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No.

☒ BCS近似の妥当性

$\varepsilon = 0$ ——●——●——●—— $m = 2j+1$ （j, m）

$$H = -G\hat{P}^{\dagger}\hat{P} = -G\Omega\hat{A}^{\dagger}\hat{A}\ ;\ \Omega = \frac{2j+1}{2}$$

$$\hat{A}^{\dagger} = \frac{1}{\sqrt{\Omega}}\sum_{m>0} a_{jm}^{\dagger} a_{j-m}^{\dagger}$$

- exact な解

$$AA^{\dagger} = \frac{1}{\Omega}\sum_{m>0}\sum_{m'>0} a_{j-m}\underbrace{a_{jm}\, a_{jm'}^{\dagger}}_{\delta_{m,m'} - a_{jm'}^{\dagger} a_{jm}}\, a_{j-m'}^{\dagger}$$

$$= \frac{1}{\Omega}\sum_{m>0}\left(a_{j-m}a_{j-m}^{\dagger} - \sum_{m'>0} a_{j-m}a_{jm'}^{\dagger}\, a_{jm}\, a_{j-m'}^{\dagger}\right)$$

$$= \frac{1}{\Omega}\sum_{m>0}\left(\underbrace{a_{j-m}a_{j-m}^{\dagger}}_{1 - a_{j-m}^{\dagger}a_{j-m}} - \sum_{m'>0} a_{jm'}^{\dagger}\, \underbrace{a_{j-m}a_{j-m'}^{\dagger}}_{\delta_{m,m'} - a_{j-m'}^{\dagger}a_{j-m}}\, a_{jm}\right)$$

$$= -\frac{1}{\Omega}\sum_{m>0}\left(a_{jm}^{\dagger}a_{jm} + a_{j-m}^{\dagger}a_{j-m}\right) + \frac{1}{\Omega}\sum_{m>0}\sum_{m'>0} a_{jm'}^{\dagger}\, a_{j-m}^{\dagger} \times a_{j-m}a_{jm}$$

$$+ \frac{1}{\Omega}\sum_{m>0} 1$$

$$= A^{\dagger}A - \frac{1}{\Omega}\hat{N} + 1$$

$\hat{A}|0\rangle = \hat{N}|0\rangle = 0$ ∴ $[\hat{N}, \hat{A}^{\dagger}] = 2\hat{A}^{\dagger}$

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$$H\hat{A}^{+}|0\rangle = -G\Omega A^{+}\underbrace{AA^{+}}_{\| \; A^{+}A+1-\frac{\hat{N}}{\Omega}}|0\rangle = -G\Omega A^{+}|0\rangle$$

$$H(A^{+})^{2}|0\rangle = -G\Omega A^{+}\underbrace{AA^{+}}_{\| \; A^{+}A+1-\frac{\hat{N}}{\Omega}}A^{+}|0\rangle$$

$$= -G\Omega\Big[(A^{+})^{2}\underbrace{AA^{+}}_{\| \; A^{+}A+1-\frac{\hat{N}}{\Omega}} + (A^{+})^{2} - \frac{1}{\Omega}\hat{A}^{+}\underbrace{\hat{N}\hat{A}^{+}}_{\| \; A^{+}N+2A^{+}}\Big]|0\rangle$$

$$= -G\Omega\left(1+1-\frac{2}{\Omega}\right)(A^{+})^{2}|0\rangle$$

$$= -2G\Omega\left(1-\frac{1}{\Omega}\right)(A^{+})^{2}|0\rangle$$

$$H(A^{+})^{\frac{N}{2}}|0\rangle = -\frac{GN}{4}(2\Omega-N+2)(A^{+})^{\frac{N}{2}}|0\rangle$$

$$E = -\frac{GN}{4}(2\Omega-N+2)$$

- BCS 近似

$$N = 2\sum_{m>0}|V_{jm}|^{2} = 2\cdot\frac{2j+1}{2}v^{2} = 2\Omega v^{2} \rightarrow v^{2} = \frac{N}{2\Omega}$$

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$$u^2 = 1 - v^2 = 1 - \frac{N}{2\Omega}$$

$$\Delta = G\Omega uv = G\Omega \sqrt{\frac{N}{2\Omega}} \sqrt{1 - \frac{N}{2\Omega}}$$

$$E_{BCS} = -\frac{\Delta^2}{G} = -\frac{G^2\Omega^2}{G} \cdot \frac{N}{2\Omega}\left(1 - \frac{N}{2\Omega}\right)$$

$$= -\frac{GN}{4}(2\Omega - N)$$

N が 大きいと $N-2 \sim N$ となり $E \sim E_{BCS}$

◀ BCS近似の例題

角運動量

ε ——— $[j]$

- - - - - - - $\lambda = 0$

$-\varepsilon$ ——— $[j]$

$$\Omega = \frac{2j+1}{2}$$

ギャップの方程式：

$$\Delta = \frac{G}{2} \sum_{\nu>0} \frac{\Delta}{\sqrt{(\varepsilon_\nu - \lambda)^2 + \Delta^2}} = \frac{G}{2}\Omega \cdot 2 \times \frac{\Delta}{\sqrt{\varepsilon^2 + \Delta^2}}$$

$$\rightarrow \quad \Delta^2 = G^2\Omega^2 \cdot \frac{\Delta^2}{\varepsilon^2 + \Delta^2}$$

解： $\Delta = 0$, $\Delta = \sqrt{G^2\Omega^2 - \varepsilon^2}$

↑ この解は $G > \frac{\varepsilon}{\Omega}$ のときのみ

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$\Delta = 0$ のとき

$v_1 = 1, \quad v_2 = 0$

$\Delta = \sqrt{G^2\Omega^2 - \varepsilon^2}$ のとき

$$v_1^2 = \frac{1}{2}\left(1 - \frac{-\varepsilon}{\sqrt{\varepsilon^2 + G^2\Omega^2 - \varepsilon^2}}\right) = \frac{1}{2}\left(1 + \frac{\varepsilon}{G\Omega}\right)$$

$$v_2^2 = \frac{1}{2}\left(1 - \frac{\varepsilon}{G\Omega}\right)$$