

β 崩壊  $K > 17$

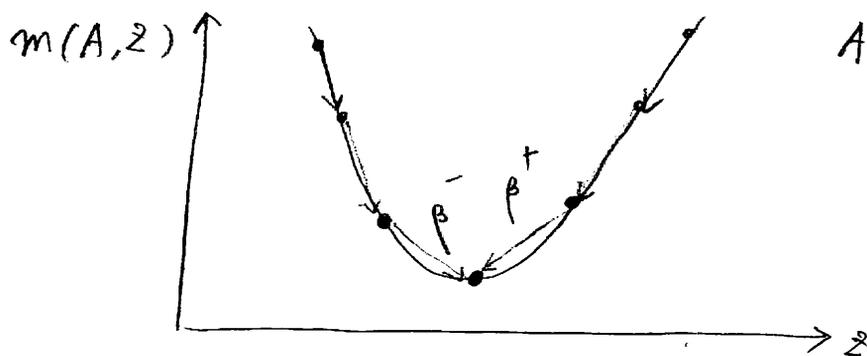
### 0. β 安定線

液滴模型

$$B(N, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - \underbrace{a_{sym} \frac{(N-Z)^2}{A}}$$

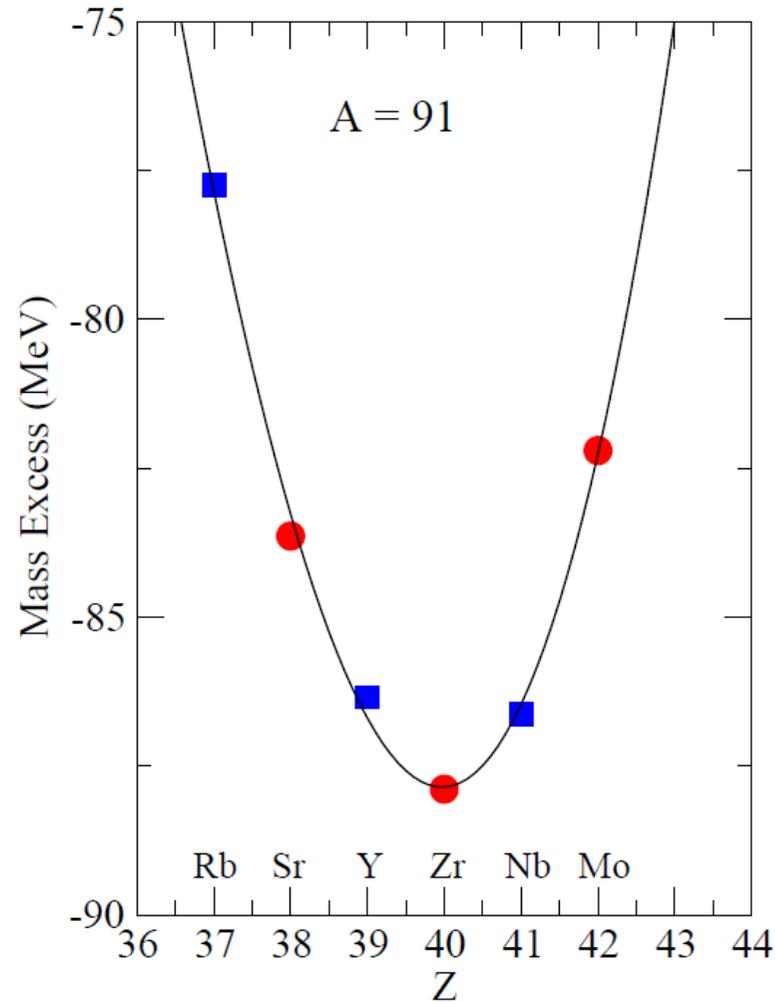
↓

$$m(A, Z) = f(A) + a_c \frac{Z^2}{A^{1/3}} + a_{sym} \frac{(A-2Z)^2}{A}$$

