◻ 集団振動運動について

1. 慣性質量に対する流体模型

原子核の変形： $R(\theta,\varphi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\varphi)\right)$.

平衡点の周りの微小振動を記述するハミルトニアン：

$$H = \frac{1}{2} \sum_{\lambda,\mu} \left\{ B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2 \right\}.$$

∘ B_λ をどう見積もるか → 流体アプローチ

古典的な渦なし (irrotational), 非圧縮性 (incompressible) 流体を仮定.

渦なし流体 → $\nabla \times \underbrace{\boldsymbol{v}(\boldsymbol{r})}_{\text{流れの速度}} = 0$

↓

$\boldsymbol{v} = -\nabla \underbrace{\Phi(\boldsymbol{r})}_{\text{速度ポテンシャル}}$

非圧縮性流体 → $\rho = \text{const}$

↓ 連続の式： $\underbrace{\frac{\partial \rho}{\partial t}}_{= 0} + \nabla \cdot (\rho \boldsymbol{v}) = 0$.

↓ $\nabla \cdot \boldsymbol{v} = 0$

↓ $$\boxed{\nabla^2 \Phi(r) = 0.}$$

解：$\Phi(r) = \sum_{\lambda\mu} d_{\lambda\mu}^{*}\, r^{\lambda}\, Y_{\lambda\mu}(\hat{r})$

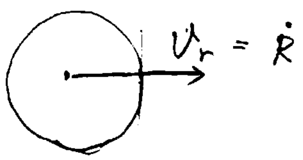
cf. 自由粒子に対するシュレーディンガー方程式

$$-\frac{\hbar^2}{2m}\nabla^2 \Psi(r) = E\,\Psi(r)$$

$$\Psi(r) = \frac{u_l(r)}{r}\, Y_{lm}(\hat{r})$$

$$u_l(r) \sim r^{l+1}$$

境界条件：



$$\frac{\partial}{\partial t} R(\theta,\varphi) = [v_r]_{r=R_0} = -\left[\frac{\partial \Phi}{\partial r}\right]_{r=R_0}$$

↓

$$R_0 \sum_{\lambda,\mu} \dot{\alpha}_{\lambda\mu} Y_{\lambda\mu}^{*} = -\sum_{\lambda\mu} \lambda R_0^{\lambda-1} d_{\lambda\mu} Y_{\lambda\mu}^{*}$$

↓ $$\boxed{d_{\lambda\mu} = -\frac{1}{\lambda} R_0^{2-\lambda}\, \dot{\alpha}_{\lambda\mu}}$$

流体全体の運動エネルギー：

$$T = \frac{m}{2}\rho \int v^2 \, d\mathbf{r} = \frac{m}{2}\rho \int |\nabla \Phi|^2 \, d\mathbf{r}$$

$$= \cdots\cdots = \frac{m}{2}\rho R_0^5 \sum_{\lambda,\mu} \frac{1}{\lambda} |\dot{\alpha}_{\lambda\mu}|^2$$

$$\updownarrow$$

$$\frac{1}{2} \sum_{\lambda,\mu} B_\lambda |\dot{\alpha}_{\lambda\mu}|^2$$

$$\swarrow$$

$$\boxed{B_\lambda = \frac{\rho m R_0^5}{\lambda} = \frac{3}{4\pi\lambda} A \cdot m R_0^2}$$

2. 表面振動の量子化

$$H = \frac{1}{2} \sum_{\lambda,\mu} \left\{ B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2 \right\}$$

$$\xrightarrow[\text{正準量子化}]{} \quad H = \sum_{\lambda\mu} \hbar \omega_\lambda \left(b^\dagger_{\lambda\mu} b_{\lambda\mu} + \frac{1}{2} \right)$$

$$\omega_\lambda = \sqrt{C_\lambda / B_\lambda}$$

$$[b_{\lambda\mu}, b_{\lambda'\mu'}] = 0, \quad [b_{\lambda\mu}, b^\dagger_{\lambda'\mu'}] = \delta_{\lambda\lambda'} \delta_{\mu\mu'}$$

・1フォノン状態

$$b^\dagger_{\lambda\mu} |0\rangle$$

角運動量 $I = \lambda,\ I_z = \mu,$
パリティ $(-)^\lambda$

・2フォノン状態

$$\frac{1}{\sqrt{2}} b^\dagger_{\lambda\mu} b^\dagger_{\lambda'\mu'} |0\rangle$$

角運動量のよい状態に組み直す

$\rightarrow$ $(I, I_z) = (\lambda, \mu)$ と $(I, I_z) = (\lambda', \mu')$
の合成

$$[b_\lambda^\dagger b_{\lambda'}^\dagger]^{(IM)} = \sum_{\mu\mu'} \underbrace{\langle \lambda\mu\,\lambda'\mu' | IM \rangle}_{\text{クレブシュ-ゴルダン係数}} b_{\lambda\mu}^\dagger\, b_{\lambda'\mu'}^\dagger$$

