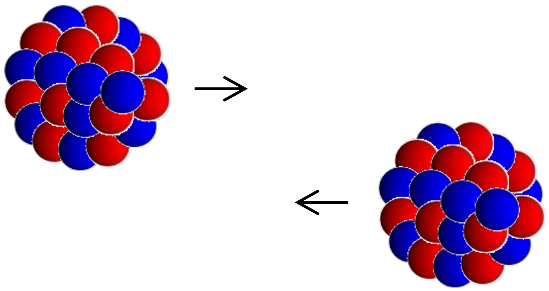
Nuclear Reaction Theory

Quantum Many-body Dynamics in low-energy heavy-ion reactions



Kouichi Hagino（萩野浩一）

Kyoto University（京都大学）

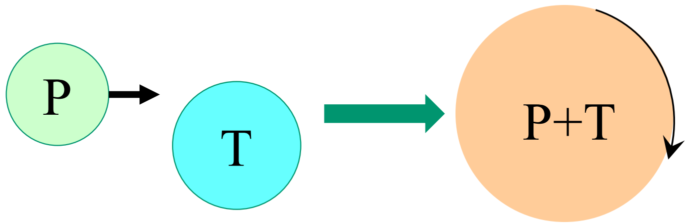
Lecture 1: Overview of low-energy heavy-ion collisions
and a single-channel description

Lecture 2: **Multi-channel scattering and heavy-ion subbarrier fusion reactions**

Lecture 3: Fusion for superheavy elements

核物理青年科学家学校 Nuclear physics School for Young Scientists (NUSYS2026), 2026.8.9-8.16, Beihang University

Fusion reactions: compound nucleus formation

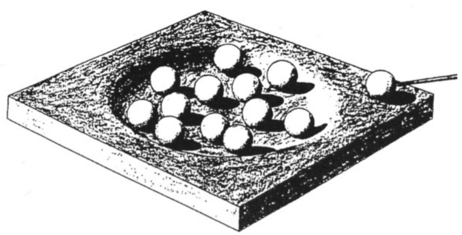


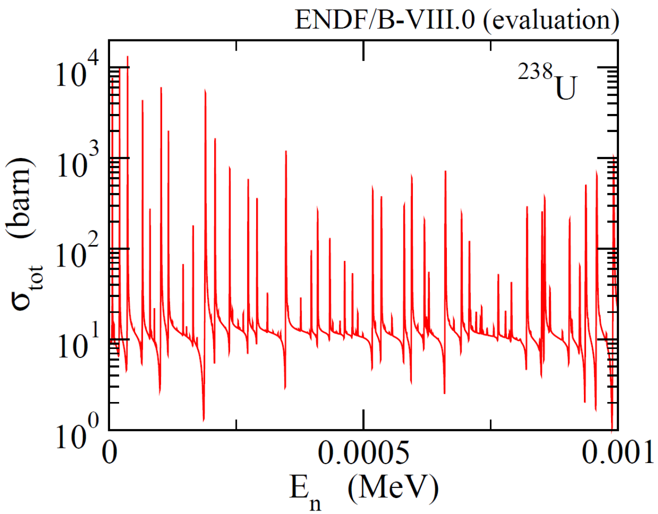
compound
nucleus（複合核）

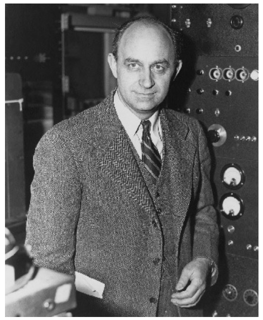
Niels Bohr (1936)

Wikipedia

neutron capture
reactions

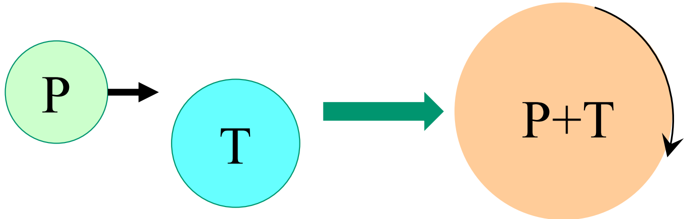




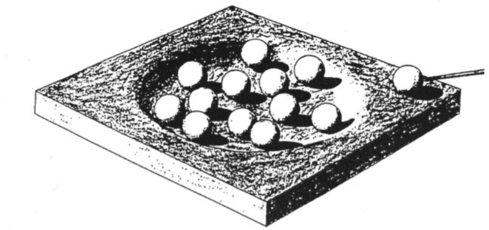


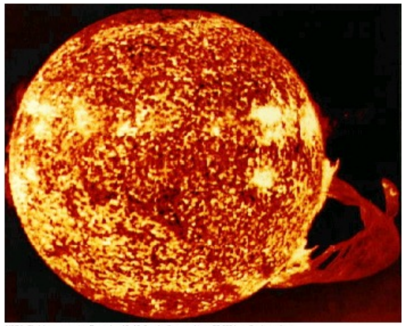
Exp: Enrico Fermi (1935)
many very narrow resonances (width ~ eV)

Fusion reactions: compound nucleus formation

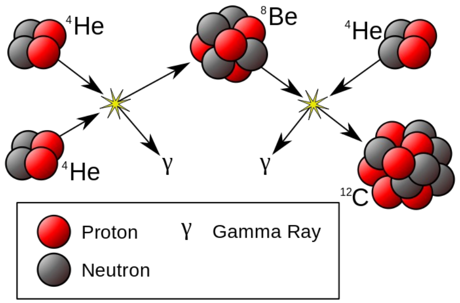


compound
nucleus（複合核）

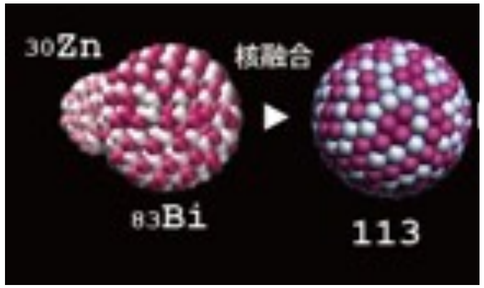




energy production
in stars (Bethe '39)



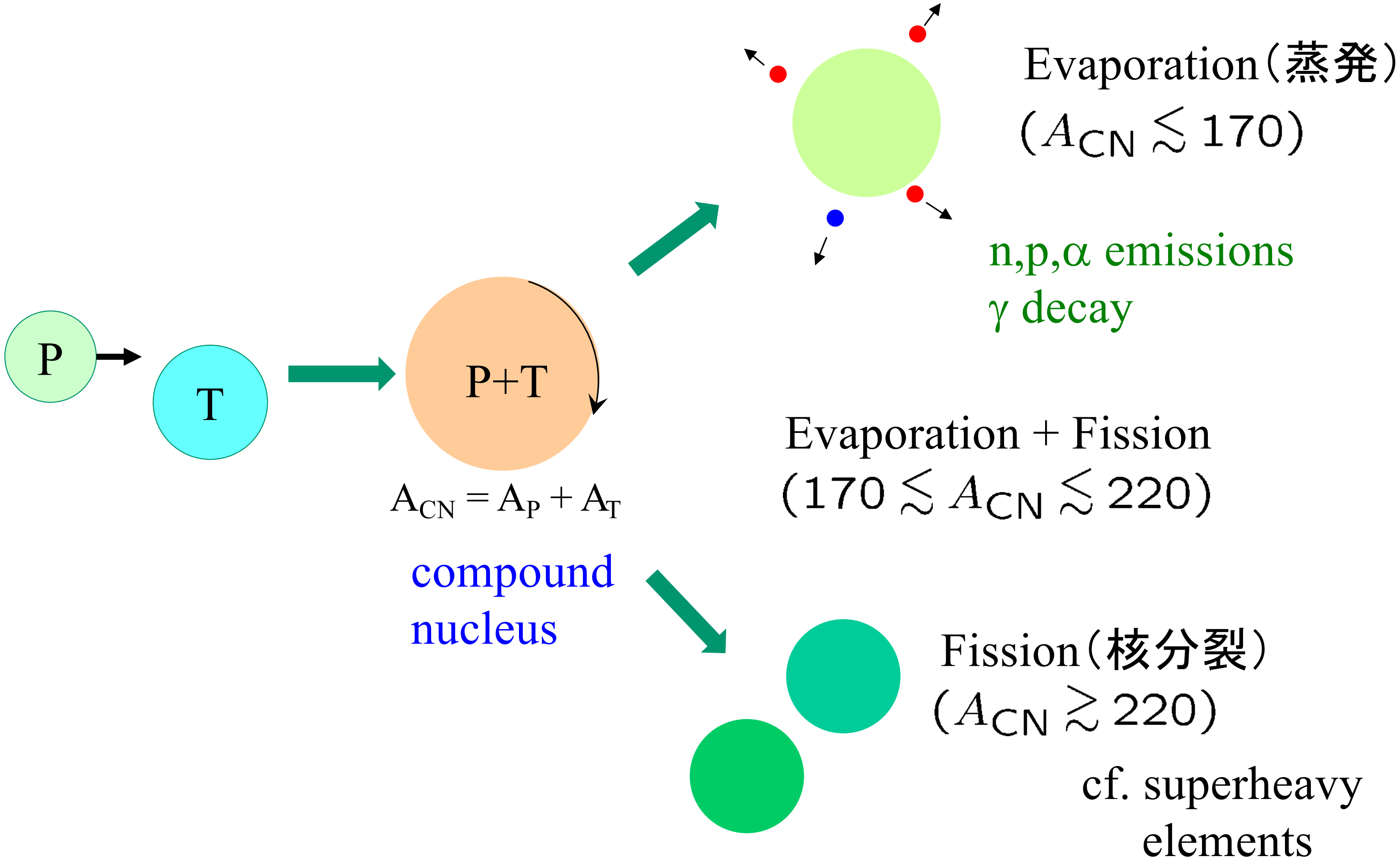
nucleosynthesis



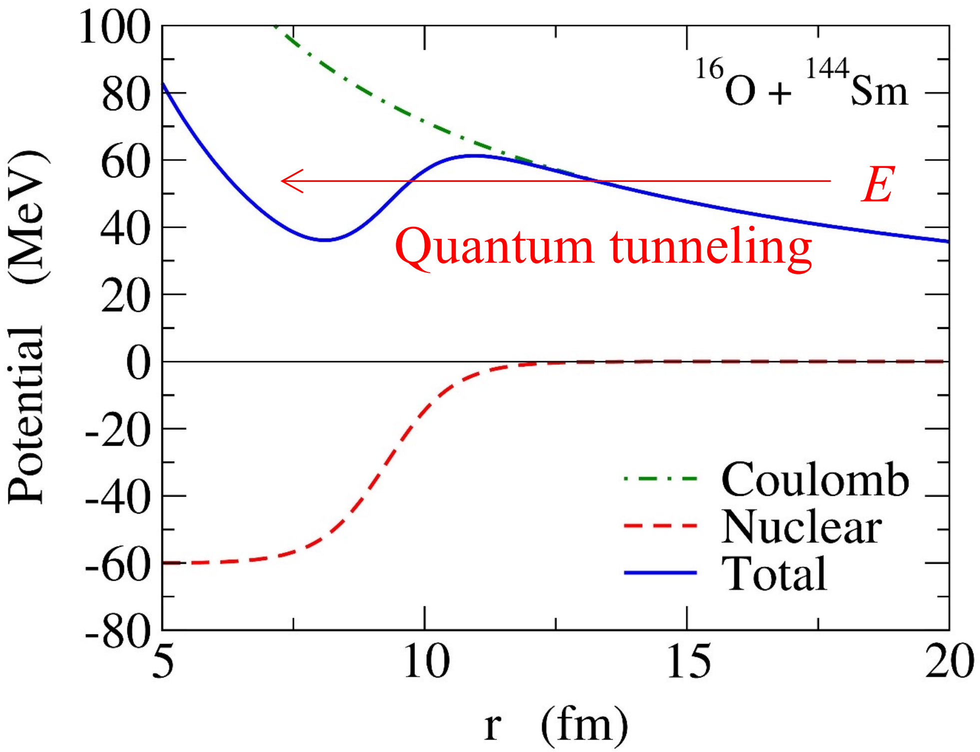
superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems
← microscopic understanding: an ultimate goal of nuclear physics

Fusion reactions: compound nucleus formation



Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

Potential barrier (Coulomb barrier)

