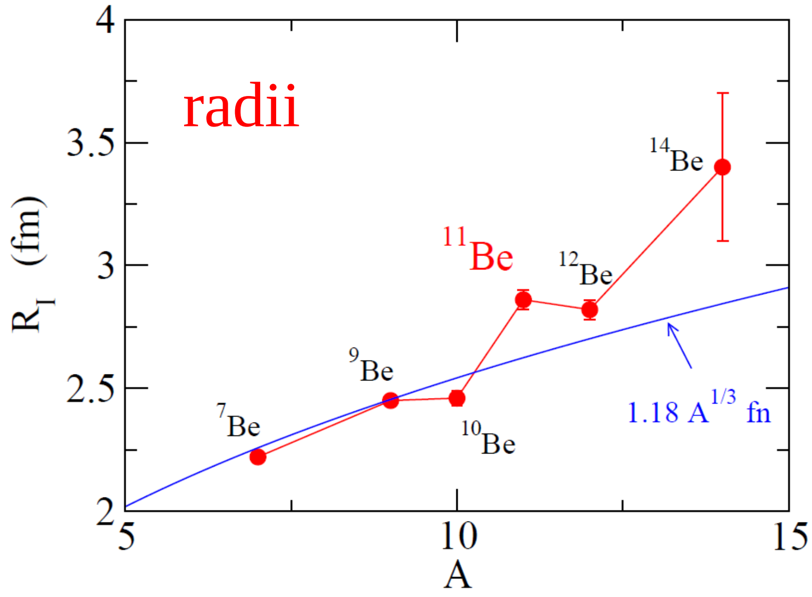
Properties of 1n halo nuclei

- bound state
- role of angular momentum
- Coulomb excitations and sum rules

what is a 1n halo nucleus?

a typical example: $^{11}_{4}Be_{7}$



I. Tanihata et al.,
PRL55('85)2676; PLB206('88)592

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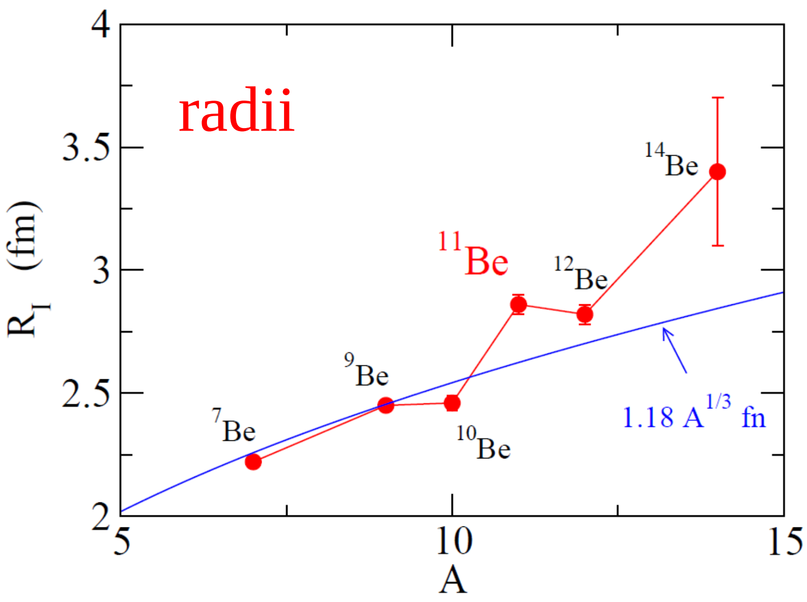

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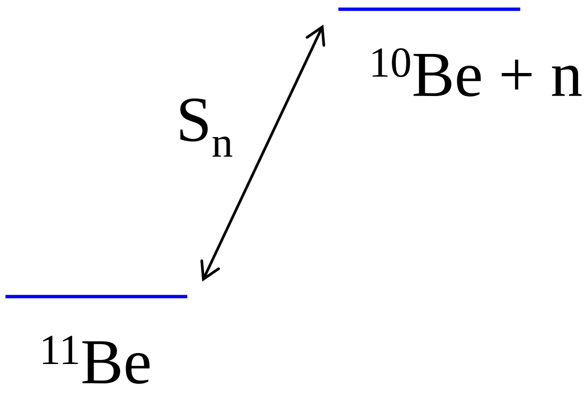
PRL55('85)2676; PLB206('88)592

1n separation energy



S_n = 504 +/- 6 keV

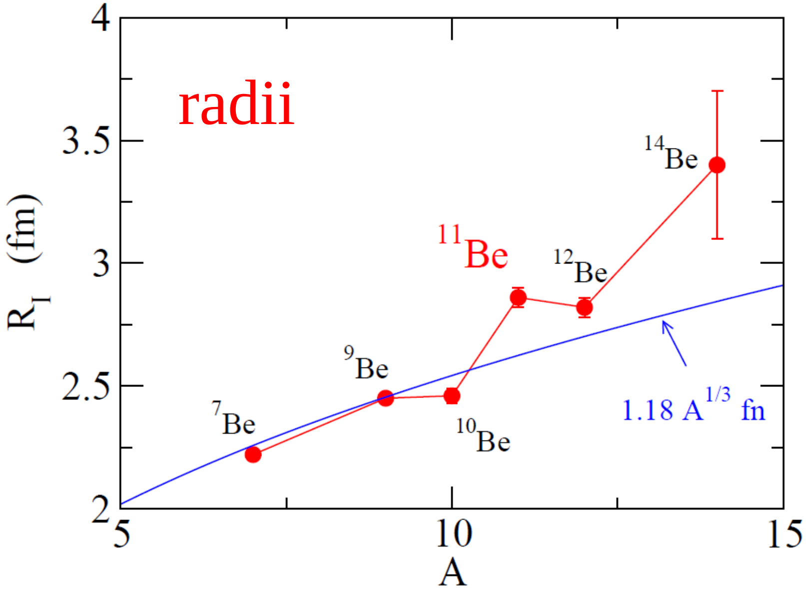
very small

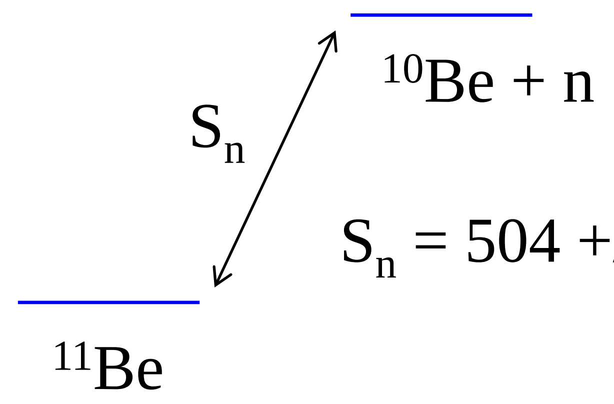
cf. S_n = 4.95 MeV for ^{13}C

1n halo nuclei

1n separation energy

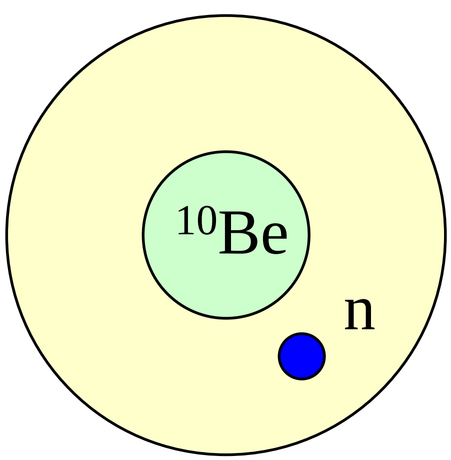
a typical example: $^{11}{}_{4}Be_{7}$





S_n = 504 +/- 6 keV

Interpretation: weakly bound neutron around ^{10}Be

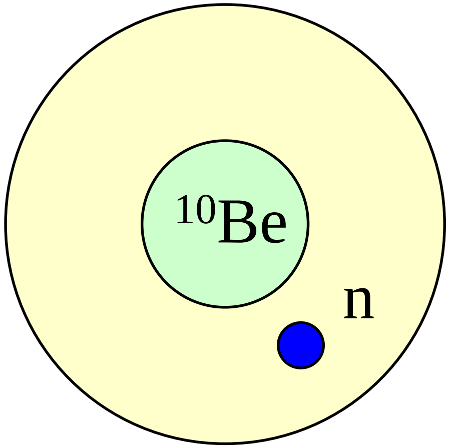


$$\psi(r) \sim \exp(-\kappa r) \qquad \kappa = \sqrt{2m|\epsilon|/\hbar^2}$$

a weakly bound system

extended spatial distribution (halo structure)

Interpretation: weakly bound neutron around ^{10}Be



$$\psi(r) \sim \exp(-\kappa r) \qquad \kappa = \sqrt{2m|\epsilon|/\hbar^2}$$

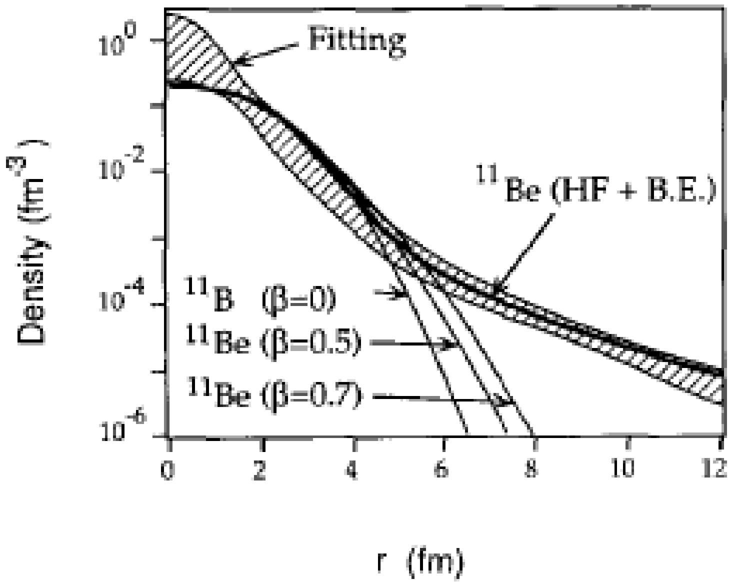
a weakly bound system

↓

extended spatial distribution (halo structure)

