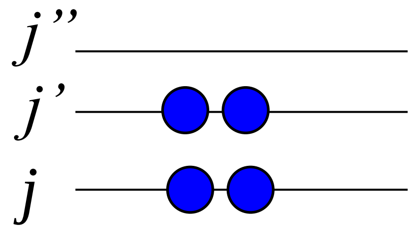
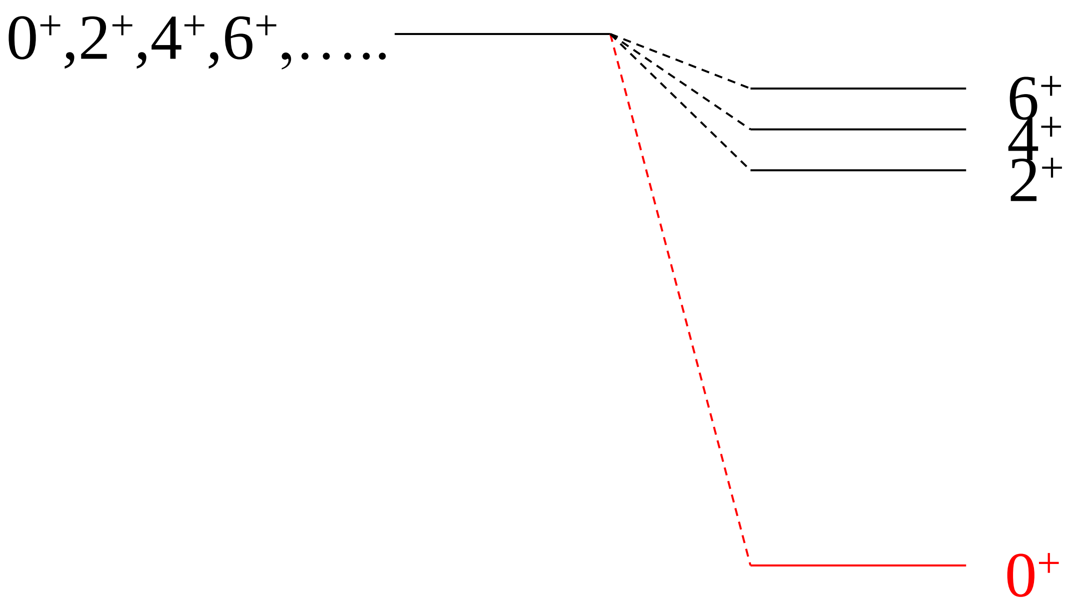
3. 対相関の理論的記述:BCS近似



複数個のレベルに
複数個のペアがある問題

$$v_{\mathrm{res}}(\boldsymbol{r}, \boldsymbol{r}') \quad \sim -g\,\delta(\boldsymbol{r} - \boldsymbol{r}')$$



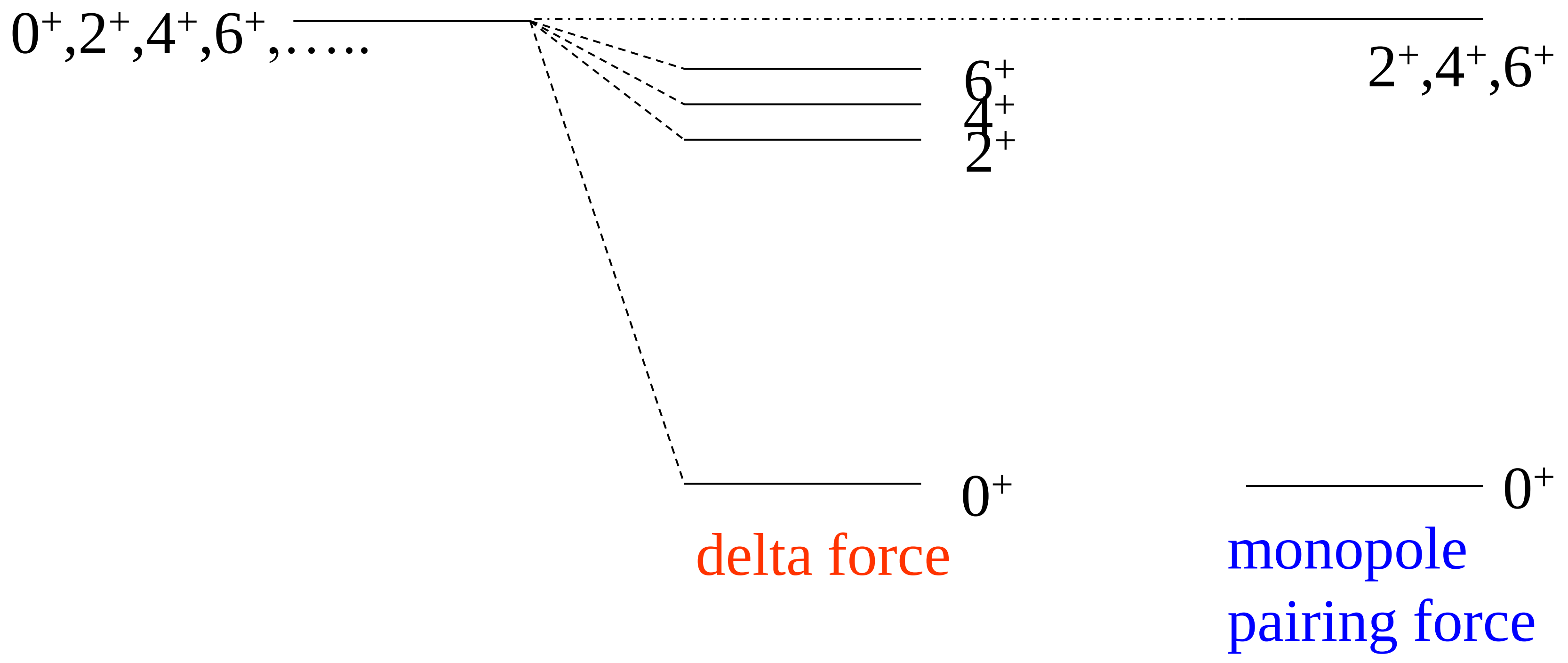
デルタ関数のままでもいいが、説明を簡単にするために
もう少し簡単にした相互作用を導入する。