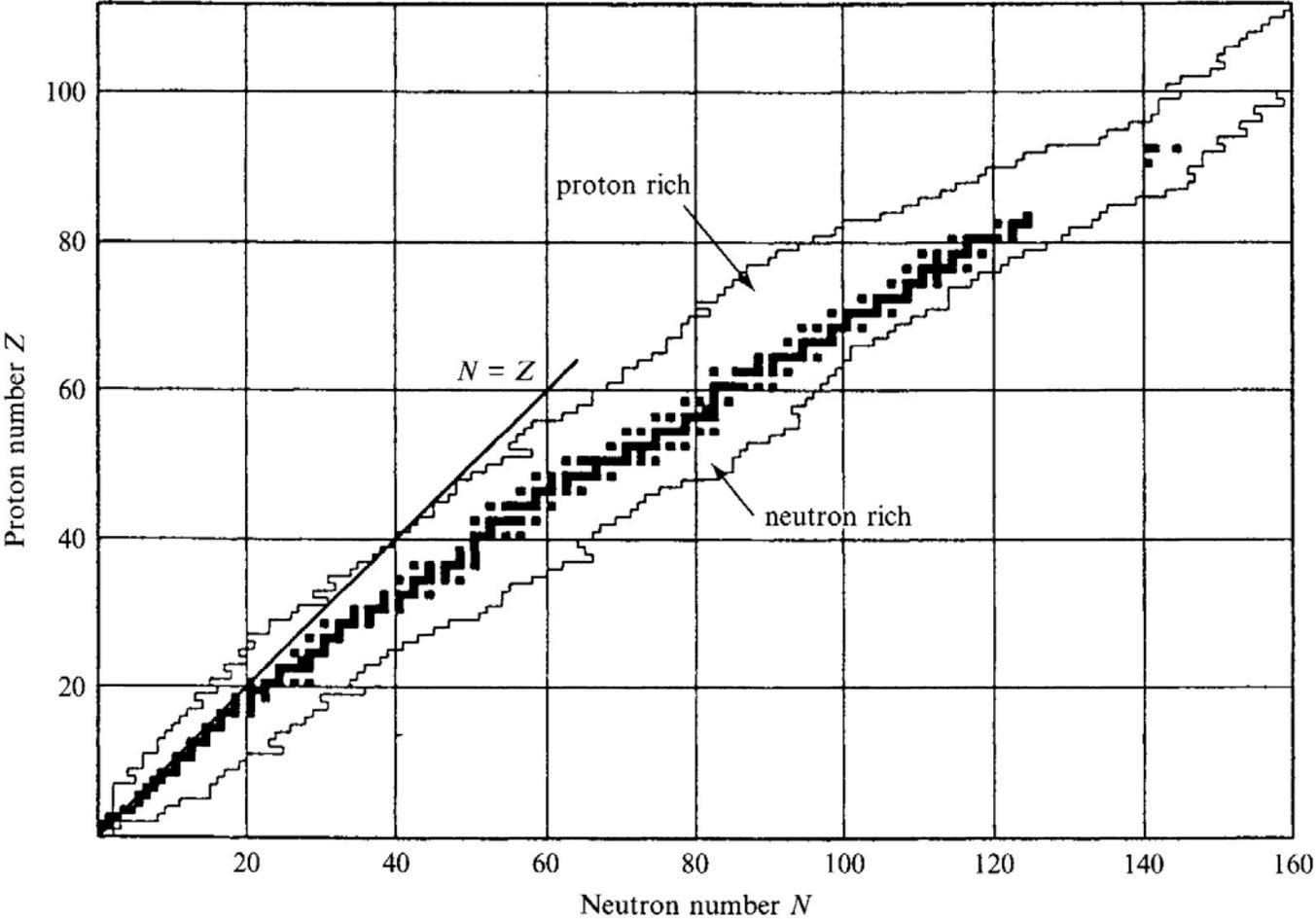


# β-安定線

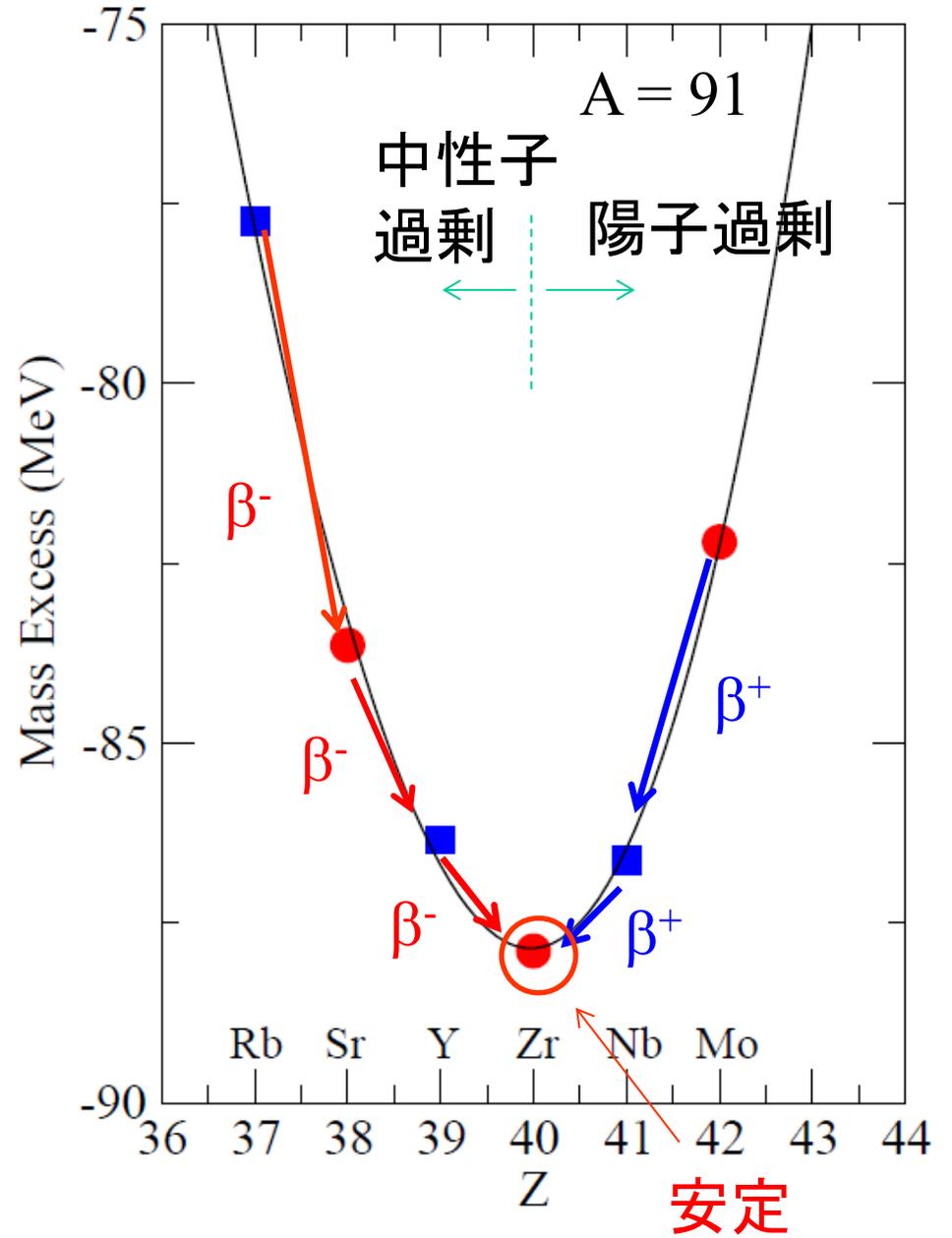
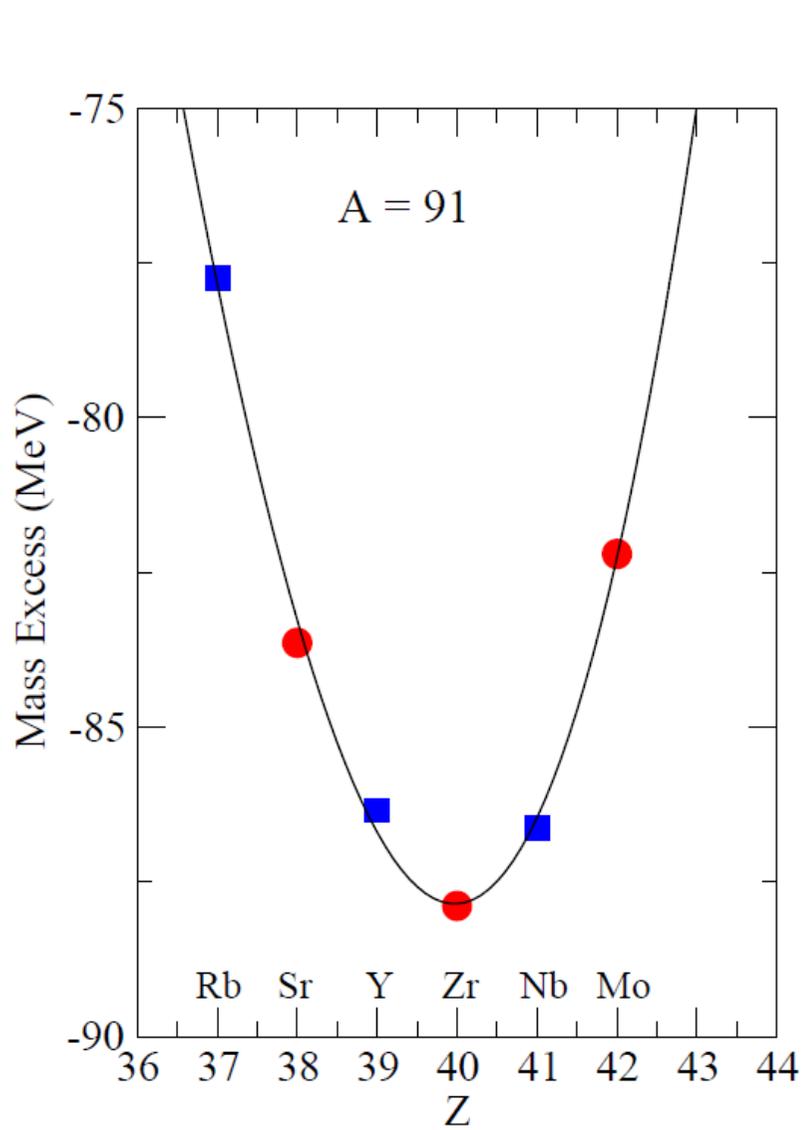
$$B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$$
$$m(A, Z) = f(A) + a_C \frac{Z^2}{A^{1/3}} + a_{\text{sym}} \frac{(A - 2Z)^2}{A}$$