Moon halo

density distribution that explains
the measured cross section



M. Fukuda et al., PLB268('91)339

momentum distribution

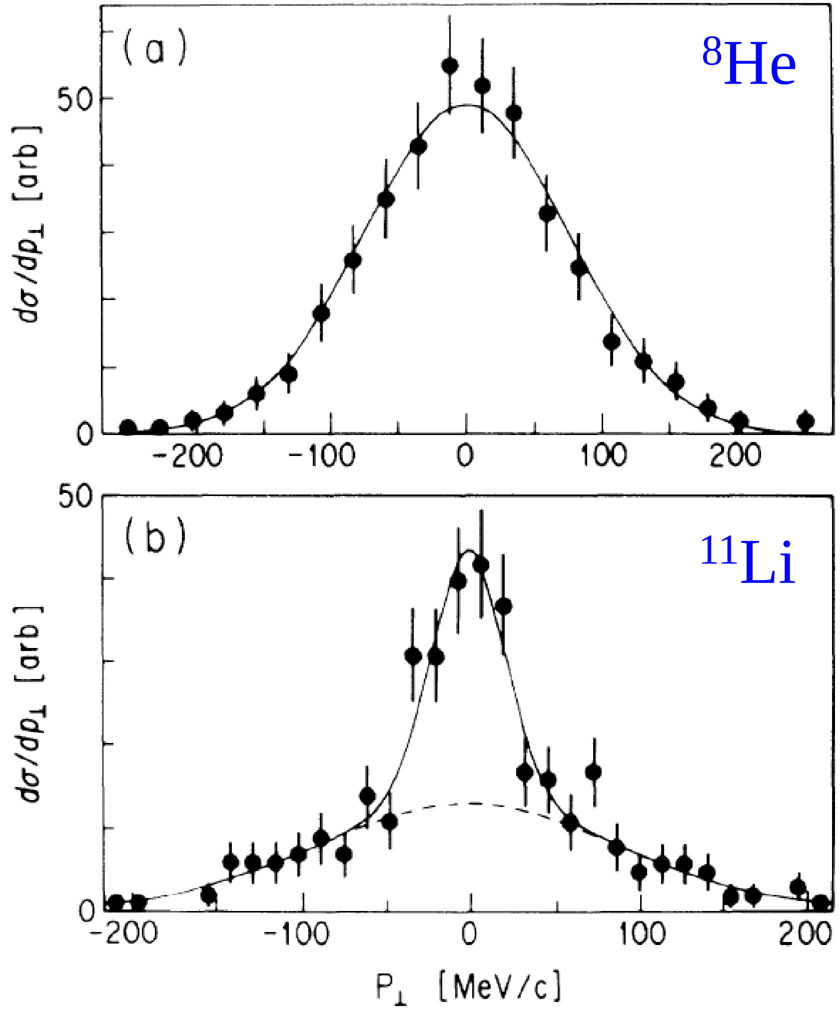


FIG. 1. Transverse-momentum distributions of (a) ^{6}He fragments from reaction ^{8}He+C and (b) ^{9}Li fragments from reaction ^{11}Li+C. The solid lines are fitted Gaussian distributions. The dotted line is a contribution of the wide component in the ^{9}Li distribution.

S_{2n} ~ 2.1 MeV

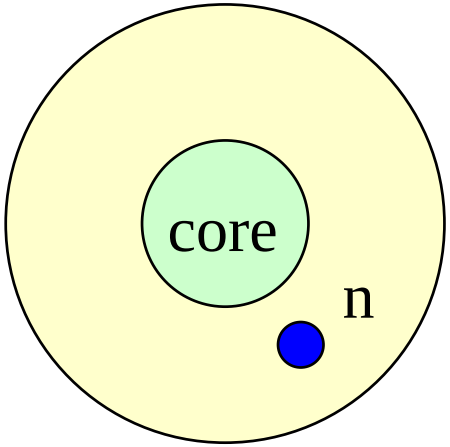
S_{2n} ~ 300 keV

spatial extension
→ narrow momentum distribution

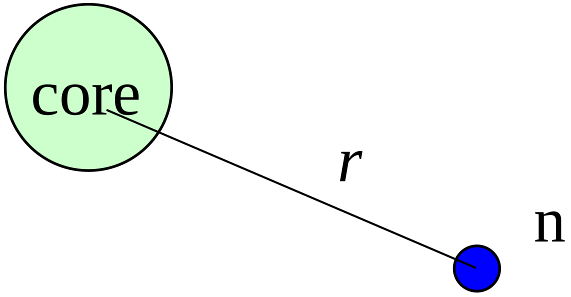
↔ neutron halo

T. Kobayashi et al., PRL60 ('88) 2599

one particle motion: a bound state



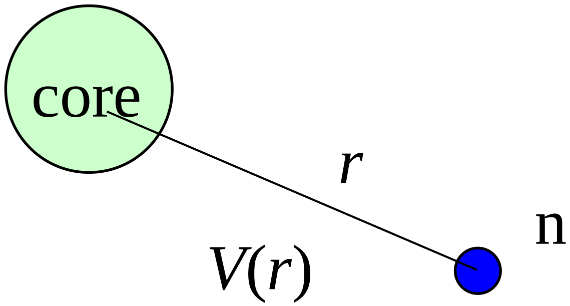
2-body problem of a core nucleus +
a valence neutron



Assume a spherical potential $V(r)$ as a function of r

The Hamiltonian for the relative motion:

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$

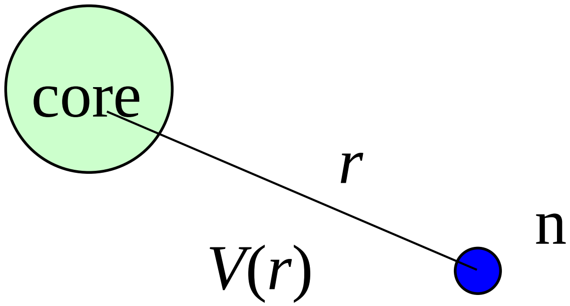


The Hamiltonian for the relative motion:

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$

For simplicity, let us ignore the spin-orbit interaction
(the essence remains the same even without the *ls* interaction)

$$\Psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}})\,\chi_{m_s}$$



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$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - \epsilon_l\right] u_l(r) = 0$$

Boundary condition (for a bound state):

$$\begin{aligned} u_l(r) &\sim r^{l+1} \qquad (r \sim 0) \\ &\to e^{-\kappa r} \qquad (r \to \infty) \end{aligned}$$

More precisely, the modified spherical Bessel function (the spherical Hankel)

angular momentum and neutron halo

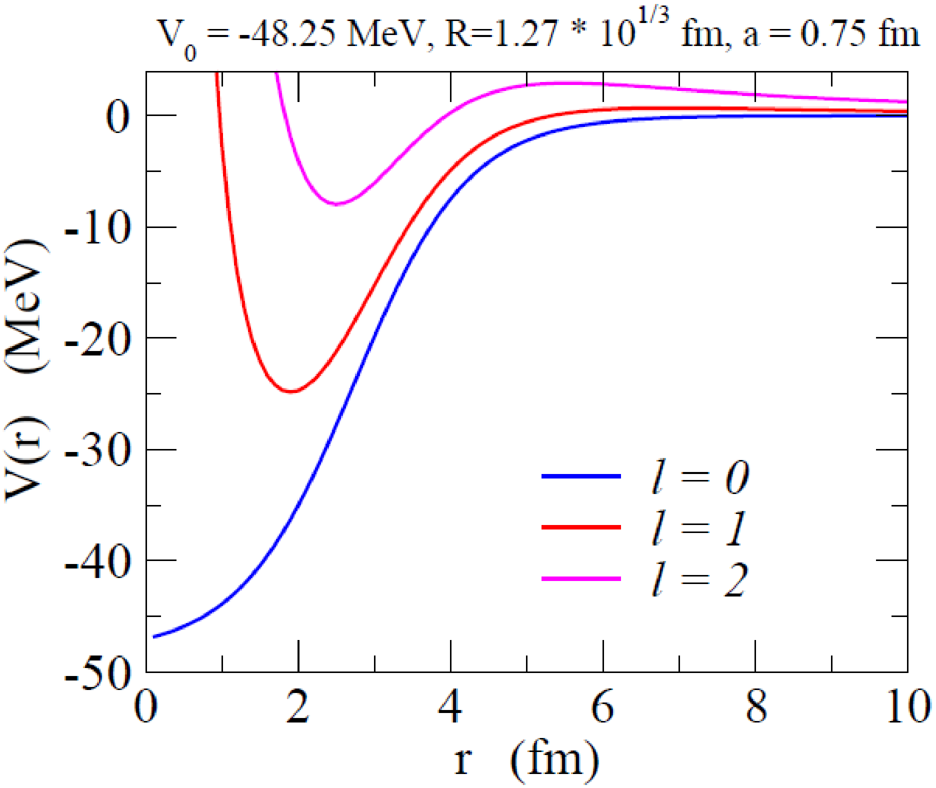
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \underline{\frac{l(l+1)\hbar^2}{2\mu r^2}} + V(r) - \epsilon_l\right] u_l(r) = 0$$