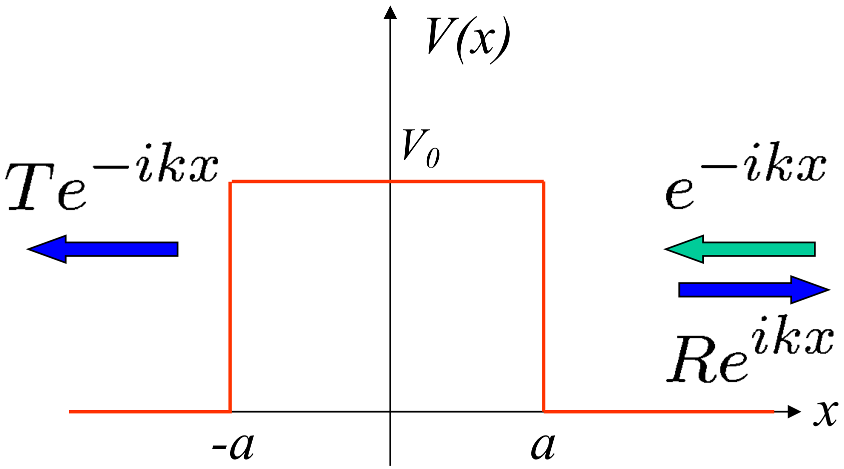
Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

Fusion reactions at energies around the Coulomb barrier

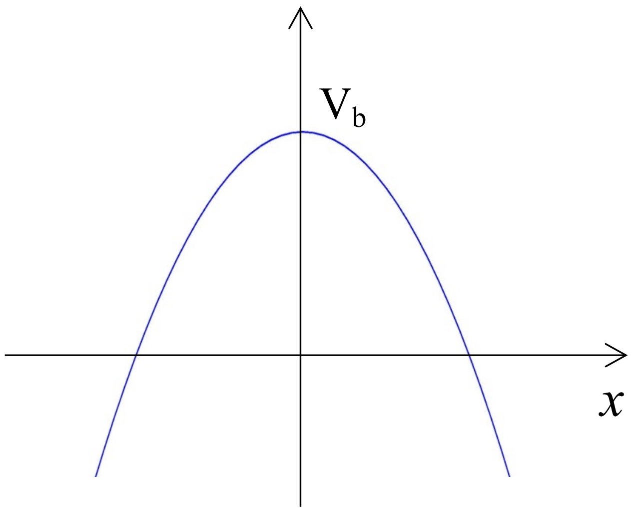
Sub-barrier fusion reactions and quantum tunneling

Fusion with quantum tunneling



for a parabolic barrier:

$$V(x) = V_b - \frac{1}{2} m\Omega^2 x^2$$



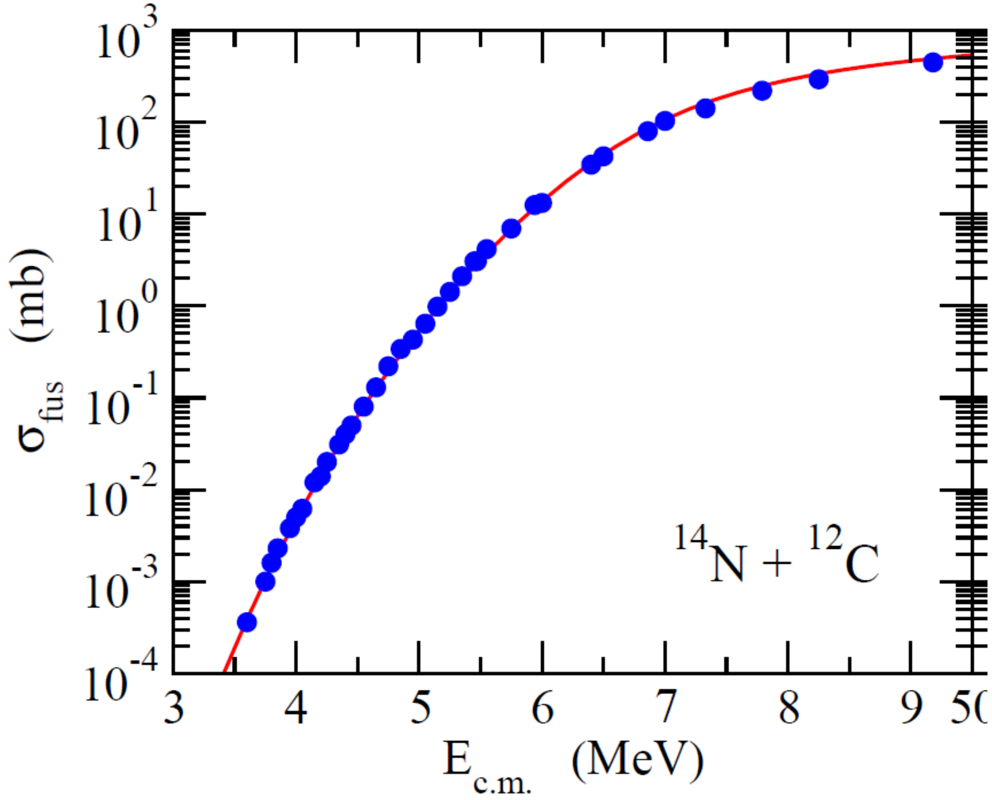
$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

$$\sim \exp\left[-\frac{2\pi}{\hbar\Omega}(V_b - E)\right] \qquad (E \ll V_b)$$

a very strong energy dependence!

the potential model: inert nuclei (no structure)

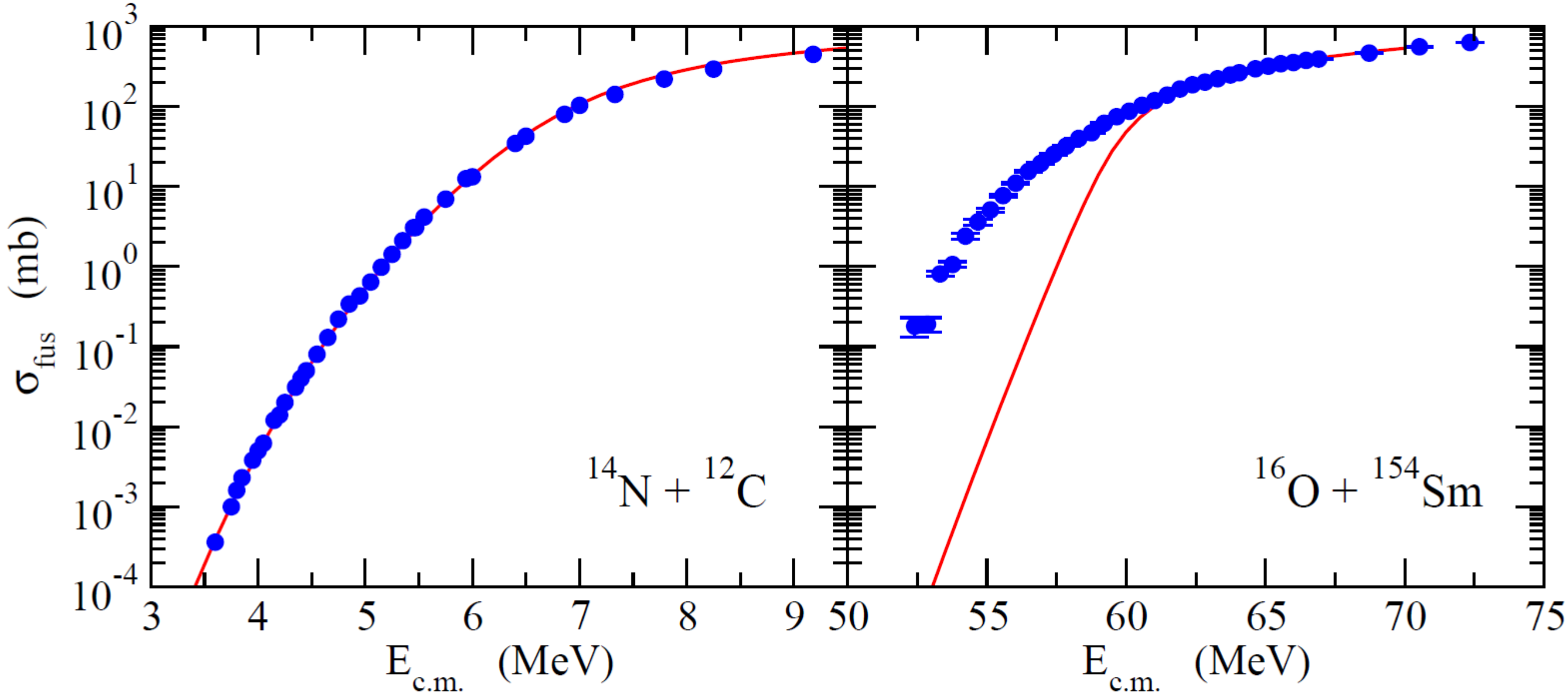
$$\boxed{\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$



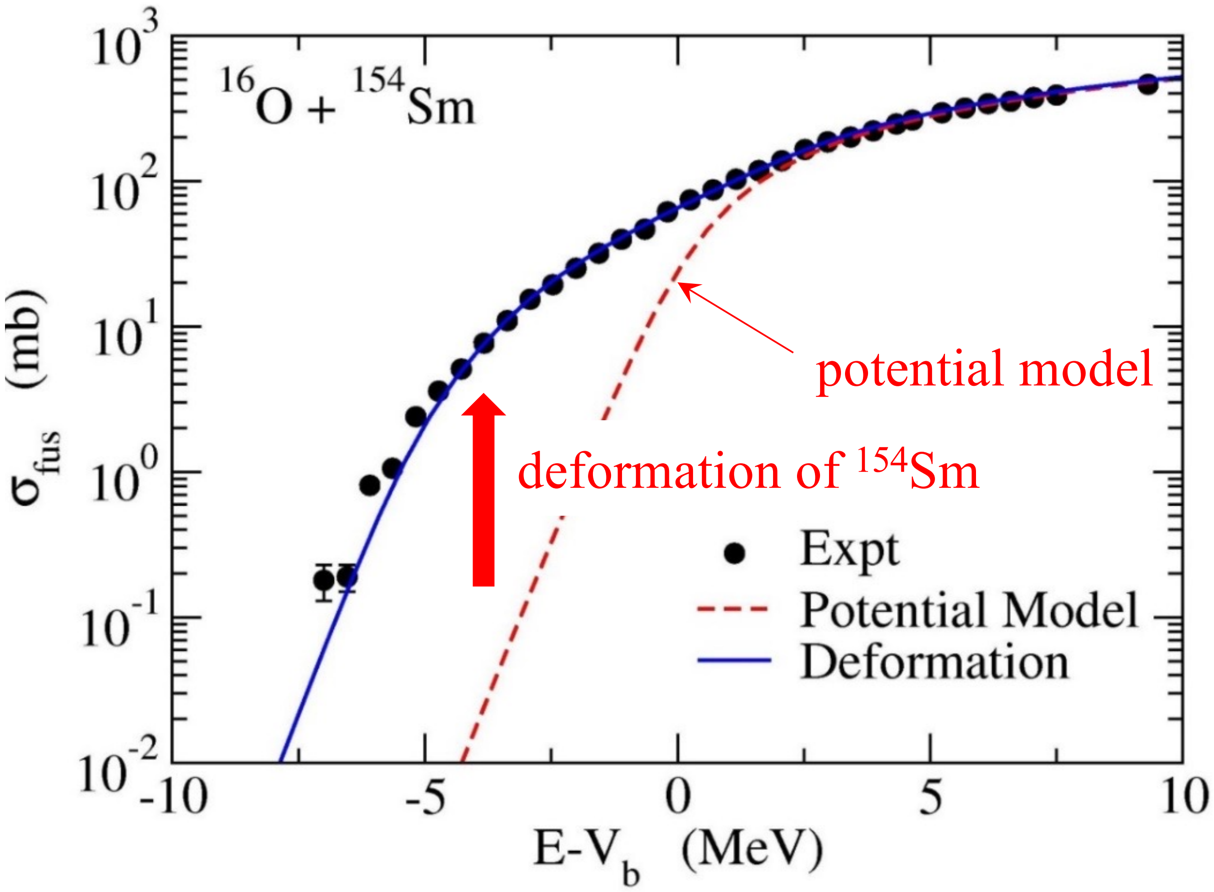
Discovery of large sub-barrier enhancement of σ_{fus} (~80's)

the potential model: inert nuclei (no structure)

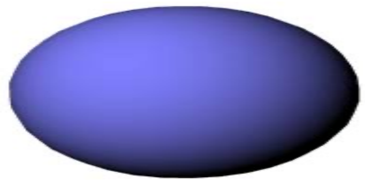
$$\sigma_{\mathrm{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$



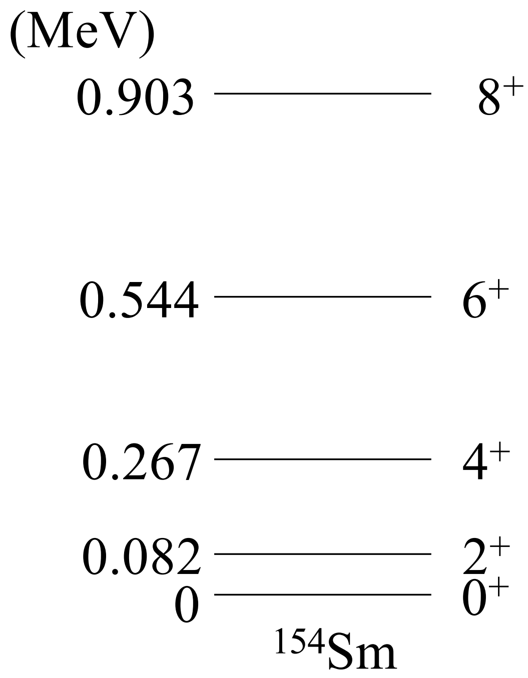
Sub-barrier fusion reactions and quantum tunneling



K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.