# 核图表

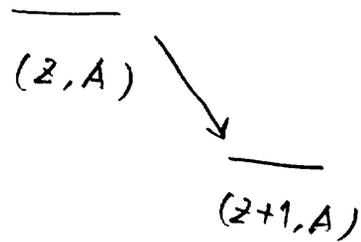
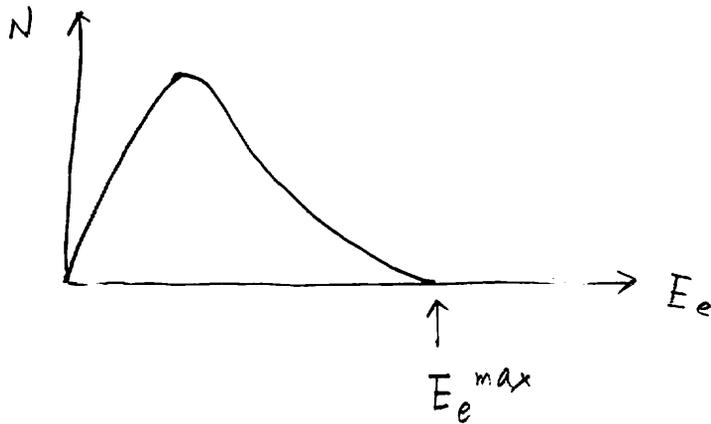
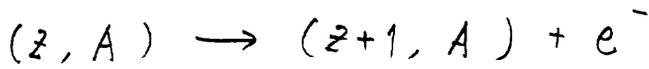


安定核:  $N \geq Z$



# β崩壊について

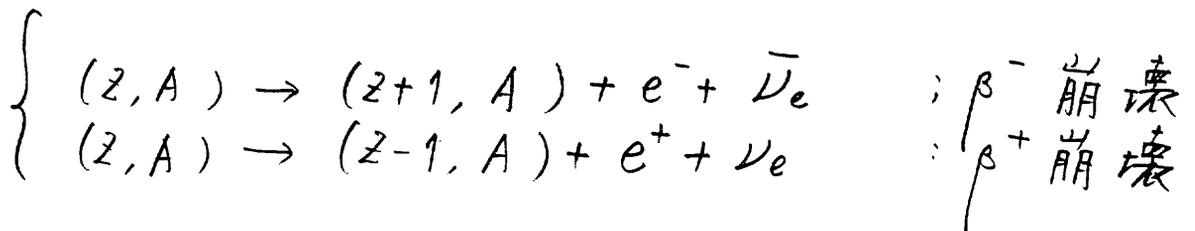
1. β線スピン外ルとニュートリノ



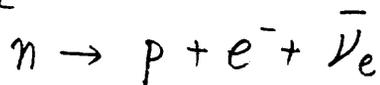
$$E_e^{max} \sim M(A, Z) c^2 - M(A, Z+1) c^2 - m_e c^2$$

