$$\left(-\frac{\hbar^2}{2m}\nabla^2 + \underbrace{V(r)}_{\text{摂動}} - E\right)\psi(r) = 0$$

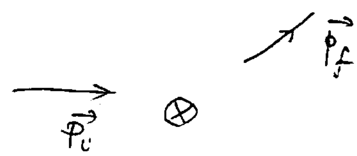
散乱問題における
5.2. Born Approximation $\left(\frac{V(r)}{E} \ll 1\right)$ 高エネルギー散乱

$$\psi_i(r) = e^{i\vec{p}_i\cdot\vec{r}/\hbar} \xrightarrow[\text{遷移}]{} \psi_f(r) = e^{i\vec{p}_f\cdot\vec{r}/\hbar}$$

$$\left(\frac{p_i^2}{2m} = \frac{p_f^2}{2m} = E\right)$$

Fermi's Golden Rule:

$$R_{i\to f} = \frac{2\pi}{\hbar}\int\frac{d^3p_f}{(2\pi\hbar)^3}\,|M_{fi}|^2\,\delta\left(\frac{p_f^2}{2m} - \frac{p_i^2}{2m}\right)$$



$$\begin{aligned}
M_{fi} &= \langle\psi_f|V|\psi_i\rangle \\
&= \int d\vec{r}\ \psi_f^*(\vec{r})\,V(\vec{r})\,\psi_i(\vec{r}) \\
&= \int d\vec{r}\ e^{i(\vec{p}_i - \vec{p}_f)\cdot\vec{r}/\hbar}\,V(\vec{r}) \\
&\equiv \int d\vec{r}\ e^{-i\vec{q}\cdot\vec{r}}\,V(\vec{r})
\end{aligned}$$

$$\vec{q} \equiv (\vec{p}_f - \vec{p}_i)/\hbar \quad : \quad \text{momentum transfer}$$

$$\equiv \tilde{V}(\vec{q}) \qquad (\text{フーリエ変換})$$

$$\downarrow$$

$$\begin{aligned}
R_{i\to f} &= \frac{2\pi}{\hbar}\int\frac{p_f^2\,dp_f\,d\Omega}{(2\pi\hbar)^3}\,|\tilde{V}(\vec{q})|^2\,\delta\left(\frac{p_f^2}{2m} - E\right) \\
&= \frac{2\pi}{\hbar}\cdot\frac{1}{(2\pi\hbar)^3}\cdot\int m p_f\, d\left(\frac{p_f^2}{2m}\right) d\Omega\ |\tilde{V}(\vec{q})|^2\,\delta\left(\frac{p_f^2}{2m} - E\right) \\
&= \frac{m p_i}{4\pi^2\hbar^4}\int d\Omega\ |\tilde{V}(\vec{q})|^2
\end{aligned}$$

$$\frac{1}{\cancel{2}} m v^2 = \frac{k^2\hbar^2}{\cancel{2}m} \quad \Downarrow \quad v = \frac{k\hbar}{m}$$

$$= \frac{\hbar}{2im}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right)$$

$$j_{in} = \frac{k\hbar}{m} = v$$

$\Downarrow$

$$d\sigma = \frac{1}{4\pi^2 \hbar^4} \cancel{\frac{1}{|v_{rel}|} m p_i} \, m^2 \, d\Omega \, |\tilde{V}(\vec{q})|^2.$$

$\Downarrow$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2} \left| \frac{1}{\hbar^2} \tilde{V}(\vec{q}) \right|^2.}$$

例)

$$V(\vec{r}) = \int \frac{\rho(r')}{|r - r'|} dr' \cdot z_p e$$

(note)
$\nabla^2 V = -4\pi \rho \cdot z_p e$
ポアソン~~方程式~~

$\downarrow$

$$\tilde{V}(\vec{q}) = \int d\vec{r} \; e^{-i\vec{q}\cdot\vec{r}} \, V(\vec{r})$$

$$= \cancel{\frac{1}{-iq} e^{-i\vec{q}\cdot\vec{r}} V(\vec{r}) \Big|_{-\infty}^{\infty}} \to 0 + \frac{1}{iq} \cdot \int e^{-i\vec{q}\cdot\vec{r}} \nabla V(r) \, dr$$

$$= \cancel{\frac{1}{q^2} \nabla V \Big|_{-\infty}^{\infty}} \to 0 - \frac{1}{q^2} \int e^{-i\vec{q}\cdot\vec{r}} \nabla^2 V(r) \, dr$$

$$= + \frac{4\pi}{q^2} \int e^{-i\vec{q}\cdot\vec{r}} \rho(\vec{r}) \, d\vec{r} \cdot z_p e$$

$\Downarrow$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{q^4 \hbar^4} |F(\vec{q})|^2 \cdot z_p^2 e^2$$

