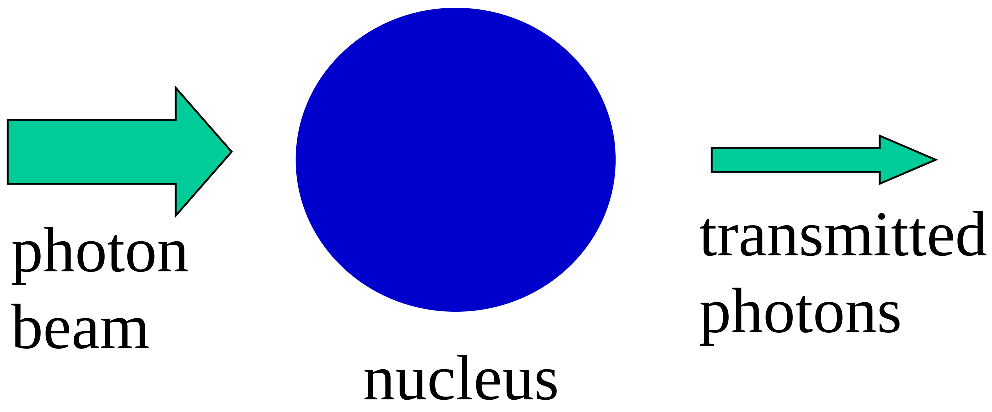
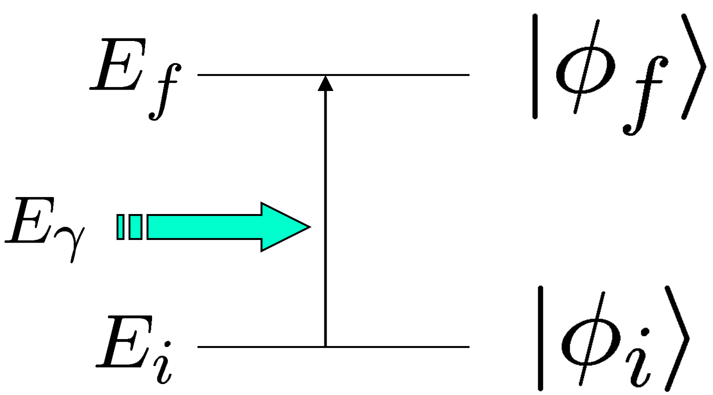
Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section





The state is strongly excited when

$$E_f - E_i = E_\gamma.$$

Giant Dipole Resonance (GDR)

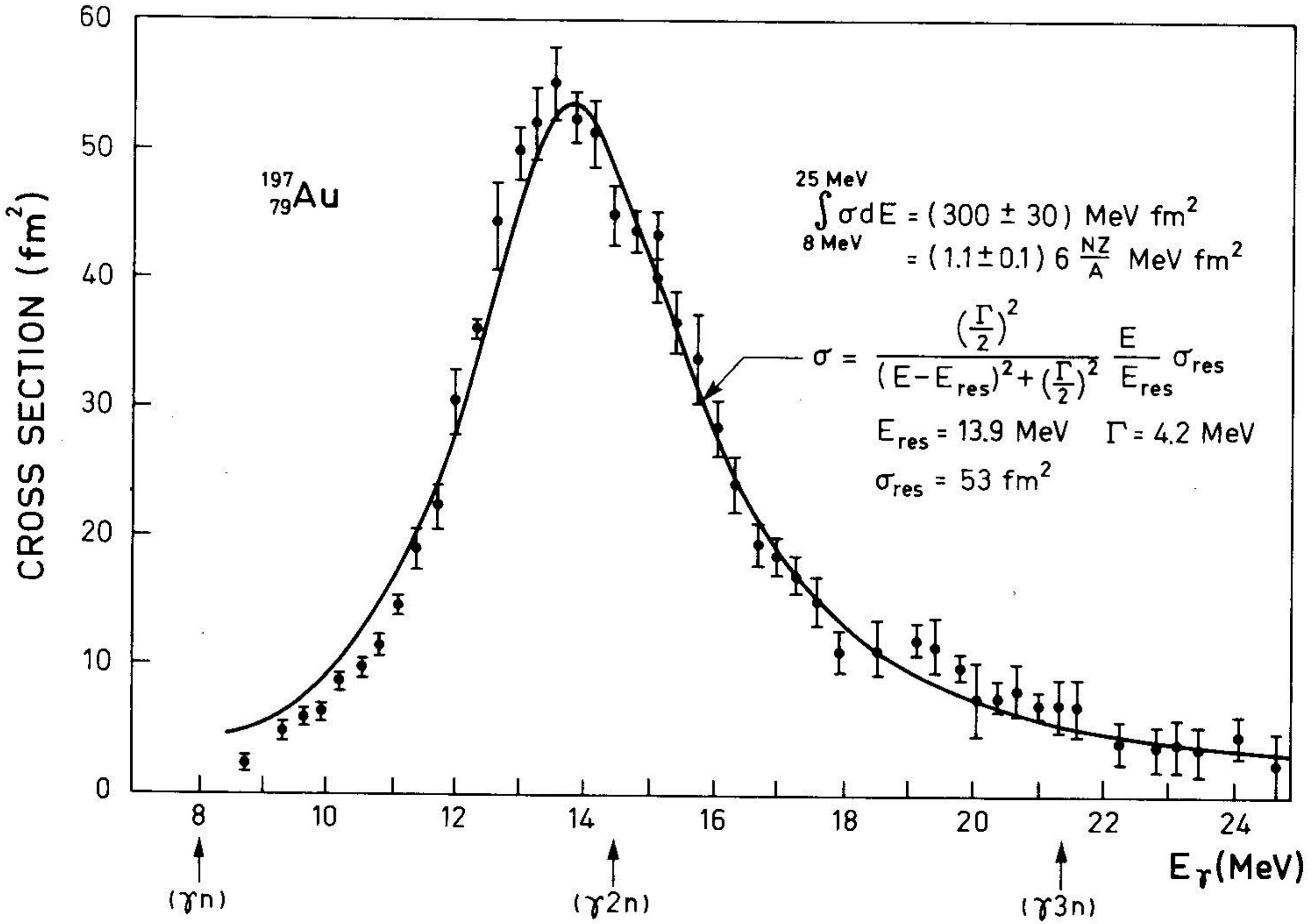


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

Remarks

i) Photon interaction $\longleftrightarrow$ dipole excitation

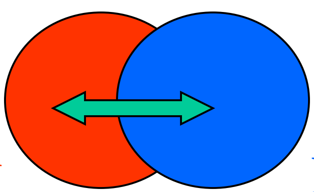
$$
\begin{aligned}
H_{\text{int}} &= \frac{1}{2m}\frac{e}{c}(\boldsymbol{p}\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\boldsymbol{p}) \\
\boldsymbol{A}(\boldsymbol{r},t) &= \sum_{\boldsymbol{k}}\sum_{\alpha=1,2}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}(a_{\boldsymbol{k}\alpha}\,\epsilon_\alpha\,e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega_k t}+h.c.)
\end{aligned}
$$

$e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\sim 1$ (dipole approximation)

$$
\boxed{\sigma_{\text{abs}}(E_\gamma)=\frac{4\pi^2e^2}{\hbar c}(E_f-E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma-E_f+E_i)}
$$

ii) Isospin

$$\tilde{z}=\sum_p(z_p-Z_{cm})$$

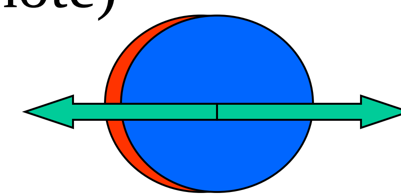


neutron

proton

Isovector type

(note)



Isoscalar dipole motion

$\longleftrightarrow$ c.m. motion (to the first order)

iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

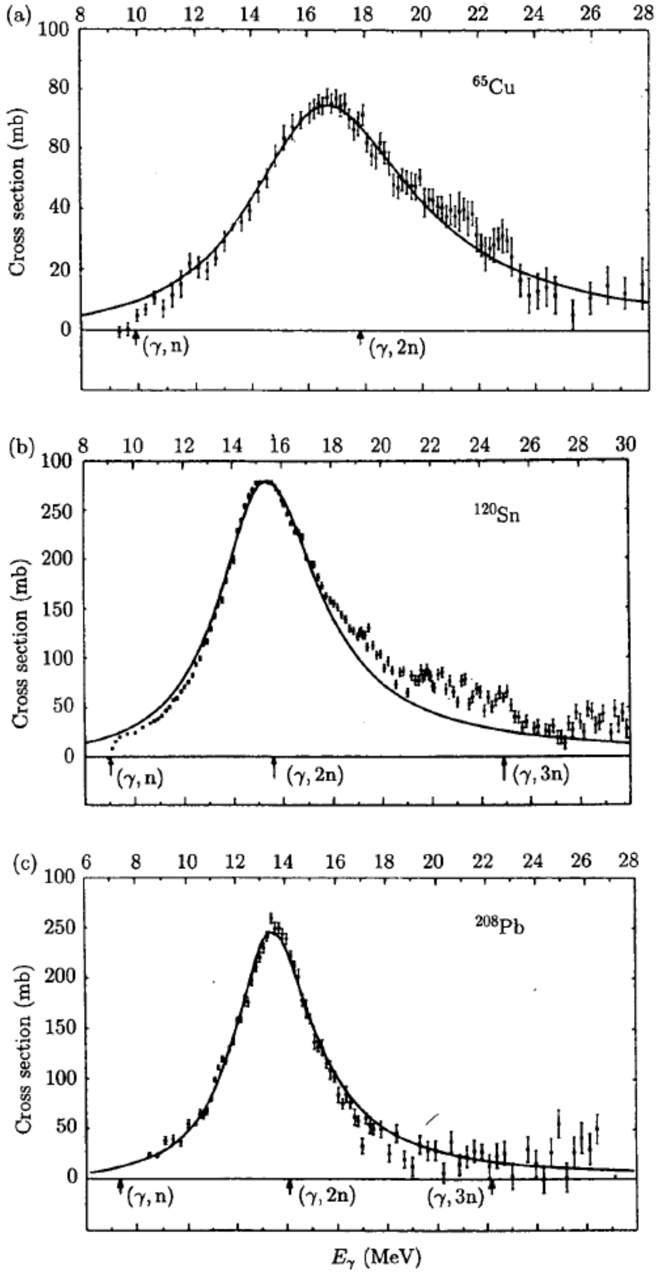
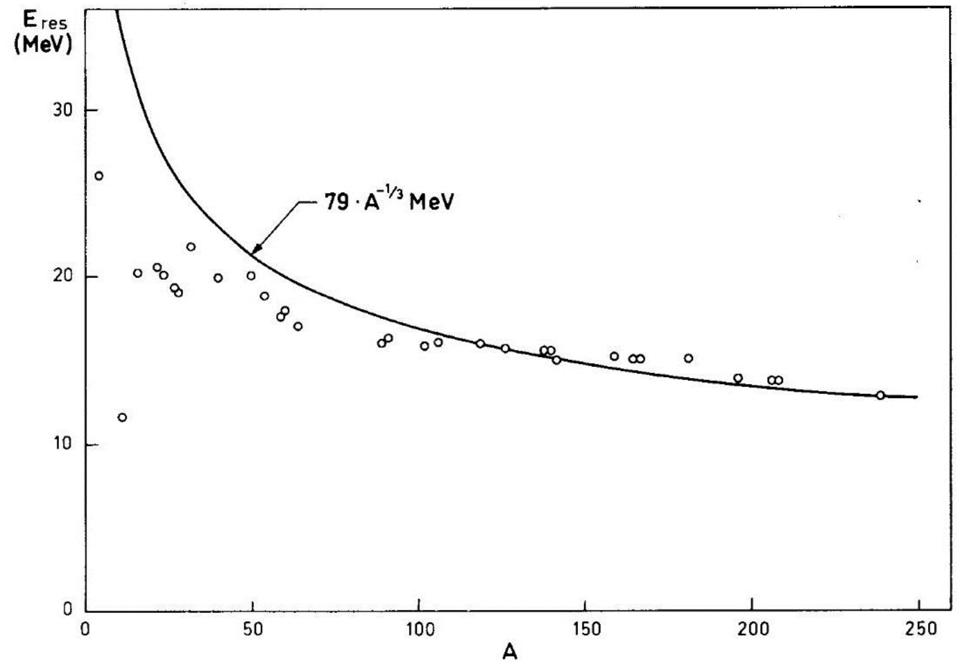
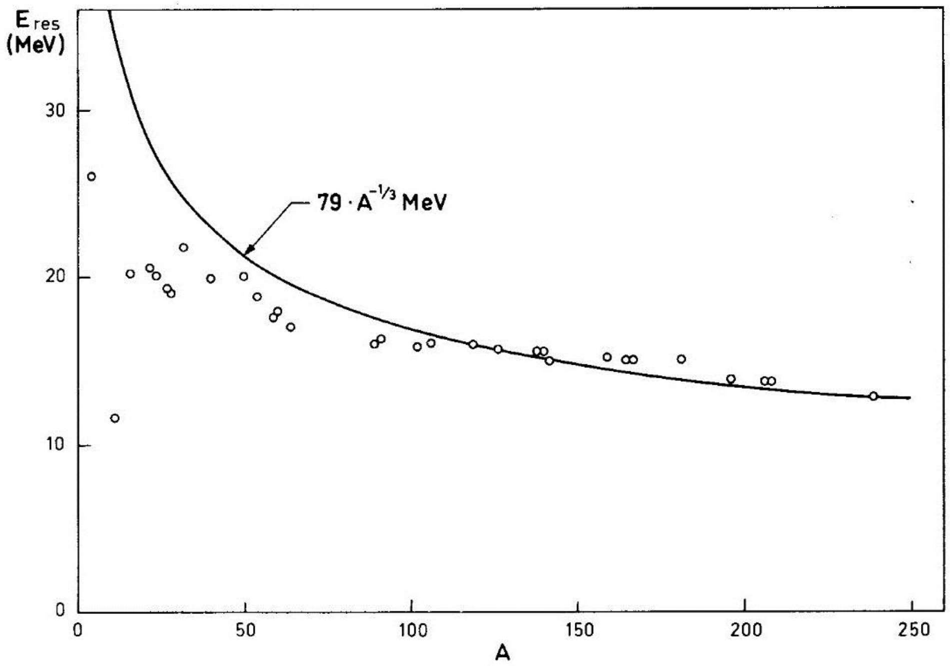