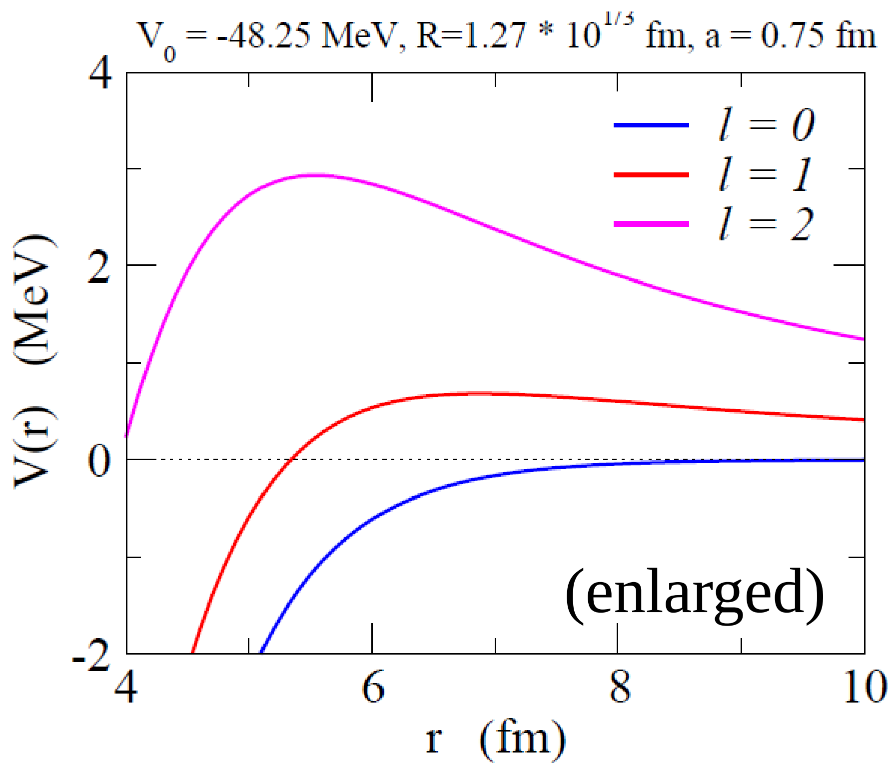
centrifugal potential

angular momentum and neutron halo

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V(r)-\epsilon_l\right]u_l(r)=0$$

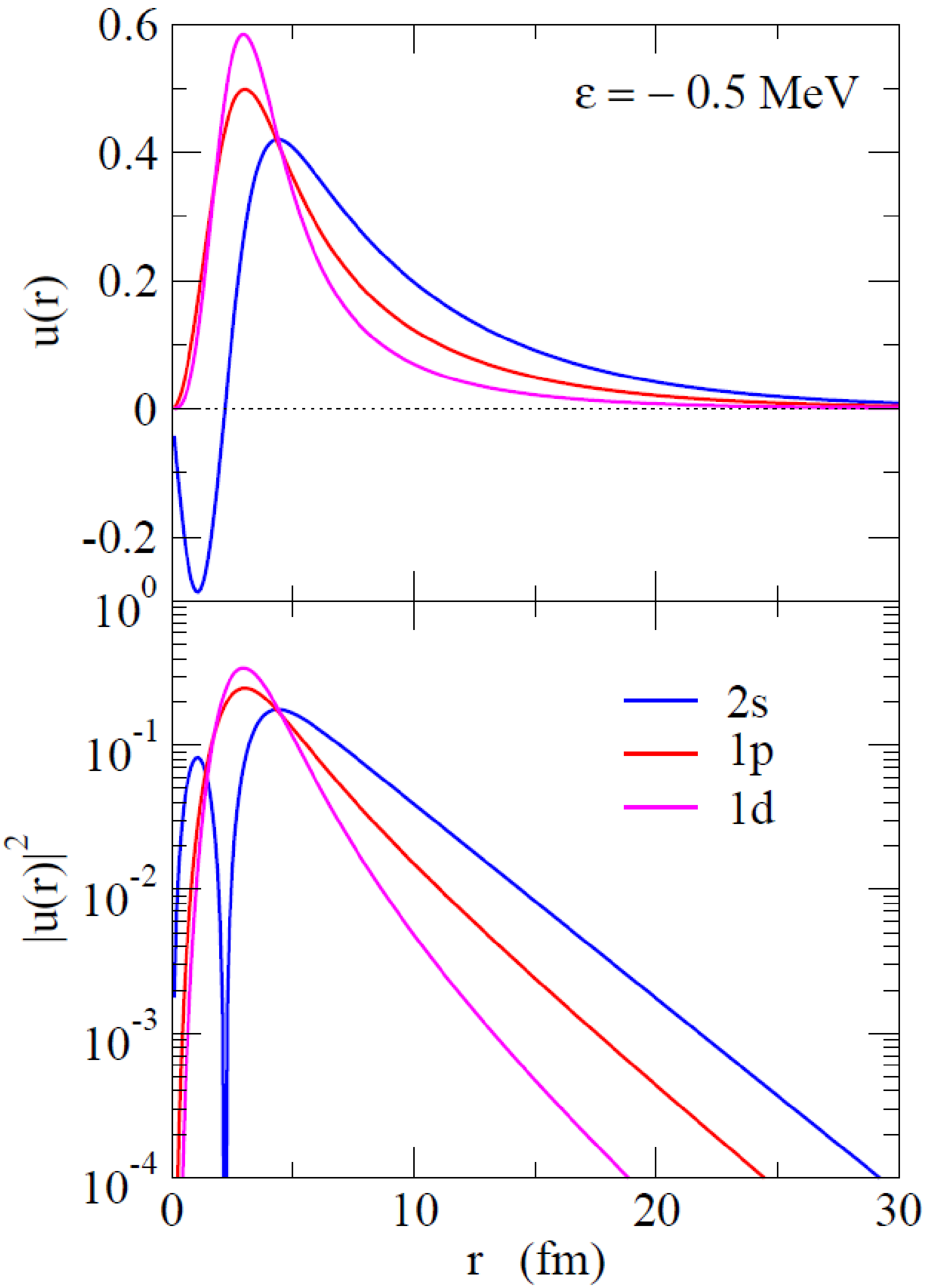
centrifugal potential





the height of the centrifugal barrier:
0 MeV (l = 0), 0.69 MeV (l = 1), 2.94 MeV (l = 2)

wave function adjust V_0 for each l so that ε = -0.5 MeV



l = 0 :a long tail

l = 2 : a localization

l = 1 : intermediate

the root-mean-square radius:

$$\sqrt{\langle r^2 \rangle} = \sqrt{\int_0^\infty dr\, r^2 u_l(r)^2}$$

7.17 fm (l = 0)

5.17 fm (l = 1)

4.15 fm (l = 2)

behavior at zero binding

K. Riisager, A.S. Jensen, and P. Moller, NPA548 ('92) 393

$$I_n \equiv \int_0^\infty dr\, r^{n+2} |R_l(r)|^2$$

$$R_l(r) = \frac{u_l(r)}{r}$$

how does this behave as $\kappa \to 0$?

$$\kappa = \sqrt{2\mu|\epsilon|/\hbar^2}$$

$r > R$: $\quad R_l(r) \sim B h_l^{(+)}(i\kappa r)$

$$I_n \sim B^2 \int_R^\infty dr\, r^{n+2}\, |h_l^{(+)}(i\kappa r)|^2 = \cdots$$

$$\sim \; |R_l(R)|^2 (\kappa R)^{2l+2}$$

$$\times \kappa^{-(n+3)} \left(\frac{\epsilon^{n-2l+1} - (\kappa R)^{n-2l+1}}{n-2l+1} + \cdots \right)$$

* ε (> κR) : a small number

behavior at zero binding

K. Riisager, A.S. Jensen, and P. Moller, NPA548 ('92) 393

$$\langle r^2 \rangle = \frac{I_2}{I_0} \sim \frac{\kappa^{-5}\left(\frac{\epsilon^{-2l+3}-(\kappa R)^{-2l+3}}{-2l+3} + \cdots\right)}{\kappa^{-3}\left(\frac{\epsilon^{-2l+1}-(\kappa R)^{-2l+1}}{-2l+1} + \cdots\right)}$$

$l = 0$

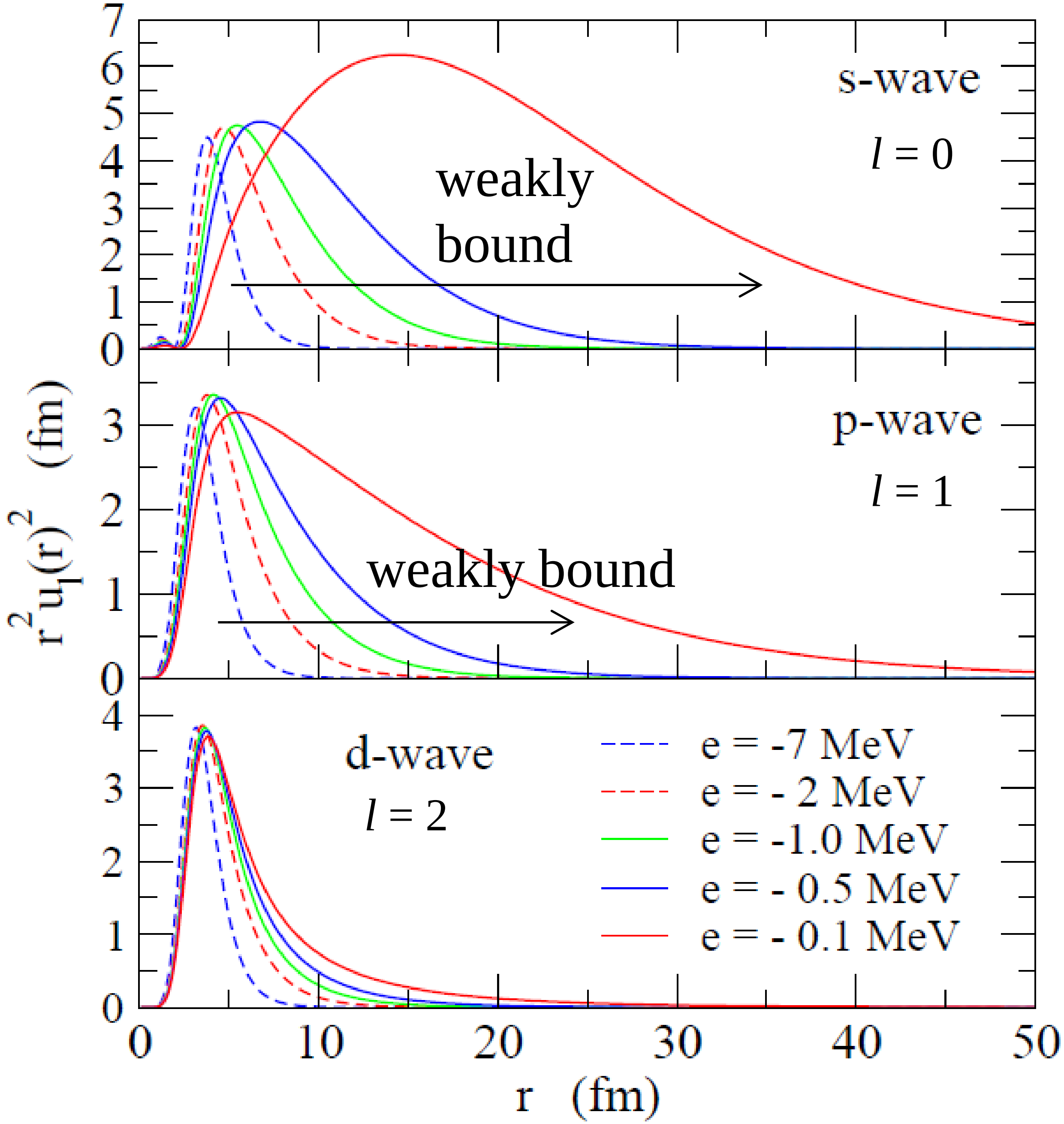
$$\langle r^2 \rangle \sim \frac{\kappa^{-5}\left(\frac{\epsilon^3}{3} + \cdots\right)}{\kappa^{-3}(\epsilon + \cdots)} \propto \kappa^{-2} \to \infty$$