Simplified pairing interaction

$$V = -G\,P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$\bar{\nu}$: the time reversed state of ν

e.g., $|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$



Simplified pairing interaction

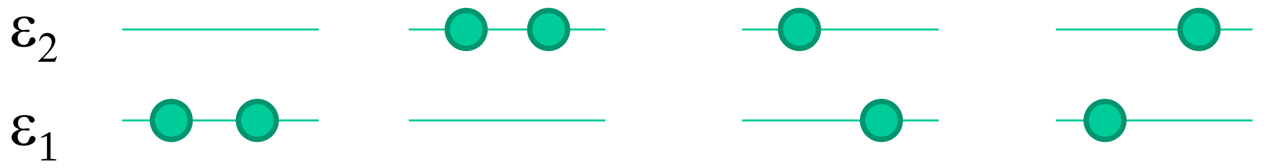
$$V = -G\,P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$$

$\bar{\nu}$: the time reversed state of ν

$$H = \sum_k \epsilon_k (a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - G \left(\sum_{k>0} a^\dagger_k a^\dagger_{\bar{k}}\right) \left(\sum_{k>0} a_{\bar{k}} a_k\right)$$

$$H = \begin{pmatrix} 2\epsilon_1 - G & -G & 0 & 0 \\ -G & 2\epsilon_2 - G & 0 & 0 \\ 0 & 0 & \epsilon_1 + \epsilon_2 & 0 \\ 0 & 0 & 0 & \epsilon_1 + \epsilon_2 \end{pmatrix}$$

$$\to \quad \Psi_{\mathrm{g.s.}} = C_1 \Psi_1 + C_2 \Psi_2$$

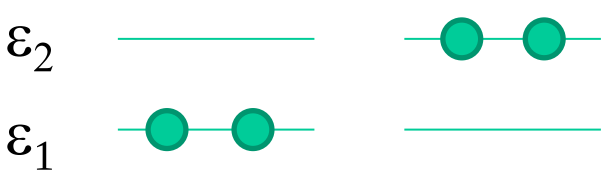


$$H = \begin{pmatrix} 2\epsilon_1 - G & -G & 0 & 0 \\ -G & 2\epsilon_2 - G & 0 & 0 \\ 0 & 0 & \epsilon_1 + \epsilon_2 & 0 \\ 0 & 0 & 0 & \epsilon_1 + \epsilon_2 \end{pmatrix}$$

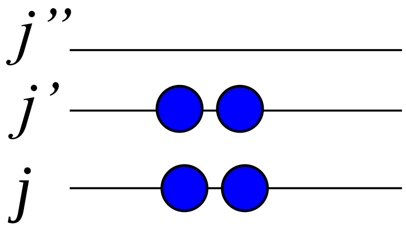
2x2 行列の対角化をすると:

$$E_{\mathrm{g.s.}} = \epsilon_1 + \epsilon_2 - G - \sqrt{(\epsilon_2 - \epsilon_1)^2 + G^2} < 2\epsilon_1 \quad (\mathrm{for}\ G > 0)$$

$$\Psi_{\mathrm{g.s.}} = C_1\Psi_1 + C_2\Psi_2$$



励起状態に入れることにより相関をかせぎ、
エネルギーが下がる→それぞれの準位は部分的に占有されている

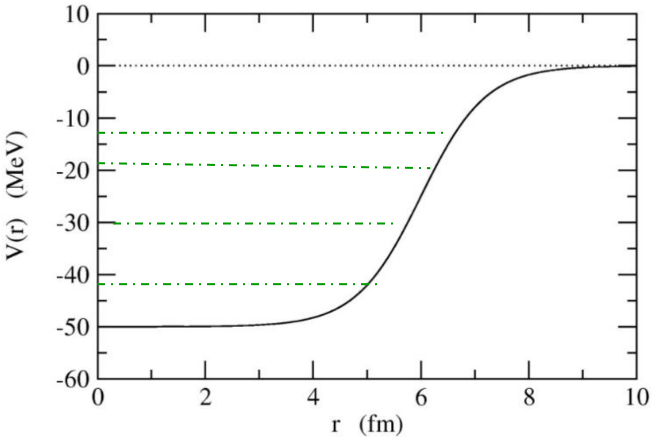


複数個のレベルに
複数個のペアがある問題

BCS理論:もともと、超伝導を説明するために Bardeen, Cooper, Schrieffer によって1957年に定式化された理論
→これを原子核の対相関現象に適用 (Bohr, Mottelson, Pines, 1958)

HF+BCS theory

①平均場近似をして核子の感じるポテンシャルを求める
（平均的な振る舞いをまず決める）



$$H = \sum_k \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \left(\sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger \right) \left(\sum_{k>0} a_{\bar{k}} a_k \right)$$

②各準位の占有確率を決める。
決め方は、残留相互作用も含めてエネルギーが最小になるようにする。

$$H = \sum_{\nu} \epsilon_{\nu} (a^{\dagger}_{\nu} a_{\nu} + a^{\dagger}_{\bar{\nu}} a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a^{\dagger}_{\nu} a^{\dagger}_{\bar{\nu}} \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_{\nu} \right)$$