FIG. 1.2. The photo-neutron cross section $\sigma(\gamma, \mathrm{n})$ as a function of the photon energy for the three nuclei ^{208}Pb, ^{120}Sn and ^{65}Cu. Note that for these nuclei $\sigma(\gamma, \mathrm{n}) \approx \sigma_{abs}(\gamma)$. From reference (BER75).



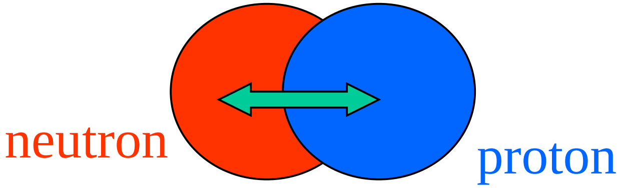
Bohr-Mottelson
“Nuclear Structure vol. II”

M.N. Harakeh and A. van der Woude,
“Giant Resonances”



$$E_{\mathrm{GDR}} \propto A^{-1/3}$$
$$\propto 1/R$$

Bohr-Mottelson
"Nuclear Structure vol. II"



Isovector type

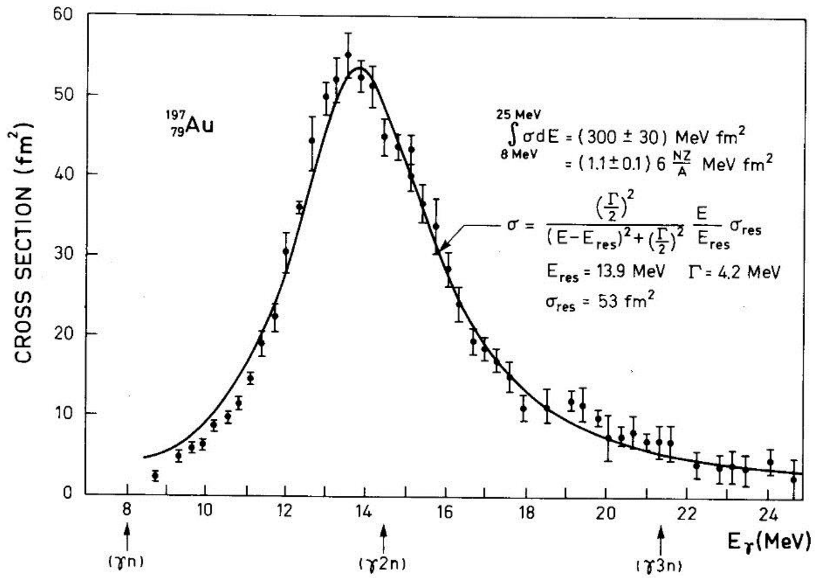
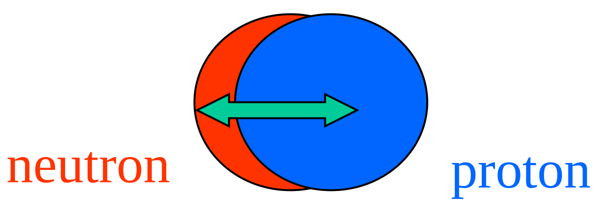
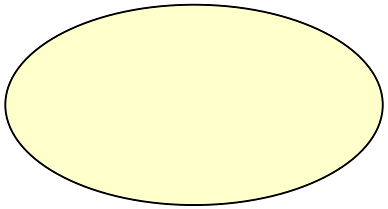


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

?

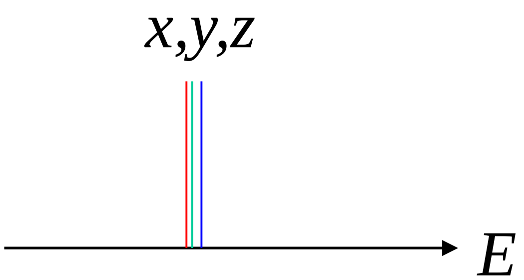
deformed nucleus

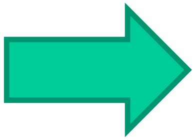


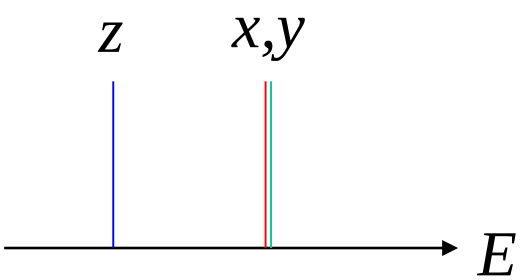


$$E_{\mathrm{GDR}} \propto 1/R$$

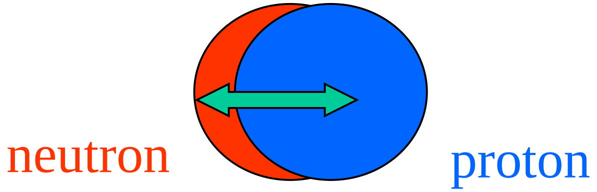
(prolate deformation)