$l = 1$

$$\langle r^2 \rangle \sim \frac{\kappa^{-5}(\epsilon + \cdots)}{\kappa^{-3}\left(\frac{-(\kappa R)^{-1}}{-1} + \cdots\right)} \propto \kappa^{-1} \to \infty$$

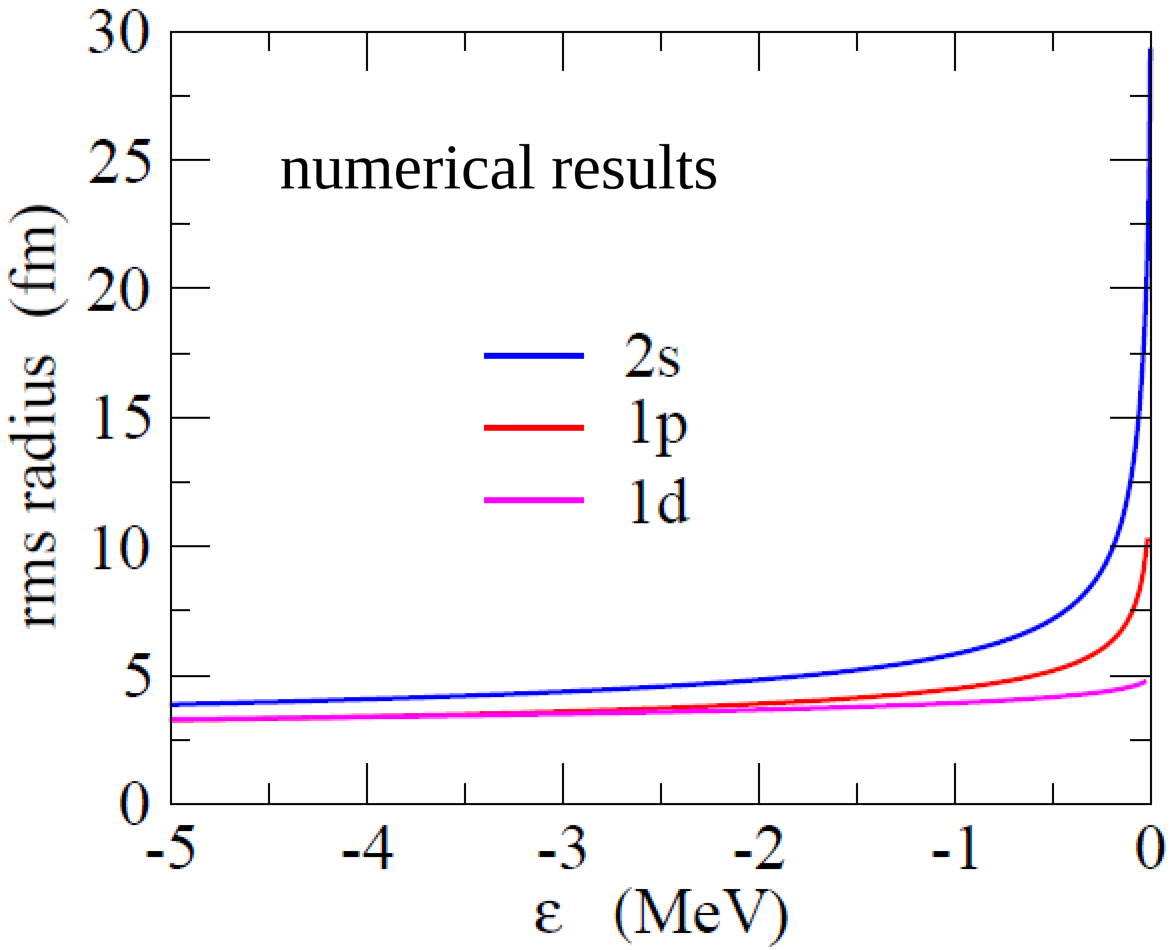
$l \geq 2$

$$\langle r^2 \rangle \sim \frac{\kappa^{-5}\left(\frac{-(\kappa R)^{-2l+3}}{-2l+3} + \cdots\right)}{\kappa^{-3}\left(\frac{-(\kappa R)^{-2l+1}}{-2l+1} + \cdots\right)} \propto \kappa^{0} \to \text{finite}$$



s-wave
l = 0
weakly
bound
p-wave
l = 1
weakly bound
d-wave
l = 2
e = -7 MeV
e = - 2 MeV
e = -1.0 MeV
e = - 0.5 MeV
e = - 0.1 MeV
r²u_l(r)² (fm)
r (fm)

$$\langle r^2 \rangle \propto \begin{cases} 1/|\epsilon_0| & (l=0) \\ 1/\sqrt{|\epsilon_1|} & (l=1) \\ \text{finite} & (l=2) \end{cases}$$



⇓

the radius diverges for
l=0 and 1
(in the zero binding limit)

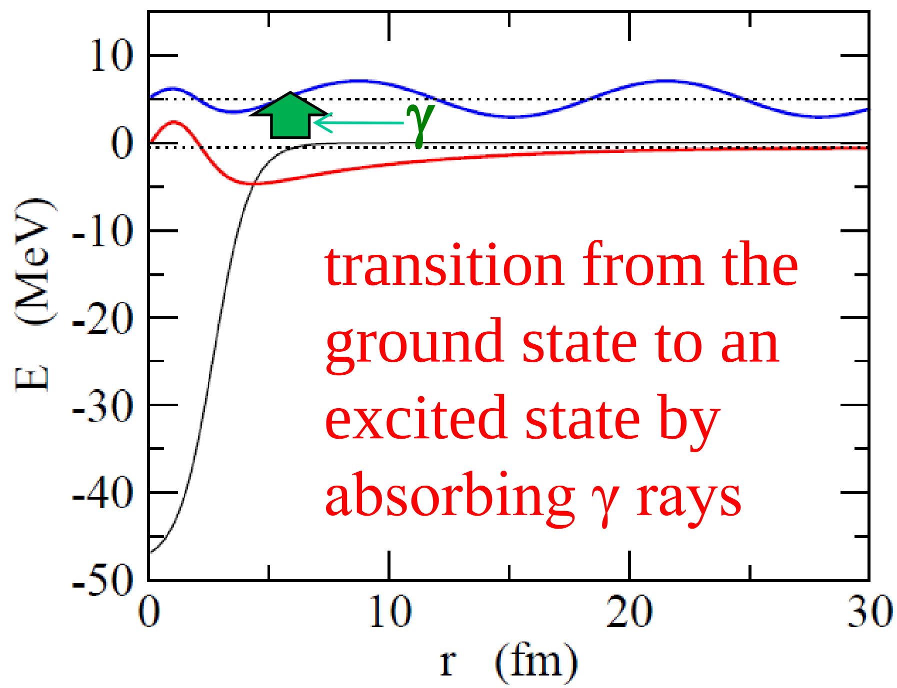
⇓

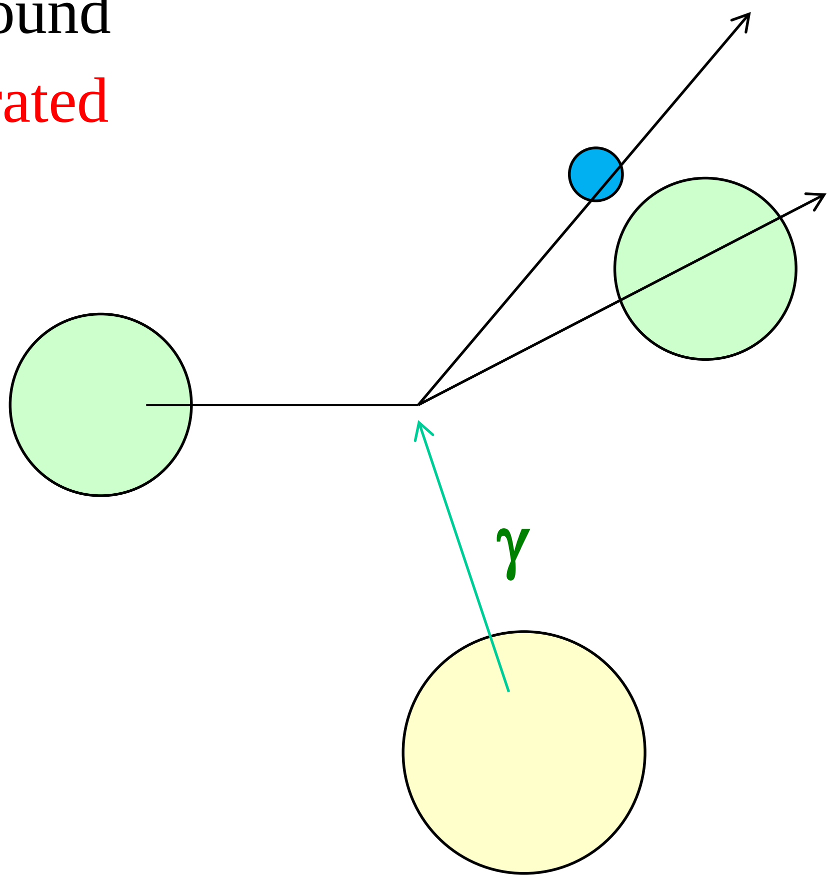
halo (a very large radius)
occurs only for *l* = 0 or 1
and for weakly binding