2体の相互作用
→ １体近似をする

cf. HF potential

$$V_H(\boldsymbol{r}) = \int v(\boldsymbol{r}, \boldsymbol{r}') \rho_{\mathrm{HF}}(\boldsymbol{r}') d\boldsymbol{r}$$

Solve the pairing Hamiltonian

$$H = \sum_{\nu} \epsilon_\nu (a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu \right)$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G\, P^\dagger P \to -G \left(\langle P^\dagger \rangle P + P^\dagger \langle P \rangle \right) = -\Delta (P^\dagger + P)$$

⟺ particle number violation

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$
\begin{aligned}
H' &= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger\hat{P} \\
&\to \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P}) \\
&= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta\sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}}a_k)
\end{aligned}
$$

we consider $H' = H - \lambda\hat{N}$ instead of H :

$$H' = \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger\hat{P}$$
$$\to \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P})$$
$$= \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta\sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

- Transform *H'* in a form of

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

→ g.s.: $\alpha_k|BCS\rangle = 0$

1st excited state: $|1_k\rangle = \alpha_k^\dagger|BCS\rangle$ at E_k

…. and so on.

Bogoliubov transformation

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

(Quasi-particle operator)

or $$a_\nu^\dagger = u_\nu \alpha_\nu^\dagger + v_\nu \alpha_{\bar{\nu}}, \qquad a_{\bar{\nu}}^\dagger = u_\nu \alpha_{\bar{\nu}}^\dagger - v_\nu \alpha_\nu$$

(note)

$$\{\alpha_\nu, \alpha_{\nu'}\} = 0, \qquad \{\alpha_\nu, \alpha_{\nu'}^\dagger\} = \delta_{\nu,\nu'}$$

$$\rightarrow \quad u_\nu^2 + v_\nu^2 = 1$$

Bogoliubov transformation

$$\alpha^\dagger_\nu = u_\nu a^\dagger_\nu - v_\nu a_{\bar{\nu}}, \qquad \alpha^\dagger_{\bar{\nu}} = u_\nu a^\dagger_{\bar{\nu}} + v_\nu a_\nu$$

(Quasi-particle operator)

or $$a^\dagger_\nu = u_\nu \alpha^\dagger_\nu + v_\nu \alpha_{\bar{\nu}}, \qquad a^\dagger_{\bar{\nu}} = u_\nu \alpha^\dagger_{\bar{\nu}} + -v_\nu \alpha_\nu$$

$$H' = \sum_{k>0} (\epsilon_k - \lambda)(a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - \Delta \sum_{k>0} (a^\dagger_k a^\dagger_{\bar{k}} + a_{\bar{k}} a_k)$$
$$\rightarrow$$

using the quasi-particle operators:

$$\begin{aligned} H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - \Delta \sum_{k>0} (a^\dagger_k a^\dagger_{\bar{k}} + a_{\bar{k}} a_k) \\ &= \sum_{k>0} [(\epsilon_k - \lambda)(u^2_k - v^2_k) + 2\Delta u_k v_k](\alpha^\dagger_k \alpha_k + \alpha^\dagger_{\bar{k}} \alpha_{\bar{k}}) \\ &\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u^2_k - v^2_k)](\alpha^\dagger_k \alpha^\dagger_{\bar{k}} + \alpha_{\bar{k}} \alpha_k) \end{aligned}$$

using the quasi-particle operators:

$$
\begin{aligned}
H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\
&= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
&\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta (u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

if $2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2) = 0$

then $H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$

with $E_k = (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k$

$$H' = \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$
$$+ \sum_{k>0} [2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)$$

$$\begin{cases} 0 = 2(\epsilon_k - \lambda)u_k v_k - \Delta(u_k^2 - v_k^2) \\ 1 = u_k^2 + v_k^2 \end{cases}$$

$$u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$
$$v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$

$$
\begin{aligned}
H' &= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
&\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

$$
\begin{aligned}
u_\nu^2 &= \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right) \\
v_\nu^2 &= \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
E_k &= (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k \\
&= \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}
\end{aligned}
$$

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$

$$|BCS\rangle \propto \prod_{\nu>0} \alpha_\nu \alpha_{\bar{\nu}} |0\rangle$$

$$= \prod_{\nu>0} v_\nu \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)\Big|\, 0\Big\rangle$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)\Big|\, 0\Big\rangle$$

Ground state wave function:

$$\alpha_k|BCS\rangle = 0$$

$$|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

(note) $\Big\langle BCS|a^\dagger_\nu a_\nu|BCS\Big\rangle = |v_\nu|^2$: occupation probability

(note)

$$E'_{\rm BCS} = \langle BCS|H'|BCS\rangle \sim 2\sum_{\nu>0}(\epsilon_\nu - \lambda)v_\nu^2 - \frac{\Delta^2}{G}$$

Ground state wave function:

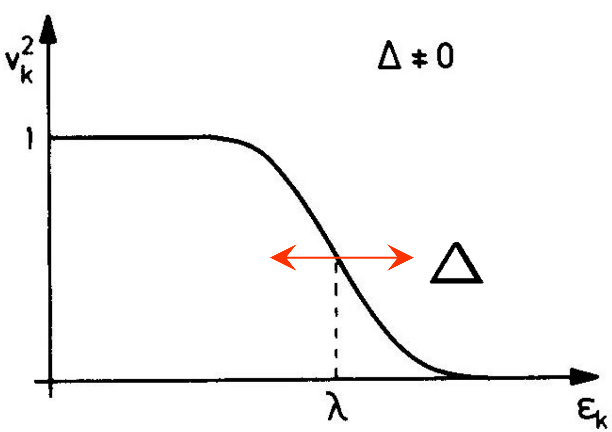
$$\alpha_k|BCS\rangle = 0$$

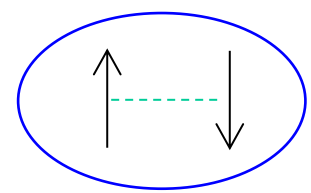
$$|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

