Nuclear Physics

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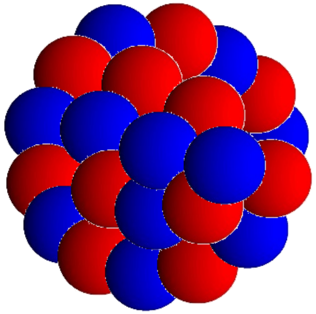
Nuclei: aggregate of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***



- Liquid drop model
- Single-particle motion and Shell structure
- Hartree-Fock approximation
- Bruckner Theory
- Pairing correlations and Superfluid Nuclei
- Angular momentum and number projections
- Random Phase Approximation
- (Generator Coordinate Method (GCM))
- (Time-dependent Hartree-Fock Method)
- (Nuclear Reactions)

References

- P. Ring and P. Schuck, "The Nuclear Many-Body Ploblem"
- A. Bohr and B.R. Mottelson, "Nuclear Structure" Vol. 1 and 2
- G.E. Brown, "Unified Theory of Nuclear Models and Forces"
- D.J. Rowe, "Nuclear Collective Motion"

- J. Lilley, "Nuclear Physics"
- R.F. Casten, "Nuclear Structure from a Simple Perspective"
- S.G. Nilsson and I. Ragnarsson, "Shapes and Shells in Nuclear Structure"

- 市村宗武、坂田文彦、松柳研一 「原子核の理論」（岩波講座・現代の物理学）
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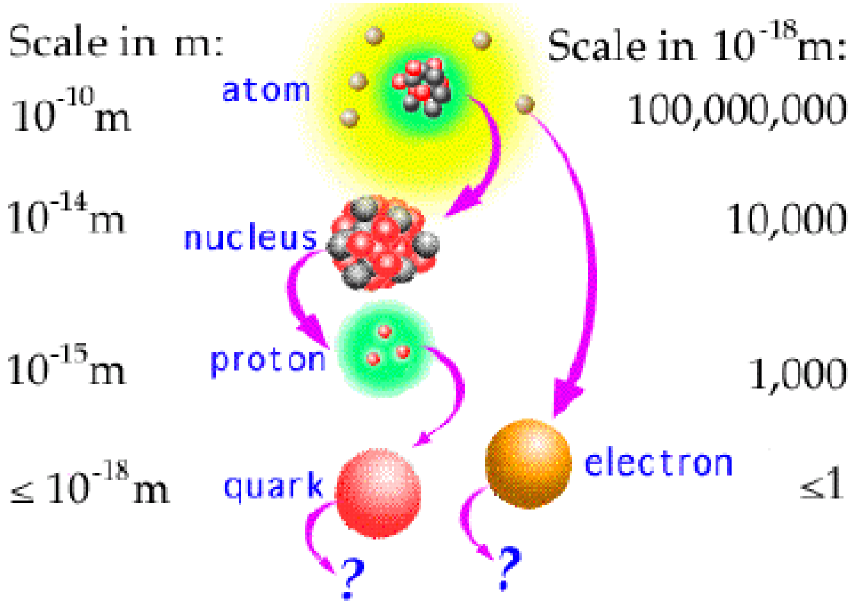
Basic Properties of Nuclei

Nuclear Physics



$1\ \text{fm} = 10^{-15}\ \text{m}$

Nucleus as a *quantum many body system*

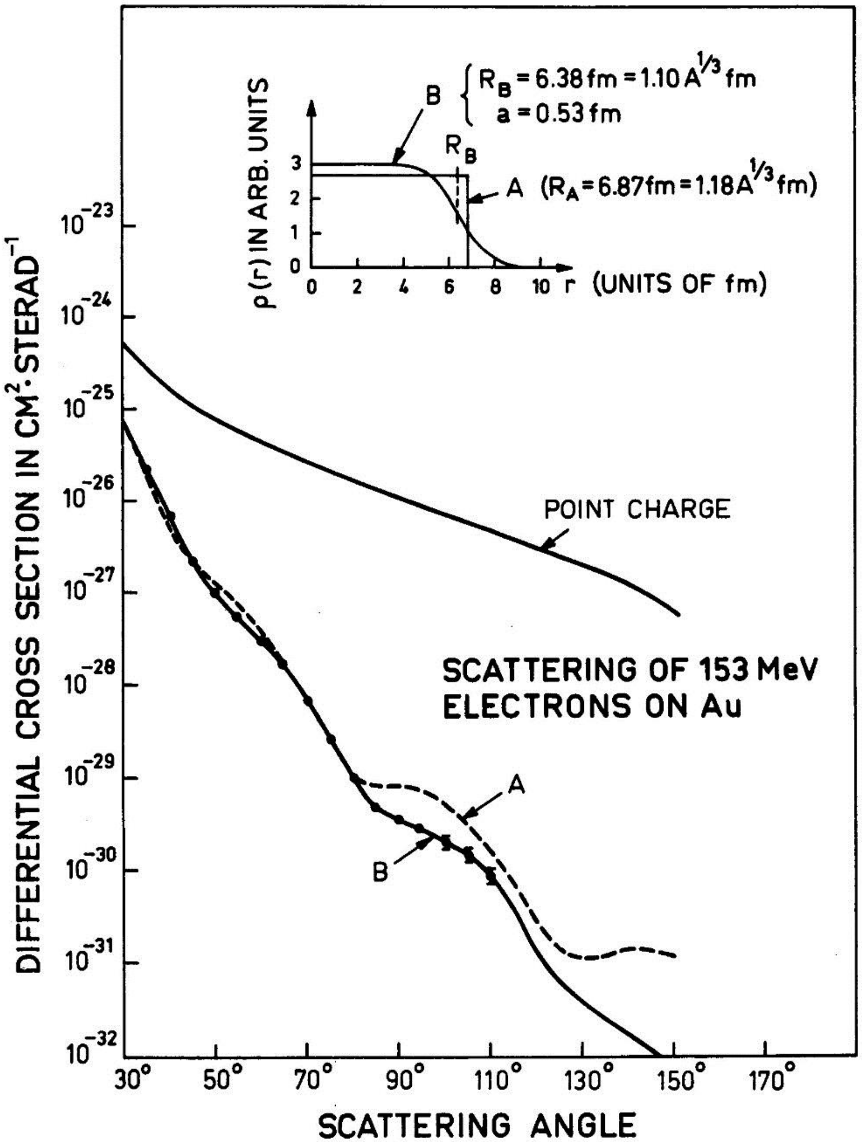
Basic ingredients:

	charge	mass (MeV)	spin
Proton	+e	938.256	½+
Neutron	0	939.550	½+

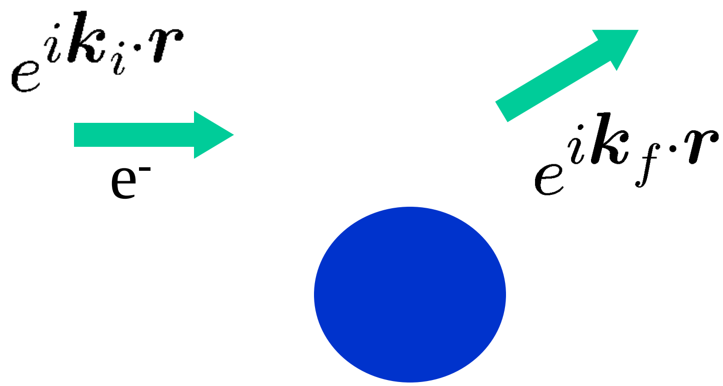
(note) $n \longrightarrow p + e^- + \bar{\nu}$ (10.4 min)

Density Distribution

High energy electron scattering



Born approximation:

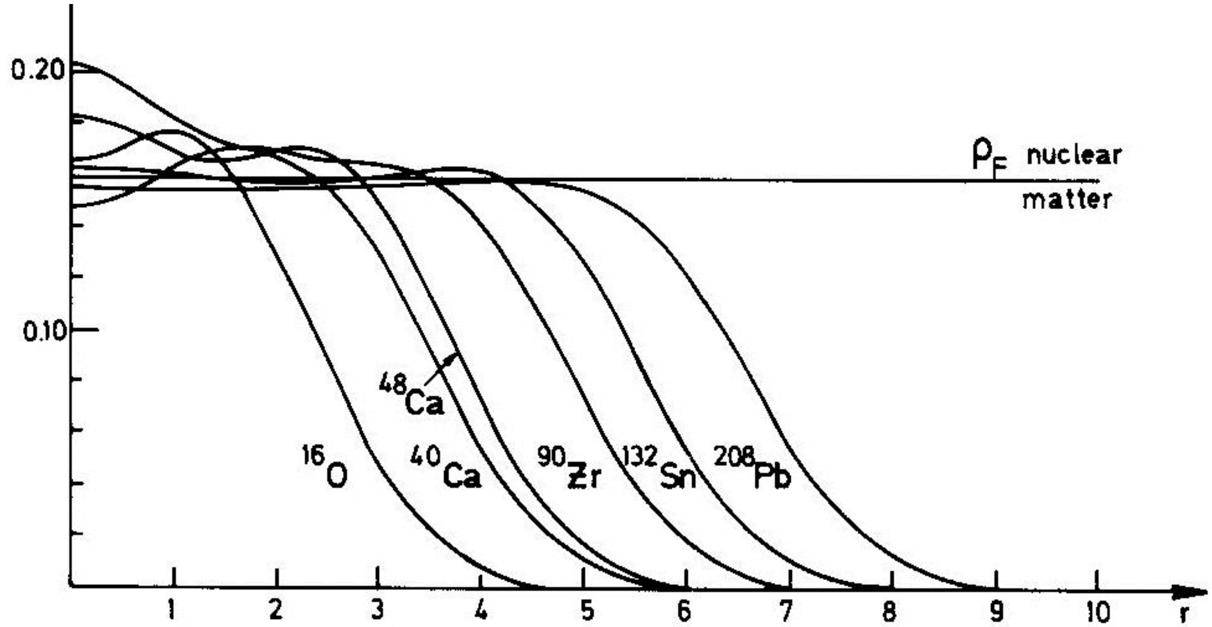


$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)



Fermi distribution

$$\rho(r) = \rho_0 / [1 + \exp((r - R_0)/a)]$$

$$\rho_0 \sim 0.17 \ (\mathrm{fm}^{-3})$$

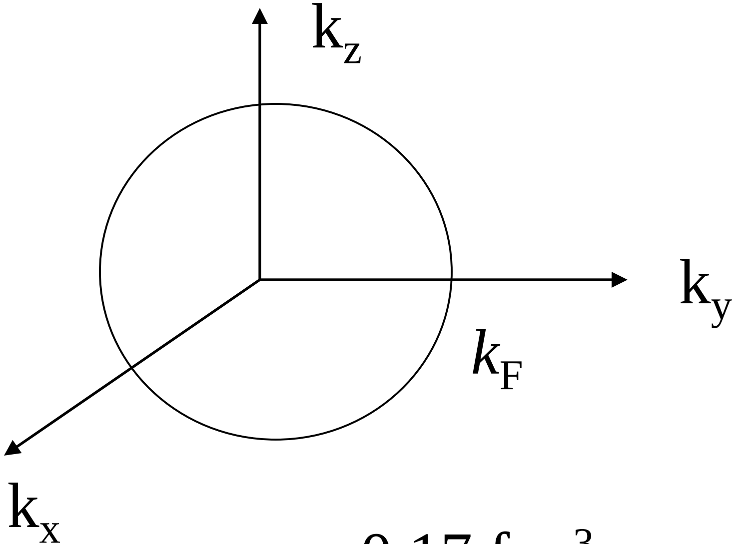
Saturation property

$$R_0 \sim 1.1 \times A^{1/3} \ (\mathrm{fm})$$

$$a \sim 0.57 \ (\mathrm{fm})$$

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3} = \frac{2}{3\pi^2} k_F^3$$

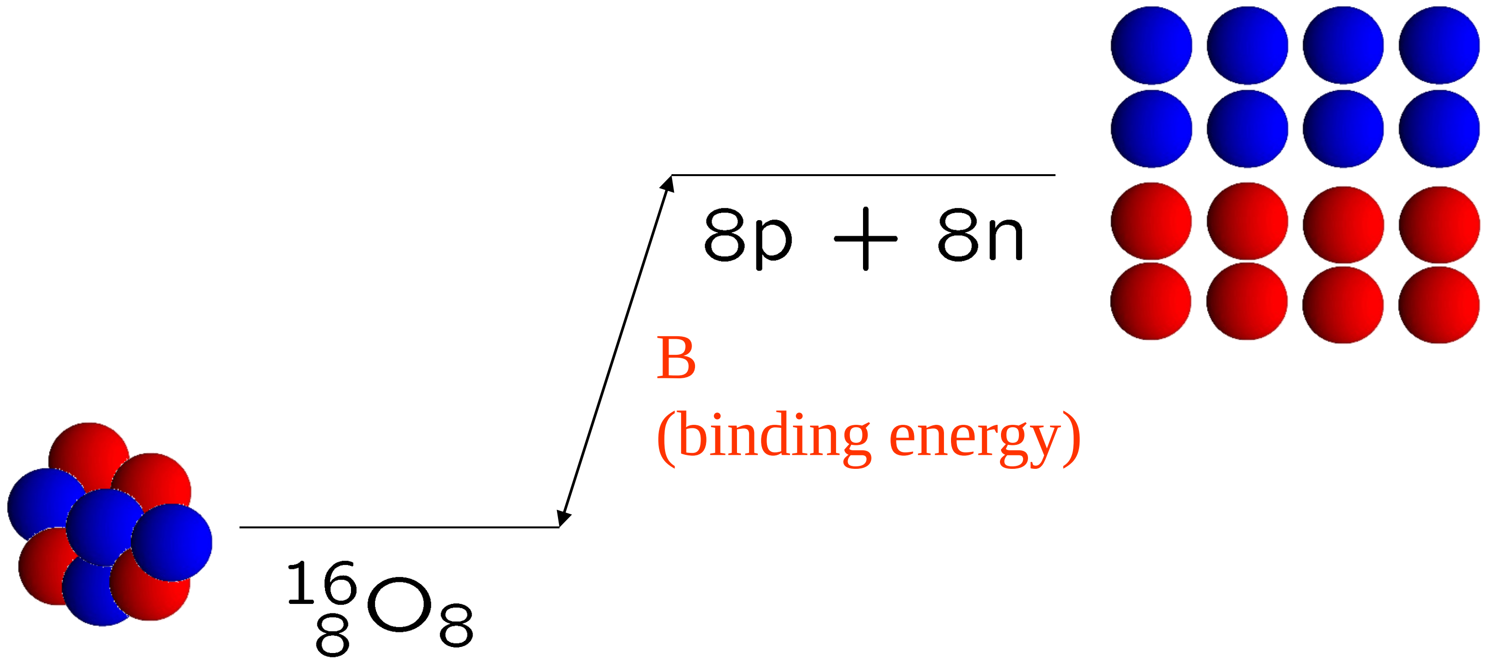
(note: spin-isospin degeneracy)