Coulomb excitations of a 1n halo nucleus

A key word of halo nuclei: weakly bound

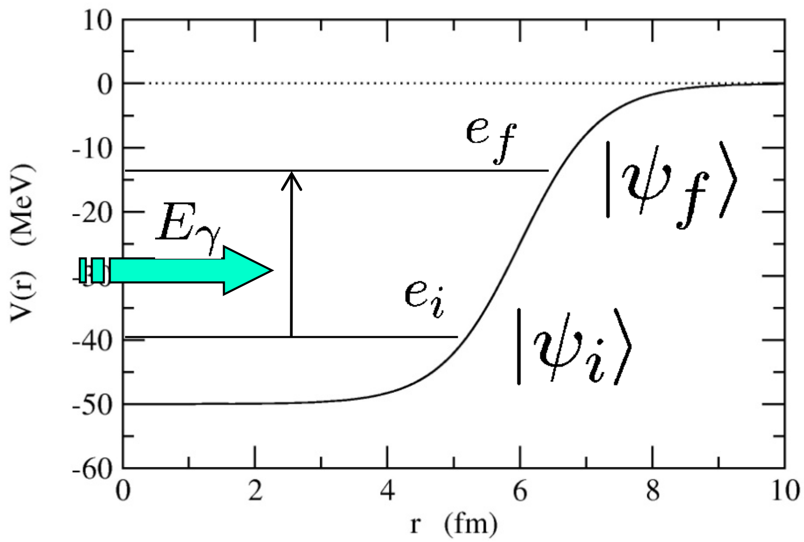
→ valence neutron(s) easily separated

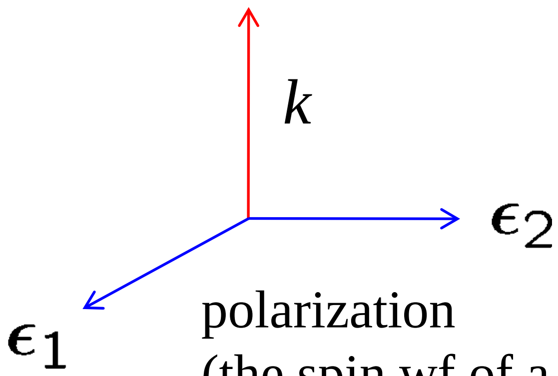




if excited to a continuum state: breakup process ⟵ excitations due to the Coulomb field of a target nucleus

electromagnetic transitions





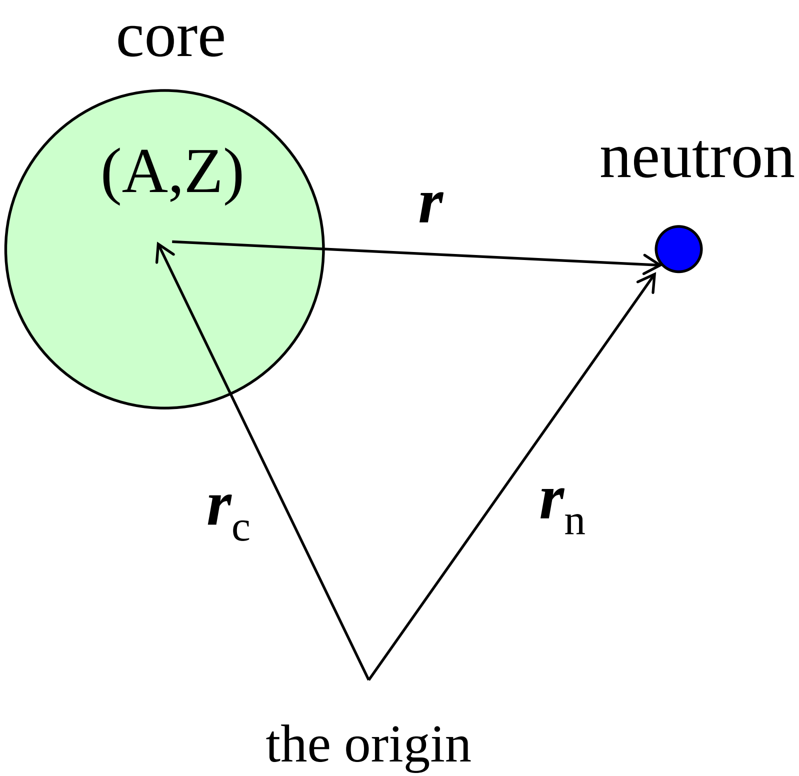
polarization
(the spin wf of a photon)

initial state: $|\psi_i\rangle|n_{k\alpha} = 1\rangle$ ⟵ a nucleus: Ψ_i, a photon with mom. *k* and polarization α (α = 1 or 2)

transition ↓ H_{int} (the interaction between a nucleus and the EM field) ⟵ the Fermi golden rule (the 1st order perturbation)

final state: $|\psi_f\rangle|n_{k\alpha} = 0\rangle$

electromagnetic transitions



$$H = \frac{\boldsymbol{p}_c^2}{2Am} + \frac{\boldsymbol{p}_n^2}{2m} + V(r)$$

the EM interaction:

$$\boldsymbol{p}_c \to \boldsymbol{p}_c - \frac{Ze}{c}\boldsymbol{A}(\boldsymbol{r}_c, t)$$

in the rest frame of c.m. motion ($\boldsymbol{P} = 0$)

$$H_{\rm int} = \frac{1}{Am}\cdot\frac{Ze}{c}\boldsymbol{A}\cdot\boldsymbol{p}$$

$$\boldsymbol{p} = \frac{1}{A+1}(A\boldsymbol{p}_n - \boldsymbol{p}_c)$$

the EM interaction

$$H_{\text{int}} = \frac{1}{Am} \cdot \frac{Ze}{c} \boldsymbol{A} \cdot \boldsymbol{p}$$

E1 approximation (absorption of an E1 photon)

$$\boldsymbol{A} = const.$$

the Fermi golden rule

$$\Gamma_{i \to f} = \frac{1}{2\pi} \left(\frac{Ze}{A} \right)^2 \frac{1}{m^2 \omega} \left| \langle \psi_f | p_z | \psi_i \rangle \right|^2 \delta(e_f - e_i - \hbar\omega)$$

(z axis: the direction of the pol. vector) (note) $[\boldsymbol{p}^2, \boldsymbol{r}] = -2i\hbar\boldsymbol{p}$

$$\langle\psi_f|p_z|\psi_i\rangle = \left\langle \psi_f \left| \frac{1}{-2i\hbar}\cdot 2\mu\left[\frac{\boldsymbol{p}^2}{2\mu}+V(r), z\right]\right| \psi_i\right\rangle$$

H_0

$$= \frac{i\mu}{\hbar}\langle\psi_f|H_0 z - zH_0|\psi_i\rangle$$

$$= \frac{i\mu}{\hbar}(e_f - e_i)\langle\psi_f|z|\psi_i\rangle$$

$$\Gamma_{i\to f} = \frac{1}{2\pi\hbar}\left(\frac{Ze}{A+1}\right)^2 (e_f - e_i)\left|\langle\psi_f|z|\psi_i\rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

cf. photoabsorption cross section if this is devided by the photon flux $c/(2\pi)^3$

$$\sigma_\gamma = \frac{4\pi^2}{\hbar c}\left(\frac{Ze}{A+1}\right)^2 (e_f - e_i)\left|\langle\psi_f|z|\psi_i\rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

E1 effective charge

$$\sigma_\gamma = \frac{4\pi^2}{\hbar c}\left(\frac{Ze}{A+1}\right)^2 (e_f - e_i)\left|\langle\psi_f|z|\psi_i\rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

$$z = r\cos\theta = \sqrt{\frac{4\pi}{3}}\, rY_{10}(\theta)$$

$$\sigma_\gamma = \frac{16\pi^3}{3\hbar c}\left(\frac{Ze}{A+1}\right)^2 (e_f - e_i)\left|\langle\psi_f|rY_{10}|\psi_i\rangle\right|^2 \delta(e_f - e_i - \hbar\omega)$$

dipole operator: $\hat{D}_\mu = e_{\rm E1}\cdot rY_{1\mu}(\theta,\phi)$

E1 effective charge $e_{\rm E1} = \frac{Z}{A+1}e$

(the charge distribution measured from c.m.)

in general: $e_{\rm E1} = \frac{Z_1A_2 - Z_2A_1}{A_1+A_2}e$

Wigner-Eckart theorem and reduced transition probabilities

$$\sigma_\gamma = \frac{16\pi^3}{3\hbar c}\left(\frac{Ze}{A+1}\right)^2 E_\gamma \left|\langle\psi_f|rY_{10}|\psi_i\rangle\right|^2 \delta(e_f - e_i - E_\gamma)$$

$$E_\gamma = e_f - e_i = \hbar\omega$$

in reality

$$\begin{cases} \psi_i(\boldsymbol{r}) = \psi_{lm}(\boldsymbol{r}) = \dfrac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \\ \psi_f(\boldsymbol{r}) = \psi_{l'm'}(\boldsymbol{r}) = \dfrac{u_{l'}(r)}{r} Y_{l'm'}(\hat{\boldsymbol{r}}) \end{cases}$$

eigenstates of the ang. mom.