(note) $$\left(1+\frac{v_\nu}{u_\nu}\,a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle = \exp\left(\frac{v_\nu}{u_\nu}\,a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

$$|\Psi\rangle \propto \exp\left(\sum_{\nu>0}\frac{v_\nu}{u_\nu}\,a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

(pair condensed wave function)





クーパー対の凝縮

Gap equation

$$\begin{cases} u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \\ v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{E_\nu}\right) \end{cases}$$

$$E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}$$

$$\Delta = G\langle BCS|\hat{P}|BCS\rangle = G\sum_{\nu>0} u_\nu v_\nu = \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu}$$

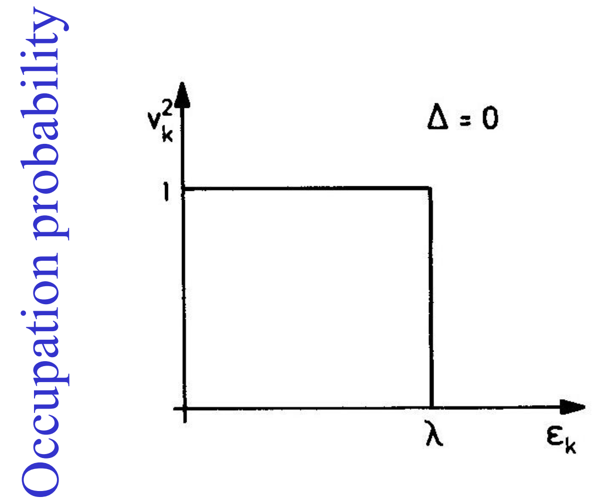
(Gap equation)

$$N = 2\sum_{\nu>0} v_\nu^2 \qquad \leftarrow \lambda$$

$$\Delta = \frac{G}{2} \sum_{\nu > 0} \frac{\Delta}{E_\nu}$$

i) Trivial solution: always exists

$$\Delta = 0$$



$$\Delta = G \sum_{\nu > 0} u_\nu v_\nu$$

$$\begin{aligned} v_\nu^2 &= 1 \quad (\epsilon_\nu \leq \lambda) \\ &= 0 \quad (\epsilon_\nu > \lambda) \end{aligned}$$

$$|\Psi\rangle = \prod_{\nu > 0} a_\nu^\dagger a_{\bar{\nu}}^\dagger |0\rangle$$

$$\Delta = \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu}$$

i) Trivial solution: always exists

$$\Delta = 0$$

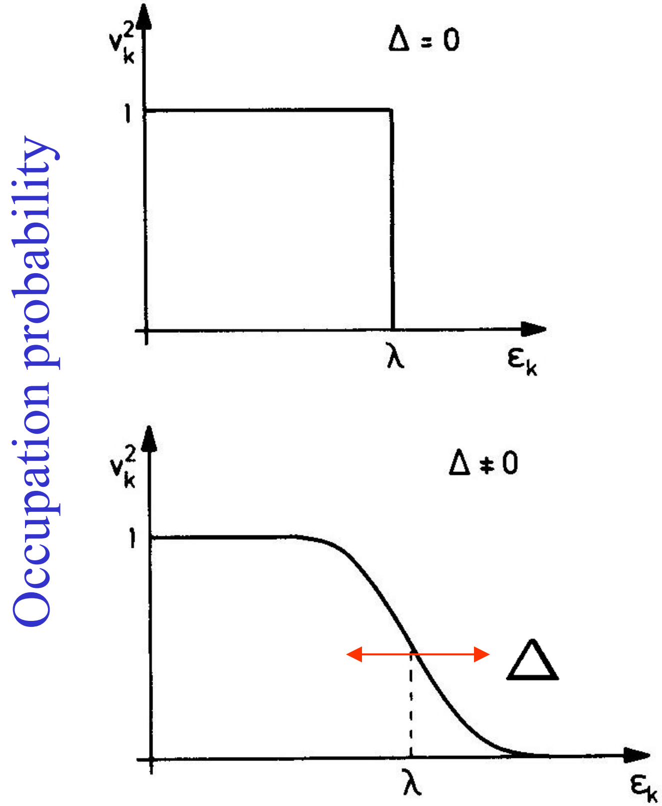
G a/o N ⟶ large

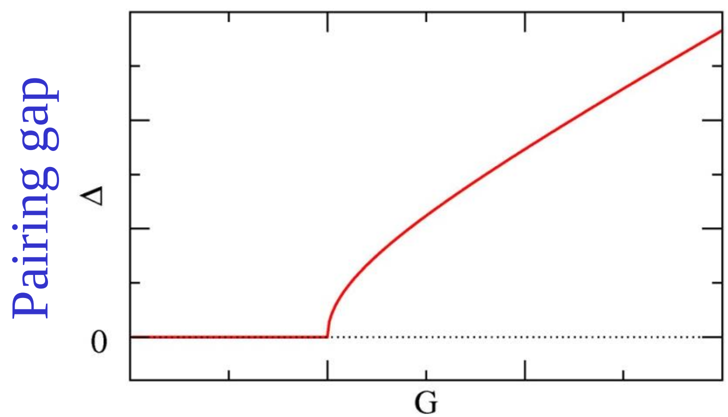
ii) Superfluid solution

$$\Delta \neq 0$$

$$v_\nu^2 < 1$$

$$|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar\nu}^\dagger\right)\big|0\big\rangle$$