$\rho = 0.17\ \mathrm{fm}^{-3}$ ⟶ $k_F \sim 1.36\ \mathrm{fm}^{-1}$

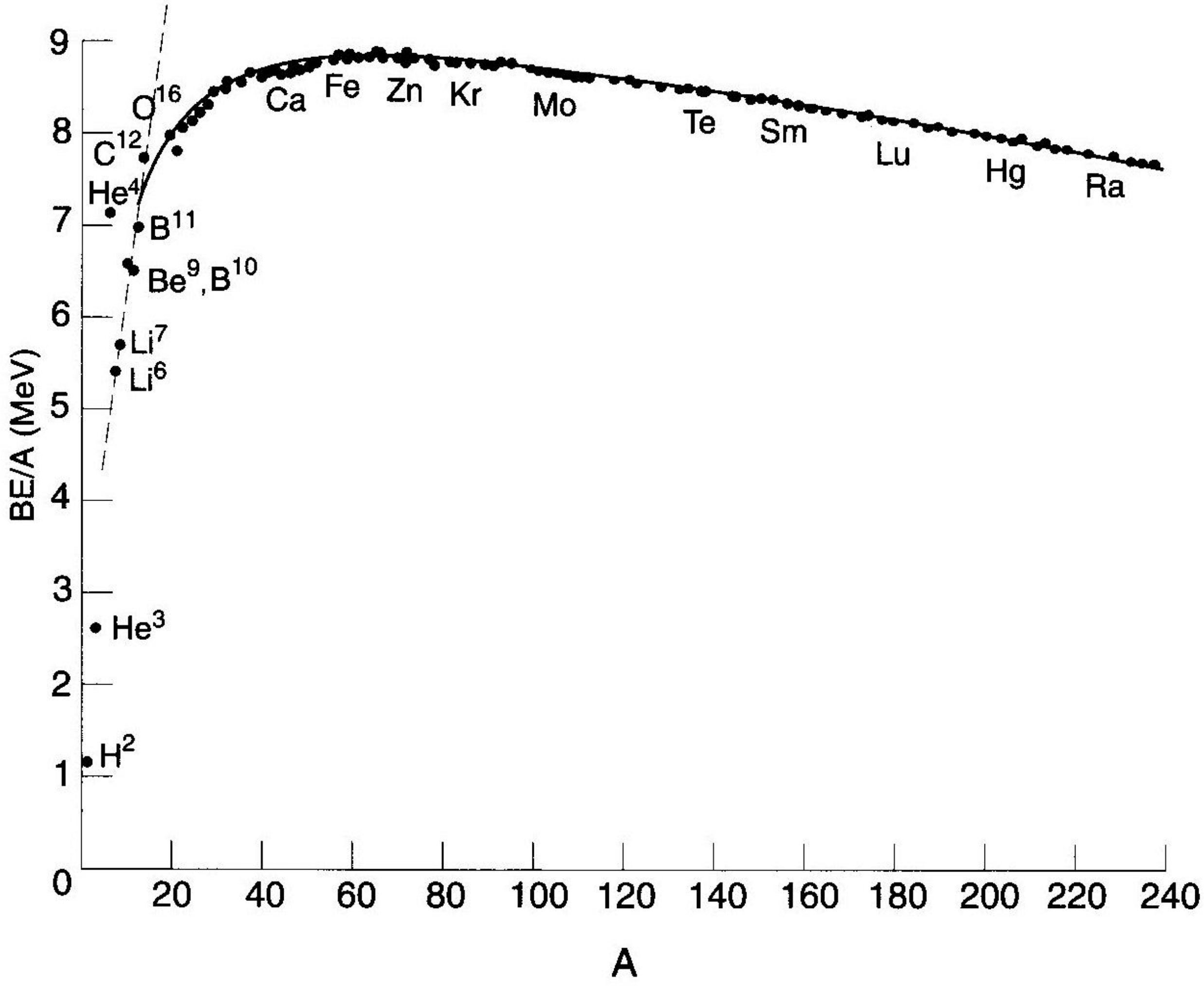
$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \dfrac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)

Nuclear Mass



$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

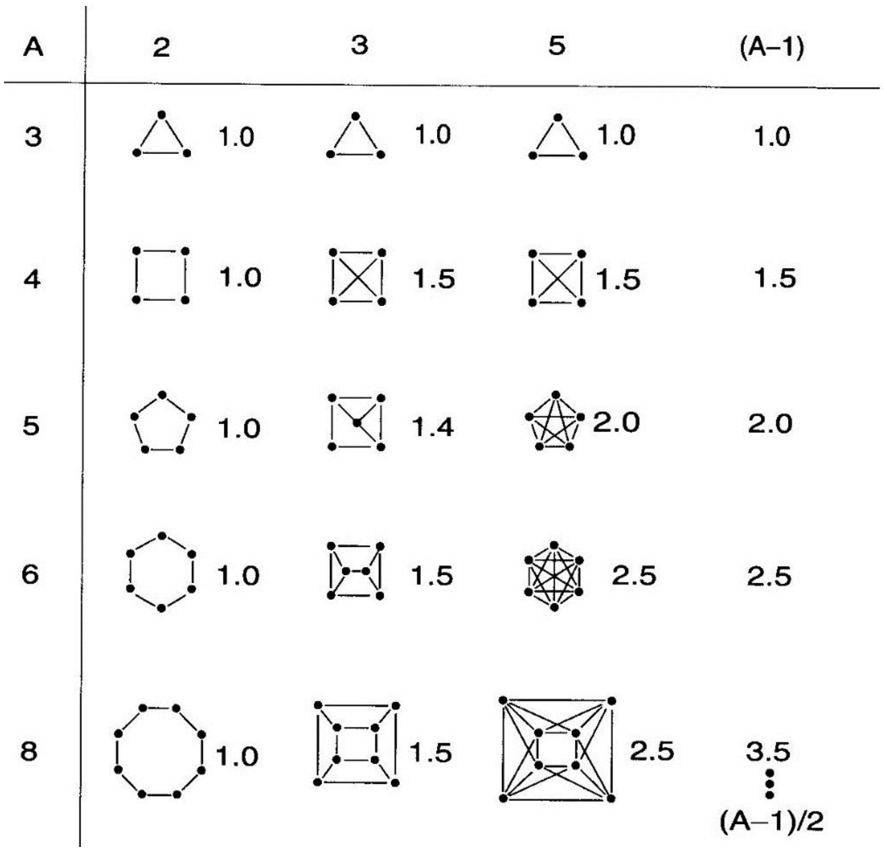


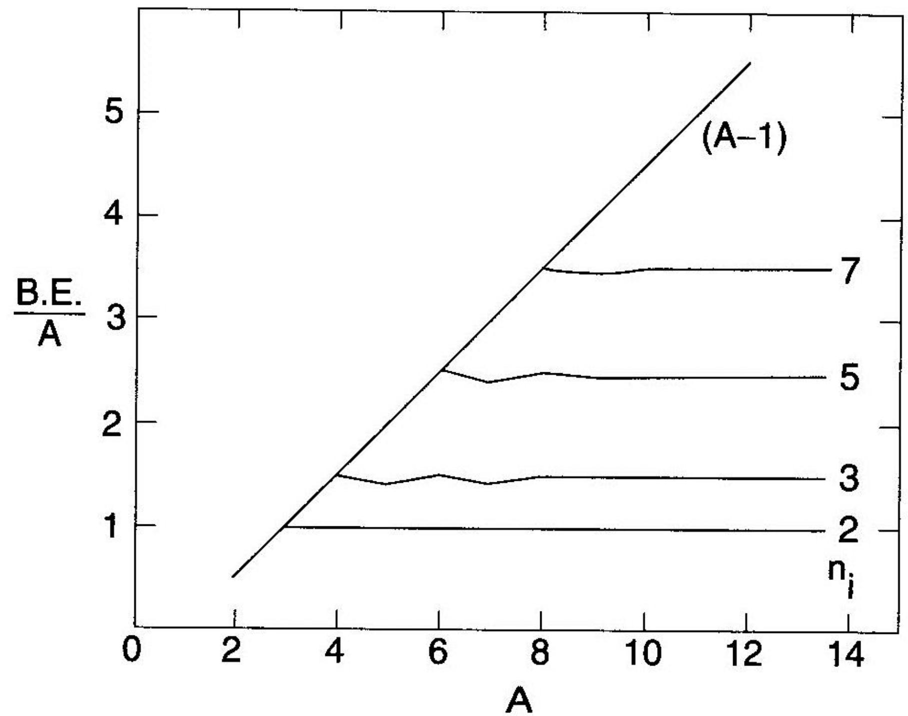
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