^{154}Sm

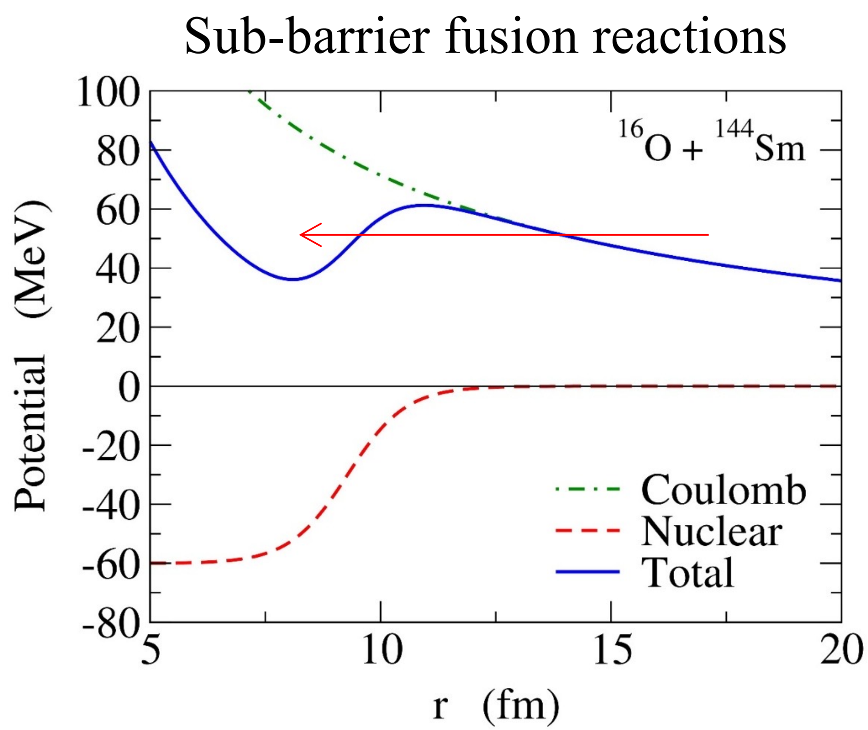


rotational spectrum

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

Sub-barrier fusion reactions and quantum tunneling

Fusion with quantum tunneling

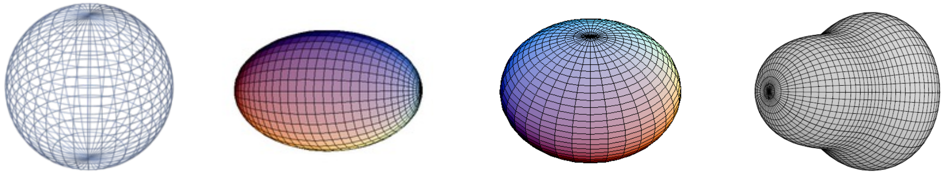


Sub-barrier fusion reactions and quantum tunneling

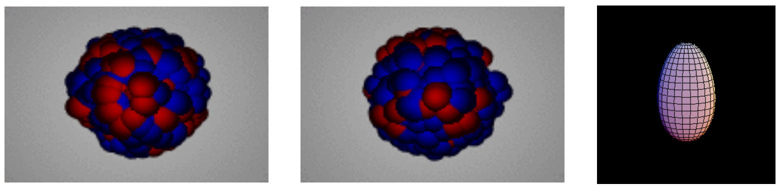
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



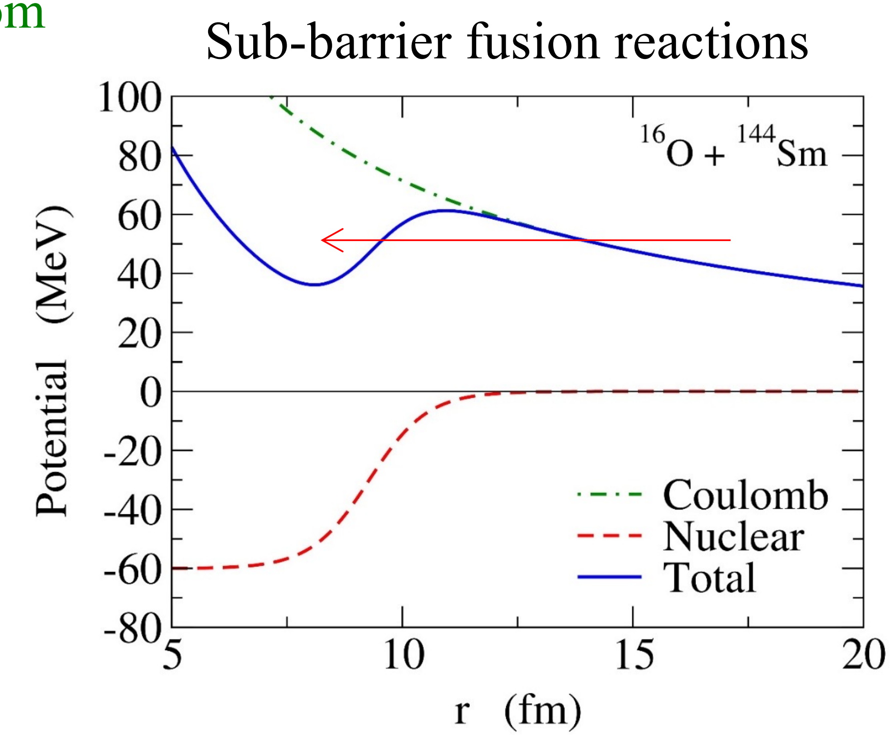
- several surface vibrations



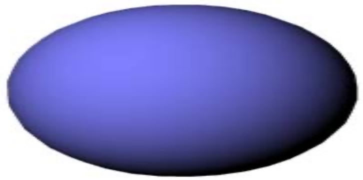
several modes and adiabaticities

- several types of nucleon transfers

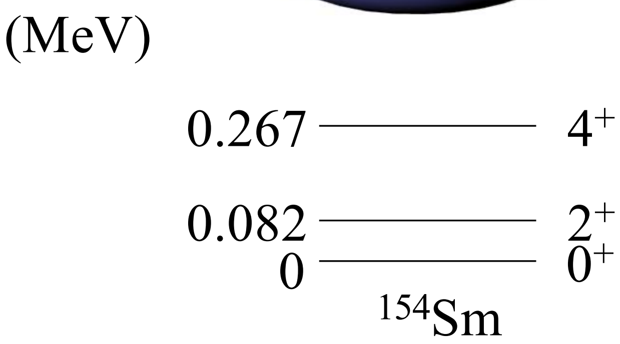
Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified



Effects of nuclear deformation on fusion



^{154}Sm



a small rotational energy

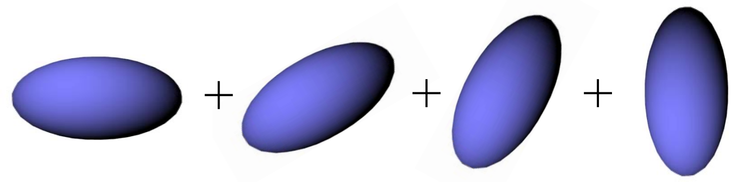
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

$\rightarrow$ a large moment of inertia J

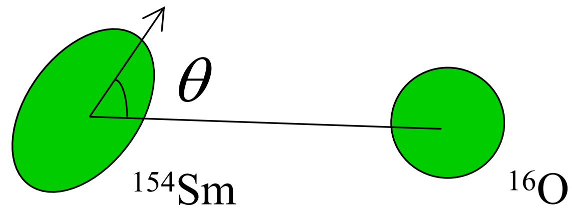
$\rightarrow$ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$



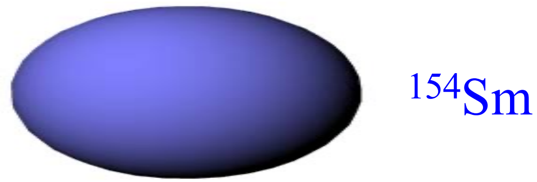
$\rightarrow$ first fix the orientation, and then take an average

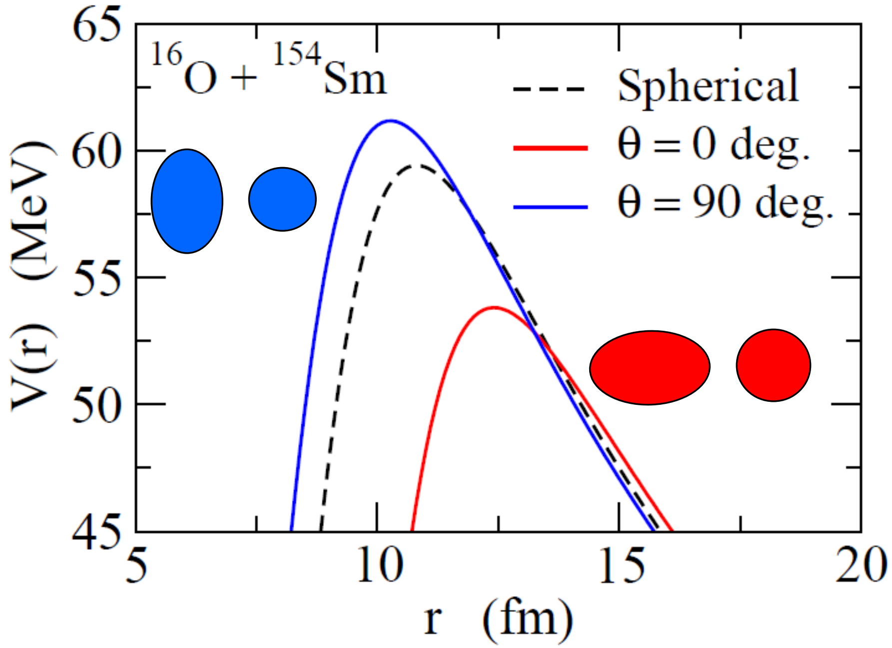


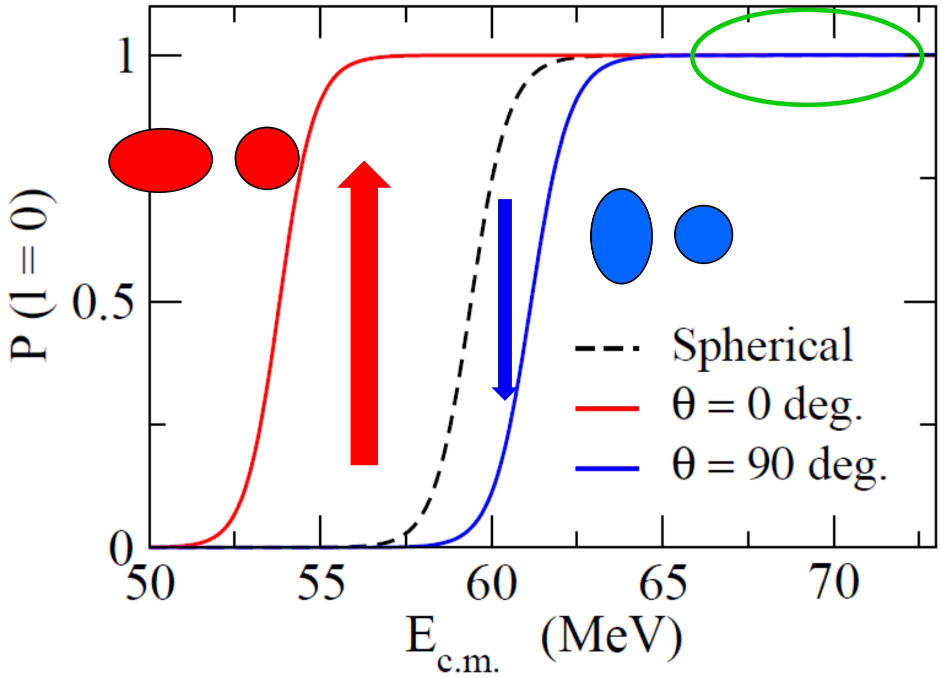
$$\boxed{\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)}$$