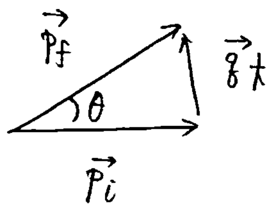
$$F(\vec{q}) = \int e^{-i\vec{q}\cdot\vec{r}} \rho(\vec{r}) d\vec{r}$$

$$\int_{-1}^{1} d(\cos\theta)\, P_\ell(\cos\theta)\, P_{\ell'}(\cos\theta) = \frac{2}{2\ell+1}\,\delta_{\ell\ell'}$$

$\frac{p_i^2}{2m} = E$

$P_0(\cos\theta) = 1$

(note)



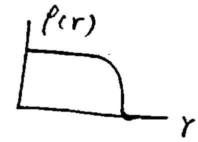
$$q\hbar = 2 p_i \sin\frac{\theta}{2}$$
$$= 2\cdot\sqrt{2mE}\,\sin\frac{\theta}{2}$$

↓

$$\frac{d\sigma}{d\Omega} = \frac{\cancel{4m^2}}{16\cdot \cancel{4m^2} E^2}\,\frac{1}{\sin^4\frac{\theta}{2}}\,|F(\vec{q})|^2\cdot z_p^2 e^2$$

$$= \frac{1}{(4E\sin^2\frac{\theta}{2})^2}\,|F(\vec{q})|^2\cdot z_p^2 e^2$$

形状因子
"form factor" ⟶ 電荷密度の決定



$\rho_0 \sim 0.17\ fm^{-3}$

$R \sim 1.1 A^{1/3}\ fm$

(note) $e^{+i\vec{q}\cdot\vec{r}} = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^\ell\, j_\ell(qr)\, P_\ell(\cos\theta)$

↓ for $\rho(\vec{r}) = \rho(r)$

$$F(\vec{q}) = 2\pi \int_{-1}^{1} d(\cos\theta) \int_0^\infty r^2 dr \sum_{\ell=0}^{\infty} (2\ell+1)\, i^\ell\, j_\ell(qr)\, P_\ell(\cos\theta) \times \rho(r)$$

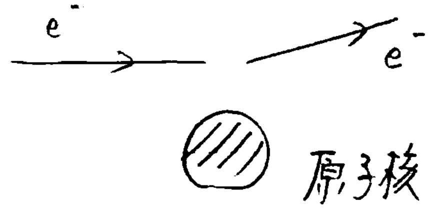
$$= 4\pi\int_0^\infty r^2 dr\, \underbrace{j_0(qr)}_{\frac{\sin(qr)}{qr}}\, \rho(r)$$

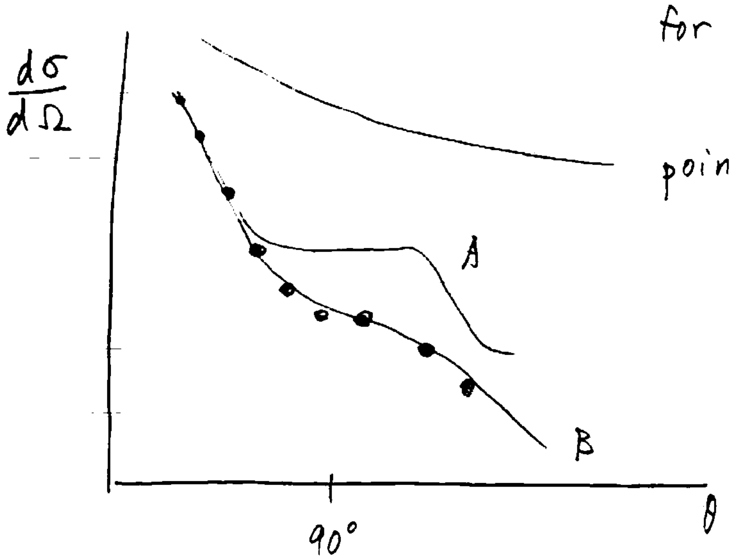
(note) if $\rho(r) = z_T e\, \delta(r)$ ⤳ $F(\vec{q}) = z_T e$

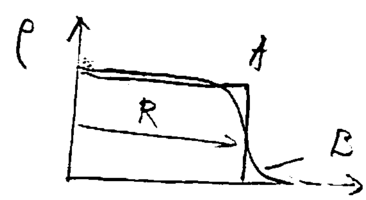
↓ $$\frac{d\sigma}{d\Omega} = \left(\frac{z_p z_T e^2}{4E\sin^2\frac{\theta}{2}}\right)^2$$

(古典的なラザフォード散乱の式と同じ)

電子散乱による原子核の密度分布の決定







$\rho_0 \sim 0.17\ \mathrm{fm}^{-3}$

$R \sim 1.1\ A^{1/3}\ \mathrm{fm}$

$a \sim 0.53\ \mathrm{fm}$