Long vs short range interaction

Long range force: $B \propto A(A-1)/2$ $\quad B/A \propto A$

Short range force: saturation



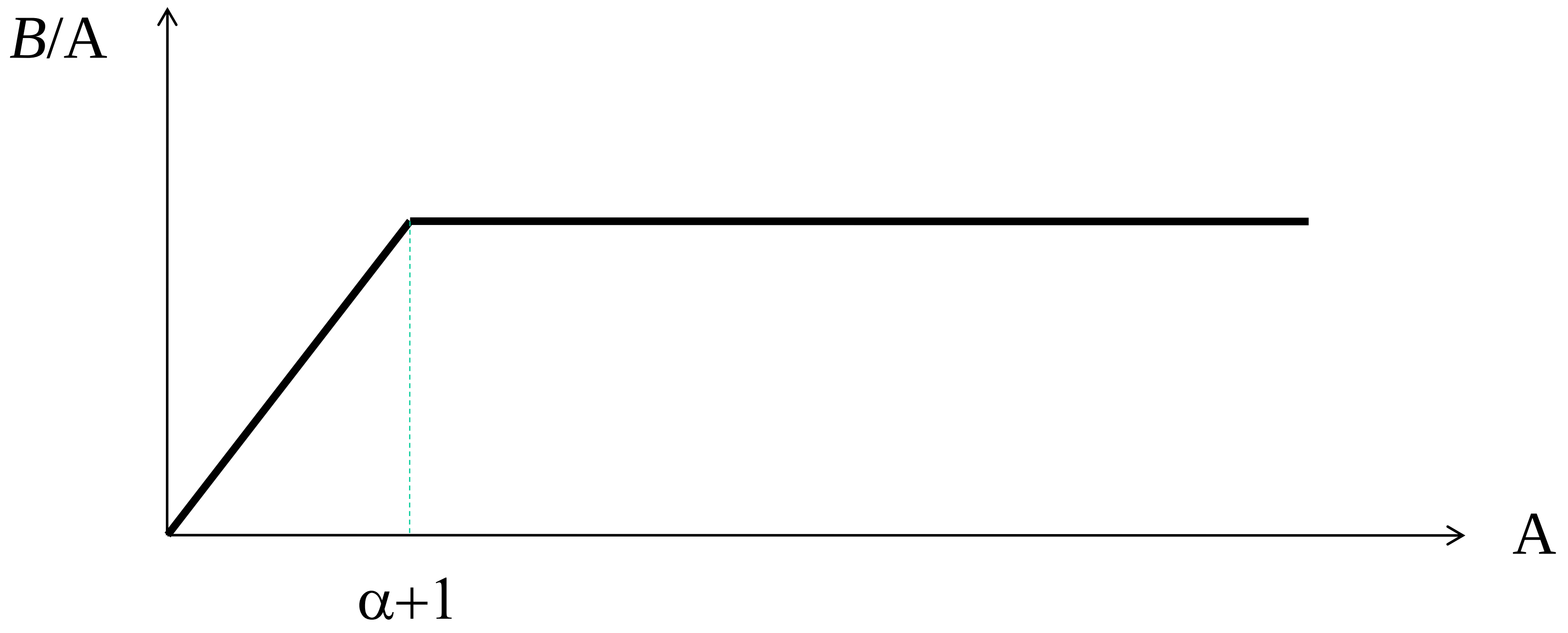


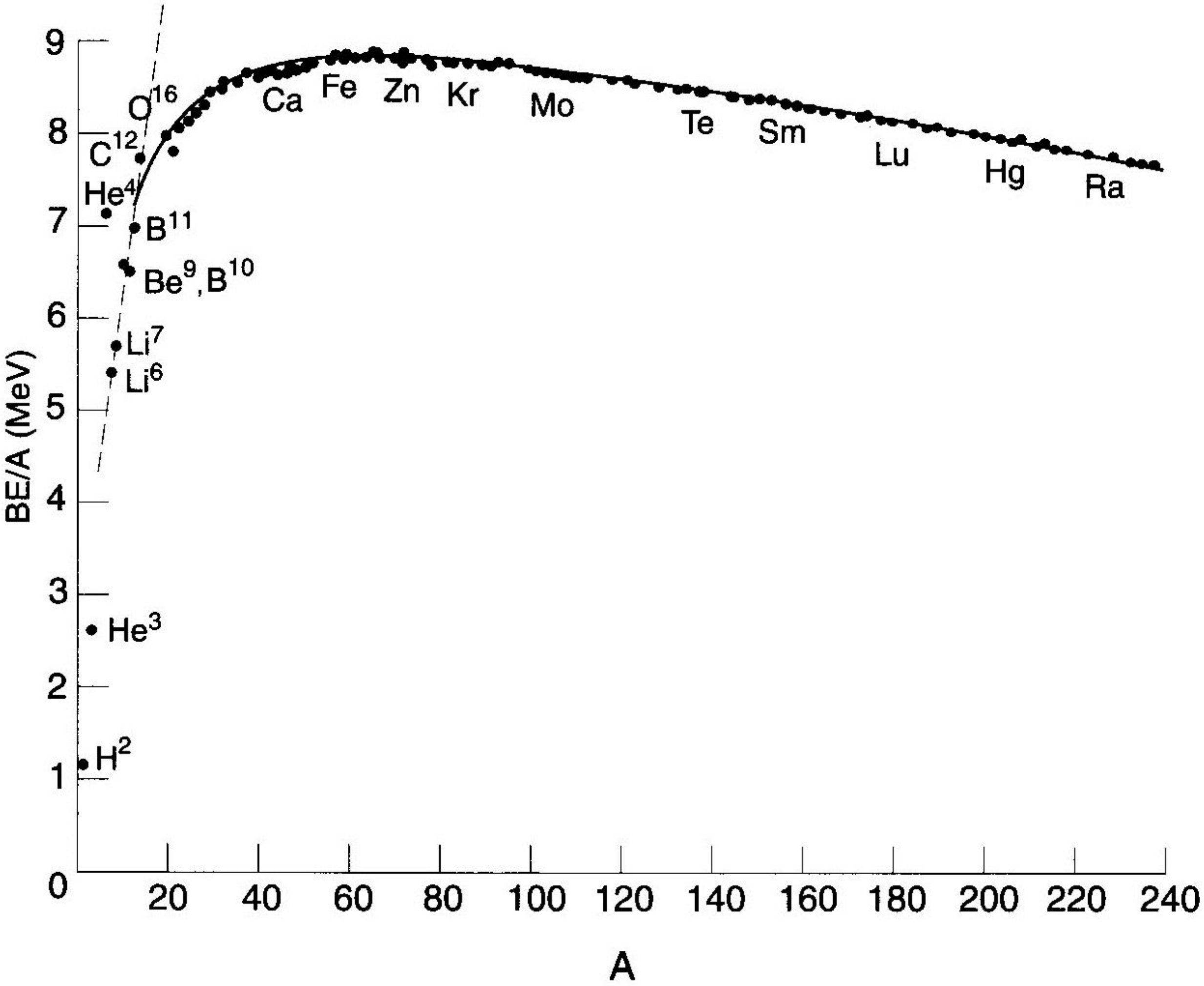
1つの核子が α 個の核子とのみ相互作用するとすると、