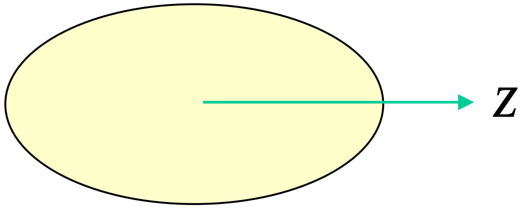




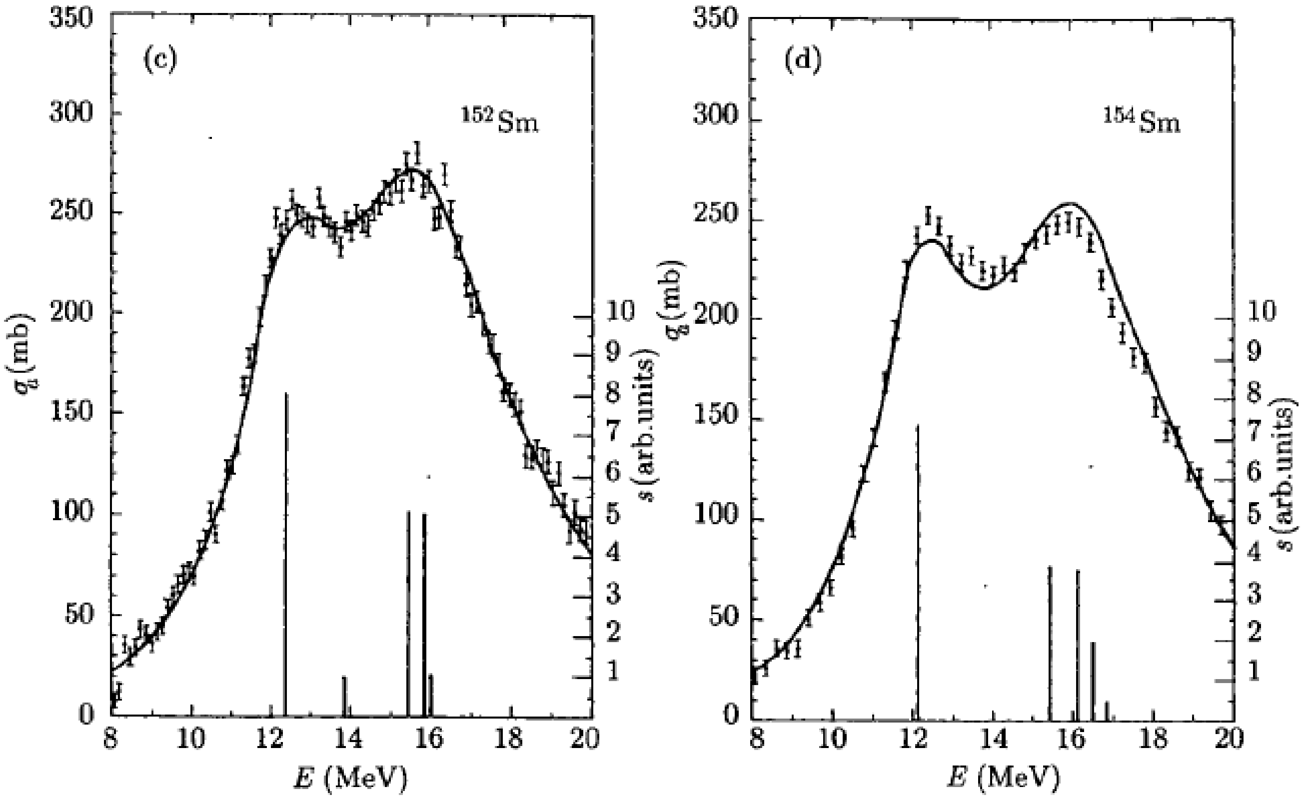
spherical nucleus



deformed nucleus

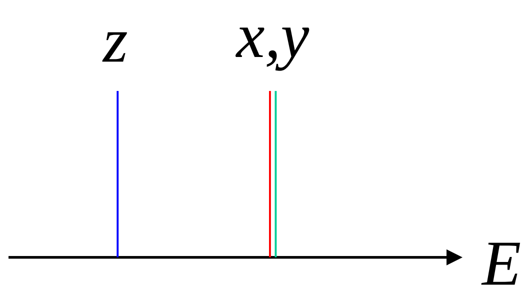


$$E_{\mathrm{GDR}} \propto 1/R$$

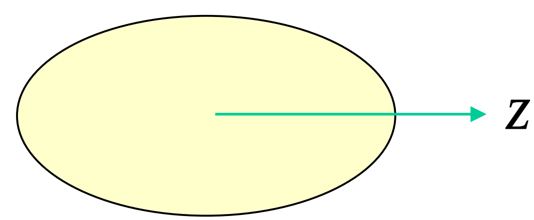


M.N. Harakeh and A. van der Woude, “Giant Resonances”

(prolate deformation)



deformed nucleus



Deformation effect

$$\hbar\omega \sim A^{-1/3} \sim 1/R$$

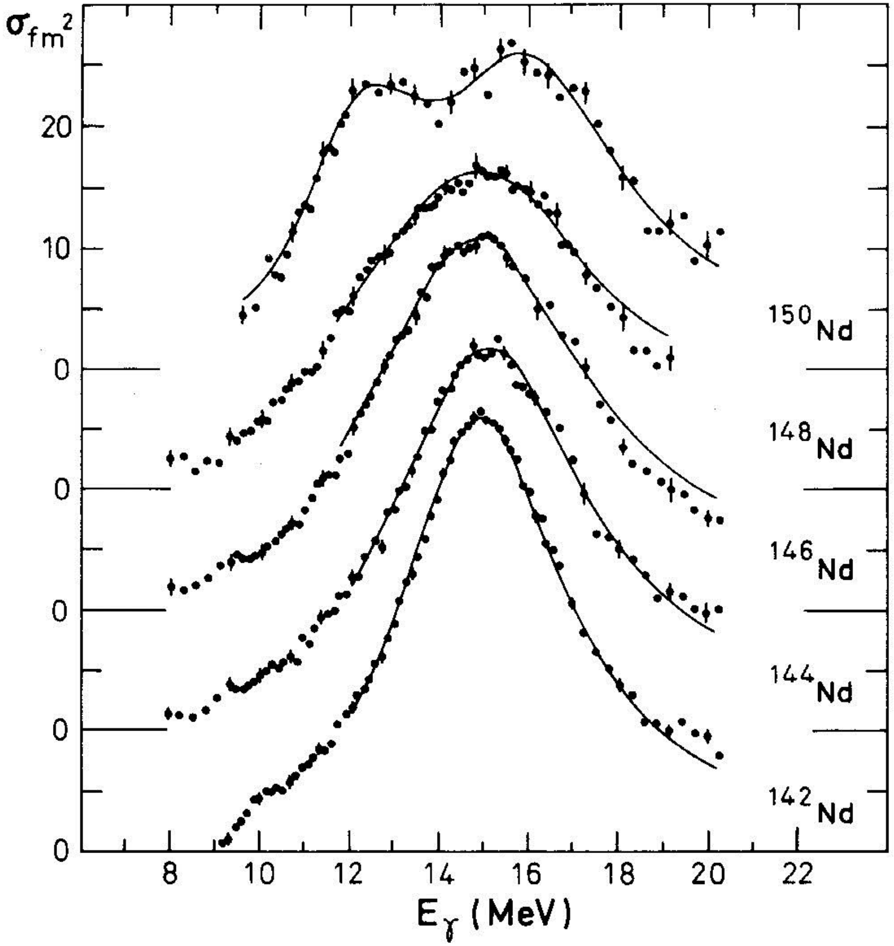
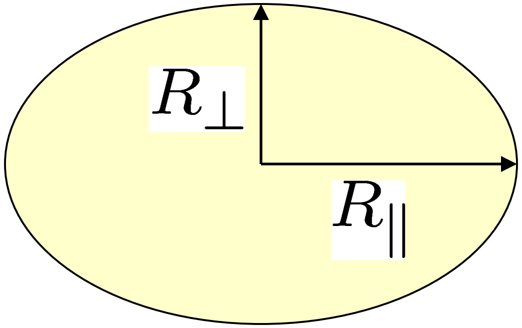
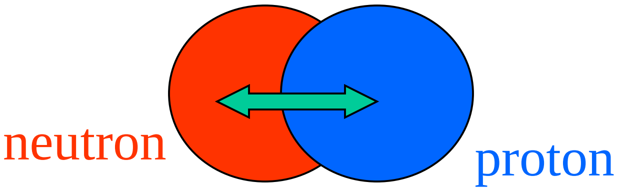


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys.* **A172**, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.



Giant Dipole Resonances

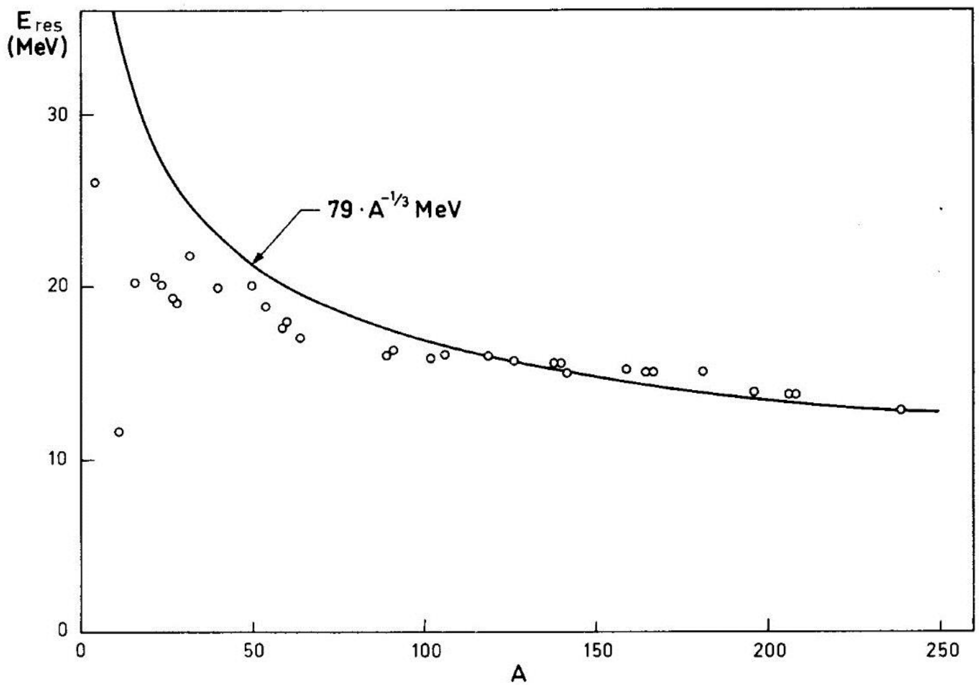
● <u>Goldhaber-Teller type</u>



$$\hat{Q} = rY_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

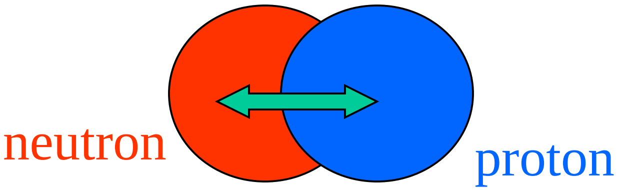
$$\Rightarrow \quad \hbar\omega \sim A^{-1/6}$$

$\Rightarrow$ Inconsistent with expt.
(except for light nuclei)



Giant Dipole Resonances

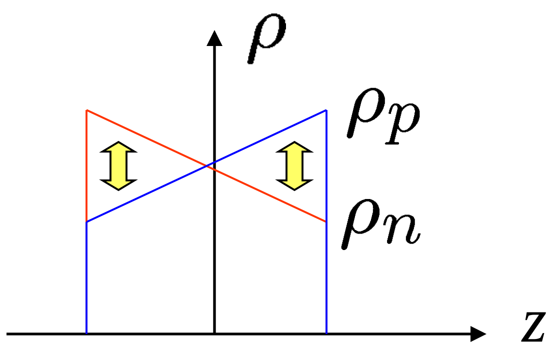
- <u>Goldhaber-Teller type</u>



$$\hat{Q} = rY_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

$$\hbar\omega \sim A^{-1/6}$$

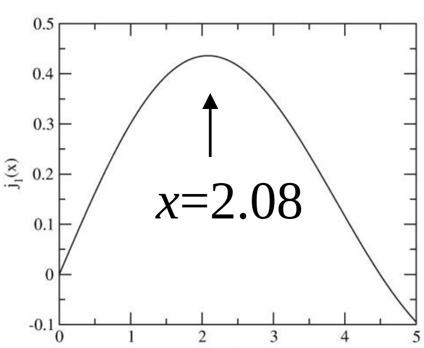
- <u>Steinwedel-Jensen type</u>



$$\hat{Q} = j_1(kr)Y_{1\mu}(\hat{\boldsymbol{r}})\tau_z$$

$$\hbar\omega \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x\cos x)/x^2$$