連続分布 → 3体崩壊を示唆

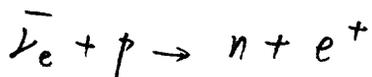
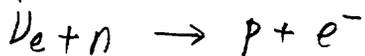
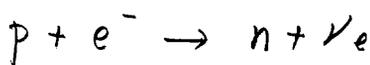
パウリ (1931): ニュートリノを仮定



素過程



ファストプロセス



(電子捕獲)

ニュートリノ-原子核反応

## β線スペクトル

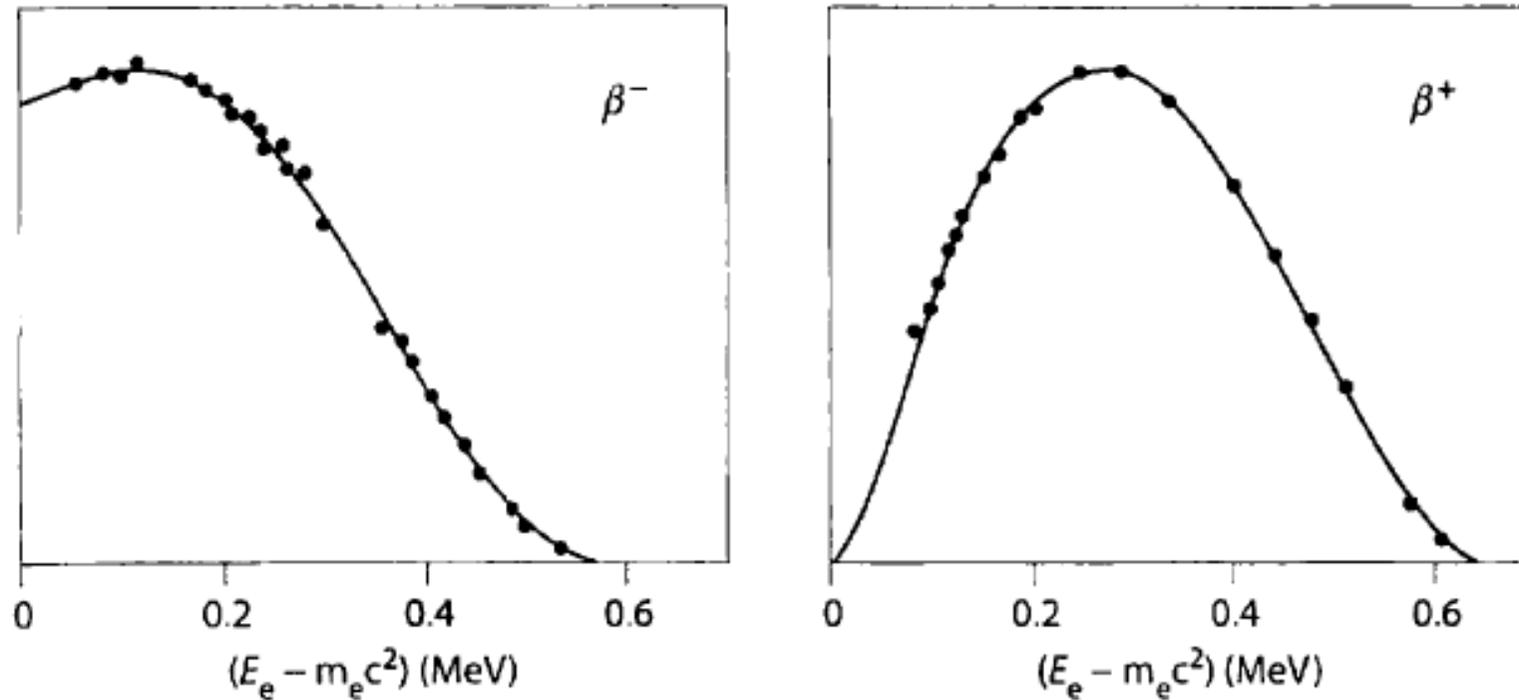


Figure 8.1 Energy distribution of the electron and positron in  $^{64}\text{Cu}$  decay. The low energy part of the electron spectrum is enhanced due to the deceleration caused by the nuclear attraction. For the positron one has the opposite effect.

連続スペクトル

2.  $\beta$ 崩壊の理論 (非相対論的, スピン無し模型)

$$H_{\beta} = g_{\beta} \int (\psi_p^{\dagger}(r) \psi_n(r)) (\psi_e^{\dagger}(r) \psi_{\bar{\nu}_e}(r)) dr + h.c.$$

A Feynman diagram for beta decay. A central vertex is marked with an 'X'. An arrow labeled 'n' points up into the vertex. From the vertex, three arrows point outwards: one labeled 'p' pointing up-left, one labeled 'e<sup>-</sup>' pointing up-right, and one labeled 'ν<sub>e</sub><sup>-</sup>' pointing right.

$\psi_a^{\dagger}(r)$ : 場所  $r$  に  $a$  を生成  
 $\bar{a}$  を消滅

} 正粒子  $a$  と 反粒子  $\bar{a}$

↓  $\psi_e^{\dagger} \psi_{\bar{\nu}_e}$ :  $e^{-} + \bar{\nu}_e$  を生成

フェルミの Golden Rule:

$$\lambda = \frac{2\pi}{\hbar} \sum_{\text{終状態}} |\langle f | H_{\beta} | i \rangle|^2$$

•  $L$  70トノ波動関数

$$\psi_e(r) = \frac{1}{\sqrt{V}} e^{iP_e \cdot r/\hbar}, \quad \psi_D(r) = \frac{1}{\sqrt{V}} e^{+iP_D \cdot r/\hbar}$$

$$P_D = -P_e$$

↓

$$\langle f | H_p | i \rangle = \frac{g_F}{V} \int [\psi_f^*(r) \textcircled{I_+} \psi_i(r)] e^{-i(P_e + P_D) \cdot r/\hbar} d^3r$$

$$e^{-i(P_e + P_D) \cdot r/\hbar} \sim 1 - \frac{i}{\hbar} (P_e + P_D) \cdot r + \dots$$