$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

ただし、A < α+1 の時は、すべての核子対が相互作用するので、

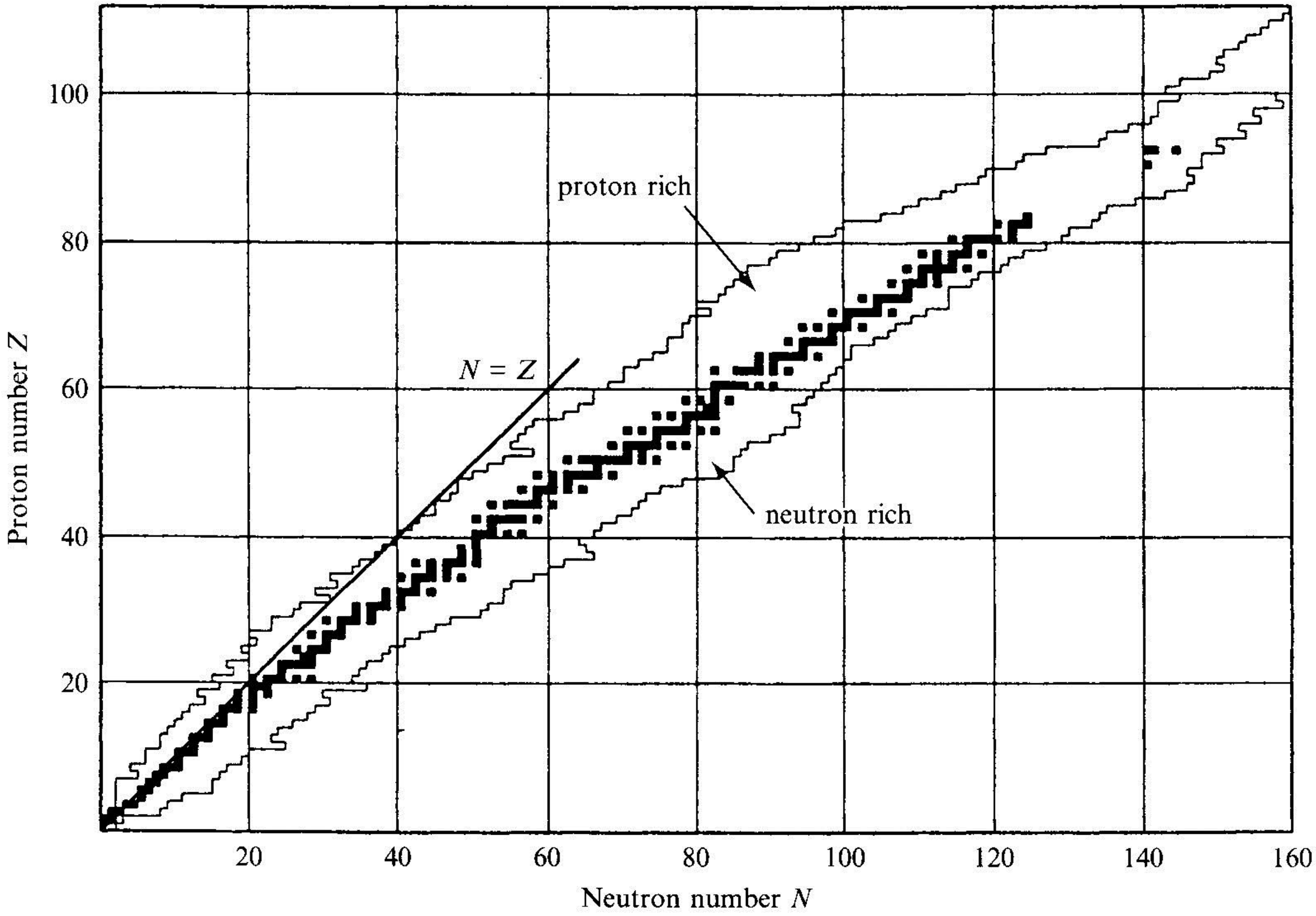
$B/A \propto A$



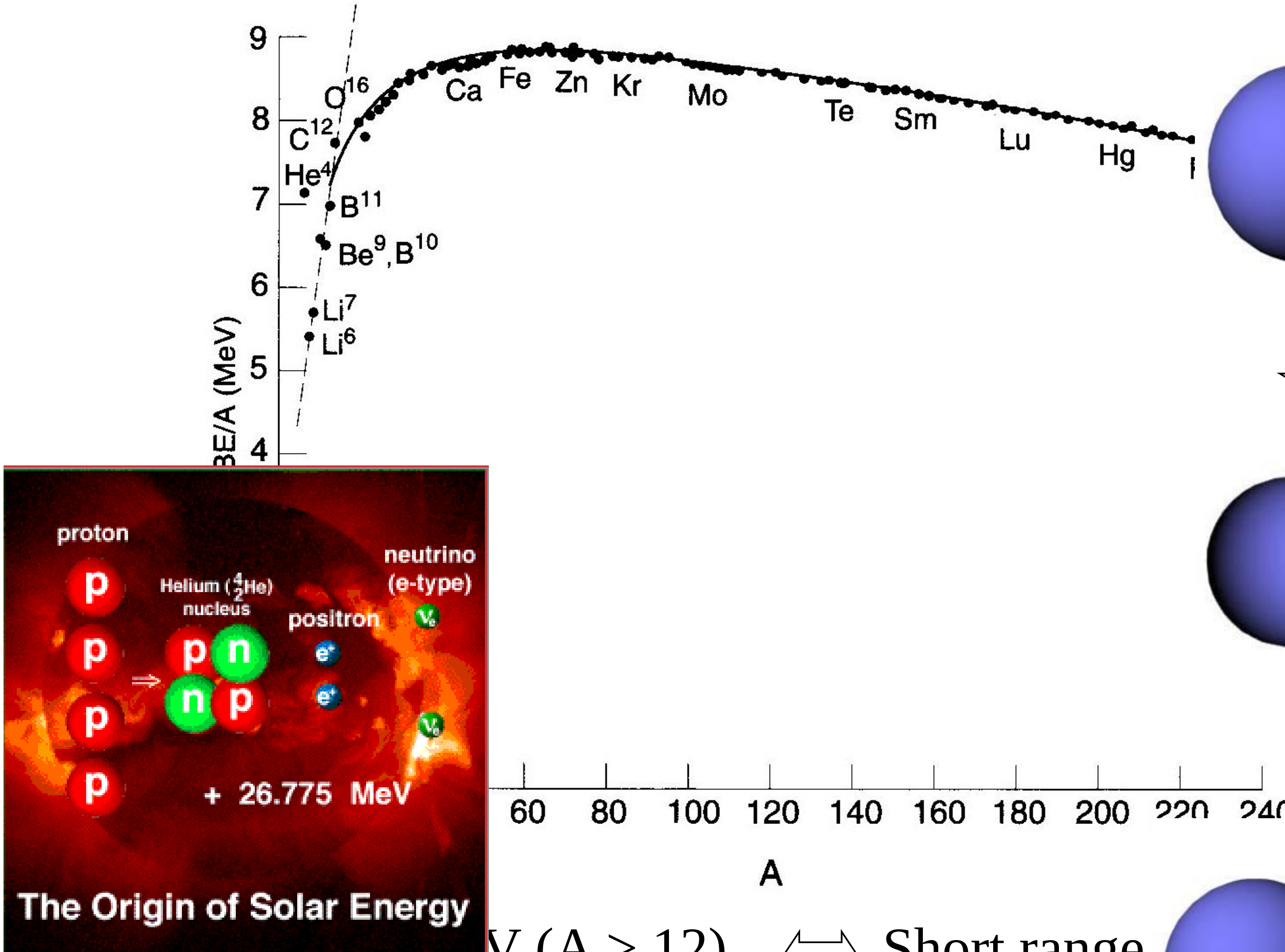


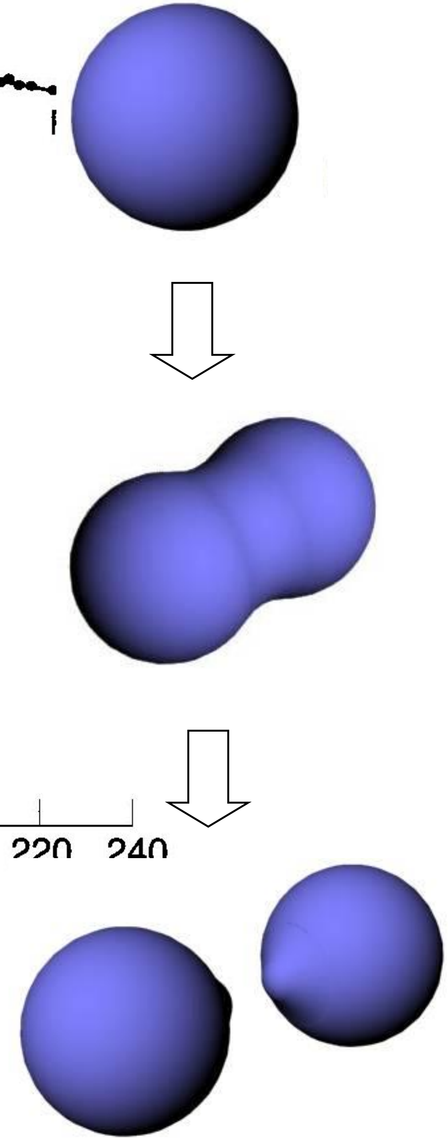
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$





