$$\alpha_{(\omega)}T = \frac{i}{\omega}\rho - \left(\mu\,\sigma_{(\omega)} + \mu_5\,\sigma_{5(\omega)}\right), \quad \alpha_{5(\omega)}T = \frac{i}{\omega}\rho_5 - \left(\mu_5\,\tilde{\sigma}_{(\omega)} + \mu\,\sigma_{5(\omega)}\right). \tag{29}$$

- The heat current is

$$\mathcal{Q}^i \equiv T^{ti} - \mu J^i - \mu_5 J_5^i = \bar{\alpha}_{(\omega)} T E_i - \bar{\kappa}_{(\omega)} \nabla_i T, \tag{30}$$

where the transport coefficients are

$$\bar{\alpha}_{(\omega)} = \alpha_{(\omega)}, \quad \bar{\kappa}_{(\omega)} T = \frac{i}{\omega}\left(\mathcal{E}+\mathcal{P}-2\rho\mu-2\rho_5\mu_5\right) + \left(\mu^2\,\sigma_{(\omega)} + \mu_5^2\,\tilde{\sigma}_{(\omega)} + 2\mu\mu_5\sigma_{5(\omega)}\right)$$

and $\bar{\kappa}_{(\omega)}$ is the low-frequency limit of thermal conductivity.

Constraint from the second law of thermodynamics

- The second law of thermodynamics (non-negativeness of divergence for the entropy current), could not fix the values of dissipative transport coefficients, it does set constraints for them,

$$\sigma_e \geq 0, \qquad \tilde{\sigma}_e \geq 0, \qquad \sigma_e \tilde{\sigma}_e \geq \sigma_{5e}^2, \tag{31}$$

- Ohmic and CESE Conductivities in The High Temperature Regime

Pre-factors	QED Plasma ['13]	QGP (u, d) ['14]	S-S Model ['14]	$U(1)_V \times U(1)_A$ ['18]
$\chi_V \equiv \sigma_e / T$	$\frac{15.7}{e^3 \ln(1/e)}$	$13.0 \frac{\mathrm{Tr}_f Q_e Q_V}{g_c^4 \ln(1/g_c)}$	$0.025 \frac{8N_c^2 g_{YM}^2}{81}$	π / q_V^2
$\chi_A \equiv \sigma_{5e}/(T \bar{\mu} \bar{\mu}_5)$	$\frac{20.5}{e^3 \ln(1/e)}$	$14.5 \frac{\mathrm{Tr}_f Q_e Q_A}{g_c^4 \ln(1/g_c)}$	$0.002 \frac{8N_c^2 g_{YM}^2}{81}$	$-\pi / q_A^2$

- We found the negative CESE coefficient in the Holographic $U(1)_V \times U(1)_A$ model.

$$\sigma_e = \frac{\sigma_Q \mu^2 + \sigma_0 \mu_5^2}{\mu^2 + \mu_5^2}, \qquad \sigma_{5e} = \tilde{\sigma}_{5e} = \mu\mu_5 \frac{\sigma_Q - \sigma_0}{\mu^2 + \mu_5^2}, \qquad \tilde{\sigma}_e = \frac{\sigma_Q \mu_5^2 + \sigma_0 \mu^2}{\mu^2 + \mu_5^2}, \tag{32}$$

Summary and Outlook

- In this work, we explored transport properties of the holographic $U(1)_V \times U(1)_A$ models in the asymptotic AdS_5 black brane.
- Our main finding is the nonzero CESE conductivity when the gravitational back-reaction effect is taken into account.

$$J_V^\mu = \rho\, u^\mu + \sigma_e\left[E^\mu - TP^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \sigma_{5e}\, TP^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi\omega^\mu + \xi_B B^\mu\right),$$

$$J_A^\mu = \rho_5\, u^\mu + \sigma_{5e}\left[E^\mu - TP^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right] - \tilde{\sigma}_e\, TP^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \left(\xi_5\omega^\mu + \xi_{5B} B^\mu\right),$$

- We confirmed our results with two complementary methods — Fluid/Gravity Duality v.s. Linear Response Analysis.
- Applications of CESE in Semi-Metals & Experimental Test.

Thanks for all your attention!