↑  
許容転移  
(allowed transition)

↑  
1次禁止転移  
(forbidden transition)

$\Psi(r)$  : 原子核の多体波動関数

• 選択則

$$\langle f | H_p | i \rangle \sim \frac{g_F}{V} \int [\psi_f^*(r) I_+ \psi_i(r)] d^3r$$

$\Delta I = 0$ , パリティ変化なし

$$0^+ \rightarrow 0^+$$

・終状態の数

$$\sum_{\text{終状態}} = V^2 \int \frac{dP_e}{(2\pi\hbar)^3} \frac{dP_{\bar{\nu}_e}}{(2\pi\hbar)^3} \underbrace{\delta(E_e + E_{\bar{\nu}_e} - Q)}_{\text{エネルギー保存}}$$

$$= V^2 \frac{(4\pi)^2}{(2\pi\hbar)^4} \int P_e^2 dP_e P_{\bar{\nu}_e}^2 dP_{\bar{\nu}_e} \delta(E_e + \underbrace{E_{\bar{\nu}_e}}_{\substack{\parallel \\ cP_{\bar{\nu}_e}}}) - Q)$$

$$= V^2 \frac{(4\pi)^2}{(2\pi\hbar)^6} \int \underbrace{P_e^2 dP_e}_{\substack{\parallel \\ \frac{1}{c^2}(E_e^2 - m_e^2 c^4)}} \cdot \frac{1}{c^3} (cP_{\bar{\nu}_e})^2 d(cP_{\bar{\nu}_e}) \delta(cP_{\bar{\nu}_e} + E_e - Q)$$

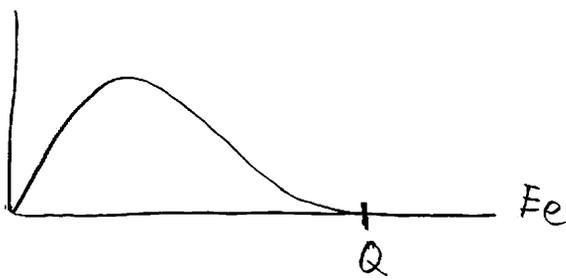
$$= V^2 \cdot \frac{(4\pi)^2}{(2\pi\hbar)^6} \cdot \int dP_e \cdot \frac{1}{c^5} (E_e^2 - m_e^2 c^4) (Q - E_e)^2$$

$$\hookrightarrow P_e = \frac{1}{c} \sqrt{E_e^2 - m_e^2 c^4} \quad \Rightarrow \quad dP_e = \frac{E_e}{c \sqrt{E_e^2 - m_e^2 c^4}}$$

$$= V^2 \cdot \frac{(4\pi)^2}{(2\pi\hbar)^6} \cdot \int dE_e \frac{1}{c^6} (Q - E_e)^2 E_e \sqrt{E_e^2 - m_e^2 c^4}$$

$$\Downarrow \quad \lambda = \int dE_e \frac{d\lambda}{dE_e}$$

$$\frac{d\lambda}{dE_e} \propto |\langle f | H_{\beta} | i \rangle|^2 \cdot \underbrace{(Q - E_e)^2 E_e \sqrt{E_e^2 - m_e^2 c^4}}_{\substack{\downarrow \\ e^- \text{ のエネルギー分布}}}$$



3. スピン導入 (ガモフ・テラ-遷移)

$$H_{\beta} = g_{GT} \int d\mathbf{r} \sum_{\text{spin}} [ \psi_p^{\dagger}(\mathbf{r}) \vec{\sigma} \tau_{+} \psi_n(\mathbf{r}) ] \\ \cdot [ \psi_e^{\dagger}(\mathbf{r}) \vec{\sigma} \psi_{\nu_e}(\mathbf{r}) ]$$

$\Delta I = 1$ , 10% 以上変化なし

$$0^{+} \rightarrow 1^{+}$$



#### 4. 相対論的な取り扱

$$H_{\beta} = \int d^4x \left\{ C_V (\psi_p^\dagger \gamma_\mu \psi_n) (\psi_e^\dagger \underbrace{\gamma^\mu}_{\text{ハリリテ}} \underbrace{(1 + \gamma_5)}_{\text{ハリリテ}} \psi_\nu) + h.c. \right. \\ \left. - C_A (\psi_p^\dagger \gamma_\mu \gamma_5 \psi_n) (\psi_e^\dagger \gamma^\mu \gamma_5 (1 + \gamma_5) \psi_\nu) + h.c. \right\}$$

"V-A 型"

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}, \quad \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$-i\gamma_k\gamma_5 = \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix}, \quad -i\gamma_0\gamma_5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

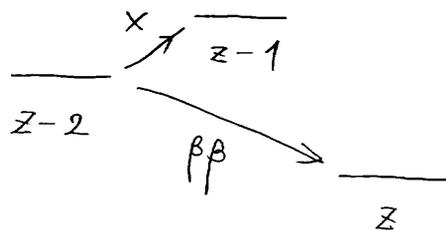
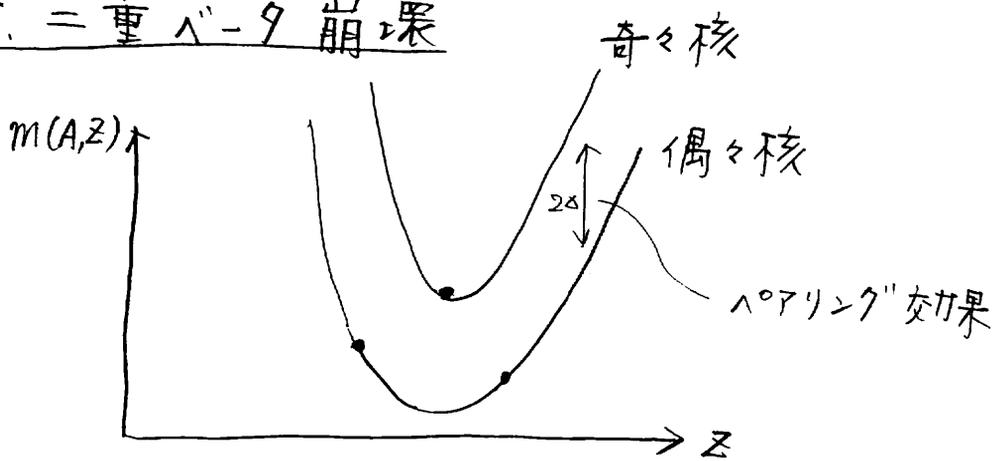
↑ 1 → "非相対論的極限":

$$\gamma_\mu \sim \delta_{\mu,0}, \quad \gamma_5 \sim 0, \quad -i\gamma_k\gamma_5 \sim \sigma_k \\ -i\gamma_0\gamma_5 \sim 0$$