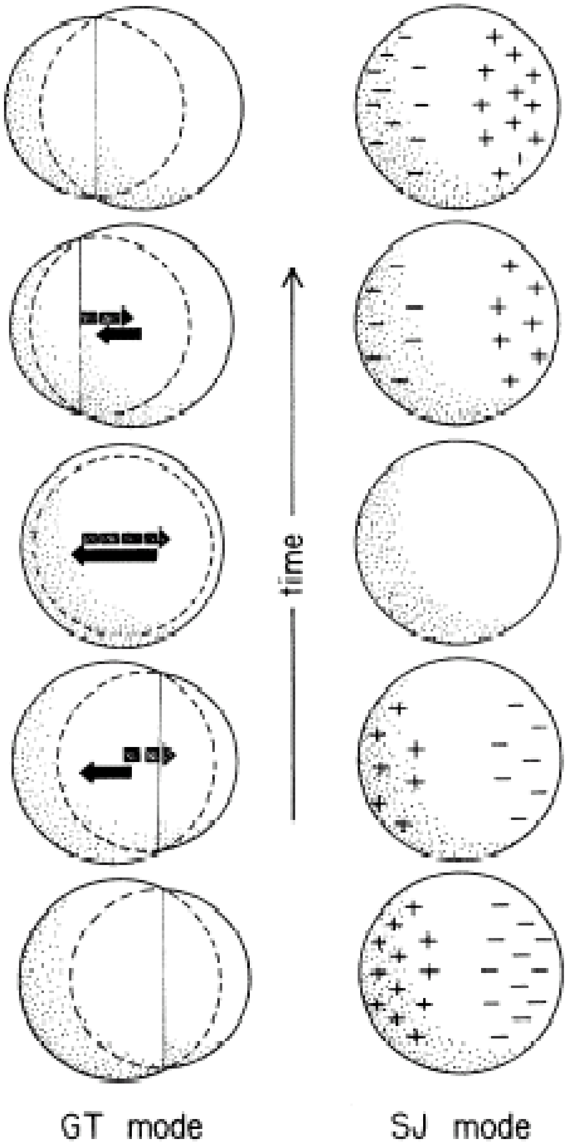
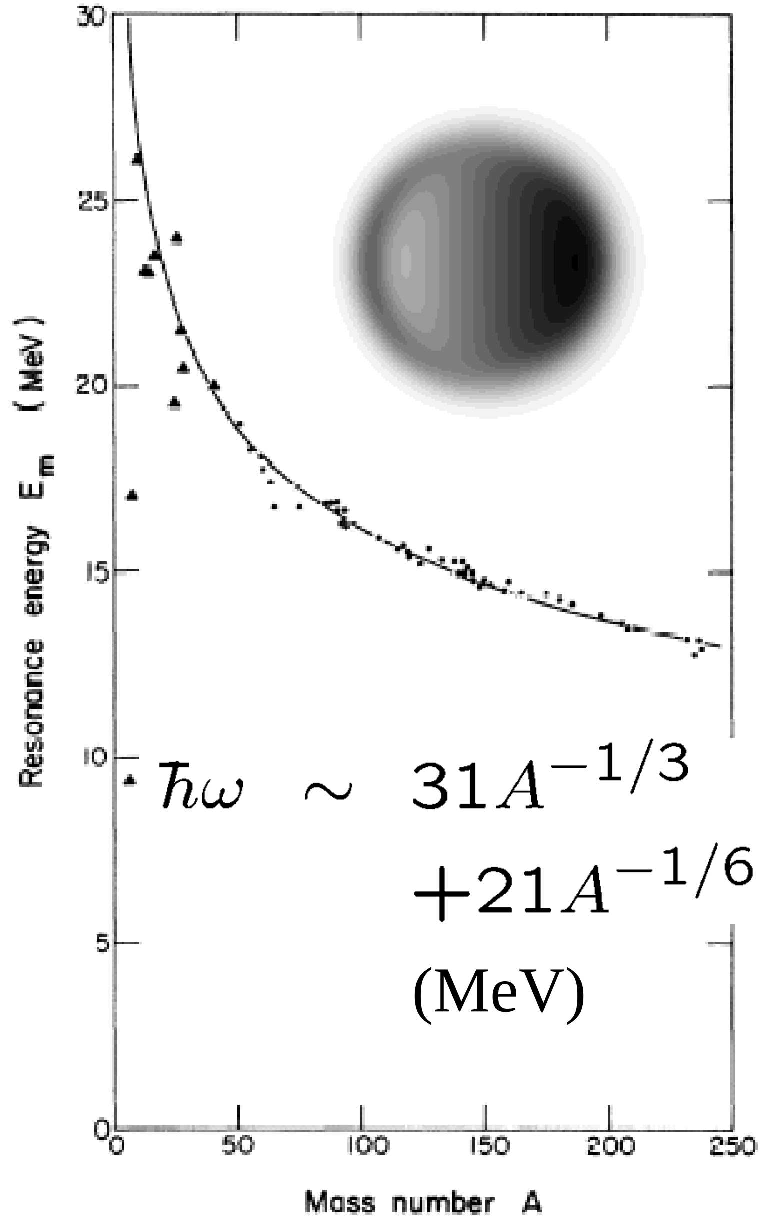


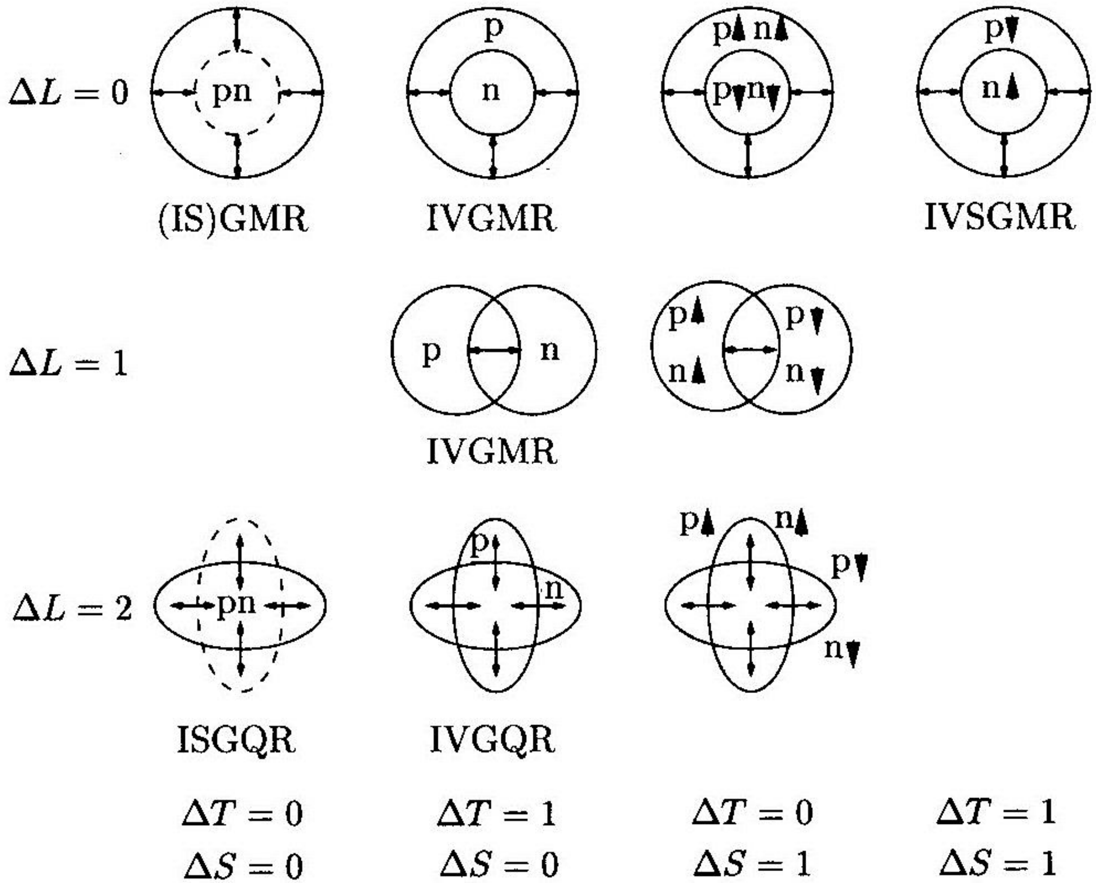
FIG. 1. Schematic drawings that serve to illustrate the the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



J.D. Myers et al., PRC15('77)2032

ii) Inelastic scattering

(e,e'), (p,p'), (α,α'), Heavy-ion $\longrightarrow$ Higher multipolarities

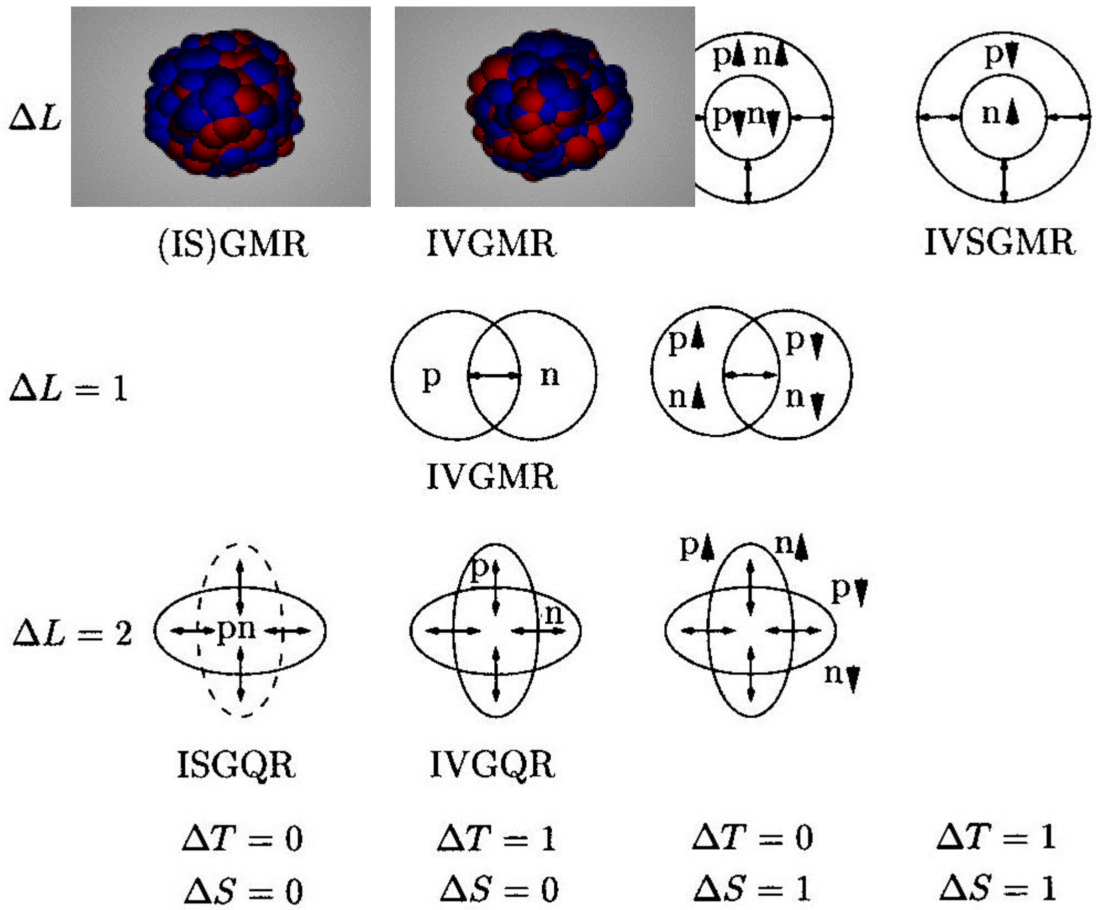


(note) $\Delta L = 2 \longrightarrow \Delta N = 2$ Giant Resonance (GQR)