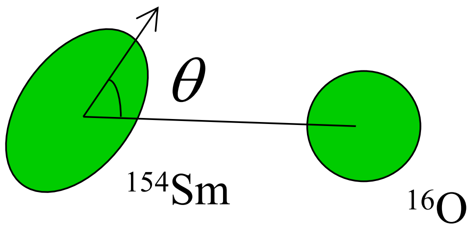
Effects of nuclear deformation on fusion

^{154}Sm : a typical deformed nucleus



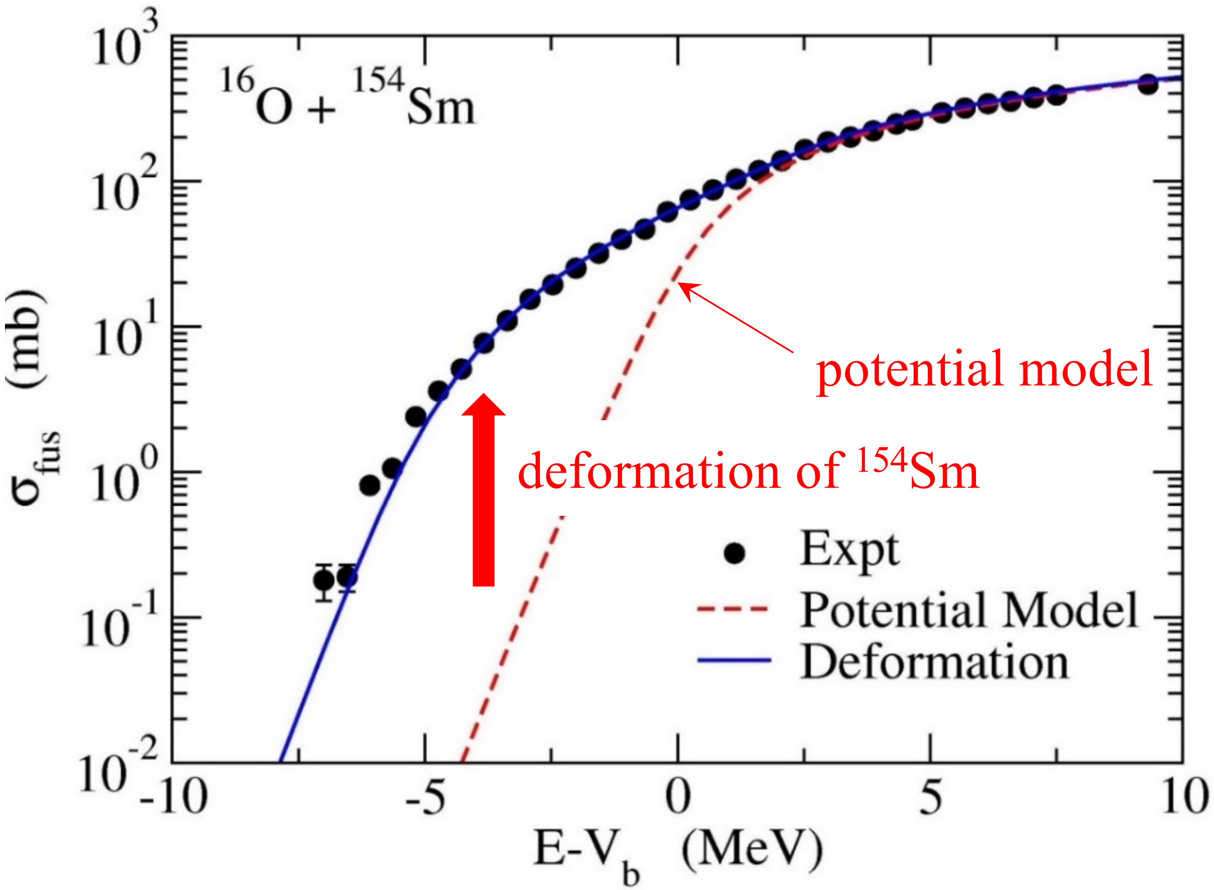




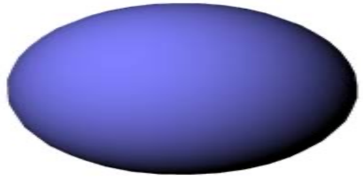


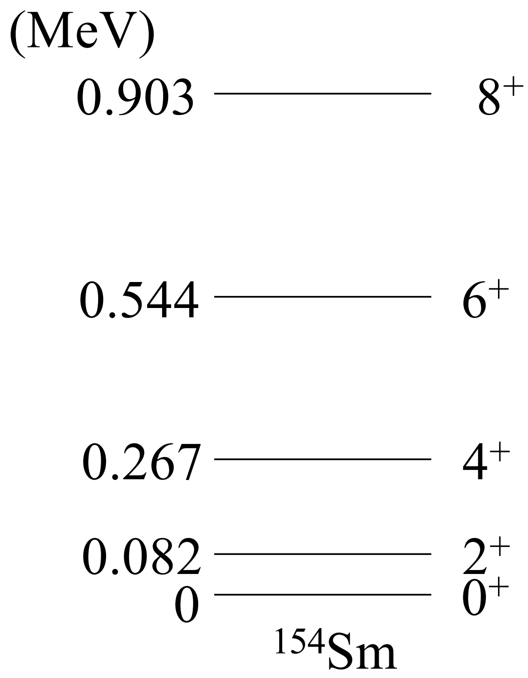
Fusion: strong interplay between nuclear structure and reaction

Sub-barrier fusion reactions and quantum tunneling



K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

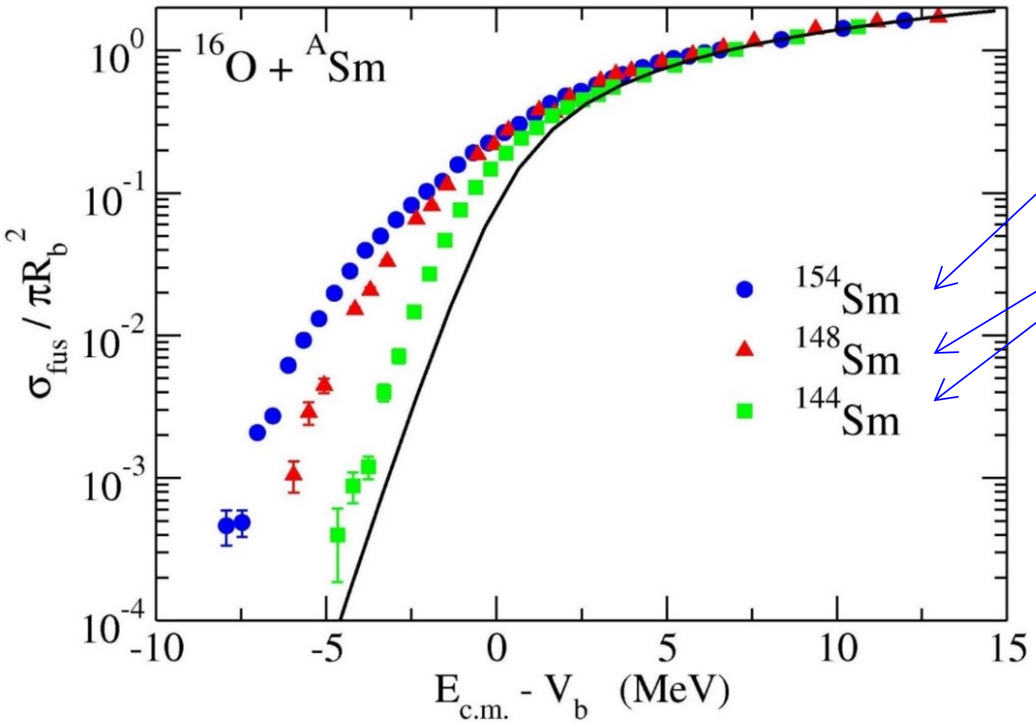




rotational spectrum

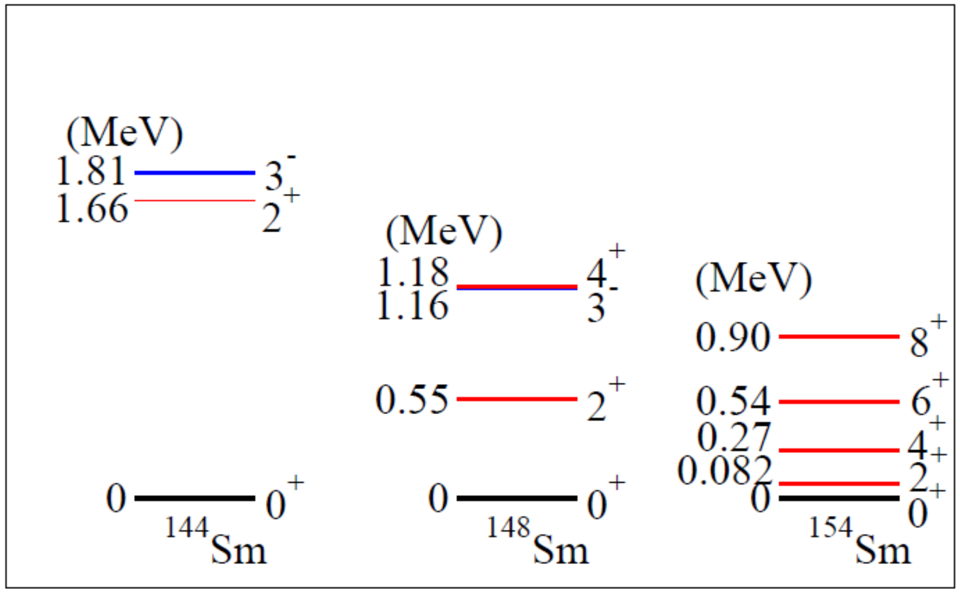
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

similar enhancement for non-deformed nuclei



static deformation

spherical, but with vibration



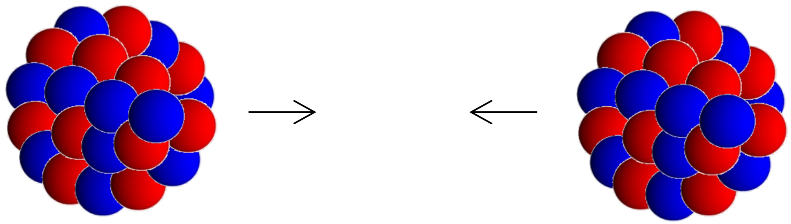
strong correlation with nuclear spectrum

→ coupling assisted tunneling phenomena

K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

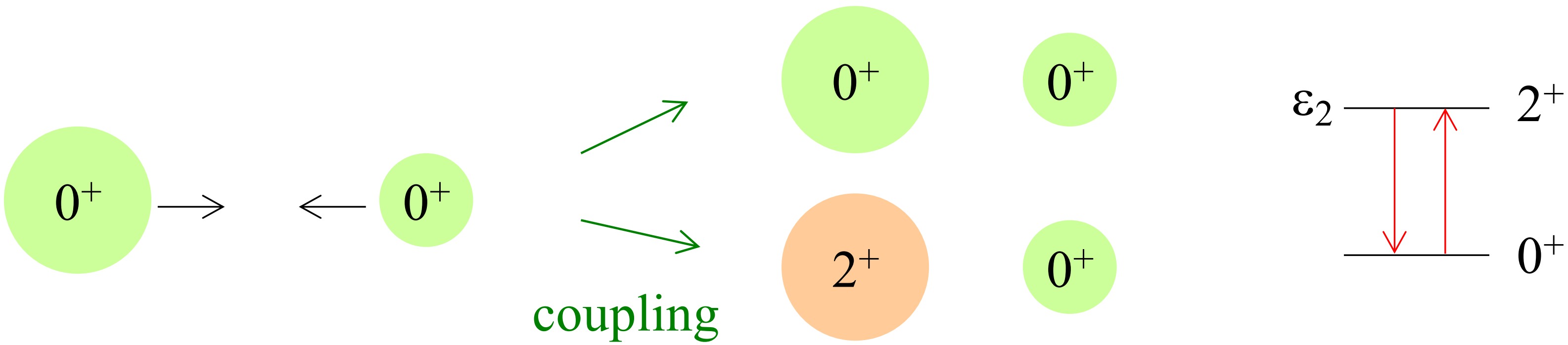
Coupled-channels method: a quantal scattering theory with excitations