$$\left|\langle\psi_f|rY_{10}|\psi_i\rangle\right|^2 \to \frac{1}{2l+1}\sum_{m,m'} |\langle\psi_{l'm'}|rY_{10}|\psi_{lm}\rangle|^2$$

$$= \frac{1}{3}\cdot\frac{1}{2l+1}|\langle\psi_{l'}||rY_1||\psi_l\rangle|^2$$

Wigner-Eckart theoretm

$$\sigma_\gamma = \frac{16\pi^3}{9\hbar c}E_\gamma \cdot \frac{1}{2l+1}\left|\langle\psi_f||e_{E1}rY_1||\psi_i\rangle\right|^2 \delta(e_f - e_i - E_\gamma)$$

$$= \frac{16\pi^3}{9\hbar c}E_\gamma \cdot \frac{dB(E1)}{dE_\gamma}$$

reduced transition prob.

cf. Wigner-Eckart theorem

$$
\begin{aligned}
\langle \psi_{l'm'} | r Y_{10} | \psi_{lm} \rangle &= (-1)^{l'-m'} \begin{pmatrix} l' & 1 & l \\ -m' & 0 & m \end{pmatrix} \langle \psi_{l'} || r Y_1 || \psi_l \rangle \\
&= \underbrace{\frac{(-1)^{l-m}}{\sqrt{3}} \langle l'm'l-m|10\rangle}_{} \underbrace{\langle \psi_{l'} || r Y_1 || \psi_l \rangle}_{}
\end{aligned}
$$

m, *m*' dep: Clebsch-Gordan coeff. independent of *m*, *m*'

$$
\begin{aligned}
\left| \langle \psi_f | r Y_{10} | \psi_i \rangle \right|^2 &\to \frac{1}{2l+1} \sum_{m,m'} |\langle \psi_{l'm'} | r Y_{10} | \psi_{lm} \rangle|^2 \\
&= \frac{1}{3} \cdot \frac{1}{2l+1} |\langle \psi_{l'} || r Y_1 || \psi_l \rangle|^2 \\
&\quad \times \sum_{m,m'} \langle l'm'l-m|10\rangle^2 \quad = 1 \\
&= \frac{1}{3} \cdot \frac{1}{2l+1} |\langle \psi_{l'} || r Y_1 || \psi_l \rangle|^2
\end{aligned}
$$

A simple analytical model for E1 transitions

consider a transition from $l = 0$ to $l = 1$:

initial wave function: $$\Psi_i(\boldsymbol{r}) = \sqrt{2\kappa}\,\frac{e^{-\kappa r}}{r} Y_{00}(\hat{\boldsymbol{r}})$$

final wave function $$\Psi_f(\boldsymbol{r}) = \sqrt{\frac{2\mu k}{\pi\hbar^2}}\, j_1(kr) Y_{1m}(\hat{\boldsymbol{r}})$$

$$\kappa = \sqrt{\frac{2\mu|E_b|}{\hbar^2}}$$

$j_1(kr)$:spherical Bessel

$$k = \sqrt{\frac{2\mu E_c}{\hbar^2}}$$

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$j_1(kr)$:spherical Bessel

$$k = \sqrt{\frac{2\mu E_c}{\hbar^2}}$$

$$\frac{dB(E1)}{dE} = \frac{3}{4\pi} e_{E1}^2 \left| \int_0^\infty r^2 dr\, r \cdot \frac{\sqrt{2\kappa}e^{-\kappa r}}{r} \cdot \sqrt{\frac{2\mu k}{\pi\hbar^2}}\, j_1(kr) \right|^2$$

the integral can be done analytically

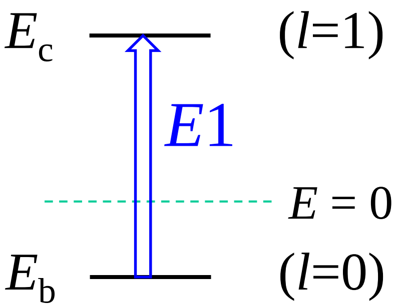
$$\frac{dB(E1)}{dE} = \frac{3\hbar^2}{\pi^2\mu} e_{E1}^2 \frac{\sqrt{|E_b|}\, E_c^{3/2}}{(|E_b| + E_c)^4}$$

Refs. (for more general cases)
- M.A. Nagarajan, S.M. Lenzi, A. Vitturi, Eur. Phys. J. A24('05)63
- S. Typel and G. Baur, NPA759('05)247

$$\frac{dB(E1)}{dE} = \frac{3\hbar^2}{\pi^2\mu} e_{E1}^2 \frac{\sqrt{|E_b|}\, E_c^{3/2}}{(|E_b| + E_c)^4}$$

the peak position: $E_c = \frac{3}{5}|E_b|$

$$\left(E_x = E_c - E_b = \frac{8}{5}|E_b|\right)$$



the peak height: $\propto 1/|E_b|^2$

the total probability:

$$B(E1) = S_0 = \frac{3\hbar^2 e_{E1}^2}{16\pi^2\mu|E_b|}$$

$$\frac{dB(E1)}{dE} = \frac{3\hbar^2}{\pi^2\mu} e_{E1}^2 \frac{\sqrt{|E_b|}\, E_c^{3/2}}{(|E_b| + E_c)^4}$$

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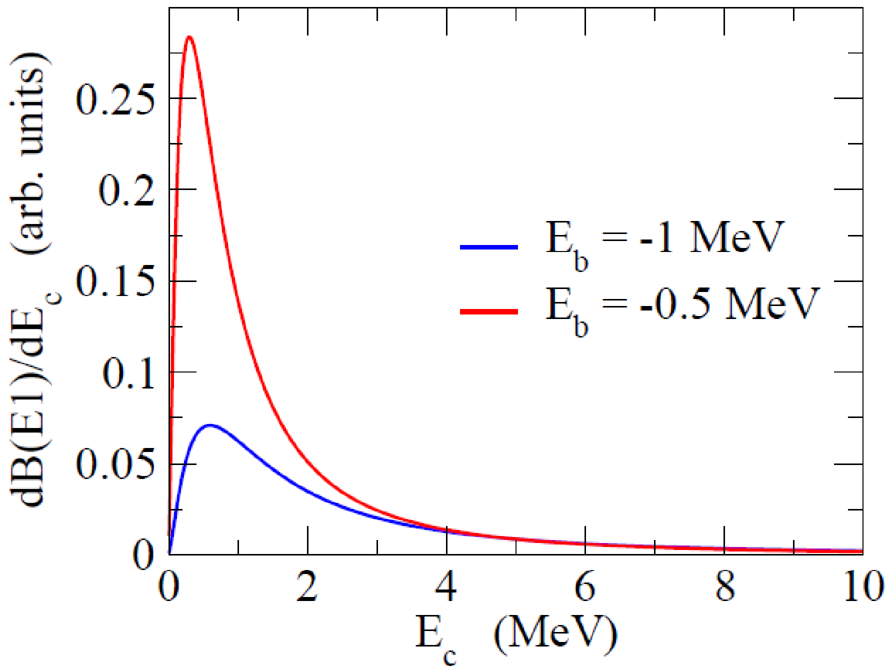
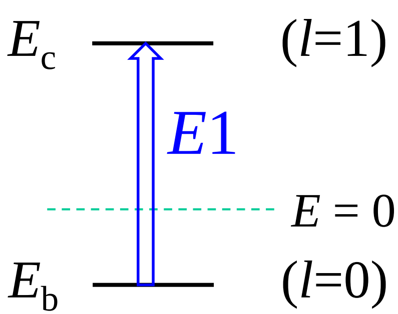
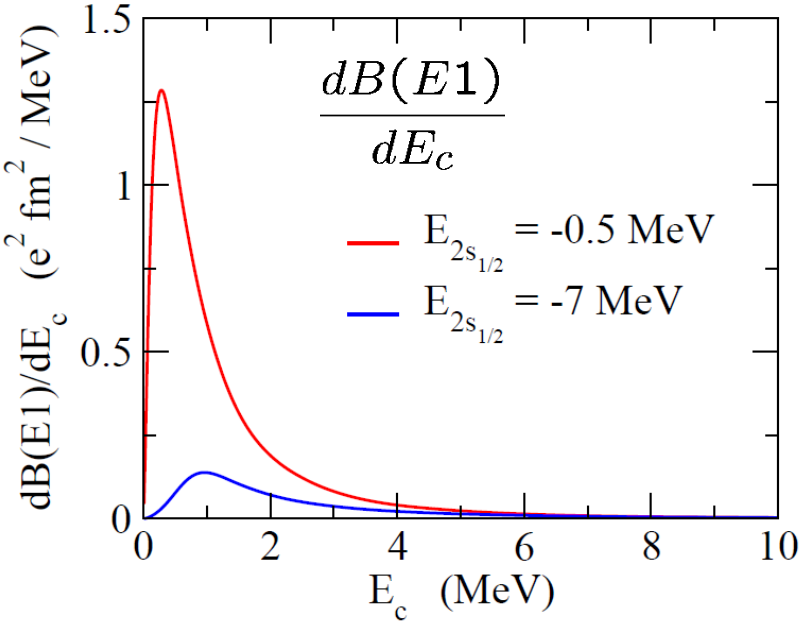


- a sharp and high peak as the binding energy gets smaller
- the peak position gets smaller as the binding energy gets smaller



E_{peak} = 0.3 MeV (E_b=-0.5 MeV)