$\Delta N = 0$ Low-lying state

ii) Inelastic scattering

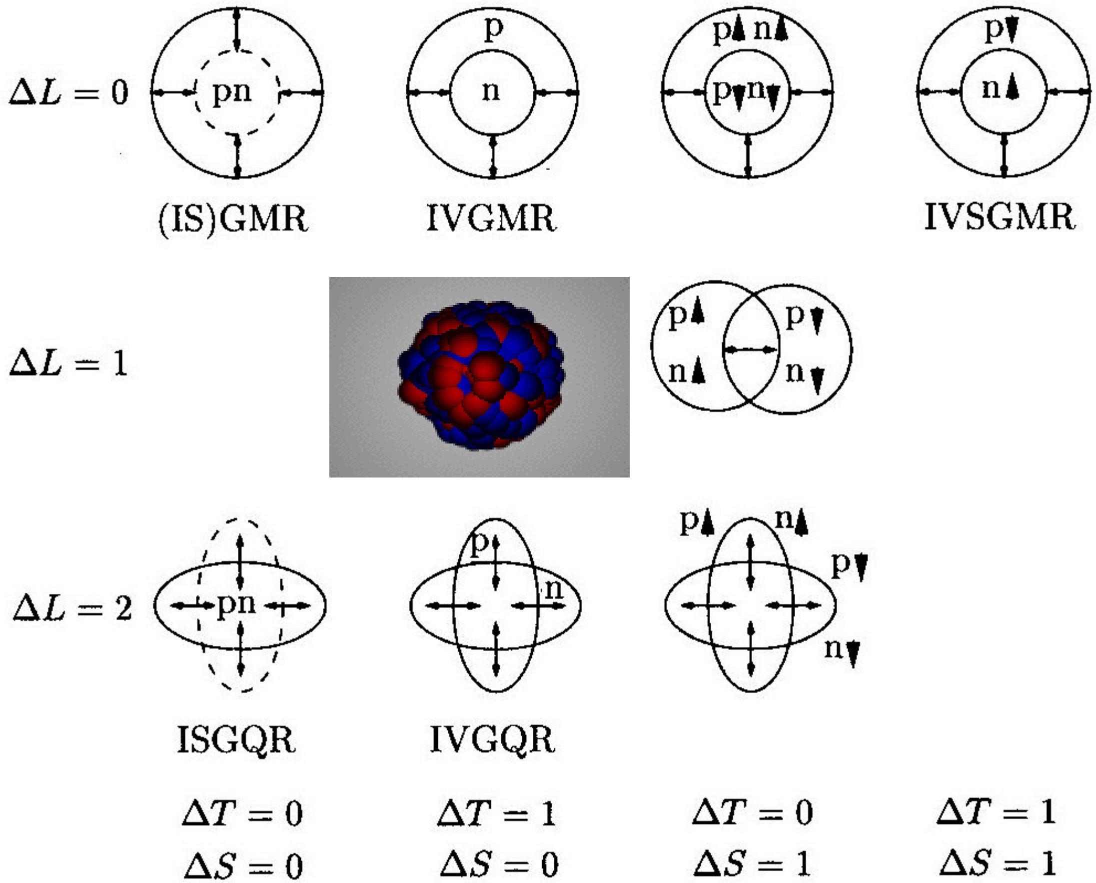
(e,e’), (p,p’), (α,α’), Heavy-ion → Higher multipolarities



movies: H.-J. Wollersheim,
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html

ii) Inelastic scattering

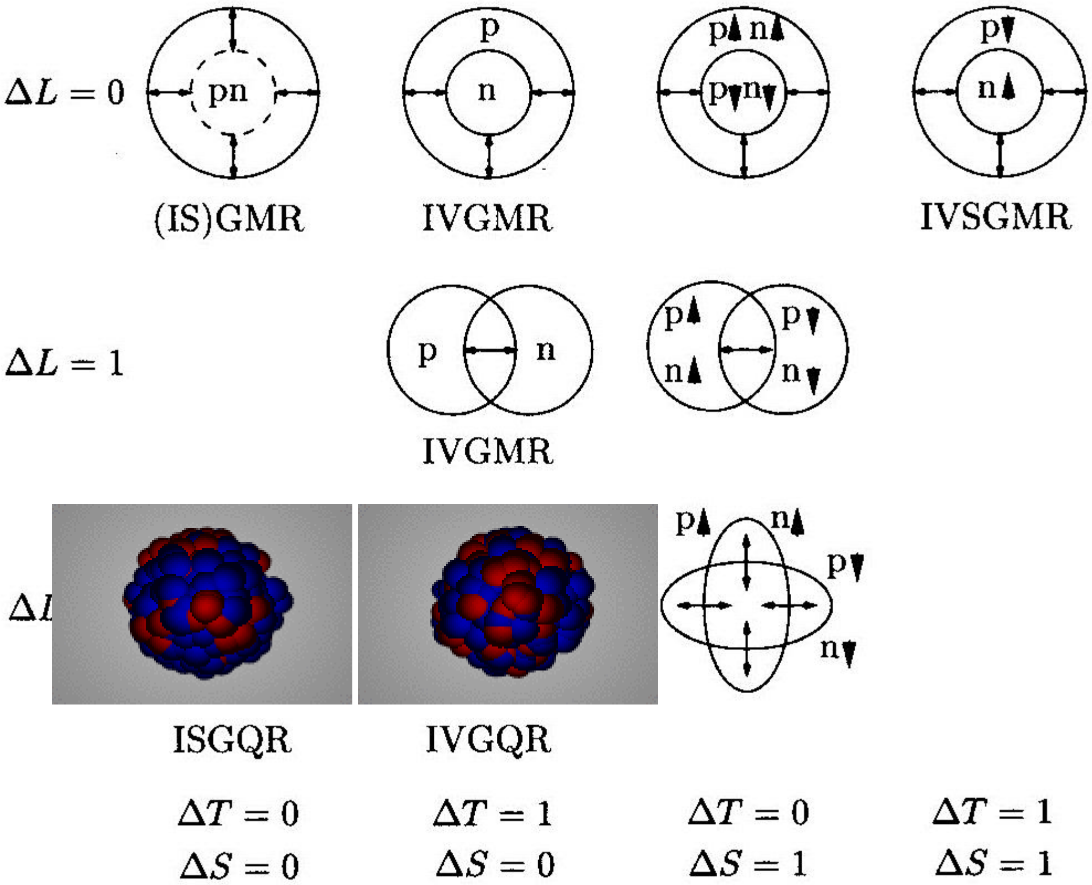
(e,e'), (p,p'), (α,α'), Heavy-ion $\rightarrow$ Higher multipolarities



movies: H.-J. Wollersheim,
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html

ii) Inelastic scattering

(e,e’), (p,p’), (α,α’), Heavy-ion → Higher multipolarities



movies: H.-J. Wollersheim,
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html

Discovery of Giant Quadrupole Resonance (GQR)

VOLUME 29, NUMBER 16 PHYSICAL REVIEW LETTERS 16 OCTOBER 1972

Giant Multipole Resonances in ^{90}Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in ^{90}Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

VOLUME 30, NUMBER 21 PHYSICAL REVIEW LETTERS 21 MAY 1973

Electroexcitation of Giant Resonances in ^{208}Pb

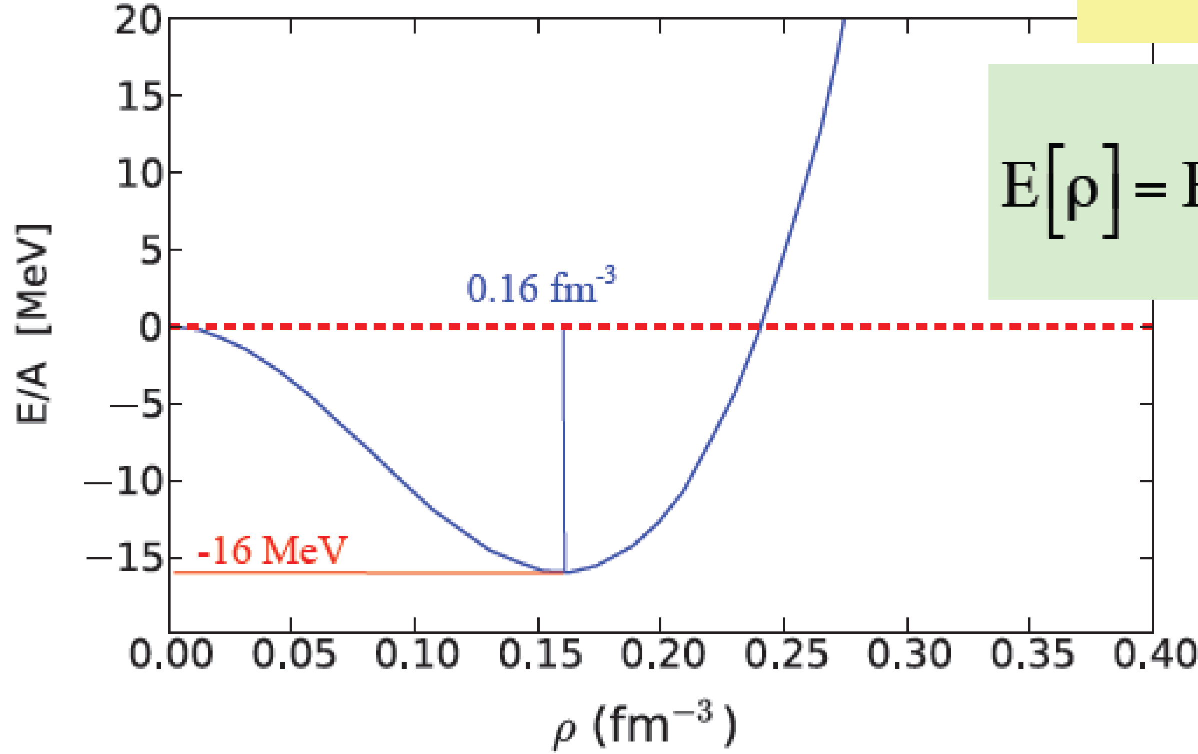
M. Nagao and Y. Torizuka
Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan
(Received 27 February 1973)

The giant-resonance region in ^{208}Pb was observed by inelastic electron scattering. We present evidence for the existences of a 2^+ (or 0^+) state at ~22 MeV and a 3^- state at ~19 MeV with giant-resonance character. The resonance states between 8.6 and 11.6 MeV are confirmed to be 2^+ (or 0^+) and the sum of their strengths exhausts about 50% of the $E2$ sum rule or 100% of $E0$.