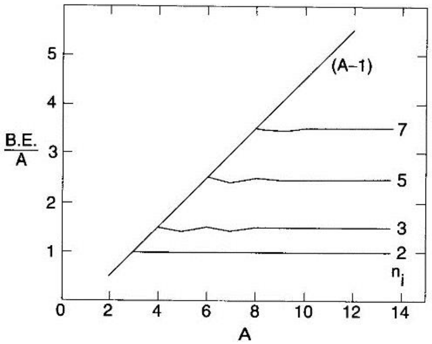
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

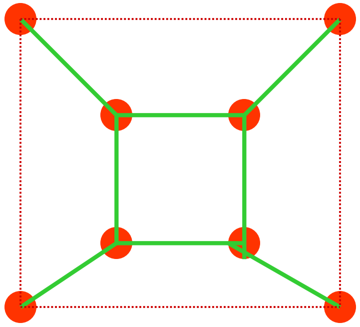
- Volume energy: $a_v A$



$$R_0 \sim 1.1 \times A^{1/3} \rightarrow V \propto A$$
$$S \propto A^{2/3}$$

- Surface energy: $-a_s A^{2/3}$

A nucleon near the surface
interacts with fewer nucleons.



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Coulomb energy: $-a_C\, Z^2/A^{1/3}$

$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_C}$$ for a uniformly charged sphere

- Symmetry energy: $-a_{\text{sym}}\,(N-Z)^2/A$

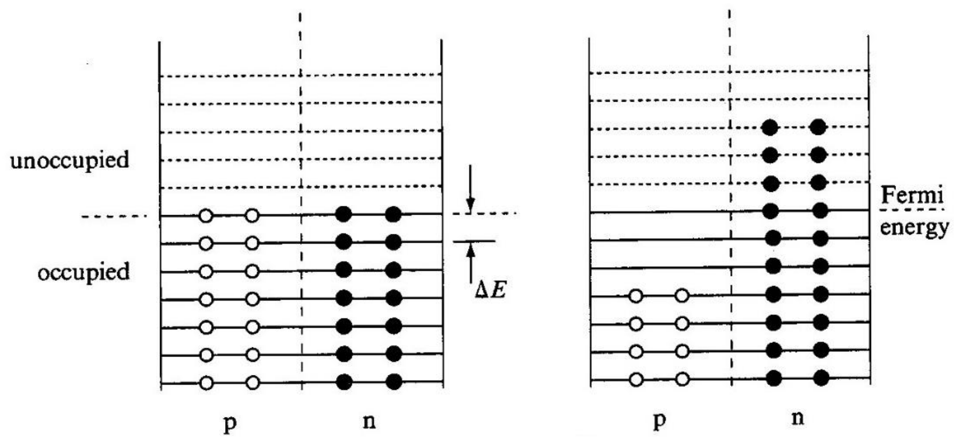
Potential energy $v_{nn} = v_{pp} = v, \quad v_{np} \sim 2v$

a nucleon interacting with nuclear matter:

$$N(v_{nn}N/A + v_{pn}Z/A) + Z(v_{pn}N/A + v_{pp}Z/A) = \frac{v}{2}(3A - (N-Z)^2/A)$$

Kinetic energy

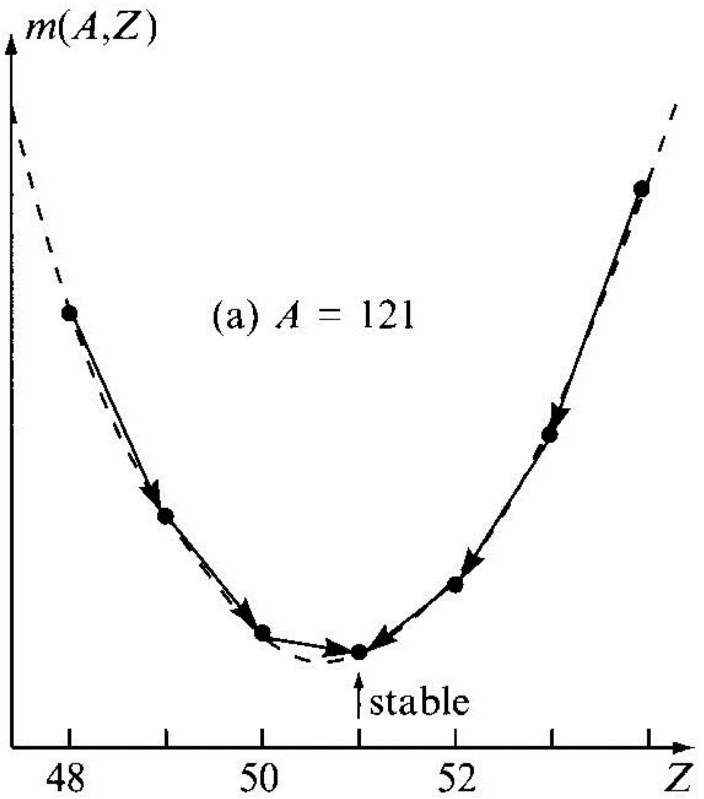
Pauli exclusion principle



β-stability line

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

$$m(A,Z) = f(A) + a_C \frac{Z^2}{A^{1/3}} + a_{\text{sym}} \frac{(A-2Z)^2}{A}$$



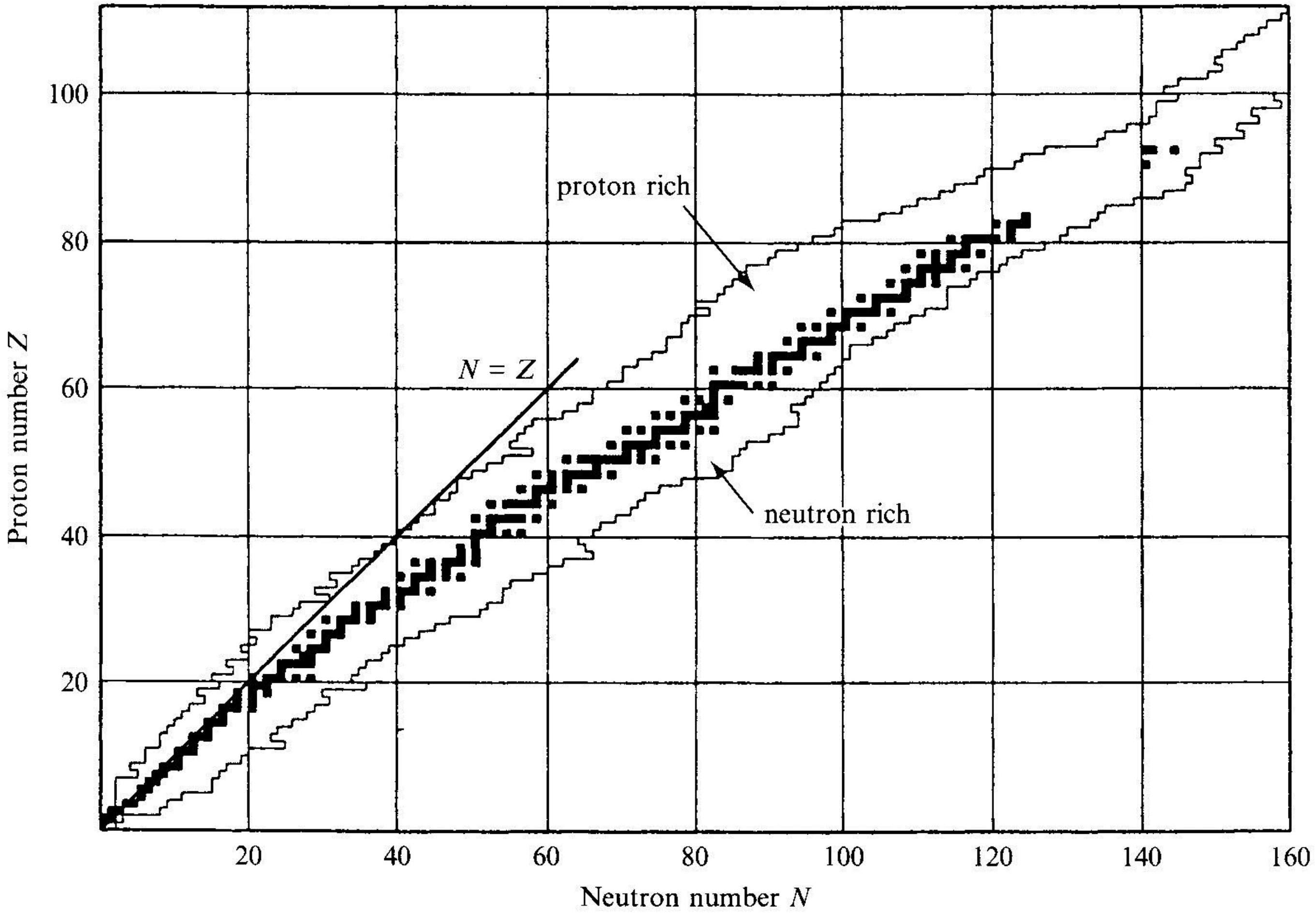
Stable nuclei (beta-stability line)