many-body problem



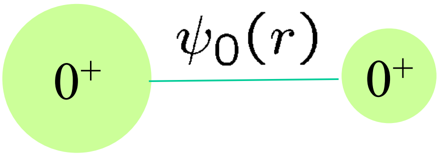
still very challenging

two-body problem, but with excitations : **the coupled-channels approach**



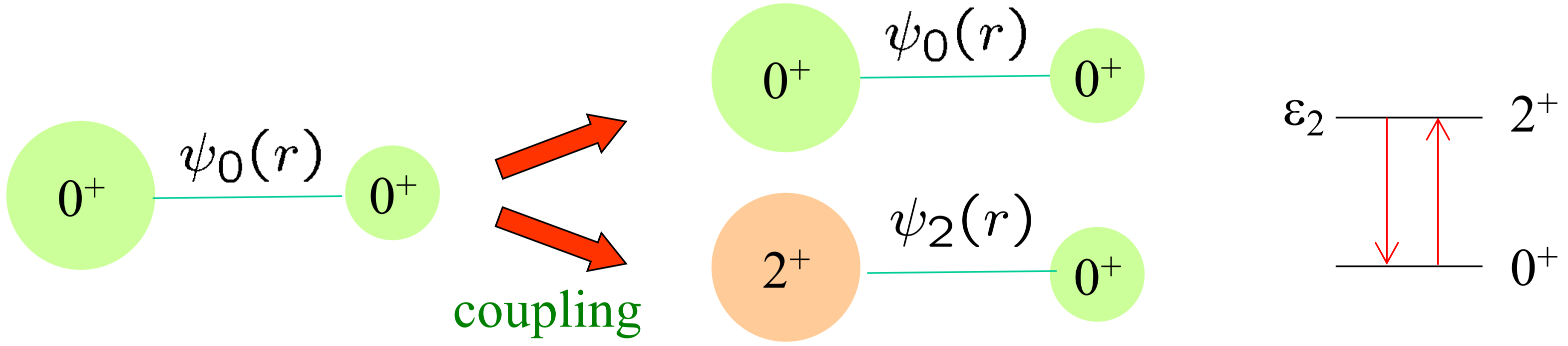
scattering theory with excitations

Coupled-channels method: a quantal scattering theory with excitations



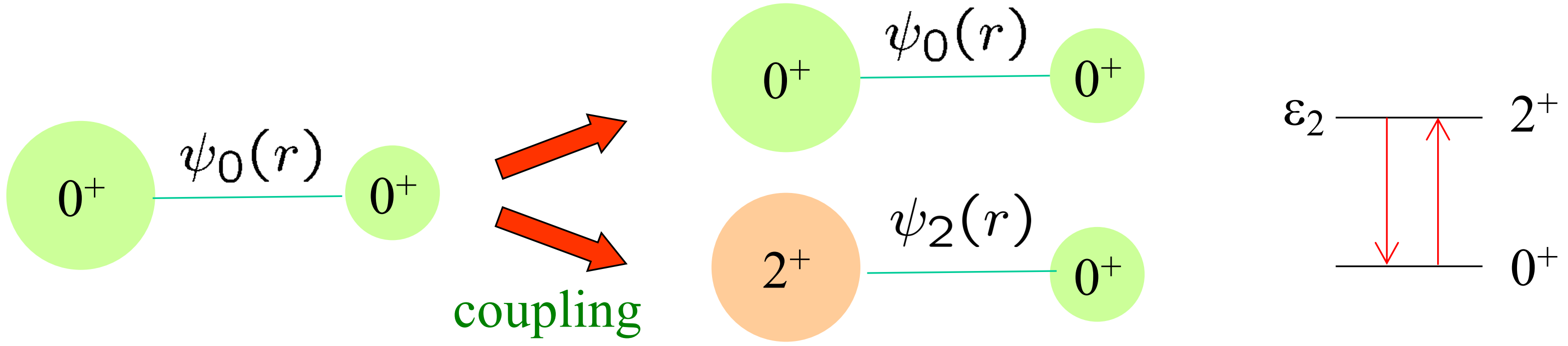
$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) - E\right]\psi_0(\boldsymbol{r}) = 0$$

Coupled-channels method: a quantal scattering theory with excitations



$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) - E\right]\psi_0(\boldsymbol{r}) = -F_{0\to 2}(r)\psi_2(\boldsymbol{r})$$

Coupled-channels method: a quantal scattering theory with excitations



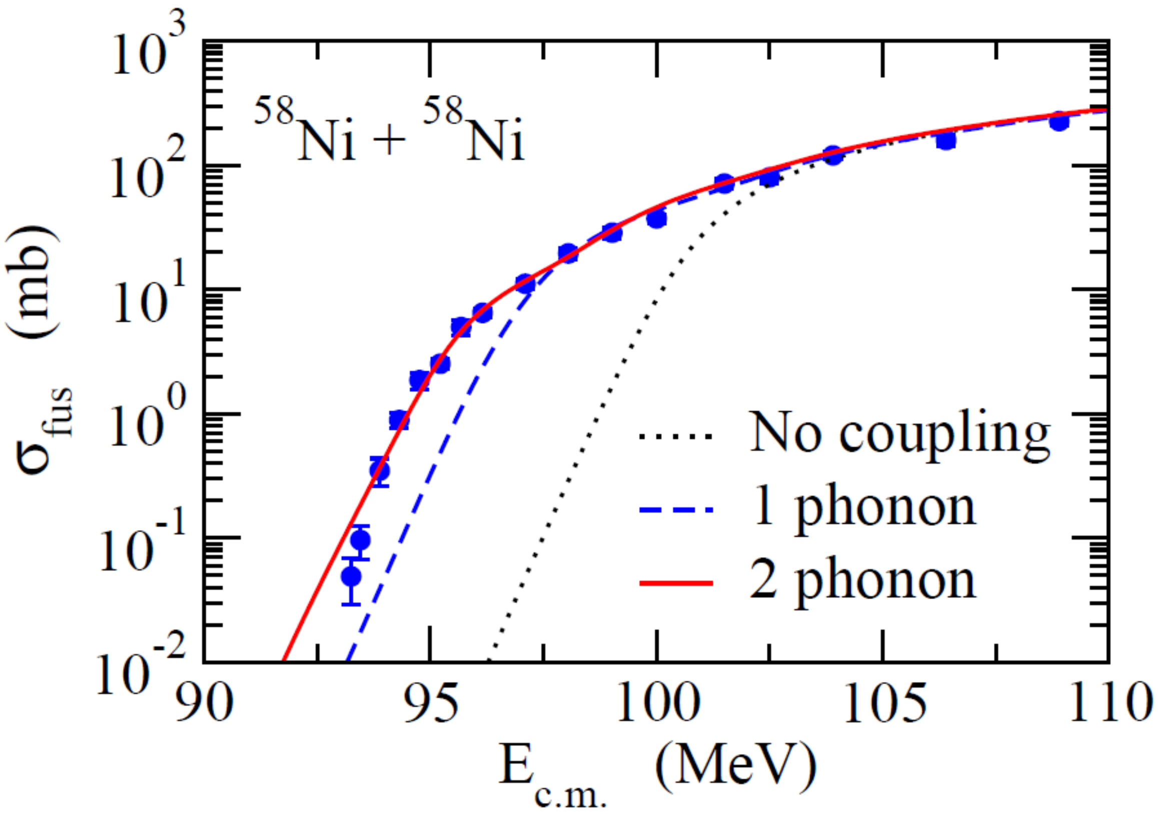
$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) - E\right]\psi_0(\boldsymbol{r}) = -F_{0\to 2}(r)\psi_2(\boldsymbol{r})$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_2(r) - (E-\epsilon_2)\right]\psi_2(\boldsymbol{r}) = -F_{2\to 0}(r)\psi_0(\boldsymbol{r})$$

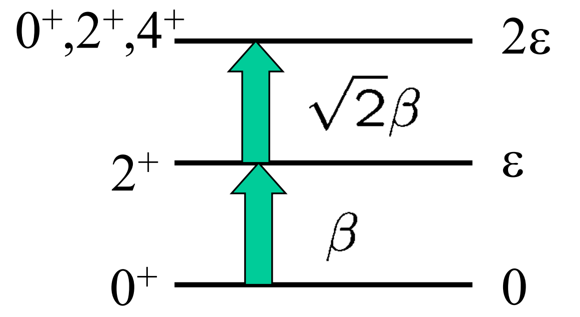
- Fusion → an absorbing potential (an optical potential)
- excitations to unbound states → breakup reactions (neutron-rich nuclei)

a recent review: K. Hagino, K. Ogata, and A.M. Moro,
Prog. in Part. and Nucl. Phys. 125, 103951 (2022).

Coupled-channels method: a quantal scattering theory with excitations



simple harmonic oscillator



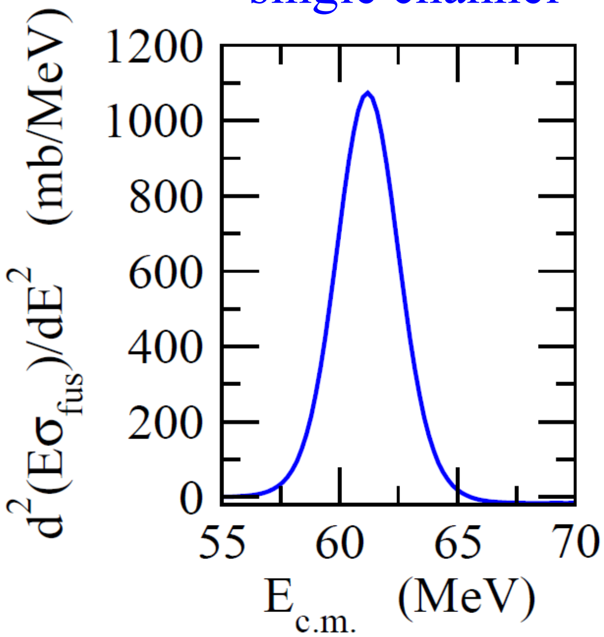
the fortran code CCFULL: K. Hagino, N. Rowley, and A.T. Kruppa, CPC123 (‘99) 143)

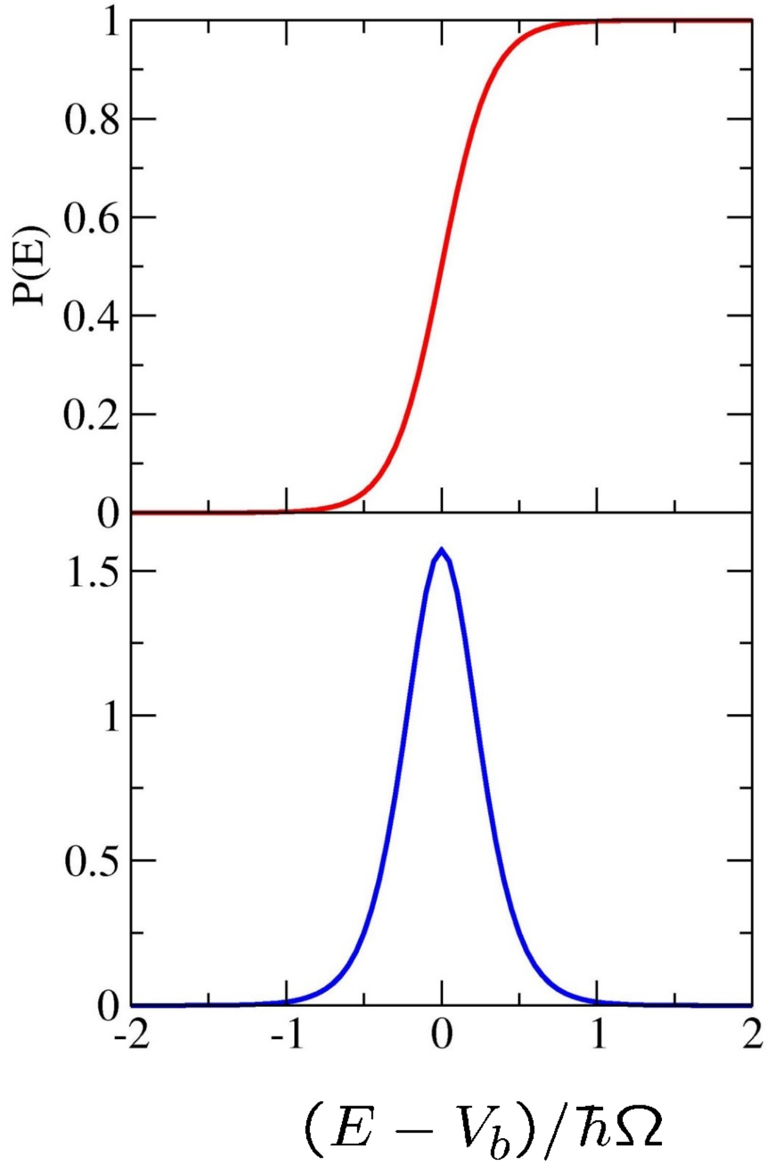
Fusion barrier distribution

$$\boxed{D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}} \quad \propto \frac{dP_{l=0}}{dE}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