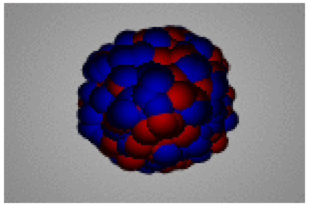
EOS of infinite nuclear matter

$$K_\infty = 9\rho^2 \frac{d^2\left[E(\rho)/\rho\right]}{d\rho^2}\Bigg|_{\rho_0}$$

$$E\left[\rho\right] = E\left[\rho_0\right] + \frac{1}{18}K_\infty\left(\frac{\rho-\rho_0}{\rho_0}\right)^2$$



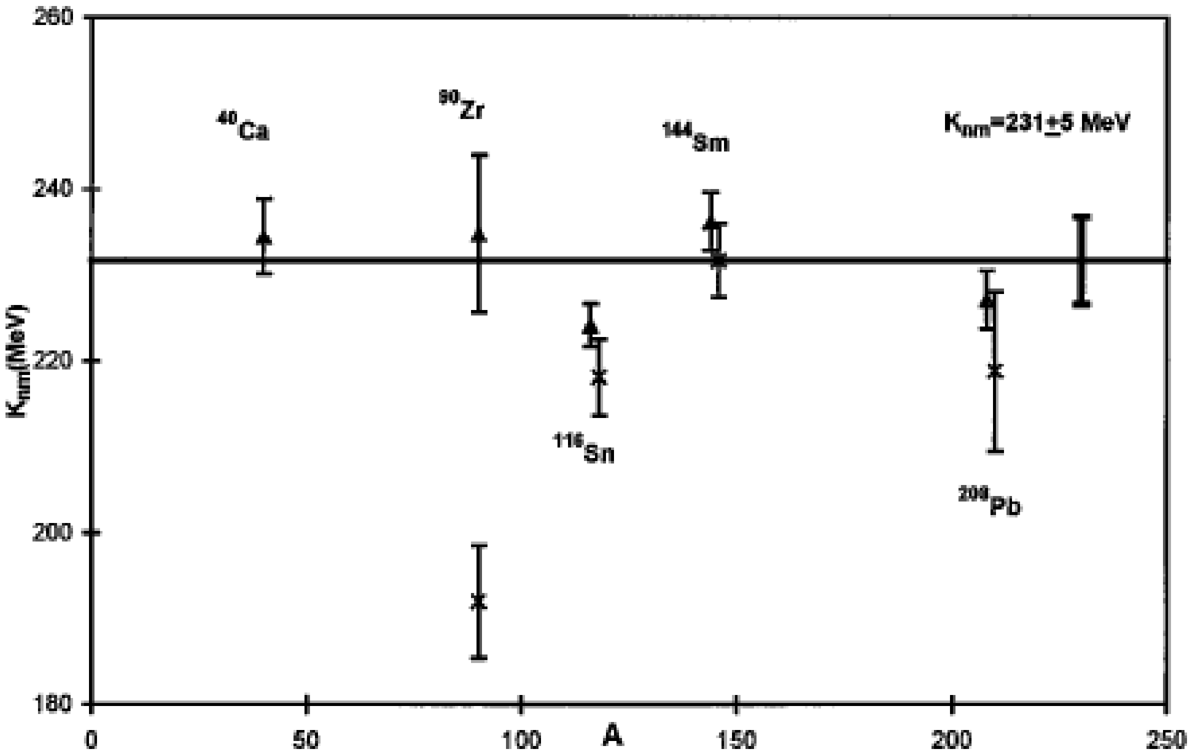
slide: Carlos Bertulani



Isoscalar giant monopole resonances (breathing mode)

$$E_{\mathrm{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m\langle r^2\rangle}}$$

J.P. Blaizot,
Phys. Rep. 64 ('80) 171

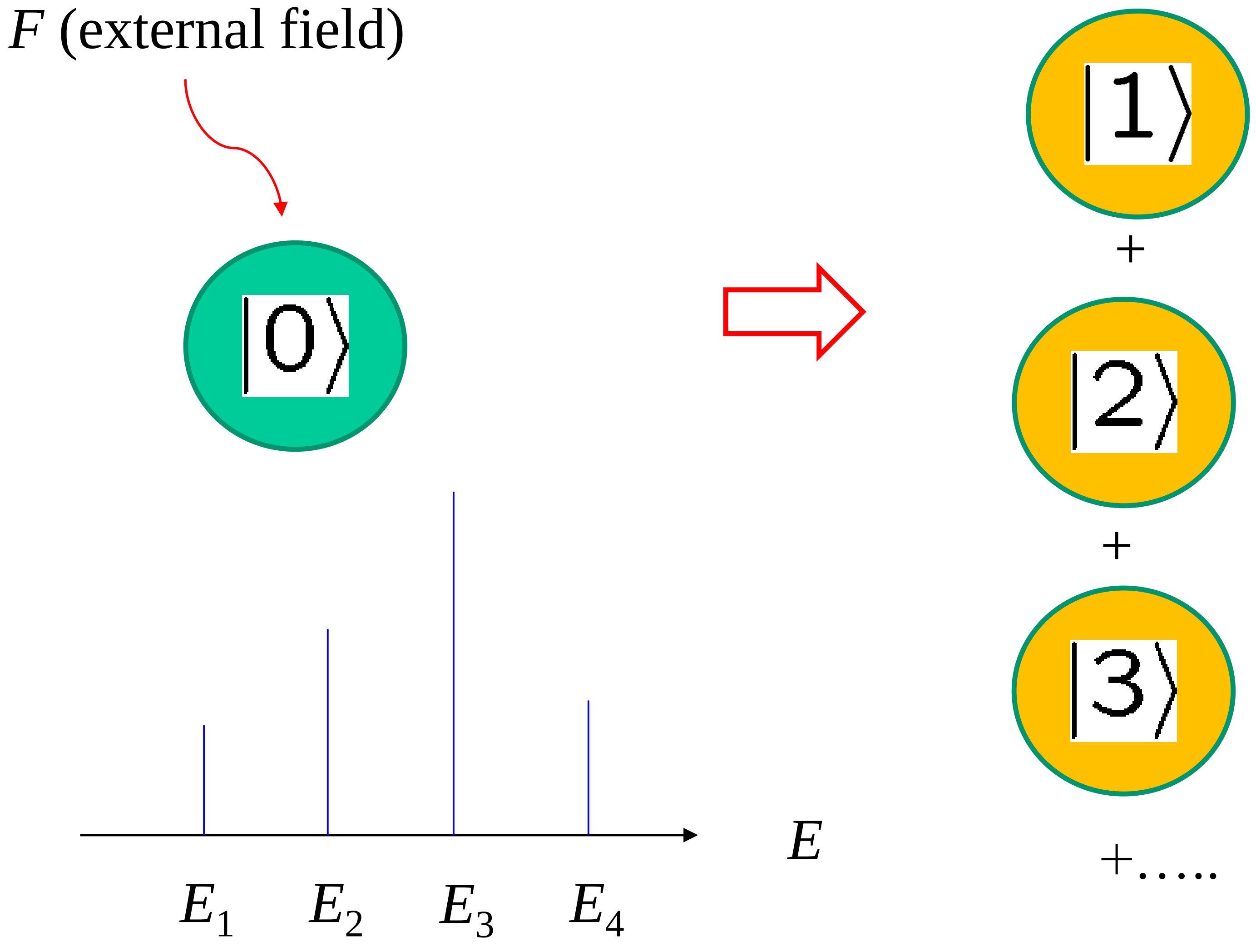


K ~ 231 +/- 5 MeV

D.H. Youngblood, H.L. Clark, and Y.-W. Lui, PRL82 ('99) 691

Sum Rule

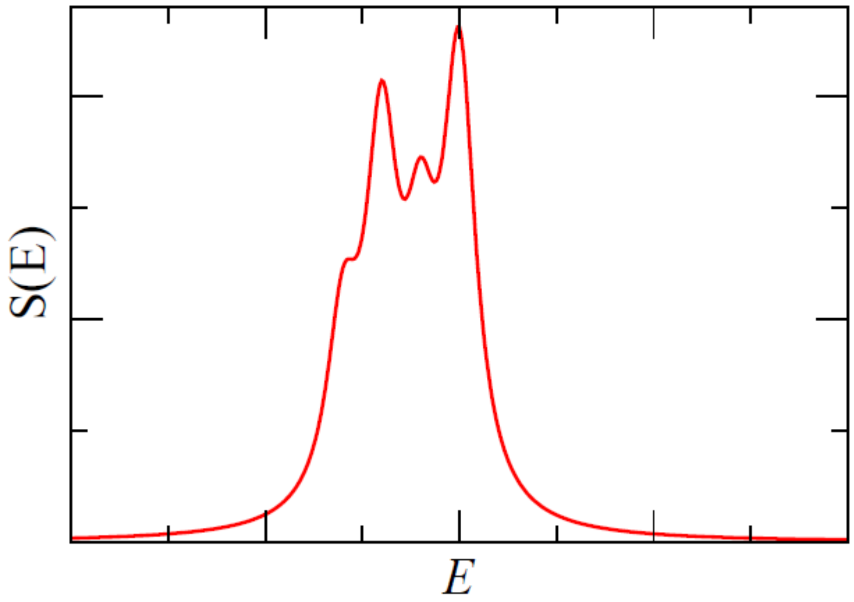
Strength function: $$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \, \delta(E_\nu - E_0 - E)$$



Sum Rule

Strength function:

$$S(E) = \sum_\nu |\langle\nu|F|0\rangle|^2 \times\delta(E_\nu - E_0 - E)$$



✓ non-energy weighted sum rule

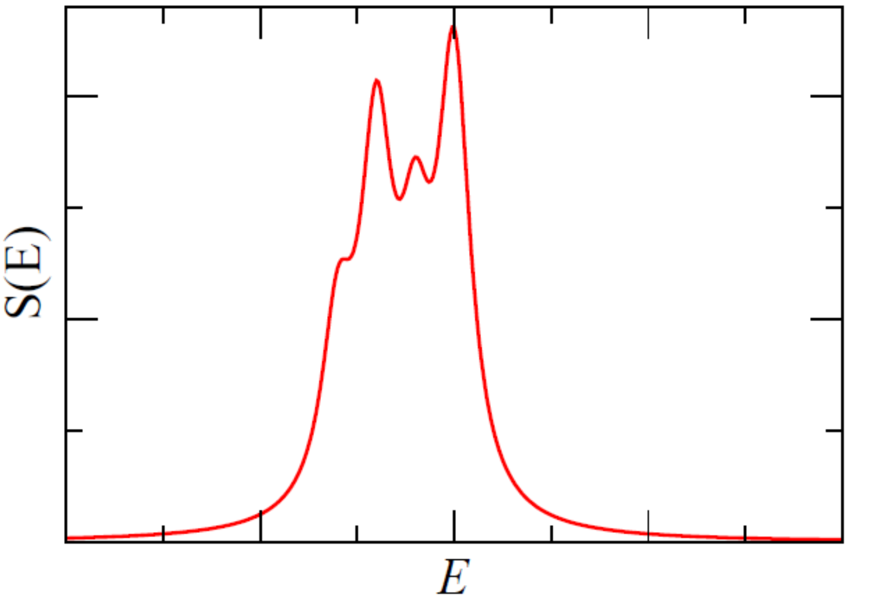
$$\boxed{S_0 \equiv \int S(E)dE = \sum_\nu |\langle\nu|F|0\rangle|^2}$$

✓ energy weighted sum rule

$$\boxed{S_1 \equiv \int ES(E)dE = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2}$$