$$\left.\frac{\partial m}{\partial Z}\right|_{A=const.} = 0$$

$$Z = \frac{4a_{\text{sym}}}{2a_C/A^{1/3} + 8a_{\text{sym}}/A}$$

⇨ Z < A/2

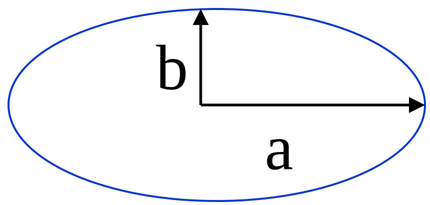
Nuclear Chart



Stable nuclei: $N \geq Z$

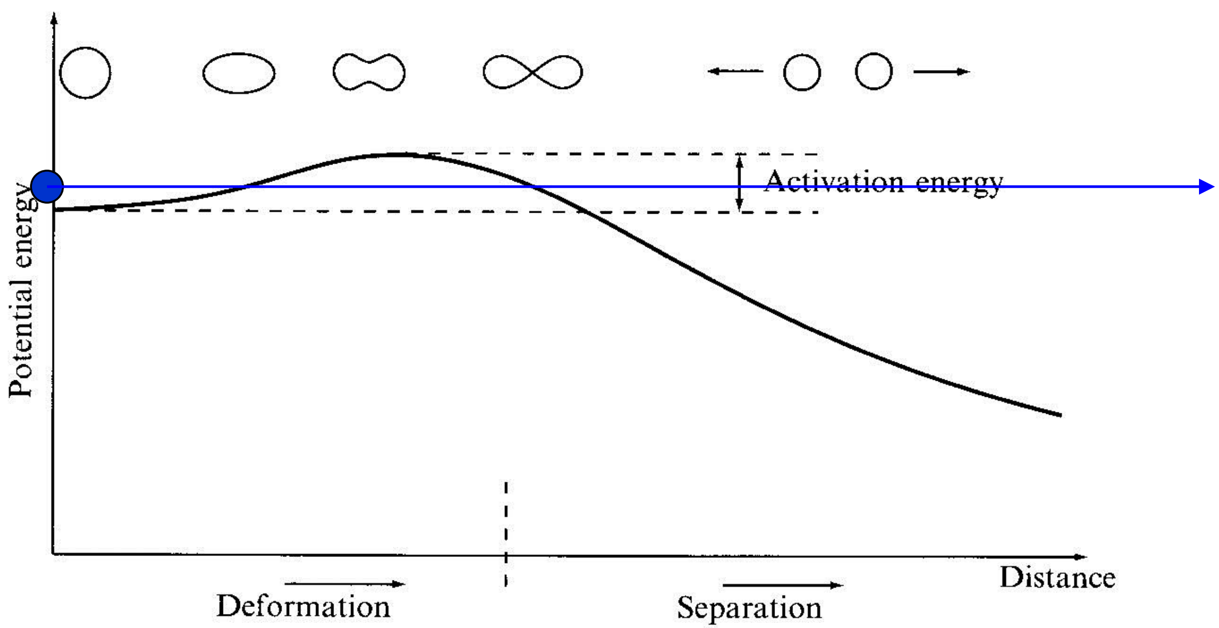
Nuclear Fission

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$



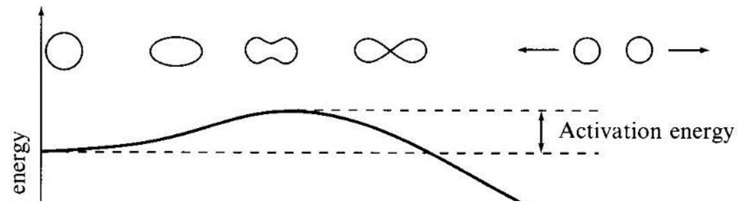
$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

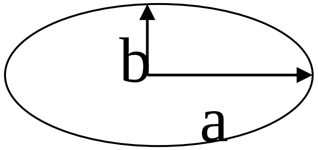
$$\begin{aligned} E_{\text{surf}} &= E_{\text{surf}}^{(0)} (1 + 2\epsilon^2/5 + \cdots) \\ E_C &= E_C^{(0)} (1 - \epsilon^2/5 + \cdots) \end{aligned}$$



Quantum tunneling

Collective Vibrations





$$a = R \cdot (1+\epsilon)$$
$$b = R \cdot (1+\epsilon)^{-1/2}$$

In general, $R(\theta,\phi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)$

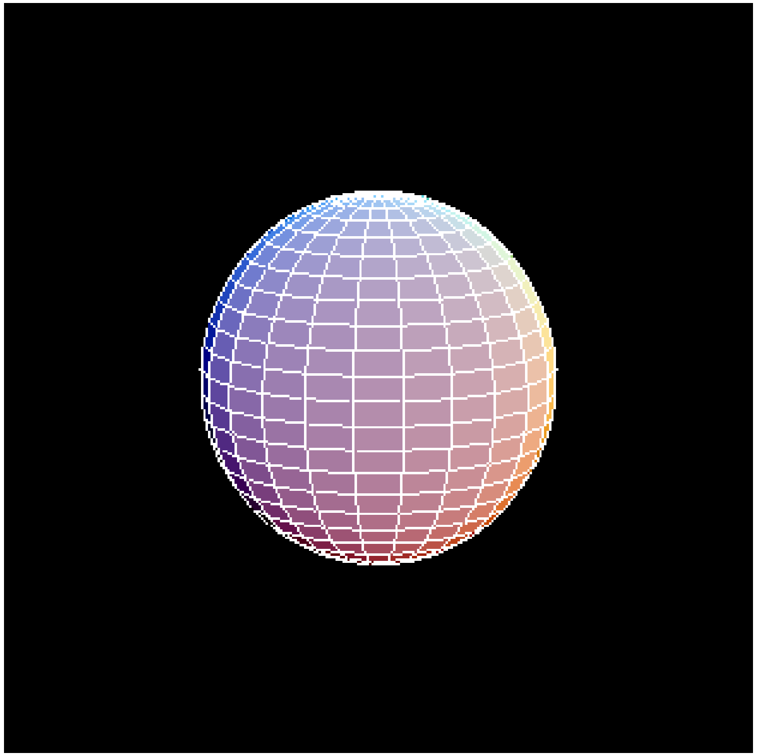
$$V = \frac{1}{2} \sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$