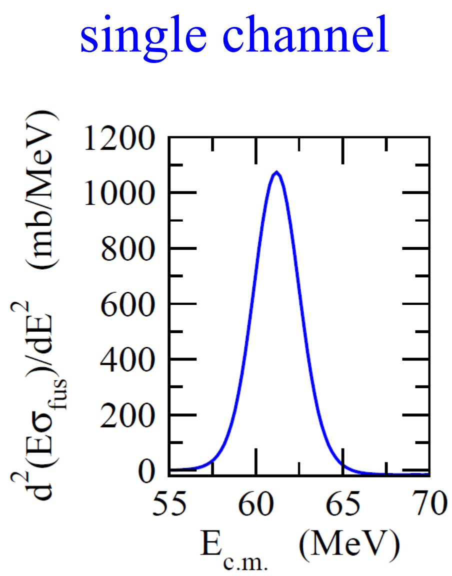
single channel

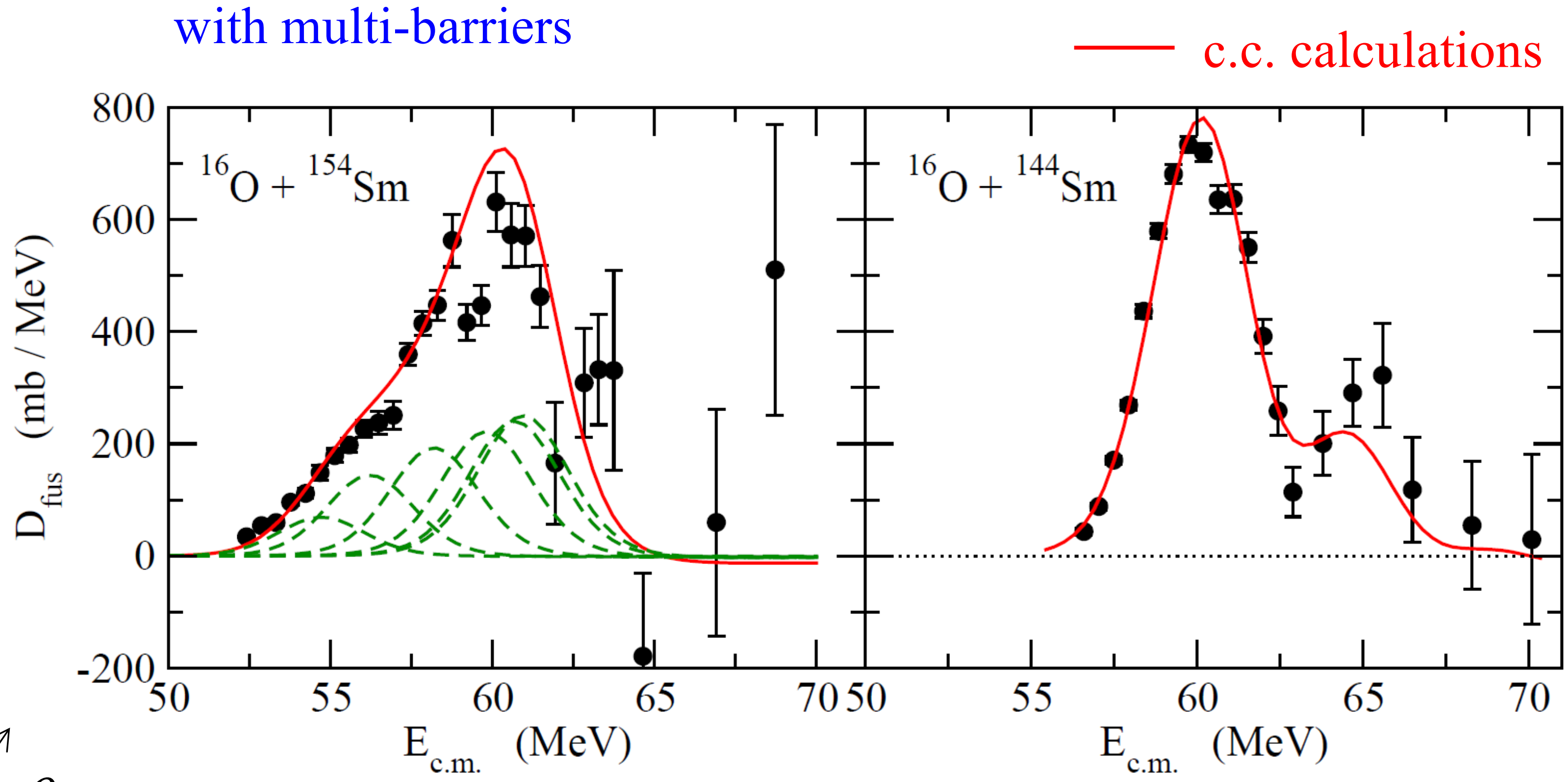


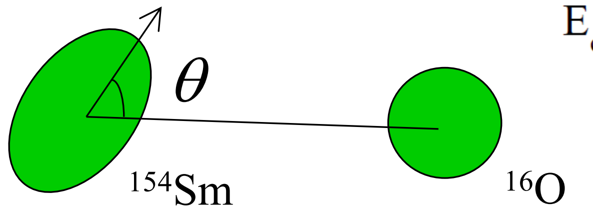


<u>Fusion barrier distribution</u>

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2} \propto \frac{dP_{l=0}}{dE}$$



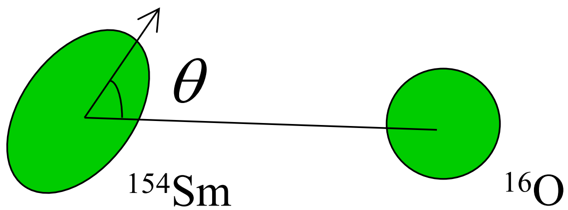


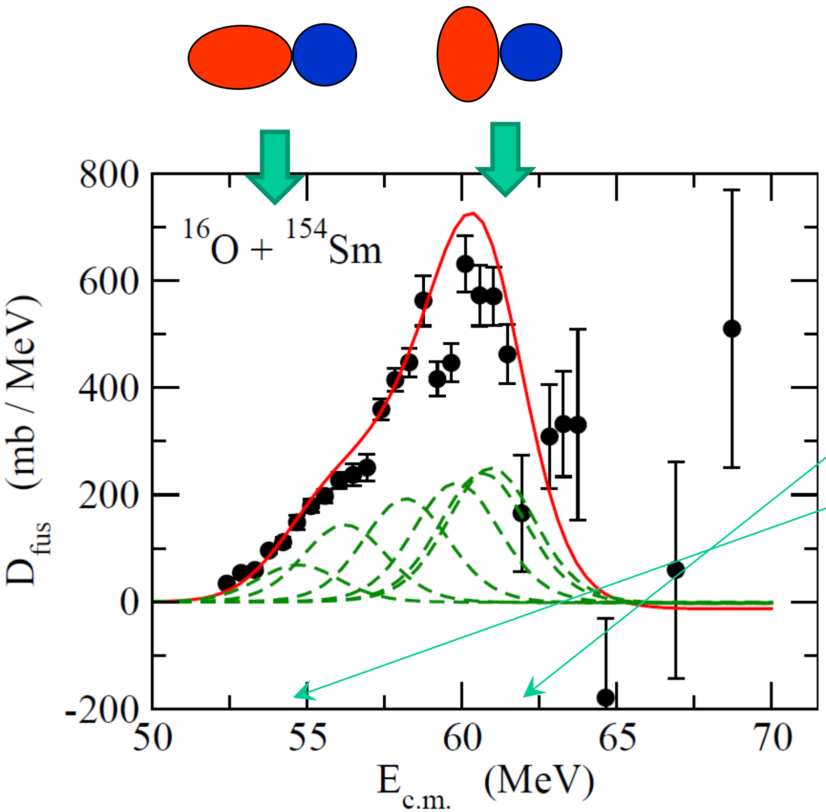


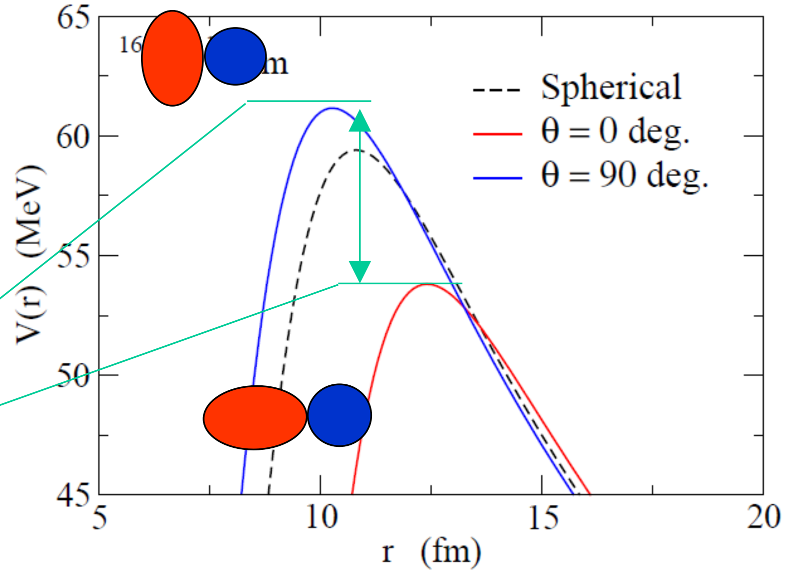
K.H. and N. Takigawa, PTP128 ('12) 1061

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91) 25)

$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2} \sim \pi R_b^2 \frac{dP_{l=0}}{dE}$$







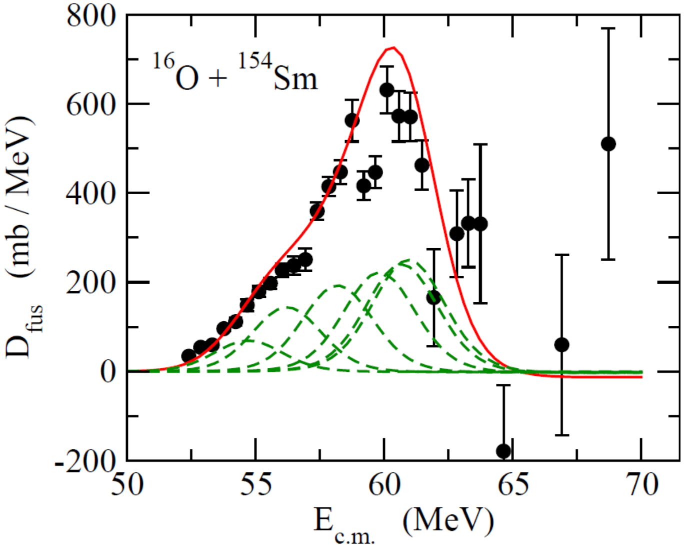
$$\boxed{\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)}$$

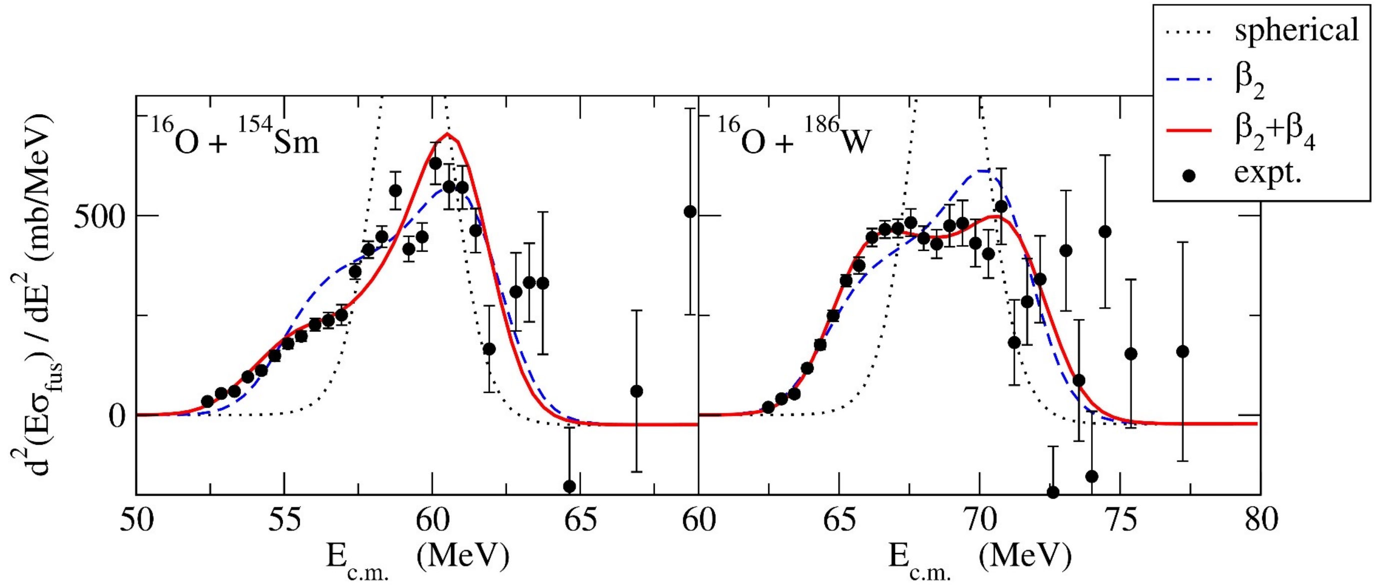
Data: J.R. Leigh et al., PRC52 (1995) 3151

