電磁遷移

1. 遷移確率

$$\Gamma_{fi}(\lambda\mu) \sim \frac{8\pi(\lambda+1)}{\hbar\lambda((2\lambda+1)!!)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} |\langle\Psi_f|\mathcal{M}_{\lambda\mu}|\Psi_i\rangle|^2 \tag{1}$$

- Eλ 遷移

$$\mathcal{M}_{\lambda\mu} = \sum_{i=1}^{Z} er_i^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}}_i) \equiv \hat{Q}_{\lambda\mu} \tag{2}$$

- Mλ 遷移

$$\mathcal{M}_{\lambda\mu} = \mu_N \sum_{i=1}^{A}\left\{g_s^{(i)}\boldsymbol{s}_i + \frac{2}{\lambda+1}g_l^{(i)}\boldsymbol{l}_i\right\}\cdot\left(\boldsymbol{\nabla}r_i^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}}_i)\right) \equiv \hat{M}_{\lambda\mu} \tag{3}$$

$\mu_N = e\hbar/2mc$, $g_l = 1$ (陽子) or 0 （中性子）, $g_s = 5.586$ (陽子) or -3.826 （中性子）.

2. 換算遷移確率

角運動量の z 成分を区別しないとき、

$$\begin{aligned}\Gamma_{fi} &= \frac{1}{2I_i+1}\sum_{M_f,M_i,\mu}\Gamma_{fi}(\lambda\mu) \quad (4)\\ &= \frac{1}{2I_i+1}\sum_{M_f,M_i,\mu}\frac{8\pi(\lambda+1)}{\hbar\lambda((2\lambda+1)!!)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}|\langle I_fM_f|\mathcal{M}_{\lambda\mu}|I_iM_i\rangle|^2 \quad (5)\end{aligned}$$

Wigner-Eckart の定理：

$$\langle I_fM_f|\mathcal{M}_{\lambda\mu}|I_iM_i\rangle = (-1)^{I_i-M_i}\frac{1}{\sqrt{2\lambda+1}}\langle I_fM_fI_i-M_i|\lambda\mu\rangle\langle I_f||\mathcal{M}_\lambda||I_i\rangle \tag{6}$$

C.G. 係数の性質：

$$\sum_{M_iM_f}\langle I_fM_fI_i-M_i|\lambda\mu\rangle^2 = 1 \tag{7}$$

を用いると

$$\Gamma_{fi} = \frac{8\pi(\lambda+1)}{\hbar\lambda((2\lambda+1)!!)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}\frac{1}{2I_i+1}|\langle I_f||\mathcal{M}_\lambda||I_i\rangle|^2 \tag{8}$$

ここで

$$\begin{aligned}B(E\lambda;I_i\to I_f) &= \frac{1}{2I_i+1}|\langle I_f||Q_\lambda||I_i\rangle|^2 \quad (9)\\ B(M\lambda;I_i\to I_f) &= \frac{1}{2I_i+1}|\langle I_f||M_\lambda||I_i\rangle|^2 \quad (10)\end{aligned}$$

を換算遷移確率という。