Quantization: Harmonic Vibrations

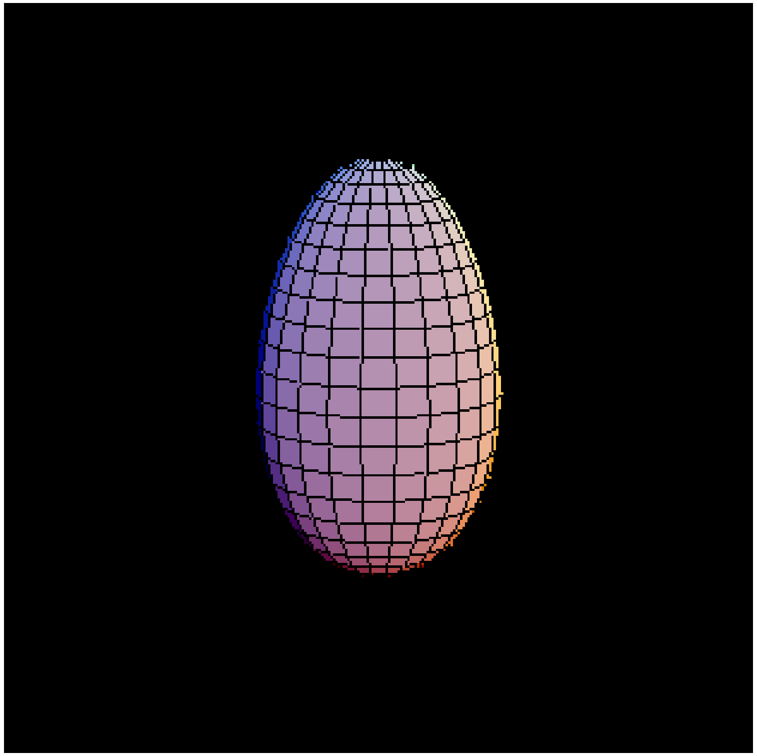
(note) moment of inertia $\Leftarrow$ incompressible and irrotational flow

$$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)$$

$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



λ=2: Quadrupole vibration



λ=3: Octupole vibration

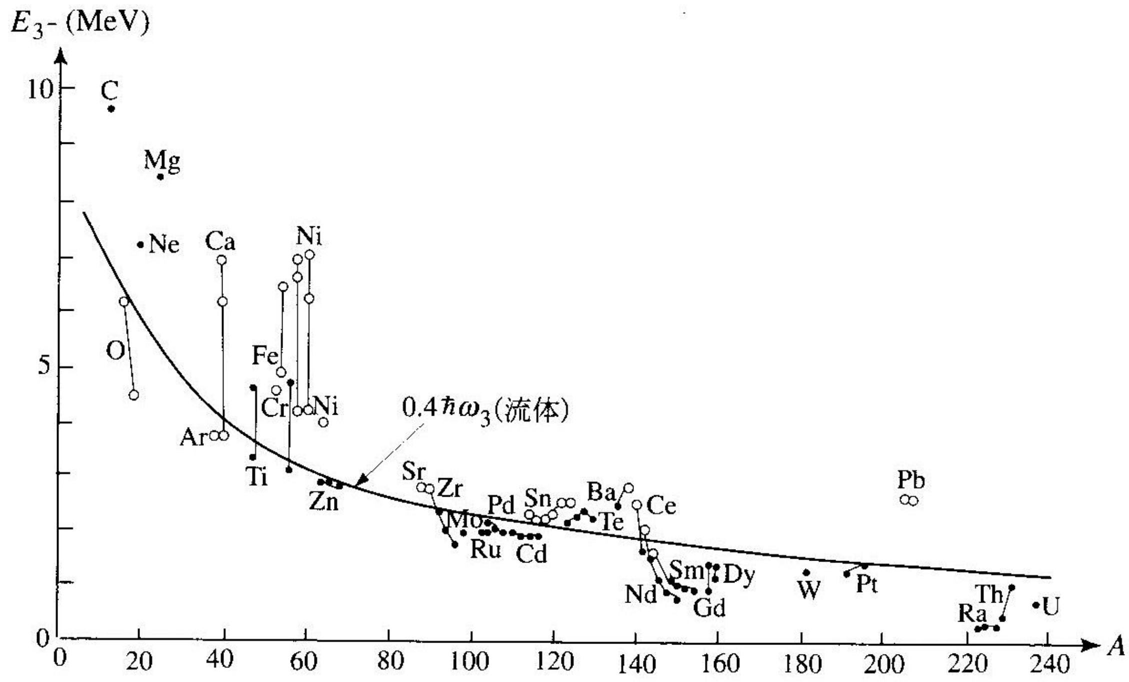


図 3.3 偶々核の第 1 励起 3⁻ 状態の励起エネルギー

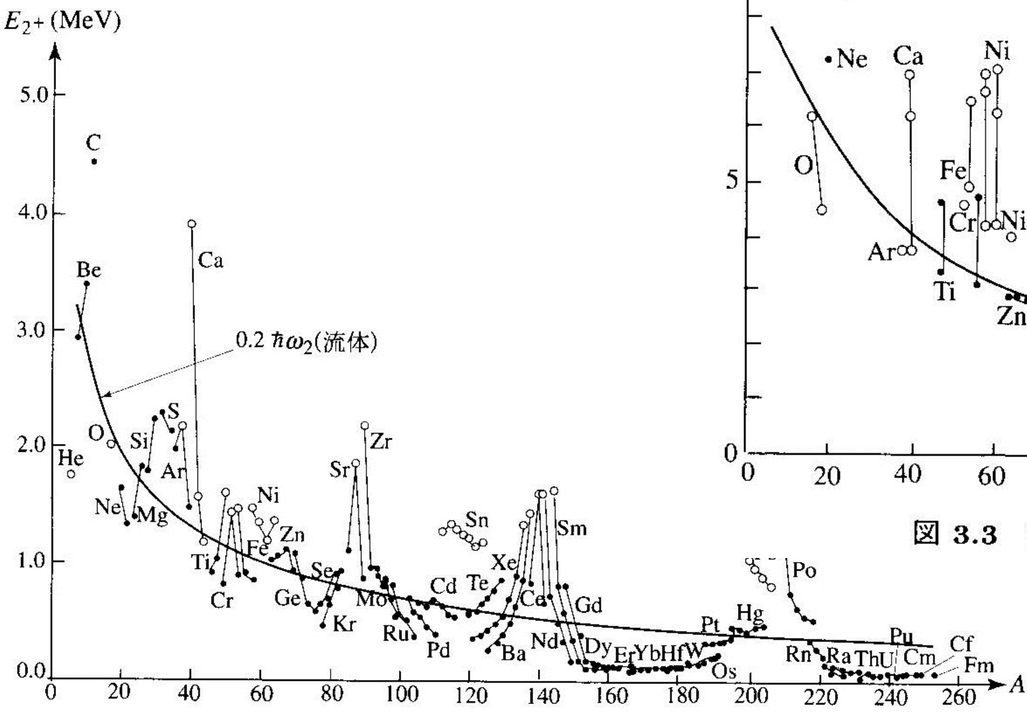
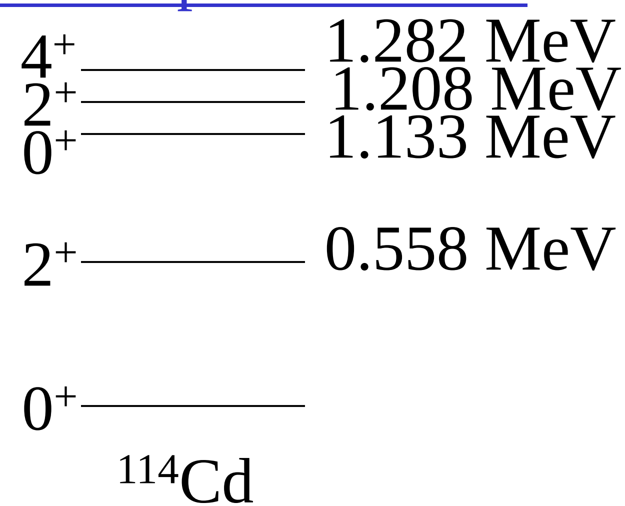


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

Double phonon states



Microscopic description

⇨ Random phase approximation (RPA)

[later in this lecture]