$\lambda = \lambda' = 2$ の場合：

$$[b_2^\dagger b_2^\dagger]^{(IM)} = \sum_{\mu\mu'} \langle 2\mu\, 2\mu' | IM \rangle\, b_{2\mu}^\dagger\, b_{2\mu'}^\dagger$$

$$= \sum_{\mu\le\mu'} \langle 2\mu\, 2\mu' | IM \rangle\, b_{2\mu}^\dagger\, b_{2\mu'}^\dagger + \underbrace{\langle 2\mu'\, 2\mu | IM \rangle}_{= (-)^I \langle 2\mu 2\mu' | IM \rangle}\, \underbrace{b_{2\mu'}^\dagger\, b_{2\mu}^\dagger}_{= b_{2\mu}^\dagger\, b_{2\mu'}^\dagger}$$

$$= \sum_{\mu\le\mu'} \left\{ 1 + (-)^I \right\} \langle 2\mu\, 2\mu' | IM \rangle\, b_{2\mu}^\dagger\, b_{2\mu'}^\dagger$$

$\Downarrow$ I：偶数のみ

$2\times\hbar\omega_2$ ———— $0^+, 2^+, 4^+$

$\hbar\omega_2$ ———— 2^+

———— 0^+

但し dipole 振動はアイソスピンに要注意,

(例)*

調和振動子

$$\varphi_0(x) \propto e^{-\alpha x^2}$$
$$\varphi_1(x) \propto x e^{-\alpha x^2}$$
$$\varphi_2(x) \propto (4\alpha x^2 - 1)\, e^{-\alpha x^2}$$

3次元調和振動子

・g.s. $(n_x, n_y, n_z) = (0,0,0)$

$$\Phi_0(x,y,z) = e^{-\alpha(x^2+y^2+z^2)} = e^{-\alpha r^2} \quad \longleftrightarrow \ell = 0$$

・1st. $(n_x, n_y, n_z) = (0,0,1),\ (0,1,0),\ (1,0,0)$

$$\varphi_{001}(x,y,z) \propto z e^{-\alpha r^2}$$
$$\varphi_{101} \propto x e^{-\alpha r^2}$$
$$\varphi_{010} \propto y e^{-\alpha r^2}$$

(note) $\varphi_{001} \propto r e^{-\alpha r^2} \cos\theta \propto r e^{-\alpha r^2} Y_{10}(\theta)$

$$\varphi_{101} \pm i \varphi_{010} \propto r e^{-\alpha r^2} \sin\theta (\cos\varphi \pm i \sin\varphi)$$
$$= r e^{-\alpha r^2} \sin\theta\, e^{\pm i\varphi} \propto r e^{-\alpha r^2} Y_{1\pm1}(\theta,\varphi)$$

· 2nd. $(n_x, n_y, n_z) = (2,0,0),\ (0,2,0),\ (0,0,2)$
$(1,1,0),\ (1,0,1),\ (0,1,1)$

$$\varphi_{200} \propto (4\alpha x^2 - 1)\, e^{-\alpha r^2}$$
$$\varphi_{020} \propto (4\alpha y^2 - 1)\, e^{-\alpha r^2}$$
$$\varphi_{002} \propto (4\alpha z^2 - 1)\, e^{-\alpha r^2}$$
$$\varphi_{110} \propto xy\, e^{-\alpha r^2}$$
$$\varphi_{101} \propto xz\, e^{-\alpha r^2}$$
$$\varphi_{001} \propto yz\, e^{-\alpha r^2}$$

$$\varphi_{200} + \varphi_{020} + \varphi_{002} \propto (4\alpha r^2 - 3)\, e^{-\alpha r^2} \Leftrightarrow l=0$$

$$2\varphi_{002} - \varphi_{200} - \varphi_{020} \propto (2z^2 - x^2 - y^2)\, e^{-\alpha r^2}$$
$$= (3z^2 - x^2 - y^2 - z^2)\, e^{-\alpha r^2}$$
$$= (3\cos^2\theta - 1)\, r^2 e^{-\alpha r^2}$$
$$\propto r^2 e^{-\alpha r^2}\, Y_{20}(\theta)$$

$$\varphi_{101} \pm i\,\varphi_{001} \propto r^2 e^{-\alpha r^2} \cos\theta \sin\theta\, (\cos\varphi \pm i \sin\varphi)$$
$$= r^2 e^{-\alpha r^2} \cos\theta \sin\theta\, e^{\pm i\varphi}$$
$$\propto r^2 e^{-\alpha r^2}\, Y_{2\pm1}(\theta, \varphi)$$

$$\varphi_{200} - \varphi_{020} + 2i \cdot 4\alpha\, \varphi_{110}$$
$$\propto 4\alpha\, (x^2 - y^2 + 2i\, xy)\, e^{-\alpha r^2}$$
$$= 4\alpha\, (\cos^2\varphi - \sin^2\varphi + 2i \cos\varphi \sin\varphi)\, r^2 \sin^2\theta\, e^{-\alpha r^2}$$
$$= 4\alpha\, (\cos 2\varphi + 2i \sin 2\varphi)\, r^2 \sin^2\theta\, e^{-\alpha r^2}$$
$$= 4\alpha\, r^2 \sin^2\theta\, e^{\pm 2i\varphi}\, e^{-\alpha r^2}$$
$$\propto r^2 e^{-\alpha r^2}\, Y_{2\pm2}(\theta, \varphi)$$