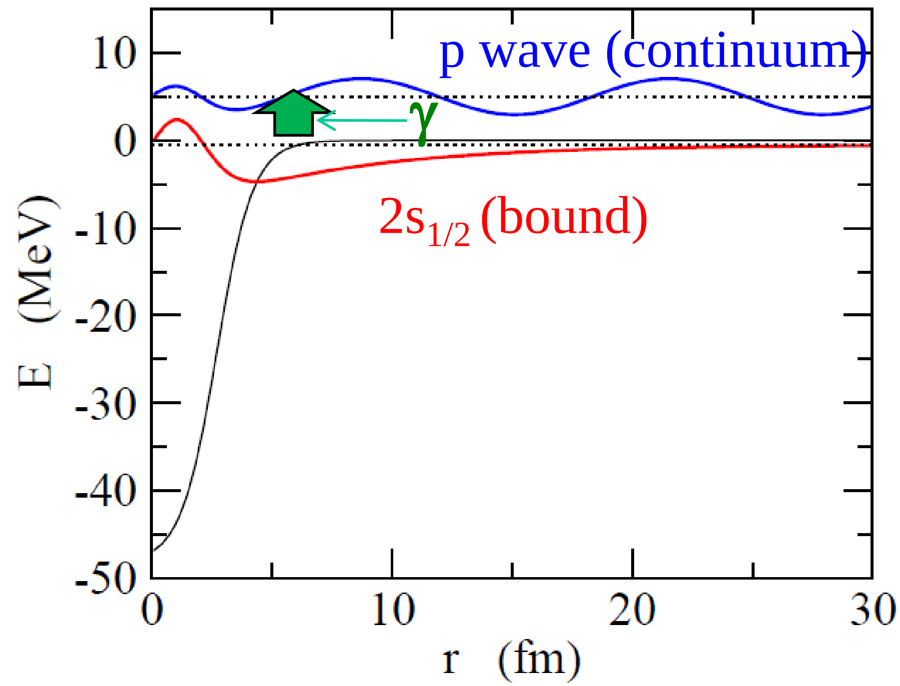
numerical calculations with a Woods-Saxon potential

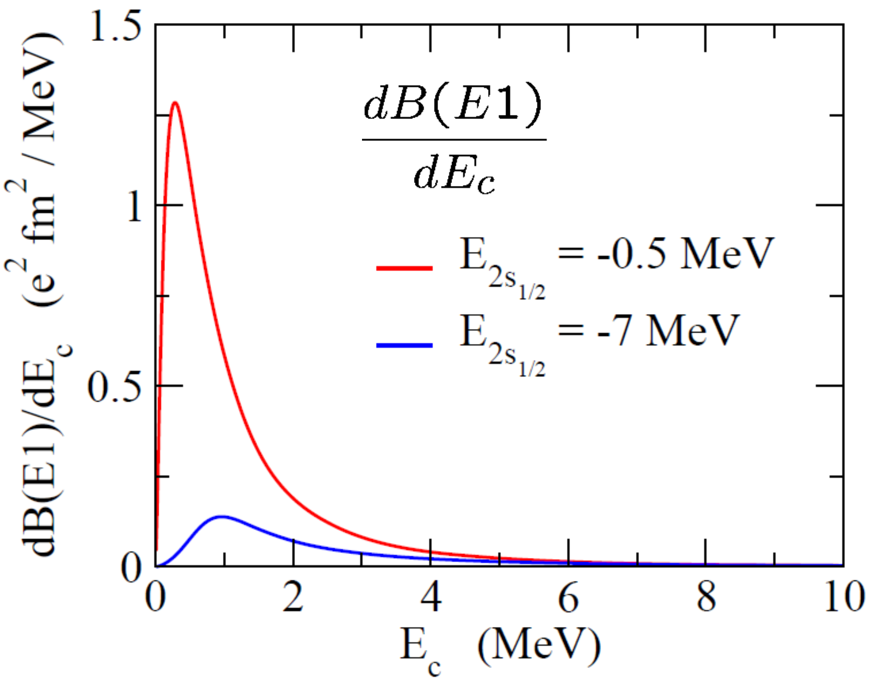


$^{11}Be = {}^{10}Be + n$

transition from the bound $2s_{1/2}$ state to continuum p-wave states (l = 1)

comparison between a weakly bound case and a strongly bound case





^{11}Be = ^{10}Be + *n* $2s_{1/2} \longrightarrow$ p state

- a sharp and high peak as the binding energy gets smaller

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c}$$

=1.53 e^2fm^2 (E_b = -0.5 MeV)
0.32 e^2fm^2 (E_b = -7 MeV)

- the peak position gets smaller as the binding energy gets smaller

E_{peak} = 0.28 MeV (E_b=-0.5 MeV)
0.96 MeV (E_b = -7MeV)

Sum Rules

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c}$$

the basic idea:

$$\sum_f |\langle f|\widehat{F}|\psi_i\rangle|^2 = \sum_f \langle\psi_i|F|f\rangle\langle f|\widehat{F}|\psi_i\rangle = \langle\psi_i|\widehat{F}^2|\psi_i\rangle$$

↑

the completeness $\sum_f |f\rangle\langle f| = 1$

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the completeness $\sum_f |f\rangle\langle f| = 1$

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} \sim \sum_f \frac{1}{2l_i+1} |\langle\psi_f||\widehat{D}||\psi_i\rangle|^2$$

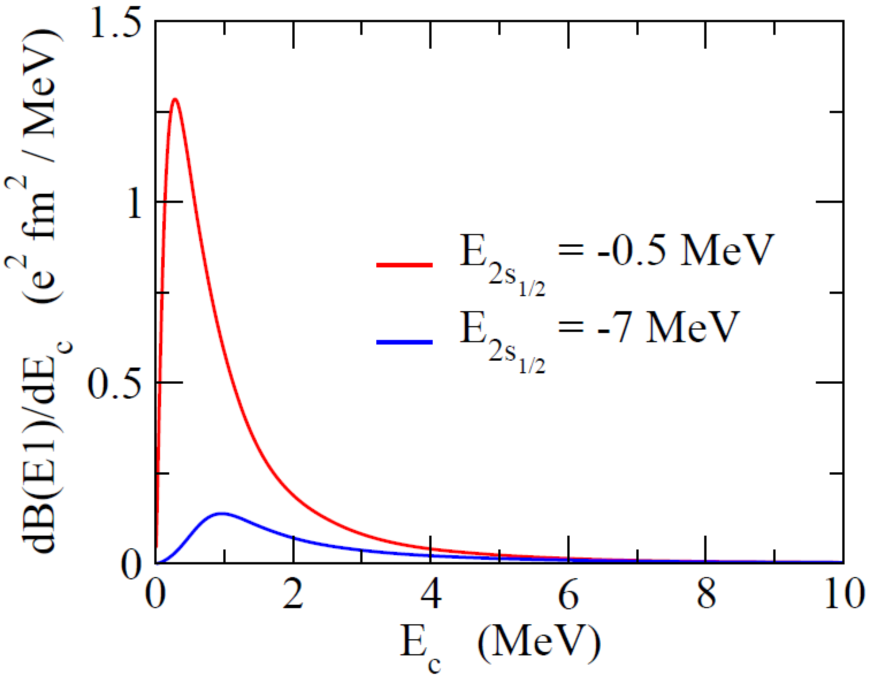
$$\widehat{D}_\mu = e_{\mathrm{E1}} r Y_{1\mu}(\widehat{\boldsymbol{r}})$$

$$= \frac{3}{4\pi} e_{\mathrm{E1}}^2 \langle r^2\rangle_i$$

Sum Rules

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$

the total E1 strength: proportional to the g.s. expectation value of r^2



$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c}$$

$=1.53\ e^2 fm^2$ (E_b = -0.5 MeV)
$0.32\ e^2 fm^2$ (E_b = -7 MeV)

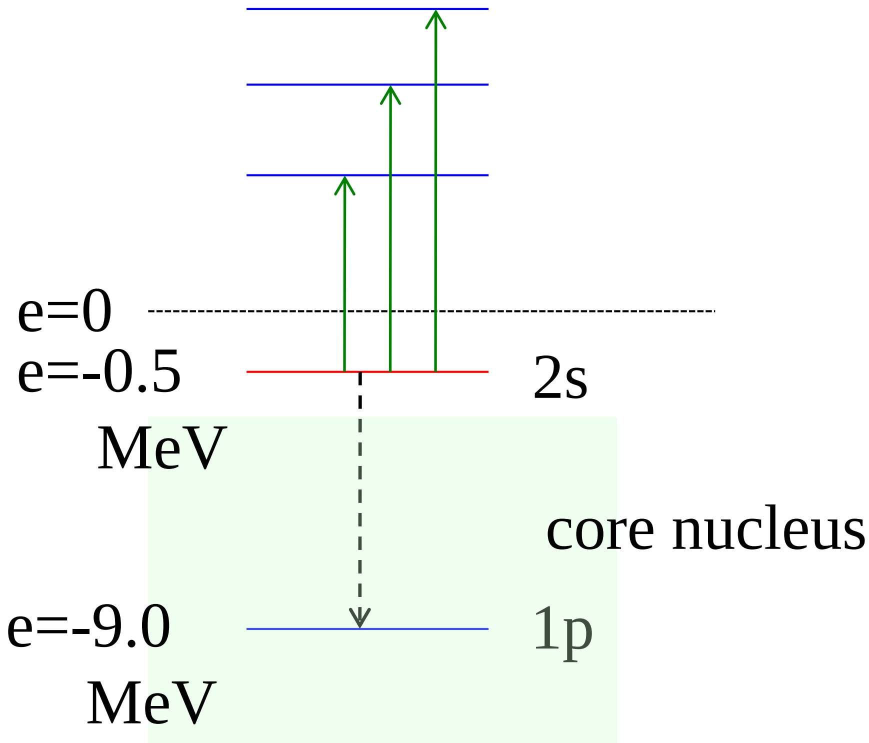
$$\frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$

$=1.62\ e^2 fm^2$ (E_b = -0.5 MeV)
$0.41\ e^2 fm^2$ (E_b = -7 MeV)