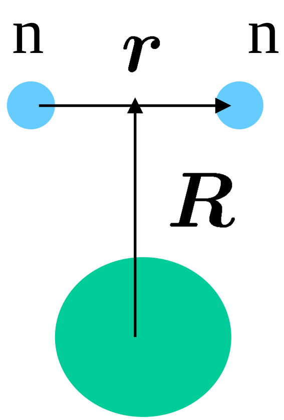
✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_\nu |\langle\nu|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$



$$S(E) = \sum_\nu |\langle\nu|F|0\rangle|^2 \times\delta(E_\nu - E_0 - E)$$

cf. geometry of Borromean nuclei



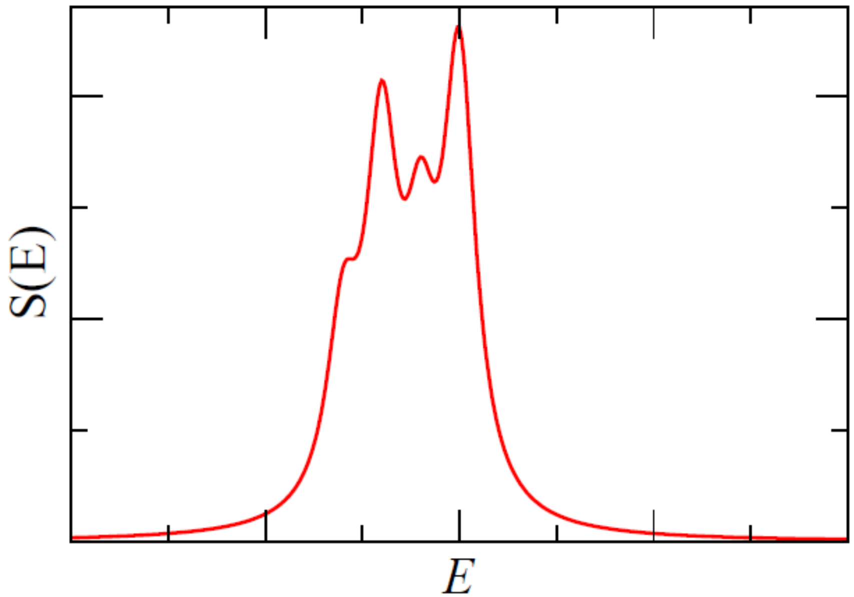
$$B(E1) = \sum_i B(E1; gs \to i)$$
$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \langle R^2\rangle$$

$$\Longrightarrow \langle\theta_{nn}\rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li})$$
$$= 74.5^{+11.2}_{-13.1} \quad (^{6}\text{He})$$

K.H. and H. Sagawa,
PRC76('07)047302

✓ energy weighted sum rule

$$
\begin{aligned}
S_1 &\equiv \int E S(E) dE \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \\
&= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle
\end{aligned}
$$



$$
\begin{aligned}
S(E) &= \sum_\nu |\langle \nu | F | 0 \rangle|^2 \\
&\quad \times \delta(E_\nu - E_0 - E)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)H \rangle \\
&= \langle FHF - E_0 F^2 \rangle \\
&= \sum_\nu E_\nu \, |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\
&= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2
\end{aligned}
$$

Energy weighted sum rule:

$$\boxed{\begin{aligned} S_1 &= \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 \\ &= \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle \end{aligned}}$$

For $F = F(\boldsymbol{r})$ (local operator)

$$\begin{aligned} [H,F] &= \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] \\ &= -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla) \end{aligned}$$

$$[F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

$$\boxed{S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2}$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For *F=z*

$$\boxed{S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

Model independent

For $F = r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$

$$\boxed{S_1 = \frac{\lambda(2\lambda+1)\hbar^2}{8\pi m} A\,\langle r^{2\lambda-2}\rangle}$$

Photo absorption cross section:

$$\sigma_{\mathsf{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\begin{aligned}\tilde{z} = \sum_p (z_p - Z_{cm}) \;&=\; \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n \right) \right\} \\ &=\; \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n \right)\end{aligned}$$

$$\boxed{\begin{aligned}\int \sigma_{\mathsf{abs}}(E_\gamma)dE_\gamma \;&=\; \frac{4\pi^2 e^2}{\hbar c}\cdot\frac{\hbar^2}{2m}\cdot\frac{NZ}{A} \\ &=\; \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}\end{aligned}}$$

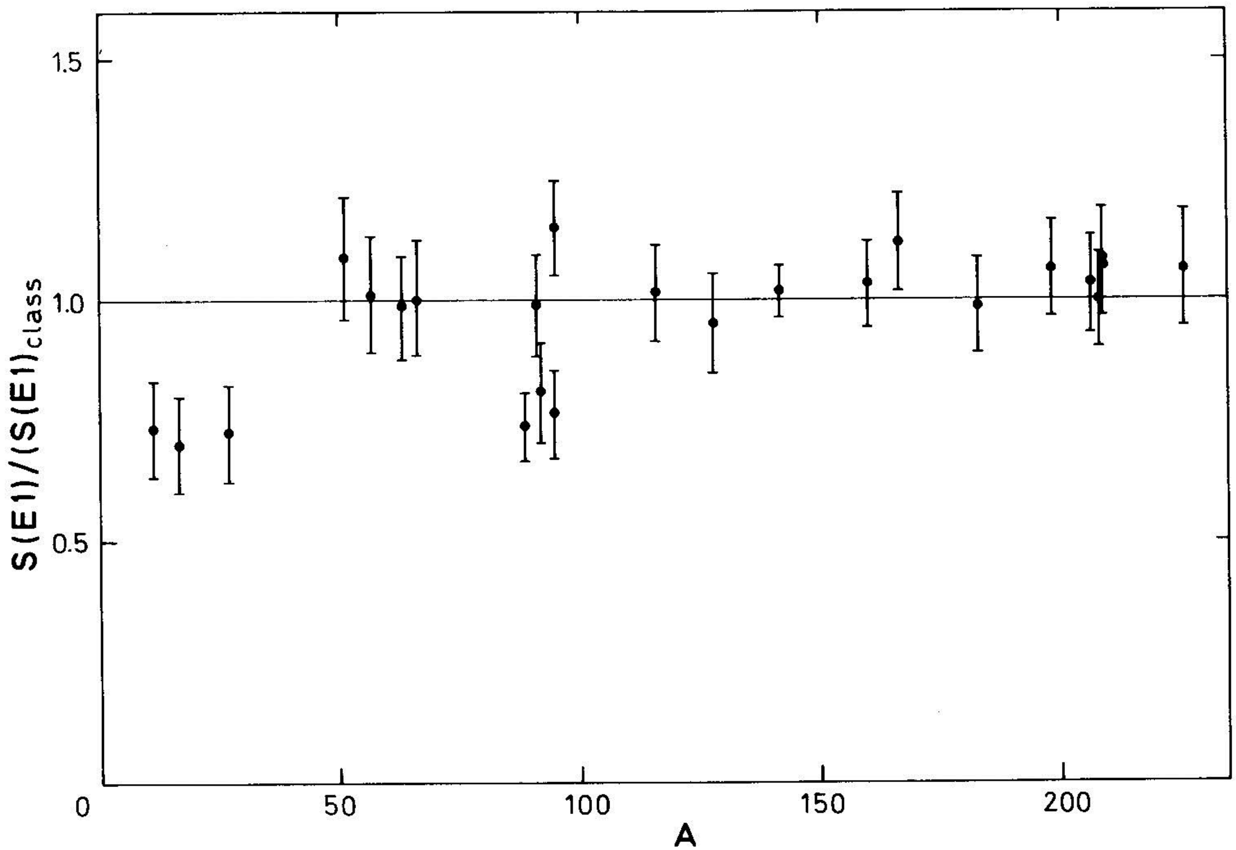
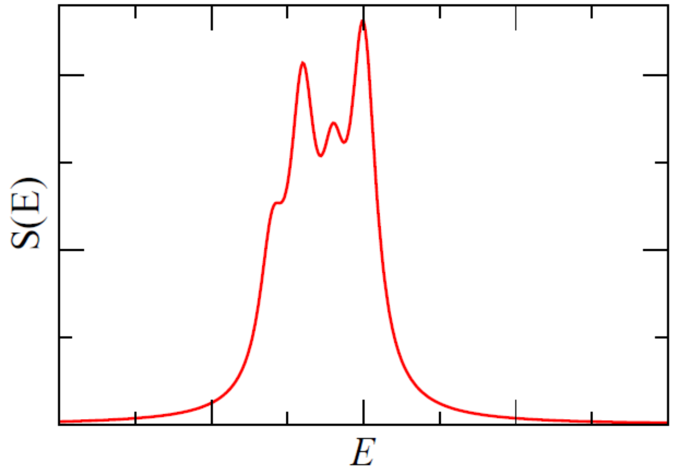


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al*., *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al*., 1970).

和則の利点

$$S_0 = \langle 0|F^2|0\rangle$$
$$S_1 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$

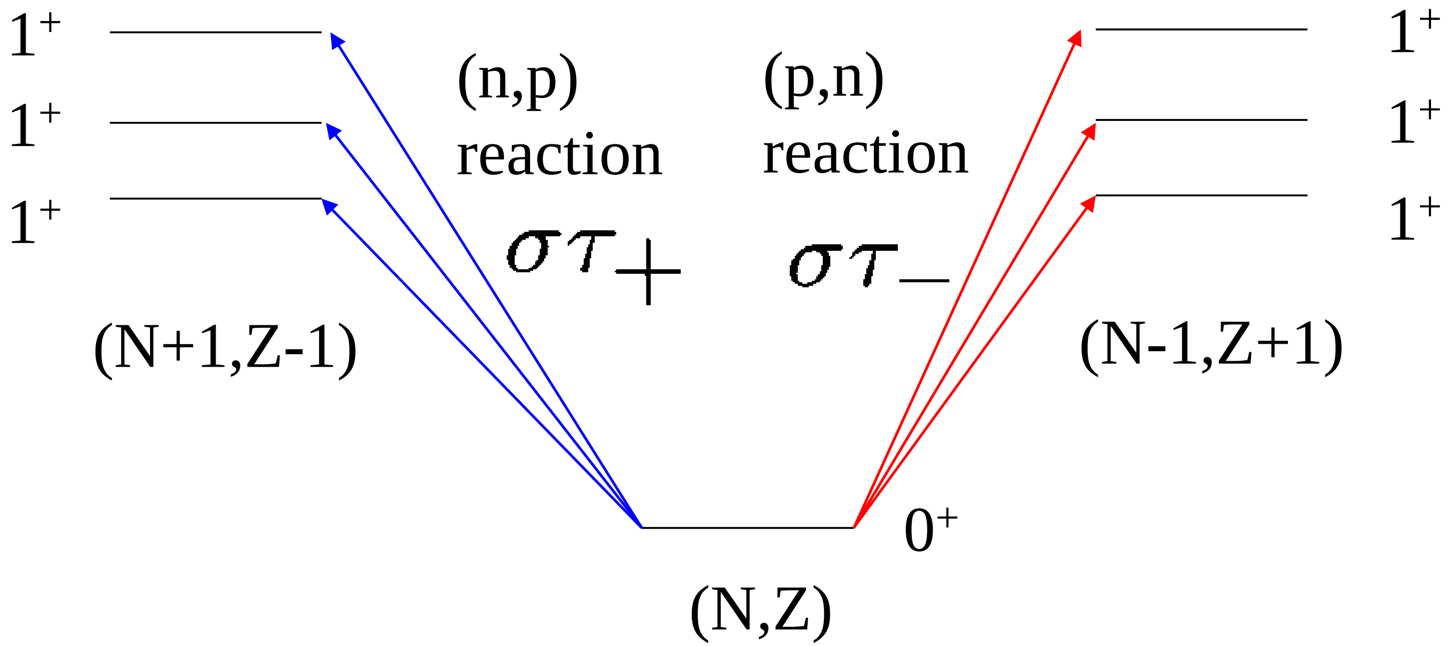


和則:
励起状態の(ある種の)情報が基底状態の
性質のみによって表わされる
(励起状態の情報を知っている必要がない)。

- 実験で強度分布が測られた時、測られた範囲外にも強度があるかどうか (missing strength) 判断できる。
- 強度分布を測ることによって原子核の半径などの情報を得られる。
- 実験データや数値計算のチェックになる。
  (和則の値よりとても大きくなると何かがおかしい)。
- 近似法の妥当性が判断できる。基本的な和則を満たす近似かどうか。
  cf. RPA は TRK sum rule を満たすが、Tamm-Dancoff 近似は満たさない。