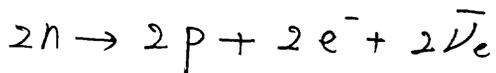
↓

$$H_{\beta} \sim \int d^4x \left\{ C_V (\psi_p^\dagger \psi_n) (\psi_e^\dagger \psi_\nu) + h.c. \right. \\ \left. + \underbrace{C_A (\psi_p^\dagger \vec{\sigma} \psi_n) \cdot (\psi_e^\dagger \vec{\sigma} \psi_\nu)}_{\text{GT}} + h.c. \right\}$$

# 5. 二重β崩壊

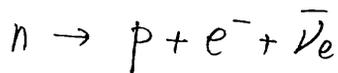


•  $2\nu\beta\beta$

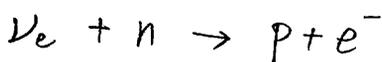


β崩壊が2回おこる

•  $0\nu\beta\beta$



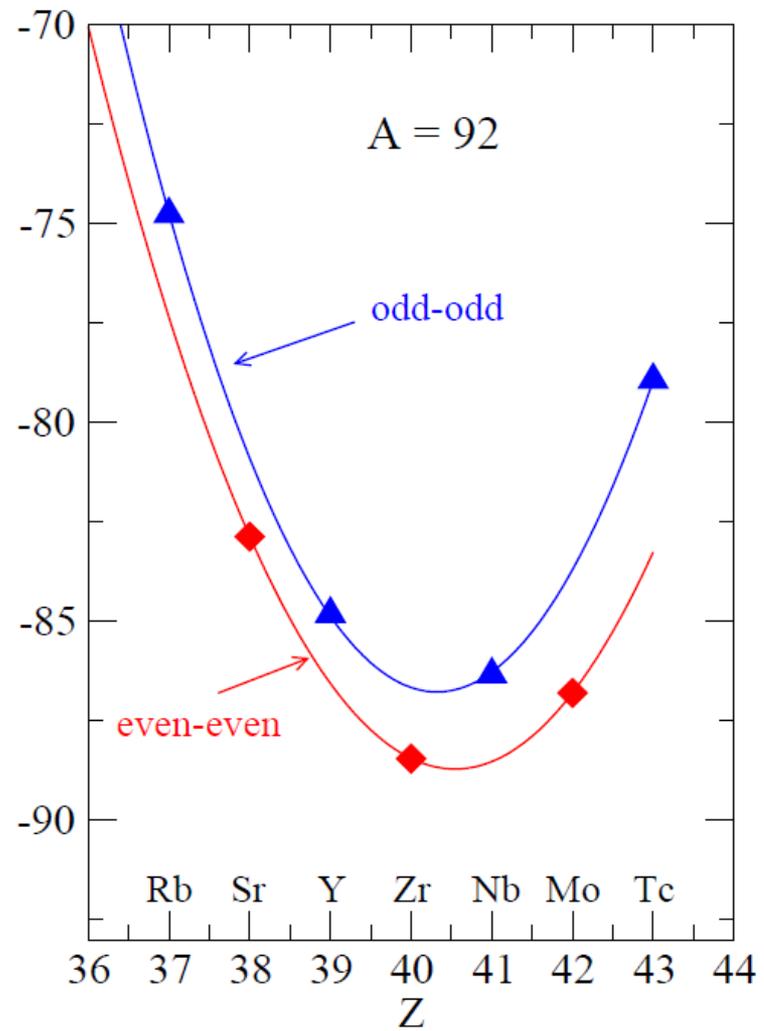
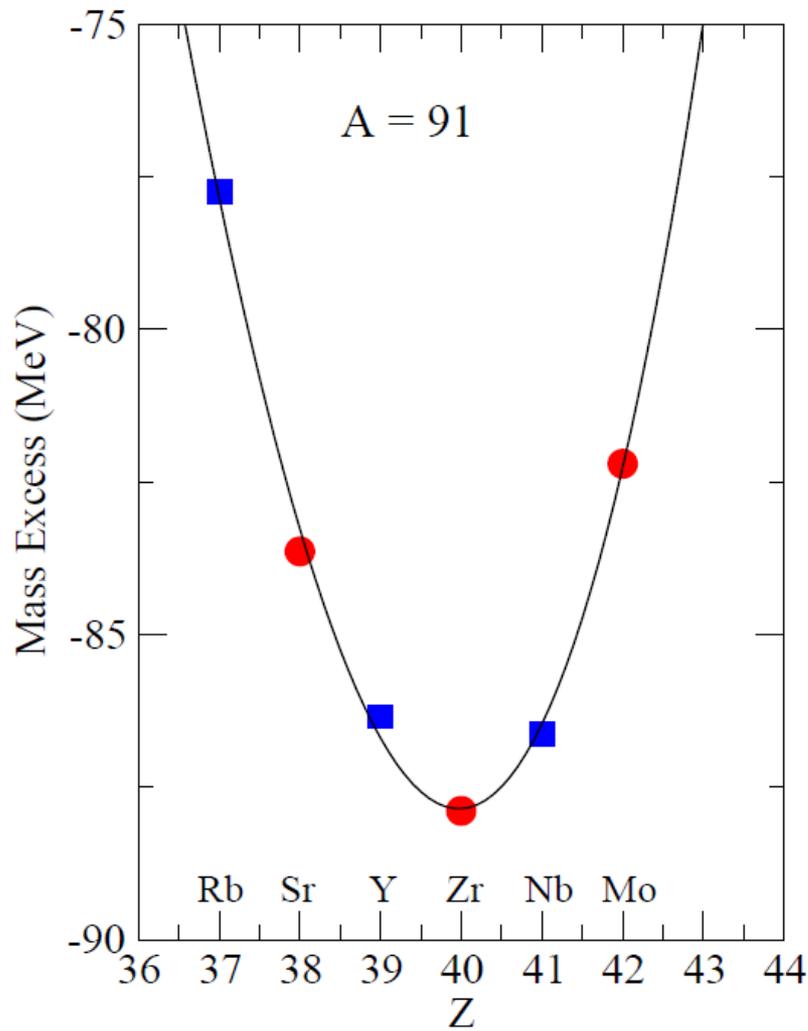
$\bar{\nu}_e = \nu_e$  反ニュートリノ (反ニュートリノ)