* almost agreement.
a slight deviation due to the Pauli forbidden transition (2s → 1p)

cf. Pauli forbidden transition

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$



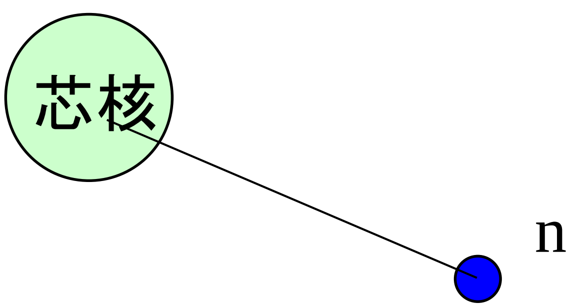
$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c}$$

$=1.5275\ e^2 fm^2$

B(E1: 2s → 1p) = 0.0967 $e^2 fm^2$

$$\frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$

$=1.6244\ e^2 fm^2$



1.5275+0.0967 = 1.6242

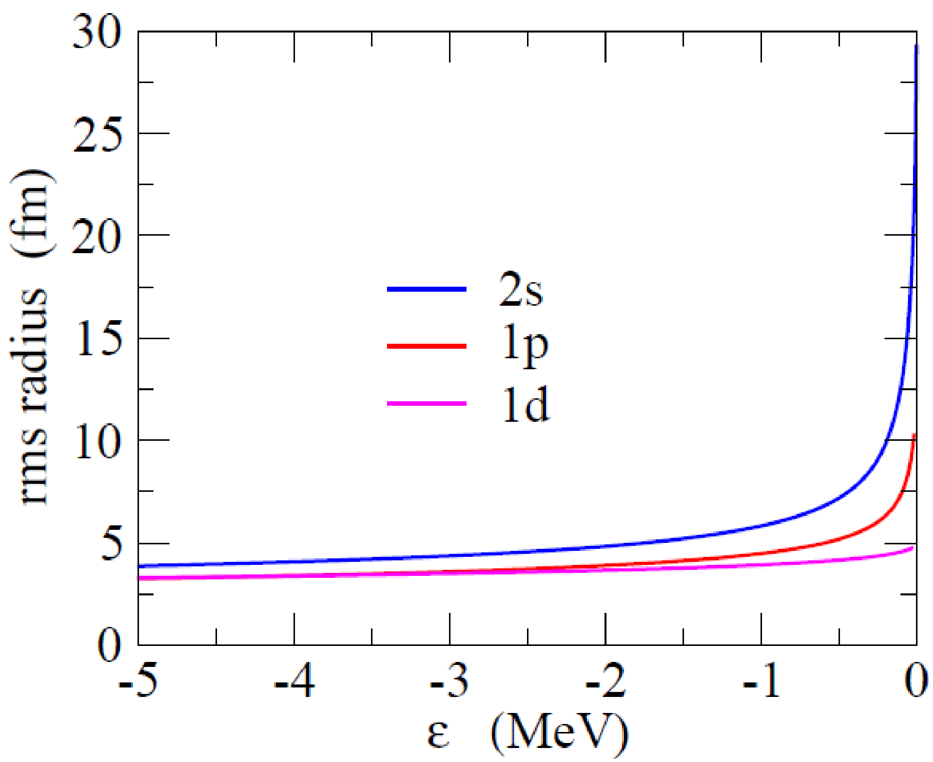
physical transition

forbidden transition

Sum Rules

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$

the total E1 strength: proportional to the g.s. expectation value of r^2



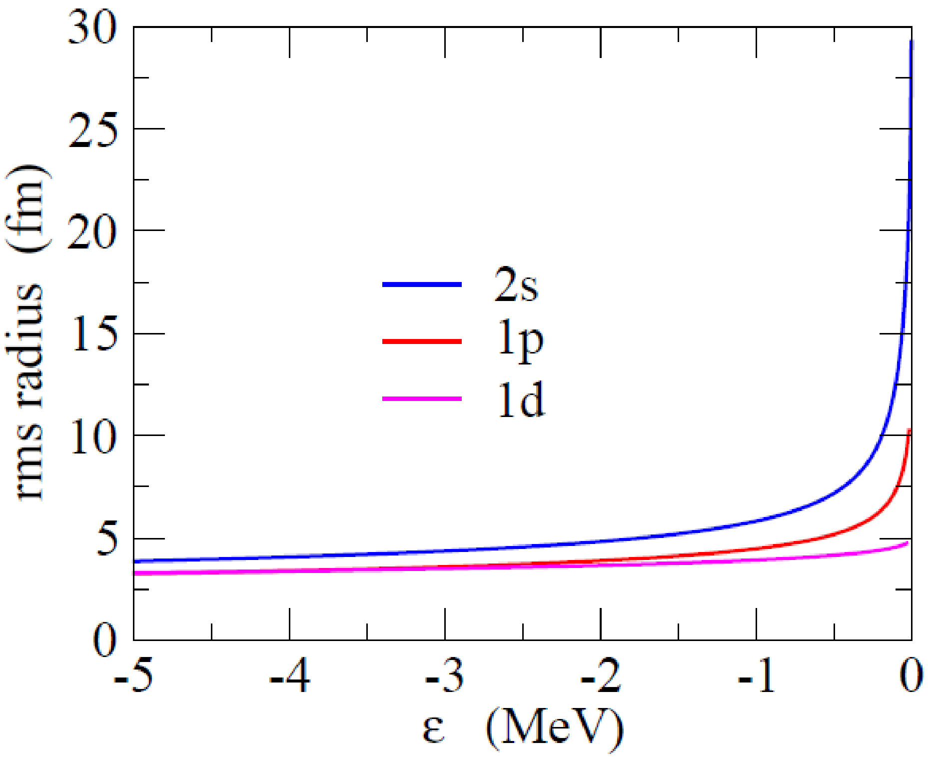
a large radius for l=0 or l=1 when the binding is weak

the total E1 strength: also increases

Sum Rules

$$S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i$$

the total E1 strength: proportional to the g.s. expectation value of r^2



a large radius for l=0 or l=1 when the binding is weak

⇩

the total E1 strength: also increases

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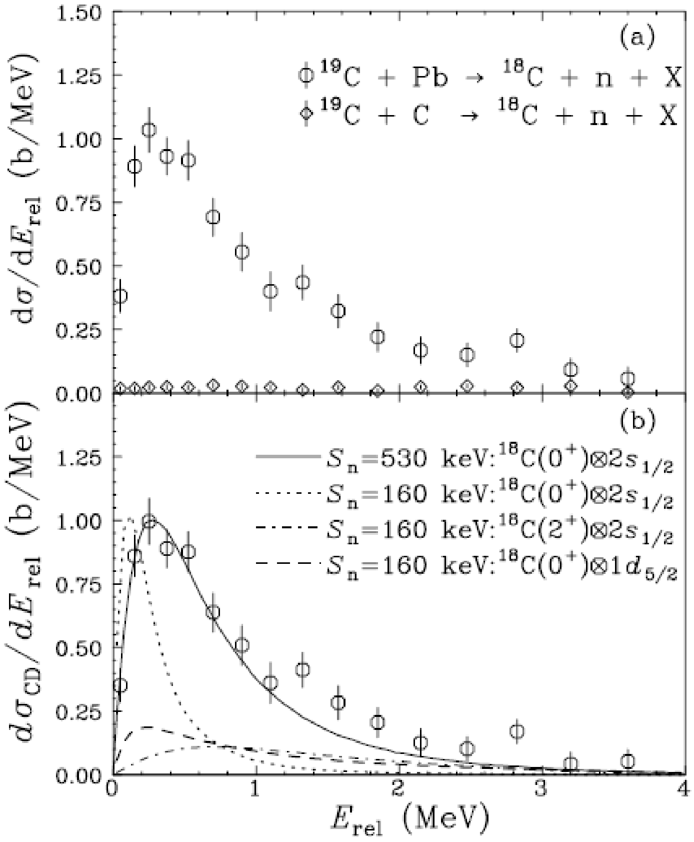
Inversely, if a large E1 strength is observed → halo structure (l=0 or 1)

candidate for 1n halo nuclei

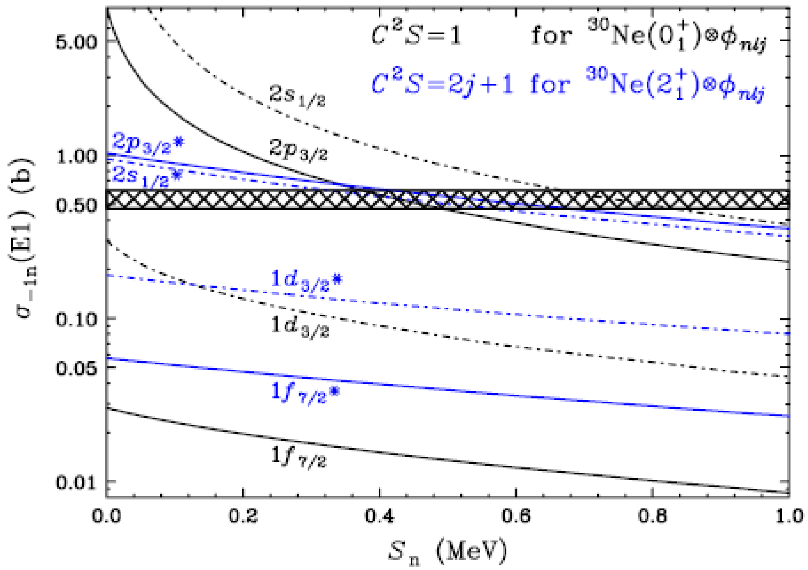
^{19}C: S_n = 0.58(9) MeV



the Coul. b.u. of ^{19}C

T. Nakamura et al., PRL83(‘99)1112

^{31}Ne: S_n = 0.29 +/- 1.64 MeV



large Coulomb breakup cross sections

T. Nakamura et al., PRL103(‘09)262501