Fusion barrier distribution

$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

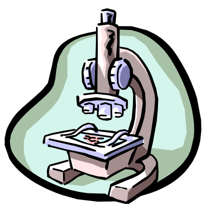


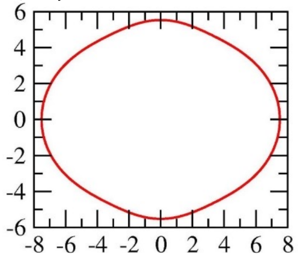


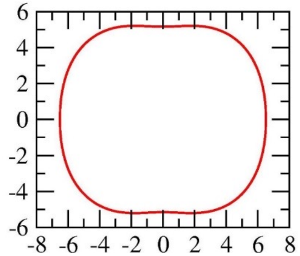
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$$\beta_2 = 0.33 \qquad \beta_2 = 0.29$$

$$\beta_4 = +0.05 \qquad \beta_4 = -0.03$$







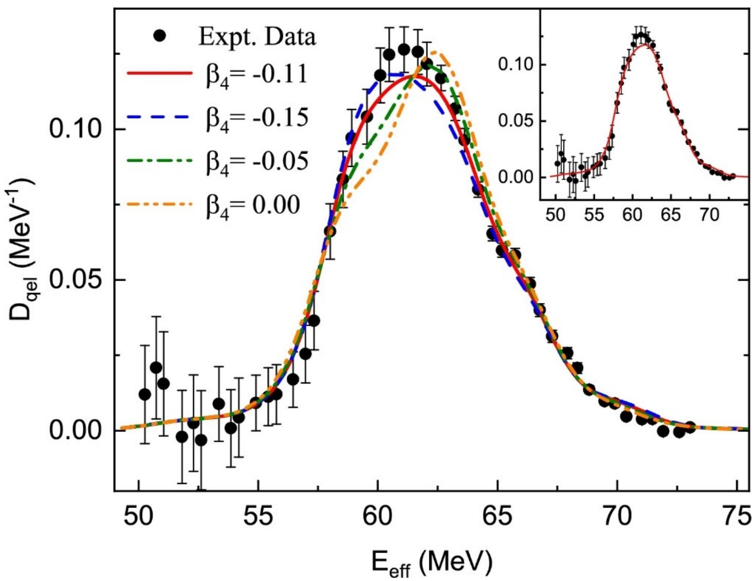
sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

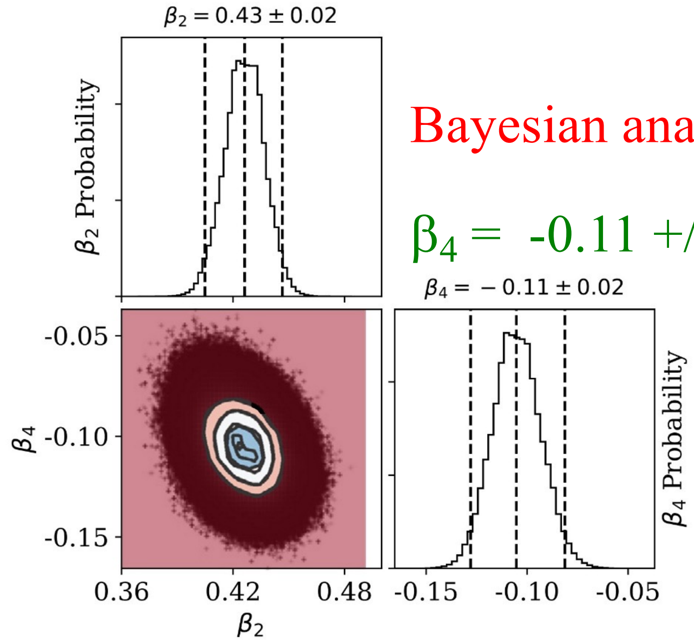
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



$\beta_2 =$ 0.43 +/- 0.02



Bayesian analysis

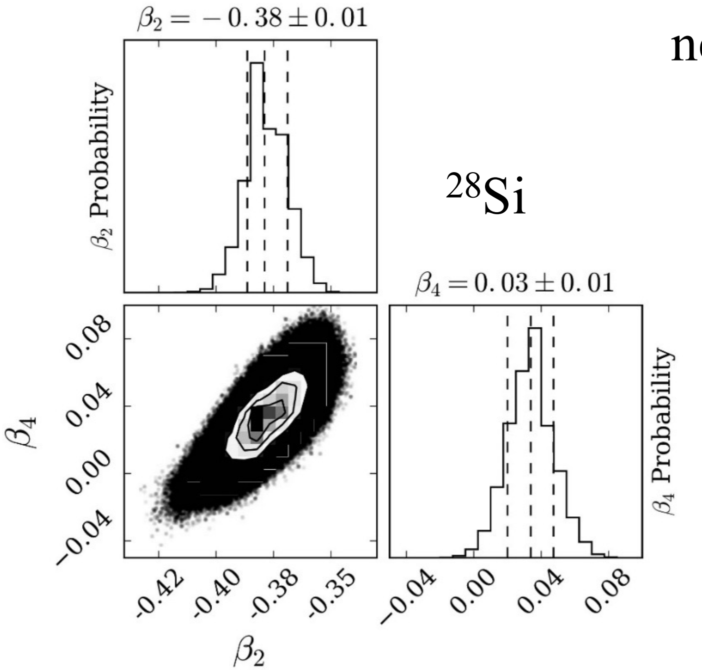
$\beta_4 =$ -0.11 +/- 0.02

high precision determination of β_4
→ for the first time

cf. (p,p'): β_4 = -0.05 +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

Emulator for multi-channel scattering



^{28}Si

Y.K. Gupta et al.,
PLB845, 138120 (2023).

needs to repeat many calculations with different (β_2,β_4)

→ an emulator to speed-up the calculations

Eigenvector continuation

➢ bound state problems:

$$H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle \qquad \Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

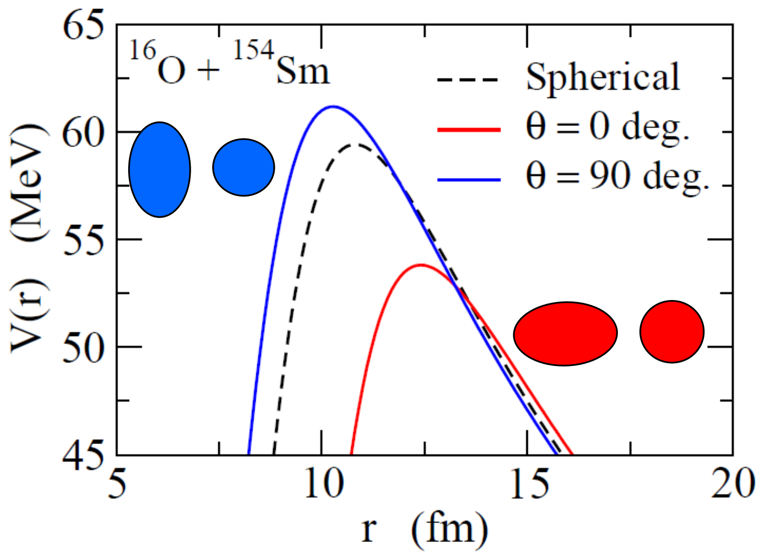
T. Duguet et al.,
Rev. Mod. Phys. 96, 031002 (2024)

➢ Extension to scattering problems:

- R. Furnstahl et al., PLB809, 135719 (2020)
- C. Drisshler et al., PLB823, 136777 (2021)
- J. Liu, J. Lei, and Z. Ren, PLB858, 139070 (2024)
- K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, Phys. Rev. C112, 024618 (2025).

Taking a snapshot of a nucleus with a "fast" nuclear reaction

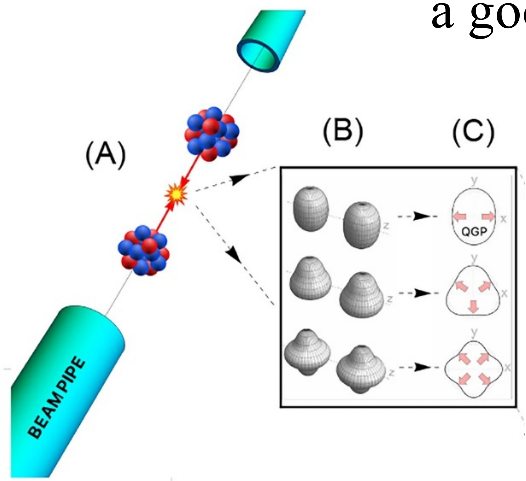
low-energy H.I. fusion reactions of a deformed nucleus



in recent years increasing interests in:

relativistic H.I. collisions with a deformed nucleus

a good probe of nuclear shapes



J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

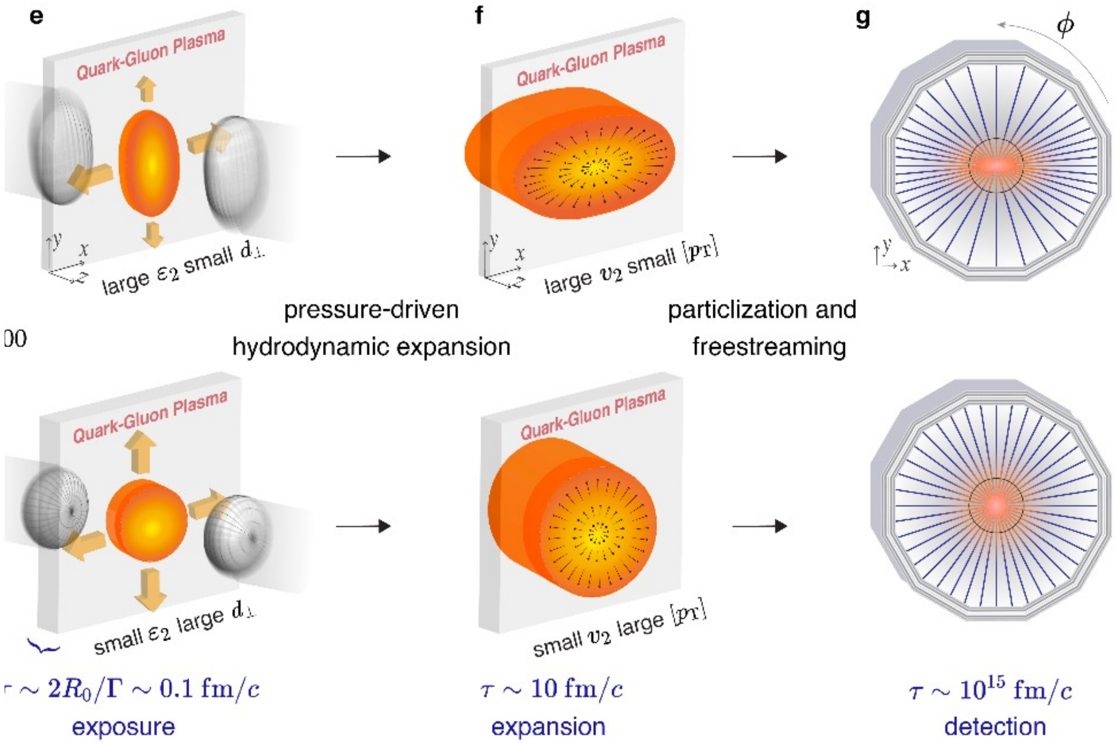
large contributions from China

Jiangyon Jia 贾江涌
Chunjian Zhang 張春健
Huichao Song 宋慧超
You Zhou 周铀
Xiaomei Li 李笑梅
....