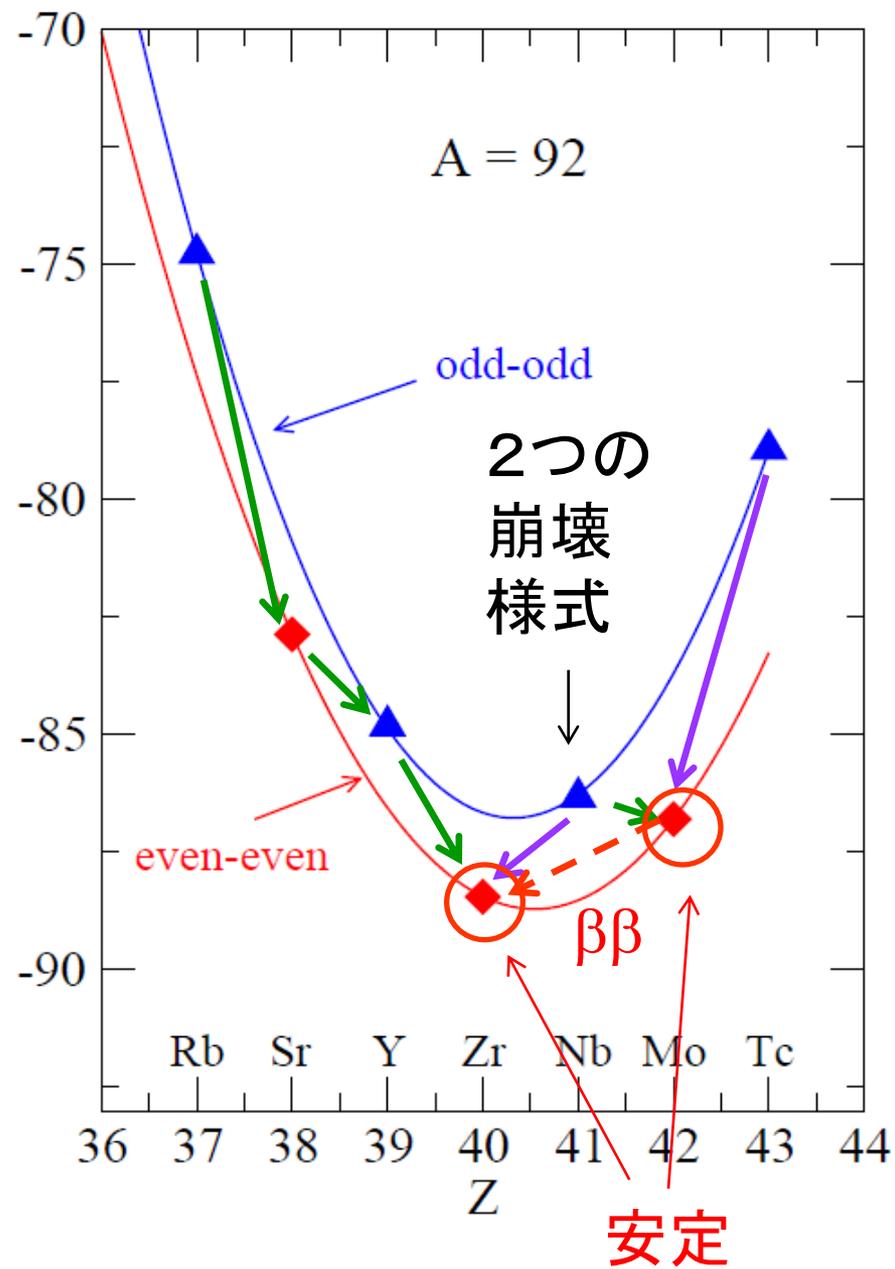
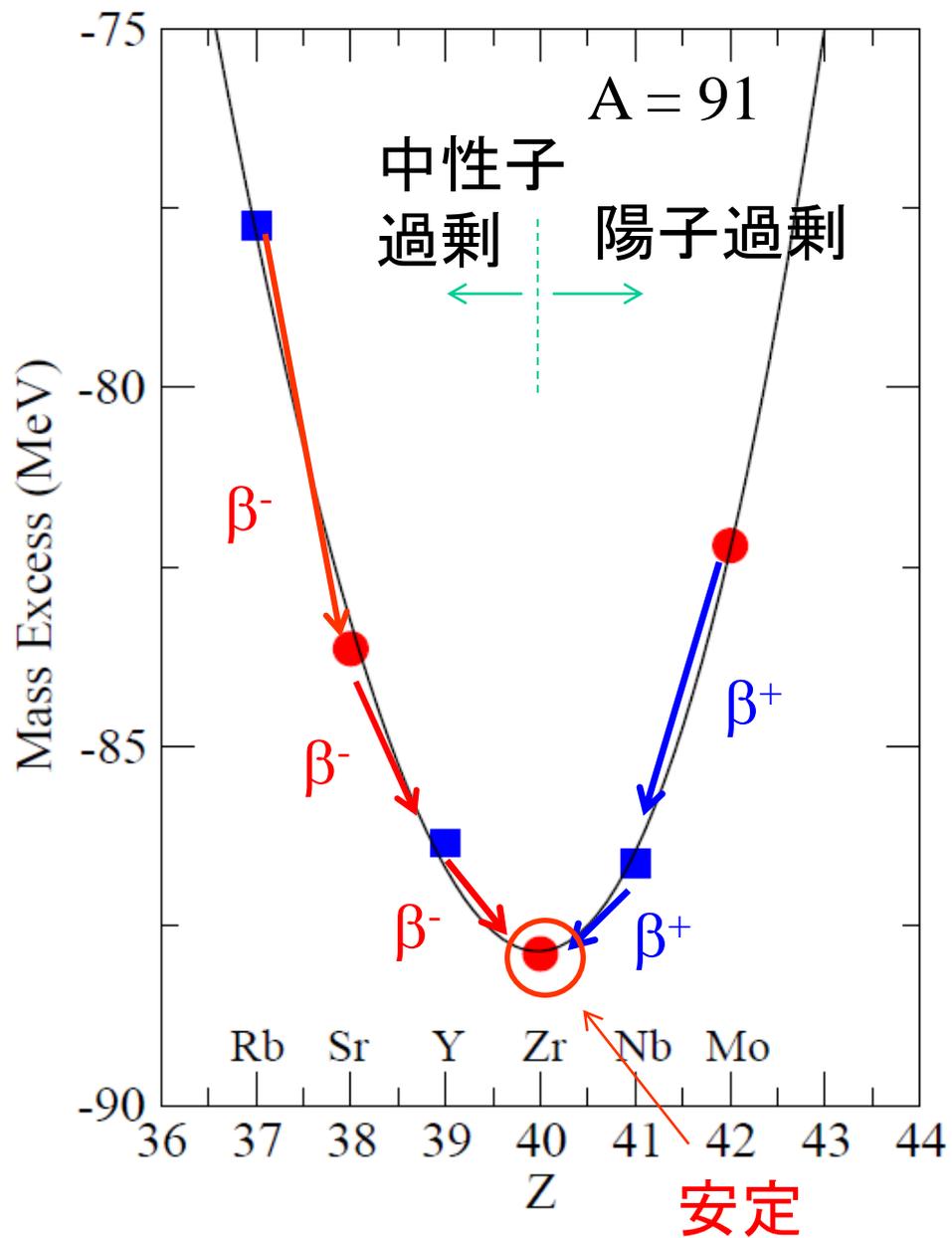