Ikeda sum rule

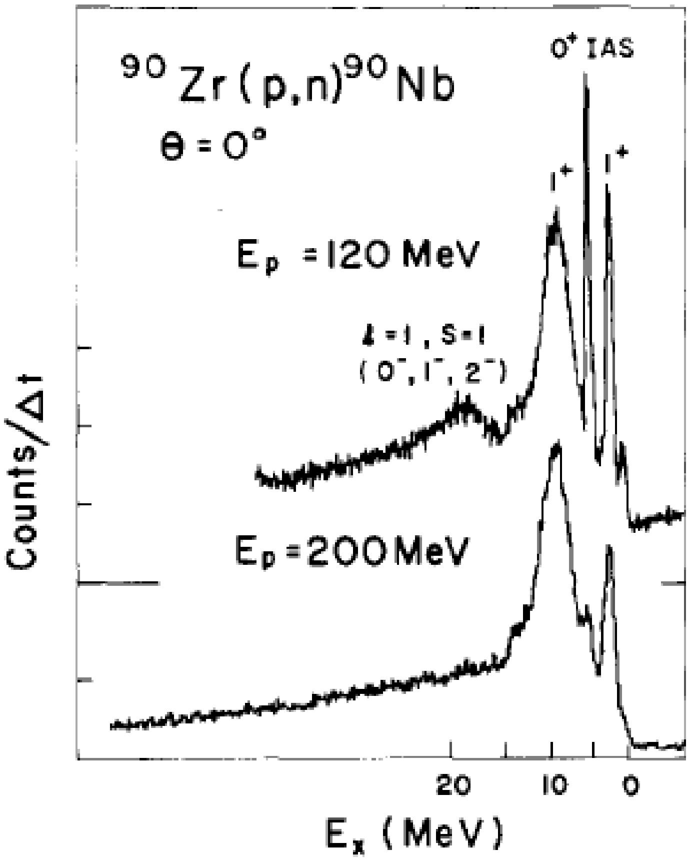
charge exchange reactions: Gamow-Teller transitions



Ikeda sum rule

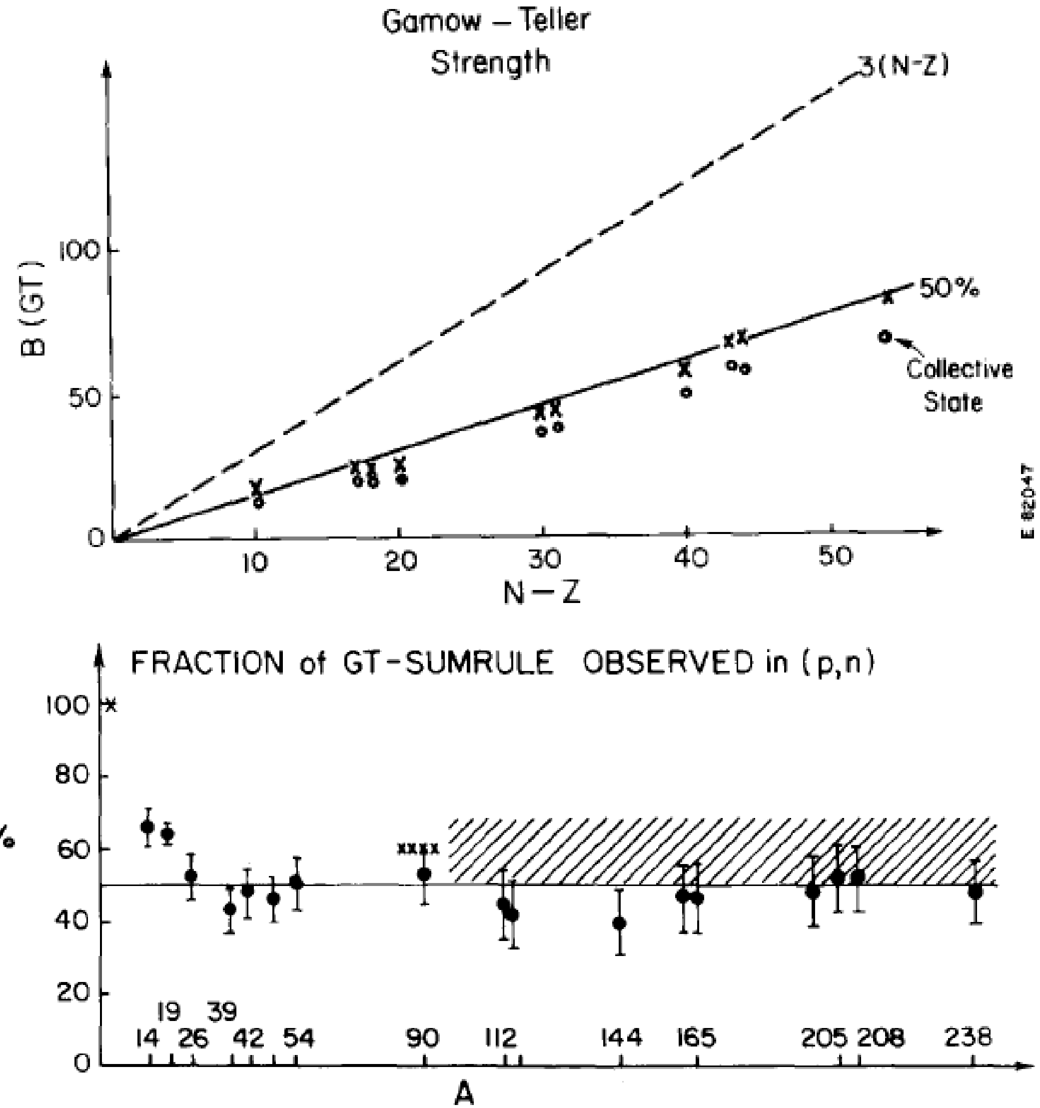
$$S_0(\sigma\tau_-) - S_0(\sigma\tau_+) = 3(N-Z)$$

situation before 1997



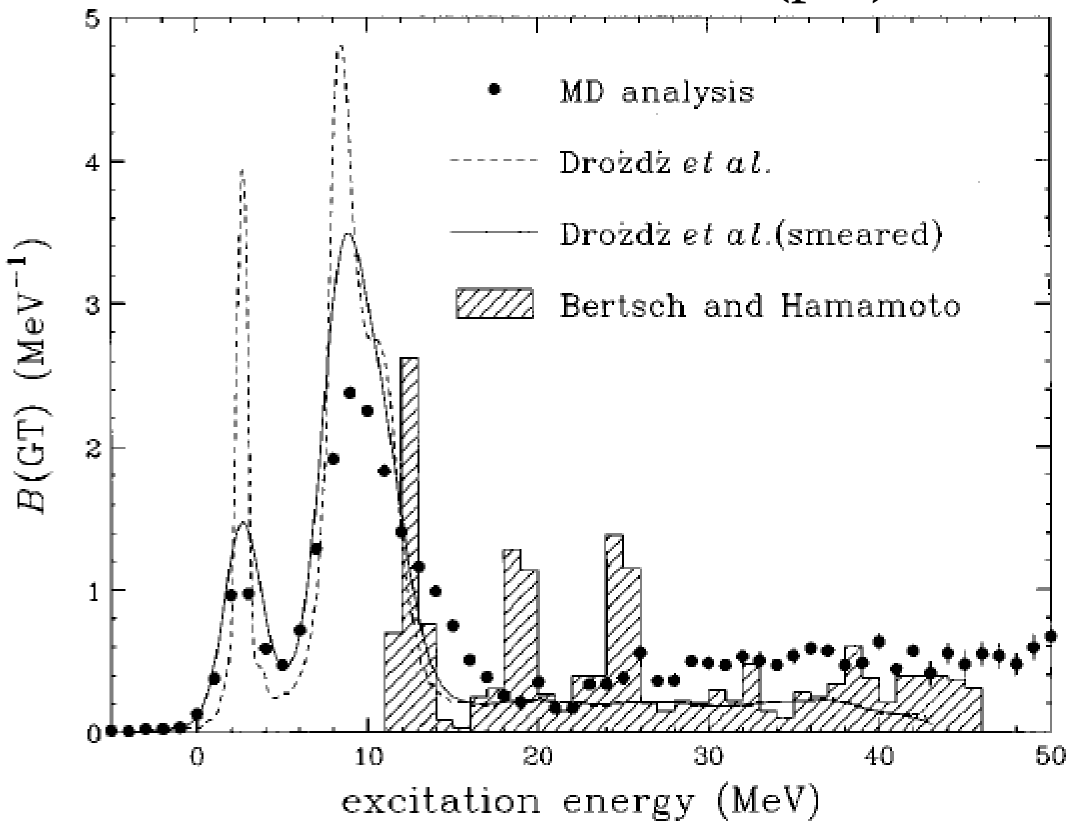
the “quenching problem”
of GT strength

quark (Δ resonance)?



C. Gaarde, Nucl. Phys. A396(‘83)127c

^{90}Zr (p,n) ^{90}Nb



T. Wakasa et al., PRC55 (‘97) 2909

$S_- - S_+ = 27.0 \pm 1.6 = (90 \pm 5)\%$ of Ikeda sum rule

$\longrightarrow$ quark contribution: small

(proof of Ikeda sum rule)

$$(Y_\pm)_\mu \equiv \sum_i \tau_\pm(i)\sigma_\mu(i)$$

$$[(Y_\pm)_\mu]^\dagger = (-)^\mu (Y_\mp)_{-\mu}$$

$$\begin{aligned}
S_- - S_+ &= \langle 0|Y_-^\dagger Y_-|0\rangle - \langle 0|Y_+^\dagger Y_+|0\rangle \\
&= \sum_\mu (-)^\mu \langle 0|[(Y_+)_\mu, (Y_-)_{-\mu}]|0\rangle \\
&= \langle 0|\sum_i [\tau_+(i), \tau_-(i)](\sum_\mu (-)^\mu \sigma_\mu \sigma_{-\mu})|0\rangle \\
&= 3\langle 0|\sum_i [\tau_+(i), \tau_-(i)]|0\rangle \\
&= 3\langle 0|\sum_i \tau_z(i)|0\rangle = 3(N-Z)
\end{aligned}$$