Number fluctuation

Normal-Superfulid phase transition

Quasi-particle excitations

$$H = \sum_{\nu} \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \left(\sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger \right) \left(\sum_{k>0} a_{\bar{k}} a_k \right)$$

ハミルトニアンを書き直すと:

$$H \sim E_{BCS} + \sum_k E_k \, \alpha_k^\dagger \alpha_k$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

基底状態: $|\mathrm{BCS}\rangle$

1準粒子状態: $\alpha_k^\dagger|\mathrm{BCS}\rangle$

奇核に対応

2準粒子状態: $\alpha_k^\dagger \alpha_{k'}^\dagger|\mathrm{BCS}\rangle$

- N +/- 2 の原子核
- 同じ原子核の励起状態に対応

$$\alpha^\dagger \alpha^\dagger \sim a^\dagger a^\dagger + aa + a^\dagger a + aa^\dagger$$

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_k E_k \alpha_k^\dagger \alpha_k$$

$$\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \qquad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu$$

（ボゴリューボフ変換）

基底状態： $|\mathrm{BCS}\rangle$

1準粒子状態： $\alpha_k^\dagger|\mathrm{BCS}\rangle$ 　　奇核に対応

2準粒子状態： $\alpha_k^\dagger \alpha_{k'}^\dagger|\mathrm{BCS}\rangle$

- N +/- 2 の原子核
- 同じ原子核の励起状態に対応

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2} \geq \Delta$$

（エネルギー・ギャップ）

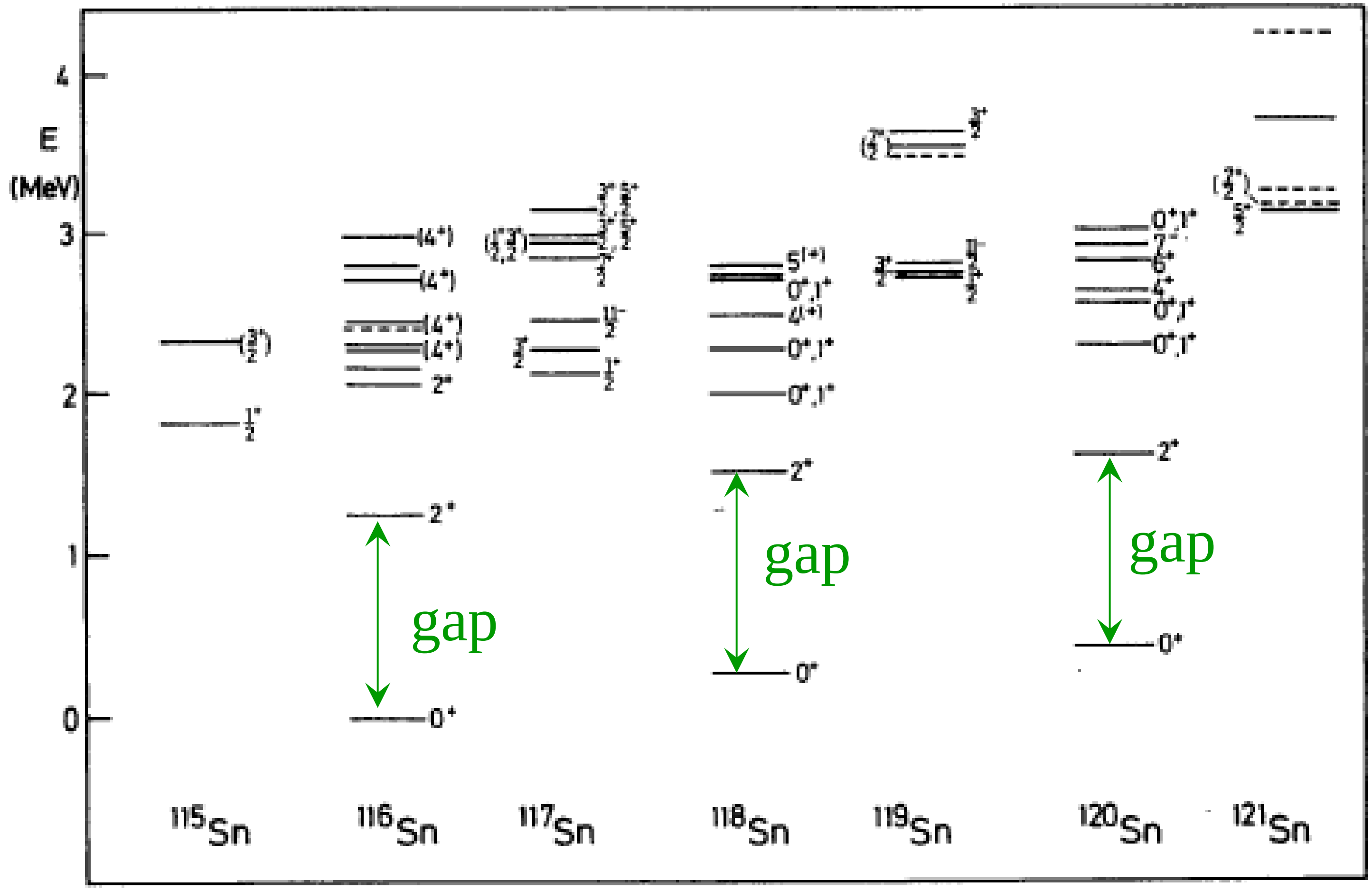
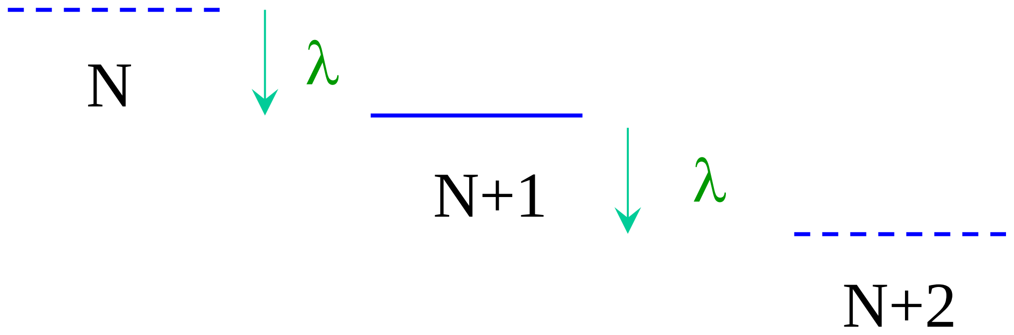