レプトン数非保存

KAMLANDで探索実験開始 (11秋~)

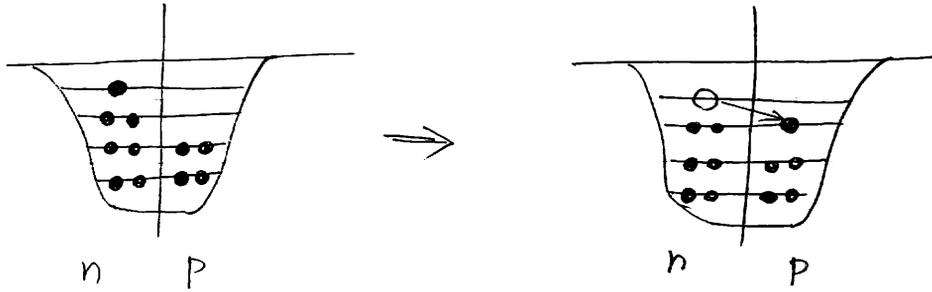
KAMLAND-ZEN



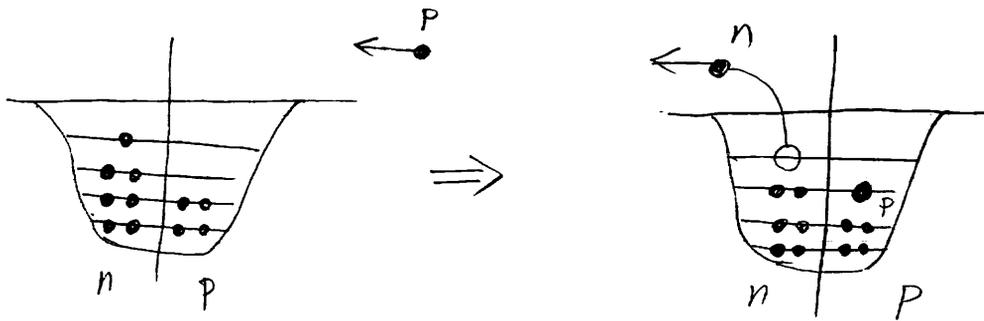


# 6. 荷電交換反応

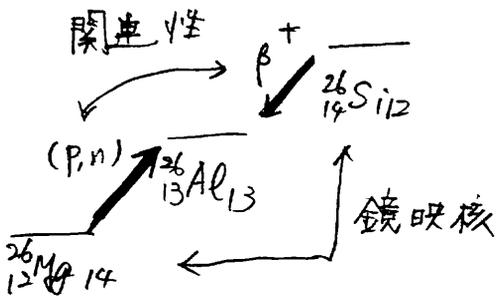
核内  $T_z$  の  $\beta$  崩壊



荷電交換反応 (p, n) 反応  
 $p$  を  $\beta^-$  の  $T_z$   $n$  を出す反応



$\beta$  崩壊と同じ  $\Psi_i, \Psi_f$   
 同じ matrix element



$$\langle \Psi_f | T_z | \Psi_i \rangle$$

$$\text{or } \langle \Psi_f | \vec{\sigma} T_z | \Psi_i \rangle$$

他にも

ニュートリノ原子核反応