Large similarities → intersection of **High *E*** and **Low *E*** HI collisions

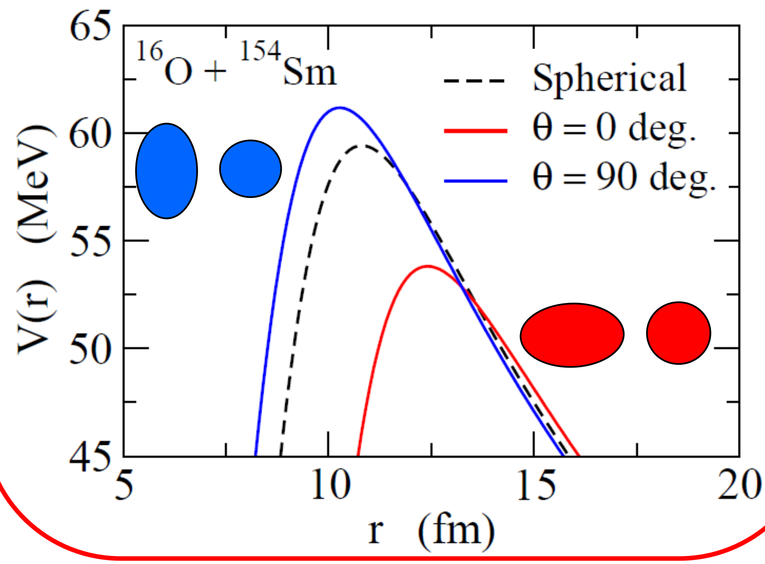
Probing nuclear shapes in Relativistic Heavy-Ion collisions

the initial shape of QGP **expansion** **detection**



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

large similarities to
low-energy H.I. fusion reactions
of deformed nuclei



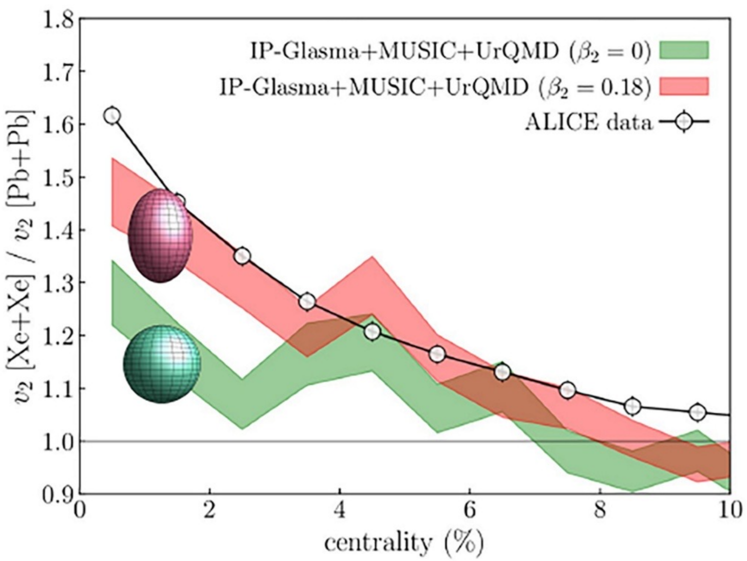
→ an intersection of
High *E* and **Low *E*** H.I. collisions

Probing nuclear shapes in Rel. H.I. collisions

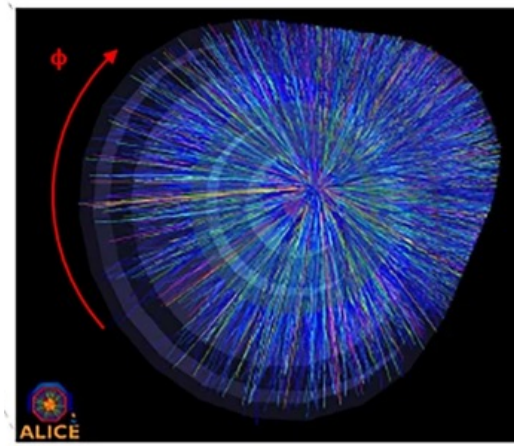
flow:
the final N-distribution

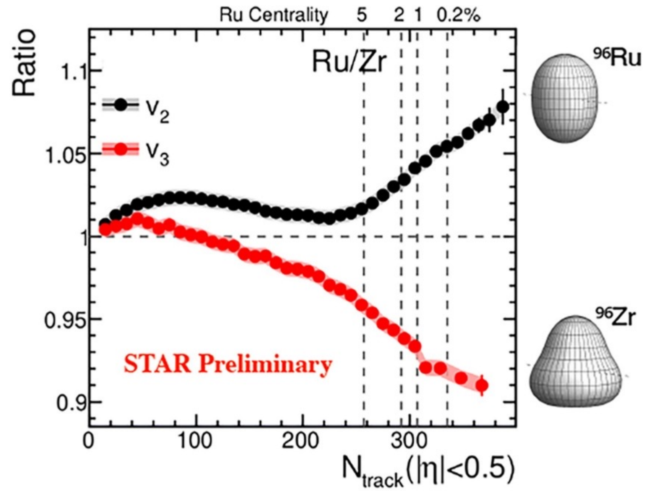
$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos n(\phi - \Psi_n)\right]$$

elliptic（楕円）flow v_2



the ratio of ^{129}Xe+^{129}Xe to ^{208}Pb+^{208}Pb
→ quadrupole deformation of ^{129}Xe





the ratio of ^{96}Ru+^{96}Ru to ^{96}Zr+^{96}Zr
→ octupole deformation of ^{96}Zr

J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

Probing nuclear shapes in Rel. H.I. collisions

flow:
the final N-distribution

$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_{n} v_n \cos n(\phi - \Psi_n)\right]$$

other examples:

✓ triaxial (γ) deformation

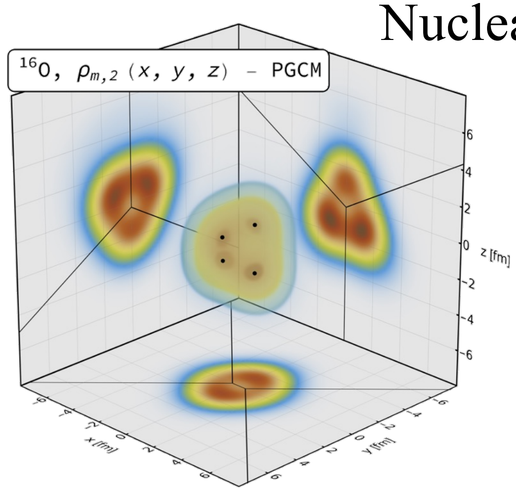
  

- G. Aad et al., PRC107, 054910 (2023)
- STAR collaboration, Nature 635, 67 (2024)

✓ α cluster

- ^{16}O+^{16}O: ALICE, preliminary
- Y. Wang et al., PRC109, L051904 (2024)

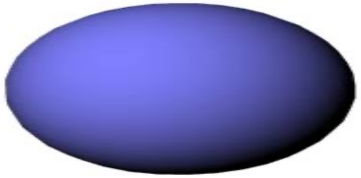
Nuclear Lattice EFT



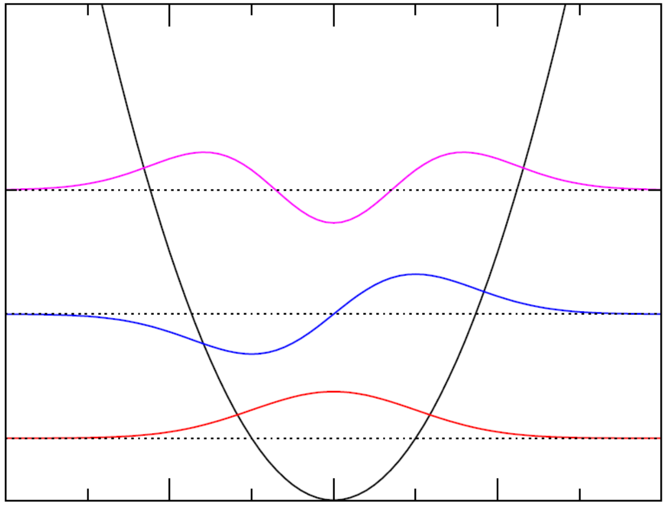
G. Giacalone et al., arXiv: 2402.05995

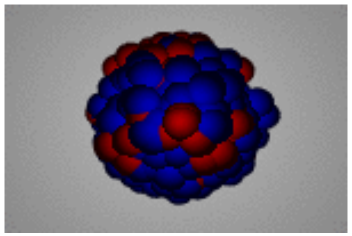
Probing nuclear shapes in Relativistic Heavy-Ion collisions

In addition to static deformation,



there also exist several <u>dynamical</u> deformations of a spherical nucleus





$$\langle \beta \rangle = 0$$

but fluctuates around β=0
(zero-point motion)

cf. 1-dim. H.O.

$$\phi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

$$\rightarrow \langle x \rangle = 0, \quad \langle x^2 \rangle = 1/2\alpha$$

a typical example:
2_1^+ at 1.45 MeV in ^{58}Ni

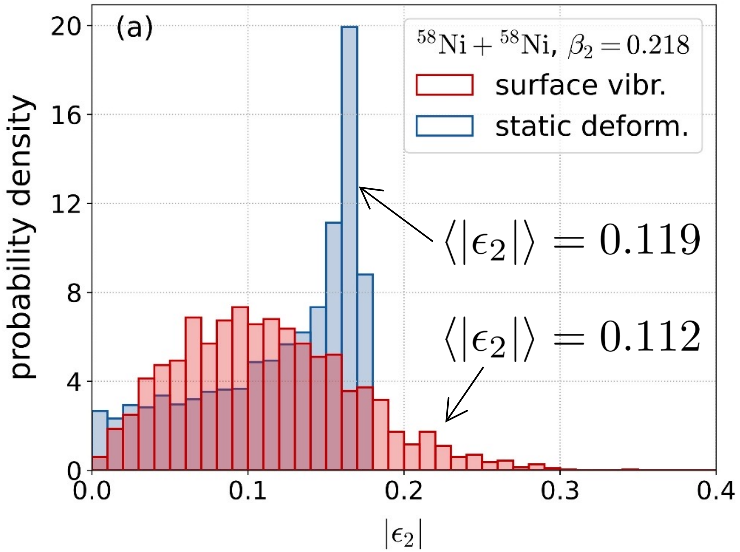
Probing nuclear shapes in Relativistic H.I. collisions

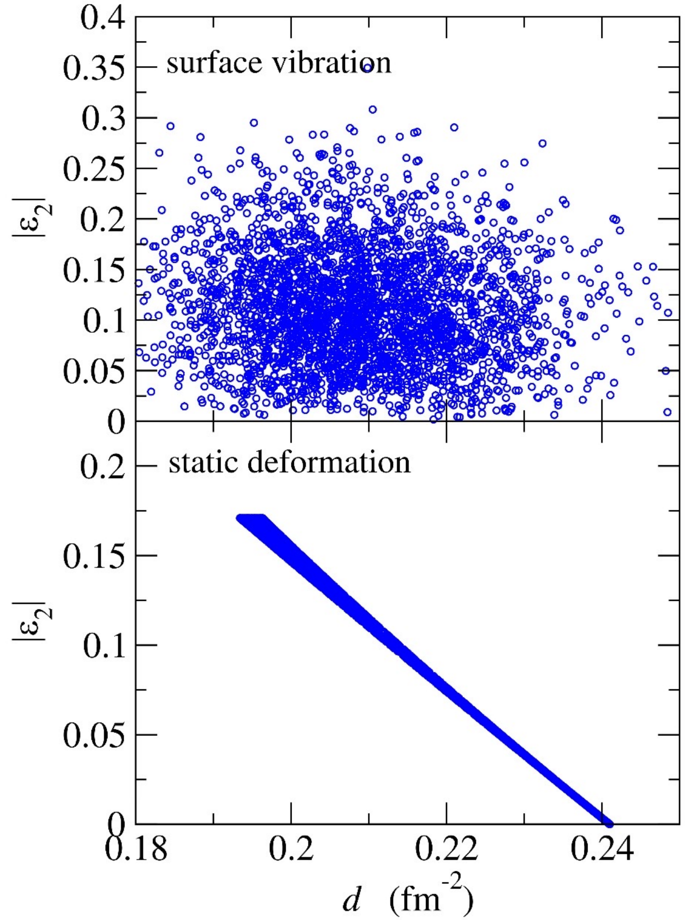
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

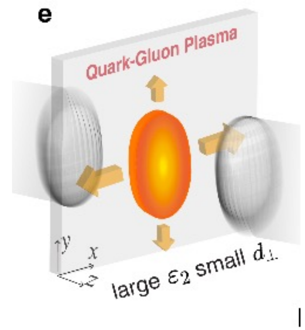
the eccentricity parameter:

$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle (x-iy)^2\rangle}{\langle x^2+y^2\rangle}$$

Monte Carlo sampling (3000 samples)







the inverse area

$$d \equiv \frac{1}{\sqrt{\langle x^2\rangle\langle y^2\rangle}} \leftrightarrow \bar{p}_T$$

K. Hagino and M. Kitazawa, Phys. Rev. C112, L041910 (2025).