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Ring-Schuck

Even-odd mass difference and pairing gap

$$E(N+2,Z) = E(N,Z) + 2\lambda$$
$$E(N+1,Z) = E(N,Z) + \lambda + \Delta$$

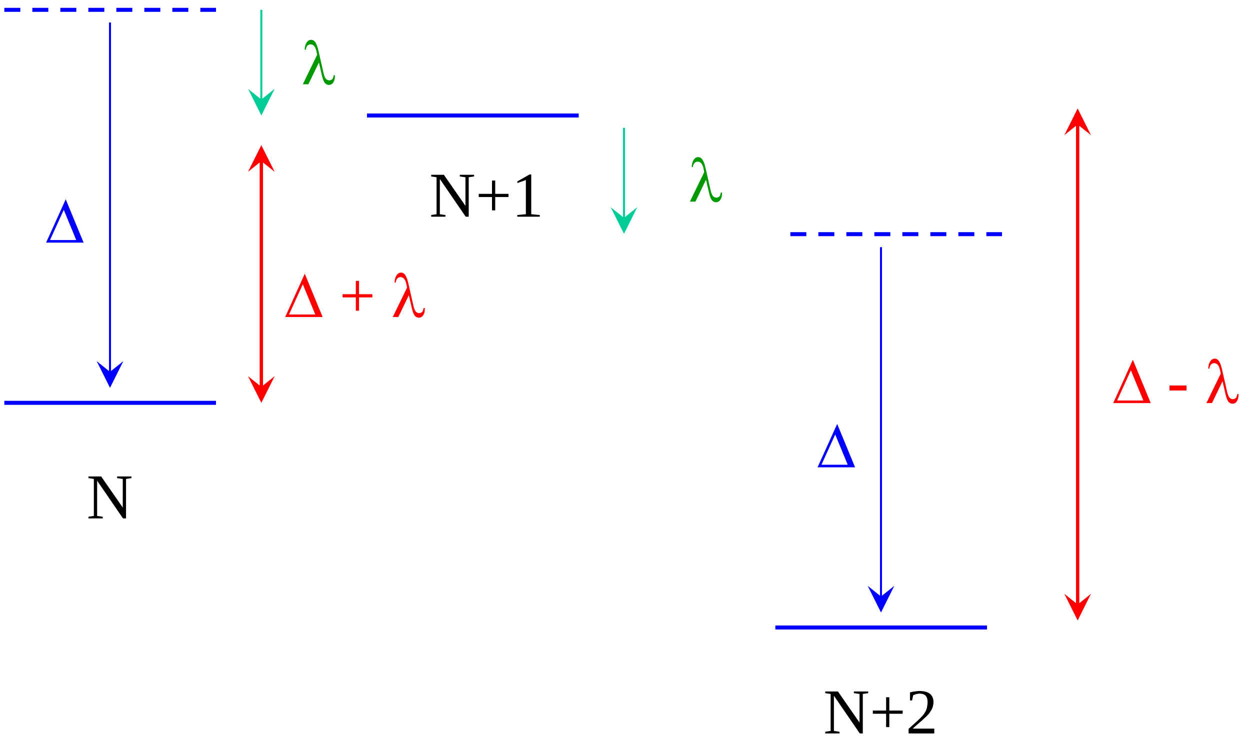


(note) $\lambda < 0$

$$-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2$$

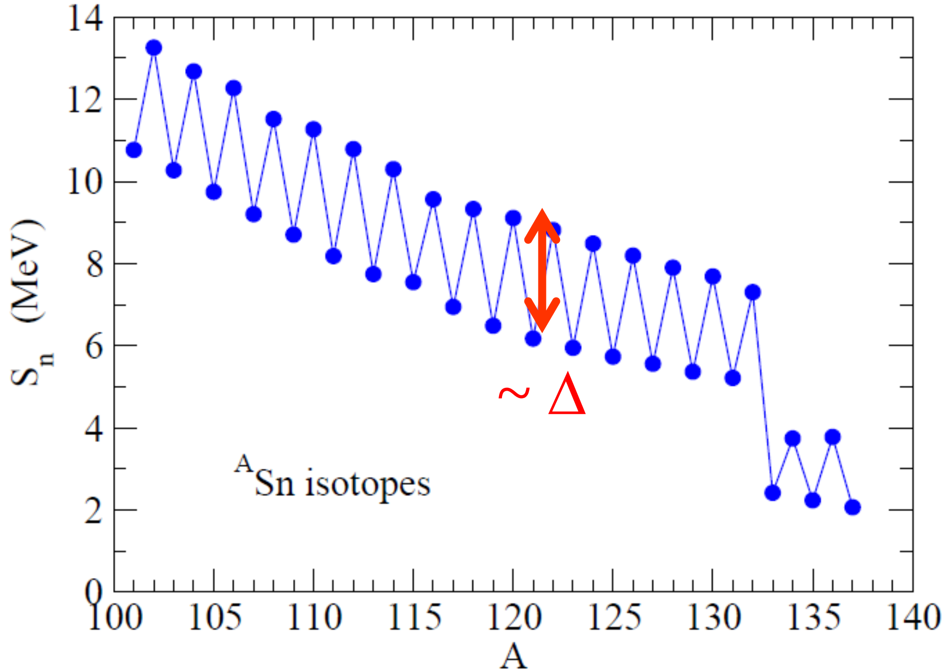
Even-odd mass difference and pairing gap

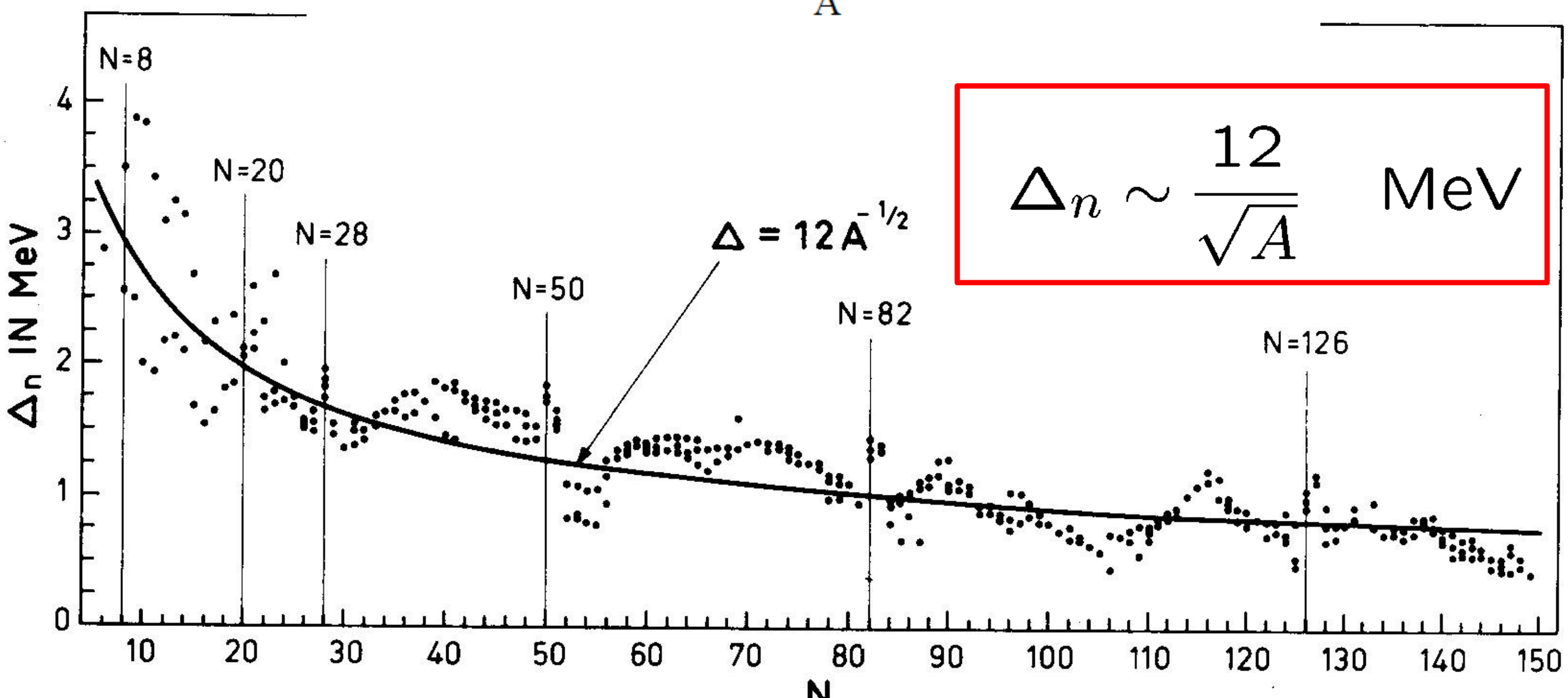
$$E(N+2,Z) = E(N,Z) + 2\lambda$$
$$E(N+1,Z) = E(N,Z) + \lambda + \Delta$$



(note) $\lambda < 0$

$$-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2$$

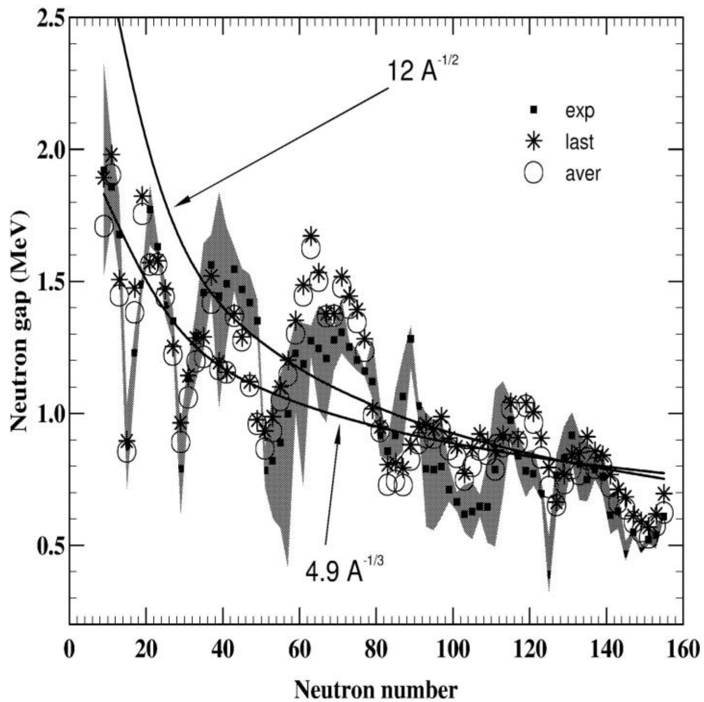




$$\Delta_n \sim \frac{12}{\sqrt{A}} \quad \text{MeV}$$

$$\Delta_n \sim \frac{12}{\sqrt{A}} \quad \text{MeV}$$

ただし、このA依存性はあまり理論的根拠のあるものではない。



S. Hilaire et al.
Phys. Lett. B531, 61 (2002).

$$\Delta \propto \frac{1}{A^{1/3}}?$$

弱結合近似:
$$\Delta \propto e^{-1/(G\rho)} \sim 1 - \frac{1}{G\rho} \propto 1 + cA^{-1/3}$$

$G \propto 1/A$ ←対相関エネルギーから

$\rho \propto A(1 + cA^{-1/3})$ ←フェルミガス近似

弱結合近似: $\Delta \propto e^{-1/(G\rho)}$

Ring-Schuck, Ch. 6.3.6

$$
\begin{aligned}
1 &= \frac{1}{2}G\int_{\epsilon''-\lambda}^{\epsilon'-\lambda}\frac{1}{\sqrt{\epsilon^2+\Delta^2}}\rho(\epsilon)d\epsilon \\
N &= 2\int_{\epsilon''-\lambda}^{\epsilon'-\lambda}\frac{1}{2}\left(1-\frac{\epsilon}{\sqrt{\epsilon^2+\Delta^2}}\right)\rho(\epsilon)d\epsilon
\end{aligned}
$$

$$\to \Delta = \frac{\epsilon'-\epsilon''}{2\sinh(1/G\rho)}\sqrt{1-(1-N/\Omega)^2} \propto e^{-1/(G\rho)}$$

$$\Omega \equiv \rho(\epsilon'-\epsilon'')$$

$$G \propto 1/A$$

$$\Delta E_L \sim -g\int_0^\infty r^2 dr\,(R_l(r))^4 \sim -g\frac{1}{R_0^3} \propto 1/A$$

$$R_l(r) \sim \sqrt{\frac{1}{R_0^3}}\theta(R_0-r)$$

$$\rho \propto A(1+cA^{-1/3})$$

$$\frac{dN}{dE} = V\frac{4\pi k^2}{(2\pi)^3}\frac{dk}{dE} = V\frac{mk_F}{2\pi^2\hbar^2} \propto V \propto A$$

BCS近似の妥当性

ノートで

粒子数非保存と粒子数射影法

$$|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

様々な粒子数の状態が混ざっている $|BCS\rangle = \sum_{N_k} C_{N_k}|N_k\rangle$

ただし、平均値だけは正しく設定されている:

$$\langle BCS|\hat{N}|BCS\rangle = 2\sum_{\nu>0} v_\nu^2 = N$$

粒子数のゆらぎの度合い:

$$(\Delta N)^2 = \langle BCS|\hat{N}^2|BCS\rangle - N^2 = 4\sum_{\nu>0} u_\nu^2 v_\nu^2$$

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粒子数射影: $\hat{P}_N|BCS\rangle = C_N|N\rangle$

$$\hat{P}_N = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(\hat{N}-N)\phi}$$

(note)

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(N'-N)\phi} = \delta_{N,N'}$$

粒子数非保存と粒子数射影法

ハミルトニアンは粒子数を保存：

$$[H,\hat{N}]=0\rightarrow U^\dagger(\phi)HU(\phi)=H;\quad U(\phi)=e^{i\phi\hat{N}}$$

U(1) 対称性

BCS状態は U(1) 対称性が自発的に破れた状態

$$|BCS\rangle=\prod_{\nu>0}\left(u_\nu+v_\nu\,a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0\rangle$$

$$|BCS(\phi)\rangle\equiv U^\dagger(\phi)|BCS\rangle=\prod_{\nu>0}\left(u_\nu+v_\nu\,e^{-2i\phi}\,a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)|0$$

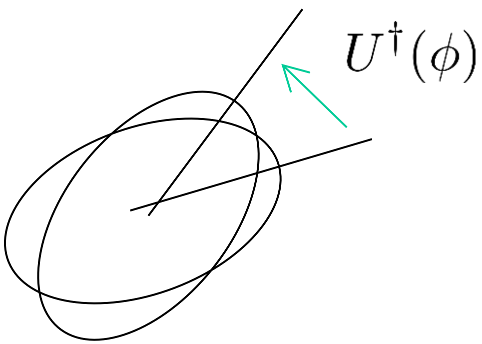
ゲージ空間で「変形」している状態

粒子数非保存と粒子数射影法

$$[H, \hat{N}] = 0 \rightarrow U^\dagger(\phi) H U(\phi) = H; \quad U(\phi) = e^{i\phi\hat{N}}$$

BCS状態は U(1) 対称性が自発的に破れた状態

$$|BCS(\phi)\rangle \equiv U^\dagger(\phi)|BCS\rangle = \prod_{\nu>0}\left(u_\nu + v_\nu e^{-2i\phi} a_\nu^\dagger a_{\bar{\nu}}^\dagger\right)|0\rangle$$



ちょうど角運動量と角度の関係のようなもの：　$\hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$

↕

（この場合、角度 φ はゲージ角）　⟶　$\hat{N} = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$

粒子数非保存と粒子数射影法

$$\hat{N} = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$ →対回転状態 $$E \sim \frac{(N-N_0)^2}{2\mathcal{J}}$$